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Electromagnetic waves in artificial periodic structures

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1. Introduction

The present talk discusses artificial periodic structures for which the propagation of electromagnetic waves is suitably described in terms of the band theory [1], originally developed in solid state physics to study de Broglie waves. Such crystals are of special interest for developing short-wavelength and, in particular, optical-wavelength devices. There is a correspondence between the respective concepts associated with these two kinds of waves (see the Table).

Devices based on such artificial crystals (also known as photonic crystals) include resonators, transmission lines, filters, signal splitters, etc. Underlying their operation are crystal inhomogeneities, which produce local oscillations analogous to impurity levels in a solid, with point inhomogeneities providing a resonator and line inhomogeneities providing a waveguide channel and other radio engineering systems.

Two major spheres of artificial crystal research are the study of wave refraction and reflection at an interface between such crystals and looking for media where waves deviate from their normal behavior. For example, if the group and phase velocities in a medium are opposite, it was shown in [2] that a beam incident from free space is deflected opposite to where it normally should be — a discovery which was followed by a flurry of research on media that are unconventional in the way they reflect and refract waves [3–9]. Most media studied were those transmitting waves in two dimensions, such as two-dimensionally periodic arrays of metallic elements [4–9], ferrite films that carry magnetostatic waves [10, 11], and cholesteric liquid crystals [12]. Plasmas also have unusual reflection and refraction properties [13, 14].

Table. Analogy between de Broglie and electromagnetic waves.

de Broglie waves	Electromagnetic waves
Electron energy $E = h\omega$	Frequency ω
Quasimomentum $\mathbf{p}$	Wave vector $\mathbf{k}$
Electron velocity $\mathbf{v}_{\mathrm{c}} = \mathrm{grad}_{\mathbf{p}}\omega$	Wave group velocity $\mathbf{v}_{\Gamma} = \mathrm{grad}_{\mathbf{k}}\omega$, equal to the energy transfer velocity
Dispersion characteristic $E(\mathbf{p})$	Dispersion characteristic $\omega(\mathbf{k})$
Energy band	Passband
Forbidden band	Stop band
Isoenergetic surface	Isofrequency
Impurity or surface levels	Local oscillations

One can study beampaths in a medium by considering the electric permittivity tensor ε and the magnetic permeability tensor μ of the medium [13, 14]. But because the calculation of these tensors requires averaging the fields, this approach applies only if the structure period is small compared to the wavelength. A more convenient and user-friendly approach, which is in addition valid for any wavelength, is the method of isofrequencies. Isofrequencies, the surfaces where the wave vector terminates for any wave directions, can be constructed based on the ε and μ tensors and can also be obtained without an averaging procedure.

The analogy between de Broglie waves in crystals and electromagnetic waves in periodic structures led to the discovery of a number of new physical phenomena, to be discussed here.

We consider structures that are periodic in three dimensions. The electromagnetic field in such a structure is expanded in a triple Fourier series of spatial harmonics with wave vectors $\mathbf{k}_{m_1}, \mathbf{k}_{m_2}, \mathbf{k}_{m_3}$ ($-\infty < m_i < \infty$, $i = 1, 2, 3$). Each harmonic corresponds to its own domain of variation (Seitz zone).

The phase velocities of the spatial harmonics are determined by the relation $\mathbf{v}_{m_i} = \omega\mathbf{k}_{m_i}/|\mathbf{k}_{m_i}|^2$ and are different, whereas the group velocities $\mathbf{v}_{\Gamma} = \mathrm{grad}_{\mathbf{k}}\omega$ are all equal. The group velocity determines the direction of the flow of energy (or the beam, or information) and is directed along the normal to the isofrequency and toward higher frequencies.

Knowing the phase velocities, wave vectors, and harmonic amplitudes of electromagnetic waves is important because these quantities determine the direction and intensity of diffraction peaks (e.g., in diffraction and antenna gratings). This knowledge is also essential in studying the interaction of electrons with a wave (the Vavilov – Cherenkov effect). In a one-dimensionally periodic medium, this interaction is strongest when the phase velocity $v = \omega/k$ is close to the electron velocity ($v_{\mathrm{e}} \approx \omega/k$), i.e., $kv_{\mathrm{e}} = \omega$. The extension of this relation to two- and three-dimensional systems is given by the Vavilov – Cherenkov condition $\mathbf{kv}_{\mathrm{e}} = \omega$, known in high-frequency technologies as the condition for electron – wave *synchronism* and used in developing electronic devices (traveling wave tubes, backward wave tubes, etc.).

2. Wave vector construction using isofrequencies

We generally distinguish between *forward* and *backward* waves. The term forward refers to a wave in which the angle θ between the phase and group velocities does not exceed $\pi/2$ ($|\theta| < \pi/2$). In the case where $\pi/2 < |\theta| \leqslant \pi$, the wave is said to be backward.

