#### METHODOLOGICAL NOTES

## High-frequency electric field amplification in a magnetized plasma

A V Timofeev

DOI: 10.1070/PU2006v049n11ABEH006057

### Contents

1. Introduction	1197
2. HF fields in a vacuum chamber	1198
3. Plasma effect on HF fields	1200
4. Simplified model of the TE mode	1201
5. Conditions at the ends of an ICR-heating system	1203
6. Conclusion	1205
References	1205

<u>Abstract.</u> In the investigation of cyclotron ion heating in systems designed for plasma isotope separation, the high-frequency (HF) electric field amplification effect was found to occur in equilibrium plasma. In the present article this effect is treated as a result of the interaction of the plasma placed in a constant external magnetic field with the HF modes of the vacuum chamber. Consistent elaboration of this approach allowed obtaining a clear interpretation of the HF electric field amplification effect and constructing a simple model of HF field excitation in a plasma column embedded in the external magnetic field.

#### 1. Introduction

Plasmas are characterized by the tendency of getting screened from both constant and alternating electric and magnetic fields. However, there are exceptions to this rule. For instance, electromagnetic fields may be amplified during propagation toward the interior of a plasma with a nonequilibrium charged-particle velocity distribution. Filling a bounded volume with the plasma may lead to the amplification of externally excited electromagnetic fields at resonance with the plasma eigenmodes (magnetoacoustic resonance, resonance with Gould-Trievelpiece waves, etc.). This note discusses one more possibility for the amplification of electromagnetic waves by plasma with an equilibrium charged-particle velocity distribution, which was discovered in the investigation of ion cyclotron resonance heating (ICR heating) of plasma placed in a constant external magnetic field. It has been largely due to this effect that it has been possible to achieve isotope separation by the ICR technique (see, for instance, Ref. [1]).

A V Timofeev Russian Research Centre 'Kurchatov Institute', pl. Kurchatova 1, 123182 Moscow, Russian Federation Tel. (7-495) 1969183 E-mail: avtim@nfi.kiae.ru

Received 26 January 2006 Uspekhi Fizicheskikh Nauk **176** (11) 1227–1236 (2006) Translated by E N Ragozin; edited by A Radzig This effect was supposedly discussed for the first time by Romesser et al. [2], who gave its qualitative interpretation. It owes its origin to the plasma response to the longitudinal (along the basic magnetic field) component  $E_{\parallel,ex}$  of the electric field excited by HF antennas. Due to high electron mobility along the basic magnetic field, even a low-density plasma is efficiently screened from this component. When the external HF field is nonuniform in the transverse direction, the redistribution of the plasma electric charge may give rise to an appreciable transverse field.

This field amplification effect was considered in greater detail by Compant La Fontaine and Pashkovsky [3] (see also Ref. [4]). The analysis in these papers was based on the notion of plasma oscillation mode - the combination of electric and magnetic fields described by the independent solution of the system of Maxwell equations. In the plasma parameter range typical for ICR ion separation systems there are two modes, which are referred to as fast and slow in Refs [3, 4]. The transverse electric field prevails in the fast mode. When excited at the plasma boundary, it relatively rapidly penetrates deep into the plasma. This mode comprises Alfvén and magnetoacoustic oscillations. The slow mode possesses a substantial longitudinal electric field and is characterized by lower velocities of propagation transverse to the magnetic field. For a low plasma density, this mode is referred to as the Gould-Trievelpiece mode. With increasing density it turns into lower-hybrid oscillations.

The HF antennas used in ICR heating systems excite both modes. In a vacuum, these modes are phased in such a way that their relatively strong transverse electric fields largely compensate each other. The screening of the slow mode 'releases' the transverse electric field of the fast mode. Calculations carried out in Ref. [3] showed that the amplification effect, for instance, in the conditions of the French ERIC facility, shows up even for a plasma number density  $n_0 \approx 10^5 - 10^6$  cm<sup>-3</sup>. Up to a density  $n_0 \approx 10^{12}$  cm<sup>-3</sup>, the amplification factor remains constant at a level of  $K \approx 10$ . In a narrow density range  $n_0 \approx 10^{12} - 10^{13}$  cm<sup>-3</sup>, the amplification coefficient initially rises by about a factor of 2.5 after which the regime of vacuum field amplification by the plasma is abruptly replaced with attenuation. The HF electric fields in a magnetized plasma column were experimentally investigated at the SIRENA facility [5]. It was found that the plasma does amplify the transverse electric field.

In earlier theoretical papers [6-8], attention was drawn to the following fact: in a relatively broad plasma density range corresponding to the 'plateau' in the  $E_{\perp}(n_0)$  dependence, which was obtained in Ref. [3], the plasma has only a slight effect on the fast mode. In this interval, the fast mode coincides with the TE mode of the cylindrical vacuum chamber, whose electric field is oriented transversely  $(E_{\parallel} = 0)$  to the chamber axis. (In ICR ion separation systems, the basic, stationary magnetic field is directed along the axis.) Another independent mode of the vacuum chamber is the TM mode, which the slow mode turns into. The longitudinal component of the HF magnetic field of this mode is equal to zero. Both the TE and TM modes possess appreciable potential components of the transverse electric field. However, when the HF fields are inductively excited (by current antennas) in a vacuum, the potential components compensate each other. As a result, there remains a relatively weak vortex electric field. The charge separation in plasma, whereby the TM mode is screened, restores the vacuum field of the TE mode. The potential component of the vacuum modes prevails over the vortical one when  $k_{||}r_A \leq 1$ , where  $r_A$  is the antenna radius. The results of Refs [6-8] suggest that in the limiting case  $k_{\parallel}r_{\rm A} \ll 1$  the amplification factor is  $K \approx (k_{\parallel}r_{\rm A})^{-2}$  to an order of magnitude.

The systematic statement of the approach elaborated in Refs [6-8] is the aim of this work. The complete picture has been obtained for HF field excitation in a plasma column embedded in an external magnetic field. In particular, the boundary conditions at the ends of the plasma column were established, which should be used in the determination of the longitudinal structure of the HF fields.

