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The Nambu – Jona-Lasinio model and its development

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Contents

Abstract. **The historical development of the Nambu – Jona-Lasinio (NJL) model is briefly reviewed. The SU(2) × SU(2) and U(3) × U(3) local quark NJL models are considered. The mechanisms responsible for spontaneous breaking of chiral symmetry and vector dominance are exhibited. The local NJL model is adequate in describing the mass spectrum and the strong and electroweak decay modes of the four ground-state meson nonets: pseudoscalar, scalar, vector, and axial-vector. The applicability of the model to mesons in a hot dense medium is discussed. It is shown that solving problems related to the description of meson radial excitations and quark confinement requires the nonlocal extension of the NJL model. The primary emphasis of this review is on the methods that are used in various versions of the NJL model. The reader is referred to the cited works for what these models predict in low-energy hadron physics.**

1. Introduction

In 1961, Nambu and Jona-Lasinio (NJL) proposed a model in which an attempt was made to explain the origin of the nucleon mass by the spontaneous breaking of chiral symmetry [1]. The model was formulated in terms of nucleons, pions, and scalar σ-mesons.[1] We should recall that the fundamental theory of strong interactions, namely, quantum chromodynamics (QCD), had not yet been constructed at that time.

Fifteen years later, the Japanese physicists T Eguchi and K Kikkawa [4, 5] reformulated this model in the language of quarks. It is opportune to emphasize that phenomenological quark models are based on the assumption that all hadrons are formed of constituent quarks with a mass $m \approx 300$ MeV, whereas the QCD theory deals with lighter current quarks with a mass $m^0 \approx 5-7$ MeV. It has been shown in Refs [4, 5] that light current quarks transform to massive constituent quarks as a result of spontaneous chiral symmetry breaking. However, their authors considered only the simplest version of the Nambu – Jona-Lasinio model in the chiral limit of $m^0 = 0$, where all masses of pseudoscalar mesons are equal to zero.

In 1982, M K Volkov and D Ebert and co-workers started to consider a more realistic version of the quark NJL model with $m^0 \neq 0$ [6 – 8]. It enabled them to describe the mass spectrum, internal properties, strong and electroweak interactions of scalar, pseudoscalar, vector, and axial-vector meson nonets [9, 10]. In 1984, T Hatsuda and T Kunihiro applied this model to the description of hadrons in a hot and dense medium [11, 12].

After 1986, the NJL model gained even greater popularity, with more than 600 papers devoted to its different aspects being published. For this reason, it is impossible to present a comprehensive list of pertinent references. We only mention here those countries where the NJL model particularly received much attention.

These include Germany and Japan, where the model of interest was applied to the description of low-energy hadron physics. It also attracted the attention of many researchers in Great Britain, Belarus, Italy, China, Portugal, USA, Uzbekistan, Ukraine, France, SAR, and other states.

In this country, the NJL model was most extensively developed in Dubna (Joint Institute for Nuclear Research), Moscow (Institute of Theoretical and Experimental Physics, M V Lomonosov Moscow State University, V A Steklov

[1] It is worth noting that two papers devoted to the solution of similar problems were published in the same year of 1961 by V G Vaks and A I Larkin [2] in *Zh. Eksp. Teor. Fiz.*, and by B A Arbuzov, A N Tavkhelidze, and R N Faustov [3] in *Dokl. Akad. Nauk SSSR*.

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It should be noted that the NJL model is now widely employed in various applications not only in elementary particle physics but also in nuclear physics.

One of the basic problems is the search for the possibility of obtaining an effective NJL Lagrangian taking advantage of the first principles of the fundamental QCD theory. An interesting attempt to this effect was undertaken in Ref. [13] (see also Ref. [14]), where the $SU(2) \times SU(2)$-symmetric four-quark interaction in the scalar and pseudoscalar channels was obtained by using the instanton model of the QCD vacuum. However, it proved difficult, with this approach, to describe vector and axial-vector mesons or to construct a $U(3) \times U(3)$-symmetric NJL model. Here, a definitive solution has yet to be found.

The present review is designed to consider the formulation and applications of the NJL model for the description of the low-energy physics of mesons. Also, the NJL model is successfully used to describe barions. These particles may be regarded as quark – diquark states [15, 16], on the one hand, and as chiral solitons [17 – 20], on the other hand.

The review outline is as follows. Section 2 gives the formulation of the simplest $SU(2) \times SU(2)$ NJL model describing a single scalar σ-meson and three pions. This model is used to demonstrate the spontaneous breaking of chiral symmetry and the fulfillment of the low-energy Goldberger – Treiman and Gell-Mann – Oakes – Renner (GMOR) relations. Then, vector (ρ) and axial-vector (a_1) mesons are included in the model and account is taken of additional renormalization of pion fields due to $\pi - a_1$ transitions. Section 3 is focused on the generalization of the NJL model and its extension to the chiral $U(3) \times U(3)$ group. The 't Hooft interaction is introduced to resolve the $U_A(1)$-problem. Section 4 illustrates the fulfillment of vector dominance in the $U(3) \times U(3)$ NJL model following the introduction of electromagnetic interactions. Section 5 deals with the NJL model in application to the description of mesons in a hot and dense medium. The first radial excitations of mesons are considered in Section 6, and a nonlocal version of the NJL model (with allowance for quark confinement) in Section 7. The review is concluded by a discussion of the feasibility of combining the QCD perturbation theory and the phenomenological NJL model for the description of internal properties and interactions of mesons in a broad energy range.

2. The $SU(2) \times SU(2)$ Nambu – Jona-Lasinio model

2.1 Pseudoscalar and scalar mesons

In order to illustrate the main properties of the NJL model, we shall first consider its simplest version describing one scalar and three pseudoscalar mesons. The initial four-quark $SU(2) \times SU(2)$-symmetric Lagrangian has the form

$$\mathcal{L}(\bar{q}, q) = \bar{q}(x)(\mathrm{i}\hat{\partial}_x - m^0)q(x) + \frac{G}{2}\Big(\big(\bar{q}(x)q(x)\big)^2 + \big(\bar{q}(x)\mathrm{i}\tau^a\gamma^5 q(x)\big)^2\Big), \quad (1)$$

where $\bar{q}(x) = \{\bar{u}(x), \bar{d}(x)\}$ are the fields of u- and d-antiquarks, m^0 is the current quark mass, G is the four-quark coupling constant, τ^a denotes the Pauli matrices, and γ^5 is the Dirac matrix.

Let us demonstrate now how meson fields are introduced and phenomenological meson Lagrangians obtained. Using the generating functional

$$Z(\bar{\eta}, \eta) = \frac{1}{N}\int \mathcal{D}\bar{q}\,\mathcal{D}q \exp\left(\mathrm{i}\int \mathrm{d}^4x\,\left[\mathcal{L}(\bar{q}, q) + \eta\bar{q} + \bar{\eta}q\right]\right), \quad (2)$$

this procedure can be implemented by means of identical transformations in three stages. The initial four-fermion interaction can be rewritten by introducing Gaussian integrals over auxiliary bosonic fields π, σ:

$$\begin{aligned}
Z(\bar{\eta}, \eta) &= \frac{1}{N'}\int \mathcal{D}\bar{q}\,\mathcal{D}q\,\mathcal{D}\sigma\,\mathcal{D}\pi \\
&\quad\times \exp\left(\mathrm{i}\int \mathrm{d}^4x\,\left[\mathcal{L}'(\bar{q}, q, \sigma, \pi) + \eta\bar{q} + \bar{\eta}q\right]\right), \\
\mathcal{L}'(\bar{q}, q, \sigma, \pi) &= \bar{q}(x)\big(\mathrm{i}\hat{\partial}_x - m^0 + \sigma(x) + \mathrm{i}\gamma^5\tau^a\pi^a(x)\big)q(x) \\
&\quad - \frac{\big(\sigma(x)\big)^2 + \big(\pi^a(x)\big)^2}{2G}, \\
Z(\bar{\eta}, \eta) &= \frac{1}{N'}\int \mathcal{D}\sigma'\,\mathcal{D}\pi \\
&\quad\times \exp\left(\mathrm{i}\int \mathrm{d}^4x\,\left[\mathcal{L}''(\sigma', \pi) - \int \mathrm{d}^4y\,\bar{\eta}(x)\,S(x, y)\,\eta(y)\right]\right), \\
\mathcal{L}''(\sigma', \pi) &= -\frac{\big(\sigma'(x)\big)^2 + \big(\pi^a(x)\big)^2}{2G} - \mathrm{i}\,\mathrm{Tr}\left[\ln S^{-1}(x, y)\right]_{x=y}, \\
S^{-1}(x, y) &= \left[\mathrm{i}\hat{\partial}_x - m + \sigma'(x) + \mathrm{i}\gamma^5\tau^a\pi^a(x)\right]\delta^{(4)}(x - y).
\end{aligned} \quad (3)$$

In the beginning, there is a purely quark Lagrangian $\mathcal{L}$ under the sign of the exponent. Then, meson fields enter $\mathcal{L}'$ along with quark ones. Finally, at the last stage of integration over the quark fields, only the observed meson fields remain in $\mathcal{L}''$.

