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Gravitational field self-limitation and its role in the Universe

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Abstract. It is shown that according to the relativistic theory of gravity, the gravitational field slows down the rate of time flow but stops doing so when the field is strong, thus displaying its tendency toward self-limitation of the gravitational potential. This property of the gravitational field prevents massive bodies from collapsing and allows a homogeneous isotropic universe to evolve cyclically.

1. Introduction

The relativistic theory of gravity (RTG) is described in detail elsewhere in *Physics – Uspekhi* [1]. Here, we briefly consider some of its basic elements.

RTG is based on special relativity, which ensures the energy – momentum and angular momentum conservation for all physical processes, including gravitational ones. RTG assumes that gravity is universal and its source is a conserved energy – momentum tensor of all matter fields, including the gravitational one. This is why the gravitational field is described by a tensor field $\phi^{\mu\nu}$. Such an approach is consistent with the idea by Einstein, who wrote as early as 1913 [2] that "...tensor $\vartheta_{\mu\nu}$ of the gravitational field acts as a field generator in the same way as the tensor $\Theta_{\mu\nu}$ of the material processes. An exceptional position of gravitational energy in comparison with all other kinds of energies would lead to untenable consequences." This idea was fundamental in developing the relativistic theory of gravity. In general relativity (GR), a gravitational field pseudotensor emerged instead of the energy – momentum tensor.

The approach to gravity accepted in RTG leads to *geometrization*: an effective Riemannian space appears *but only endowed with simple topology*. As a result, the motion of a test body in the Minkowski space under the action of a gravitational field turns out to be equivalent to the motion of this body in the effective Riemannian space created by the gravitational field. Gravitational forces are physical and cannot vanish due to the choice of a coordinate frame. It is this property of the theory that allows separating gravitational forces from inertial ones.

In the present paper, we do not consider the process of emission of gravitational waves. We have not studied this process. But in connection to the problem of ghost states with negative energy, we note that, indeed, the inclusion of a nonzero graviton mass in the linear theory of the gravitational field leads to the wave flux being not positive definite due to spin 0. However, for the nonzero rest mass of a graviton, the scalar curvature R in the linear approximation is given by

$$R = \frac{m^2}{2}\,\phi\,.$$

This means that the wave moves in the effective Riemannian space and therefore the flux density has the form

$$R\phi^{0i} = \frac{m^2}{2}\,\phi\,\phi^{0i}\,.$$

This value can be positive or negative depending on the overall sign.

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It was not clear for us how to take this flux into account, but later we specially examined this problem in paper [3]. The peculiarity of the geometrized theory of gravity is that the gravitational field energy–momentum tensor density, defined in this theory according to Hilbert as the variation of the gravitational field Lagrangian density with respect to the metric tensor $g_{\mu\nu}$, is exactly zero, in contrast to other theories, because outside the source this variation is given by the equation for the gravitational field.

In the field approach to gravity, an effective Riemannian space appears, but only with simple topology. That is why the field concept cannot lead to GR, where the topology is not simple in general.

The above notions allow writing the following complete system of equations [1, 4, 5]:

$$\left(R^{\mu\nu} - \frac{1}{2}\, g^{\mu\nu} R\right) + \frac{m^2}{2}\left[g^{\mu\nu} + \left(g^{\mu\alpha} g^{\nu\beta} - \frac{1}{2}\, g^{\mu\nu} g^{\alpha\beta}\right)\gamma_{\alpha\beta}\right] = 8\pi G T^{\mu\nu}\,, \tag{1}$$

$$D_\nu \tilde{g}^{\nu\mu} = 0\,. \tag{2}$$

Here, $T^{\mu\nu}$ is the energy–momentum tensor of matter,[1] D_ν is the covariant derivative in the Minkowski space, $\gamma_{\alpha\beta}$ is the metric tensor of the Minkowski space, $g_{\alpha\beta}$ is the metric tensor of the effective Riemannian space, $m = m_{\mathrm{g}} c/\hbar$, m_{g} is the graviton mass, $\tilde{g}^{\nu\mu} = \sqrt{-g}\, g^{\nu\mu}$ is the density of the metric tensor $g^{\nu\mu}$, and $g = \det g_{\mu\nu}$.

Due to nonzero graviton rest mass in Eqns (1), Eqns (2) follow from gravitational field equations (1) and matter equations.

The effective metric $g^{\mu\nu}$ of the Riemannian space is related to the gravitational field $\phi^{\mu\nu}$ as

$$\tilde{g}^{\mu\nu} = \tilde{\gamma}^{\mu\nu} + \tilde{\phi}^{\mu\nu}\,,$$

where

$$\tilde{\gamma}^{\mu\nu} = \sqrt{-\gamma}\,\gamma^{\mu\nu}\,, \qquad \tilde{\phi}^{\mu\nu} = \sqrt{-\gamma}\,\phi^{\mu\nu}\,, \qquad \gamma = \det \gamma_{\mu\nu}\,.$$

The system of equations (1), (2) is generally covariant with respect to arbitrary coordinate transformations and form-invariant with respect to Lorentz transformations. It directly follows from the least action principle with the Lagrangian density

$$L = L_{\mathrm{g}}(\gamma_{\mu\nu}, \tilde{g}^{\mu\nu}) + L_{\mathrm{M}}(\tilde{g}^{\mu\nu}, \phi_A)\,,$$

where

$$L_{\mathrm{g}} = \frac{1}{16\pi}\, \tilde{g}^{\mu\nu}(G^{\lambda}_{\mu\nu} G^{\sigma}_{\lambda\sigma} - G^{\lambda}_{\mu\sigma} G^{\sigma}_{\nu\lambda}) - \frac{m^2}{16\pi}\left(\frac{1}{2}\,\gamma_{\mu\nu}\tilde{g}^{\mu\nu} - \sqrt{-g} - \sqrt{-\gamma}\right),$$

$$G^{\lambda}_{\mu\nu} = \frac{1}{2}\, g^{\lambda\sigma}(D_\mu g_{\sigma\nu} + D_\nu g_{\sigma\mu} - D_\sigma g_{\mu\nu})\,,$$

and ϕ_A are matter fields.

For time-like and isotropic intervals in the effective Riemannian space not to extend outside the original Minkowski space cone, it suffices that the conditions

$$\gamma_{\mu\nu} v^\mu v^\nu = 0\,, \qquad g_{\mu\nu} v^\mu v^\nu \leqslant 0 \tag{3}$$

be satisfied, where v^ν is the four-velocity vector.

Thus, the motion of test bodies under the action of a gravitational field always occurs *inside* both the Riemannian cone and the Minkowski-space cone, which ensures the geodesic completeness.

In the inertial coordinate frame, the tensor field $\tilde{\phi}^{\mu\nu}$ is decomposed into irreducible representations corresponding to spins 2, 1, 0, and 0′. Equation (2) eliminates spins 1 and 0′, and hence only spins 2 and 0 survive. In RTG, the graviton rest mass is introduced when spins 2 and 0 are present. Introducing a nonzero rest mass only for spin 2 would contradict observational effects in the Solar system (light ray deflection by the Sun, the Mercury perihelion advance, etc.), as shown in papers [6].

The nonzero rest mass of a graviton necessarily emerges in the theory because only when it is introduced can the gravitational field be treated as a physical field in the Minkowski space, with its source being the total conserved energy–momentum tensor of all matter. But it is the nonzero mass of a graviton that fundamentally changes both the collapse process and the evolution of the Universe.

When Einstein in 1913 connected the gravitational field with the metric tensor of a Riemannian space, it turned out that such a field causes time dilation in physical processes. In particular, it can be illustrated in the Schwarzschild metric, for example, by comparing the flow of time in the vicinity of a gravitating body with that at infinity. But GR involves only the metric tensor of the Riemannian space in general, and therefore no signatures of the inertial time of the Minkowski space can be found in the Hilbert–Einstein equations.