Figure 1 depicts the cubic elementary cells of two artificial media and shows plots, in the k_x, k_y plane, of their respective isofrequencies for the zero zone in the second transmission band. The numbers labeling the curves are proportional to the frequencies and represent the values of a/λ, where a is the size

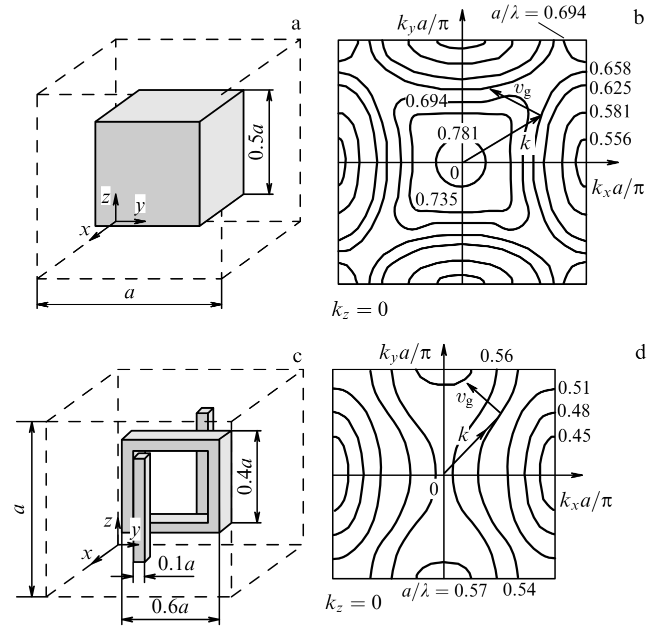

Figure 1. Examples of elementary cells of three-dimensional periodic structures and their isofrequencies constructed in coordinates k_x, k_y in the zero zone and calculated in the second passband.

of the cell and λ is the wavelength in free space. Also shown are the wave vectors and group velocity directions (bold and thin arrows, respectively).

We use isofrequencies ($a/\lambda = \text{const}$) to consider the refraction of a wave at the interface of two isotropic media and the path of beams ($\mathbf{v}_b$) for waves that have passed through a parallel-plane plate. In Fig. 2, panels a, b, and e correspond to a forward wave, and c, d, and f to a backward wave in a refracting medium. Shown dashed in Fig. 2 and all the

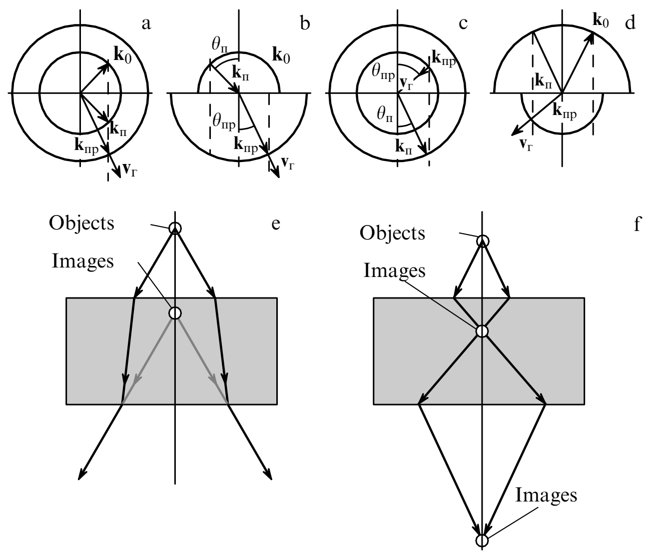

Figure 2. Wave vectors **k** and beams $\mathbf{v}_b$, and objects and images in the cases where a wave passes through the interface between two media (a, b, c, d) and through a plate (e, f). Diagrams a, b, e: a forward wave traveling in a refractive medium and a plate; diagrams c, d, f: the same for a backward wave. Constructions in diagrams a and c are rigorous but difficult to grasp; those in b and d are easy to grasp but not rigorous.

following figures are the projections of the vectors **k** on the abscissa. We note that the energy of the refracted wave is transferred from the interface between the two media and that the projections on the interface of the incident ($\mathbf{k}_{inc}$), reflected ($\mathbf{k}_{reflec}$), and refracted ($\mathbf{k}_{refrac}$) waves are equal (Figs 2a – d).