The passage from $\mathcal{L}'$ to $\mathcal{L}''$ resulted in vacuum restructuring due to the spontaneous violation of chiral symmetry, with the current quark mass matrix being substituted by the matrix of constituent quark masses. This substitution is due to the fact that the vacuum expectation of the initially introduced scalar field has a nonzero vacuum average: $\langle\sigma\rangle_0 = \sigma_0 \neq 0$. Therefore, it is necessary to make a shift of this field in order to obtain the physical scalar field: $\sigma' = \sigma + \sigma_0$. Exclusion of the terms linear in the field $\sigma'(x)$ from the Lagrangian produces the mass gap equation

$$\left.\frac{\delta\mathcal{L}''}{\delta\sigma'}\right|_{\sigma'=0} = 0\,, \quad \Rightarrow \quad m^0 = m + \sigma_0 = m\big(1 - 8GI_1^\Lambda(m)\big)\,. \quad (4)$$

This equation describes a spontaneous breaking of chiral symmetry. As a result, light current quarks with a mass m^0 add on to the vacuum expectation of the field $\sigma(x)$ and transform to massive constituent quarks with a mass m. Notice that all basic physical quantities in the NJL model are expressed through quadratically and logarithmically divergent integrals $I_1^\Lambda(m)$ and $I_2^\Lambda(m)$ emerging in the

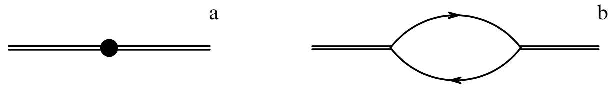

Figure 1. Diagrams defining mass and renormalization of a pion and σ-meson.

consideration of quark loops:

$$
\begin{aligned}
I_1^{\Lambda}(m) &= \frac{N_c}{(2\pi)^4}\int \frac{d_E^4 k\,\theta(\Lambda^2-k^2)}{m^2+k^2} \\
&= \frac{N_c}{(4\pi)^2}\left[\Lambda^2 - m^2\ln\left(\frac{\Lambda^2}{m^2}+1\right)\right], \\
I_2^{\Lambda}(m) &= \frac{N_c}{(2\pi)^4}\int \frac{d_E^4 k\,\theta(\Lambda^2-k^2)}{(m^2+k^2)^2} \\
&= \frac{N_c}{(4\pi)^2}\left[\ln\left(\frac{\Lambda^2}{m^2}+1\right) - \left(1+\frac{m^2}{\Lambda^2}\right)^{-1}\right],
\end{aligned}
\tag{5}
$$

where the integrals are given in Euclidean space, $N_c = 3$ is the number of quark colors, and Λ is the ultraviolet (UV) cut-off parameter that defines the range of applicability of the local NJL model.

The diagrams shown in Fig. 1 define the free meson Lagrangian for pseudoscalar and scalar fields:

$$
\begin{aligned}
&\left(-\frac{1}{2G} + 4I_1^{\Lambda}(m) + 2p^2 I_2^{\Lambda}(m)\right) \\
&\quad\times \left(\pi_a(p)\,\pi_a(-p) + \sigma'(p)\,\sigma'(-p)\right) \\
&\quad - 8m^2 I_2^{\Lambda}(m)\,\sigma'(p)\,\sigma'(-p) \\
&= \frac{1}{2}(p^2 - M_\pi^2)\,\pi_a^R(p)\,\pi_a^R(-p) \\
&\quad + \frac{1}{2}(p^2 - M_\sigma^2)\,\sigma^R(p)\,\sigma^R(-p), \\
&\pi_a^R(p) = g_{\pi qq}\pi_a(p), \qquad \sigma^R(p) = g_{\sigma qq}\sigma(p), \\
&M_\pi^2 = g_{\pi qq}^2\left(\frac{1}{G} - 8I_1^{\Lambda}(m)\right), \qquad M_\sigma^2 = M_\pi^2 + 4m^2,
\end{aligned}
\tag{6}
$$

where $g_{\pi qq} = g_{\sigma qq} = \left(4I_2^{\Lambda}(m)\right)^{-1/2}$ are the renormalization constants of meson fields that provide correct coefficients of the kinetic terms in the meson Lagrangian. In the local NJL model, only divergent parts of the integrals $I_1^{\Lambda}(m)$ and $I_2^{\Lambda}(m)$ are taken into account in the derivation of the free Lagrangian; the momentum dependence of these integrals is discarded in conformity with the assumption of interaction locality. It is only under this condition that an opportunity appears to conserve the chiral-symmetric structure of the Lagrangian describing meson – meson interactions [see formula (26) in Section 3].

The weak pion decay $\pi \to \mu\nu$ is described by the quark loop presented in Fig. 2. This loop is expressed through the integral $I_2^{\Lambda}(m)$, resulting in the Goldberger – Treiman (GT) relation at the quark level [9]:

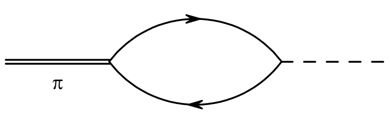

Figure 2. Weak pion decay.

$$
F_\pi = g_{\pi qq} m\left[4I_2^{\Lambda}(m) = \frac{1}{g_{\pi qq}^2}\right] = \frac{m}{g_{\pi qq}}, \tag{7}
$$

where $F_\pi = 93$ MeV is the weak pion decay constant [21].

It can be shown that formulas (6) give the GMOR relation according to which the pion mass squared is proportional to the first power of the current quark mass. Indeed, the following expression for the pion mass can be obtained from Eqns (4), (6), and (7):

$$
M_\pi^2 = g_{\pi qq}^2\frac{m^0}{Gm} = \frac{m^0 m}{GF_\pi^2} = 2\,\frac{m^0\langle\bar{q}q\rangle}{F_\pi^2} + O\left((m^0)^2\right), \tag{8}
$$

where $\langle\bar{q}q\rangle = -4mI_1^{\Lambda}(m)$ is the quark condensate. Thus, the pion becomes a Goldstone particle with zero mass in the chiral limit ($m^0 = 0$).

2.2 Vector and axial-vector mesons

The quark Lagrangian corresponding to the vector and axial-vector mesons has the form

$$
\mathcal{L}(\bar{q}, q) = -\frac{G_V}{2}\left(\left(\bar{q}(x)\,\gamma^\mu\tau^a q(x)\right)^2 + \left(\bar{q}(x)\,\gamma^\mu\gamma^5\tau^a q(x)\right)^2\right), \tag{9}
$$

where G_V is the respective four-quark interaction constant.

This interaction is possible to bosonize by introducing Gaussian integration over vector and axial fields interacting with vector and axial quark currents, respectively. At the second stage of bosonization, this Lagrangian takes the form

$$
\begin{aligned}
\mathcal{L}'(\bar{q}, q, \rho, a_1) &= \bar{q}(x)\left(\gamma^\mu\tau^a\rho_a^\mu(x) + \gamma^\mu\gamma^5\tau^a {a_1}_a^\mu(x)\right)q(x) \\
&\quad + \frac{\left(\rho_a^\mu(x)\right)^2 + \left({a_1}_a^\mu(x)\right)^2}{2G_V},
\end{aligned}
\tag{10}
$$

where ρ_a^μ, ${a_1}_a^\mu$ are the fields of vector (ρ) and axial-vector (a_1) mesons. When passing to the second bosonization stage (during integration over the quark fields), the vector and axial-vector fields are combined with the scalar and pseudoscalar fields in the fermion determinant

$$
\begin{aligned}
\mathcal{L}''(\sigma', \pi, \rho, a_1) &= -\frac{\left(\sigma'(x)\right)^2 + \left(\pi^a(x)\right)^2}{2G} \\
&\quad + \frac{\left(\rho_a^\mu(x)\right)^2 + \left({a_1}_a^\mu(x)\right)^2}{2G_V} - \mathrm{i}\,\mathrm{Tr}\left[\ln S^{-1}(x,y)\right]_{x=y}, \\
S^{-1}(x,y) &= \left[\mathrm{i}\hat{\partial}_x - m + \sigma'(x) + \mathrm{i}\gamma^5\tau^a\pi^a(x)\right. \\
&\quad \left. + \gamma^\mu\tau^a\rho_a^\mu(x) + \gamma^\mu\gamma^5\tau^a {a_1}_a^\mu(x)\right]\delta^{(4)}(x-y).
\end{aligned}
$$

