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LETTERS TO THE EDITORS

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Fractional charge: a new trend in electronics

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Abstract. Comparatively recent theoretical and experimental research, which indicates the possible existence of electronic objects with a charge equal to a fraction of the elementary electron charge ($e/2$, $e/3$, etc.), is briefly reviewed.

The question of the existence of electronic objects with a fractional charge, i.e., with a charge equal to $e/2$, $e/3$, etc., is to a degree a fundamental question, but for the present we cannot say that it has been completely solved. That is why the title of the paper makes use of the word 'trend': this avenue of research should draw the reader's attention as precisely a trend.

We direct our attention to some facts related to electrons. It should be noted that we are going to deal, strictly speaking, with well-known things.

Figure 1 shows the 2p state of atomic hydrogen. Where the transition, say, from the 2p state to the 1s state is involved, the well-known leading line of the Lyman series in hydrogen corresponds to this transition. These states have been comprehensively investigated and repeatedly observed, and thus do not surprise anybody. Nevertheless, as is evident from the drawing, the wave function of this state is divided into two parts. If a formal approach is adopted, it is valid to say that the upper part of the electron distribution contains half the electron charge, and the other half is in the lower part of the distribution.

Wave functions with several maxima pertaining to one electron are the subject of considerable speculation in the literature. They are sometimes referred to as 'Schrödinger-cat' type states, although this actually bears little relation to Schrödinger's original paper.

Figure 1 also gives the expression for the electron wave function ψ and depicts the section of ψ and the square of the modulus of ψ in the same section. One can see that, owing to the dependence on the cosine of the polar angle, the wave function amplitude vanishes for $\vartheta = \pi/2$, and the wave function is split into two parts.

Should the upper part of the wave function be shifted up in some way, say by 1 cm, and then the lower half similarly shifted down, thereby letting these halves, each with a charge of $e/2$, exist independently, this would most likely provoke objections.

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Why should this provoke objections? In this connection we quote an extract from the well-known textbook on quantum mechanics written by D I Blokhintsev [1]: "In reality, a particle *always* acts as a single whole, and discovered in a device is the whole particle rather than its fraction." And further: "...The simplest particles *always* act as a single whole, and therein lies the atomism observed in microworld phenomena. That is why the notion of particles as objects made of de Broglie waves is at variance with atomism and should be rejected."

By way of example, we have cited an extract from Blokhintsev's textbook because it clearly states the basic idea. As a matter of fact, this is the standpoint of the majority of physicists with the exception of a small group of 'dissidents', among which, however, was A Einstein. I think that Blokhintsev differs from other authors in that he nevertheless expressed surprise at this fact, while others make the same statement as a fact which is beyond any doubt. (Blokhintsev expressed his surprise not in the textbook cited but in a separately published brochure entitled *Printsipial'nye Voprosy Kvantovoi Mekhaniki* (Fundamental Problems of Quantum Mechanics) [2]).

The word '*always*' is italicized in the extract quoted from Blokhintsev's book. Is it always? Strictly speaking, below we are dealing with precisely this question.

First, we consider some low-temperature experiments with liquid helium. Liquid helium is not a typical object for electronic research. But our subsequent discussion deals with rather simple matters.

It is well known that liquid helium transforms to a superfluid state at a temperature of about 2 K, and in doing so it loses viscosity. Less well known is another property: in the superfluid state, helium does not bear any impurities and all impurities dissolved in it immediately settle on the walls of the helium-containing vessel upon transition to the superfluid state. An exception to the rule is the He^3 isotope, which may be contained in superfluid helium in small quantities. Therefore, superfluid helium is one the purest substances on Earth. This circumstance attracted the attention of our famous scientist Aleksandr Iosifovich Shal'nikov in his time. (The author of this paper took a post-graduate course under Shal'nikov's supervision. In this connection, the author is pleased to remember him and emphasize his outstanding contribution to science.)

Shal'nikov ask himself the following question: liquid helium does not bear any atomic or molecular impurities, but how would it behave if electrons were the impurity? Electrons are easy to inject into liquid helium: it would suffice to place a radioactive source beside liquid helium and electrons will travel through the vessel walls to find themselves in the liquid helium.