Two different vectors $\mathbf{k}_{refrac}$ may have equal projections on the interface in a refractive medium. One of the vectors (Fig. 2a) is directed from the interface downward and corresponds to a forward wave, whereas the other (Fig. 2c) is directed upward and corresponds to a backward wave. Of these two, the one that corresponds to the group velocity (beam) directed from the interface belongs to the refracted wave (compare Figs 2a, b and Fig. 2c, d). For each case in Fig. 2, two constructions are made. One of them, in which the vectors are always directed from the origin (Figs 2a, c), is rigorous and always yields the directions of all the vectors in a unique way. The rigorous construction is rather involved, however. In an approach used more frequently (Figs 2b, d), the wave vectors of the incident wave are depicted in the upper half-plane and those of the refracted wave in the lower half-plane. This construction, however, although easier to grasp, sometimes leaves one uncertain as to the direction of the beam.

Figures 2e, f demonstrate the course of beams for the forward (Fig. 2e) and backward (Fig. 2f) waves in a plate and demonstrate the images of objects in these plates. In the case of a forward wave, the object and its image are on the same side of the plate. For a backward wave, two images are possible, one of them lying on the other side of the plate.

Backward waves and their associated propagation media and refraction laws have been known for a long time now [3, 4, 8]. In 1940 – 1966, these waves were widely used in backward wave tubes and antennas. Pafomov [1] showed [13] that backward waves are possible in an isotropic medium with both a negative electric permittivity ε and a negative magnetic permeability μ. Such a phenomenon can be observed in plasma [13, 14]. It was only in 2000, i.e., about forty years after Refs [3, 4, 8], that Ref. [16] suggested a medium — a periodic array of metal rods and rings — that can support a backward wave in one dimension and in the second passband only. The isofrequencies of this structure are similar to those shown in Fig. 1d.

The discussion above is limited to isotropic media, in which isofrequencies form spherical or circular shapes centered at the origin. In reality, however, propagation media only behave isotropically in a certain limited frequency band (Fig. 1b). The construction of wave vectors and beams, i.e., group velocities, for isofrequencies of different forms is illustrated in Fig. 3. There are two diagrams corresponding to each shape: for the passage of waves and beams through the interface between two media and for the passage through a plate; in the former case, free-space isofrequencies (origin-centered semicircles) and medium isofrequencies are shown in the upper and lower half-planes, respectively. Figure 3a corresponds to media assumed to support a forward wave.

The situation in Fig 3a, with the interface parallel to the $\mathbf{k}_x$ axis, may occur for the isofrequency $a/\lambda = 0.57$ shown in Fig. 1d. An unusual feature is here that the object and its image lie on different sides of the plate even though the wave

[1] Although the proof of the existence of backward waves in $\varepsilon < 0$, $\mu < 0$ media is often mistakenly credited to V G Veselago [15], it is in fact V E Pafomov who first did it.

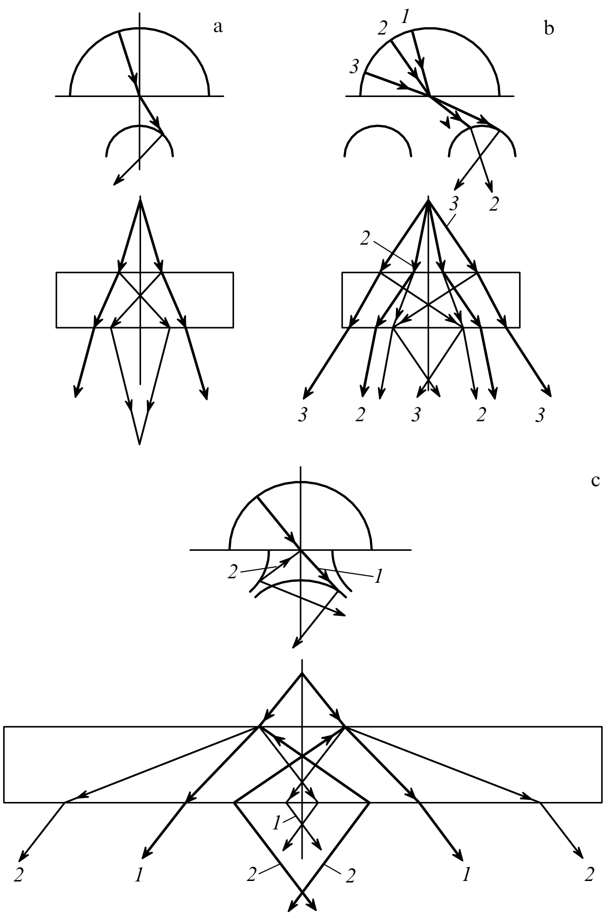

Figure 3. Wave vectors (bold arrows) and beams (thin arrows) corresponding to wave passage from free space through the interface between two media and through parallel-plane plates of materials with different isofrequency shapes.

is forward. The opposite should be observed when the corresponding isofrequency ($a/\lambda = 0.556-0.658$ in Fig 1b, $a/\lambda = 0.45-0.56$ in Fig. 1d) is that of a backward wave. In this case, the beam is deflected from the normal to the same side as in a dielectric, and the object and its image lie on the same side of the lower boundary of the plate (as in Fig. 2e). The situation shown in Fig. 3b is possible for isofrequencies $a/\lambda = 0.694$ (Fig. 1b) when the interface normal is parallel to either the $\mathbf{k}_x$ or $\mathbf{k}_y$ axis. At small incidence angles (wave vector *1*), the wave does not penetrate into the artificial medium because no isofrequencies at which the wave vector can terminate exist in this medium. As the incidence angle increases, the wave starts to penetrate the medium, and the refraction angle first increases and then changes sign.

