Physics – Uspekhi **49** (12) 1289 – 1306 (2006)

METHODOLOGICAL NOTES

PACS numbers: **72.70.+m**, **85.30.–z**, 85.60.Dw

Photoinduced and thermal noise in semiconductor p – n junctions

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DOI: 10.1070/PU2006v049n12ABEH006134

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Abstract. **A brief review of the literature on the theory of thermal and photoinduced noise in semiconductor p – n junctions is given. The coordinate and frequency dependences of photoinduced noise in a $p^+ - n$ junction with a locally irradiated n-region are calculated. In contrast to vacuum tubes, where the physical sources of current distribution noise are still unknown, it is established that in $p^+ - n$ junctions, this noise is produced by fluctuations in the local hole recombination and diffusion rates in the n-region. White high-frequency spectra of thermal and photoinduced noise arise when the hole concentration increases linearly outside the space charge region and occur because the diffusion noise currents and the effective length for collecting noise from the n-region compensate each other in terms of frequency dependence.**

1. Introduction

Inherent noise sets a limit on the measurement accuracy of electronic and optoelectronic systems and on their sensitivity to weak signals. Importantly, inherent noise in system components (such as semiconductor diodes, transistors, or photodetectors) contains information not only on the perturbations they randomly (or stochastically) produce but also on the way these perturbations relax — i.e., on all the electrophysical processes that are involved and which, importantly, occur at small deviations from the thermodynamically equilibrium or steady-state conditions. Therefore, it clearly becomes necessary to understand the noise formation mechanism and how component noise characteristics can be calculated.

Although noise in semiconductor diodes and photodiodes has been a subject of study for more than 50 years now, Kurbatov argues in his recently published textbook [1] that "shot noise in a semiconductor photodiode cannot be considered a simple problem." Rogalski is even more categorical in his review monograph [2]: "the general theory of noise in semiconductor photodiodes ... has not yet been developed."

The aim of this paper is to fill this gap, at least partially.

2. Literature review

Various approaches to assessing photoinduced and thermal noise in semiconductor diodes are reviewed here.

2.1 Analogy to vacuum tube noise

Photo-induced noise belongs to the well-known class of statistical physics problems that involve a fluctuating flux of particles passing through a system with an independently fluctuating transmission coefficient. A typical example of such a system is a multielectrode vacuum tube with a cathode, an anode, and a screening grid that captures part of the electron flux emitted by the cathode. Such fluxes obey Burgess's dispersion theorem [3 – 5]

$$\overline{\Delta m^2} = (\overline{X_i})^2\, \overline{\Delta n^2} + \overline{n}\, \overline{\Delta X_i^2}\,, \tag{1}$$

where n is the (fluctuating) number of particles per unit time carried by the primary flux (for example, by the cathode current in a vacuum tube) and m is the number of particles carried at the same time by the output flux (for example, by the reduced electron flux to the anode). The appearance of the ith input particle changes the number of particles in the reduced flux by $X_i < 1$, where different values of the X_i are

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Received 2 June 2006
Uspekhi Fizicheskikh Nauk **176** (12) 1321 – 1339 (2006)
Translated by E G Strel'chenko; edited by A M Semikhatov

statistically independent. Then

$$m = \sum_{i=1}^{n} X_i \quad \text{and} \quad \overline{m} = \overline{n}\,\overline{X_i}\,,$$

where, as usual, the bar denotes mean values; for example, $\overline{\Delta m^2}$ is the mean square of the deviation of the number of particles in the output flux.

Using Burgess's theorem and the fact that the division of the electron flux leaves an electron indivisible, it is possible to calculate the so-called current distribution noise that arises in an electronic vacuum tube due to the stochastic manner in which the primary cathode current is reduced. We assume that $X_i = 1$ if the electron reaches the anode and $X_i = 0$ if it does not. Letting $\eta = \overline{X_i} = \overline{X_i^2}$ denote the probability that the electron does reach the anode (with X_i^2, like X_i, being either 1 or 0), we obtain

$$\overline{m} = \overline{n}\eta \quad \text{and} \quad \overline{\Delta X_i^2} = \overline{(X_i - \overline{X_i})^2} = \overline{X_i^2} - (\overline{X_i})^2 = \eta - \eta^2$$

and Eqn (1) becomes

$$\overline{\Delta m^2} = \eta^2\,\overline{\Delta n^2} + \overline{n}(\eta - \eta^2)\,. \tag{2}$$

From Eqn (2), we determine noise current spectral densities at angular frequencies lower than the inverse correlation time between the primary and the output particle fluxes (or the inverse flight time of an electron from the cathode to the anode). For example, the spectral density of the anode current noise $S_{I_a}(\omega)$ is given by [3, 4]

$$S_{I_a}(\omega) = S_{w,\,I_a}(\omega) + S_{s,\,I_a}(\omega) = \eta^2 S_{I_c}(\omega) + 2e\overline{I}_c(\eta - \eta^2)\,, \tag{3}$$

where e is the electron charge, $\overline{I_a} = e\overline{m}$ is the anode current, $\overline{I_c} = e\overline{n}$ is the cathode current, and $S_{I_c}(\omega)$ is the spectral density of the cathode current noise. The first term in Eqn (3), $S_{w,\,I_a}(\omega)$, is the contribution made by fluctuations in the primary current I_c to the spectral density of the anode current noise (or the reduced primary current noise), and the second term, $S_{s,\,I_a}(\omega)$, is the current distribution noise (or the inherent tube noise induced by the primary current).

In particular, if the primary cathode current has shot noise ($S_{I_c}(\omega) = 2e\overline{I_c}$), then

$$S_{I_a}(\omega) = \eta^2 2e\overline{I_c} + 2e\overline{I_c}(\eta - \eta^2) = 2e\overline{I_c}\eta = 2e\overline{I_a}\,, \tag{4}$$

and the anode current also contains shot noise: the Poisson distribution of a stochastic current of indivisible particles remains Poisson in passing through a system with independently fluctuating transmission.

The generalized relations above fully apply to semiconductor photodiodes if there are no RC restrictions on their frequency characteristics — the assumption that also extends to all other approaches to be described below. For a photodiode, n is the photon flux incident on a photosensitive area, η is the quantum efficiency (or collection efficiency) of the photodiode, I_a is the photocurrent in the outside current, and $(1/e^2)S_{I_c}(\omega)$ and $S_{I_a}(\omega)$ are the respective spectral densities of the photon flux and photocurrent noise.

In calculating photoinduced photodiode noise — in the case where the diode is exposed to blackbody radiation, for example — it is necessary to introduce the degeneracy coefficient of a photonic gas, k_d, characteristic of particles obeying the Bose–Einstein statistics [1]:

$$\overline{\Delta n^2} = \overline{n}k_d = \overline{n}\,\frac{1}{1 - \exp(-h\nu/kT)}\,,$$

where $\overline{n}$ is the mean density of the flux of photons of energy $h\nu$.

For $h\nu \gg kT$, $k_d = 1$ and $\overline{\Delta n^2}$ assumes the form classically known from the Poisson statistics; namely, the mean-square deviation from the mean is equal to the square root of the mean. The inequality $h\nu \gg kT$ is valid in the short-wave region and close to the maximum of Planck's radiation curve. In the long-wavelength part of Planck's curve, blackbody photons group together and k_d increases. The spectral density $S_{I_c}(\omega)$ of radiation noise in Eqn (3) then increases by a factor of k_d, and the output noise has the spectrum

$$S_{I_a}(\omega) = 2e\overline{I_a}\big[1 + (k_d - 1)\eta\big] \tag{5}$$

at low frequencies.

Based on the theoretical considerations mentioned above, the equivalent noise circuit of a vacuum tube has the current distribution noise generator connected between the anode and the screen grid, the physical source of the noise remaining unidentified.

Unfortunately, the generalized approach (or the vacuum tube analogy) fails to provide the value of η and, most importantly, to determine photoinduced and thermal noise in semiconductor diodes at angular frequencies higher than the inverse flight time of the photocarriers.

2.2 Analogy to the diffusion of generated ideal gas particles

In calculating the spectrum of the flux of particles of a classical ideal gas as they diffuse to a screen to be absorbed by it, Rabinovich [6] took fluctuations in the particle generation rate into account and used the Fokker–Planck equation to describe the random walk of current carriers. He then extended his results to an idealized semiconductor photodiode by assuming that the carriers photogenerated in the neutral region of the p–n junction do not subsequently recombine either in the bulk or at the surface [7]. It was shown that if only fluctuations in the carrier generation rate and those in the speed of the particle diffusion to the space charge region are considered, then, at the frequencies at which the photocurrent weakly decreases with increasing the radiation modulation frequency, the expression for the spectral density of photoinduced noise becomes identical to Eqns (3) and (5) with $\eta = 1$. At higher frequencies, however, where the photocurrent all but ceases and relations (3)–(5) are no longer valid, the spectral noise density is determined by a relation similar to Eqn (4), also with $\eta = 1$, and is again independent of the frequency. The frequency dependence of photoinduced noise in a diode exposed to a nonmodulated radiation flux with $k_d > 1$ has two plateaus, a fact which Rabinovich and M A Trishenkov [8] suggest can be used to assess the inertial properties of semiconductor photodiodes.

For $k_d \approx 1$, in accordance with Ref. [7], a photodiode with no loss to recombination turns out to have a white spectrum at low and high frequencies (similar to what had been calculated in Refs [9–11] for thermal noise in a reverse-biased p–n junction) — which was consistent with the experimental data available at the time [5, 12].

The boundary between the low- and high-frequency spectral regions is taken throughout this paper to be the angular frequency equal to the inverse value of the photocurrent (or dark current) attenuation time; it depends on the parameters of the structure and irradiation conditions.

Unfortunately, the theory Rabinovich developed (which, to repeat, exploits the analogy to the diffusion of generated ideal gas particles and predicts two plateaus in the frequency characteristic of photoinduced noise in an idealized p – n junction for $k_d > 1$) did not allow calculating photoinduced noise in real photodiodes, with the bulk and surface recombination not negligible.

It also remained unexplained

• whether the absence of the bulk and surface carrier recombination is necessary for photoinduced noise to have a white spectrum at high frequencies, and

• how fluctuations in the generation and diffusion of current carriers relate to current distribution noise in a semiconductor photodiode.

2.3 Analogy to an *RC*-distributed transmission line

Inherent thermal noise in a semiconductor diode was first calculated by van der Ziel [9 – 11] using the diffusion – recombination model that W Shockley [13] had shortly before proposed to explain the I – V characteristics of p – n junctions.

The Shockley model — with minority carrier diffusion in neutral p- and n-regions of a p – n junction, which is accompanied by the linear recombination of minority carriers in the case of a forward junction current, or results from thermal carrier generation during reverse currents — is a universal model. It is valid for any semiconductor and determines both the minimum achievable level of reverse currents in 'thin' p – n junctions (thin meaning that the space charge region is thinner than the carrier diffusion length) and the minimum values of over-barrier injection currents in transistor structures or semiconductor emitters.

Noise calculations in Refs [9 – 11] used the Langevin equation [3, 4, 11], obtained from the differential equation for the diffusion – recombination model of a p – n junction by first linearizing it with respect to small deviations from the steady state and then introducing random perturbing factors to represent fluctuations. Besides being easy to understand and applicable to any frequency range, this approach is also advantageous in allowing the use of the mathematical technique developed to analyze the relaxation of a p – n junction following dynamic influences. We also note that the Langevin equation is amenable to the Fourier method, which allows determining the spectral density of the response of a p – n junction to random perturbations.

To simplify the analysis of the diffusion – recombination mechanism of noise perturbation relaxation, van der Ziel used the *RC*-distributed electric signal transmission line as the equivalent circuit, noting [11] that the equivalent *RC*-circuit with distributed parameters had already been used by others — specifically, by Petrits and North — in the analysis of p – n junctions.

The inherent noise in a distributed parameter circuit is caused by thermal noise in the circuit active elements. The p – n junction analogues of such sources are noise generators in local, thermodynamically equilibrium generation and recombination fluxes of minority carriers, as well as local fluctuations in their thermodynamically equilibrium diffusion fluxes in the neutral regions of the p – n junction. It is a remarkable achievement on the part of van der Ziel — and one that led to new physical results — that relations for generation – recombination and diffusion noise were extended to steady-state conditions, where applying an external voltage across a p – n junction changes the minority concentration relative to the thermodynamic equilibrium.

It is well known that generation, recombination, and diffusive scattering events involving electron – hole pairs do not, in themselves, cause changes in the charge of semiconductor regions and do not produce a current in the external circuit of the p – n junction. There are, however, perturbations in the minority carrier distribution due to the random nature of their generation, recombination, and diffusion, and these do reach the space charge region as they propagate in the diffusion – recombination flux of minority carriers. The subsequent flight of the carriers through the space charge region produces current pulses in the diode external circuit, while at the same time broadening them (an effect that can be neglected if the noise analysis is restricted to angular frequencies lower than the inverse flight time of carriers through the space charge region).

Now classic, the van der Ziel relations explain many observed features of the noise characteristics of p – n junctions and place a fundamental limit on how much their inherent noise can be reduced.