It is worth noting that the description of vector and axial-vector mesons requires the use of gauge-invariant regularization [9, 10]. For the Lagrangian of free vector and axial-vector mesons, it gives

$$
\begin{aligned}
&\left(-\frac{1}{2G_V} + \sqrt{\frac{2}{3}}\,p^2 I_2^{\Lambda}(m)\right)\left(g^{\mu\nu} - \frac{p^\mu p^\nu}{p^2}\right) \\
&\quad\times\left(\rho_a^\mu(p)\,\rho_a^\nu(-p) + {a_1}_a^\mu(p){a_1}_a^\nu(-p)\right) \\
&\quad + \sqrt{6}\,I_2^{\Lambda}(m)\,{a_1}_a^\mu(p)\,{a_1}_a^\mu(-p).
\end{aligned}
\tag{11}
$$

The quark loop with two vector vertices defines the kinetic term of the vector meson and the renormalization constant of vector field $g_{\rho qq}$. As a result, a simple equation relating $g_{\sigma qq}$ to $g_{\rho qq}$ emerges [5–7, 9]:

$$g_{\rho qq} = \sqrt{6}\, g_{\sigma qq}\,. \tag{12}$$

The ρ-meson mass is given by

$$M_\rho^2 = \frac{g_{\rho qq}^2}{4G_V}\,. \tag{13}$$

Renormalization of the a_1-meson field yields $g_{a_1 qq} = g_{\rho qq}$, and the a_1-meson mass is expressed as

$$M_{a_1}^2 = M_\rho^2 + 6m^2\,. \tag{14}$$

2.3 π – a_1 transitions

The NJL model comprises nontrivial quark loops with pseudoscalar and axial-vector vertices, which describe π – a_1 transitions [9, 22–25]. These transitions lead to the appearance of nondiagonal terms of the type $\sqrt{6}\, m\, a_{1a}^{\mu}(x)\, \partial_x^{\mu} \pi_a(x)$ in the meson Lagrangian. In order to exclude these terms, it is necessary to redefine the axial-vector field as

$$a_{1a}^{\mu}(x) = a_{1a}^{\prime\mu}(x) - \frac{\sqrt{6}\, m}{M_{a_1}^2}\, \partial_x^{\mu} \pi_a(x)\,. \tag{15}$$

This gives rise to an additional contribution to the kinetic terms and to the modification of constant $g_{\pi qq}$. Now, this constant is not equal to $g_{\sigma qq}$:

$$g_{\pi qq} = Z^{1/2} g_{\sigma qq}\,, \qquad Z = \left(1 - \frac{6m^2}{M_{a_1}^2}\right)^{-1}. \tag{16}$$

Interestingly, the allowance for π – a_1 transitions does not affect the Goldberger – Treiman relation. Indeed, the diagram shown in Fig. 2 acquires an additional factor Z that is, however, cancelled in the calculation of an additional process with an intermediate a_1-meson [9].

2.4 Numerical estimations

Let us define the model parameters. Equations (7), (12), and (16) may be used to find a constituent quark mass from the observables $F_\pi = 93$ MeV, $g_{\rho qq} = 6.14$ [this value corresponds to the experimental width of the ρ-meson: $\Gamma_{\rho \to \pi\pi} = g_{\rho qq}^2 (M_\rho^2 - 4M_\pi^2)^{3/2} / 48\pi M_\rho = 150$ MeV], and $M_{a_1} = 1.26$ GeV [21]:

$$g_{\rho qq} = \sqrt{6}\, g_{\sigma qq} = \sqrt{\frac{6}{Z}}\, g_{\pi qq} = \sqrt{\frac{6}{Z}}\, \frac{m}{F_\pi} = \sqrt{6\left(1 - \frac{6m^2}{M_{a_1}^2}\right)}\, \frac{m}{F_\pi}\,. \tag{17}$$

Whence it follows that

$$m^2 = \frac{M_{a_1}^2}{12}\left(1 - \sqrt{1 - \frac{4 g_{\rho qq}^2 F_\pi^2}{M_{a_1}^2}}\right) \quad \Rightarrow \quad m = 280 \text{ MeV}\,. \tag{18}$$

The parameter Λ can be found from the equation

$$g_{\rho qq} = \sqrt{\frac{6}{4 I_2}} \quad \Rightarrow \quad \Lambda = 1.25 \text{ GeV}\,, \tag{19}$$

while G and G_V from the equations (6), (13) for pion and ρ-meson masses that gives the values of $G = 4.9$ GeV^{-2} and $G_V = 16$ GeV^{-2}. The value of the current quark mass is determined from the mass gap equation (4) as $m^0 = 3$ MeV. The quark condensate equals $\langle \bar{q} q \rangle = -(305 \text{ MeV})^3$. Notice that the thus obtained overestimated value for the quark condensate is consistent with the underestimated current quark mass; hence, the correct pion mass value is in conformity with the GMOR relation (8). Thus, these two unobservable quantities do not distort the correct relation between the observable quantities M_π and F_π.

It appears from this line of reasoning that the σ-meson mass is 580 MeV, and the width of its decay into two pions[2] equals 500 MeV.

3. The U(3) × U(3) Nambu – Jona-Lasinio model

In order to introduce strange mesons into the model, it is necessary to replace the Pauli matrices τ_i ($i = 1, 2, 3$) by the Gell-Mann matrices λ_i ($i = 0, \ldots, 8$, where $\lambda_0 = \sqrt{2/3}\, \mathbf{1}$). It appears appropriate to recall the $U_A(1)$-problem related to the correct description of η- and η′-meson masses. Indeed, by using a U(3) × U(3)-symmetric Lagrangian, one obtains 'ideal' singlet – octet mixing for pseudoscalar, isoscalar mesons. Then, one of these states contains only u- and d-quarks, and the other a strange quark alone. This situation is in conflict with experimental findings.

In order to solve this problem, it is necessary to add the 't Hooft interaction [27] to the NJL Lagrangian [15, 16, 28, 29]. The addition yields a model describing scalar and pseudoscalar meson nonets, which is composed of two Lagrangians[3]: the standard U(3) × U(3)-symmetric (in the chiral limit) Lagrangian $\mathcal{L}^{\rm NJL}$, and the 't Hooft six-quark Lagrangian $\mathcal{L}^{\rm tH}$:

$$\begin{aligned} \mathcal{L}^{\rm NJL} &= \bar{q}(\mathrm{i}\hat{\partial} - m^0) q + \frac{G}{2} \sum_{i=0}^{8} \left[(\bar{q}\lambda_i q)^2 + (\bar{q}\mathrm{i}\gamma_5 \lambda_i q)^2 \right], \\ \mathcal{L}^{\rm tH} &= -K\left(\det\left[\bar{q}(1+\gamma_5) q\right] + \det\left[\bar{q}(1-\gamma_5) q\right]\right), \end{aligned} \tag{20}$$

where $\bar{q} = \{\bar{u}, \bar{d}, \bar{s}\}$ are the antiquark fields, and m^0 is the diagonal current quark mass matrix with the elements m_u^0, m_d^0, m_s^0 ($m_u^0 \approx m_d^0$).

It follows from numerical estimations (see below) that the principal interaction here is the four-quark Nambu – Jona-Lasinio interaction, whereas the analogous diagonal terms in the 't Hooft interaction provide only small corrections to the principal NJL interaction. However, the 't Hooft interaction comprises nondiagonal terms responsible for u-, d-, and s-quark mixing. These terms are crucial for the correct description of η- and η′-meson masses, taking into account the deviation from their ideal mixing and thus facilitating the solution to the $U_A(1)$-problem. Therefore, only terms containing two quark – antiquark pairs may be retained in

[2] It is noteworthy that this value may be much smaller if π – a_1 transitions are taken into account [26].