The answer to this question turned out to be rather simple. It was found that liquid helium stayed true to its nature and

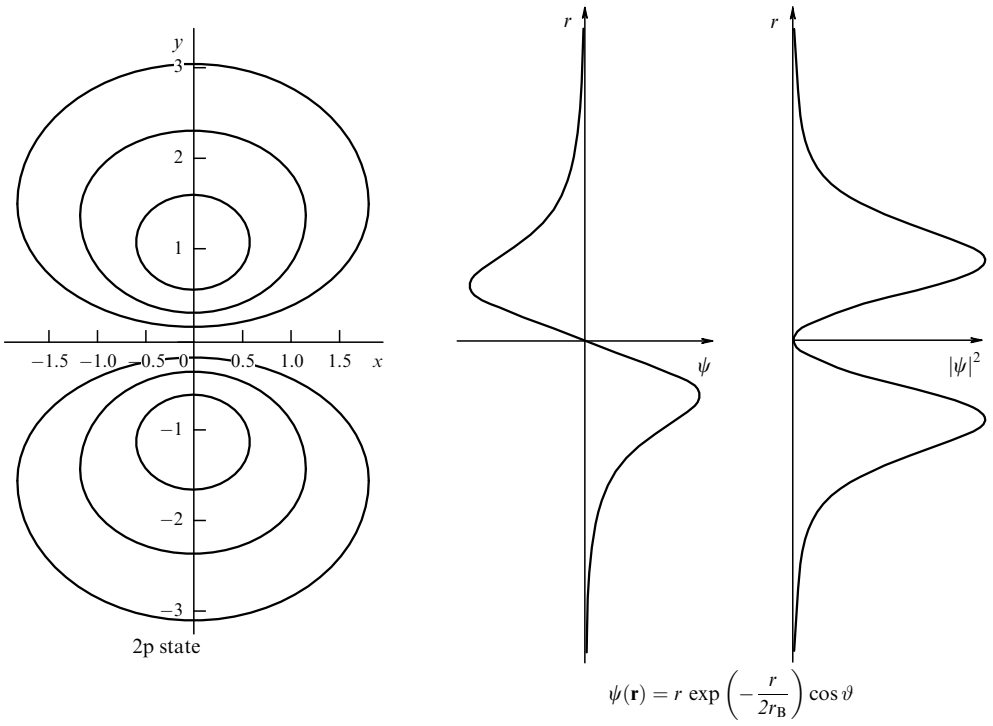

Figure 1. 2p state of atomic hydrogen.

allotted a small bubble to each electron or, in other words, some empty space was allotted to accommodate the electron. The main part of the liquid helium remained pure.

Physically, a bubble in helium constitutes a potential well approximately 1 eV deep (see Fig. 2). It is easy to find the fundamental, spherically symmetric solution to the Schrödinger equation for this well:

$$\psi(\mathbf{r}) = A\,\frac{\sin kr}{r}\,.$$

A scientific area of its own has formed in connection with these charged bubbles. A start was made on the measurements of their mobility; to do this, the whole system was embedded in an alternating electric field. It turned out that the bubbles behaved like particles with a rather large mass, on the order of two hundred atomic helium masses. It has been possible to determine the bubble diameter which turned out to be equal to 38 Å.

More recently, an ultrasonic field was excited in superfluid helium containing these charged bubbles, and the bubbles were found to possess deformation resonances. Theoretical papers in this area also followed. By and large, theoretical inferences and experimental examinations agreed closely with each other.

Since the bubble constitutes a potential well, it should have, apart from the ground state, excited states as well, which have been possible to observe with the aid of laser radiation sources.

As regards the theory, researchers were concerned primarily with the dimension and shape of the bubbles. The theory is conceptually simple: the bubble dimension and shape are defined by the minimum condition for the total energy, although determination of this condition in some cases is a rather arduous task.

The bubble energy consists of three parts: the electron energy, the surface energy, and the energy related to the

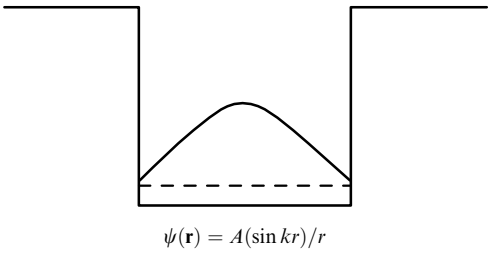

Figure 2. Bubble in superfluid helium as a potential well for the electron; electron wave function is also shown.

variation in the bubble volume; it can be written down as

$$E = E_{\rm el} + \alpha \int {\rm d}S + \int {\rm d}VP\,,$$

where $E_{\rm el}$ is the electron energy which may be found as the solution to the corresponding Schrödinger equation, α is the surface tension, P is the pressure, S is the bubble surface area, and V is the bubble volume. In the spherically symmetric case, this functional has the following form

$$E = \frac{h^2}{8mR^2} + 4\pi R^2\alpha + \frac{4}{3}\,\pi R^3 P\,,$$

where R is the bubble radius. In this case, the functional minimum is trivially found, and for the diameter of a spherically symmetric bubble there results precisely the above value of 38 Å determined from experiments.