The work of van der Ziel [9 – 11] evoked sharp criticism, however, to the point that his 'correct results' were declared to have been 'obtained by accident' (see, e.g., Ref. [14]). The main point of objection was the way van der Ziel represented fluctuations that occur in diffusion – recombination fluxes and produce current noise in the external circuit. He represented these fluctuations as noise in two independent currents that traverse the space charge region. One of these is the algebraic sum of the external current and the saturation current (although taken to flow in the forward direction), the other is the junction saturation current. The present-day view [13, 15], however, is that there are two nearly equal and opposite electron diffusion fluxes and two similar hole fluxes flowing through the space charge region, which ensure thermodynamic equilibrium between electron and hole concentrations in the neutral p- and n-regions at their boundaries with the space charge region, and which are several orders of magnitude greater than the external current (at least if the forward and reverse biases are small). It remained unclear why relatively small diffusion – recombination currents produce noise in the external circuit, whereas large diffusion currents flowing through the same space charge region, and also exhibiting shot noise, do not.

It was not until 1974 that Buckingham and Faulkner [16] (followed later by Robinson [17]) showed that as minority carriers move via diffusion – recombination processes in the neutral regions, 'bottlenecking' the throughout fluxes of electrons and holes there, fluctuations in the diffusion fluxes through the space charge region cause charges to buildup on the boundaries of this region, which changes the voltage across it (usually by less than kT/e). This change, in turn, modulates the diffusion fluxes of holes and electrons from the respective p- and n-regions in such a way as to compensate the original fluctuations. In the equivalent noise circuit of a p – n junction, the compensation of diffusion fluxes is represented by diffusion conductances for electrons and holes (in Fig. 1, for a p^+ – n junction in which the electron components of the currents and noise are negligible, only the diffusion conductance for holes, G_{dif}, is shown). The diffusion conductance shunts diffusion current fluctuations from the in-series connected external short circuit and the diffusion – recombination conductance G_{dr} of the n-region (the ratio G_{dif}/G_{dr} at

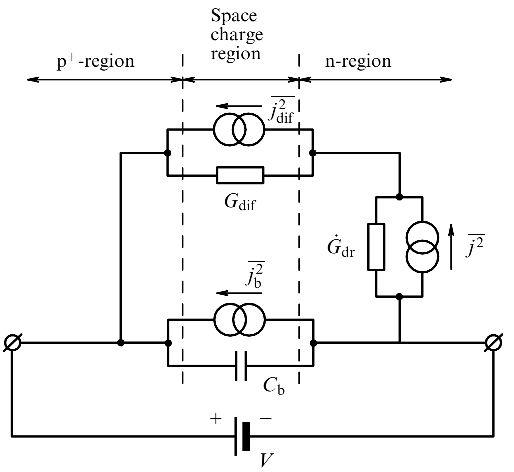

Figure 1. Equivalent noise scheme of a p^+-n junction in which the electron components of currents and noise are neglected.

low frequencies is approximately equal to the ratio of the hole diffusion length to the hole mean free path, which is several orders of magnitude).

Importantly, when external currents flow through a p–n junction, voltage drops across the conductance are small enough to be left out of account in the Shockley model when calculating the I–V characteristics (which corresponds to the assumption of constant Fermi quasilevels for electrons and holes in the space charge region).

It is only when the equivalent noise circuit of the diode is modified by adding the diffusion conductance $G_{\rm dif}$ to shunt the space charge region of the p^+-n junction that both relaxation paths that return the hole distribution to the steady state (via $G_{\rm dif}$ for diffusion – recombination perturbations in the neutral n-region and via the same $G_{\rm dif}$ and bypassing the external circuit for diffusion perturbations in the space charge region) become closed, and the p^+-n junction model is physically complete. An important finding that emerged in this work was that the noise in the diffusion electron and hole fluxes through the space charge region can be considered to have no effect up to very high frequencies in calculating noise currents in the external short circuit of the p^+-n junction.

Also, the work virtually invalidated claims that the equivalent circuit for the diffusion – recombination mechanism of perturbation relaxation in the neutral regions of the p–n junction cannot be used in what is apparently the easiest-to-understand form of the *RC*-distributed transmission line.

Thus, in calculating noise in an external short circuit of a p^+-n junction with no generation and no recombination in the space charge region, the diffusion conductance $G_{\rm dif}$ should be short-circuited, and the noise in diffusion fluxes $\overline{j^2_{\rm dif}}$ shown in Fig. 1 can be left out of consideration (as can the barrier capacitance $C_{\rm b}$ of the p^+-n junction and the generator of noise $\overline{j^2_{\rm b}}$ that arises due to the generation or recombination of current carriers in the space charge region). All of them are nevertheless shown in Fig. 1 to provide the correct picture of relations between the elements in the full equivalent noise scheme of a p^+-n junction and to fix the approximations that have been made.

There are, however, some aspects of thermal noise in p–n junctions that need further discussion, including:

• how does the white spectrum of the reverse saturation current result from the frequency-dependent diffusion – recombination transfer of minority carriers in the neutral regions of p–n junctions? (Especially noting that at high frequencies, the inherent thermal noise of the equivalent *RC* transmission line increases with the frequency [11]);

• why do local currents generated or recombining in a p–n junction at different distances from the space charge region contribute equally to shot noise?

• how does the analogy to the *RC* line produce p–n junction saturation current that the line does not contain?

On the other hand, it is becoming evident that the diffusion – recombination model by Shockley and its analogous distributed-parameter line model used by van der Ziel are most adequate for describing the physical processes in a p–n junction without current carriers being generated and recombining in the space charge region, and should therefore be preferred in calculating photoinduced noise in p–n junctions.

For example, the diffusion – recombination model was used in [18] to calculate the spectral noise density of an irradiated reverse-biased diode, a situation in which both the dark current and photocurrents are collected only from the 'long' p region ('long' meaning relative to the electron diffusion length there), $k_{\rm d} = 1$, and the photogeneration rate decreases exponentially away from the space charge.

A further point to note is that in a semiconductor diode (in contrast to an electronic vacuum tube), the minority carrier generation – recombination flows transform into the external circuit current either over an extended portion of the neutral region (at distances of the order of the minority carrier diffusion length from its boundary with the space charge region) or throughout the entire neutral region thickness (if it is less than the diffusion length). In these cases, the possibility exists in principle of controlling noise formation conditions in the desired manner — for example, by probe irradiation of the neutral region at various distances from its boundary with the space charge region.

In what follows, we therefore consider the noise model of a p^+-n junction without space-charge-region generation and recombination processes in detail and, using the *RC*-distributed analogue, calculate and analyze photoinduced noise and thermal noise in such a p^+-n junction. Based on the comparative analysis of photoinduced noise and thermal noise (the former including the local illumination arrangement), the questions listed above are answered, and additional information on noise due to the diffusion – recombination transport of minority carriers in the neutral region of a p–n junction is obtained.[1]

3. Problem formulation

We follow van der Ziel's treatment [11] and consider a planar p^+-n junction (Fig. 2) conveniently taken to have a unit area. We take the x axis to be perpendicular to the p^+-n junction

[1] The problem of how noise arises in p–n junctions caught the author's attention when he was preparing an undergraduate textbook on solid-state photoelectronics. Given more than half a century of research in p–n junction noise, some of the results obtained in this paper may have already been obtained and published. The author, however, has found none of them, either in the original literature or monographs or in college or university texts.

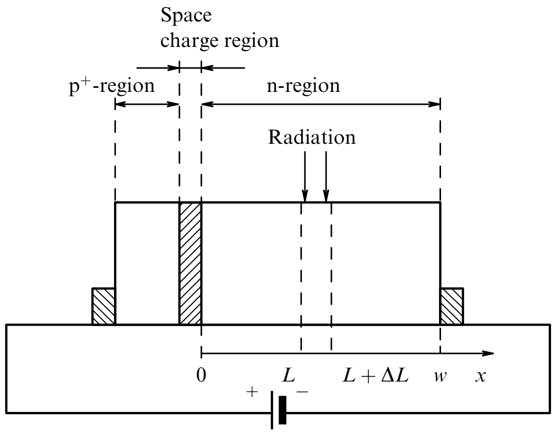

Figure 2. Structure and irradiation scheme of a $p^+ - n$ junction.

plane and directed to the n-region, and choose its origin to be at the boundary between the neutral n-region and the space charge region.

We neglect such things as the generation and recombination of current carriers in the space charge region, surface leakage and generation – recombination processes on the side faces of the crystal perpendicular to the junction, and the electronic component of the current through the space charge region. There are no difficulties in extending our results to p – n junctions in which electronic currents and noise are not small [11, 13].

We assume that the n-region of the diode is homogeneous and that the distance w_l between the Ohmic contact to it and the space charge region greatly exceed the hole diffusion length L_p, allowing the hole component of the current through the contact to be neglected. This is the so-called long diode case; the opposite extreme, a short diode, with length $w_s \ll L_p$ and a small contact area, is considered in Section 6.

We further restrict ourselves to the case in which hole injection into the n-region of a forward-biased or irradiated $p^+ - n$ junction is small. Then the hole recombination in the n-region is linear and is characterized by the hole lifetime $\tau_p = L_p^2/D_p$ (D_p is the hole diffusion coefficient), and the conduction in the n-region can be considered unmodulated. We also neglect the voltage drop across the n-region and across the Ohmic contact to it when an external current runs through the $p^+ - n$ junction.

Next, because the series resistance of the n-region is assumed to be small and the external circuit of the diode to be short-circuited (the internal resistance of the source of the constant diode voltage V is also negligible), the RC limitations on the frequency characteristics of the $p^+ - n$ junction can be ignored in further analyses.

In calculating photoinduced currents in a $p^+ - n$ junction, a radiation harmonically modulated at an angular frequency $\omega \geqslant 0$ is assumed to be projected on the n-region of the short-circuited diode under external voltage V, which is shaped into a narrow strip (of width $\Delta L \ll L$) parallel to the $p^+ - n$ junction plane and located at a distance $L \leqslant L_p$ from the space charge region boundary, and which generates electron – hole pairs in the n-region, uniformly over the section of the $p^+ - n$ junction and at rate g cm^{-3} s^{-1}. There is no difficulty in estimating the pair generation rate in the case where radiation losses due to reflection from the semiconductor surface or the photodiode entrance window are important.

The spectrum of photoinduced noise in a $p^+ - n$ junction is calculated by keeping the voltage constant and by assuming local radiation to be unmodulated.

Clearly, whatever the irradiation distribution pattern along the x axis, its effect on the diode can be represented as a linear superposition of solutions obtained under the conditions described above. It should only be remembered that in the problem under consideration, no radiation comes either into the space charge region of the $p^+ - n$ junction or the p^+ region.

Because the nonequilibrium holes in the n-region are charge-neutralized by the electrons pulled out of the contact (it takes a time comparable to the semiconductor's dielectric relaxation time — of the order of 10^{-12} s — for the regions involved to restore electric neutrality), and because there are no electric fields in the neutral n-region, the only possible transport mechanism for holes is their thermal diffusion in the presence of a concentration gradient.

Given the assumptions above, the motion of minority carriers in the n-region is one-dimensional and the diffusion current density is given by

$$j_p(x, t) = -eD_p \frac{\partial \Delta p(x, t)}{\partial x}, \tag{6}$$

where $\Delta p(x, t)$ is the excess electron concentration relative to the equilibrium concentration p_n in an n-type material.

The electron current gradient under linear recombination conditions is determined by the non-steady-state continuity condition [15]

$$\frac{\partial j_p(x, t)}{\partial x} = -e \frac{\Delta p(x, t)}{\tau_p} - e \frac{\partial \Delta p(x, t)}{\partial t}. \tag{7}$$

Under the conditions outlined above, it has been shown [13, 15] that $j_p(x = 0, t)$ is equal to the external circuit current of the $p^+ - n$ junction.

The thermal concentration of the excess holes in the n-region at the boundary with the space charge region follows changes in the voltage $V(t)$,

$$\Delta p(x = 0, t) = p_n \left[\exp\left(\frac{eV(t)}{kT}\right) - 1\right]. \tag{8}$$

As proved in Ref. [15], for relation (8) to hold, it suffices that the recombination and generation of current carriers be negligible in the space charge region.

The second boundary condition for the concentration of nonequilibrium holes in a nonirradiated n-region of a long diode has the form

$$\Delta p(x = w_l, t) = 0.$$

For a 'short' diode, the boundary condition at the contact-free surface of the n-region (the typical thin-base diode topology) is written as

$$j_p(x = w_s, t) = s\,\Delta p(x = w_s, t),$$

where s is the surface recombination rate.

At an arbitrary section x, the thermal generation and recombination of the minority carriers occur locally at the combined rate

$$\frac{2p_{\mathrm{n}} + \Delta p(x,t)}{\tau_{\mathrm{p}}}\,,$$

where $p_{\mathrm{n}}/\tau_{\mathrm{p}}$ is the equilibrium recombination rate of the holes, equal to their thermal generation rate. Then the shot noise of the Poisson processes of hole generation and recombination over the length Δx in the frequency range Δf is given by

$$\overline{j^2(x)} = 2e^2\,\frac{2p_{\mathrm{n}} + \Delta p(x)}{\tau_{\mathrm{p}}}\,\Delta x\,\Delta f\,. \tag{9}$$

The total hole concentration in the x section is $p_{\mathrm{n}} + \Delta p(x,t)$, and its fluctuations due to diffusion can be written as

$$\overline{\Delta p^2(x)} = 4\,\frac{p_{\mathrm{n}} + \Delta p(x)}{D_{\mathrm{p}}}\,\Delta x\,\Delta f\,. \tag{10}$$

We note that the generation – recombination and diffusion generators on the length Δx can be considered independent of each other as long as Δx exceeds the hole mean free path. The proof of relation (10) is given in Section 4.

