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Joint scientific session of the Physical Sciences Division of the Russian Academy of Sciences and the Joint Physical Society of the Russian Federation "Strongly correlated electrons in two-dimensional systems" (26 October 2005)

A joint scientific session of the Physical Science Division of the Russian Academy of Sciences and the Joint Physical Society of the Russian Federation was held on 26 October 2006 at the conference room of the Lebedev Physics Institute, RAS. The following reports were presented at the session.

(1) **Pudalov V M** (Lebedev Physics Institute, RAS) "Metal – insulator transitions and related phenomena in a strongly correlated two-dimensional electron system";

(2) **Iordanskii S V, Kashuba A** (Landau Institute for Theoretical Physics, RAS) "Two-dimensional multicomponent electron gas as a model for silicon heterostructures";

(3) **Olshanetskii E B** (Institute of Semiconductor Physics (ISP), RAS SB, Novosibirsk), **Renard V** (GHML, MPI-FKF/CNRS, Grenoble, France), **Kvon Z D** (ISP, RAS SB, Novosibirsk), **Gornyi I V** (Institut für Nanotechnologie, Karlsruhe, Germany; Ioffe Physical Technical Institute, RAS, St. Petersburg), **Toropov A I** (ISP, RAS SB, Novosibirsk), **Portal J C** (GHML, MPI-FKF/CNRS, Grenoble, France) "Interaction effects in the transport and magnetotransport of two-dimensional electrons in AlGaAs/GaAs and Si/SiGe heterojunctions."

Summaries of the reports are given below.

PACS numbers: **71.27.+a**, **71.30.+h**, 72.15.Rn
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Metal – insulator transitions and related phenomena in a strongly correlated two-dimensional electron system

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1. Introduction: competition of quantum interference and dimensionality

Understanding the properties of two-dimensional (2D) electron systems in the presence of both strong interparticle interactions and disorder is an outstanding problem in condensed matter physics that is still far from being solved. The late 1970s witnessed the completion of the theory of quantum (i.e., wave) interference corrections for noninteracting electrons, according to which the quantum interference correction $\delta\sigma_{\rm qi}$ to the semiclassical Drude – Boltzmann value $\sigma_{\rm D}$ acts to decrease the conductivity σ and can be regarded as 'backscattering' [1 – 3]:

$$\sigma = \sigma_{\rm D} + \delta\sigma_{\rm qi} = \sigma_{\rm D} - \frac{e^2}{\pi h} \ln\left(\frac{\tau_\varphi}{\tau}\right) \approx \sigma_{\rm D} + \frac{e^2}{\pi h} \ln T .$$

While small in three dimensions, the quantum interference correction is of fundamental importance in 2D systems, where the conductance G is independent of size. As temperature decreases, the quantum correction increases logarithmically until it becomes comparable to $\sigma_{\rm D}$ and leads to localization, which implies an exponential decrease in the conductivity.

These were the ideas that led to the creation of the scaling theory of conductivity in 1979 [4]. According to this theory, localization develops as the quantum length scale increases and is a result of competition between quantum interference, disorder, and dimensionality. In particular, a 2D system of noninteracting electrons cannot have metallic conduction and must become an insulator at $T = 0$. Figure 1a below shows an example of such conductivity behavior for a strongly disordered (low-mobility) sample. It can be seen that at low electron concentration, the conductivity decreases exponentially with decreasing temperature, suggesting a strong localization in the system. As the density increases, a transition occurs to the regime in which the conductivity varies approximately logarithmically with the temperature. Just as the scaling theory of conductivity predicts, at no concentrations or temperatures does the conductivity show evidence for metallic behavior ($d\sigma/dT < 0$).

The theory of quantum corrections led to a major revision of the classical understanding of conductivity and (as reviewed in Refs [1, 2]) was brilliantly confirmed in numerous experimental studies on 3D metallic samples and metallic films that were available in the 1970 – 1980s and in which the electron – electron interaction is weak. Indeed, the nearly 15-year dominance of the scaling theory misled researchers to believe that it is universal and works equally well when the interparticle interaction is strong.