The approach taken in Refs [6-8] is valid provided  $N_{||}^2 \gg \varepsilon_{\perp}$ , where  $N_{||}$  is the longitudinal component of the refractive index, and  $\varepsilon_{\perp}$  is the transverse permittivity of the plasma. In ICR ion separation systems, this condition is violated when the plasma density exceeds the limiting value  $n_0 \approx 10^{12-14} \text{ cm}^{-3}$ . The exact value of the limiting density depends on the frequency of the HF field, the composition of the plasma ion component, etc. When it is exceeded, the plasma influence modifies the vacuum TE mode: it transforms into Alfvén or magnetoacoustic oscillations. The resonance with the intrinsic Alfvén oscillations of the plasma column gives rise to a peak in the dependence of gain on plasma density (see Ref. [3]). Upon a further increase in plasma density, however, the transparency region of the Alfvén oscillations shifts to the plasma periphery, and therefore the electric field decreases at the center of the plasma column. As regards the magnetoacoustic oscillations, because of the specific character of their polarization they can be employed for ICR heating only when the principal and heated ions differ greatly in concentrations and masses (the impurity ion method).

#### 2. HF fields in a vacuum chamber

We analyze the excitation of HF electromagnetic fields in a cylindrical vacuum chamber and restrict ourselves to the longitudinal direction. The chamber walls are assumed to be perfectly conducting.

The geometry of current antennas commonly employed for ICR plasma heating is such that they simultaneously excite both the TE and TM modes. In a vacuum, the electric field of the TE mode is directed transversely  $(E_{\parallel} = 0)$  to the chamber axis; the same is true of the magnetic field orientation of the TM mode  $(B_{\parallel} = 0)$ . That is why it is convenient to describe the TE mode by the longitudinal component  $B_{\parallel}$  of the alternating magnetic field, and the TM mode by the electric field component  $E_{\parallel}$ . Hereinafter, the subscripts 'parallel' and 'perpendicular' indicate the field orientation relative to the system's axis.

The TE mode is excited by the transverse electric current, and the TM mode by the longitudinal current, as well as by the charge, provided its density is modulated in the longitudinal direction. These processes are described by the equations (see, for instance, Refs [6-9])

$$\hat{L}B_{\parallel} = -\frac{4\pi}{c} \frac{1}{r} \frac{\mathrm{d}}{\mathrm{d}r} r j_{\theta} , \qquad (1)$$

$$\hat{L}E_{||} = -\frac{4\pi i\omega}{c^2} \left( j_{||} - \frac{c^2 k_{||}}{\omega} \rho \right), \qquad (2)$$

where

$$\hat{L} = \frac{1}{r} \frac{d}{dr} r \frac{d}{dr} - \frac{m^2}{r^2} - k_{||}^2 + \frac{\omega^2}{c^2}.$$

In Eqns (1) and (2) use is made of the cylindrical coordinate system with the 0Z-axis aligned with the axis of the vacuum chamber. The chamber is assumed to possess axial and azimuthal symmetry, and the alternating quantities are therefore expanded into the Fourier series in azimuth  $\theta$  (*m* is the azimuthal wave number) and longitudinal coordinate *z* ( $k_{\parallel}$  is the axial longitudinal wave number). The antennas employed for ICR heating are inductive (current-excited), with  $\rho = 0$  in them. They consist of conductors located on some cylindrical surface ( $r = r_A$ ), and the radial component of electric current is therefore absent in Eqns (1) and (2).

The transverse components of electric and magnetic fields are expressed by the following relations in terms of  $B_{\parallel}$  and  $E_{\parallel}$ :

$$\mathbf{E}_{\perp} = \frac{\mathbf{i}}{1 - N_{\parallel}^2} \frac{c}{\omega} \left( \nabla \times (\mathbf{b}B_{\parallel}) + N_{\parallel} \nabla_{\perp} E_{\parallel} \right), \qquad (3)$$

$$\mathbf{B}_{\perp} = \frac{\mathbf{i}}{1 - N_{\parallel}^2} \frac{c}{\omega} \left( N_{\parallel} \nabla_{\perp} B_{\parallel} - \nabla \times (\mathbf{b} E_{\parallel}) \right), \tag{4}$$

where  $N_{||} = ck_{||}/\omega$ , and **b** is the unit vector aligned with the system's axis. The first terms in parentheses in relations (3) and (4) describe the transverse electromagnetic fields of the TE mode, and the second terms describe those of the TM mode.

ICR heating is due to the electric field component which is perpendicular to the basic magnetic field and has the same sense of rotation as the ions rotating in the Larmor circle (the left-polarized component  $E_+$ ). It is precisely this component that is responsible for systematic changes in Larmor gyration energy, when the cyclotron resonance condition  $\omega = \omega_i (\omega_i \text{ is}$ the ion cyclotron frequency) is fulfilled. At the same time, the right-polarized component  $E_-$  of the electric field gives rise, under resonance conditions, to merely ion energy oscillations with a frequency  $2\omega$ . For the temporal dependence of alternating quantities of the form  $\propto \exp(-i\omega t)$ , the circularly polarized components of the electric field are given by the expressions

$$E_{\pm} = \frac{1}{\sqrt{2}} \left( E_r \pm i E_{\theta} \right)$$
$$= \frac{i}{\sqrt{2}(1 - N_{\parallel}^2)} \frac{c}{\omega} \left( \pm \frac{d}{dr} - \frac{m}{r} \right) \left( B_{\parallel} \pm i N_{\parallel} E_{\parallel} \right).$$
(5)

Owing to the small thickness of current-antenna conductors, the radial distribution of current in them may be approximated by the  $\delta$  function. In the region inside the antenna ( $r < r_A$ ), we obtain the following expressions from Eqns (1) and (2):

$$B_{||} = -\frac{\rho_{\rm A}}{I'_{\rm m}(\rho_{\rm B})} \, \Phi_{\rho_{\rm A},\rho_{\rm B}}''(\rho_{\rm A},\rho_{\rm B}) \, I_{\rm m}(\rho) \, \frac{4\pi}{c} \, J_{\theta} \,, \tag{6}$$

$$E_{||} = \frac{i}{\sqrt{N_{||}^2 - 1}} \frac{\rho_{\rm A}}{I_{\rm m}(\rho_{\rm B})} \Phi(\rho_{\rm A}, \rho_{\rm B}) I_{\rm m}(\rho) \frac{4\pi}{c} J_{||} , \qquad (7)$$

where  $\mathbf{J} = (0, J_{\theta}, J_{\parallel})$  is the surface current density in the antenna,  $I_{\rm m}$  is the modified Bessel function,  $K_{\rm m}$  is the Macdonald function,  $\rho = (r\omega/c)\sqrt{N_{\parallel}^2 - 1}$ ,  $\Phi(\rho_{\rm A}, \rho_{\rm B}) = I_{\rm m}(\rho_{\rm A}) K_{\rm m}(\rho_{\rm B}) - I_{\rm m}(\rho_{\rm B}) K_{\rm m}(\rho_{\rm A})$ , and  $r_{\rm B}$  is the radius of the vacuum chamber.