In what follows, the thermal and photoinduced noise in a short-circuited $\mathrm{p}^+ - \mathrm{n}$ junction are calculated, as in Ref. [11], by representing the junction as an *RC*-distributed electric line.

4. *RC*-distributed line

Differential equations (6) and (7) describing the diffusion and recombination of holes are analogous (apart from the dimensionality) to the equations for the time-varying voltages $u(x,t)$ and currents $i(x,t)$ along an electric line with distributed parameters (Fig. 3), containing a series resistance R, a shunt conductance B, and a shunt capacitor C (all three, per unit length of the line):

$$i(x,t) = -\frac{1}{R}\,\frac{\partial u(x,t)}{\partial x}\,, \tag{11}$$

$$\frac{\partial i(x,t)}{\partial x} = -Bu(x,t) - C\,\frac{\partial u(x,t)}{\partial t}\,. \tag{12}$$

The general solution of system (11), (12) for a harmonic perturbation of frequency ω has the form

$$\dot u(x) = \dot C_1 \exp\left(\frac{x}{\dot L_L}\right) + \dot C_2 \exp\left(-\frac{x}{\dot L_L}\right), \tag{13}$$

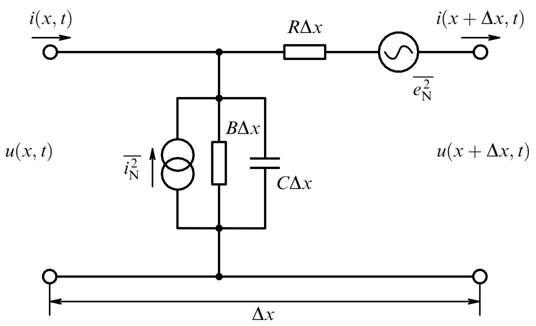

Figure 3. Element of an *RC*-distributed transmission line.

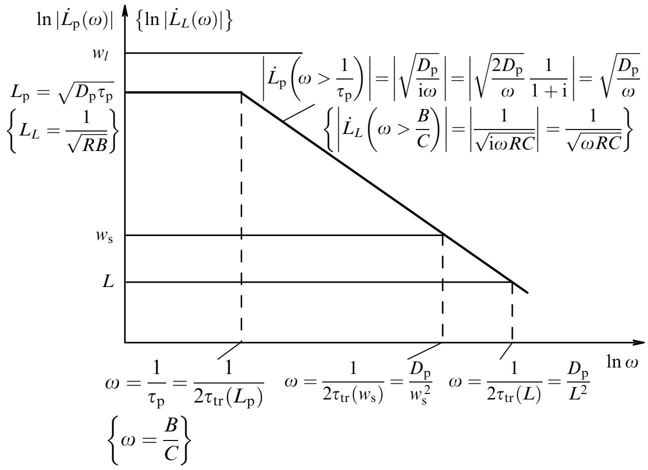

Figure 4. Absolute effective diffusion length of holes in an n-type semiconductor,

$$|\dot L_{\mathrm{p}}(\omega)| = \left|\sqrt{\frac{D_{\mathrm{p}}\tau_{\mathrm{p}}}{1+\mathrm{i}\omega\tau_{\mathrm{p}}}}\right| = \frac{\sqrt{D_{\mathrm{p}}\tau_{\mathrm{p}}}}{\sqrt[4]{1+\omega^2\tau_{\mathrm{p}}^2}} = \frac{L_{\mathrm{p}}}{\sqrt{2a^2-1}}\,,$$

as a function of the frequency in the log – log scale. The braces contain the corresponding quantities for an *RC*-distributed transmission line. L is the distance from the n-region/space charge region boundary to the narrow ($\Delta L \ll L$) radiation strip, w_{l} (w_{s}) is the n-region thickness of a $\mathrm{p}^+ - \mathrm{n}$ diode with a long (short) n-region; and τ_{tr} is the duration of photohole diffusion flight over a distance L.

$$\begin{aligned} i(x) &= -\frac{1}{R}\,\frac{\partial \dot u(x,t)}{\partial x} \\ &= -\frac{1}{R\dot L_L}\left[\dot C_1 \exp\left(\frac{x}{\dot L_L}\right) - \dot C_2 \exp\left(-\frac{x}{\dot L_L}\right)\right] \\ &= -\dot Y\left[\dot C_1 \exp\left(\frac{x}{\dot L_L}\right) - \dot C_2 \exp\left(-\frac{x}{\dot L_L}\right)\right], \end{aligned} \tag{14}$$

where the complex quantity

$$\dot L_L = \frac{1}{\sqrt{RB}\sqrt{1+\mathrm{i}\omega C/B}} = \frac{L_L}{\sqrt{1+\mathrm{i}\omega C/B}}$$

of the dimension of length characterizes the effective propagation length of a perturbation of the frequency ω in the line and has a low-frequency component L_L (Fig. 4). The characteristic, or total, conductance of a distributed *RC* line, $\dot Y$ (of dimension Ω^{-1}), is given by

$$\begin{aligned} \dot Y &= \frac{1}{R\dot L_L} = \sqrt{\frac{B}{R}}\sqrt{1+\mathrm{i}\omega\,\frac{C}{B}} = Y\sqrt{1+\mathrm{i}\omega\,\frac{C}{B}} \\ &= Y\left[\sqrt{\frac{\left[1+\omega^2(C/B)^2\right]^{1/2}}{2}+\frac{1}{2}} \right.\\ &\left.\quad +\mathrm{i}\sqrt{\frac{\left[1+\omega^2(C/B)^2\right]^{1/2}}{2}-\frac{1}{2}}\right] \\ &= Y(a+\mathrm{i}b) = Y_{\mathrm{a}} + \mathrm{i}\,Y_{\mathrm{r}}\,, \end{aligned} \tag{15}$$

where Y, Y_{a}, and Y_{r} are the respective low-frequency, active, and reactive components of $\dot Y$, and $b^2 = a^2 - 1$.

If the line length w is much larger than $|\dot L_L|$, then $\dot C_1 = 0$ and the input conductance of the line is

$$\frac{i(x=0)}{u(x=0)} = \dot Y\,.$$

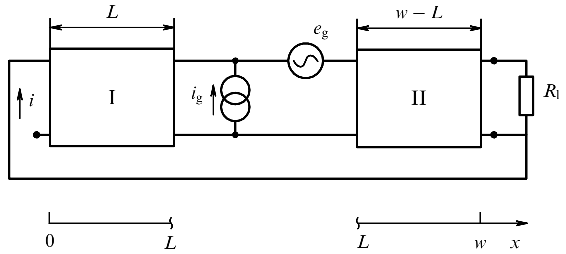

Figure 5. The calculation of the signal and noise contribution to the input current of a distributed parameter line from sources at a distance L from the line's short-circuited input.

The inherent noise in a distributed parameter line is due to thermal noise in its active elements (see Fig. 3) and is given in the frequency band Δf by

$$\overline{e_{\rm N}^2} = 4kTR\,\Delta x\,\Delta f = S_{\rm e}\,\Delta f\,, \tag{16}$$

$$\overline{i_{\rm N}^2} = 4kTB\,\Delta x\,\Delta f = S\,\Delta f\,, \tag{17}$$

where $S_{\rm e}$ and S are the respective spectral noise densities of the voltage and current. These noise sources correlate neither with one another nor with similar sources in other line elements. We recall that the Nyquist formula was derived under the condition that electrons and holes have a fixed charge e and that the diffusive scattering events of current carriers are independent of one another.

We use the circuit shown in Fig. 5 to calculate the contribution of the source mentioned above to the noise current density $\overline{i^2}$ of the short-circuited input of a distributed parameter line. The linear property of the transmission line is then used to independently calculate the input currents from all signal or noise sources.

From system (13), (14), the output impedance of segment I of a short input line $\left[u_1(x=0)=0\right]$ is given by

$$\dot Z_{\rm out,\,1}(x=L) = -R\dot L_L\,\frac{\exp\left(L/\dot L_L\right)-\exp\left(-L/\dot L_L\right)}{\exp\left(L/\dot L_L\right)+\exp\left(-L/\dot L_L\right)} = -R\dot L_L\tanh\left(\frac{L}{\dot L_L}\right). \tag{18}$$

For a line with $w/|\dot L_L| \gg 1$, the total input resistance of segment $R_{\rm l}$ for any value of the load resistance is

$$\dot Z_{\rm in,\,2}(x=L) = \frac{1}{\dot Y}\,.$$

It is now a simple matter to calculate the input current of the line due to the generator $i_{\rm g}$ at a distance L from the input. We obtain

$$i = -i_{\rm g}\exp\left(-\frac{L}{\dot L_L}\right). \tag{19}$$

The coordinate dependence of the voltage across segment I (for $0 \leqslant x < L$) is given by

$$\dot u_1(x,L) = i_{\rm g}\,\frac{R\dot L_L}{2}\exp\left(-\frac{L}{\dot L_L}\right)\times\left[\exp\left(\frac{x}{\dot L_L}\right)-\exp\left(-\frac{x}{\dot L_L}\right)\right], \tag{20}$$

and that across segment II (for $L \leqslant x \leqslant w$) by

$$\dot u_2(x,L) = i_{\rm g}\,\frac{R\dot L_L}{2}\left[\exp\left(\frac{L}{\dot L_L}\right)-\exp\left(-\frac{L}{\dot L_L}\right)\right]\exp\left(-\frac{x}{\dot L_L}\right). \tag{21}$$

Accordingly, the current produced by the generator $e_{\rm g}$ in the short-circuited input of the line is

$$i = e_{\rm g}\,\dot Y\exp\left(-\frac{L}{\dot L_L}\right), \tag{22}$$

and the transmission coefficients to the input of a long line of similarly arranged noise sources are found to be

$$\begin{aligned}\overline{i^2} &= \overline{i_{\rm N}^2}\left|\exp\left(-\frac{2L}{\dot L_L}\right)\right| = \overline{i_{\rm N}^2}\left|\exp\left(-\frac{2L}{L_L}\sqrt{1+{\rm i}\omega\frac{C}{B}}\right)\right| \\ &= \overline{i_{\rm N}^2}\left|\exp\left[-\frac{2L}{L_L}(a+{\rm i}b)\right]\right| \\ &= \overline{i_{\rm N}^2}\exp\left(-\frac{2aL}{L_L}\right)\left|\exp\left(-\frac{{\rm i}2bL}{L_L}\right)\right| = \overline{i_{\rm N}^2}\exp\left(-\frac{2aL}{L_L}\right),\end{aligned} \tag{23}$$

$$\overline{i^2} = \overline{e_{\rm N}^2}\left|\dot Y^2\exp\left(-\frac{2L}{\dot L_L}\right)\right| = \overline{e_{\rm N}^2}\,Y^2(2a^2-1)\exp\left(-\frac{2aL}{L_L}\right), \tag{24}$$

where $2a^2-1=\sqrt{1+\omega^2(C/B)^2}$ and $Y^2=B/R$.

The coefficient of transformation to the external short circuit current, $\overline{e_{\rm N}^2}$, includes the factor $|\dot Y|^2$, in addition to the factor $\exp\left(-2aL/L_L\right)$ (also present is the transformation coefficient $\overline{i_{\rm N}^2}$), which accounts for the coordinate and frequency dependences of the contribution of local noise currents and which determines the effective region for collecting noise to the external circuit, $L_L/2a$. This additional factor clearly serves the purpose of recalculating $\overline{e_{\rm N}^2}$ to noise currents.

The spectral density of the inherent noise current in the short-circuited input of a long distributed-parameter line, given by

$$\begin{aligned}S(\omega) &= \int_0^w\left[S\left|\exp\left(-\frac{x}{\dot L_L}\right)\right|^2+S_{\rm e}\left|\dot Y\exp\left(-\frac{x}{\dot L_L}\right)\right|^2\right]{\rm d}x \\ &= 4kT\int_0^\infty\left[B+RY^2(2a^2-1)\right]\exp\left(-\frac{2ax}{L_L}\right){\rm d}x \\ &= 4kTa\sqrt{\frac{B}{R}} = 4kTaY = 4kTY_{\rm a}\,,\end{aligned} \tag{25}$$

at all frequencies correlates with the active component of the line's input conductance, which increases with frequency for $a > 1$.

Analysis of Eqn (25) shows that at lower frequencies ($a \approx 1$), the thermal noise contributions from R and B [Eqns (16) and (17)] to the transmission line's input noise current relate as $Y^2(2a^2-1):1 \approx Y^2:1$. However, at frequencies $\omega \gg B/C$, where $a \approx \sqrt{(\omega C/B)/2}$ and the active part of the transverse conductance B becomes negligible compared with the reactive conductance ωC, the quantity $|\dot Y|^2 = Y^2(2a^2-1) \approx \omega C/R$ ceases to depend on B and increases linearly with the frequency, whereas noise in the external circuit is determined by voltage fluctuations alone. But because the effective length for collecting noise decreases with the frequency as $L_L/2a$, the spectral noise density at the line input, which is due to voltage fluctuations, increases with the frequency in proportion to a, or $\sqrt{\omega}$, but not to a^2.