Over the years, with advances in semiconductor technologies and the fabrication of increasingly pure samples with 2D electron layers, 2D electron systems with much lower concentrations have become available to study. In the low-concentration limit, electrons in an ideally pure system must form a Wigner crystal [5 – 7]. At higher densities, the electron system remains in the 'liquid' state and is characterized by strong electron – electron correlations. Such a strongly 2D system can exhibit new quantum states different from those

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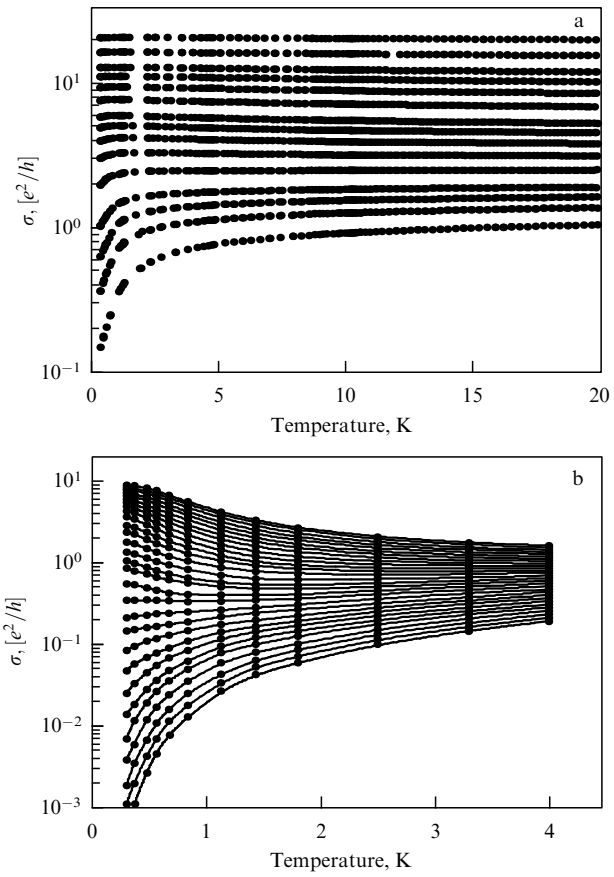

Figure 1. The temperature dependence of the conductivity of a 2D electron system in silicon in the absence of a magnetic field: (a) for a sample with a low mobility of 0.15 m^2 $(V\ s)^{-1}$, the curves, from bottom up, are for the following electron densities (in units of 10^{11} cm^{-2}): 3.85, 4.13, 4.83, 5.53, 6.23, 7.63, 9.03, 10.4, 11.8, 13.2, 16.0, 18.8, 21.6, 24.4, 30.0, 37.0; (b) for a sample with a high mobility of 3.62 m^2 $(V\ s)^{-1}$ [16], the curves are for electron densities that vary (from bottom up) from 0.717 to 1.326 in steps of 0.0218.

known for noninteracting electrons — which is very attractive to both theorists and experimenters.

The universal applicability of scaling theory was directly compromised in the early 1990s when studying the transition of an electron liquid from the state with quantized Hall resistance to the insulator state [8 – 11]. The experiments revealed, namely, that localized states that lie at the centers of the disorder-broadened Landau levels in the strong-magnetic-field quantum-Hall-effect regime merge and remain in a finite energy interval as the magnetic field H decreases. This contradicts the scaling theory prediction that delocalized states rise in energy and move across the Fermi level as $H \to 0$ [12, 13]. Furthermore, in a direct challenge to the one-particle scaling theory, the subsequent studies of conductivity in the absence of a magnetic field provided evidence for metallic transport and the metal-to-insulator transition in 2D systems of electrons in high-mobility Si structures.

Figure 1b shows typical temperature behavior of conductivity for pure (high-mobility) samples [16]. It is seen that at low densities, the curves $\sigma(T)$ differ little from the corresponding curves in Fig. 1a and demonstrate hopping conductivity, the usual behavior for a strongly localized state. But above a certain critical density n_c (in our case, $n_c = 0.96 \times 10^{11}$ cm^{-2}), $\sigma(T)$ shows a sharply different temperature dependence: similarly to what happens in an ideal metal, the conductivity starts to increase rapidly with decreasing temperature. Two branches of the dependence $\sigma(T)$ (the 'metallic' for $n > n_c$, and the 'insulating' for $n < n_c$) are mirror-symmetric with respect to $\sigma(n_c)$ [15] — akin to the symmetry the conductivity of 3D systems shows at the metal – insulator transition. A standard scaling analysis of these curves shows [15] that the experimental $\sigma(T)$ curves for $n > n_c$ and those for $n < n_c$ can indeed be reduced to two universal dependences $\sigma(T)_{i=1,2} \propto \exp(\pm T_0/T)$ in which the scaling parameter T_0 shows the critical behavior $T_0 \propto (n - n_c)^{-p}$ near n_c.