In the practically significant wave spectrum domain employed for ICR heating, the characteristic longitudinal wavelength is far greater than the transverse dimension of the chamber and at the same time is small in comparison to the wavelength in a vacuum, so that the following inequalities are valid:  $N_{||} \ge 1$ ,  $k_{||}r_A \ll 1$  (ordinarily,  $N_{||} \approx 10^3 - 10^4$ , and  $k_{||}r_A \approx 0.1 - 0.3$ ).

In these approximations, from expressions (6) and (7) we obtain

$$B_{||} \approx \frac{|m|}{2} \left(\frac{r}{r_{\rm A}}\right)^{|m|} \left[ \left(\frac{r_{\rm A}}{r_{\rm B}}\right)^{2|m|} - 1 \right]$$
$$\times \frac{4\pi}{c} J_{\theta} \left[ 1 + O_1 \left( (k_{||}r_{\rm A})^2 \right) \right], \tag{8}$$

$$E_{||} \approx \frac{\mathrm{i}r_{\mathrm{A}}\omega}{2c} \left(\frac{r}{r_{\mathrm{A}}}\right)^{|m|} \left[ \left(\frac{r_{\mathrm{A}}}{r_{\mathrm{B}}}\right)^{2|m|} - 1 \right]$$
$$\times \frac{4\pi}{c} J_{||} \left[ 1 + O_2\left((k_{||}r_{\mathrm{A}})^2\right) \right], \tag{9}$$

$$E_{\pm} \approx -\frac{\mathrm{i}\pi\sqrt{2}}{\omega N_{||}^{2}} \left(\frac{r}{r_{\mathrm{A}}}\right)^{|m|-1} \left[ \left(\frac{r_{\mathrm{A}}}{r_{\mathrm{B}}}\right)^{2|m|} - 1 \right] (\pm |m| - m) \\ \times \left\{ \frac{|m|}{r_{\mathrm{A}}} J_{\theta} \left[ 1 + O_{1} \left( (k_{||}r_{\mathrm{A}})^{2} \right) \right] \mp k_{||} J_{||} \left[ 1 + O_{2} \left( (k_{||}r_{\mathrm{A}})^{2} \right) \right] \right\}.$$
(10)

In a plasma-filled chamber, the plasma density is highest on the axis (r = 0). Here, the amplitude of the left-polarized component of the electric field is nonzero only for the first mode which travels in an azimuth in the direction of ion gyration in the magnetic field (m = -1), while the amplitude of the right-polarized component is nonzero only for the first mode traveling in the direction of electron gyration (m = 1).

For all azimuthal modes, in view of the antenna current continuity condition

$$\frac{m}{r_{\rm A}}J_{\theta} + k_{\parallel}J_{\parallel} = 0 \tag{11}$$

the zero-order terms of expansion in  $(k_{||}r_A)^2$  in expression (10) cancel out. In accordance with Eqns (1) and (2), the term in this expression that is proportional to the azimuthal current takes into account the TE-mode contribution, and the term proportional to the longitudinal current takes into account the TM-mode contribution. The resultant field is due to the fact that the transverse electric field of the TE mode in the first order in  $(k_{||}r_A)^2 \ll 1$  exceeds the field of the TM mode.

The main role in ICR heating is played by the leftpolarized electric field of the first azimuthal harmonic traveling in the direction of ion gyration. For this field at the chamber center, from expression (10) we obtain

$$E_{+} \approx \frac{\pi i}{\sqrt{2}} \left(\frac{r_{A}}{c}\right)^{2} \omega k_{\parallel} J_{\parallel} \left[1 - \left(\frac{r_{A}}{r_{B}}\right)^{2} - 4\ln\left(\frac{r_{A}}{r_{B}}\right)\right].$$
(12)

This integral field is  $K \approx (k_{||}r_A)^{-2} \gg 1$  times weaker in the modulus than the fields of the TE and TM modes taken separately, which are, correct to factors on the order of  $(k_{||}r_A)^2$ , equal to

$$E_{+} \approx \pm \frac{4\pi i}{\sqrt{2}} \frac{\omega}{k_{\parallel}c^{2}} J_{\parallel} \left[ 1 - \left(\frac{r_{\rm A}}{r_{\rm B}}\right)^{2} \right],\tag{13}$$

where the plus sign corresponds to the TE mode, and the minus sign to the TM mode.

The total field of the current antennas in a vacuum is vortical, while all main components of the transverse electric field of the TE and TM modes are inherently potential. For  $N_{||} \ge 1$ , the latter statement is an obvious corollary of expression (3), in which the second term defines the transverse electric field of the TM mode. The quantity  $(-i/k_{||}) E_{||}$  plays the part of electric potential in it. It should also be noted that Eqn (1) outside of the antenna is, under the assumption  $k_{||}r_A \ll 1$ , of the form  $\nabla_{||} \times \mathbf{E} = 0$ , which lets us treat the transverse electric field of the TE mode as a potential one.

The electromagnetic oscillations employed for ICR heating are relatively low in frequency. Vacuum chambers are evanescent waveguides for them. The spatial structure of the electric field of these oscillations is determined by antenna shape. With an increase in frequency, the propagation of electromagnetic oscillations along the waveguide becomes possible — these oscillations are natural. The spatial structure of the transverse electric field of cylindrical-waveguide eigenmodes shown in Fig. 1 confirms the thesis about the existence of a substantial potential component of the electric field of the TE and TM modes. Indeed, the electric field lines in Fig. 1 approach the surface of a perfectly conducting waveguide along the normal. This testifies to the presence of electric charges on the surface of the perfectly conducting



Figure 1. Electric field lines of the lowest radial eigenmodes of the vacuum waveguide (m = 1): (a) the TE<sub>11</sub> mode, and (b) the TM<sub>11</sub> mode.

waveguide, the potential component of the transverse electric field being due to these charges. It is pertinent to note that the TM-mode electric field is three-dimensional and its lines of force can change their inclination to the axis and, in particular, become parallel to it. In the vicinity of such a point, the projections of the lines of force onto the plane orthogonal to the axis terminate in Fig. 1b.