[3] $\mathcal{L}^{\rm NJL}$ and $\mathcal{L}^{\rm tH}$ are of different natures. Indeed, the effects of $U_A(1)$-symmetry breaking must disappear in the limit $N_c \to \infty$ [30], in agreement with a N_c-fold suppression of the 't Hooft interaction compared with the four-quark one. It is worth noting that such a model may pose a problem ensuing from vacuum instability. Nevertheless, such an approximation may be justified [31].

the 't Hooft six-quark interaction for the further description of the U(3) × U(3) NJL model. Such interaction can be realized by dint of a single pairing of quark lines in all possible ways in the six-quark interaction. The Lagrangian possessing all the aforementioned qualities necessary for further calculations can be written down in the form[4]

$$\mathcal{L} = \bar{q}(\mathrm{i}\hat{\partial} - \bar{m}^0)q + \frac{1}{2}\sum_{i=1}^{9}\left[G_i^{(-)}(\bar{q}\lambda_i' q)^2 + G_i^{(+)}(\bar{q}\mathrm{i}\gamma_5\lambda_i' q)^2\right] + G_{\mathrm{us}}^{(-)}(\bar{q}\lambda_{\mathrm{u}}q)(\bar{q}\lambda_{\mathrm{s}}q) + G_{\mathrm{us}}^{(+)}(\bar{q}\mathrm{i}\gamma_5\lambda_{\mathrm{u}}q)(\bar{q}\mathrm{i}\gamma_5\lambda_{\mathrm{s}}q)\,, \qquad (21)$$

where the following notation was utilized:

$$\lambda_i' = \lambda_i\,, \quad i = 1, \ldots, 7\,; \quad \lambda_8' = \lambda_{\mathrm{u}} = \frac{\sqrt{2}\,\lambda_0 + \lambda_8}{\sqrt{3}}\,,$$

$$\lambda_9' = \lambda_{\mathrm{s}} = \frac{-\lambda_0 + \sqrt{2}\,\lambda_8}{\sqrt{3}}\,,$$

$$G_1^{(\pm)} = G_2^{(\pm)} = G_3^{(\pm)} = G \pm 4Km_{\mathrm{s}}\,I_1^{\Lambda}(m_{\mathrm{s}})\,, \qquad (22)$$

$$G_4^{(\pm)} = G_5^{(\pm)} = G_6^{(\pm)} = G_7^{(\pm)} = G \pm 4Km_{\mathrm{u}}\,I_1^{\Lambda}(m_{\mathrm{u}})\,,$$

$$G_8^{(\pm)} = G_{\mathrm{u}}^{(\pm)} = G \mp 4Km_{\mathrm{s}}\,I_1^{\Lambda}(m_{\mathrm{s}})\,, \quad G_9^{(\pm)} = G_{\mathrm{s}}^{(\pm)} = G\,,$$

$$G_{\mathrm{us}}^{(\pm)} = \pm 4\sqrt{2}\,Km_{\mathrm{u}}I_1^{\Lambda}(m_{\mathrm{u}})\,,$$

and $\bar{m}^0$ is the diagonal matrix of modified current quark masses $m_{\mathrm{u}}^0, m_{\mathrm{d}}^0, m_{\mathrm{s}}^0$:

$$\bar{m}_{\mathrm{u}}^0 = m_{\mathrm{u}}^0 - 32m_{\mathrm{u}}m_{\mathrm{s}}K\,I_1^{\Lambda}(m_{\mathrm{u}})\,I_1^{\Lambda}(m_{\mathrm{s}})\,,$$

$$\bar{m}_{\mathrm{s}}^0 = m_{\mathrm{s}}^0 - 32K\left(m_{\mathrm{u}}\,I_1^{\Lambda}(m_{\mathrm{u}})\right)^2.$$

It should be emphasized that the 't Hooft interaction increases the pseudoscalar four-quark constants, and decreases the scalar ones, which brings about an additional violation of chiral symmetry.

The 't Hooft interaction gives rise to an additional term in the gap equations for the u- and s-quark masses that take the form

$$m_{\mathrm{u}} = m_{\mathrm{u}}^0 + 8m_{\mathrm{u}}G\,I_1^{\Lambda}(m_{\mathrm{u}}) + 32m_{\mathrm{u}}m_{\mathrm{s}}K\,I_1^{\Lambda}(m_{\mathrm{u}})\,I_1^{\Lambda}(m_{\mathrm{s}})\,,$$

$$m_{\mathrm{s}} = m_{\mathrm{s}}^0 + 8m_{\mathrm{s}}G\,I_1^{\Lambda}(m_{\mathrm{s}}) + 32K\left(m_{\mathrm{u}}\,I_1^{\Lambda}(m_{\mathrm{u}})\right)^2. \qquad (23)$$

Mass formulas for pseudoscalar and scalar mesons may be derived in the same way as in Section 2. It should be recalled that, in this case, the momentum dependence in all divergent integrals is neglected, which corresponds to the truncation of all the terms with higher derivatives in the effective meson Lagrangian,[5] with meson masses being determined by their quark content alone.

The cumbersome mass formulas are presented in the Appendix. Qualitatively, the picture looks like the following: the calculation of isovector and strange pseudoscalar meson masses will result in the strong compensation for large terms associated with the contributions from contact and two-vertex quark loop diagrams (see Fig. 1). This produces relatively small pion and kaon mass values. Masses of scalar mesons would exceed those of pseudoscalar ones by virtue of two effects. First, 't Hooft interactions being taken into account, the scalar four-quark constants turn out to be smaller than the pseudoscalar constants. Moreover, in the expression for scalar masses squared a substantial additional contribution arises from quark loop diagrams of the form $4m_{\mathrm{u}}^2$ for a_0, and $(m_{\mathrm{u}} + m_{\mathrm{s}})^2$ for K_0^*.

A more complicated situation occurs in the calculation of masses of η- and η′-mesons, and of σ- and $\mathrm{f}_0(980)$-mesons. Here, the 't Hooft interaction leads to the appearance of additional nondiagonal terms responsible for mixing strange and nonstrange quarks. Thus, it requires diagonalization of the free Lagrangian. This makes the formulas for the masses of these particles even more complicated (they are also compiled in the Appendix).

Two additional arbitrary parameters enter the U(3) × U(3) NJL model, namely, the constituent mass of a strange quark m_{s}, and the 't Hooft interaction constant K. These parameters can be determined from the kaon mass and the difference between η- and η′-meson masses. The result is

$$m_{\mathrm{s}} = 425\ \mathrm{MeV}\,, \qquad K = 13.3\ \mathrm{GeV}^{-5}\,. \qquad (24)$$

Parameters m_{u} and Λ retain their previous values, $m_{\mathrm{u}} = 280$ MeV and $\Lambda = 1.25$ GeV. The same refers to the parameter $G_\pi = 4.9\ \mathrm{GeV}^{-2}$ found from the pion mass [the SU(2) model contained simple G]. In this case, the contributions of the 't Hooft terms to constituent quark masses and four-quark coupling constants do not exceed 10%. Indeed, the term related to the 't Hooft interaction in the mass gap equation for u-quarks is 27 MeV, and the NJL term amounts to 250 MeV. For strange quarks, these contributions are 21 MeV and 330 MeV, respectively. A similar situation takes place in the determination of four-quark coupling constants $G_i^{(\pm)}$.

Estimates for scalar and pseudoscalar meson masses are presented in the table below. The parameters of the model were fixed by the masses of pseudoscalar mesons. At the same time, the scalar meson masses qualitatively agree with experimental data.

Table. Pseudoscalar, scalar, and vector meson masses in the U(3) × U(3) NJL model versus experimental values [21].

Meson mass	NJL model	Experiment
M_{π^0}	135	134.9766 ± 0.0006
M_{K^0}	495	497.648 ± 0.022
M_η	520	547.75 ± 0.12
$M_{\eta'}$	1000	957.78 ± 0.14
M_σ	550	400 – 1200
M_{f_0}	1130	980 ± 10
M_{a_0}	810	985.1 ± 1.3
$M_{\mathrm{K}_0^*}$	960	~ 800
M_ρ	770	775.8 ± 0.5
M_{K^*}	930	896.10 ± 0.27
M_ϕ	1090	1019.460 ± 0.019

Vector and axial-vector mesons are introduced in U(3) × U(3) models as in the SU(2) × SU(2) version [see Lagrangian (9)]. As a result, bosonization of the quark Lagrangian leads to the following expressions for vector meson masses [9]:

$$M_\rho^2 = \frac{g_\rho^2}{4G_{\mathrm{V}}}\,, \quad M_{\mathrm{K}^*}^2 = \frac{g_{\mathrm{K}^*}^2}{4G_{\mathrm{V}}} + \frac{3}{2}(m_{\mathrm{s}} - m_{\mathrm{u}})^2\,, \quad M_\phi^2 = \frac{g_\phi^2}{4G_{\mathrm{V}}}\,, \qquad (25)$$

[4] Here, we follow Ref. [28].

[5] It should be once again noted that only in this way is it possible to obtain the chiral-symmetric Lagrangian of meson – meson interactions [see formula (26)].

where $g_\rho = \sqrt{6}\, g_{a0}$, $g_{K^*} = \sqrt{6}\, g_{K_0^*}$, and $g_\phi = \sqrt{6}\, g_{\sigma_s}$. Notice that loop diagrams contribute to the mass of the K^*-meson alone; as regards the remaining mesons, the loop diagrams make contributions only to their kinetic terms. The resultant particle masses are in fairly good agreement with experimental values (see the table).