The appearance of the effective Riemannian space in the field theory of gravity with the Minkowski space kept as the underlying space *allows comparing the flow of time in the gravitational field with that in the inertial frame of the Minkowski space in the absence of gravity*.

To show how the change of the flow of time leads to the appearance of a force, we consider the Newton equation

$$m\,\frac{\mathrm{d}^2 x}{\mathrm{d}t^2} = F\,.$$

By formally passing from the inertial time t to a time τ using the rule

$$\mathrm{d}\tau = U(t)\,\mathrm{d}t\,,$$

we easily obtain

$$m\,\frac{\mathrm{d}^2 x}{\mathrm{d}\tau^2} = \frac{1}{U^2}\left(F - m\,\frac{\mathrm{d}x}{\mathrm{d}t}\,\frac{\mathrm{d}}{\mathrm{d}t}\ln U\right).$$

This shows that the time dilation determined by the function U leads to the appearance of an effective force. However, the argument is here purely formal, because there is no physical reason for changing the time flow rate in this case. But this formal example suggests that any time dilation process in nature necessarily produces effective field forces that must be taken into account in the theory. The physical gravitational field changes both the flow of time and spatial parameters compared to their values in the inertial Minkowski frame in the absence of gravity.

[1] Matter is assumed to include all material fields except the gravitational field.

The field approach allows more deeply understanding the nature of the gravitational field and leads to the conclusion that the gravitational field can both slow down the flow of inertial time and stop time dilation. This underlies both the ability of the field to limit its own value (self-limitation) and the impossibility of stopping the flow of time by the gravitational field. *Therefore, according to RTG, the slowing down of the flow of inertial time and the stopping of time dilation represent one common property of the gravitational field.* Only the first part of this property has been manifest in GR.

This confirms the statement by A S Eddington: "The star has to go on radiating and radiating and contracting and contracting until, I suppose, it gets down to a few km. radius, when gravity becomes strong enough to hold in the radiation, and the star can at last find peace.... I felt driven to the conclusion that this was almost a *reductio ad absurdum* of the relativistic degeneracy formula. Various accidents may intervene to save the star, but I want more protection than that. I think there should be a law of Nature to prevent a star from behaving in this absurd way!"[2]

It turns out that in the field concepts of gravity, all the above features are contained in the physical property of the gravitational field to stop the *process* of time dilation and hence to restrict its potential, which stops the *process* of collapse.

In what follows, using the gravitational collapse and evolution of a homogeneous isotropic Universe as examples, we consider how the *self-limitation* of the gravitational field potential emerges. This self-limitation stops both the slowing down of the flow of time and the process of the collapse of matter.

2. Equations for a spherically symmetric static gravitational field

In the inertial frame (Minkowski space), the interval in spherical coordinates is given by

$$\mathrm{d}\sigma^2 = (\mathrm{d}x^0)^2 - (\mathrm{d}r)^2 - r^2(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2)\,, \tag{4}$$

where $x^0 = ct$. In the effective Riemannian space, the interval for a spherically symmetric static field is written as

$$\mathrm{d}s^2 = U(r)(\mathrm{d}x^0)^2 - V(r)\,\mathrm{d}r^2 - W^2(r)(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2)\,. \tag{5}$$

RTG equations (1) and (2) can be expressed as

$$R^\mu_\nu - \frac{1}{2}\,\delta^\mu_\nu R + \frac{m^2}{2}\left(\delta^\mu_\nu + g^{\mu\alpha}\gamma_{\alpha\nu} - \frac{1}{2}\,\delta^\mu_\nu g^{\alpha\beta}\gamma_{\alpha\beta}\right) = \varkappa T^\mu_\nu\,, \tag{6}$$

$$D_\mu \tilde{g}^{\mu\nu} = 0\,. \tag{7}$$

In the expanded form, Eqn (7) is

$$\partial_\mu \tilde{g}^{\mu\nu} + \gamma^\nu_{\lambda\sigma}\tilde{g}^{\lambda\sigma} = 0\,, \tag{8}$$

where $\gamma^\nu_{\lambda\sigma}$ are the Christoffel symbols for the Minkowski space.

[2] *The Observatory* **58** (729) 38 (1935).

For a spherically symmetric static source, the components of T^μ_ν are

$$T^0_0 = \rho(r)\,, \qquad T^1_1 = T^2_2 = T^3_3 = -\frac{p(r)}{c^2}\,, \tag{9}$$

where ρ is the mass density and p is the isotropic pressure.

To determine metric coefficients U, V, and W, we can use Eqns (6) with the indices $\mu = 0$, $\nu = 0$; $\mu = 1$, $\nu = 1$:

$$\frac{1}{W^2} - \frac{1}{VW^2}\left(\frac{\mathrm{d}W}{\mathrm{d}r}\right)^2 - \frac{2}{VW}\frac{\mathrm{d}^2W}{\mathrm{d}r^2} - \frac{1}{W}\frac{\mathrm{d}W}{\mathrm{d}r}\frac{\mathrm{d}}{\mathrm{d}r}\left(\frac{1}{V}\right) + \frac{1}{2}\,m^2\left[1 + \frac{1}{2}\left(\frac{1}{U} - \frac{1}{V}\right) - \frac{r^2}{W^2}\right] = \varkappa\rho\,, \tag{10}$$

$$\frac{1}{W^2} - \frac{1}{VW^2}\left(\frac{\mathrm{d}W}{\mathrm{d}r}\right)^2 - \frac{1}{UVW}\frac{\mathrm{d}W}{\mathrm{d}r}\frac{\mathrm{d}U}{\mathrm{d}r} + \frac{1}{2}\,m^2\left[1 - \frac{1}{2}\left(\frac{1}{U} - \frac{1}{V}\right) - \frac{r^2}{W^2}\right] = -\varkappa\,\frac{p}{c^2}\,. \tag{11}$$

Equation (8) becomes

$$\frac{\mathrm{d}}{\mathrm{d}r}\left(\sqrt{\frac{U}{V}}\,W^2\right) = 2r\sqrt{UV}\,. \tag{12}$$

Taking the identity

$$\frac{\mathrm{d}r}{\mathrm{d}W}\frac{1}{W^2}\frac{\mathrm{d}}{\mathrm{d}r}\left(\frac{W}{V}\left(\frac{\mathrm{d}W}{\mathrm{d}r}\right)^2\right) = \frac{1}{VW^2}\left(\frac{\mathrm{d}W}{\mathrm{d}r}\right)^2 + \frac{2}{VW}\frac{\mathrm{d}^2W}{\mathrm{d}r^2} + \frac{1}{W}\frac{\mathrm{d}W}{\mathrm{d}r}\frac{\mathrm{d}}{\mathrm{d}r}\left(\frac{1}{V}\right)$$

into account and passing from derivatives with respect to r to derivatives with respect to W, we rewrite Eqns (10)–(12) as

$$1 - \frac{\mathrm{d}}{\mathrm{d}W}\left(\frac{W}{V(\mathrm{d}r/\mathrm{d}W)^2}\right) + \frac{1}{2}\,m^2\left[W^2 - r^2 + \frac{W^2}{2}\left(\frac{1}{U} - \frac{1}{V}\right)\right] = \varkappa W^2\rho\,, \tag{13}$$

$$1 - \frac{W}{V(\mathrm{d}r/\mathrm{d}W)^2}\frac{\mathrm{d}}{\mathrm{d}W}(\ln UW) + \frac{1}{2}\,m^2\left[W^2 - r^2 - \frac{W^2}{2}\left(\frac{1}{U} - \frac{1}{V}\right)\right] = -\varkappa W^2\,\frac{p}{c^2}\,, \tag{14}$$

$$\frac{\mathrm{d}}{\mathrm{d}W}\left(\sqrt{\frac{U}{V}}\,W^2\right) = 2r\sqrt{UV}\,\frac{\mathrm{d}r}{\mathrm{d}W}\,. \tag{15}$$