Comparison of relations (6) and (7) with system (11), (12) shows that $u(x,t)$ in the latter pair corresponds to $\Delta p(x,t)$ in the former. Similarly, $i(x,t)$ corresponds to $j_p(x,t)$, R to $1/eD_p$, B to e/τ_p, and C to e. There is also a correspondence between $\dot{L}_L$ for a transmission line and the complex diffusion length of holes in the n-region:

$$\dot{L}_p = \sqrt{\frac{D_p\tau_p}{1+i\omega\tau_p}} = \frac{L_p}{\sqrt{1+i\omega\tau_p}} = \sqrt{D_p\dot{\tau}_p}\,,$$

where for a harmonic perturbation of frequency ω, $\dot{\tau}_p = \tau_p/(1+i\omega\tau_p)$ and $L_p = \sqrt{D_p\tau_p}$ is the low-frequency value of $\dot{L}_p$ (see Fig. 4). Also, it is readily seen that $\dot{Y}$ corresponds to the ratio

$$\frac{eD_p}{\dot{L}_p} = \frac{eD_p\sqrt{1+i\omega\tau_p}}{L_p} = \frac{eL_p\sqrt{1+i\omega\tau_p}}{\tau_p}$$

in the $p^+ - n$ junction. The condition of a long line, $w \gg |\dot{L}_L|$, in the case of a $p^+ - n$ junction transforms into the similar condition mentioned above, that the n-region length be much larger than $|\dot{L}_p|$.

The noise current generator $\overline{j^2}$ in a $p^+ - n$ junction [Eqn (9)] represents the shot noise due to local hole generation and recombination currents over the length Δx in the frequency band Δf. In thermodynamic equilibrium, $\Delta p(x,t) = 0$ and

$$\overline{j^2} = 4e^2\,\frac{p_n}{\tau_p}\,\Delta x\,\Delta f\,.$$

The diffusion–recombination transport of minority current carriers in the neutral regions of a diode on the one hand and currents in a distributed parameter line on the other hand have a deeper analogy between them than revealed in Ref. [11], and one that extends to equilibrium noise. One further analogy is between the equilibrium hole concentration p_n and the diffusion potential kT/e in Eqns (16) and (17) for thermal noise in an RC line.

Given the last-listed point of correspondence, it immediately follows from Eqn (16) that under thermodynamic equilibrium, the hole concentration is characterized by the mean-square deviation

$$\overline{\Delta p^2} = 4\,\frac{p_n}{D_p}\,\Delta x\,\Delta f$$

due to the random nature of hole diffusion.

Relation (10) can also be proved in general for $\Delta p(x) \neq 0$ [11]. We let the mean number of holes in an element of length Δx be denoted by $P = p\,\Delta x$. For diffusive scattering, the mean square deviation of the number of holes $\overline{\delta P^2}$ is equal to their mean number, $\overline{\delta P^2} = P = p\,\Delta x$, and hence the mean square deviation of hole concentration in an element Δx is

$$\overline{\delta p^2} = \frac{\overline{\delta P^2}}{(\Delta x)^2} = \frac{p}{\Delta x}\,.$$

But if a fluctuating quantity $X(t)$ is characterized by the normalized correlation function of the form $R(\tau) = \exp(-\tau/\tau_0)$, then the Wiener–Khinchin theorem [3, 11] tells us that the spectral density of this function at relatively low frequencies is given by $S(\omega) = 4\overline{X^2(t)}\tau_0$.

The frequency characteristic of an element of a distributed parameter line (see Fig. 3) for $\Delta x \to 0$ and the short-circuited output is expressed by

$$i_{out} = \frac{i_{in}}{(B\Delta x + i\omega C\Delta x + 1/R\Delta x)R\Delta x} = \frac{i_{in}}{1 + RB(\Delta x)^2 + i\omega RC(\Delta x)^2} \approx \frac{i_{in}}{1 + i\omega RC(\Delta x)^2} = \frac{i_{in}}{1 + i\omega\tau_0}\,.$$

Using the relations above, we obtain $\tau_0 = (\Delta x)^2/D_p$, $S(\omega) = 4p\,\Delta x/D_p$, and

$$\overline{\Delta p^2} = 4\,\frac{p}{D_p}\,\Delta x\,\Delta f = 4\,\frac{p_n + \Delta p(x)}{D_p}\,\Delta x\,\Delta f\,.$$

The same relation was obtained earlier by Petritz from the solution of the Fokker–Planck equation (see Ref. [11], p. 141).

Thus, $\overline{j^2}$ and $\overline{\Delta p^2}$ both prove to be proportional to the local hole concentration. However, if the generation–recombination noise $\overline{j^2}$ transforms into noise in the diode's external short circuit with the factor $\exp(-2ax/L_p)$ (as does photon noise in the case of local irradiation to the point x), then the factor

$$\left|\frac{e\dot{L}_p}{\dot{\tau}_p}\right|^2 = \left|\frac{eD_p}{\dot{L}_p}\right|^2 = \frac{e^2D_p^2(2a^2-1)}{L_p^2} = \frac{e^2D_p(2a^2-1)}{\tau_p}$$

is added to recalculate concentration fluctuations $\overline{\Delta p^2}$, which is equal to e^2D_p/τ_p at low frequencies ($\omega\tau_p \ll 1$) and is equal to $\omega e^2 D_p$ and increases with the frequency for $\omega \gg 1/\tau_p$, when $a \approx \sqrt{\omega\tau_p/2} \gg 1$.

By the last two equations, the diffusive fluctuations in the local minority carrier concentration in a long diode turn out to be equivalent to shot noise due to their local generation and recombination fluxes, either of which is

$$\frac{p_n + \Delta p(x)}{|\dot{\tau}_p|}\,\Delta x\,.$$

In diffusion fluctuations of the minority carrier concentration, in contrast to fluctuations due to the diffusion of majority carriers, which relax on the time scale of dielectric (or Maxwellian) relaxation, only the charge of the minority carriers is neutralized by the majority carriers during the Maxwellian time. As regards the local fluctuations in the minority carrier concentration (and it is their relaxation that gives rise to diffusion noise in the diode's external circuit), they relax by the minority carriers' recombination and thermal generation. As the effective minority carrier lifetime decreases, local diffusion relaxation currents increase, with the diffusion noise dominating at angular frequencies larger than the inverse lifetime of the minority carriers.

Noise transmission coefficients to the input of a distributed parameter line are found in Section 6.

One further point of correspondence between the $p^+ - n$ junction and the transmission line is worth noting: the load resistance R_l in the line (see Fig. 5) corresponds in the $p^+ - n$ junction to the quantity $1/es$, where s is the recombination rate at the surface of the n-region that is parallel to the $p^+ - n$ junction.

5. Long $p^+ - n$ junction

For the n-region length $w_1 \gg L_p$ (which imposes no limitations on the frequency range studied, see Fig. 4) and for a constant external voltage V across the $p^+ - n$ junction, it follows from Eqns (8), (13), and (14) that

$$\Delta p_{th}(x) = p_n \left[\exp\left(\frac{eV}{kT}\right) - 1 \right] \exp\left(-\frac{x}{L_p}\right), \qquad (26)$$

where the subscript 'th' indicates that the hole concentration deviates from its equilibrium value due to the electric bias applied to the diode. Combining this with Eqns (6) and (26) yields the well-known expression for the static I – V characteristic of a nonirradiated long $p^+ - n$ junction [13]:

$$j = j_p(x=0) = \frac{ep_n D_p}{L_p} \left[\exp\left(\frac{eV}{kT}\right) - 1 \right] = \frac{ep_n L_p}{\tau_p} \left[\exp\left(\frac{eV}{kT}\right) - 1 \right] = j_s \left[\exp\left(\frac{eV}{kT}\right) - 1 \right], \qquad (27)$$

where j_s is the saturation current of the $p^+ - n$ junction.

Assuming a $p^+ - n$ junction to be exposed to a time-constant voltage V and a harmonically varying small perturbation $dV < kT/e$, the specific (per cm^2 of the cross section) differential conductance of the junction can be written as

$$\dot{G} = \frac{d j_p(x=0)}{dV} = \frac{d j_p(x=0)}{d\Delta p_{th}(x=0)} \frac{d\Delta p_{th}(x=0)}{dV}.$$

The derivative

$$\frac{d j_p(x=0)}{d\Delta p_{th}(x=0)}$$

corresponds to the characteristic conductance $\dot{Y}$ of an equivalent transmission line, and its value for a $p^+ - n$ junction is $(eL_p/\tau_p)\sqrt{1 + i\omega\tau_p}$. From Eqn (8),

$$\frac{d\Delta p_{th}(x=0)}{dV} = \frac{ep_n}{kT} \exp\left(\frac{eV}{kT}\right).$$

We then find that

$$\dot{G} = \frac{eL_p}{\tau_p} \sqrt{1 + i\omega\tau_p} \frac{ep_n}{kT} \exp\left(\frac{eV}{kT}\right) = \frac{ej_s}{kT} \exp\left(\frac{eV}{kT}\right) \sqrt{1 + i\omega\tau_p} = \frac{e}{kT}(j + j_s)\sqrt{1 + i\omega\tau_p} = G\sqrt{1 + i\omega\tau_p}$$
$$= G \left[\sqrt{\frac{(1+\omega^2\tau_p^2)^{1/2}}{2} + \frac{1}{2}} + i\sqrt{\frac{(1+\omega^2\tau_p^2)^{1/2}}{2} - \frac{1}{2}} \right] = G(a + ib) = G_a + G_r, \qquad (28)$$

where G, G_a, and G_r are the respective low-frequency value, active component, and reactive component of the complex conductance $\dot{G}$ of a $p^+ - n$ junction. Clearly, relation (28) can also be obtained from the I – V characteristic of a long diode using the frequency characteristics of $\dot{L}_p$ and $\dot{\tau}_p$.

For low ($\omega \ll 1/\tau_p$) frequencies, the specific dynamic resistance of a zero-biased $p^+ - n$ junction can be written as

$$r = \frac{1}{G} = \frac{kT}{ej_s}. \qquad (29)$$

We note that for $\omega \ll 1/\tau_p$,

$$iG_r = iG\sqrt{\frac{(1+\omega^2\tau_p^2)^{1/2}}{2} - \frac{1}{2}} \approx iG\sqrt{\frac{\omega^2\tau_p^2}{4}} = i\omega\frac{G\tau_p}{2} = i\omega\frac{\tau_p}{2}\frac{ej_s}{kT}\exp\left(\frac{eV}{kT}\right) = i\omega C_{eff}. \qquad (30)$$

The effective capacitance detected in the external circuit of a $p^+ - n$ junction at low frequencies turns out to be half its diffusion capacitance:

$$C_{dif} = \frac{\partial Q}{\partial V} = \frac{\partial}{\partial V}\left\{ e\int_0^\infty p_n \left[\exp\left(\frac{eV}{kT}\right) - 1 \right] \exp\left(-\frac{x}{L_p}\right) dx \right\} = 2C_{eff}.$$

The discussion above suggests that the reason for the decrease in the effective capacitance is the diffusion – recombination transport of holes in the n-region, a process that produces the charge and discharge currents of the diffusion capacitance (the dynamics of change in the hole concentration corresponds to a distributed parameter equivalent line).

The frequently cited explanation of the twice reduced diffusion capacitance — that electrons and holes are not separated spatially [15] — is not sufficient and is at odds with the familiar notion that capacity increases as the condenser plates are brought closer together.

We recall that the diffusion capacitance at forward bias is orders of magnitude larger than the barrier capacitance C_b of a $p^+ - n$ junction.

If the radiation locally incident on the $p^+ - n$ junction is intensity-modulated at a frequency ω, then it follows from Eqn (19) that the photocurrent of the external short circuit is

$$j_{ph} = -eg\Delta L \exp\left(-\frac{L}{\dot{L}_p}\right), \qquad (31)$$

and its amplitude – frequency characteristic is given by

$$|j_{ph}| = eg\Delta L \left| \exp\left(-\frac{L}{\dot{L}_p}\right) \right| = eg\Delta L \left| \exp\left(-\frac{L(a+ib)}{L_p}\right) \right| = eg\Delta L \exp\left(-\frac{aL}{L_p}\right) = eg\Delta L \exp\left(-\frac{L}{L_p}\right) \exp\left[-(a-1)\frac{L}{L_p}\right], \qquad (32)$$

because $|\exp(-ibL/L_p)| = 1$.