#### 3. Plasma effect on HF fields

We now analyze the plasma effect on the electric field of current antennas. The plasma is assumed to be placed in a longitudinal magnetic field. Owing to high electron longitudinal mobility, particularly active is the plasma interaction with the TM mode which possesses a substantial longitudinal electric field. At the same time, since  $E_{||} = 0$  for the TE mode, it experiences a weaker influence of the plasma. There is a broad plasma density range, which is approximately determined by the conditions  $\varepsilon_{||} \ge 1$  and  $N_{||}^2 \ge \varepsilon_{\perp}$ , in which the plasma influence significantly modifies the TM mode, while the TE mode still remains a 'vacuum' mode. (Here,  $\varepsilon$  is the plasma heating, this interval ranges from  $n_0 \approx 10^5 - 10^6$  to  $\approx 10^{12} - 10^{13}$  cm<sup>-3</sup>. The plasma influence results here in a substantial amplification of the total transverse electric field.

To understand the amplification mechanism, we consider the plasma oscillations near the lower boundary of the abovespecified plasma density interval. The differential equation taking into account the interaction of the longitudinal electric field of the TM mode with the plasma is of the form of Eqn (2) with a modified expression for the  $\hat{L}$  operator:

$$\hat{L} = \frac{1}{r} \frac{d}{dr} r \frac{d}{dr} - \frac{m^2}{r^2} - k_{||}^2 \varepsilon_{||} + \frac{\omega^2}{c^2},$$

in which

$$\mathbf{E}_{||} \approx \begin{cases} 1 + 2\left(\frac{\omega_{\mathrm{pe}}}{k_{||}v_{\mathrm{Te}}}\right)^{2} \left(1 + \mathrm{i}\sqrt{\pi} \, \frac{\omega}{k_{||}v_{\mathrm{Te}}}\right) & (\omega \ll k_{||}v_{\mathrm{Te}}), \\ \\ 1 - \left(\frac{\omega_{\mathrm{pe}}}{\omega}\right)^{2} & (\omega \gg k_{||}v_{\mathrm{Te}}), \end{cases}$$

where  $v_{Te} = (2T_e/m_e)^{1/2}$ , and  $\omega_{pe} = (4\pi e^2/m_e)^{1/2}$  is the electron plasma frequency.

In the vacuum chamber in the region inside the antenna  $(r < r_A)$ , one finds

$$E_{\parallel} \propto I_{\rm m} \left( \frac{r\omega}{c} \sqrt{N_{\parallel}^2 - 1} \right).$$

When the electrons are hot ( $\omega \leq k_{||}v_{\text{Te}}$ ), Re  $\varepsilon_{||} > 1$  and the plasma influence is equivalent to the increase in  $N_{||}$ . This causes a sharper decrease in  $|E_{||}|$  from the antenna towards the center of the plasma column. The absorption of electromagnetic energy, which is accounted for by the imaginary part of  $\varepsilon_{||}$ , has a similar effect. Due to the decrease in  $E_{||}$ , the difference between the contributions of the TE and TM modes to expression (5) increases, which leads to an increase in the total circularly polarized electric field.

In the case of cold electrons ( $\omega \ge k_{||}v_{\text{Te}}$ ), the plasma influence decreases the longitudinal dielectric response  $\varepsilon_{||}$  and, consequently, strengthens the longitudinal electric field of the TM mode. That is why for a very low plasma density the difference between the TE- and TM-mode contributions

to expression (5) decreases, resulting in a lowering of the total field. However, on further density increase, the transverse electric field of the TM mode becomes much higher than the transverse electric field of the TE mode. As a result, the total circularly polarized field strengthens. When the longitudinal dielectric plasma response becomes negative, the plasma waveguide is no longer evanescent for the TM modes — they turn into potential plasma oscillations (the Gould – Trievelpiece modes). If the antenna current parameters are close to the Gould – Trievelpiece eigenmode parameters, the transverse electric field of the TM mode is far stronger than the transverse field of the vacuum TE mode. In this case, the total transverse electric field is determined by the field of plasma waveguide eigenmodes.

It is pertinent to note that with an increase in ion mass, when the resonance HF-field frequency lowers, the progressively lower-density plasma comes to significantly affect the HF field. Simultaneously lowered is the temperature at which the electrons may be considered 'hot'.

These statements are borne out by the numerical solution of the wave equations which take into complete account the plasma influence both on the TE mode and on the TM mode (Figs 2, 3). These equations, as well as their comprehensive analysis, are reported in Ref. [7]. As in Figs 2 and 3, calculated in this work was the HF field excited in a plasma column by an



**Figure 2.** Electromagnetic fields of an individual axial harmonic of helical current for a low plasma density in the case of 'hot' electrons:  $T_e = 5 \text{ eV}$ ,  $T_i = 0.5 \text{ eV}$ ,  $\lambda_{||} = 10^2 \text{ cm}$ ,  $m_i = 200 m_p$ ,  $m_p$  is the proton mass,  $\omega = 5 \times 10^5 \text{ s}^{-1}$ ,  $\omega_i = \omega/2 \text{ s}^{-1}$ , the current amplitude is 1 A,  $r_A = 2.55 \times 10^{-4} \omega/c$ , and  $r_B = 3.7 \times 10^{-4} \omega/c$ , and  $r_{pl} = 2\Delta_{pl} = 0.85 \times 10^{-4} \omega/c$ ; I - vacuum,  $2 - n_0 = 10^2 \text{ cm}^{-3}$ ,  $3 - n_0 = 10^3 \text{ cm}^{-3}$ , and  $4 - n_0 = 10^4 \text{ cm}^{-3}$ .



**Figure 3.** The same as in Fig. 2 for  $10^2$  times higher frequencies ('cold' electrons); all dimensionless lengths are accordingly increased  $10^2$ -fold: *I* — vacuum, *2* —  $n_0 = 10^4$  cm<sup>-3</sup>, 3 —  $n_0 = 2 \times 10^4$  cm<sup>-3</sup>, and 4 —  $n_0 = 10^5$  cm<sup>-3</sup>.

individual Fourier harmonic of the helical current flowing through the antenna cylinder. The plasma density distribution was assumed to obey the law

$$n_0(r) = n_0(0) \left[ 1 - \tanh\left(\frac{r^2 - r_{\rm pl}^2}{2\Delta_{pl}r_{\rm pl}}\right) \right] \left[ 1 + \tanh\left(\frac{r_{\rm pl}}{2\Delta_{\rm pl}}\right) \right]^{-1}$$

The dependences shown in Fig. 2 were obtained on the supposition of 'hot' electrons ( $\omega \ll k_{\parallel} v_{Te}$ ), which is usually correct for ICR-heating systems. In this case, even a lowdensity plasma screens the TM mode in a thin surface layer. The characteristic scale of screening is, to the order of magnitude, equal to the electron Debye radius. The screening of the TM mode 'releases' the electric field of the TE mode. With an increase in plasma density, when the condition  $\varepsilon_{\perp} \ge 1$  is fulfilled, the plasma interaction with the transverse HF electric field also becomes significant. In this case, the scale of screening ceases to vary with plasma density. The scale length depends on such parameters as the ion composition and the ratio between the HF-field frequency and the ion cyclotron frequencies. At the same time, as long as the inequality  $N_{||}^2 \gg \varepsilon_{\perp}$  is valid, the plasma influence on the TE mode can be neglected.