Thus far, we have discussed particle masses. To conclude this section, here is an expression for the interaction Lagrangian that describes strong interactions of four meson nonets, obtained in the one-loop quark approximation:

$$\mathcal{L}^{\rm int} = \frac{1}{4}\,{\rm Tr}\left\{ g^2\left(\left[\left(\bar\sigma - \frac{M}{g}\right), \bar\phi\right]_-^2 - \left[\left(\bar\sigma - \frac{M}{g}\right)^2 + \bar\phi^2\right]^2\right) - \frac{1}{2}\left(G_V^{\mu\nu}G_V^{\mu\nu} + G_A^{\mu\nu}G_A^{\mu\nu}\right) + \left[D_\mu\left(\bar\sigma - \frac{M}{g}\right)^2 + \frac{g_\rho}{2}\{\bar A_\mu, \bar\phi\}_+\right]^2 + \left[D_\mu\bar\phi - \frac{g_\rho}{2}\left\{\bar A_\mu, \left(\bar\sigma - \frac{M}{g}\right)\right\}_+\right]^2\right\}, \quad (26)$$

where

$$G_V^{\mu\nu} = \partial_\mu \bar V^\nu - \partial_\nu \bar V^\mu - {\rm i}\,\frac{g_\rho}{2}\left([\bar V^\mu, \bar V^\nu]_- + [\bar A^\mu, \bar A^\nu]_-\right),$$
$$G_A^{\mu\nu} = \partial_\mu \bar A^\nu - \partial_\nu \bar A^\mu - {\rm i}\,\frac{g_\rho}{2}\left([\bar V^\mu, \bar A^\nu]_- + [\bar A^\mu, \bar V^\nu]_-\right), \quad (27)$$
$$\bar a = \lambda_i a^i, \qquad D_\mu \bar a = \partial_\mu \bar a - {\rm i}\,\frac{g_\rho}{2}\,[\bar V_\mu, \bar a].$$

Electroweak interactions are introduced in the model in a gauge-invariant manner on the basis of the original quark Lagrangian (1). This makes it possible to describe not only strong processes (strong decays, ππ- and πK-scattering, etc.) but also different electroweak processes, such as electromagnetic and weak decays, radii, polarizabilities, and various rare processes (e.g., $\eta \to \pi^0\gamma\gamma$).

4. Vector dominance

After introducing electromagnetic interactions into the NJL Lagrangian, photons can interact with charged mesons only via quark loops. In contrast to the Lagrangian terms related to the mesons, which are composite objects, the kinetic term for photons is independently introduced in the Lagrangian. Taking the quark loops into consideration only results in renormalization of both the electromagnetic fields and the charge.

The part of the Lagrangian describing electromagnetic interactions has the form

$$\mathcal{L}_{\rm em} = -\frac{1}{4}(F_{\mu\nu})^2 - {\rm i\,Tr\,ln}\left[1 - \frac{e}{{\rm i}\hat\partial - m}\,Q\hat A\right], \quad (28)$$

where

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu, \quad (29)$$

and $Q = (\lambda_3 + \lambda_8/\sqrt{3})/2$ is the operator of the quark electric charge.

The calculation of the divergent self-energy photon diagram (Fig. 3a) yields the following expression for $\mathcal{L}_{\rm em}$:

$$\mathcal{L}_{\rm em} = -\frac{1}{4}(F'_{\mu\nu})^2 - {\rm i\,Tr\,ln}\left[1 - \frac{e'}{{\rm i}\hat\partial - M}\,Q\hat A\right]', \quad (30)$$

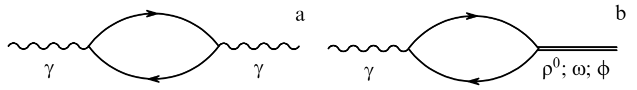

Figure 3. (a)) Divergent quark loops with external photons, and (b) vector mesons ρ^0, ω, ϕ.

where

$$A'_\mu = \left(1 + \frac{4}{3}\frac{e^2}{g_\rho^2}\right)^{1/2} A'_\mu, \qquad e' = \left(1 + \frac{4}{3}\frac{e^2}{g_\rho^2}\right)^{-1/2} e. \quad (31)$$

Besides self-energy diagrams involving a photon, there are mixed-type divergent diagrams describing transitions $\gamma - \rho^0$, $\gamma - \omega$, and $\gamma - \phi$ (Fig. 3b). With the inclusion of these diagrams, the Lagrangian acquires terms of the form

$$\frac{1}{2}\frac{e'}{g_\rho}\,F'_{\mu\nu}\left(\rho^0_{\mu\nu} + \frac{1}{3}\,\omega_{\mu\nu} + \frac{\sqrt{2}}{3}\,\phi_{\mu\nu}\right), \quad (32)$$

where $\rho^0_{\mu\nu}$, $\omega_{\mu\nu}$, and $\phi_{\mu\nu}$ are tensors similar to that in formula (29).

As a consequence, the part of the Lagrangian that describes electromagnetic interactions of mesons and quarks takes the form

$$\mathcal{L}_{\rm em} = \frac{M_\rho^2}{2}\left(\omega_\mu^2 + \rho_\mu^{0\,2}\right) + \frac{M_\phi^2}{2}\,\phi_\mu^2 - \frac{1}{4}\left(\rho_{\mu\nu}^{0\,2} + \omega_{\mu\nu}^2 + \phi_{\mu\nu}^2 + F'^2_{\mu\nu}\right) + \frac{1}{2}\frac{e'}{g_\rho}\,F'_{\mu\nu}\left(\rho^0_{\mu\nu} + \frac{1}{3}\,\omega_{\mu\nu} + \frac{\sqrt{2}}{3}\,\phi_{\mu\nu}\right) - {\rm i\,Tr\,ln}\left[1 + \frac{1}{{\rm i}\hat\partial - M}\left(\frac{g_\rho}{2}(\gamma_\mu\lambda^i V^i_\mu) - e'Q\hat A\right)\right]', \quad (33)$$

where V^i_μ denotes the fields of vector mesons.

The kinetic terms can be diagonalized by means of the following replacements of fields:

$$\rho^0_\mu = \tilde\rho^0_\mu + \frac{e'}{g_\rho}\,A'_\mu,$$
$$\omega_\mu = \tilde\omega_\mu + \frac{e'}{3g_\rho}\,A'_\mu, \quad (34)$$
$$\phi_\mu = \tilde\phi_\mu + \frac{\sqrt{2}\,e'}{3g_\rho}\,A'_\mu.$$

The electromagnetic field and charge e' are then renormalized as follows:

$$\tilde A_\mu = \left(1 - \frac{4}{3}\frac{e'^2}{g_\rho^2}\right)^{1/2} A'_\mu,$$
$$\tilde e = \left(1 - \frac{4}{3}\frac{e'^2}{g_\rho^2}\right)^{-1/2} e' = \left[\left(1 - \frac{4}{3}\frac{e'^2}{g_\rho^2}\right)\left(1 + \frac{4}{3}\frac{e^2}{g_\rho^2}\right)\right]^{-1/2} e \to \tilde e = e. \quad (35)$$

It is easy to see that the two renormalizations, Eqns (30) and (35), result in the electric charge taking the initial value. The final Lagrangian has the form

$$\begin{aligned}\mathcal{L}_{\rm em} = {} & \frac{M_\rho^2}{2}\left(\tilde{\omega}_\mu^2 + \tilde{\rho}_\mu^{0\,2}\right) + \frac{M_\phi^2}{2}\,\tilde{\phi}_\mu^2 \\ & - \frac{1}{4}\left(\tilde{\rho}_{\mu\nu}^{0\,2} + \tilde{\omega}_{\mu\nu}^2 + \tilde{\phi}_{\mu\nu}^2 + \tilde{F}_{\mu\nu}^{\prime\,2}\right) \\ & + \left(\frac{e}{3g_\rho}\right)^2 \left(5M_\rho^2 + m_\phi^2\right) \tilde{A}_\mu^2 \\ & + \frac{e}{g_\rho}\left[M_\rho^2\left(\tilde{\rho}_\mu^0 + \frac{\tilde{\omega}_\mu}{3}\right) + \frac{\sqrt{2}}{3}\,M_\phi^2 \tilde{\phi}_\mu\right] A_\mu \\ & - \mathrm{i}\,\mathrm{Tr}\ln\left[1 + \frac{1}{\mathrm{i}\hat{\partial} - M}\,\frac{g_\rho}{2}\left(\gamma_\mu \lambda^i V_\mu^i\right)\right]. \qquad (36)\end{aligned}$$

It is easy to verify now that photons can interact with charged particles only through the agency of neutral vector mesons. This automatically brings about a model describing vector dominance. Under the sign of the logarithm, the term with photons is completely absorbed by vector mesons.