Thus, a certain science arose around these charged bubbles, whose foundation was laid, as noted above, by Shal'nikov, and rather many researchers have been involved in it, including the famous expert in this area H Maris of Brown University (USA). His paper "Properties of electron bubbles in liquid helium" [3], which was published in the

reliable *Journal of Low Temperature Physics* in 1998, deserves special mention.

Maris asked himself the question about the bubble shape in the case when the electron state in the bubble is found excited, i.e., about the state that has lost spherical symmetry. Naturally, the absence of spherical symmetry in the electron state should also influence the bubble shape. The first excited electronic state in the potential well constituted by the bubble is reminiscent of the excited 2p state of atomic hydrogen, depicted in Fig. 1.

Although the problem theoretically consists, as before, in the minimization of the above functional, it is now much more complicated. Indeed, the solution procedure now involves the solution to the Schrödinger equation: first for a given shape of the potential well, and then the shape of the potential well (i.e., of the bubble) is somewhat deformed, following which it remains to be seen whether the total bubble energy decreases or increases. A decrease in total bubble energy signifies that the bubble shape was deformed in the right direction. And this is repeated several times until the total energy stabilizes at some minimal level. The bubble shape corresponding to this energy is precisely the equilibrium bubble shape for the excited electronic state selected.

The results of Maris's calculations are depicted in Fig. 3. The equilibrium bubble shapes corresponding to various electronic states are shown. The ground, spherically symmetric 1s state is shown at the upper left. Naturally, the bubble shape is also spherical in this case. Shown to the right of the spherical state is the 2p state (the 1p state in Maris's notation, since the axial symmetry is utilized in his work and not the spherical one, as is usually the case with atoms), which in fact bears strong resemblance to the hydrogenic 2p state (see Fig. 1). To this state there corresponds the wave function displayed above it. The section in the transverse direction is also shown. As is evident from Fig. 3, the wave function turns out to be slightly elongated in the vertical direction; accordingly, the bubble also stretches in this direction. An even more elongated wave function corresponding to a higher energy state resides at the upper right corner of Fig. 3 — the bubble also strongly stretches in the same direction.

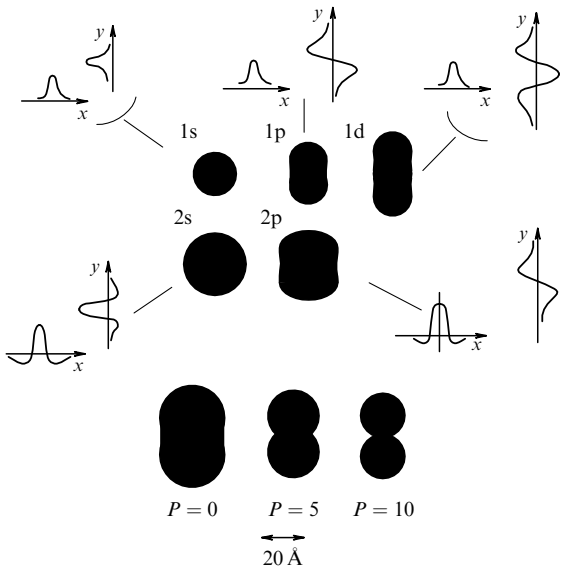

Figure 3. Shapes of charged bubble for various electronic states and different pressures *P*.

One can also see another spherically symmetric state — 2s; the bubble in this case is also spherically symmetric but its diameter is somewhat longer than for the 1s state.

Maris also noticed that the bubble corresponding to the 1p electronic state has a small waist. This is quite natural, because the waist corresponds to the zero of the wave function, which is reflected in the bubble shape.

Next, Maris asked himself the question: what will happen if the pressure is increased? Varying the pressure in calculations is rather simple. The bubble shapes which he calculated are plotted in the lower row in Fig. 3, with the corresponding pressures indicated underneath.

The shape of the bubble at the lower left of Fig. 3 is precisely the same as for the bubble in the middle of the upper row, although on a somewhat different scale. We emphasize that these pictures are the result of a highly complicated calculation, because both the potential well shape and the wave function are varied, and these variations are interrelated. Despite the simplicity of the functional, the calculation is rather arduous.

One can see that increasing the pressure to 5 atm decreases the waist diameter, which becomes even smaller at a pressure of 10 atm; at higher pressures, the bubble simply disintegrates into two separate bubbles of smaller size. We emphasize that the charge of either little bubble is equal to $e/2$, i.e., half the elementary electron charge.