Moving to the case of cold electrons ( $\omega \ge k_{\parallel}v_{Te}$ ), the HF-field frequency was increased 10<sup>2</sup>-fold, all other parameters remaining unchanged. In accordance with the aforesaid, initially with the increase in density the transverse electric field lowers (see Fig. 3). However, later on the excitation of the

Gould – Trievelpiece modes, which the TM mode turns into, becomes the governing factor. As this takes place, the electric field strengthens sharply.

#### 4. Simplified model of the TE mode

The phenomenon of TM-mode screening may be treated as its excitation in antiphase with the action of the longitudinal component of the electric current flowing through the antenna. (According to Eqn (2), only this component excites the TM mode.) When the distance from the plasma surface to the antenna is small, it may be crudely assumed that the screening layer coincides with the antenna cylinder. In this case, the mutual neutralization condition for the longitudinal current and the charge screening the TM mode should, as follows from Eqn (2), be of the form

$$j_{\parallel} = \frac{c^2 k_{\parallel}}{\omega} \rho \,. \tag{14}$$

Since we assume that both the longitudinal current and the charge are localized on the antenna surface  $r = r_A$  $(j_{\parallel} = J_{\parallel}\delta(r - r_A)$ , and  $\rho = \sigma\delta(r - r_A)$ ), a relationship similar to formula (14) should also hold for the surface densities of these quantities:

$$J_{\parallel} = \frac{c^2 k_{\parallel}}{\omega} \,\sigma \,. \tag{15}$$

It should be noted that the assumption about the localization of the electric charge neutralizing the TM mode on the antenna surface merely makes calculations easier without affecting the result. Indeed, the role of the charge simply consists in the 'reestablishment' of the electric field of the TE mode. This electric field is 'vacuumlike' in all those places where the TM mode does not penetrate.

According to the aforesaid, it is precisely the charge neutralizing the action of longitudinal current that is responsible for the potential electric-field component of the TE mode. Therefore, relationship (15) may also be obtained employing Eqns (1), (3), and (6). Indeed, the tangential components of the alternating magnetic field experience a discontinuity at the antenna surface, in particular, one has

$$B_{||}|_{r_{A^{-}}}^{r_{A^{+}}} = -\frac{4\pi}{c} J_{\theta} \,. \tag{16}$$

On the strength of relationship (3), the radial component of the electric field should also experience a discontinuity:

$$E_r|_{r_{A^-}}^{r_{A^+}} = \frac{1}{N_{\parallel}^2 - 1} \frac{c}{\omega} \frac{m}{r_A} B_{\parallel}|_{r_{A^-}}^{r_{A^+}}.$$
 (17)

In accordance with the Poisson equation, this requires that there be a charge with a density

$$\sigma = \frac{1}{1 - N_{\parallel}^2} \frac{1}{\omega} \frac{m}{r_{\rm A}} J_{\theta} \tag{18}$$

on the surface  $r = r_A$ .

Taking into account the current continuity condition (11), we find that in the limit  $N_{\parallel} \ge 1$  expression (18) coincides with expression (15). (To also reconcile corrections to expression (18), which are  $\propto N_{\parallel}^{-2}$ , account should be taken of the currents required to produce the charge, see below.) Therefore, by taking either of the independent equations [Eqn (1) or Eqn (2)] as the starting point of our analysis, we arrive at the same results. It should be noted that the Poisson equation employed in obtaining expression (18) is a corollary of the Maxwell equations and the charge conservation equation.

We take advantage of relationship (15) for an approximate calculation of the left-polarized component of the electric field excited in ICR-heating systems. As noted above, it is precisely this component that is responsible for ion heating. The simplest antenna employed in ICR-heating systems consists of two helical conductors located opposite each other on the antenna cylinder. The conductors make a half turn around the plasma column (a helical half-wave antenna). The ends of the helical conductors are bridged by circular conductors (see Fig. 4). The longitudinal current distribution in this antenna is defined by the expression

$$j_{||}(\mathbf{r},t) = \left[\delta\left(\theta - \frac{\pi}{2} - A\pi \frac{z}{L}\right) - \delta\left(\theta + \frac{\pi}{2} - A\pi \frac{z}{L}\right)\right] \\ \times \cos(\omega t) \frac{1}{r_{\rm A}} \,\delta(r - r_{\rm A}) \,I \quad (|z| < L) \,, \tag{19}$$

where *L* is one-half the antenna length, and *I* is the current in each of the conductors, which are assumed to be infinitely thin. In the half-wave antenna, one has A = 1/2.

In accordance with expressions (15) and (19), the screening charge distribution over the longitudinal coordinate is described by a discontinuous function which undergoes abrupt changes at the conductors. For a fixed z value, the longitudinal currents flowing through the conductors located on opposite sides of the antenna cylinder are opposite in sense, and so the screening charges are also opposite in sign. This establishes a transverse electric field. The left-polarized field on the axis of the plasma column is defined by the first azimuthal harmonic traveling in the direction of ion gyration (see above). We separate it out from expression (19):

$$j'_{||} = \sin\left(\theta + \omega t - A\pi \frac{z}{L}\right) \frac{1}{\pi r_{\rm A}} I\delta(r - r_{\rm A}).$$
<sup>(20)</sup>

Hereinafter, the prime indicates the quantities characterizing the azimuthal mode with m = -1.



Figure 4. Helical half-wave antenna (the arrows indicate the instantaneous direction of current).