For $\omega \ll 1/\tau_p$ or an unmodulated radiation flux, $j_{ph} = -eg\Delta L \exp(-L/L_p)$, and, in addition to $\Delta p_{th}(x)$ [see Eqn (26)], photogenerated excess holes appear in the n-region, whose concentration can be written, using Eqns (20) and (21), as

$$\Delta p_{ph1}(x) = g\Delta L \frac{\tau_p}{2L_p} \exp\left(-\frac{L}{L_p}\right) \times \left[\exp\left(\frac{x}{L_p}\right) - \exp\left(-\frac{x}{L_p}\right) \right] \qquad (33)$$

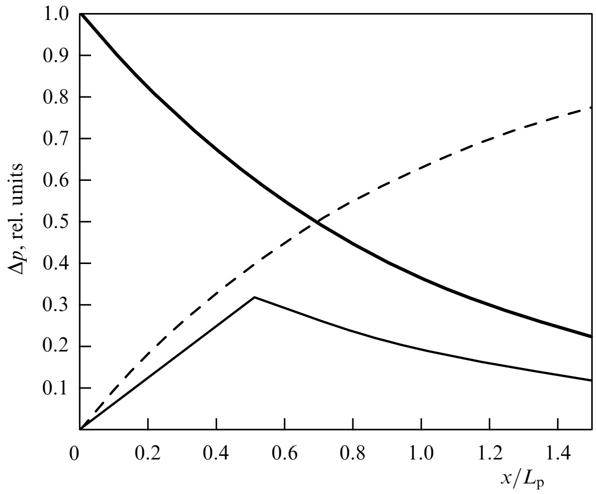

Figure 6. Distribution of the steady-state hole concentration in the n-region of a long p^+-n junction. Thin solid line: photohole concentrations $\Delta p_{ph,1}(x, L = L_p/2)$ and $\Delta p_{ph,2}(x, L = L_p/2)$ (in units of $g\tau_p \Delta L/L$) for local irradiation at the point $L = L_p/2$. Dashed line: (1) photohole concentration $\Delta p_{ph}(x)$ in units of $g\tau_p$ for uniform irradiation and (2) hole concentration (in units of p_n) for reverse saturation current j_s. Thick solid line: hole concentration $\Delta p_{th}(x)$ (in units of p_n) for the direct current j_s through the p^+-n junction.

for $0 \leqslant x \leqslant L$, and

$$\Delta p_{ph2}(x) = g\Delta L \frac{\tau_p}{2L_p}\left[\exp\left(\frac{L}{L_p}\right) - \exp\left(-\frac{L}{L_p}\right)\right] \times \exp\left(-\frac{x}{L_p}\right) \qquad (34)$$

for $0 \leqslant x \leqslant w_l$.

The thin solid line in Fig. 6 shows the hole distribution in the case where the n-region of a long diode is exposed to steady-state radiation shaped into a narrow strip located at a distance of half the low-frequency diffusion length from the space charge region.

The spectral noise density of an unmodulated photon flux is [1]

$$S_{f,ph} = 2e^2 g\Delta L k_d \,. \qquad (35)$$

With Eqns (23) and (32), the spectral density component of external photoinduced noise that is due to the noise in the incident photon flux (the spectral density of the reduced radiation noise power) is given by

$$S_{w,ph}(\omega) = 2e^2 g\Delta L k_d \left|\exp\left(-\frac{L}{\dot{L}_p}\right)\right|^2 = 2e^2 g\Delta L k_d \left|\exp\left(-\frac{L\sqrt{1+i\omega\tau_p}}{L_p}\right)\right|^2 = 2e^2 g\Delta L k_d \exp\left(-\frac{2aL}{L_p}\right). \qquad (36)$$

Here, the factor that expresses the coordinate and time dependences is the same as in Eqn (32), but squared.

In the external circuit, the spectral density of the total noise current (which includes thermal noise and photoinduced noise) is calculated from a formula similar to Eqn (25), but with the addition of the reduced radiation noise power,

$$S(\omega) = \int_0^{\infty}\left[2e^2\,\frac{\Delta p_{th}(x) + \Delta p_{ph}(x) + 2p_n}{\tau_p} + 4\,\frac{\Delta p_{th}(x) + \Delta p_{ph}(x) + p_n}{D_p}\,\frac{e^2 L_p^2}{\tau_p^2}(2a^2 - 1)\right] \times \exp\left(-\frac{2ax}{L_p}\right)dx + 2e^2 g\Delta L k_d \exp\left(-\frac{2aL}{L_p}\right), \qquad (37)$$

where, for $w_l \gg L_p$, the upper integration limit can clearly be replaced by infinity.

In Eqn (37), $\Delta p_{th}(x)$ is defined by Eqn (26), $\Delta p_{ph}(x)$ is defined by Eqns (33) and (34), and $|\dot{Y}|^2$ corresponds to

$$\frac{e^2 L_p^2}{\tau_p^2}\left|(1 + i\omega\tau_p)\right| = \frac{e^2 L_p^2}{\tau_p^2}(2a^2 - 1)\,.$$

The first term in the brackets in the integrand in Eqn (37) involves the sum of the spectral densities of the local shot noise from the thermal generation of holes, p_n/τ_p, and from their recombination,

$$\frac{\Delta p_{th}(x) + \Delta p_{ph}(x) + p_n}{\tau_p}\,.$$

The expression

$$\frac{4\left[\Delta p_{th}(x) + \Delta p_{ph}(x) + p_n\right]}{D_p}$$

is the spectral density of noise due to the thermal diffusion of holes.

5.1 Photoinduced noise in a long p^+-n junction

We begin by calculating the spectral density of photoinduced noise in a long p^+-n junction:

$$S_{ph}(\omega) = S_{s,ph}(\omega) + S_{w,ph}(\omega) = \int_0^{\infty}\left[2e^2\,\frac{\Delta p_{ph}(x)}{\tau_p} + 4\,\frac{\Delta p_{ph}(x)}{D_p}\,\frac{e^2 L_p^2}{\tau_p^2}(2a^2 - 1)\right] \times \exp\left(-\frac{2ax}{L}\right)dx + 2e^2 g\Delta L k_d \exp\left(-\frac{2aL}{L_p}\right)$$

$$= 2e^2 g\Delta L\left[\frac{1 + 2(2a^2 - 1)}{4a^2 - 1} \times\left\{\exp\left(-\frac{L}{L_p}\right) - \exp\left(-\frac{2aL}{L_p}\right)\left[\left(a + \frac{1}{2}\right) - \left(a - \frac{1}{2}\right)\right]\right\} + k_d \exp\left(-\frac{2aL}{L_p}\right)\right]$$

$$= 2e^2 g\Delta L\left\{\exp\left(-\frac{L}{L_p}\right) + (k_d - 1)\exp\left(-\frac{2aL}{L_p}\right)\right\}$$

$$= 2e^2 g\Delta L \exp\left(-\frac{L}{L_p}\right)\left\{1 + (k_d - 1)\exp\left[-\frac{(2a-1)L}{L_p}\right]\right\}$$

$$= 2e j_{ph}\left\{1 + (k_d - 1)\exp\left[-\frac{(2a-1)L}{L_p}\right]\right\}, \qquad (38)$$

where $S_{s,ph}$ is the radiation-induced inherent noise of the diode.

To trace the origin of some terms in the final expressions, a number of intermediate calculation steps are kept in Eqn (38).

Comparing Eqns (32) and (38) reveals different algorithms by which the photocurrent and photoinduced noise form during the diffusion–recombination transfer of holes photogenerated in the n-region.

As photoholes diffuse from their generation place to the space charge region in a diode irradiated locally by an unmodulated or low frequency flux, they form a steady-state concentration profile $\Delta p_{ph}(x)$ in the n-region, dependent on L and L_p [see Eqns (33) and (34) and the thin solid line in Fig. 6]. Photohole recombination at the local rate $\Delta p_{ph}(x)/\tau_p$ results in some loss in the photocurrent: its collecting coefficient is $\exp(-L/L_p)$ according to Eqn (32). Increasing the radiation modulation frequency decreases the effective lifetime and diffusion length of the holes, as well as reduces the hole collecting coefficient, which is now $\exp(-aL/L_p)$.

Hence, the spectral density of the reduced radiation noise (reduced noise power) in the external circuit [Eqn (36)] is proportional to $\exp(-2aL/L_p)$.

However, noise in the external circuit of a p^+–n junction is also created by fluctuations that occur in the local recombination and diffusion rates and are determined by the steady-state profile of the photohole concentration. Importantly, the transmission coefficient of the local recombination and diffusion noise to the external circuit is $\exp(-2ax/L_p)$, whereas the transmission coefficient of local diffusion noise increases unlimitedly with the frequency due to the reduction in the effective diffusion length of the holes.

It follows from Eqns (36) and (38) that the larger the ratio L/L_p and the higher the modulation frequency, the greater the inherent noise of the diode, $S_{s,ph}$, contributes to the photoinduced noise relative to the reduced radiation noise, $S_{w,ph}$.

Equation (38) also implies that local recombination and diffusion fluctuations are in the proportion $1:2(2a^2-1)$ to the inherent photonoise of a long diode. At low frequencies, $a \approx 1$ and this ratio is expectedly 1:2, whereas at high frequencies, for $\omega \gg 1/\tau_p$, the photoinduced noise is produced by diffusion noise.

For $k_d \approx 1$, according to Eqn (38), the spectral density of photoinduced noise turns out to be frequency independent, and, furthermore, its value corresponds to the shot noise of the constant component of the photocurrent

$$S_{ph}(\omega) = 2e^2 g\Delta L \exp\left(-\frac{L}{L_p}\right) = 2ej_{ph}\,.$$

This conclusion holds at least for angular frequencies lower than the inverse time of flight of current carriers through the space charge region of the p^+–n junction.

It follows from the last equation, as from the generalized equation (4) above, that $S_{ph}(\omega)$ is determined only by the constant photocurrent component j_{ph} and is independent of where they are generated or how efficiently they are collected.

Finally, by comparing Eqns (32) and (38) for $k_d \gg 1$, we see that the amplitude of photonoise due to Eqn (36), which is enhanced relative to shot noise, decreases in the same frequency ranges where the photocurrent decreases. Thus, independently of k_d, it turns out that at high frequencies (when the photocurrent virtually disappears), photoinduced noise in a p^+–n junction again corresponds to the shot noise of the constant photocurrent component and is due to the diffusion noise transferred to the external circuit.

When $L \approx L_p$, it follows from Eqn (32) that the photocurrent decreases to the level of $1/\sqrt{2} = 0.707$ at the frequency $\omega = 10/\tau_p = 5/\tau_{tr}(L_p)$, with the proportion of the reduced photon noise being already less than 2.5% at this frequency for $k_d = 1$. Here, $\tau_{tr}(L) = L^2/2D_p$ is the photohole diffusion time of flight over a distance L.

For $L \ll L_p$, the photocurrent decreases to the same level at $a \approx 0.35(L_p/L) \gg 1$ (i.e., $a \approx \sqrt{\omega\tau_p/2}$). In this case, relation (32) becomes

$$|j_{ph}| = eg\Delta L \exp\left(-\sqrt{\frac{\omega\tau_p}{2}}\,\frac{L}{L_p}\right) = eg\Delta L \exp\left(-\sqrt{\omega\tau_{tr}(L)}\,\right). \tag{39}$$

Now $|j_{ph}|$ drops to the level of 0.707 at the much higher frequency $\omega \approx 1.82/\tau_{tr}(L)$. It is readily seen that also in this case, for $\omega \ll 1/\tau_{tr}(L)$, the photoinduced noise in the external circuit is mainly due to the reduced radiation noise, and for $\omega \gg 1/\tau_{tr}(L)$, due to noise caused by photohole diffusion in the n-region.

We note that for an equivalent RC-distributed line, the frequency range $\omega \gg 1/\tau_p$ corresponds to frequencies $\omega \gg B/C$, for which the value of B is no longer significant (nor is the presence of a limit L_p in the frequency dependence of $|\dot{L}_p(\omega)|$).

For a uniformly irradiated n-region with the bulk photocarrier generation rate g ($cm^{-3}\ s^{-1}$), as before, the photocurrent in the external short circuit of a p^+–n junction is obtained by integrating Eqn (31) over the n-region, yielding

$$j_{ph} = -eg\int_0^\infty \exp\left(-\frac{L}{\dot{L}_p}\right) dL = -eg\dot{L}_p\,. \tag{40}$$

The absolute value of the photocurrent is

$$|j_{ph}| = \frac{egL_p}{\sqrt[4]{1+\omega^2\tau_p^2}} = \frac{egL_p}{\sqrt{2a^2-1}}\,.$$

The spectral density of the photon flux noise transferred to the external circuit (reduced radiation noise) is

$$S_{w,ph}(\omega) = 2e^2 g k_d \int_0^\infty \exp\left(-\frac{2aL}{L_p}\right) dL = 2e^2 g L_p k_d\,\frac{1}{2a}$$

for a uniform irradiation of the entire n-region, with the spectral density of the inherent photonoise of the p^+–n diode being

$$S_{s,ph}(\omega) = 2e^2 g\int_0^\infty \left[\exp\left(-\frac{L}{L_p}\right) - \exp\left(-\frac{2aL}{L_p}\right)\right] dL = 2e^2 g L_p\left(1-\frac{1}{2a}\right)$$

in accordance with Eqn (38).