Actually, Maris realized (theoretically, of course, by way of simulations) the situation discussed in the foregoing, when we discussed the hypothetical possibility of splitting the 2p state of atomic hydrogen into two parts in such a way that either part has a charge of $e/2$. Upon formation, the small bubbles drift apart in the liquid and subsequently exist independently of each other. It is pertinent to note that this picture results from a quantum-mechanical calculation and in no way contradicts quantum mechanics.

All the above may be attributed to the fact that this is all pure theory, while only experiment can show how matters actually stand. Maris suggests that the corresponding experiments should be carried out, but so far such experiments have not been reported. Nevertheless, we all live in an age when no doubts concerning the validity of quantum mechanics persist. In fact, matter develops exactly as quantum mechanics predicts. As regards Maris's paper [3], it was written quite accurately and completely in the framework of conventional quantum mechanics. That is why all this is rather serious and, as they say, gives us food for thought. The disintegration of a bubble into two parts actually shows that the wave function describes a materially existing wave, at variance with the above-cited statement from Blokhintsev's textbook.

We now turn our attention to the work in a different area, though in the low-temperature field as well. We will deal with two-dimensional electronic systems. The corresponding research was reported in three papers, for which a Nobel Prize in Physics was awarded in 1998. This is the work on the so-called fractional Hall effect. One paper, written by Tsui, Störmer, and Gossard [4], is experimental, while two other papers, by Laughlin [5], are theoretical.

The sample used in experiments is structurally complex. It consists of several thin layers of different compositions (Fig. 4). A certain number of electrons are accumulated at the interface between two layers to form a two-dimensional electron gas, since here a potential well for electrons exists. In other words, the electrons cannot move in the direction

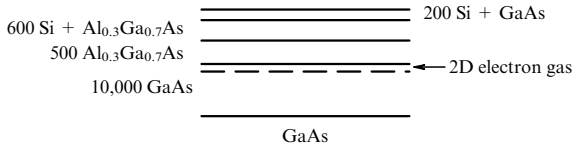

Figure 4. Multilayer sample for the investigation of a two-dimensional electron gas.

perpendicular to the layers. At the same time, the electrons are perfectly free to move along the layer.

However, a strong magnetic field is applied perpendicular to these layers. Referring to Fig. 5, the field varies up to 200 kG, i.e., up to rather high values. An electric current is allowed to pass through the sample during measurements; owing to the strong transverse magnetic field applied to the sample, the Hall effect occurs, i.e., there appears a transverse electric potential difference.

In the layer which accommodates the electrons that make up the two-dimensional electron gas, the ordinary Hall effect occurs for relatively weak magnetic fields, i.e. the departure of the electrons from a rectilinear trajectory and the emergence of the Hall 'transverse' voltage which increases in proportion to the magnetic field intensity. The results of measurements of the Hall effect are depicted in Fig. 5. The upper part of the figure shows the sample structure and the dependence of the Hall effect on the magnetic field; the lower part portrays the sample resistance under the same conditions.

In a two-dimensional electron gas, Landau states occur for a sufficiently high transverse magnetic field. These states possess a certain dimension, and it is possible to calculate how many Landau states fall in a given area. When the number of electrons coincides with the number of Landau states,

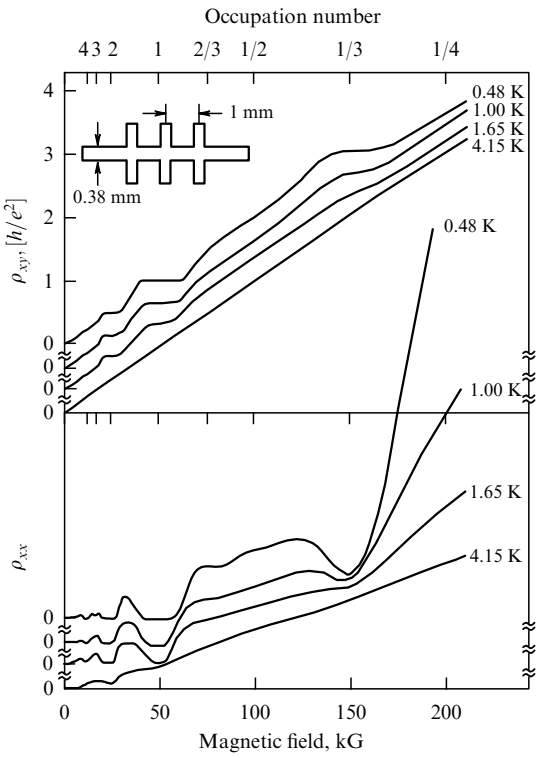

Figure 5. Measurement data according to Ref. [4].