The density of charge screening the TM mode is found from the differential analogue of Eqn (15):

$$\frac{\partial j'_{||}}{\partial t} = -c^2 \frac{\partial \rho'}{\partial z} \,. \tag{21}$$

In this case, we obtain

$$\rho' = f(t,\theta,z) \frac{1}{A\pi^2} \frac{\omega L}{r_{\rm A}c^2} I\delta(r-r_{\rm A}).$$
(22)

For convenience in the subsequent calculations, the factor which takes into account the spatio-temporal charge structure, namely

$$f(t, \theta, z) = \sin\left(\theta + \omega t - A\pi \frac{z}{L}\right),$$

is represented as

$$f(t, \theta, z) = \sin(\theta + \omega t) f_{\rm ev}(z) - \cos(\theta + \omega t) f_{\rm od}(z), \quad (23)$$

where the quantity  $f_{\rm ev}(z) = \cos(A\pi z/L)$  for |z| < L describes the structure of the z-coordinate-even part of the screening charge, and the quantity  $f_{\rm od}(z) = \sin(A\pi z/L)$  the odd part. They are displaced in phase  $\theta' = \theta + \omega t$  by  $\pi/2$  relative to each other.

Due to TM mode screening, the electric field amplification effect under discussion is significant for antennas sufficiently elongated along the magnetic field ( $L \gg r_A$ ). In this case, the longitudinal-coordinate dependences of the electric charge and potential are close to each other, and as an approximation it may be assumed that  $\varphi(z) \propto \rho'(z)$ , omitting the term  $\partial^2 \varphi / \partial z^2$  in the Poisson equation:

$$\frac{1}{r} \frac{\mathrm{d}}{\mathrm{d}r} r \frac{\mathrm{d}}{\mathrm{d}r} \varphi - \frac{1}{r^2} \varphi = -4\pi \rho'.$$
(24)

Here, as in the foregoing, the azimuthal wave number is taken to be m = -1.

The solution to Eqn (24) takes the form

$$\varphi = 2\pi\sigma' \frac{r_{\rm A}^2}{r_{\rm B}} \begin{cases} \frac{r}{r_{\rm B}} \left[ \left( \frac{r_{\rm B}}{r_{\rm A}} \right)^2 - 1 \right] & (r < r_{\rm A}), \\ \frac{r_{\rm B}}{r} - \frac{r}{r_{\rm B}} & (r > r_{\rm A}), \end{cases}$$
(25)

where

$$\sigma' = \frac{\omega LI}{A\pi^2 r_{\rm A} c^2} f_{\rm ev(od)}$$

is the surface density of the even (odd) part of the charge.

Using expression (25), we find that in the region inside the antenna ( $r < r_A$ )

$$E_{+}(z) = -\frac{1}{\sqrt{2}} \frac{\omega}{c} \left(\frac{\mathrm{d}}{\mathrm{d}r} + \frac{1}{r}\right) \varphi(r, z)$$
$$= -2\sqrt{2} \pi \sigma'(z) \left[1 - \left(\frac{r_{\mathrm{A}}}{r_{\mathrm{B}}}\right)^{2}\right]. \tag{26}$$

This formula relates the potential component of the leftpolarized electric field to the screening charge density, and thereby to the longitudinal electric current in the antenna. It is easily seen that formula (26) is equivalent to expression (13). Indeed, if we put [in accordance with formula (20)]

$$k_{||} = \frac{A\pi}{L} , \quad J_{||} = \frac{I}{\pi r_{\rm A}} ,$$

in expression (13), the latter will define the amplitude of the Fourier harmonic  $\propto \sin(\theta + \omega t - A\pi z/L)$  in expression (26).

By comparing the total vacuum field (12) with the field of the TE mode [see expression (13) or (26)], we find the amplification factor

$$K = \frac{1}{\left(k_{\parallel} r_{\rm A}\right)^2} \,\chi(q)\,,\tag{27}$$

where  $\chi(q) = 4(1-q^2)(1-q^2-4\ln q)^{-1}$ , and  $q = r_A/r_B$ ; if  $r_B - r_A \ll r_A$ , then  $\chi(q) \approx 4/3$ .

The amplification factor (27) can be given a simple qualitative interpretation. The electric field of the plasma charges is potential, and its components therefore obey the relation  $(E_{\perp}/E_{\parallel})_{\rm pl} = k_{\perp}/k_{\parallel}$ . In a vacuum, due to the low frequency of the processes under consideration, the inductive electric field may be considered as being directed along the helical conductors, and therefore its inclination to the basic magnetic field is equal approximately to  $(E_{\perp}/E_{\parallel})_{\rm vac} = \Theta r_{\rm A}/L$ , where  $\Theta$  is the conductor twist angle. By assuming that the electric field of the plasma charges compensates for the longitudinal vacuum field  $(|E_{\parallel,\rm pl}| = |E_{\parallel,\rm vac}|)$ , we obtain

$$K = \frac{E_{\perp, \text{pl}}}{E_{\perp, \text{vac}}} = \frac{\Theta r_{\text{A}} k_{\perp}}{L k_{\parallel}} .$$
<sup>(28)</sup>

There is good reason to believe that the quantities  $k_{\perp}$  and  $k_{\parallel}$  are equal approximately to  $r_A^{-1}$  and  $L^{-1}$ , respectively. Under these assumptions, expressions (27) and (28) are in good agreement with each other.

# 5. Conditions at the ends of an ICR-heating system

In the screening charge density  $\sigma'$  entering into expression (26) there appear the quantities  $f_{\text{ev(od)}}$  which were defined above only within the bounds of the antenna (|z| < L). The size of ICR-heating systems normally exceeds the antenna length. To extend function (26) beyond the antenna requires making specific the boundary conditions at the ends of the system. The approach elaborated in our work permits establishing the form of the boundary conditions by taking into consideration only the general character of the electric coupling of the plasma to the system's ends, omitting a detailed analysis of the processes in this region.

For simplicity we assume that the ends are located symmetrically about the antenna center at  $z = \pm L_1$  $(L_1 > L)$ . In this case, the portion of longitudinal antenna current even in the z-coordinate is related via the electric charge [see Eqn (21)] to the odd part of the distribution of transverse electric field, and the odd portion of the current to the even part of the field.

When the condition  $k_{\parallel}r_{\rm p} \ll 1$  is fulfilled, the  $E_+(z)$  and  $\rho(z)$  dependences are identical, i.e.,  $E_+(z) \propto \rho(z)$ , with  $j_{\parallel}(z) \propto \partial \rho/\partial z$ . This information is sufficient to establish the boundary conditions at the ends of the plasma column. Boundary conditions  $E_+(\pm L_1) = 0$  of the first kind permit

the existence of a nonzero derivative  $dE_+/dz$  at the boundary. In this case,  $d\rho/dz \neq 0$  at the boundary and, therefore,  $j_{\parallel} \neq 0$  in accordance with Eqn (21).