In the Solar system, the effect of a nonzero graviton rest mass m_g can be neglected with a high precision, and the system of equations (13)–(15) outside the source in the inertial frame takes the form

$$1 - \frac{\mathrm{d}}{\mathrm{d}W}\left(\frac{W}{V(\mathrm{d}r/\mathrm{d}W)^2}\right) = 0\,,$$

$$1 - \frac{W}{V(\mathrm{d}r/\mathrm{d}W)^2}\frac{\mathrm{d}}{\mathrm{d}W}(\ln UW) = 0\,,$$

$$\frac{\mathrm{d}}{\mathrm{d}W}\left(\sqrt{\frac{U}{V}}\,W^2\right) = 2r\sqrt{UV}\,\frac{\mathrm{d}r}{\mathrm{d}W}\,.$$

The first two equations are identical to GR equations, but are written in harmonic coordinates. The last equation follows from Eqn (2), which in the inertial frame exactly coincides with the harmonic coordinate condition. Such a system of GR equations was considered by V A Fock. It is easy to see that for a source with mass M, this system of equations has the solution

$$\mathrm{d}s^2 = \frac{r - r_\mathrm{g}/2}{r + r_\mathrm{g}/2}\, c^2\, \mathrm{d}t^2 - \frac{r + r_\mathrm{g}/2}{r - r_\mathrm{g}/2}\, \mathrm{d}r^2 - \left(r + \frac{r_\mathrm{g}}{2}\right)^2 (\mathrm{d}\theta^2 + \sin^2\theta\, \mathrm{d}\phi^2)\,,$$

where $r_\mathrm{g} = 2GM/c^2$ is the Schwarzschild radius.

For the light ray deflection by the Sun and for the Mercury perihelion advance, we then obtain exactly the same expressions as in GR in harmonic coordinates.

In what follows, we investigate the system of equations (13)–(15) for different state equations of matter. Using these equations, we show in Sections 3–5 that the gravitational field has the property of *self-limitation* that sets a limit for the time dilation by gravitational field.

3. External solution for a spherically symmetric static body

We show that a nonzero graviton rest mass quantitatively changes the character of the solution in the region close to the Schwarzschild sphere.

Subtracting Eqn (14) from Eqn (13) and introducing the new variable

$$Z = \frac{UW^2}{V\dot{r}^2}\,, \qquad \dot{r} = \frac{\mathrm{d}r}{\mathrm{d}t}\,, \qquad t = \frac{W - W_0}{W_0}\,, \tag{16}$$

we obtain

$$\frac{\mathrm{d}Z}{\mathrm{d}W} - \frac{2Z}{U}\frac{\mathrm{d}U}{\mathrm{d}W} - 2\,\frac{Z}{W} - \frac{m^2W^3}{2W_0^2}\left(1 - \frac{U}{V}\right) = -\varkappa\,\frac{W^3}{W_0^2}\left(\rho + \frac{p}{c^2}\right)U\,. \tag{17}$$

Adding Eqns (13) and (14), we find

$$1 - \frac{1}{2}\,\frac{W_0^2}{W}\,\frac{1}{U}\,\frac{\mathrm{d}Z}{\mathrm{d}W} + \frac{m^2}{2}\,(W^2 - r^2) = \frac{1}{2}\,\varkappa W^2\left(\rho - \frac{p}{c^2}\right). \tag{18}$$

We consider Eqns (17) and (18) outside matter in the region determined by the inequalities

$$\frac{U}{V} \ll 1\,, \qquad \frac{1}{2}\,m^2(W^2 - r^2) \ll 1\,. \tag{19}$$

In this region, Eqn (18) is rewritten as

$$U = \frac{1}{2}\,\frac{W_0^2}{W}\,\frac{\mathrm{d}Z}{\mathrm{d}W} = \frac{1}{2}\,\frac{W_0}{W}\,\frac{\mathrm{d}Z}{\mathrm{d}t}\,. \tag{20}$$

Taking Eqn (20) into account, we transform Eqn (17) into the form

$$Z\,\frac{\mathrm{d}^2Z}{\mathrm{d}W^2} - \frac{1}{2}\left(\frac{\mathrm{d}Z}{\mathrm{d}W}\right)^2 + \frac{1}{4}\,m^2\,\frac{W^3}{W_0^2}\,\frac{\mathrm{d}Z}{\mathrm{d}W} = 0\,. \tag{21}$$

We introduce the variable t in accordance with (16). Then Eqn (21) becomes

$$Z\ddot{Z} - \frac{1}{2}\dot{Z}^2 + \alpha(1 + t)^3\dot{Z} = 0\,, \tag{22}$$

where $\alpha = m^2W_0^2/4$ and $\dot{Z} = \mathrm{d}Z/\mathrm{d}t$. For values of t from the interval

$$0 \leqslant t \ll \frac{1}{3}\,, \tag{23}$$

Eqn (22) is simplified:

$$Z\ddot{Z} - \frac{1}{2}\,\dot{Z}^2 + \alpha\dot{Z} = 0\,. \tag{24}$$

It has the solution

$$\lambda\sqrt{Z} = 2\alpha\ln\left(1 + \frac{\lambda\sqrt{Z}}{2\alpha}\right) + \frac{\lambda^2}{2}\,t\,, \tag{25}$$

where λ is an arbitrary constant.

Based on (20) and (16), we have

$$U = \frac{1}{2}\,\frac{W_0}{W}\,\dot{Z}\,, \qquad V\dot{r}^2 = \frac{1}{2}\,W_0W\,\frac{\dot{Z}}{Z}\,. \tag{26}$$

Using (25), we find

$$\dot{Z} = 2\alpha + \lambda\sqrt{Z}\,. \tag{27}$$

Substituting (27) in (26), we obtain

$$U = \frac{W_0}{W}\left(\alpha + \frac{\lambda}{2}\,\sqrt{Z}\right), \qquad V\dot{r}^2 = W_0W\,\frac{\alpha + \lambda\sqrt{Z}/2}{Z}\,. \tag{28}$$

For $\alpha = 0$, it follows from (25) that

$$\sqrt{Z} = \frac{\lambda}{2}\,t\,. \tag{29}$$

Substituting this formula in (28), we obtain

$$U = \left(\frac{\lambda}{2}\right)^2\,\frac{W - W_0}{W}\,. \tag{30}$$

But this expression must exactly coincide with the Schwarzschild solution

$$U = \frac{W - W_\mathrm{g}}{W}\,, \qquad W_\mathrm{g} = \frac{2GM}{c^2}\,. \tag{31}$$

Comparing (30) and (31), we obtain[3]

$$\lambda = 2\,, \qquad W_0 = W_\mathrm{g}\,. \tag{32}$$

We thus find

$$U = \frac{W_\mathrm{g}}{W}(\alpha + \sqrt{Z})\,, \qquad V\dot{r}^2 = W_\mathrm{g}W\,\frac{\alpha + \sqrt{Z}}{Z}\,. \tag{33}$$

Next, we must determine the dependence of r on W with the help of (15).

[3] Strictly speaking, the constant λ depends on the parameter α. But due to the smallness of α this dependence is insignificant.