Experimental finding of new physics in the field where it was not expected provided a major stimulus for subsequent studies, both theoretical and experimental, of strongly correlated 2D systems. Metallic conductivity and the metal – insulator transition were similarly found in a variety of 2D systems, including 2D electron layers in GaAs/AlGaAs, InAs/GaAs, *n*-AlAs, Si/SiGe and inverted silicon metal – insulator – semiconductor (Si-MOS) structures, and 2D hole layers in GaAs/AlGaAs and Si/SiGe (see reviews [17 – 21] and the references therein). In all the cases studied, the metal – insulator transition occurs with decreasing the electron density, when the system's conductance reaches a value of the order of e^2/h [17 – 19]. Is the observed effect a true quantum transition and is the metallic state of a 2D system its ground state (at $T = 0$)? Or does the conventional physics of disordered and interacting electrons suffice to settle things? These fundamental questions are stimulating interest in this field.

2. Quantitative study of the e – e interaction

Because the critical behavior of conductivity as shown in Fig. 1b contradicts the expectations for noninteracting electrons, various types of interaction were analyzed theoretically for their possible effects. It was found that the spin – orbit interaction — even though it affects scaling behavior even on the one-particle level [1 – 3] — is not renormalized with decreasing the electron density and does not have a strong effect on transport processes. The electron – phonon interaction is also negligible in the low-temperature range $T \ll T_F$, especially for a monatomic crystal such as Si, in which the electron – phonon coupling is only via the deformation potential, with no piezoelectric component. Therefore, by the method of exclusion, the only interaction left to consider is the electron – electron (e – e) interaction.

In this case, it becomes clear where the high value of the carrier mobility μ comes into play. According to the Ioffe – Regel criterion, electrons become localized when their Fermi wavelength becomes equal to the mean free path, $\lambda_F \approx l_{tr}$. Hence, at the localization threshold, the Fermi energy is inversely proportional to the carrier mobility μ: $E_F \propto 1/\tau_{tr} \propto 1/\mu$. We note that for a 2D system, the Fermi energy is proportional to the electron concentration, $E_F \propto n$. It then follows that the higher the mobility (purity) in the 2D system, the lower the electron density that can be achieved in the 'metallic' phase and the stronger the electron – electron interaction, which is usually characterized by the dimensionless ratio of the Coulomb potential energy E_{ee} to the Fermi kinetic energy E_F,

$$r_s = \frac{E_{ee}}{E_F} = \frac{e^2 2m}{\varkappa \pi^{1/2} \hbar^2 n^{1/2}} \propto n^{-1/2}\,.$$

In the relevant experiments, the critical values of the density n_c corresponded to the value $r_s \approx 10$, making it obvious that such a system is by no means an ideal Fermi gas.

Realizing the importance of the electron–electron interaction led to the intense theoretical and experimental studies of its role in 2D systems in recent years. Experimental studies were made of the e–e interaction-induced renormalization of the parameters such as the g-factor $g^*/g = 1/(1 + F_0^a)$, the effective mass $m^*/m = 1 + F_1^s/2$, the compressibility $\varkappa^*/\varkappa = (m^*/m)/(1 + F_0^s)$, and the spin susceptibility $\chi^*/\chi = (m^*/m)/(1 + F_0^a)$. Here, g, m, $\varkappa$, and χ are the corresponding bare (band) values, and $F_i^{a(s)}$ is the lowest-order, antisymmetric (symmetric) Fermi-liquid constant.