The longitudinal current distribution independent of the *z*-coordinate  $(j_{||}(z) = \text{const})$  satisfies the continuity equation for an arbitrary value of the azimuthal wave number *m*. If the electric charge linearly distributed in the coordinate oscillates simultaneously with this current,  $\rho(z) = z (c^2 k/\omega) j_{||}$ , the TM mode is not excited, in accordance with Eqn (21).

Another combination of the longitudinal current and the charge that satisfies Eqns (11) and (21) is  $j_{\parallel} = 0$ ,  $\rho(z) = \text{const.}$ 

Neither the TM mode nor the TE mode is excited under the two above distributions of the longitudinal current and charge. However, they affect the dependence of the TE-mode electric field on the longitudinal coordinate via the boundary conditions.

On the strength of the identity of the  $E_+(z)$  and  $\rho(z)$  dependences, the electric field

 $E_{+}(z) = C_1 + C_2 z \,. \tag{29}$ 

corresponds to these charge distributions.

The constants  $C_1$  and  $C_2$  may be selected in such a way as to satisfy the boundary conditions  $E_+(\pm L_1)=0$ . In this case, by selecting  $C_1$  it is possible to make the even part of  $E_+(z)$ vanish at the boundary, and by selecting  $C_2$  to make the odd part of  $E_+(z)$  vanish there.

The presence of arbitrariness in the dependence of the leftpolarized electric field on the longitudinal coordinate also follows from relationship (5). Its differential analogue for  $E_{\parallel} = 0$  is the second-order equation

$$\frac{\mathrm{d}^2 E_+}{\mathrm{d}z^2} + \left(\frac{\omega}{c}\right)^2 E_+ = \mathrm{i}\,\frac{\omega}{c}\left(\frac{\mathrm{d}}{\mathrm{d}r} - \frac{m}{r}\right)B_{||}\,.\tag{30}$$

In the low-frequency limit  $(N_{||} \ge 1)$ , the second term on the left-hand side of equation (30) may be omitted. In this approximation, it defines  $E_+$  up to the expressions of the form (29).

In the above reasoning, no account was taken of the current  $j''_{||}$  needed to produce the required charge distributions. This current is produced by longitudinal electron motion and it may be found from the charge conservation equation

$$-\omega
ho+k_{||}j_{||}^{\prime\prime}=0$$
 .

In this case,  $j_{||}''/j_{||} \approx N_{||}^{-2}$ , in accordance with relation (14). In real-life conditions,  $N_{||} \approx 10^3 - 10^4$ , and the influence of the longitudinal current flowing through the plasma can therefore be neglected.

In Ref. [8], the electromagnetic fields of the TE mode were determined by Eqns (1), (6), and (9), which were solved with the aid of the Fourier transform in the *z*-coordinate. For boundary conditions  $E_+(\pm L_1) = 0$  of the first kind, the even part of solutions describing the dependences  $B_{||}(z)$  and  $E_+(z)$  was expanded in terms of the functions

$$\cos\left[\left(n+\frac{1}{2}\right)\pi \frac{z}{L_1}\right]$$

and the odd part in terms of the functions

$$\sin\left(n\pi\,\frac{z}{L_1}\right).$$

In this case, the relevant selection of the constants  $C_{1,2}$  in expression (29) occurs automatically.

In view of relationships (15) and (26), meeting the requirement  $E_{\perp}(\pm L_1)=0$  entails the necessity of electric coupling to the ends. In the plasma density range under discussion (see above), longitudinal currents are produced by the electron motion between the plasma and the ends. Since the currents periodically change direction, their flow through the ends is possible only in the presence of electron emission. When the ends are insulated from the plasma (the emission is absent), both  $\partial \rho / \partial z$  and  $\partial E_+ / \partial z$  vanish at the ends, along with the longitudinal current. To satisfy the boundary conditions of the second kind in the case of insulated ends in the solution of Eqns (1), (6), and (9) in Ref. [8], the even part of the transverse electric field was expanded in terms of the functions

$$\cos\left(n\pi\,\frac{z}{L_1}\right),\,$$

and the odd part in terms of the functions

$$\sin\left[\left(n+\frac{1}{2}\right)\pi\frac{z}{L_1}\right].$$

The situation is quite realistic where highly conducting ends do not emit electric current. Since the left-polarized field is parallel to the end planes, in the perfect-conductivity approximation it should vanish on their surface. This requirement is satisfied due to the charges induced on the ends. However, the influence of these charges extends only over the interval  $|z - z_{\rm B}| \leq r_{\rm B}$ . The electric field distribution outside this interval may be found by solving the electrodynamic problem subject to the condition  $dE_+/dz|_{z=\pm L_1} = 0$ .

The issue of boundary conditions is of prime importance for the problem of ICR plasma heating because the coupling between the electric field of the TE mode and its magnetic field contains a factor  $1/(N_{||}^2 - 1)$  [see Eqn (5)]. For  $N_{||} \ge 1$  it increases steeply with an increase in longitudinal wavelength. If the boundary conditions are disregarded, which is equivalent to the assumption that the system is unbounded, waves with arbitrary lengths become admissible (see, for instance, Refs [3, 4]). The inclusion of waves with  $k_{||} \le L^{-1}$ in the analysis of an electric field excitation results in an overestimation of the field amplitude.

Having elucidated the possible form of boundary conditions, we continue the electric field to the boundary, initially assuming the ends to be insulated from the plasma. In this case, use should be made of the boundary conditions  $dE_+/dz|_{z=\pm L_1} = 0$  of the second kind. They can be satisfied by adding expressions of the form of Eqn (29) to the  $E_+(z)$ dependence. The additions may be different in different z-coordinate intervals; however, the total charge density should be continuous in accordance with equation (21). In view of these considerations, we arrive at the following expression for the  $f_{od}$  function which defines the distribution of the odd portion of charge in the z-coordinate:

$$f_{\rm od} = \begin{cases} \sin\left(\alpha \frac{z}{L}\right) & (|z| < L),\\ \sin(\alpha) \operatorname{sgn}(z) & (|z| > L). \end{cases}$$
(31)

For the even portion of the charge we additionally take into account that the charge is, in the case of insulated ends, conserved on each magnetic field line (for the odd portion this requirement is automatically satisfied):

$$f_{\rm ev} = \begin{cases} \cos\left(\alpha \, \frac{z}{L}\right) - \cos\alpha + \frac{L}{L_1} \left(\cos\alpha - \frac{1}{\alpha}\sin\alpha\right) & (|z| < L) \,, \\ \frac{L}{L_1} \left(\cos\alpha - \frac{1}{\alpha}\sin\alpha\right) & (|z| > L) \,. \end{cases}$$
(32)