5. Mesons in a hot and dense medium

In the last few years, researchers have become increasingly interested in the search for a new state of matter, namely, quark – gluon plasma (QGP). New data for the processes of hadron matter transition to QGP are already coming from experiments on heavy ion collisions, carried out in a number of large physical centres, such as Brookhaven and CERN. New facilities are about to be commissioned (LHC, SIS-300) for a deeper insight into the problem. QGP is expected to manifest itself through modified properties of hadronic reactions and their products.

The NJL model provides a very convenient tool for the investigation of meson behavior in a hot and dense medium. The very first calculations of this kind in the framework of the NJL model were reported in Refs [11, 12].

A variety of methods are available for the study of meson behavior in a hot and dense medium. The most popular one is the Matsubara technique [33] in which 'imaginary time' formalism implies the replacement of integration over the zero component of the momentum by the summation of frequencies:

$$p^0 \to \mathrm{i}\omega_n + \mu\,, \qquad (37)$$

$$\int \frac{\mathrm{d}^4 p}{(2\pi)^4} \to \mathrm{i}T \sum_n \int \frac{\mathrm{d}^3 p}{(2\pi)^3}\,, \qquad (38)$$

where ω_n stands for the Matsubara frequencies: $\omega_n = (2n+1)\pi T$ for fermions, and $\omega_n = 2n\pi T$ for bosons; μ is the chemical potential, and T is the temperature.

In certain approximations, however, it is more convenient to utilize a simpler method[6] with which the quark propagator is represented in a medium in the 'real time' formalism [34, 37]:

$$\begin{aligned}S(p, T, \mu) = (\hat{p} + m)\Bigg[& \frac{1}{p^2 - m^2 + \mathrm{i}\epsilon} \\ & + \mathrm{i}2\pi\delta(p^2 - m^2)\big(\theta(p^0)\,n(\mathbf{p},\mu) + \theta(-p^0)\,n(\mathbf{p},-\mu)\big)\Bigg], \qquad (39)\end{aligned}$$

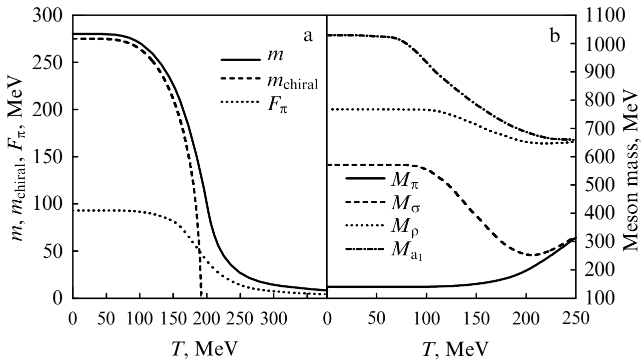

Figure 4. Quark masses, constants F_π of weak pion decay (a) and meson masses M_π, M_σ, M_ρ, M_{a_1} (b) as the functions of T.

where

$$n(\mathbf{p},\mu) = \left(1 + \exp\frac{E-\mu}{T}\right)^{-1} \qquad (40)$$

is the Fermi – Dirac function for quarks, and $E = \sqrt{\mathbf{p}^2 + m^2}$. This leads to the following method for the calculation of integrals $I_1^{\Lambda_3}(m,T,\mu)$ and $I_2^{\Lambda_3}(m,T,\mu)$. First, contour integration is performed in the complex p_0 plane. It is followed by regularization of the remaining integral with the use of three-dimensional cut-off Λ_3. As a result, the divergent integrals $I_1^{\Lambda_3}(m,T,\mu)$ and $I_2^{\Lambda_3}(m,T,\mu)$ take the forms

$$\begin{aligned}I_1^{\Lambda_3}(m,T,\mu) &= \frac{N_c}{(2\pi)^2}\int_0^{\Lambda_3} \mathrm{d}p\,\frac{p^2}{E}\big(1 - \eta(\mathbf{p},\mu) - \eta(\mathbf{p},-\mu)\big)\,, \\ I_2^{\Lambda_3}(m,T,\mu) &= \frac{N_c}{2(2\pi)^2}\int_0^{\Lambda_3} \mathrm{d}p\,\frac{p^2}{E^3}\big(1 - \eta(\mathbf{p},\mu) - \eta(\mathbf{p},-\mu)\big)\,. \qquad (41)\end{aligned}$$

We determine the values of model parameters in a vacuum[7] under the same conditions as in Section 2.4. Furthermore, we assume that model parameters G, G_V, m^0, and Λ_3 do not depend on T and μ. At the same time, the constituent quark mass m, as well as the quark condensate, exhibits the dependence on temperature and chemical potential. This dependence can be calculated from the mass gap equation. Thereafter, T- and μ-dependences of the basic integrals $I_1^{\Lambda_3}(m,T,\mu)$ and $I_2^{\Lambda_3}(m,T,\mu)$ are computed. This, in turn, permits determining the T- and μ-dependences of all physical quantities.

The behavior of $m(T)$ is illustrated in Fig. 4. The critical value of temperature T_c in the chiral limit $m^0 = 0$ is that at which the constituent quark mass turns to zero. At this point, the chiral symmetry is restored and the quark condensate completely 'evaporates' — that is, the order parameter vanishes. The critical value of the chemical parameter is determined in a similar way. When $m^0 \neq 0$, the sharp phase boundary disappears. The same figure shows the behavior of F_π for $m^0 \neq 0$, and the behavior of meson masses M_π, M_σ, M_ρ, M_{a_1} as the functions of T (see also Ref. [37]). As T increases, the mass of the σ-meson rapidly decreases, exactly

[6] It should be noted that the Matsubara technique provides a more universal approach, while a simplified method requires a special prescription in the case of certain more complex diagrams. These two methods are described at length in Ref. [33]. In addition, Refs [35, 36] present a detailed description of real time formalism.

[7] It should be emphasized that three-dimensional regularization leads to a change in three model parameters, viz., $\Lambda_3 = 1.03$ GeV, $G = 3.48\ \mathrm{GeV}^{-2}$, and $m^0 = 2$ MeV. The two remaining parameters are unaltered: $m = 280$ MeV, and $G_V = 16\ \mathrm{GeV}^{-2}$.

as the constituent quark mass does. On the other hand, the pion mass remains unaltered until the critical conditions for the chiral restoration are reached, beyond which it begins to grow. The M_ρ mass shows a weak dependence on T, whereas M_{a_1} decreases like M_σ. Above the critical temperature, one has $M_\pi \approx M_\sigma$ and $M_\rho \approx M_{a_1}$ as would be expected for the chirally symmetric phase. In Refs [37, 38], the critical temperature was found to be $T_c \approx 200$ MeV. At the same time, lattice calculations gave a lower T_c value, namely, $T_c \approx 170$ MeV [39].

Recently, very interesting results have been obtained in the investigation of strongly interacting quark matter in the color superconducting phase. We do not consider these issues here and confine ourselves to references to the original works [40, 41].

Concluding this section, we would like to emphasize that the properties of some particles can change considerably when approaching the phase transition boundary. Specifically, the σ-meson which presents a very broad resonance in a vacuum may become a very sharp resonance at a certain temperature and chemical potential. As a consequence, some processes in which the σ-meson is involved as an intermediate state may be sharply intensified at close-to-critical values of μ and T. These processes are exemplified by ππ-scattering [42] and $\pi\pi \to \gamma\gamma$ [43]. Such intensification, if observed in heavy ion collision experiments, can be interpreted as indicating the approach to the QGP region.

6. First radial excitations of mesons

It is impossible to describe radial excitations of mesons in a local version of the NJL model. Therefore, not only the standard local Lagrangian $\mathcal{L}$ (1) but also an additional nonlocal Lagrangian $\mathcal{L}^{\text{nonloc}}$ needs to be considered if both the ground and the first radially excited states are to be described. Form factors for each quark – antiquark current are introduced in the Lagrangian $\mathcal{L}^{\text{nonloc}}$:

$$J_I(x) = \iint d^4x_1\, d^4x_2\, \bar{q}(x_1)\, F_I(x; x_1, x_2)\, q(x_2)\,. \tag{42}$$