Different groups used different experimental techniques to measure the spin susceptibility and effective mass renormalization [22–37] (for brief reviews, see Refs [20, 21]). Figure 2a shows the results of two independent measurements of χ^* for 2D electrons in Si-MOS structures [22, 23] and a GaAs/AlGaAs heterojunction [24]. As one can see, the results are in quite good agreement despite the difference in the carrier effective mass between Si and GaAs by a factor of three, the difference in the 'thickness' of the 2D layer by a factor of six, and the difference in the character and amount of disorder between samples prepared by different techniques. The dependence $\chi^*(r_s)$ turns out to be universal for samples of the same type with different disorder: for example, for Si-MOS structures, there is good agreement in results for samples with different mobilities [20, 23].

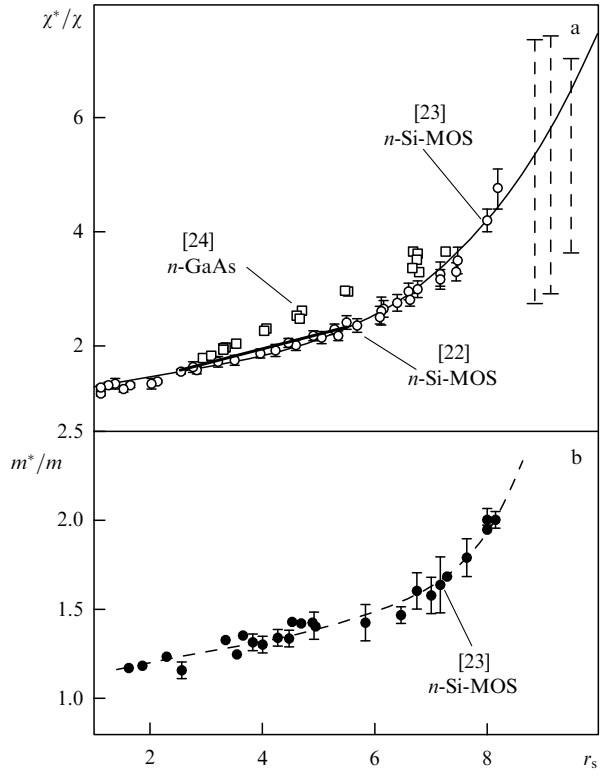

Figure 2. Renormalized spin susceptibility (a) and renormalized mass (b) as measured from Shubnikov–de Haas oscillations. White and black circles: measurements in Si-MOS structures in crossed fields [23], straight solid line: Si-MOS in a tilted field [22], squares: n-GaAs [24]. Horizontal bars on vertical dashed straight lines indicate the upper and lower estimates for χ^* as obtained from the period and phase of Shubnikov–de Haas oscillations [20].

Experimental determination of the renormalized effective mass renormalization is a more difficult task experimentally because it requires a detailed theory. While the experiments on Si-MOS structures all suggest a strong mass renormalization, there is only qualitative agreement among them [20]. One explanation for the quantitative spread is disagreement between the models the researchers used to extract the effective mass from the experimental data. It is also possible that the effective mass renormalization is different in different effects — in kinetics and thermodynamics, for example. Finally, the effective mass may be strongly temperature-dependent, and hence measurements in different temperature ranges may lead to different results.

The first major result from the experimental study of the renormalized parameters of 2D electrons was that, based on the measured values of renormalized parameters and using theoretical predictions for quantum interaction corrections [38, 39], it allowed a satisfactorily qualitative (and, in some cases, even quantitative) description of a) the 'metallic' temperature dependence of conductivity in the absence of a field and b) magnetoresistivity in a parallel magnetic field [25–28, 40]. Figure 3 gives an example of how the measured dependences $\sigma(T)$ compare with the calculated quantum interaction corrections [28]. It can be seen that over a wide range of electron densities (but for $n \gg n_c$), the agreement between theory and experiment is good if one uses the renormalized parameters $g^*(n)$ and $m^*(n)$ measured in independent experiments [23].