After substituting (33) in Eqn (15) and changing the variables as

$$l = \frac{r}{W_g}\,, \tag{34}$$

we obtain

$$\frac{\mathrm{d}}{\mathrm{d}\sqrt{Z}}\left((1+t)\,\frac{\mathrm{d}Z}{\mathrm{d}t}\,\frac{\mathrm{d}l}{\mathrm{d}\sqrt{Z}}\right) = 4l\,. \tag{35}$$

Taking (27) into account and differentiating with respect to $\sqrt{Z}$ in (35), we have

$$(1+t)(\alpha+\sqrt{Z})\,\frac{\mathrm{d}^2 l}{(\mathrm{d}\sqrt{Z})^2} + (1+t+\sqrt{Z})\,\frac{\mathrm{d}l}{\mathrm{d}\sqrt{Z}} - 2l = 0\,. \tag{36}$$

Because we are interested in values of t from interval (23), Eqn (36) is simplified in this region to

$$(\alpha+\sqrt{Z})\,\frac{\mathrm{d}^2 l}{(\mathrm{d}\sqrt{Z})^2} + (1+\sqrt{Z})\,\frac{\mathrm{d}l}{\mathrm{d}\sqrt{Z}} - 2l = 0\,. \tag{37}$$

The general solution of Eqn (37) can be written as

$$l = Al_1 + Bl_2\,, \tag{38}$$

where

$$l_1 = F\big(-2,\,1-\alpha,\,-(\alpha+\sqrt{Z})\big)\,,$$

$$l_2 = (\alpha+\sqrt{Z})^{\alpha} F\big(-2+\alpha,\,1+\alpha,\,-(\alpha+\sqrt{Z})\big)\,,$$

A and B are arbitrary constants, and F is the degenerate hypergeometric function.

The analysis of solution (38) in the region defined by inequalities (19) and (23) yields the equality

$$\dot{r} = W_g\,. \tag{39}$$

We consider the limit case where

$$\sqrt{Z} \gg \alpha\,. \tag{40}$$

With (32), it then follows from Eqn (25) that

$$\sqrt{Z} = t\,. \tag{41}$$

Substituting this expression in (28) and using (32) and (39), we obtain the Schwarzschild solution

$$U = \frac{W - W_g}{W}\,, \qquad V = \frac{W}{W - W_g}\,. \tag{42}$$

This implies that because the effect of the graviton rest mass can be neglected in the Solar system, using the equation for geodesic motion of a test body allows easily explaining all observational effects in the Solar system (the light ray deflection by the Sun, the Mercury perihelion advance, etc.).

We now consider another limit case, where the nonzero graviton rest mass is significant. We now let the inequality

$$\sqrt{Z} \ll \alpha \tag{43}$$

hold. In this approximation, with (32) taken into account, it follows from Eqn (25) that

$$Z = 2\alpha t\,. \tag{44}$$

Substituting this expression in (28) and using (32) and (39), we obtain [7]

$$U = \alpha\,\frac{W_g}{W}\,, \qquad V = \frac{1}{2}\,\frac{W}{W - W_g}\,. \tag{45}$$

From Eqns (43) and (45), this solution is valid in the region

$$t \ll \frac{\alpha}{2}\,,\ \text{i.e.,}\ \ W - W_g \ll \frac{1}{2}\,W_g\left(\frac{m_g c}{\hbar}\,\frac{W_g}{2}\right)^2.$$

It follows from (45) that the graviton mass m_g does not allow the vanishing of U. *For any body a nonzero graviton mass sets a limit on time dilation*, which is determined by a linear function of the Schwarzschild radius, i.e., of the mass of the body, and is equal to

$$\frac{1}{2}\,\frac{m_g c}{\hbar}\,W_g\,.$$

There is no such limit in GR. This feature of the gravitational field drastically changes the motion of any body in the gravitational field.

The motion of a test body occurs along a geodesic in the Riemannian space

$$\frac{\mathrm{d}v^{\mu}}{\mathrm{d}s} + \Gamma^{\mu}_{\alpha\beta}\,\frac{\mathrm{d}x^{\alpha}}{\mathrm{d}s}\,\frac{\mathrm{d}x^{\beta}}{\mathrm{d}s} = 0\,, \tag{46}$$

where $v^{\mu} = \mathrm{d}x^{\mu}/\mathrm{d}s$ is the four-velocity; it satisfies the condition

$$g_{\mu\nu}v^{\mu}v^{\nu} = 1\,. \tag{47}$$

We first consider the radial motion

$$v^{\theta} = v^{\phi} = 0\,, \qquad v^{r} = \frac{\mathrm{d}r}{\mathrm{d}s}\,. \tag{48}$$

If we recall the Christoffel symbol

$$\Gamma^{0}_{01} = \frac{1}{2U}\,\frac{\mathrm{d}U}{\mathrm{d}r}\,, \tag{49}$$

from Eqn (46), we find

$$\frac{\mathrm{d}v^0}{\mathrm{d}s} + \frac{1}{U}\,\frac{\mathrm{d}U}{\mathrm{d}r}\,v^0 v^r = 0\,. \tag{50}$$

Solving Eqn (50), we obtain

$$\frac{\mathrm{d}}{\mathrm{d}r}\ln(v^0 U) = 0\,. \tag{51}$$

From here, we find

$$v_0 = \frac{\mathrm{d}x^0}{\mathrm{d}s} = \frac{U_0}{U}\,, \tag{52}$$

where U_0 is the integration constant. Setting the velocity of a test body equal to zero at infinity, we obtain $U_0 = 1$. From

formula (47) we derive

$$\frac{\mathrm{d}r}{\mathrm{d}s} = -\sqrt{\frac{1-U}{UV}}\,. \tag{53}$$

If we substitute (45) in this equation and take (39) into account, we obtain

$$\frac{\mathrm{d}W}{\mathrm{d}s} = -\frac{\hbar}{m_{\mathrm{g}}c}\,\frac{2}{W_{\mathrm{g}}}\sqrt{2\,\frac{W}{W_{\mathrm{g}}}\left(1-\frac{W_{\mathrm{g}}}{W}\right)}\,. \tag{54}$$

This shows that a turning point appears. Differentiating (54) with respect to s, we find

$$\frac{\mathrm{d}^2W}{\mathrm{d}s^2} = 4\left(\frac{\hbar}{m_{\mathrm{g}}c}\right)^2\frac{1}{W_{\mathrm{g}}^3}\,. \tag{55}$$

It follows that the acceleration is positive at the turning point, i.e., repulsion with an appreciable magnitude occurs. Integrating (54), we obtain the expression

$$W = W_{\mathrm{g}} + 2\left(\frac{\hbar}{m_{\mathrm{g}}c}\right)^2\frac{(s-s_0)^2}{W_{\mathrm{g}}^3}\,,$$

which shows that the *test body cannot cross the Schwarzschild sphere*.

In accordance with (45), the scalar quantity g/γ, where $g = \det g_{\mu\nu}$ and $\gamma = \det\gamma_{\mu\nu}$, develops a singularity at the point $W = W_{\mathrm{g}}$, which cannot be eliminated by the choice of the coordinate frame. That is why the presence of such a singularity in the vacuum is *unacceptable*, because otherwise the outer solution cannot be matched with the solution inside the body. It follows that the radius of the body is greater than the Schwarzschild radius. Thus, a *self-limitation* of the field value emerges in RTG and the reason for the 'Schwarzschild singularity' disappears. This is fully consistent with Einstein's opinion, which he expressed as early as 1939 in paper [8]: "The essential result of this investigation is **a clear understanding of why the 'Schwarzschild singularities' do not exist in physical reality**" (highlighted by the authors). And further: "The 'Schwarzschild singularity' does not appear for the reason that matter cannot be concentrated arbitrarily. And this is because otherwise the constituting particles **would reach the velocity of light**" (highlighted by the authors).