The form factors $F_I(x; x_1, x_2)$ can be written down in a covariant form [44]. They are not discussed in detail here; suffice it to point out that the form factors for the ground and the first radially excited states may be presented in a very simple form in momentum space:

$$\begin{aligned} f_1(k_\perp) &= \theta\big(\Lambda_3 - |k_\perp|\big)\,, \\ f_2(k_\perp) &= c\big(1 + d\,|k_\perp|^2\big)\,\theta\big(\Lambda_3 - |k_\perp|\big)\,, \end{aligned} \tag{43}$$

where k is the relative momentum of a quark – antiquark pair, and $k_\perp$ is the part of k transversal to the total momentum P:

$$k_\perp \equiv k - \frac{kP}{P^2}\, P\,. \tag{44}$$

The step function $\theta\big(\Lambda_3 - |k_\perp|\big)$ is nothing but a covariant generalization of the three-dimensional cut-off in the NJL model. For $d < -\Lambda_3^{-2}$, the form factor $f_2(k_\perp)$ has the form of a wave function of the radially excited state with a node in the interval $0 < |k_\perp| < \Lambda_3$. Form factors (43) are the first terms in a series expansion in terms of $k_\perp^2$; the inclusion of higher radially excited states in the model would require the introduction of higher-degree polynomials. Factor c describes changes in the four-quark interaction force in radially excited channels relative to the interaction force in the ground-state channel with constant G. This factor is determined from the mass of a radially excited pion: $M'_\pi = 1300$ MeV. Parameter d may be found from the condition [8]

$$I_1^f = -\mathrm{i}N_c \int_{\Lambda_3} \frac{d^4k}{(2\pi)^4}\, \frac{f_1(\mathbf{k})}{m^2 - k^2} = 0\,. \tag{46}$$

The physical meaning of this condition can be explained as follows. The model contains two gap equations

$$\begin{aligned} \frac{\delta\mathcal{W}}{\delta\sigma_1} &= -\mathrm{i}N_c\,\mathrm{Tr}\int_{\Lambda_3} \frac{d^4k}{(2\pi)^4}\, \frac{1}{\hat{k} - m^0 + \langle\sigma_1\rangle_0 + \langle\sigma_2\rangle_0 f_2(\mathbf{k})} \\ &\quad - \frac{\langle\sigma_1\rangle_0}{G} = 0\,, \\ \frac{\delta\mathcal{W}}{\delta\sigma_2} &= -\mathrm{i}N_c\,\mathrm{Tr}\int_{\Lambda_3} \frac{d^4k}{(2\pi)^4}\, \frac{f_2(\mathbf{k})}{\hat{k} - m^0 + \langle\sigma_1\rangle_0 + \langle\sigma_2\rangle_0 f_2(\mathbf{k})} \\ &\quad - \frac{\langle\sigma_2\rangle_0}{G} = 0\,. \end{aligned} \tag{47}$$

In the general case, the solution of these equations is $\langle\sigma_2\rangle_0 \neq 0$; then, the constituent quark mass $-\langle\sigma_1\rangle_0 - \langle\sigma_2\rangle_0 f_1(\mathbf{k}) + m^0$ proves to be momentum-dependent. Condition (46) leads to a trivial solution $\langle\sigma_2\rangle_0 = 0$ for the second gap equation. This brings us back to the standard gap equation of the NJL model with a constant constituent quark mass.

The free part of the effective action for pions takes the form

$$\mathcal{W} = \frac{1}{2}\int \frac{d^4P}{(2\pi)^4} \sum_{i,j=1}^{2} \pi_i^a(P)\, K_{ij}^{ab}(P)\, \pi_j^b(P)\,, \tag{48}$$

where $K_{ij}^{ab}(P) \equiv \delta^{ab} K_{ij}(P)$,

$$\begin{aligned} &K_{11}(P) = Z_1(P^2 - M_1^2)\,, \qquad K_{22}(P) = Z_2(P^2 - M_2^2)\,, \\ &K_{12}(P) = K_{21}(P) = \sqrt{Z_1 Z_2}\,\Gamma P^2 \end{aligned} \tag{49}$$

and

$$\begin{aligned} M_1^2 &= \frac{1}{Z_1}\left(\frac{1}{G} - 8I_1\right) = \frac{m^0}{Z_1 G m}\,, \\ M_2^2 &= \frac{1}{Z_2}\left(\frac{1}{G} - 8I_1^{ff}\right)\,, \\ Z_1 &= 4I_2\,, \quad Z_2 = 4I_2^{ff}\,, \quad \Gamma = \frac{4}{\sqrt{Z_1 Z_2}}\, I_2^f\,. \end{aligned} \tag{50}$$

In order to determine the physical π- and π′-meson states, it is necessary to diagonalize the quadratic part of the action. This operation can be performed with the help of the orthogonal transformation of fields π_1 and π_2 (see Ref. [44] for details on this procedure). It should be noted that the a series expansion in terms of small current quark masses m^0

[8] Here, I_n, I_n^f, and I_n^{ff} denote loop integrals with no, one, and two form factors $f(k_\perp) \equiv f_2(k_\perp)$ in the numerator, respectively:

$$I_n^{f..f} \equiv -\mathrm{i}N_c\,\mathrm{Tr}\int_{\Lambda_3} \frac{d^4k}{(2\pi)^4}\, \frac{f(\mathbf{k})\ldots f(\mathbf{k})}{(m^2 - k^2)^n}\,. \tag{45}$$

yields for physical states[9]:

$$M_\pi^2 = M_1^2 + O(M_1^4)\,,$$
$$M_{\pi'}^2 = \frac{M_2^2}{1-\Gamma^2}\left[1 + \Gamma^2\,\frac{M_1^2}{M_2^2} + O(M_1^4)\right]. \qquad (51)$$

Thus, in the chiral limit, the effective meson Lagrangian actually describes a massless Goldstone boson (pion π) and a heavy pseudoscalar meson π′. The ratio of the weak π- and π′-decay constants can be directly expressed in terms of the meson mass ratio:

$$\frac{F_{\pi'}}{F_\pi} = \frac{\Gamma}{\sqrt{1-\Gamma^2}}\,\frac{M_\pi^2}{M_{\pi'}^2}\,. \qquad (52)$$

It is worth noting that the matrix element of the pseudoscalar meson, resulting from the divergence of the axial-vector current, must disappear in the chiral limit for both π and π′. In the case of the ground-state pion, the matrix element vanishes as the pion mass turns to zero, as it likewise does in the excited state because $F_{\pi'}$ becomes zero.

Here, we considered only pions. It has been shown in Refs [45 – 47] that this approach can be extended to the chiral $U(3)\times U(3)$ group for pseudoscalar, scalar, and vector mesons. In the framework of this model, the main strong decays of radially excited mesons have been described [46, 48]. One of the most interesting physical results obtained in this model concerns the identification of 19 experimentally examined scalar states with masses 0.4 – 1.7 GeV. These states can be interpreted as two scalar nonets and a scalar glueball with a mass of ~ 1.5 GeV [49]. The first nonet consists of ground-state scalar mesons with masses between 0.4 and 1 GeV, and the second one of radially excited scalar mesons with masses 1.3 – 1.7 GeV. The four scalar states and the scalar glueball mixes because they possess the same quantum numbers.

7. The nonlocal Nambu – Jona-Lasinio model and quark confinement

NJL models have two chief drawbacks. They contain ultraviolet (UV) divergences and do not allow quark confinement. Usually, UV divergences are removed with the help of the cut-off parameter Λ on the order of 1 GeV. The employment of only the lowest power of momentum expansion of quark loops makes it possible to avoid the appearance of unphysical quark – antiquark thresholds in the amplitudes of different processes.

These drawbacks in the standard NJL model can be rectified only in the framework of nonlocal models. There are many different nonlocal versions of the NJL model (see, for instance, Refs [50 – 54]). In what follows, one version based on instanton interactions will be demonstrated. Similar models are considered in Refs [55 – 59].

The $SU(2)\times SU(2)$-symmetric action with the nonlocal four-quark interaction has the form

$$\mathcal{S}(\bar q, q) = \int \mathrm{d}^4x \left\{ \bar q(x)(\mathrm{i}\hat\partial_x - m^0)q(x) + \frac{G}{2}\left[J_\sigma(x)\,J_\sigma(x) + J_\pi^a(x)\,J_\pi^a(x)\right] - \frac{G_\mathrm{V}}{2}\left[J_\rho^{\mu a}(x)\,J_\rho^{\mu a}(x) + J_{a_1}^{\mu a}(x)\,J_{a_1}^{\mu a}(x)\right]\right\}. \qquad (53)$$

[9] It follows from Eqn (50) that $M_1^2 \sim m^0$.

The nonlocal quark currents $J_I(x)$ are expressed as

$$J_I(x) = \iint \mathrm{d}^4x_1\,\mathrm{d}^4x_2\; f(x_1)\,f(x_2)\,\bar q(x-x_1)\,\Gamma_I\,q(x+x_2)\,, \qquad (54)$$

where functions $f(x)$ are normalized by the condition $f(0)=1$. In Eqn (53), matrices Γ_I are defined as $\Gamma_\sigma = \mathbf{1}$, $\Gamma_\pi^a = \mathrm{i}\gamma^5\tau^a$, $\Gamma_\rho^{\mu a} = \gamma^\mu\tau^a$, and $\Gamma_{a_1}^{\mu a} = \gamma^5\gamma^\mu\tau^a$.