Finally, the spectral density of the total photoinduced noise is given by

$$S_{ph}(\omega) = S_{w,ph}(\omega) + S_{s,ph}(\omega) = 2e^2 g L_p\left[1 + (k_d - 1)\,\frac{1}{2a}\right] = 2ej_{ph}\left[1 + (k_d - 1)\,\frac{1}{2a}\right] \tag{41}$$

or, for $k_d = 1$,

$$S_{ph}(\omega) = 2ej_{ph}\,.$$

It follows from Eqn (40) that the photocurrent in the case of a uniformly irradiated n-region decreases to 0.707 at the frequency $\omega = \sqrt{3}/\tau_p$. We note that in the case of a uniform irradiation, for $\omega \ll 1/\tau_p$ (or $a \approx 1$) and $k_d \approx 1$, the reduced radiation noise is equal to the inherent photonoise of the diode. But for $\omega \gg 1/\tau_p$ (or $a = \sqrt{\omega\tau_p/2} \gg 1$), we have

$$\left|j_{ph}(\omega)\right| \approx \frac{\left|j_{ph}(\omega \approx 0)\right|}{\sqrt{\omega\tau_p}} = eg\sqrt{\frac{L_p^2}{\omega\tau_p}} = eg\sqrt{\frac{D_p}{\omega}} \ll j_{ph}(\omega = 0)\,.$$

The photocurrent is now collected from a small high-frequency diffusion length, and photoinduced noise in the external circuit is fully determined by the photohole diffusion noise transferred to this circuit and again corresponds to the shot noise of the constant photocurrent component.

The hole concentration (or steady-state diffusion profile) in a uniformly irradiated n-region is obtained by integrating Eqns (33) and (34) over L, yielding

$$\Delta p_{ph}(x) = \int_0^x \Delta p_{ph2}(x, L)\,dL + \int_x^\infty \Delta p_{ph1}(x, L)\,dL = g\tau_p\left[1 - \exp\left(-\frac{x}{L_p}\right)\right]. \tag{42}$$

The dashed line in Fig. 6 shows the hole distribution in the case of an entirely and uniformly irradiated n-region. If the external circuit is short-circuited, then in both the local [Eqn (33)] and uniform cases, the hole concentration at the boundary with the space charge region is zero, and the slope of steady-state curves is determined by the constant component of the diffusion flux.

Thus, the fact that the steady-state concentration of photoinduced holes [Eqns (33) and (34) for arbitrary L and Eqn (42) for uniform photogeneration] and the corresponding generators of inherent diffusion and recombination photonoise change their distribution along the n-region depending on the excitation conditions (with or without recombination) leads, for $k_d = 1$, to the formation in the external circuit of white noise corresponding to the shot noise of a low-frequency photocurrent. For $k_d > 1$, at relatively low frequencies (lower than the inverse photocurrent attenuation time), a low-frequency plateau of photonoise with amplitudes corresponding to the generalized expression (3) forms.

Moreover, Eqn (38) can be written in the form

$$S_{ph}(\omega) = 2e^2 g\Delta L k_d \exp\left(-\frac{2aL}{L_p}\right) + 2e^2 g\Delta L\left[\exp\left(-\frac{L}{L_p}\right) - \exp\left(-\frac{2aL}{L_p}\right)\right],$$

which is totally equivalent to Eqn (3) for the expected value of quantum yield for a long diode at low frequencies, $\eta = \exp(-L/L_p)$.

Thus, seemingly abstract current distribution noise in semiconductor p^+-n junctions is found to arise physically from local fluctuations in the bulk recombination and diffusion of photoholes that form steady-state distribution profiles in the electrically neutral n-region. This conclusion is consistent with the fact that the inherent photonoise of a p^+-n junction can be represented in the form of noise generators distributed along the n-region, with their power proportional to the input influence g [see Eqns (9), (10), (33), (34), and (42)].

A point to emphasize here is that the Langevin method used in the calculation has allowed the current distribution noise and the spectral power of noise in a p^+-n junction to be determined at high angular frequencies (larger than the inverse photocurrent attenuation time).

In particular, comparing Eqns (3) and (38) yields the following relations for a long diode, in which the probability η depends both on L and on the frequency in general:

$$\overline{X_i} = \overline{X_i^2} = \exp\left(-\frac{L}{L_p}\right) = \eta(L, \omega = 0)\,, \qquad (\overline{X_i})^2 = \exp\left(-\frac{2aL}{L_p}\right) = \eta^2(L, \omega)\,. \tag{43}$$

The following aspects give an insight into the formation at high frequencies of a photoinduced noise current plateau that is insensitive to excitation details and corresponds to the shot noise of the constant photocurrent component.

It follows from Eqn (38) that at high frequencies, when $a \gg 1$ and the output noise current is dominated by diffusion noise, the incoherent collection of noise in the n-region occurs starting with the length $L_p/2a$, which decreases with the frequency. Here (recall Fig. 6), steady-state dependences $\Delta p_{ph1}(x)$ for local irradiation [Eqn (33)] and $\Delta p_{ph}(x)$ for uniform irradiation [Eqn (42)] are linearized. Because the photocurrent j_{ph} is due to diffusion [Eqn (6)], these linearized dependences are expressed by

$$\Delta p_{ph} \approx \frac{j_{ph}}{eD_p}\,x\,.$$

Substituting this in Eqn (38) for $a \gg 1$ leads, at high frequencies, to the frequency-independent noise current that corresponds to the shot noise of the constant photocurrent component:

$$S_{ph}(\omega) \approx \int_0^\infty 4\,\frac{\Delta p_{ph}(x)}{D_p}\,\frac{e^2L_p^2}{\tau_p^2}\,2a^2\exp\left(-\frac{2ax}{L_p}\right)dx = 2ej_{ph}\,.$$

Thus, spectrally white high-frequency noise arises when the photohole concentration increases linearly from the space charge region to the interior of the n-region because diffusion noise currents (increasing with the frequency and proportional to a^2) and the effective noise collection length (decreasing with the frequency and inversely proportional to a) compensate each other.

When the radiation degeneracy coefficient is unity, the spectral densities of photoinduced currents of both plateaus are also the same in the presence of bulk recombination.

However, if photoinduced noise at low frequencies is the radiation noise transformed by a p^+-n junction and the quantum yield is close to unity, then at high frequencies noise is produced in the p^+-n junction itself after radiation has been absorbed, and is always the shot noise of the constant photocurrent component.

That the noise production mechanism changes at high frequencies is apparently a general feature, not specific to semiconductor $p-n$ devices but also occurring in other information transformers with inherent inertia and noise.

We recall that the inherent noise of a distributed parameter line that is uniform along its length increases with increasing the frequency.

5.2 Thermal noise in a long $p^+ - n$ junction

The spectral density of thermal noise in a $p^+ - n$ junction can also be calculated from Eqn (37). In this case, for the short circuit of a biased diode irradiated locally on its n-region side, the total noise is written, using Eqn (38), in the form

$$S(\omega) = \frac{2eL_p}{\tau_p}\left[p_n + p_n \exp\left(\frac{eV}{kT}\right)(2a-1)\right] + 2ej_{ph}\left[1 + (k_d - 1)\exp\left(-\frac{(2a-1)L}{L_p}\right)\right]$$
$$= 2e(j+j_s)(2a-1) + 2e\left\{j_s + j_{ph}\left[1 + (k_d-1)\exp\left(-\frac{(2a-1)L}{L_p}\right)\right]\right\} \quad (44)$$

or, for $k_d = 1$,

$$S(\omega) = 2e(j+j_s)(2a-1) + 2e(j_s + j_{ph}) \,. \quad (45)$$

For $k_d = 1$, the spectral noise density of an irradiated semiconductor diode at relatively low frequencies ($\omega \ll 1/\tau_p$ or $a \approx 1$) can be represented as shot noise in two currents with no correlation between them. One of these is the algebraic sum of the dark current in the external circuit of the diode and a current that is equal in magnitude to the diode's saturation current but flows in the forward direction through the device. The other current flows through the diode in the backward direction and is the sum of the saturation current and the photocurrent.

At large reverse biases, $j = -j_s$ and the first term in Eqns (44) and (45) is zero, whereas with a reverse bias and no irradiation ($j_{ph} = 0$), the backward current noise is equal to the shot noise of the saturation current over the entire frequency range.

In this last case, Eqn (37) shows that the holes that were generated in the n-region by thermal vibrations of the crystal lattice and then reached the space charge region via diffusion – recombination transport behave similarly to the photoholes generated in that region by uniform irradiation. To see this, we note that for reverse biases, from Eqn (26), the thermal hole concentration distribution in the n-region is

$$p_n(x) = p_n + \Delta p_{th}(x) = p_n\left[1 - \exp\left(-\frac{x}{L_p}\right)\right], \quad (46)$$

which is the same as for photoholes uniformly generated over the n-region at the thermal generation rate p_n/τ_p [see Eqn (42) and the dashed line in Fig. 6]. In particular, by analogy with Eqn (40), Eqn (46) immediately gives

$$j_s = \frac{ep_n L_p}{\tau_p} = \frac{ep_n D_p}{L_p}$$

for the reverse saturation current of a $p^+ - n$ junction, implying that all the conclusions in the preceding subsection concerning photocurrent noise fully apply to the noise of the reverse saturation current of a $p^+ - n$ junction, the only difference being that $k_d = 1$ is assumed for thermal generation, following Refs [9 – 11].

Fluctuations in the recombination and diffusion of the thermal hole concentration $p_n(x)$ [Eqn (46)] are another source of current distribution noise. As a result of adding these noises with the reduced noise of the thermal hole generation in the n-region, the spectral noise density of the reverse-biased diode is independent of the frequency and corresponds to the shot noise of the reverse saturation current. For $\omega \gg 1/\tau_p$, the diode noise is, as before, dominated by the diffusion noise of the profile $p_n(x)$, and the low-frequency saturation current of the $p^+ - n$ junction determines the hole concentration gradient as $x \to 0$.

It therefore follows that optimizing the frequency filtration of photodiode signals requires that the high-frequency noise of a diode be cut off.

Reverse-biased semiconductor diodes and photodiodes are also used as shot noise generators, although high-frequency noise is in this case produced in a different way than, e.g., the noise of nondegenerate radiation.

In the absence of radiation, assuming a forward-biased $p^+ - n$ junction and for $\omega \gg 1/\tau_p$, the coefficient $(2a-1) \approx \sqrt{2\omega\tau_p} \gg 1$, and we obtain from Eqn (45) that

$$S_{th}(\omega) = 2e(j+j_s)\sqrt{2\omega\tau_p} \,, \quad (47)$$

that is, the noise current spectral density of a $p^+ - n$ junction increases with the frequency as $\sqrt{\omega}$, similarly to the inherent noise of a parameter distributed line. High-frequency noise is here caused by external and saturation currents running through the $p^+ - n$ junction in the transmission direction. We note that for a forward bias, as $x \to 0$, the steady-state concentration of holes in the n-region is determined by the sum of p_n and $\Delta p(x=0)$, Eqn (8) for $V > 0$, i.e., exceeds p_n and does not tend to zero (see the thick solid line in Fig. 6).

By analogy to the noise of an RC-distributed line [see Eqn (25)], Eqn (45) can be written in the form

$$S_{th}(\omega) = 2e(j+j_s) + 2e(j_s + j_{ph}) + 4e(j+j_s)(a-1)$$
$$= 2e(j+j_s) + 2e(j_s+j_{ph}) + 4kT(Y_a - Y)\,. \quad (48)$$

It follows from Eqn (48) that, as with a distributed parameter line, noise increase in the direct current of the external circuit and in the direct saturation current correlates with the magnitude of the high-frequency term added to the active conductance of a $p^+ - n$ junction.

Some of the holes (those that were injected into the n-region when forward currents passed through the $p^+ - n$ junction and which did not have enough time at high frequencies to diffuse into the n-region and to recombine there) diffuse to the $p^+ - n$ junction when it is biased in reverse and return to the p^+-region. Thus, at high frequencies, the active part of the diode conductance increases, and so does high-frequency noise.

In thermodynamic equilibrium (when there is no radiation and $V = 0$), $\Delta p(x) = 0$ and $p_n \neq f(x)$ in the n-region. It is well known, however, that the constant dark hole concentration p_n within the distance of the diffusion length from the space charge region is the result of thermodynamic balance between two factors, the outflow of holes to the p^+-region due to the driving field (just these holes form the backward saturation current), and the diffusion flux of holes (of the same magnitude and the opposite direction) from the heavily doped p^+-region. The opposing currents cancel each other, but because the noise of both the forward and backward currents has fluctuations in the generation, recombination, and diffusion of thermal holes as their origin, and because it is exactly in the n-region where these fluctuations occur, these fluctuations show up as two identical noise currents in Eqns (44) and (45).

If there is no radiation ($j_{\text{ph}} = 0$) and the $p^+ - n$ junction is zero-biased, then, for $\omega \ll 1/\tau_{\text{p}}$, the noise current spectral density of an external-short-circuited diode can be expressed in accordance with Eqns (29) and (44) by the Nyquist formula

$$S_{\text{th}}(\omega) = 4ej_{\text{s}} = \frac{4kT}{r} \,,$$

which is valid for any element of an electric circuit with an active resistance r if the circuit is in thermal equilibrium with the environment. Hence, in thermodynamic equilibrium, the shot noise arising due to the diffusion – recombination transport of minority carriers in a $p^+ - n$ junction shows up as the thermal noise of the junction's active conductance. At the same time, Eqn (37) shows that diffusion-related thermal noise gives rise to shot noise in a $p^+ - n$ junction.

As is known, both shot noise and thermal noise are generally due to the random nature of diffusion scattering, which results, for example, in current carriers being random in their free-path directions and lengths (thermal noise), and which randomizes the times at which the carriers reach potential barriers (shot current noise), recombination centers (recombination flux noise), etc.