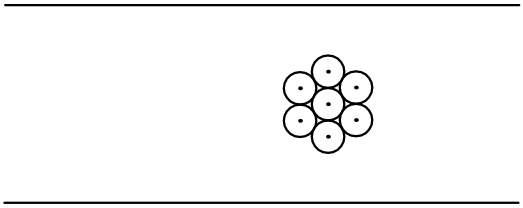

Figure 6. Ordering according to von Klitzing.

electron condensation (or crystallization) occurs in the sample (Fig. 6). That is, in the layer there forms an electron solid body of a sort, which is hard to shift in the transverse direction by a magnetic field, with the result that a plateau in the Hall voltage-vs-magnetic field curve appears (see Fig. 5). This plateau in the dependence of the Hall voltage on the magnetic field intensity was first discovered by Klaus von Klitzing, which brought him the Nobel Prize in Physics 1985. This is the left lower plateau in Fig. 5.

Actually, von Klitzing was awarded the Nobel Prize in Physics for the discovery of the aforementioned electron condensation, in other words, for the discovery of electron crystals (see Fig. 6). In the experiments by Tsui, Störmer, and Gossard, such von Klitzing condensation takes place for a 50-kG field.

What will happen if the magnetic field intensity is further increased? With strengthening magnetic field, the Landau states decrease in size and free space between them appears (Fig. 7). Due to the thermal motion this structure breaks down, and the ordinary Hall effect manifests itself once again. However, one more Landau state may be accommodated between these states for three times as high a magnetic field.

It is easy to understand that the total number of states is tripled in this case, and it turns out that the possibility of the electron crystal forming appears once again or, in other words, rigid electron solid is possible. [1] Since the number of electrons remained as before and the number of states was tripled, each state is now occupied by one third the original number of electrons. The charge of each of these little Landau states is equal to one third of the elementary electron charge. Accordingly, for a magnetic field three times higher, the dependence of the Hall voltage on the magnetic field intensity once again exhibits a plateau. It was precisely this plateau that was discovered in the experiments by Tsui, Störmer, and Gossard [4].

This is how the notion of a fractional charge made its first appearance in physics (not counting quarks, which bear no

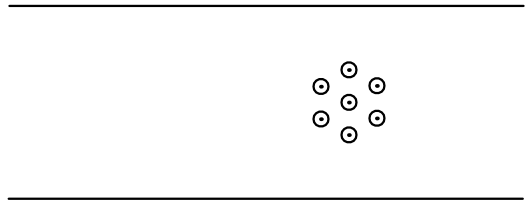

Figure 7. Ordering according to Tsui, Störmer, and Gossard.

[1] In Laughlin's paper it is stated that an 'electron liquid' forms in the two-dimensional electron system under these conditions, i.e., not the whole system is included in one crystal and only the short-range order is settled in the system, which is typical for ordinary liquids.

relation to this issue). Moreover, as noted above, the 1998 Nobel Prize was awarded for this work. There is good reason to be respectful of the data shown in Fig. 5: two Nobel Prizes in Physics have been received for them.

The problem of such objects with a charge of $e/3$ has been discussed in the scientific literature where it caused some surprise among theorists. By and large, the effect has been regarded as a collective one and, hence, has actually been attributed to the manifestation of quasiparticles. As for the charge of a quasiparticle, it need not coincide with the elementary charge.

More recently, however, a research group from Israel has stood out in supposedly disagreeing with this approach and going further. Two papers by this group were published in the journal *Nature*: "Direct observation of a *fractional* charge" (1997) by R de-Picciotto, M Reznikov, M Heiblum, V Umansky, G Bunin, and D Mahalu [6], and "Observation of quasiparticles with *one-fifth* of an electron charge" (1999) by M Reznikov, R de-Picciotto, T G Griffiths, M Heiblum, and V Umansky [7] (author's italics — V B).

They investigated a sample structurally similar to the samples used in the preceding experiments. However, the authors of Refs [6, 7] did not undertake to investigate the Hall effect, because they already knew of electron crystal formation. Instead, they made a device which they termed a 'quantum gate' (Fig. 8), which comprised electrodes laid on the sample. A negative electric potential could be applied to the electrodes. The resultant electron crystal (or the structures with the aforementioned short-range order) cannot pass undestroyed through this gate. That is why the electron crystal breaks up at the quantum gate, and the electron plasma goes further simply as an ordinary chaotic current.

The noise of ordinary currents has long been measured, and it has long been known that it is proportional, first, to the current itself and, second, to the charge of the elementary current carrier, with the result that measuring the current noise has come to be one of the conventional ways of determining the elementary current carrier. In ordinary metals, this carrier is the elementary electron charge e.