After bosonization, the scalar field σ has a nonzero vacuum expectation. In order to obtain a physical scalar field with a zero vacuum expectation, it is necessary to make a shift of the scalar field. This leads to the appearance of the nonlocal quark mass $m(p^2)$ instead of the current quark mass m^0, and the former can be found from the gap equation

$$m(p^2) = m^0 + G\,\frac{2N_\mathrm{c}}{(2\pi)^4}\,f^2(p^2)\int \mathrm{d}^4k\,\frac{f^2(k^2)m(k^2)}{k^2+m^2(k^2)} = m^0 + \big(m(0)-m^0\big)\,f^2(p^2)\,. \qquad (55)$$

The quark propagator takes the form

$$S(p) = \big(\hat p - m(p^2)\big)^{-1}\,. \qquad (56)$$

We use only the simple ansatz for the quark propagator. Namely, following Refs [50, 60] we demand that the vector part of the quark propagator be free from polar singularities:

$$\frac{1}{m^2(p^2)+p^2} = \frac{1-\exp\left(-p^2/\Lambda^2\right)}{p^2}\,. \qquad (57)$$

Then, the expression for $m(p^2)$ becomes

$$m(p^2) = \left(\frac{p^2}{\exp\left(p^2/\Lambda^2\right)-1}\right)^{1/2}. \qquad (58)$$

The mass function $m(p^2)$ contains a single arbitrary parameter Λ. This function possesses no singularities on the real axis and exponentially drops as $p^2\to\infty$ in the Euclidean region. It follows from Eqn (55) that nonlocal form factors exhibit a similar behavior, thus providing for the absence of UV divergences in the model. At $p^2=0$, the mass function is equal to the cut-off parameter Λ: $m(0)=\Lambda$. It follows from the mass gap equation that the relation between the four-quark interaction constant G and the nonlocality parameter Λ takes the form

$$G = \frac{2\pi^2}{N_\mathrm{c}}\,\frac{1}{\Lambda^2}\,. \qquad (59)$$

Moreover, the expression for the pion renormalization constant has a very simple form

$$g_\pi^{-2}(0) = \frac{N_\mathrm{c}}{4\pi^2}\left(\frac{3}{8}+\frac{\zeta(3)}{2}\right), \qquad (60)$$

where ζ is the Riemann zeta function. In the chiral limit, there are only two arbitrary parameters Λ, G_V. Their values are fixed with the aid of the weak pion decay constant $F_\pi = 93$ MeV and the ρ-meson mass $M_\rho = 770$ MeV. The use of the Goldberger – Treiman relation $g_\pi(0) = m(0)/F_\pi$ leads to $\Lambda = m(0) = 340$ MeV.

This simple model allows for sensible predictions of σ-meson mass: $M_\sigma = 420$ MeV, and of strong decay $\rho \to \pi\pi$: $\Gamma_{\rho\pi\pi} = 135$ MeV.

This nonlocal model, unlike the local NJL model, can be successfully applied to the description of both the constant parts of the meson process amplitudes and the momentum dependences. Such a possibility was demonstrated by the computation of the pion radius of the $F_{\gamma^*\pi^+\pi^-}$-process form factor [61]. In the nonlocal model, the contributions of contact diagrams and diagrams with intermediate mesons to the radius have reasonable values (specifically, the contribution from the vector mesons is markedly reduced), whereas in the local model these contributions are comparable [62]. At the same time, the vector meson diagrams play a very important role in the description of the $F_{\gamma^*\pi^+\pi^-}$ form factor in the time-like region. These diagrams can be employed to describe not only the ρ-meson resonance but also the process form factor in an energy region of up to 1 GeV.

8. Conclusion

Let us recall once again that the fundamental theory of strong interactions, viz., QCD, was nonexistent when the first version of the NJL model was proposed [1] in 1961. For this reason, different versions of phenomenological hadron models were used at that time to describe low-energy meson physics, whereas the description of hadron interactions at large energies looked highly problematic. However, the construction of QCD theory and the discovery of the phenomenon of asymptotic freedom made it possible to describe hadron interactions at large energies by means of the perturbation theory. True, it turned out that the perturbation theory is applicable only at energies in excess of 1 GeV, when the strong QCD coupling constant is smaller than unity. It again required the employment of phenomenological theories for the description of the low-energy region.

The NJL model proved to be one of the most attractive models of this kind. This model is underlain by chiral symmetry of strong interactions, which also forms the basis of QCD. The joint application of these two theories permits us to describe the entire energy region of strong interactions between elementary particles.

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9. Appendix. Mass formulas in the U(3) × U(3) Nambu – Jona-Lasinio model

Mass formulas for isovector and strange mesons have the form

$$M_\pi^2 = g_\pi^2\left(\frac{1}{G_\pi} - 8I_1^\Lambda(m_u)\right),$$

$$M_K^2 = g_K^2\left(\frac{1}{G_K} - 4\left[I_1^\Lambda(m_u) + I_1^\Lambda(m_s)\right]\right) + Z(m_s - m_u)^2, \qquad (61)$$

$$M_{a_0}^2 = g_{a_0}^2\left(\frac{1}{G_{a_0}} - 8I_1^\Lambda(m_u)\right) + 4m_u^2,$$

$$M_{K_0^*}^2 = g_{K_0^*}^2\left(\frac{1}{G_{K_0^*}} - 4\left[I_1^\Lambda(m_u) + I_1^\Lambda(m_s)\right]\right) + (m_u + m_s)^2,$$

where the following notation is used:

$$G_\pi = G_1^{(+)}, \quad G_K = G_4^{(+)}, \quad G_{a_0} = G_1^{(-)}, \quad G_{K_0^*} = G_4^{(-)},$$

$$g_{a_0}^2 = \left[4I_2^\Lambda(m_u)\right]^{-1}, \quad g_{K_0^*}^2 = \left[4I_2^\Lambda(m_u, m_s)\right]^{-1},$$

$$I_2^\Lambda(m_u, m_s) = \frac{N_c}{(2\pi)^4}\int d_E^4 k\, \frac{\theta(\Lambda^2 - k^2)}{(k^2 + m_u^2)(k^2 + m_s^2)}$$

$$= \frac{3}{(4\pi)^2(m_s^2 - m_u^2)}\left[m_s^2 \ln\left(\frac{\Lambda^2}{m_s^2} + 1\right) - m_u^2 \ln\left(\frac{\Lambda^2}{m_u^2} + 1\right)\right],$$

$$g_\pi = Z_\pi^{1/2} g_{a_0}, \quad g_K = Z_K^{1/2} g_{K_0^*}, \quad Z_\pi \approx Z_K \approx 1.44.$$

Mass formulas for nonstrange and strange isoscalar mesons have a more complicated form

$$M_{(\eta,\eta')}^2 = \frac{1}{2}\left(M_{ss}^P + M_{uu}^P \mp \sqrt{(M_{ss}^P - M_{uu}^P)^2 + 4(M_{us}^P)^2}\right), \qquad (62)$$

$$M_{(\sigma,f_0)}^2 = \frac{1}{2}\left(M_{ss}^S + M_{uu}^S \mp \sqrt{(M_{ss}^S - M_{uu}^S)^2 + 4(M_{us}^S)^2}\right), \qquad (63)$$

where

$$\begin{aligned}
M_{ss}^P &= g_{\eta_s}^2\left(\frac{1}{2}(T^P)_{ss}^{-1} - 8I_1^\Lambda(m_s)\right),\\
M_{us}^P &= \frac{1}{2} g_{\eta_u} g_{\eta_s} (T^P)_{us}^{-1},\\
M_{uu}^S &= g_{\sigma_u}^2\left(\frac{1}{2}(T^S)_{uu}^{-1} - 8I_1^\Lambda(m_u)\right) + 4m_u^2,\\
M_{ss}^S &= g_{\sigma_s}^2\left(\frac{1}{2}(T^S)_{ss}^{-1} - 8I_1^\Lambda(m_s)\right) + 4m_s^2,\\
M_{us}^S &= \frac{1}{2} g_{\sigma_u} g_{\sigma_s} (T^S)_{us}^{-1},\\
g_{\sigma_u} &= g_{\sigma\bar{q}q}, \quad g_{\sigma_s} = \left[4I_2^\Lambda(m_s)\right]^{-1/2},\\
g_{\eta_u} &= g_{\pi\bar{q}q}, \quad g_{\eta_s} = Z^{1/2} g_{\sigma_s},\\
T^{P(S)} &= \frac{1}{2}\begin{pmatrix} G_u^{(\pm)} & G_{us}^{(\pm)} \\ G_{us}^{(\pm)} & G_s^{(\pm)} \end{pmatrix}.
\end{aligned} \qquad (64)$$

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