Thus, by systematically taking diffusion – relaxation processes in a semiconductor diode into account, it is possible not only to calculate its static, dynamic, and noise characteristics but also to show the common nature of thermal and shot noise.

With minority current carriers transported by diffusion – recombination processes, individual recombination or generation events involving minority carriers that create an external circuit current of the p – n diode make the same effective contribution to noise currents, no matter how far from the space charge region the minority carriers pass. For the backward current and the photocurrent, this immediately follows from the correlation between the noise level and the shot noise of the saturation current or photocurrent. For forward currents, this follows from the fact that the integrals

$$\int_0^{\infty} \exp\left(-\frac{x}{L_{\text{p}}}\right) \mathrm{d}\left(\frac{x}{L_{\text{p}}}\right) = \int_0^{\infty} \exp\left(-\frac{2a \pm 1}{L_{\text{p}}}\, x\right) \mathrm{d}\left(\frac{2a \pm 1}{L_{\text{p}}}\, x\right)$$

are equal when local forward currents or noise sources are summed in the external circuit.

We note that relations (44) and (45), without j_{ph}, as well as Eqns (47) and (48), date back to the work of van der Ziel. Finally, Neustroev and Osipov [18] showed that the spectral noise density of a reverse-biased and irradiated long p – n junction corresponds to the shot noise of the total current through the p – n junction with $k_{\text{d}} = 1$ and the photogeneration rate decreases exponentially away from the space charge.

6. Short $p^+ - n$ junction

The conclusions in the preceding section — that at high frequencies the thermal noise of a reverse-biased semiconductor $p^+ - n$ junction and its photonoise are due to the minority carrier diffusion noise in the n-region, and that they are identical to the shot noise of the dark current or of the constant photocurrent component, respectively — are valid for any value of the ratio w/L_{p}. This fact seems to have a positive effect on the agreement between the van der Ziel theory and the wide range of experimental data.

This section is concerned with the so-called $p^+ - n$ junction with $w_{\text{s}} \ll L_{\text{p}}$, a structure which is typical of thin-based semiconductor photodiodes with a low longitudinal resistance of the irradiated n-region. Although the diffusion hole current in a junction clearly remains one-dimensional, analyzing a thin-base $p^+ - n$ junction requires including the surface generation and recombination of holes on its contact-free surface, which is located at $x = w_{\text{s}}$ and constitutes most of the area of the $p^+ - n$ junction.

For $L/|\dot{L}_{\text{p}}| \ll 1$, we find from Eqn (18) that

$$\dot{Z}_{\text{out},\,1}(x = L) = -RL\left(1 - \frac{L^2}{3\dot{L}_{\text{p}}^2}\right).$$

If the recombination rate on the photosensitive surface is reasonably limited by $s \ll D_{\text{p}}/w_{\text{s}}$ or $w_{\text{s}}(R/R_{\text{l}}) \ll 1$ (for $D_{\text{p}} = 100$ cm^2 s^{-1} and $w_{\text{s}} = 10^{-4}$ cm, the surface recombination rate s is bounded by the value 10^6 cm s^{-1}), then for relatively low frequencies $\omega \ll 1/2\tau_{\text{tr}}(w_{\text{s}})$ (or $w_{\text{s}} \ll |\dot{L}_{\text{p}}|$, see Fig. 4), we obtain

$$\dot{Z}_{\text{in},\,2} = \frac{R}{(w_{\text{s}} - L)/\dot{L}_{\text{p}}^2 + R/R_{\text{l}}} \,. \qquad (49)$$

The short-diode analogues of Eqns (19) – (24) then become

$$i = -i_{\text{g}}\left[1 + \frac{L(w_{\text{s}} - L/2)}{\dot{L}_{\text{p}}^2} + L\,\frac{R}{R_{\text{l}}}\right]^{-1}, \qquad (50)$$

$$\dot{u}_{\text{ph},1}(x, L) = i_{\text{g}}Rx\left[1 - \frac{L(w_{\text{s}} - L/2) - x^2/6}{\dot{L}_{\text{p}}^2} - L\,\frac{R}{R_{\text{l}}}\right] \approx i_{\text{g}}Rx\,, \qquad (51)$$

$$\dot{u}_{\text{ph},2}(x, L) = i_{\text{g}}RL\left[1 - \frac{x(w_{\text{s}} - x/2) - L^2/6}{\dot{L}_{\text{p}}^2} - x\,\frac{R}{R_{\text{l}}}\right] \approx i_{\text{g}}RL\,, \qquad (52)$$

$$\overline{i^2} = \overline{i_{\text{N}}^2}\left[1 - \frac{L(2w_{\text{s}} - L)}{L_{\text{p}}^2} - 2L\,\frac{R}{R_{\text{l}}} - \omega^2\tau_{\text{tr}}^2(w_{\text{s}})\,\frac{L^2(2w_{\text{s}} - L)^2}{w_{\text{s}}^4}\right], \qquad (53)$$

$$i = \frac{e_{\text{g}}}{R}\left(\frac{w_{\text{s}} - L}{\dot{L}_{\text{p}}^2} + \frac{R}{R_{\text{l}}}\right), \qquad (54)$$

$$\overline{i^2} = \frac{\overline{e_{\text{N}}^2}}{R^2}\left[\left(\frac{w_{\text{s}} - L}{L_{\text{p}}^2} + \frac{R}{R_{\text{l}}}\right)^2 + \omega^2\tau_{\text{tr}}^2(w_{\text{s}})\,\frac{4(w_{\text{s}} - L)^2}{w_{\text{s}}^4}\right]. \qquad (55)$$

The steady-state I – V characteristic of a 'short diode' is expressed in terms of $\dot{Z}_{\text{in},\,2}$ [Eqn (49)] for $L = 0$,

$$i(x = 0) = \frac{\dot{u}(x = 0)}{\dot{Z}_{\text{in},\,2}(L = 0)} = \frac{V}{R}\left(\frac{w_{\text{s}}}{\dot{L}_L^2} + \frac{R}{R_{\text{l}}}\right)$$

and, given the parallels mentioned above between the distributed parameter line and the $p^+ - n$ junction, is written in the form

$$j = \frac{ep_n w_s}{\tau_p}\left(1+\frac{s\tau_p}{w_s}\right)\left[\exp\left(\frac{eV}{kT}\right)-1\right] = ep_n\left(\frac{w_s}{\tau_p}+s\right)\left[\exp\left(\frac{eV}{kT}\right)-1\right] = j_{ss}\left[\exp\left(\frac{eV}{kT}\right)-1\right]. \tag{56}$$

We note that the limitation imposed on the surface recombination rate, $s \ll D_p/w_s$, does not exclude the possibility of the short diode I–V characteristic being determined only by its surface conditions ($j_{ss} \approx ep_n s$ for $s > w_s/\tau_p$).

It is also easily shown, similarly to Section 5, that the specific differential conductance (per 1 cm^2 of the area of the p^+-n junction) is

$$\dot{G} = \frac{e}{kT}(j+j_{ss})\left(1+i\omega\,\frac{\tau_p}{1+s\tau_p/w_s}\right) = G\left(1+i\omega\,\frac{\tau_p}{1+s\tau_p/w_s}\right). \tag{57}$$

It follows from Eqn (57) that the differential conductance starts to increase with frequency at the angular frequencies $\omega \approx (1+s\tau_p/w_s)/\tau_p$. For s tending to zero, the characteristic frequency is, as in the long case, $\omega = 1/\tau_p$. But as s approaches D_p/w_s, this frequency already tends to a much larger value $1/2\tau_{tr}(w_s)$.

The distribution of the steady-state concentration of excess holes in the n-region of a p^+-n junction under a bias V is given by a relation similar to Eqn (52) for $L=0$:

$$\Delta p_{th}(x,V) = p_n\left[\exp\left(\frac{eV}{kT}\right)-1\right]\left[1-\frac{x(w_s-x/2)}{L_p^2}-x\,\frac{s}{D_p}\right]. \tag{58}$$

It follows from Eqns (56) and (58) that with the limitations on w_s and s imposed above, the excess concentration of dark holes differs little from $p_n[\exp(eV/kT)-1]$ over the entire n-region. Therefore, for a forward bias, the recombination rate is close to

$$\frac{p_n[\exp(eV/kT)-1]}{\tau_p}\,w_s$$

over the entire n-region, and to

$$p_n s\left[\exp\left(\frac{eV}{kT}\right)-1\right]$$

on the surface. In a reverse biased p^+-n diode, $\Delta p(x) \approx -p_n$, the dark current from the volume of the n-region is $(p_n/\tau_p)w_s$, and the current from the surface is $p_n s$.

Simply substituting $eg\Delta L$ for i_g in Eqn (50) gives the p^+-n junction photocurrent in the case where the n-region is irradiated locally by a narrow strip (projected at a distance $L \leqslant w_s$ from and parallel to the boundary with the space charge region) and the electron–hole pair generation intensity is g ($cm^{-3}\,s^{-1}$).

For $\omega \ll 1/2\tau_{tr}(w_s)$, the absolute photocurrent in the local irradiation case is given by

$$|j_{ph}| = eg\Delta L\left[1-\frac{L(w_s-L/2)}{L_p^2}-L\,\frac{s}{D_p}\right] \times\left[1+\omega^2\,\frac{L^2(2w_s-L)^2}{4D_p^2}\right]^{-1/2}. \tag{59}$$

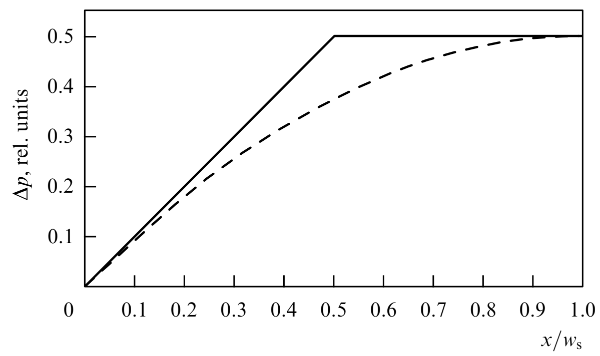

Figure 7. Distribution of steady-state photohole concentrations in the n-region of a short p^+-n junction, obtained by neglecting losses to bulk and surface recombination. Solid line: photohole concentrations $\Delta p_{ph,1}(x, L=w_s/2)$ and $\Delta p_{ph,2}(x, L=w_s/2)$ for local irradiation at the point $x=w_s/2$, in units of $gw_s\Delta L/D_p$. Dashed line: photohole concentration $\Delta p_{ph}(x)$ for uniform irradiation in units of gw_s^2/D_p.

Replacing $i_g R$ by $g\Delta L/D_p$ in Eqns (51) and (52) then yields the distributions of the excess hole concentrations in the n-region, i.e., $\Delta p_{ph,1}(x,L)$ and $\Delta p_{ph,2}(x,L)$, respectively (solid line in Fig. 7).

We also calculate the low-frequency $[\omega \ll 1/2\tau_{tr}(w_s)]$ photocurrent in the external circuit of the diode in the case where carriers are photogenerated uniformly with intensity g over the entire volume of the short n-region. We find

$$j_{ph} = -eg\int_0^{w_s}\left[1-\frac{L(w_s-L/2)}{L_p^2}-L\,\frac{s}{D_p}\right]dL = -egw_s\left[1-\frac{w_s^2}{3L_p^2}-\frac{w_s s}{2D_p}\right]$$

or

$$|j_{ph}| = egw_s\left[1-\frac{w_s^2}{3L_p^2}-\frac{w_s s}{2D_p}\right]\left[1+\omega^2\left(\frac{w_s^2}{3D_p}\right)^2\right]^{-1/2}, \tag{60}$$

and the hole distribution in the short diode (dashed line in Fig. 7) can be written as

$$\Delta p_{ph}(x) = \int_0^x \Delta p_{ph,2}(x,L)\,dL + \int_x^{w_s}\Delta p_{ph,1}(x,L)\,dL = \frac{g}{D_p}\,x\left(w_s-\frac{x}{2}\right)\left[1-\frac{8w_s^3+12w_s x^2-15x^3}{12L_p^2(w_s-x)} - \frac{s}{D_p}\,\frac{w_s^2}{2w_s-x}\right] \approx \frac{g}{D_p}\,x\left(w_s-\frac{x}{2}\right). \tag{61}$$

Unlike the noise of opposite saturation currents (see the discussion in the preceding section), noise sources for surface generation and recombination are not included in the distributed parameter equivalent line. In what follows, we assume that the generation of minority carriers from surface levels in a reverse-biased p^+-n junction and their recombination via surface levels in a forward-biased or irradiated junction are also accompanied by shot noise.

The table lists expressions, obtained using Eqns (9), (10), (53), (55), (59), and (60), for the low-frequency photocurrent and its shot noise, as well as for the individual components of short diode photonoise, due to the reduced noise of the

Table. Photocurrents and noise spectral densities of short p^+-n diodes at low frequencies [$\omega \ll 1/2\tau_{tr}(w_s)$].