As an example, we consider the gravitational field in the contracting (synchronous) reference frame. The transition to that frame from the inertial frame is achieved by the transformations

$$\mathrm{d}t = \frac{1}{U}\left[\mathrm{d}\tau - \mathrm{d}R(1-U)\right], \qquad \mathrm{d}W = \sqrt{\frac{1-U}{UV}}\,(\mathrm{d}R - \mathrm{d}\tau)\,.$$

In the synchronous frame, the intervals of the Riemannian and pseudo-Euclidean space – time have the form

$$\mathrm{d}s^2 = \mathrm{d}\tau^2 - \left[1-U(X)\right]\mathrm{d}R^2 - W^2(X)(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2)\,,$$

$$\mathrm{d}\sigma^2 = \mathrm{d}\tau^2\,\frac{1-\dot r^2U^2}{U^2} + 2\,\mathrm{d}R\,\mathrm{d}\tau\,\frac{\dot r^2U^2-(1-U)}{U^2} - \mathrm{d}R^2\,\frac{\dot r^2U^2-(1-U)^2}{U^2} - r^2(\mathrm{d}\theta^2+\sin^2\theta\,\mathrm{d}\phi^2)\,,$$

where $X = R - \tau$ and $\dot r = \mathrm{d}r/\mathrm{d}X$.

The RTG equations

$$R_{\mu\nu} = 8\pi G\left(T_{\mu\nu} - \frac{1}{2}\,g_{\mu\nu}T\right) + \frac{m^2}{2}(g_{\mu\nu}-\gamma_{\mu\nu})\,, \tag{56a}$$

$$D_\nu\tilde g^{\mu\nu} = 0$$

for the problem determined by intervals $\mathrm{d}s^2$ and $\mathrm{d}\sigma^2$ lead outside the matter to the equations

$$R_{01} = \frac{2\ddot W}{W} + \frac{1}{(1-U)W}\,\dot U\dot W = \frac{m^2}{2}\left(\frac{1-U}{U^2} - \dot r^2\right), \tag{56b}$$

$$R_{00} + R_{01} = \frac{1}{1-U}\left[\frac{1}{2}\,\ddot U + \frac{\dot U^2}{4(1-U)} + \frac{1}{W}\,\dot U\dot W\right] = -\frac{m^2}{2}\,\frac{1-U}{U}\,. \tag{56c}$$

In the range of the variable X, where the graviton mass can be neglected due to its smallness, it follows from these equations that

$$W = W_{\mathrm{g}}^{1/3}\left(\frac{3}{2}\,X\right)^{2/3}, \qquad 1-U = \left(\frac{2}{3}\,W_{\mathrm{g}}\right)^{2/3}X^{-2/3}\,. \tag{56d}$$

Equation (56d) for the function U implies that its value decreases as X decreases and its derivative $\dot U$ is positive. This decrease of U continues at smaller values of X, because the value of $\dot U$ remains positive.

In approximation (19), it follows from Eqn (56a) outside the matter that

$$R_{22} = -\frac{UW}{1-U}\,\ddot W - \frac{U}{1-U}\,\dot W^2 - \frac{W(2-U)}{2(1-U)^2}\,\dot U\dot W + 1 = 0\,.$$

For small values $0 < U \ll 1$, this equation somewhat simplifies and takes the form

$$UW\ddot W + U\dot W^2 + W\dot U\dot W - 1 = 0\,.$$

This equation has the solution

$$\dot W = \frac{X}{UW}\,.$$

At the stopping point

$$\dot W = 0$$

in accordance with Eqns (56b) and (56c), the second derivative $\ddot W$ is positive for small U, which indicates the presence of a repulsion force. It is from this point that the process of expansion starts; it stops in the region X where inequalities (56d) hold. In that region, $\ddot W$ is negative:

$$\ddot W = -\frac{1}{2}\,W_{\mathrm{g}}^{1/3}\left(\frac{3}{2}\,X\right)^{-4/3},$$

and therefore the case of attraction occurs. Therefore, if the stopping point were outside the matter, the expansion would be followed by contraction, followed by stopping and subsequent contraction, etc. But the real gravitational field excludes this regime of motion. In GR, the formula

$$W = \left[\frac{3}{2}(R-c\tau)\right]^{2/3}W_{\mathrm{g}}^{1/3}$$

pertains to this problem, but in our case we obtain the expression

$$W = W_{\mathrm{g}} + 2\left(\frac{\hbar}{m_{\mathrm{g}}c}\right)^2 \frac{(R - c\tau)^2}{W_{\mathrm{g}}^3}\,,$$

which excludes motion toward the point $W = 0$. This implies that a repulsion force emerges:

$$\frac{\mathrm{d}^2 W}{\mathrm{d}\tau^2} = \frac{4c^2}{W_{\mathrm{g}}^3}\left(\frac{\hbar}{m_{\mathrm{g}}c}\right)^2 .$$

Because the gravitational field is created by matter and self-limits its potential, the above example implies that obtaining the physical solution requires matching the solution inside matter with the outside solution, but for this the absolute value of the potential at the body surface must be limited by the inequality

$$\frac{|\phi|}{c^2} < 1\,.$$

It is this solution, which corresponds to the real gravitational field, that leads to the stopping point not being allowed in the vacuum. Therefore, world lines of particles at rest relative to the contracting frame collide with matter of the field source. These collisions occur within a finite time for any observer. This excludes the regime of motion mentioned above. At the same time, this also prohibits the formation of 'black holes.'

We now turn to the analysis of the inner solution.

4. Inner Schwarzschild-like solution

In paper [9], Schwarzschild found a spherically symmetric stationary inner solution of the GR equations. For a *homogeneous* solid ball of radius a, it is described by the interval

$$\mathrm{d}s^2 = c^2\left(\frac{3}{2}\sqrt{1 - qa^2} - \frac{1}{2}\sqrt{1 - qW^2}\right)^2 \mathrm{d}t^2 - (1 - qW^2)^{-1}\,\mathrm{d}W^2 + W^2(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2)\,, \quad (57)$$

where

$$q = \frac{1}{3}\varkappa\rho = \frac{2GM}{c^2a^3}\,, \qquad \varkappa = \frac{8\pi G}{c^2}\,, \qquad \rho = \frac{3M}{4\pi a^3}\,.$$

The common property of the inner and outer solutions in GR is manifested in vanishing metric coefficients at the differential $\mathrm{d}t^2$ for a certain value of W, which means that the gravitational field is able not only to slow down the flow of time but also to *stop it*. In the outer solution, the metric coefficient U vanishes when $W = W_{\mathrm{g}}$. To eliminate such a possibility, which is not forbidden by the theory, one usually has to assume that the radius of a body satisfies the inequality

$$a > W_{\mathrm{g}}\,. \quad (58)$$

For the inner solution, this occurs when

$$W^2 = 9a^2 - 8\,\frac{a^3}{W_{\mathrm{g}}}\,. \quad (59)$$

To exclude the possibility of the vanishing of the metric coefficient U inside the body, it is necessary to assume that

$$a > \frac{9}{8}\,W_{\mathrm{g}}\,. \quad (60)$$

We emphasize that inequalities (58) *and* (60) *are not consequences of GR.*

The inner Schwarzschild solution bears a somewhat formal character, but it is interesting mainly because it represents an exact solution of the GR equations. In Section 3, using the outer Schwarzschild solution as an example, we showed that in the relativistic theory of gravity as a field theory, inequality (58) emerges exactly due to time dilation. Below, we consider the inner Schwarzschild-like solution in the framework of RTG.