To summarize, the metallic temperature dependence of conductivity is now well understood — at least away from the transition, for $\sigma \gg e^2/h$, $n \gg n_c$, and $T \ll T_F$ — and is primarily determined by corrections in the triplet channel of the e–e interaction, which increase with decreasing the density. For the two-valley system of carriers in a Si-MOS

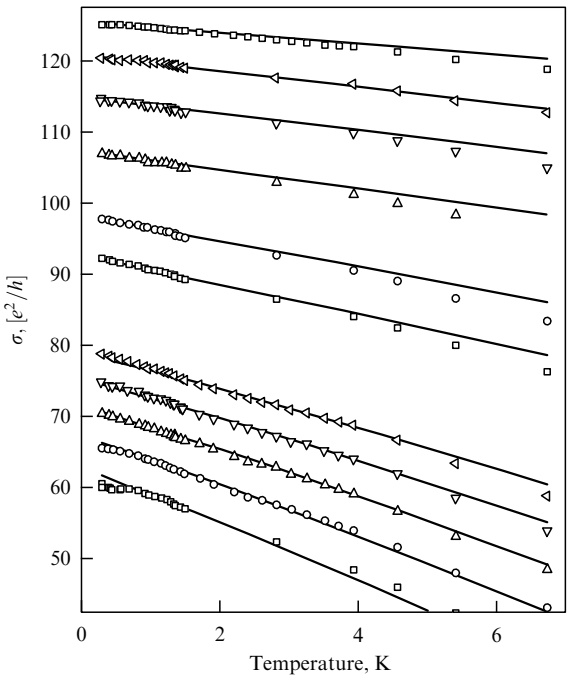

Figure 3. Comparison of the measured temperature dependence of conductivity [28] for the Si-MOS structure Si-22 (symbols) with predicted first-order corrections (solid curves) [38]. Concentrations (from top down) are 21.3, 18.9, 16.5, 14.1, 11.7, 10.5, 8.1, 7.5, 6.9, 6.3 (in units of 10^{11} cm^{-2}).

structure, the number of triplet terms increases by a factor of five [41] — explaining why the metallic behavior of conductivity is that strong in Si structures. But this picture cannot be extended to the critical regime of small $\sigma \sim e^2/h$ (i.e., $k_F l \sim 1$) near the transition ($n \approx n_c$), where the theory of quantum corrections is invalid.

Another major experimental result is that the renormalized parameters are very much the same, whether measured at the Fermi level from Shubnikov – de Haas oscillations [22 – 24] or, alternatively, over a wider energy range from spin polarization [31] or from magnetoresistance scaling in a strong magnetic field [30]. This implies that the interaction of 2D electrons in the strongly correlated regime is not sensitive to the Zeeman energy, i.e., that it occurs via spinless excitations over a wide energy range rather than in the narrow $k_B T$-neighborhood of the Fermi energy (which, we note, is exactly the kind of interaction typical of bosons). Fully consistently with this experimental fact, Iordanskii and Kashuba [42] considered a 2D system of fermions in the limit case of an infinite number of valleys, $n_v \to \infty$. In this approximation, the exchange occurs via high-energy plasmons, leading to the renormalization of the polaron-type effective mass and of the spin susceptibility; we note that the g-factor remains unchanged in the process.

A third key finding is that χ^* increases dramatically (by a factor of five or more) with decreasing density (see Fig. 2). In some papers, this increase was interpreted as a signature of the developing spontaneous spin polarization. If that were the case, the metal – insulator transition might result from a spontaneous ferro- or antiferromagnetic transition.

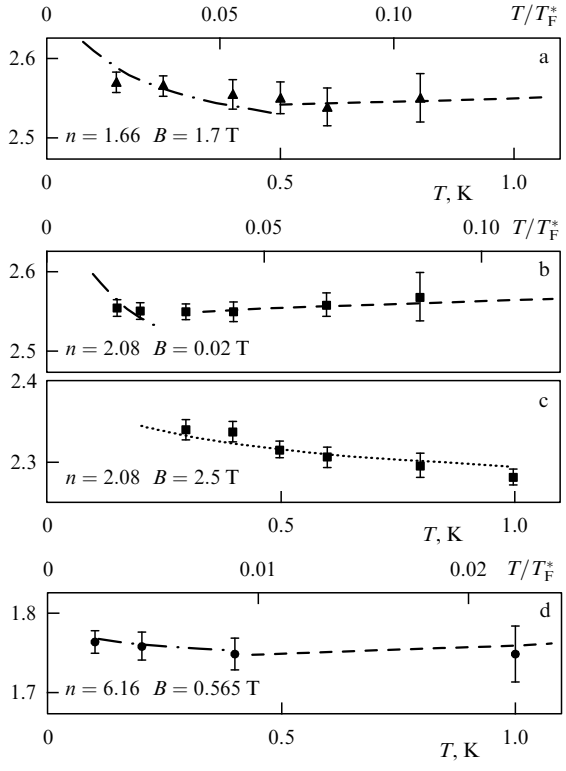

Figure 4. Temperature dependence of χ^* for three values of concentration (shown in units of 10^{11} cm^{-2}) for samples Si6-14 (a – c) and Si3-10 (d) [43]. Upper abscissa scales show temperature in units of the renormalized Fermi energy. Dot-dashed lines are the calculated quantum corrections in the diffusive regime [1], dashed lines are the same for the ballistic regime.