Irradiation	Absolute photocurrent $\lvert j_{ph}\rvert$	Noise spectral densities in the external circuit of a p^+-n junction			
		Photocurrent shot noise	Reduced radiation noise	Bulk recombination noise	Surface recombination noise
Strip at a distance $L \leqslant w_s$	$eg\Delta L\left[1-\frac{L(2w_s-L)}{2L_p^2}-\frac{Ls}{D_p}\right]$	$2e^2g\Delta L\left[1-\frac{L(2w_s-L)}{2L_p^2}-\frac{Ls}{D_p}\right]$	$2e^2g\Delta Lk_d \times\left[1-\frac{L(2w_s-L)}{L_p^2}-2\frac{Ls}{D_p}\right]$	$2e^2g\Delta L\,\frac{L(2w_s-L)}{2L_p^2}$	$2e^2g\Delta L\,\frac{Ls}{D_p}$
Uniform	$egw_s\left[1-\frac{w_s^2}{3L_p^2}-\frac{w_s s}{2D_p}\right]$	$2e^2gw_s\left[1-\frac{w_s^2}{3L_p^2}-\frac{w_s s}{2D_p}\right]$	$2e^2gw_sk_d\left[1-\frac{2}{3}\frac{w_s^2}{L_p^2}-\frac{w_s s}{D_p}\right]$	$2e^2g\,\frac{w_s^3}{3L_p^2}$	$2e^2g\,\frac{w_s^2 s}{2D_p}$

photon flux and to the bulk and surface recombination noise.

Bulk recombination noise can be calculated to the required level of accuracy by using approximate relations (51), (52), and (61). The effect of diffusion noise in a short diode at low frequencies can be ignored altogether: the transmission coefficient of diffusion fluctuations in local concentration to the external circuit decreases by the factor $(w_s/L_p)^2$ as $s \to 0$ because holes in the n-region recombine and are thermally generated at a smaller distance than the diffusion length in the long diode.

In the approximations adopted above, the low-frequency noise of a diode at $k_d = 1$ always corresponds to the shot noise of the constant photocurrent component. For $k_d > 1$, the spectral density of reduced photogeneration noise increases by the factor k_d.

Clearly, in the low-frequency range, dark current noise is also determined by the shot noise ratio

$$S_{th}(\omega) = 2e^2 p_n\left(\frac{w_s}{\tau_p}+s\right)\exp\left(\frac{eV}{kT}-1\right). \tag{62}$$

For high frequencies, $\omega \gg 1/2\tau_{tr}(w_s)$ or

$$w_s \gg |\dot{L}_p| = \sqrt{\frac{2D_p}{\omega}}\left|\frac{1}{1+i}\right| = \sqrt{\frac{D_p}{\omega}}$$

(see Fig. 4), the collection of photocurrent and thermal and photoinduced noise in a short diode is determined by long-diode relations (19) and (22)–(24). The absolute local photocurrent can be represented as

$$|j_{ph}| = \left|-eg\Delta L\exp\left(-\frac{L}{\dot{L}_p}\right)\right| = eg\Delta L\exp\left(-\sqrt{\omega\tau_{tr}(L)}\right), \tag{63}$$

and the spectral density of the reduced photocurrent noise normalized to the external circuit is given by

$$S_{w,ph}(\omega) = 2e^2 g\Delta L k_d\exp\left(-2\sqrt{\omega\tau_{tr}(L)}\right). \tag{64}$$

To avoid cumbersome calculations, we first consider the simpler case of a short diode with no bulk or surface recombination. This approximation corresponds with the discussion in Ref. [7] and, according to Eqns (59) and (60), occurs for $L \ll w_s$. In the absence of recombination, the photohole concentration is given by simplified equations (51), (52), and (61), and the inherent photoinduced noise of a p^+-n junction (i.e., diffusion noise normalized to the external circuit) is given by

$$\begin{aligned} S_{s,ph}(\omega) &= \int_0^L 4\,\frac{g\Delta Lx}{D_p^2}\,e^2\omega D_p\exp\left(-\frac{2ax}{L_p}\right)dx \\ &+ \int_L^{w_s} 4\,\frac{g\Delta LL}{D_p^2}\,e^2\omega D_p\exp\left(-\frac{2ax}{L_p}\right)dx \\ &\approx 2e^2g\Delta L\left[1-\exp\left(-2\sqrt{\omega\tau_{tr}(L)}\right)\right]. \end{aligned} \tag{65}$$

For $\omega \gg 1/2\tau_{tr}(L)$, in a short diode, the coefficient for recalculating the diffusion fluctuations in the hole concentration to currents is $e^2\omega D_p$, as in the long diode.

There are two extreme cases to consider:

(1) $\omega\tau_{tr}(L) \ll 1$, i.e., the distance from the radiation strip to the space charge region is much smaller than w_s. In this case,

$$|j_{ph}| = eg\Delta L\left(1-\sqrt{\omega\tau_{tr}(L)}\right) \approx eg\Delta L,$$

$$S_{w,ph}(\omega) = 2e^2g\Delta Lk_d\left(1-2\sqrt{\omega\tau_{tr}(L)}\right) \approx 2e^2g\Delta Lk_d,$$

$$S_{s,ph}(\omega) = 2e^2g\Delta L\,2\sqrt{\omega\tau_{tr}(L)} \ll 2e^2g\Delta L;$$

the photocurrent and photoinduced noise due to the reduced radiation noise turn out to be practically the same as those for low frequencies.

(2) $\omega\tau_{tr}(L) \gg 1$. Then,

$$|j_{ph}| = \frac{eg\Delta L}{\exp\left(\sqrt{\omega\tau_{tr}(L)}\right)} \ll eg\Delta L,$$

$$S_{w,ph}(\omega) = \frac{2e^2g\Delta Lk_d}{\exp\left(2\sqrt{\omega\tau_{tr}(L)}\right)} \ll 2e^2g\Delta L,$$

$$S_{s,ph}(\omega) \approx 2e^2g\Delta L.$$

For $k_d = 1$, the photoinduced noise of a short p^+-n junction with no loss to recombination at low or high frequencies is equal to the shot noise of the constant component of the photocurrent. For $k_d > 1$, there are two noise plateaus, with the spectral density the low-frequency one being k_d times the spectral density of the high-frequency one. These results agree with the conclusions in Ref. [7].

To assess the influence of bulk and surface recombination in a short p^+-n junction, it is of interest to consider the case $\omega\tau_{tr}(L) \gg 1$. The second integral in Eqn (65) can then be neglected, and using Eqns (51) yields

$$S_{\mathrm{s,ph}}(\omega) = \int_0^L 4\,\frac{g\Delta L x}{D_{\mathrm{p}}^2}\left\{x\left[1-\frac{L(w_{\mathrm{s}}-L/2)}{L_{\mathrm{p}}^2}-\frac{Ls}{D_{\mathrm{p}}}\right]+\frac{x^3}{3L_{\mathrm{p}}^2}\right\} \times e^2\omega D_{\mathrm{p}}\exp\left(-\frac{2ax}{L_{\mathrm{p}}}\right)\mathrm{d}x = 2e^2 g\Delta L\left[1-\frac{L(w_{\mathrm{s}}-L/2)}{L_{\mathrm{p}}^2}-\frac{Ls}{D_{\mathrm{p}}}\right]. \quad (66)$$

In the presence of bulk and surface recombination, the photoinduced noise of a locally irradiated short $p^+ - n$ junction remains white in the high-frequency range $[\omega \gg 1/2\tau_{\mathrm{tr}}(w_{\mathrm{s}})]$, corresponding, as before, to the shot noise of the constant photocurrent component.

A similar situation occurs for a short $p^+ - n$ junction under uniform irradiation conditions. In exactly the same way, the high-frequency thermal noise of a reverse-biased $p^+ - n$ junction is also white and expressed by relation (62).

Thus, the basic conclusions regarding the physical sources and formation mechanisms of noise currents, formulated for long diodes, remain valid for the short $p^+ - n$ junction — remembering only the considerable reduction in the time of flight of minority carriers from their place of generation to the space charge region. Notably, the presence of a surface less than the diffusion length away from the space charge region does not prevent white noise formation if the generation and recombination of current carriers on the surface are Poisson processes.

For both long and short diodes, it is found that at low frequencies, the noise from the thermal and photogeneration of minority carriers, reduced due to bulk and surface recombination, adds to the current distribution noise that arises in the $p^+ - n$ junction. As a result, dark current shot noise and (if the radiation degeneracy coefficient k_{d} is unity) the constant photocurrent component appear in the external circuit of the $p^+ - n$ junction. At high frequencies, the spectral density of external circuit noise in a reverse-biased $p^+ - n$ transition remains the same as that in the low-frequency range, although here the only source of noise is the diffusion component of the current distribution noise.

If the radiation degeneracy coefficient is greater than unity, then the spectral density of the low-frequency photoinduced noise increases by the factor k_{d}, but the high-frequency noise remains unchanged.

Apparently, the situation is to remain unchanged by adding an $n - n^+$ contact to the short n-region, if the hole current through the contact is accompanied by shot noise. However, the influence of various types of contacts on the noise characteristics of short $p^+ - n$ junctions requires further study, even if the area of the contact is small compared to that of the $p^+ - n$ junction.

7. Conclusion

Photoinduced noise in a semiconductor $p^+ - n$ junction has been calculated as a function of coordinates and frequency with the bulk and surface recombination of photocarriers generated in the electrically neutral n-region under local irradiation taken into account.

It is found that the physical sources of current distribution noise in semiconductor $p^+ - n$ junctions are fluctuations in the local recombination and diffusion rates of holes that form steady-state distribution profiles as they flow away diffusively from their generation place in the n-region.

A physical mechanism is identified whereby a semiconductor $p^+ - n$ junction produces high-frequency thermal noise and photoinduced noise equal to the shot noise of the dark current and the constant photocurrent component, respectively. This noise exists when the hole (photohole) concentration increases linearly from the space charge region into the interior of the n-region, and results from the mutual compensation of the frequency dependences of diffusion noise currents, on the one hand, and the effective length for collecting noise from the n-region, on the other.

Photoinduced noise at low frequencies is close to the radiation noise transformed by the $p^+ - n$ junction, but at high frequencies, photoinduced noise forms in the $p^+ - n$ junction itself following the absorption of the radiation.

A change in the noise formation mechanism in passing to higher frequencies is apparently a general feature, occurring not only in semiconductor $p - n$ junctions but also in other information transformers that are inherently inertial and noisy.

Understanding the sources and formation mechanism of inherent noise is apparently essential for the design of low-noise semiconductor structures, including multilayer ones. This makes it even more surprising that over half a century after van der Ziel's publications, despite the ever increasing interest in ultra-low noise electronic and optoelectronic devices, no knowledge has been gained of the physical sources of current distribution noise or of the mechanisms underlying the formation of white noise in semiconductor $p - n$ junctions.

Acknowledgements
The author is grateful to A M Filachev for facilitating this research and to M A Trishenkov for helpful discussions.

References

1. Kurbatov L N *Optoelektronika Vidimogo i Infrakrasnogo Diapazonov Spektra* (Optoelectronics in the Visible and Infrared) (Moscow: Izd. MFTI, 1999) p. 147
2. Rogalski A *Infrared Detectors* (Electrocomponent Sci. Monographs, Vol. 10) (Amsterdam: Gordon & Breach, 2000) [Translated into Russian (Novosibirsk: Nauka, 2003) p. 202]
3. Van der Ziel A *Noise in Measurements* (New York: Wiley, 1976) [Translated into Russian (Moscow: Mir, 1979)]
4. Luk'yanchikova N B *Fluktuatsionnye Yavleniya v Poluprovodnikakh i Poluprovodnikovykh Priborakh* (Fluctuation Phenomena in Semiconductors) (Moscow: Radio i Svyaz', 1990)
5. Spescha G, Strutt M J O *Sci. Electrica* **4** 121 (1959)
6. Rabinovich A I *Zh. Eksp. Teor. Fiz.* **54** 239 (1968) [*Sov. Phys. JETP* **27** 128 (1968)]
7. Rabinovich A I *Fiz. Tekh. Poluprovodn.* **3** 424 (1969)
8. Rabinovich A I, Trishenkov M A, USSR Author's Certificate No. 1348241/26-25 Priority of 9.07.1969
9. Van der Ziel A *Proc. IRE* **43** 1639 (1955)
10. Van der Ziel A *Proc. IRE* **45** 1011 (1957)
11. Van der Ziel A *Fluctuation Phenomena in Semi-conductors* (New York: Academic Press, 1959) [Translated into Russian (Moscow: IL, 1961)]
12. Kozel S M, Kolachevskii N N, Noginov A M *Radiotekh. Elektron.* **11** 1616 (1966)
13. Shockley W *Bell Syst. Tech. J.* **28** 435 (1949)
14. Buckingham M J *Noise in Electronic Devices and Systems* (Chichester: E. Horwood, 1983) [Translated into Russian (Moscow: Mir, 1986) p. 75]

15. Pikus G E *Osnovy Teorii Poluprovodnikovykh Priborov* (Fundamentals of the Theory of Semiconductor Devices) (Moscow: Nauka, 1965)
16. Buckingham M J, Foulkner E A *Radio Electron. Eng.* **44** 125 (1974)
17. Robinson F N H *Noise and Fluctuations in Electronic Devices and Circuits* (Oxford: Clarendon Press, 1974) [Translated into Russian (Moscow: Atomizdat, 1980)]
18. Neustroev L N, Osipov V V *Fiz. Tekh. Poluprovodn.* **15** 2186 (1981)