The Israeli researchers measured this noise to show that the elementary current carrier in this case is the charge $e/3$. This is evident from the dependence shown in Fig. 8. The experimental data points fall on the curve corresponding to the $e/3$ charge, while the dependence corresponding to the charge e lies higher. These plots are functions of the parameter t, i.e., the transparency of the quantum gate, which is defined by the potential applied to it. The authors of Ref. [6] describe this as the direct observation of a fractional charge. The words 'direct observation' should apply, of course, only to the previous experiments, in which the fractional nature of the charge was 'veiled' by the collective character of the effect, whereas in their experiment the fractional nature of charge already manifests itself. Almost the same group showed that elementary current carriers with a charge of $e/5$ may emerge under slightly different conditions.

Therefore, there is both theoretical and experimental evidence for the formation of physical objects with a fractional charge.

We are reminded once again that objects with partitioned wave functions are rather numerous in physics, in particular, among these is the ammonia molecule. It has been known that the potential curve for the nitrogen ion has two minima (Fig. 9). The similar potential is sometimes referred to as 'Lifshitz's hat'. The two-minimum potential signifies that the nitrogen ion may be located on either side of the plane going through the hydrogen atoms (Fig. 9).

As is commonly known, the so-called inversion spectrum is observed in this case, i.e., the ammonia molecule comprises symmetric and antisymmetric states of the nitrogen ion.

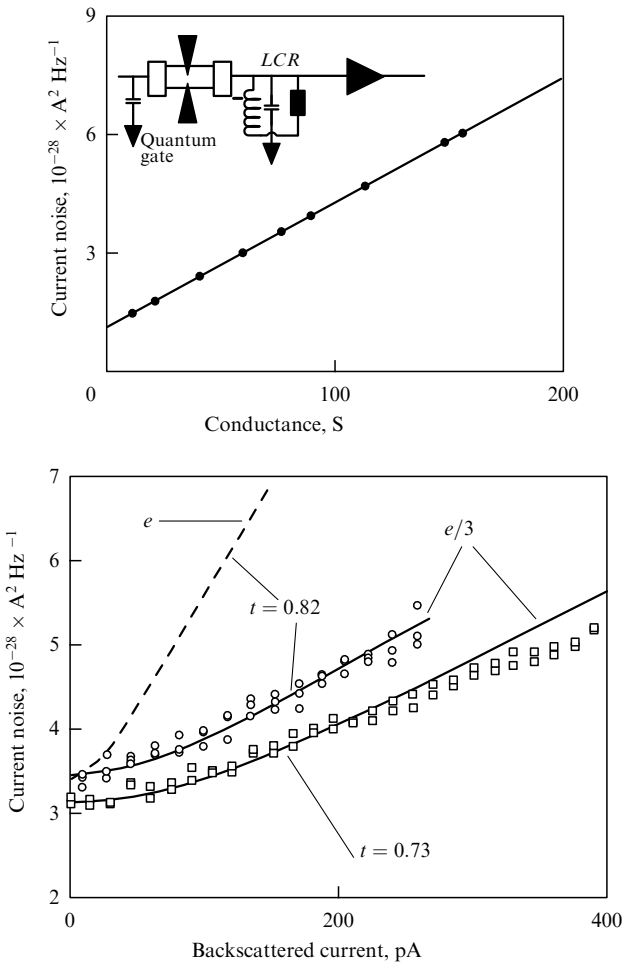

Figure 8. Quantum shot noise as a function of current flowing through the quantum gate. (Taken from Ref. [6].)

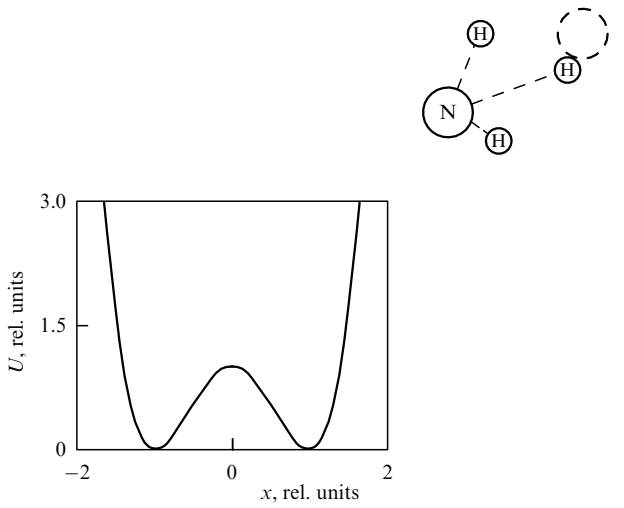

Figure 9. Potential with two minima and an ammonia molecule.