This intriguing possibility — the occurrence of a spontaneous magnetic transition — was tested in Ref. [20] by analyzing the frequency and phase of Shubnikov – de Haas oscillations at low densities. It was found that the frequency of the oscillations does not double until the very moment of the metal – insulator transition, thereby not confirming the doubling of the Fermi energy, which inevitably results from a spontaneous transition of electrons to one spin band. The analysis of the oscillation phase in the same study [20] also shows that the spin splitting in a weak field is more than half of the full cyclotron splitting but does not exceed it — an experimental fact that imposes the upper and lower limits of the spin susceptibility, as shown in Fig. 2a by the short horizontal bars.

In principle, the spin susceptibility could diverge in accordance with a power law with decreasing temperature, signaling the non-Fermi-liquid behavior of a strongly correlated 2D system. This possibility was examined by measuring the temperature variation of spin susceptibility in Refs [20, 43]. Typical $\chi^*(T)$ curves are shown in Fig. 4, which demonstrates that the temperature dependence of the susceptibility is much slower than a power law and is in qualitative agreement with the calculated quantum interaction corrections [1] $\propto \ln T\tau$ for $T\tau \ll \hbar$. We note that taken together, the data above and those from other experiments have not yet revealed any deviations from the Fermi-liquid theory.

3. Interplay of disorder and interaction

The theory of quantum corrections is not applicable near n_c, where $\sigma \sim e^2/h$. In the presence of impurities, fluctuations in local charge, spin, and valley densities show a decay on large length scales, which is equivalent to the propagation of paired electron – hole and electron – electron modes known as diffusons and cooperons [1, 2, 44]. Diffusing electrons become 'more correlated' after spending much time close to each other, which leads to disorder-dependent corrections to the interaction amplitudes $\gamma_2 = F_0^a/(1 + F_0^a)$ and γ_c that characterize the scattering of the diffuson and cooperon modes [44]. In a 2D system, all these corrections diverge logarithmically as temperature decreases [1, 2].

In the 1980s, a technique for resummation of logarithmically divergent diagrams (in fact, an extension of the nonlinear sigma-model) was developed by Finkel'stein [44] and subsequently by Castellani and Di Castro [45, 46], which starts from a weakly interacting system and allows approaching the strong-coupling regime corresponding to the metal – insulator transition. It turns out that even in the lowest-order small-resistance (small-disorder) approximation, the temperature diagram calculated in Ref. [41] (Fig. 5) in terms of the temperature logarithm versus disorder (i.e., resistance) is in qualitative agreement with the observed $\rho(T)$ behavior [16] in the critical regime. As the temperature decreases, the resistivity first increases and then, in accordance with the theory, its behavior starts to be determined by the developing renormalization of γ_2 — with the result that the resistivity passes a maximum and starts decreasing.

In the corresponding two-parameter scaling theory [41, 44 – 47], the renormalization of disorder with decreasing temperature (the increase in the quantum length scale) leads to an increase in interaction, which in turn affects the resistivity (disorder). Therefore, the metal – insulator transi-