We next consider Fig. 3b, which corresponds to the isofrequencies shown in Fig. 1b ($a/\lambda = 0.658$). In a refractive medium, there are two wave vectors with equal projections on the interface, and therefore two refracted beams may correspond to one beam in the incidence wave (*birefrigence*), with the result that the object turns out to have two images (beams *1* and *2* converge at different points). Importantly, the polarization of either beam remains the same here, in contrast to birefrigence in conventional optics.

Of greater interest are waves in ferrite films that are placed between metallic planes and subjected to a magnetic field (Fig. 4). In such systems, there exist isofrequencies that are intersected only once by a straight line perpendicular to the

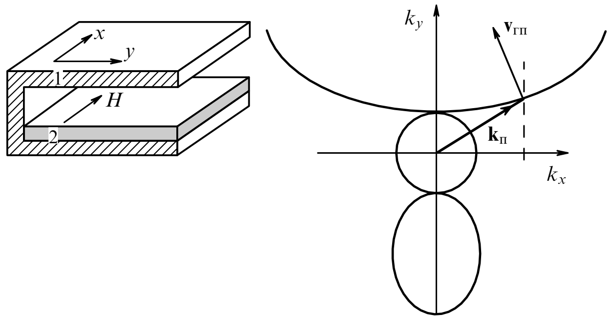

Figure 4. Isofrequencies of a ferrite film on a dielectric substrate, and incident vectors $\mathbf{k}_{\text{п}}$ and $\mathbf{v}_{\text{гп}}$ for the case where no reflected and no refracted wave are present.

interface, suggesting that something seemingly unreasonable may happen: an incident wave from which neither a refracted wave nor a reflected wave arises. This effect was predicted in Ref. [10] and observed experimentally in Ref. [11]. (We note parenthetically that the energy that comes to the interface is carried away by an edge wave).

3. Backward-wave supporting media

Two-dimensionally periodic lattices capable of supporting backward waves were described back in 1959 in Refs [3, 4], the former of which proposed a structure where, unlike those in today's publications, backward waves exist in the first passband.

According to Rayleigh, who proved the possibility of backward waves in 1877, the phase and group velocities of such a wave are related by

$$n_{\text{г}} = n - \lambda \frac{\mathrm{d}n}{\mathrm{d}\lambda},$$

where $n = c/v$ and $n_{\text{г}} = v/v_{\text{г}}$ are the phase and group velocity retardation factors. At sufficiently high dispersion $\mathrm{d}n/\mathrm{d}\lambda > 0$, the quantities $n_{\text{г}}$ and n are opposite in sign, thus giving rise to a backward wave.

Structures in which backward waves are allowed to run in only one or two dimensions have been known since 1904 (Lamb) and have been widely used in electronic and antenna engineering, starting from 1952. Backward waves also exist in cholesteric liquid crystals [12], partially dielectric waveguides, and many other systems.

Backward waves of the zero spatial harmonic in the fundamental (i.e., longest-wavelength) passband have been detected in one- and two-dimensionally periodic structures [3–8], but no three-dimensionally periodic structures with such properties are known. A second-band backward wave is relatively easy to obtain (see Ref. [5] and Fig. 1).

4. Conclusion

Artificial crystals are of particular interest for application in and around the optical part of the spectrum. Using such crystals allows developing integrated circuits, electronic instruments, and devices with unusual optical properties. In particular, not only a parallel-plane lens but also a plate can be given a property of being opaque to a normally incident wave while transmitting an obliquely incident one. Another possibility is a plate that exhibits birefrigence the amount of which depends on the frequency and the angle of incidence rather than the polarization of the beam. For a ferrite film in a magnetic film, a wave incident on the edge of the film neither is reflected from nor passes through the film. All these results are readily obtained using the isofrequency concept.

In conclusion, we note that the existence of backward waves in media with $\varepsilon < 0$ and $\mu < 0$ was first proved by V E Pafomov [13], not by V G Veselago in Ref. [15]. Backward waves and their unusual refraction properties in artificial crystal were first discussed by the present author [3, 4], not by Smith et al. in Ref. [16].

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