The inner Schwarzschild solution appeared based on the Hilbert – Einstein equations

$$1 - \frac{\mathrm{d}}{\mathrm{d}W}\left(\frac{W}{V}\right) = \varkappa W^2\rho\,,$$
$$1 - \frac{1}{V} - \frac{W}{UV}\frac{\mathrm{d}U}{\mathrm{d}W} = -\varkappa\,\frac{W^2}{c^2}\,p\,. \quad (61)$$

Because the metric coefficients coincide in accordance with (57), it follows that

$$U = \left(\frac{3}{2}\sqrt{1 - qa^2} - \frac{1}{2}\sqrt{1 - qW^2}\right)^2, \qquad V = (1 - qW^2)^{-1}. \quad (62)$$

We then find

$$\frac{U'}{U} = qW\left[\sqrt{1 - qW^2}\left(\frac{3}{2}\sqrt{1 - qa^2} - \frac{1}{2}\sqrt{1 - qW^2}\right)\right]^{-1},$$
$$U' = \frac{\mathrm{d}U}{\mathrm{d}W}\,. \quad (63)$$

Substituting (62) and (63) in Eqn (61), we obtain the expression for pressure:

$$\frac{p}{c^2} = \frac{\rho}{2}\,\frac{\sqrt{1 - qW^2} - \sqrt{1 - qa^2}}{\sqrt{U}}\,. \quad (64)$$

This shows, in particular, that if equality (59) had been allowed, the pressure inside the body at the circle determined by this equality would have become infinite. The singularity that emerges due to the metric coefficient U vanishing cannot be eliminated by choosing a reference frame because the scalar curvature R also has this singularity:

$$R = -8\pi G\,\frac{3\sqrt{1 - qa^2} - 2\sqrt{1 - qW^2}}{\sqrt{U}}\,. \quad (65)$$

We now show in the example of the inner Schwarzschild-like solution that the situation is radically different in RTG because the time dilation process stops. The same mechanism of the field self-limitation that in RTG led to inequality (58) for the outer Schwarzschild solution leads to an inequality of type (60) for the inner Schwarzschild solution.

We obtain equations for this problem from Eqns (13) and (14). Introducing the new variable

$$Z = \frac{UW^2}{Vr'^2}\,, \qquad r' = \frac{\mathrm{d}r}{\mathrm{d}W}$$

and adding Eqns (13) and (14) yields

$$1-\frac{1}{2UW}\,Z'+\frac{m^2}{2}(W^2-r^2)=\frac{1}{2}\,\varkappa W^2\left(\rho-\frac{p}{c^2}\right). \quad (66)$$

Subtracting Eqn (14) from Eqn (13) yields

$$Z'-2Z\,\frac{U'}{U}-2\,\frac{Z}{W}-\frac{m^2}{2}\,W^3\left(1-\frac{U}{V}\right)=-\varkappa W^3\left(\rho+\frac{p}{c^2}\right)U. \quad (67)$$

In our problem, the components of the matter energy–momentum tensor are

$$T_0^0=\rho\,,\qquad T_1^1=T_2^2=T_3^3=-\frac{p(W)}{c^2}\,.$$

The matter equation

$$\nabla_\nu(\sqrt{-g}\,T_\mu^\nu)=\partial_\nu(\sqrt{-g}\,T_\mu^\nu)+\frac{1}{2}\,\sqrt{-g}\,T_{\sigma\nu}\,\partial_\mu g^{\sigma\nu}=0$$

for this problem reduces to the equation

$$\frac{1}{c^2}\,\frac{\mathrm{d}p}{\mathrm{d}W}=-\left(\rho+\frac{p}{c^2}\right)\frac{1}{2U}\,\frac{\mathrm{d}U}{\mathrm{d}W}\,. \quad (68)$$

Because the pressure increases toward the center of the ball, the inequality

$$\frac{\mathrm{d}U}{\mathrm{d}W}>0 \quad (69)$$

holds, implying that the function U decreases in approaching the center of the ball, and therefore time is slowing down with respect to the inertial time. Because the density ρ is assumed to be *constant* in the inner Schwarzschild problem, Eqn (68) is easy to solve:

$$\rho+\frac{p}{c^2}=\frac{\alpha}{\sqrt{U}}\,. \quad (70)$$

Comparing (64) and (70), we find the constant α:

$$\alpha=\rho\sqrt{1-qa^2}\,. \quad (71)$$

Equations (66) and (67), under the assumption that

$$m^2(W^2-r^2)\ll 1\,,\qquad \frac{U}{V}\ll 1\,,$$

and after introducing the independent variable $y=W^2$, take the form

$$Z'=U(1-3qy)+\frac{\alpha\varkappa}{2}\,y\sqrt{U}\,, \quad (72)$$

$$\sqrt{U}\,Z'-\frac{1}{y}\,Z\sqrt{U}-4Z(\sqrt{U})'+\frac{\alpha\varkappa}{2}\,yU-\frac{m^2}{4}\,y\sqrt{U}=0\,. \quad (73)$$

Here and below, we use the notation $Z'=\mathrm{d}Z/\mathrm{d}y$.

In the analysis of the outer spherically symmetric Schwarzschild solution in Section 3, we have seen that due to the effective gravitational repulsion force, the metric coefficient U, which determines the time dilation with respect to the inertial time, does not vanish even in a strong gravitational field.

That is why we study the behavior of the solution of these equations for small y in what follows. From formula (62), for the zero graviton rest mass, we find

$$\sqrt{U}\simeq\frac{1}{2}\left(3\sqrt{1-qa^2}-1\right)+\frac{qy}{4}+\frac{1}{16}\,q^2y^2 \quad (74)$$

for small y. This expression also shows that the function $\sqrt{U}$ can vanish for the inner Schwarzschild solution if

$$3\sqrt{1-qa^2}=1\,, \quad (75)$$

which leads to an infinite value of both the pressure p and the scalar curvature R at the center. Because Eqns (72) and (73) stop time dilation if the graviton rest mass is nonzero, it is natural to expect that equality (75) cannot hold in the physical (real) region for the function $\sqrt{U}$. Based on (74), we seek the solution of Eqns (72) and (73) for the function $\sqrt{U}$ in the form

$$\sqrt{U}=\beta+\frac{qy}{4}+\frac{1}{16}\,q^2y^2\,, \quad (76)$$

where β is an unknown constant to be determined using Eqns (72) and (73).

Substituting expression (76) in Eqn (72) and integrating, we find

$$Z=\beta^2y+\frac{y^2}{2}\left(\frac{\beta q}{2}-3\beta^2q+\frac{\alpha\varkappa\beta}{2}\right)+\frac{y^3}{3}\left[\frac{q^2}{8}\left(\beta+\frac{1}{2}\right)-\frac{3\beta}{2}\,q^2+\frac{\alpha\varkappa q}{8}\right]. \quad (77)$$

Taking formulas (76) and (77) into account in Eqn (73) and ignoring small terms of the order of $(my)^2$, we obtain the equation

$$2\beta^2q+\beta(q-\alpha\varkappa)+\frac{m^2}{3}=0\,. \quad (78)$$

for the unknown constant β. We remark that the term containing y^2 is given by

$$-\frac{qy^2}{48}\left\{7\left[2\beta^2q+\beta(q-\alpha\varkappa)\right]+3m^2\right\}.$$

With Eqn (78), it can be rewritten as

$$-\frac{q}{72}\,m^2y^2\,.$$

By definition, we have

$$\alpha\varkappa-q=\frac{\varkappa\rho}{3}\left(3\sqrt{1-qa^2}-1\right),$$

and therefore Eqn (78) yields

$$\beta=\frac{3\sqrt{1-qa^2}-1+\left[(3\sqrt{1-qa^2}-1)^2-8m^2/\varkappa\rho\right]^{1/2}}{4}\,. \quad (79)$$

Hence, the metric coefficient U that determines the time dilation with respect to the inertial time is *nonzero*.