There is good reason to recall that the operation of the first-ever device of quantum electronics, the so-called ammonia maser, utilizes precisely the transition between these symmetric and antisymmetric electronic states. This provides one more confirmation that, strictly speaking, these effects have, in principle, been known.

Reverting for a moment to Maris, what, properly speaking, was he doing? He considered a rather simple system — two potential wells separated by some space. As with the ammonia molecule, in this case, too, the symmetric and antisymmetric states are possible. All this is entirely within the framework of the conventional quantum mechanics.

Is it possible to find something simpler than the object considered by Maris? Simpler is the molecular H_2^+ ion, which has been repeatedly investigated. It is well known that H_2^+ theoretically constitutes a relatively simple system: for instance, variables separate in the Schrödinger equation for the electron in the field of two spatially separated protons. It is valid to say that the H_2^+ ion, owing to its simplicity, plays the same part in molecular spectroscopy as the hydrogen atom in atomic spectroscopy.

The molecular H_2^+ ion forms a system with two potential wells: the Coulomb field near one proton, and the Coulomb field near the other one.

Such a system – two protons and one electron — was investigated in our paper "Light scattering by the products of H_2^+ molecular ion dissociation" written by V P Bykov and E Nahvifard [8]. In this paper we considered light scattering by the products of H_2^+ ion dissociation. We proceeded from the well-known plot of electronic terms (Fig. 10), which shows, in particular, how, in the view of some theorists, dissociation of this molecule occurs. We borrowed this picture from a small review written by the French authors A Giusti-Suzor et al. [9].

One can see the term which possesses a minimum and corresponds to the symmetric electronic state. There are vibrational levels above the bottom of the term. Located higher in energy is the repulsive term which corresponds to the antisymmetric electronic state.

So far, it is impossible to transfer the ion from the lower term to the upper one with the aid of a single photon, because lasers with such photons do not exist. However, this may be effected by way of a three-quantum process depicted in Fig. 10.

The term is referred to as repulsive because the ion fragments fly apart when the H_2^+ ion resides in this term, i.e., molecular dissociation occurs. This picture, properly speaking, is familiar to everyone, because it is reproduced in many textbooks on quantum mechanics.

However, it is worth drawing our attention to the following significant circumstance. At the upper left of Fig. 10 there is a symbolic expression $2p\sigma_u$ alongside the repulsive term, where 2p signifies that the corresponding state passes into the hydrogenlike 2p state when the protons are brought together, and the symbol σ_u signifies that the electronic state of the ion in this term is antisymmetric.

As noted above, the ion fragments fly apart, and the time they take to separate by a distance of 1 μm is equal to about 10^{-12} s, i.e., the fragments fly apart rather rapidly. While this time is short from the standpoint of everyday life, this disintegration is, in reality, adiabatic with respect to the electronic states, i.e., is actually a slow process. The electron state symmetry cannot change in an adiabatic process — that is, the initially antisymmetric state is bound to remain

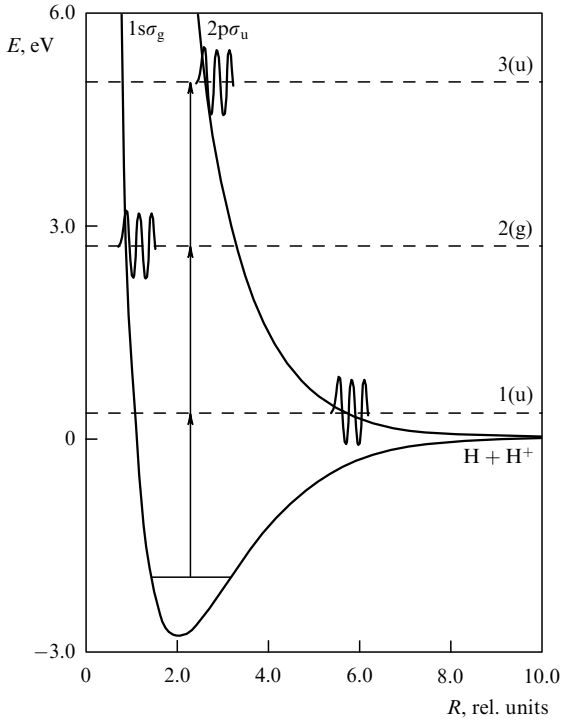

Figure 10. Electronic terms of the molecular hydrogen ion as a function of the distance between the protons.

antisymmetric upon completion of disintegration. However, at the lower right of Fig. 10 it is indicated: $H + H^+$, i.e., a neutral hydrogen atom and a free proton. In other words, the state is by no means a symmetric one, which is impossible in the case of an adiabatic separation.