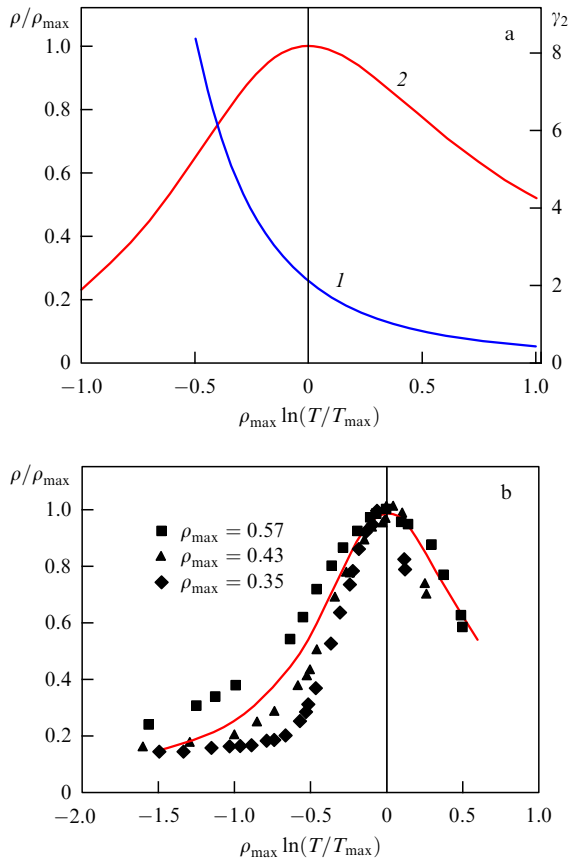

Figure 5. (a) Phase diagram for two-parameter scaling in the one-loop approximation [41]. The abscissa is proportional to the length scale (temperature logarithm), the ordinate is the resistance (disorder) normalized to the maximum value ρ. Curve *1* corresponds to the coupling constant γ_2 in the triplet channel; curve *2*, to resistance. (b) Experimental data from Ref. [16] on $\rho(T)$ (symbols) for three values of concentration are compared with the solution of the renormalization-group equation for a two-valley system [41] (solid curve). The $\rho(T)$ data are normalized by the corresponding maximum values [41].

tion occurs not as a result of the competition between dimensionality and interference (as in the 3D case) but because of the competition between disorder and interaction. Similarly to the theory of quantum corrections [38], the large number (15) of the triplet terms in a two-valley system that facilitate delocalization greatly increases the chances of the system for delocalization compared with those of a one-valley system.

Even though there is qualitative similarity between the theoretical and observed behavior of the resistivity $\rho(T)$, so far experiments have not yet supported the theoretical prediction of a strong increase in γ_2 — demonstrating only a slight variation in the susceptibility with temperature instead (see Fig. 4). A possible reason for this is that the temperature variation of the susceptibility (see Fig. 3) was probably measured not in the critical regime $n \approx n_c$ but for $n \geqslant 1.3n_c$ and in a finite magnetic field; we note that it is not yet possible experimentally to measure susceptibility in a weak field $H < T/g\mu_B$, $H < \pi T/eD$ (D is the diffusion coefficient) [1], as needed for a strict comparison with theory [17].

The result in Ref. [41] derived in the lowest order in resistivity only shows the 'metallization' trend in a 2D system and is not applicable near the transition, where the change in the resistivity with temperature is not minor. Nor does this result apply at the temperature at which γ_2 diverges, and even though this temperature is vanishingly low, $\sim \exp(-\exp((2n_v)^2))$ Kelvin [47], the failure of the scaling equations at sufficiently low temperatures is a drawback of this approximation [41].

Recently, Punnoose and Finkel'stein [47] used the approximation of an infinite number of valleys $n_v = \infty$ in Ref. [42] to obtain a two-loop solution of the renormalization group equations and to show the existence of a quantum critical repulsive point for a metal–insulator transition in the phase diagram of an interacting 2D system. Figure 6 shows a fragment of a phase diagram in Ref. [47], where the repulsive point is shown as a white circle and the arrows indicate the direction of flow as the temperature decreases. Seen in the figure are two classes of trajectories above and below the critical point, which correspond to the insulator and metal, respectively. The dashed line that starts almost horizontally from high temperatures (zero on the abscissa axis) is the separatrix between the metallic phase (bottom left) and the insulator (top left). The second separatrix [the one falling to the point (0, 1)] and the two other phases (top right and bottom right) that it separates have not yet been seen experimentally and may be due to the approximation used ($n_v = \infty$). Varying disorder (for example, the collision frequency $1/\tau$) at a fixed temperature results in motion along a vertical trajectory that intersects the separatrix (shown dashed in Fig. 6), thus leading to a metal–insulator quantum transition. Treating the electron concentration as a parameter implies a simultaneous variation of the bare disorder and bare interaction r_s, such that the system undergoes a metal–insulator transition again, but with the difference this time that it goes along a tilted rather than vertical trajectory when intersecting the separatrix.