In the case of conducting ends, advantage should be taken of the boundary conditions  $E(\pm L_1) = 0$  of the first kind. In this case, the continuity conditions at  $z = \pm L$  and the vanishing conditions at  $z = \pm L_1$  will be fulfilled for the even portion of the charge density if it is assumed that

$$f_{\rm ev} = \begin{cases} \cos\left(\alpha \, \frac{z}{L}\right) - \cos \alpha & (|z| < L), \\ 0 & (|z| > L). \end{cases}$$
(33)

To satisfy the boundary conditions for the odd portion of the charge density we introduce a term which linearly depends on the z-coordinate:

$$f_{\rm od} = \begin{cases} \sin\left(\alpha \frac{z}{L}\right) - \sin(\alpha) \frac{z}{L_1} & (|z| < L),\\ \sin\alpha \left(\operatorname{sgn}(z) - \frac{z}{L_1}\right) & (|z| > L). \end{cases}$$
(34)



**Figure 5.** Axial dependence of the left-polarized electric field of a helical half-wave antenna (A = 1/2)  $(r_A = 2.5 \times 10^{-4} \omega/c, r_B = 3.4 \times 10^{-4} \omega/c, L = 8.35 \times 10^{-4} \omega/c$ , and the width of the antenna conductor is 0.1*L*): (a) insulated ends; (b) conducting ends. In both figures: *I*—the even part; 2—the odd part; solid lines—the total left-polarized electric field; dashed lines— its potential part;  $L_1 = 2L$ , and the amplitude of the total current in the antenna is 100 A.

Expressions (31) - (34) define the longitudinal distribution of space charge density. The left-polarized electric field, which is of immediate interest to us, can be found with the help of relationships (22) - (26).

Figure 5 taken from Ref. [8] shows that the potential component of the left-polarized electric field calculated by the above-outlined scheme turns out to be close to the total field of the TE mode. In Ref. [8], the distribution of the electric current flowing through the helical antenna depicted in Fig. 2 was represented as the sum of axial Fourier harmonics. The fields of individual harmonics are given by expressions (5) and (6).

A half-wave helical antenna, in which either of the two conductors describes half a turn around the system's axis [A = 1/2 in expression (19)], was considered above. The electric field of this antenna extends beyond its bounds to occupy the entire system up to the ends. This may be undesirable, for instance, in ICR isotope separation systems. The electric field outside the antenna may be substantially weaken by replacing the half-wave antenna with a one-wave antenna (A = 1). In the latter case, both helical conductors appear on either side of the antenna cylinder, the currents  $J_{\parallel}(z)$  carried by these conductors being opposite in direction. In this case, the screening charge density undergoes abrupt changes of different signs at the conductors [see Eqns (19)-(21)]. As a result, the charge density outside the antenna (|z| > L) will be zero, and the potential component of the transverse electric field should be absent. This leads to weakening the transverse electric field outside the antenna (see Fig. 6).

The HF-field – plasma interaction model proposed above may be verified without the plasma. It may be replaced with a hollow cylinder made up of conducting wires insulated from each other, which should be oriented along the external magnetic field. Such a cylinder imitates the surface of a plasma column possessing anisotropic conductivity. The plasma conducts the electric current directed along the magnetic field very well and shows weak conductivity for the current flowing across the magnetic field. In this device it is easy to realize different versions of electric coupling between the conducting cylinder and the ends.



**Figure 6.** Axial dependence of the left-polarized electric field of a helical one-wave antenna (A = 1); all other parameters are the same as in Fig. 5: l — even part; 2 — odd part; solid lines — insulated ends; dashed lines — conducting ends;  $L_1 = 2L$ , and the total current in the antenna is 100 A.

#### 6. Conclusion

We have analyzed the interaction of HF fields excited inductively (by a current antenna) in a plasma column embedded in an external magnetic field. We have discussed the amplification mechanism of the transverse (relative to the external magnetic field) component of the HF electric field. The amplification is due to longitudinal field screening caused by electron flow along the external magnetic field. The electric charge bunches arising in this case enhance the transverse HF field. This process can be adequately described by treating the HF field of the current antenna as the superposition of the TE and TM modes. In this case, the screening of the longitudinal electric field turns out to be equivalent to TM-mode screening. The condition for TM-mode screening by the plasma relates the longitudinal component of the antenna electric current to the screening plasma charge. To calculate the electric field in the plasma requires knowing, along with the electric charge density, the conditions at the ends of the plasma column. The latter have been shown to be determined by the character of electric coupling between the plasma and the ends. The spatial distribution of an electric field in the plasma has been calculated in the two limiting cases: complete insulation of the plasma from the ends and perfect electric contact between the plasma and the ends. It is shown that the proposed simplified model of plasma interaction with the HF fields of the current antennas permits calculating the electric field with a high accuracy.

Acknowledgments. The author wishes to express his appreciation to V A Zhil'tsov and Yu A Muromkin for their helpful discussions.

#### References

- Karchevskii A I, Muromkin Yu A, in *Izotopy: Svoistva, Poluchenie, Primenenie* (Isotopes: Properties, Production, Application) (Ed. V Yu Baranov) (Moscow: IzdAT, 2000) p. 237
- 2. Romesser T E et al., Patent Cooperation Treaty No. WO 84/02803 (1984)
- Compant La Fontaine A, Pashkovsky V G Phys. Plasmas 2 4641 (1995)
- 4. Compant La Fontaine A et al. J. Phys. D: Appl. Phys. 31 847 (1998)
- Dolgolenko D A et al., in Proc. of the 7th Workshop on Separation Phenomena in Liquids and Gases (Ed. V D Borisevich) (Moscow: Moscow State Eng. Phys. Inst., 2000) p. 157
- Timofeev A V Fiz. Plazmy 25 232 (1999) [Plasma Phys. Rep. 25 207 (1999)]
- Timofeev A V Fiz. Plazmy 30 795 (2004) [Plasma Phys. Rep. 30 740 (2004)]
- Timofeev A V Fiz. Plazmy 31 1087 (2005) [Plasma Phys. Rep. 31 1012 (2005)]
- Zvonkov A V, Timofeev A V Fiz. Plazmy 13 282 (1987) [Sov. J. Plasma Phys. 13 158 (1987)]