We shall not discuss what this all signifies from the standpoint of H_2^+ ion dissociation and leave it to the authors of the paper cited [9].

All the foregoing actually leads to the central question. In a system of two protons and one electron, different electronic states are possible for large internuclear distances: stationary (symmetric and antisymmetric) (Fig. 11), and nonstationary (asymmetric) (Fig. 12), when the entire wave function is concentrated at one of the protons. Neither state is by any means at variance with quantum mechanics.

It is pertinent to note that the transition between these states for an internuclear distance of, say, about 1 μm takes

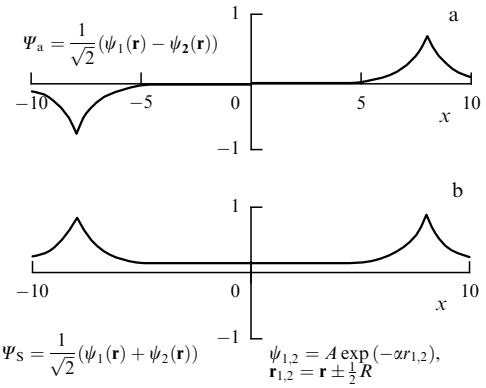

Figure 11. Symmetric and antisymmetric stationary electronic states for large internuclear distances.

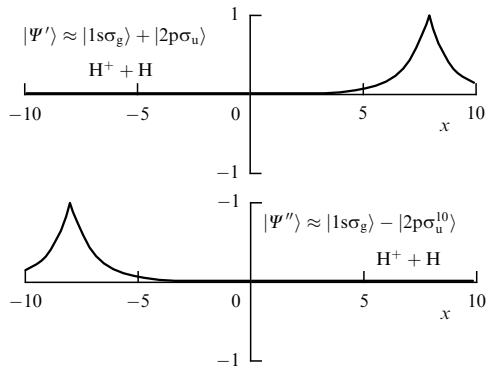

Figure 12. Nonstationary asymmetric electronic states for large internuclear distances.

about 10^{4000} years, which is many orders of magnitude longer than the lifetime of the Universe. In other words, such a transition is in fact impossible.

This brings up the question: is it possible to distinguish these electronic states by means of some physical experiment? This is, properly speaking, the *central* question. In Ref. [9] cited above it was silently assumed that these states are indistinguishable.

To answer this question we have considered the light scattering by the H_2^+ ion dissociation products. The details of our calculations can be found in our paper [8]. Briefly, the conclusion is as follows. Some simple experiments (for instance, experiments on laser radiation scattering) do not permit distinguishing these states. However, more subtle correlation experiments are possible (see Fig. 13), which involve observation of the interference in forward- and sideward-scattered radiation. In these experiments, it *is possible* to distinguish the states depicted in Figs 11 and 12! Figure 13 portrays the correlation functions corresponding to the states presented in Figs 11 and 12 — they are different. Recall that each fragment of the system shown in Fig. 11 carries a charge of $e/2$. Consequently, these objects may exist and are amenable to observation.

It would be of interest to investigate a situation in which, under the same conditions, the interference of radiation intensities occurs, which would perhaps be easier to realize from the experimental point of view. The corresponding estimate will hopefully be made in the future.

What will be the practical outcome of the scientific direction related to fractional charges? First of all, we would like to emphasize that mastering the fractional-charge domain will show up as a lowering of noise of electronic devices, which is equivalent to an increase in sensitivity of many devices. This has actually manifested itself: one can see a lowering of current noise in Fig. 8.

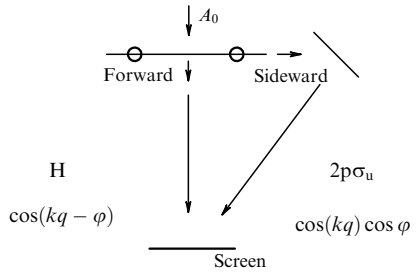

Figure 13. Layout of the correlation experiment on light scattering.

It is pertinent to note that the integer charge e considered as the electron–electromagnetic field coupling constant remains perfectly invariable. The corresponding calculations (in the framework of, say, quantum electrodynamics) remain invariable by reason of fractional-charge observations. The charge of some object is one matter, and the field interaction constant is quite another matter.

As noted in the opening paragraph, it is invalid to say that the fractional-charge problem has been completely elucidated. It invites further investigation and primarily new experiments. Evidence of fractional charge will supposedly enable devising devices of higher sensitivity and less noise.

The above-discussed fractional-charge problem is related to the famous old discussion between N Bohr and A Einstein on the genetical level. In our opinion, however, of greater interest is the prospect of raising the sensitivity of electronic devices and lowering their noise, which opens up in connection with the ideas outlined above.

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