If the graviton rest mass is set to zero, expression (79), as expected, exactly coincides with the constant term in expression (74). From formula (79), we can find the minimum value

of β:

$$\beta_{\min} = \left(\frac{m^2}{2\varkappa\rho}\right)^{1/2} . \tag{80}$$

The value of β in the function $\sqrt{U}$ sets the bound for the process of time dilation by a gravitational field. This means that a further *slowing down* of the flow of time by the gravitational field is *impossible*. This is why the scalar curvature determined by expression (65) is finite everywhere, in contrast to GR. Thus, the gravitational field itself stops time dilation because of a nonzero graviton rest mass.

According to (79), equality (75), due to the smallness of the graviton rest mass, *cannot hold* because of the inequality

$$3\sqrt{1-qa^2} - 1 \geqslant 2\sqrt{2}\left(\frac{m^2}{\varkappa\rho}\right)^{1/2} . \tag{81}$$

With the equality

$$qa^2 = \frac{W_{\mathrm{g}}}{a}$$

which holds by definition, we use inequality (81) with $\varkappa\rho \gg m^2$ to find

$$a \geqslant \frac{9}{8}\, W_{\mathrm{g}}\left[1 + \left(\frac{m^2}{2\varkappa\rho}\right)^{1/2}\right] . \tag{82}$$

This is the *limitation on the radius of the body* that occurs in inspecting the inner solution. This constraint is *more stringent* than the bound (58) obtained in Section 3 in analyzing the outer solution. Inequality (82), as we see, directly follows from the theory, while in order to avoid infinite pressure inside a body in GR, inequality (60) must be additionally imposed. Using (70) and (71), we find the pressure as

$$\frac{p}{c^2} = \frac{-\rho\sqrt{U} + \rho\sqrt{1-qa^2}}{\sqrt{U}} .$$

Taking formula (80) into account gives the maximum pressure at the center of a ball:

$$\frac{p}{c^2} \simeq \rho\left[\frac{2\varkappa\rho}{m^2}(1-qa^2)\right]^{1/2} .$$

The pressure at the center is finite, while in GR it is infinite according to (57).

The self-limitation of the gravitational field that appears in the relativistic theory of gravity makes it principally different from Einstein's GR and Newton's theory of gravity, which involve only *attraction forces*. In the field theory of gravity, the nonzero graviton mass and the property of stopping the time dilation result in the possibility that the gravitational force not only can be an attraction force but also, under certain conditions (in strong fields), can manifest itself as an effective braking force. It is this force that stops the time dilation by the gravitational field. Thus the gravitational field cannot stop the flow of time in a physical process in principle because it is endowed with the fundamental property of *self-limitation*.

In Sections 3 and 4, we have seen that the metric coefficient U that determines the time dilation by the gravitational field can vanish in GR. This feature was noticed by R Feynman, who wrote [10]: "...if our formula for time dilation were correct, physical processes should stop at the center of the universe, since the time would not run at all. This is not the only physically unacceptable prediction; since we might expect that matter near the edge of the universe should be interacting faster, light from distant galaxies should be violet-shifted. Instead, it is well known to be shifted toward lower, redder frequencies. Thus, our formula for the time dilation obviously needs to be discussed further in connection with possible models of the universe. The following discussion is purely qualitative and is meant only to stimulate wiser thoughts on this subject."

The self-restriction of the potential, as we have seen, is an important property of the gravitational field. It is this property that sets a limit to time dilation. Such a limit must necessarily exist, otherwise physically unacceptable consequences follow. Therefore, any metric field theory of the gravitational field must accept this general statement as a *physical principle*.

5. Is the Minkowski space observable?

We now consider the question: is the Minkowski space observable in principle? For this, we write Eqns (1) as

$$\frac{m^2}{2}\,\gamma_{\mu\nu} = 8\pi G\left(T_{\mu\nu} - \frac{1}{2}\,g_{\mu\nu}T\right) - R_{\mu\nu} + \frac{m^2}{2}\,g_{\mu\nu} .$$

This shows that the right-hand side of these equations contains only geometrical characteristics of the effective Riemannian space and quantities determining the distribution of matter in this space.

We now use the Weyl–Lorentz–Petrov theorem [1], which states: "Knowing ... equations of all time-like and all isotropic geodesic lines allows the metric tensor to be determined up to a constant factor." This implies that by experimentally studying the motion of particles and light in the Riemannian space, one can *in principle* determine the *metric tensor* $g_{\mu\nu}$ of the effective Riemannian space. Then by substituting $g_{\mu\nu}$ in the above equation, one can determine the *metric tensor* of the Minkowski space. After that, using coordinate transformations, one can make a transition to an *inertial* Galillean reference frame. Therefore, the Minkowski space is principally *observable*.

Here, the words of V A Fock are relevant [12]: "How one should determine the straight line: as the light ray or the straight line in that Euclidean space where harmonic coordinates x_1, x_2, x_3 serve as the cartesian ones? We believe the second definition is only correct. In fact, we have used it when said that the light near the Sun propagates along a hyperbola, — and further on this: — considerations that the straight line, as the light ray, is more directly observable, have no significance: in definitions, the crucial is not the direct observability but the correspondence to nature, even if this correspondence is inferred from indirect considerations."

The *inertial* reference frame, as we have seen, is related to the distribution of matter in the Universe. Hence, RTG gives us the possibility *in principle* to determine the *inertial* frame.

6. Evolution of a homogeneous isotropic universe

6.1 Equations of the scale factor evolution

In a homogeneous and isotropic universe, the interval in the effective Riemannian space can be written in the form of the

Friedmann – Robertson – Walker metric:

$$\mathrm{d}s^2 = c^2 U(t)\,\mathrm{d}t^2 - V(t)\left[\frac{\mathrm{d}r^2}{1-kr^2} + r^2(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2)\right], \tag{83}$$

and the Minkowski space interval is

$$\mathrm{d}\sigma^2 = c^2\,\mathrm{d}t^2 - \mathrm{d}r^2 - r^2(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2)\,. \tag{84}$$

We write Eqns (1) and (2) of RTG in the form

$$\frac{m^2}{2}\,\gamma_{\mu\nu} = 8\pi G\left(T_{\mu\nu} - \frac{1}{2}\,g_{\mu\nu}T\right) - R_{\mu\nu} + \frac{m^2}{2}\,g_{\mu\nu}\,, \tag{85}$$

$$\partial_\mu \tilde{g}^{\mu\nu} + \gamma^{\nu}_{\lambda\sigma}\tilde{g}^{\lambda\sigma} = 0\,. \tag{86}$$

Taking into account that

$$\begin{aligned}
&\gamma^1_{22} = -r\,, \quad \gamma^1_{33} = -r\sin^2\theta\,, \quad \gamma^2_{12} = \gamma^3_{13} = r^{-1}\,,\\
&\gamma^2_{33} = -\sin\theta\cos\theta\,, \quad \gamma^3_{23} = \cot\theta\,,\\
&\tilde{g}^{00} = V^{3/2}U^{-1/2}(1-kr^2)^{-1/2}r^2\sin\theta\,,\\
&\tilde{g}^{11} = -V^{1/2}U^{1/2}(1-kr^2)^{1/2}r^2\sin\theta\,,\\
&\tilde{g}^{22} = -V^{1/2}U^{1/2}(1-kr^2)^{-1/2}\sin\theta\,,\\
&\tilde{g}^{33} = -V^{1/2}U^{1/2}(1-kr^2)^{-1/2}(\sin\theta)^{-1}\,,
\end{aligned} \tag{87}$$

we can rewrite Eqns (86) with $\nu = 0$ and $\nu = 1$ as

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{V}{U^{1/3}}\right) = 0\,, \tag{88}$$

$$-\frac{\mathrm{d}}{\mathrm{d}r}\left[(1-kr^2)^{1/2}r^2\right] + 2(1-kr^2)^{-1/2}r = 0\,. \tag{89}$$

For the components $\nu = 2$ and $\nu = 3$, Eqns (86) are satisfied identically. From Eqns (88) and (89), it follows that

$$\frac{V}{U^{1/3}} = \mathrm{const} = \beta^4 \neq 0\,, \qquad k = 0\,. \tag{90}$$

Thus, because the set of RTG equations is complete, it *leads to a unique solution: the flat* (*Euclidean*) *geometry of the Universe, in contrast to GR*.