In the limit as $T \to 0$, the flow lines in Fig. 6 converge to a point corresponding to an ideal metal, seemingly violating the Fermi-liquid picture [4, 48]. But it is reasonable to expect that when the system is cooled such that T becomes less than both $\hbar/\tau_s$ and $\hbar/\tau_{vv}$ (where τ_s and τ_{vv} are the spin flip and intervalley scattering times, respectively), the contribution of the triplet (delocalizing) terms sharply diminishes, restoring the singlet-to-triplet ratio common for a single-component Fermi liquid [1, 2]. The corresponding cut-off temperatures depend on the parameters of the scatterers present in the

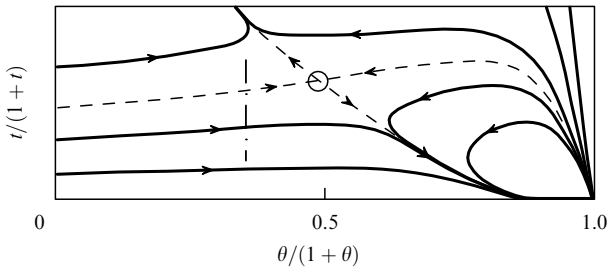

Figure 6. Phase diagram of two-parametric scaling in the approximation of an infinite number of valleys [47]. The vertical and horizontal axes represent the disorder t ($t = 1/(2\pi)^2 vD$, resistivity per valley) and interaction θ, respectively. Arrows indicate where the data point moves as the temperature decreases. The circle depicts the quantum critical repulsive point. Shown dashed is the separatrix between the metallic and insulating phases. Vertical dash-dot line depicts an order-changing trajectory for the metal–insulator transition at the temperature chosen.

sample. By way of estimate, we note that $\hbar/\tau_s \sim 10^{-6}$ K; the experimental data on $\hbar/\tau_{vv}$ in the range from 10^{-1} to 10^{-4} K are as yet of low reliability. This question is purely academic, however, because such low temperatures are many orders of magnitude below the currently accessible level.

In summary, theory suggests an extraordinary picture, where the metal–insulator transition occurs at a finite temperature and is a true quantum transition, but, strictly speaking, the metallic state that results does not survive the $T = 0$ limit (provided the 2D system does not make a spontaneous transition to another universality class — due to the formation of local magnetic moments [48] or of a two-phase microemulsion state [49], for example). Other remaining questions are whether the phase diagram of a 2D metal–insulator transition will be valid, at least in general terms, for realistic cases such as $n_v = 6$, 2 or 1, and down to what temperatures a 2D metal can exist in real-life systems.

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Two-dimensional multicomponent electron gas as a model for silicon heterostructures

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1. Introduction

Two-dimensional electron gas in a Si-heterostructure can vary very widely in density [1]. Effects due to the electron–electron Coulomb interaction are determined by the dimensionless ratio of the average Coulomb energy to the electron kinetic energy, $r_s = e^2 m/\sqrt{\pi n}\,\hbar^2$, with n being the electron density. For relatively large r_s, $1 < r_s < 10$, Si-MOSFETs (metal-oxide-semiconductor field-effect transistors) undergo a transition from the metallic (growing) to dielectric (falling) conductivity with decreasing temperature [2] and demonstrate an increase in the effective mass and magnetic susceptibility with increasing r_s [3, 4]. Because of the lack of exactly solvable models for large r_s, various phenomenological models have come to the fore. The electron–hole plasma observed in a three-dimensional (3D) electron–hole droplet in Si and Ge is also characterized by comparatively large values of r_s. As shown in the pioneering work of Ref. [5], the multivalley band structure leads to the existence in these semiconductors of a 'metallized' electron–hole plasma in the region of relatively large r_s (see also Ref. [6]). One would expect that allowing for many valleys in two-dimensional (2D) Si-heterostructures would lead to better agreement with experiment compared with Landau's Fermi-liquid theory with its small-r_s corrections to the theory of a dense electron gas. The Fermi-liquid theory predictions for Si-heterostructures are in quantitative disagreement with experiment even