Setting

$$a^2 = U^{1/3}\,, \tag{91}$$

we obtain

$$\mathrm{d}s^2 = \beta^6\left[c^2\,\mathrm{d}\tau_{\mathrm{g}}^2 - \left(\frac{a}{\beta}\right)^2(\mathrm{d}r^2 + r^2\,\mathrm{d}\theta^2 + r^2\sin^2\theta\,\mathrm{d}\phi^2)\right]. \tag{92}$$

Here, the quantity

$$\mathrm{d}\tau_{\mathrm{g}} = \left(\frac{a}{\beta}\right)^3\mathrm{d}t \tag{93}$$

characterizes time dilation in the gravitational field relative to the inertial time t. The overall numerical constant β^6 in the interval $\mathrm{d}s^2$ equally increases both the time and the space variables. It reflects the global dynamics of the Universe, as an integral of motion. Time in the Universe is determined by $\mathrm{d}\tau$, which is the time-like part of the interval $\mathrm{d}s^2$:

$$\mathrm{d}\tau = \beta^3\,\mathrm{d}\tau_{\mathrm{g}} = a^3\,\mathrm{d}t\,, \tag{94}$$

$$\mathrm{d}s^2 = c^2\,\mathrm{d}\tau^2 - \beta^4 a^2(\tau)(\mathrm{d}r^2 + r^2\,\mathrm{d}\theta^2 + r^2\sin^2\theta\,\mathrm{d}\phi^2)\,. \tag{95}$$

The matter energy – momentum tensor in the effective Riemannian space has the form

$$T_{\mu\nu} = (\rho + p)U_\mu U_\nu - g_{\mu\nu}p\,, \tag{96}$$

where ρ and p are the density and pressure of matter in its rest-mass frame and U_μ is its velocity. Because g_{0i} and R_{0i} vanish for interval (95), Eqn (85) implies that

$$T_{0i} = 0 \quad \text{and} \quad U_i = 0\,. \tag{97}$$

This means that in the *inertial frame* determined by interval (84), *matter is at rest* during the evolution of the Universe. Matter being at rest in the homogeneous isotropic Universe (ignoring peculiar velocities of galaxies) in some sense corresponds to the early (pre-Friedmann) concepts of Einstein about the Universe.

The so-called 'expansion of the Universe,' as inferred from observations of red shifts, is due *not to the motion of matter but to the change in gravitational field with time*. This should be borne in mind when using the commonly accepted term 'the expansion of the Universe.'

When describing interval (95) in terms of the proper time τ, the interval of the original Minkowski space (84) takes the form

$$\mathrm{d}\sigma^2 = \frac{c^2}{a^6}\,\mathrm{d}\tau^2 - \mathrm{d}r^2 - r^2(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2)\,. \tag{98}$$

Using (95) and (98) and taking into account that

$$R_{00} = -3\,\frac{\ddot{a}}{a}\,, \qquad R_{11} = \beta^4(a\ddot{a} + 2\dot{a}^2)\,, \tag{99}$$

$$\begin{aligned}
&T_{00} - \frac{1}{2}\,g_{00}T = \frac{1}{2}(\rho + 3p)\,,\\
&T_{11} - \frac{1}{2}\,g_{11}T = \frac{1}{2}\,\beta^4 a^2(\rho - p)\,,
\end{aligned} \tag{100}$$

we derive equations for the scale factor from Eqn (85) as

$$\frac{1}{a}\frac{\mathrm{d}^2 a}{\mathrm{d}\tau^2} = -\frac{4\pi G}{3}\left(\rho + \frac{3p}{c^2}\right) - \frac{1}{6}(mc)^2\left(1 - \frac{1}{a^6}\right), \tag{101}$$

$$\left(\frac{1}{a}\frac{\mathrm{d}a}{\mathrm{d}\tau}\right)^2 = \frac{8\pi G}{3}\,\rho(\tau) - \frac{1}{12}(mc)^2\left(2 - \frac{3}{a^2\beta^4} + \frac{1}{a^6}\right). \tag{102}$$

In a Universe without matter and gravitational waves, Eqns (101) and (102) have the trivial solution $a = \beta = 1$, i.e., the empty Universe does not evolve and the effective Riemannian space coincides with the Minkowski space. *We note that in our theory, the absolute value of the scale factor a acquires a physical interpretation*. For $m = 0$, Eqns (101) and (102) coincide with the Friedmann equations for a flat Universe. However, the terms with $m \neq 0$ significantly change the evolution at small and large scale factors.

The appearance of additional terms with $m^2 \neq 0$ in Eqns (101) and (102) (in particular, terms $\sim m^2/a^6$) is due to the difference in the inertial time t and the physical time τ in (94). Because gravity affects the flow of time, these terms turn

out to be large enough to influence evolution in strong gravitational fields (despite the smallness of the graviton mass). Due to the change in the inertial time in the gravitational field, forces emerge that manifest themselves as repulsion forces when the Universe contracts or attraction forces at final stages of the expansion of the Universe. The proportionality of terms in the right-hand side of Eqns (101) and (102) to the graviton mass squared is a manifestation of the fact that only when $m^2 \neq 0$ does the effective Riemannian space retain the relation to the underlying Minkowski space.

6.2 The absence of the cosmological singularity

From the covariant conservation law for the energy–momentum tensor density $\tilde{T}^{\mu\nu} = \sqrt{-g}\, T^{\mu\nu}$,

$$\nabla_\mu \tilde{T}^{\mu\nu} = \partial_\mu \tilde{T}^{\mu\nu} + \Gamma^{\nu}_{\alpha\beta} \tilde{T}^{\alpha\beta} = 0$$

(where ∇_μ is the covariant derivative and $\Gamma^{\nu}_{\alpha\beta}$ are the Christoffel symbols in the Riemannian space), which follows from Eqns (1) and (2) and expression (96), we can derive the equation

$$-\frac{1}{a}\frac{\mathrm{d}a}{\mathrm{d}\tau} = \frac{1}{3(\rho + p/c^2)}\frac{\mathrm{d}\rho}{\mathrm{d}\tau}\,. \tag{103}$$

With the equation of state $p = f(\rho)$, formula (103) determines the dependence of the matter density on the scale factor. For the equation of state in the form

$$\frac{p}{c^2} = \omega\rho\,,$$

this dependence is

$$\rho = \frac{\text{const}}{a^{3(\omega+1)}}\,.$$

For cold matter, including dark matter and baryonic matter, $\omega_{\mathrm{CDM}} = -1$; for the radiation density, $\omega_{\mathrm{r}} = 1/3$; and for the quintessence, $\omega_{\mathrm{q}} = -1 + \nu$, $\nu < 2/3$. Thus, the total density of matter in Eqns (101) and (102) is given by

$$\rho = \frac{A_{\mathrm{CDM}}}{a^3} + \frac{A_{\mathrm{r}}}{a^4} + \frac{A_{\mathrm{q}}}{a^{3\nu}}\,, \tag{104}$$

where A_{CDM}, A_{r}, and A_{q} are constants. According to (104), the radiation-dominated stage occurs in the Universe at small scale factors ($a \ll 1$):

$$\rho \approx \rho_{\mathrm{r}} = \frac{A_{\mathrm{r}}}{a^4}\,.$$

Turning to Eqn (102), we note that for $a \ll 1$, the absolute value of the negative term in the right-hand side increases as $1/a^6$. Because the left-hand side is positive definite, a minimum scale factor must exist:

$$a_{\min} = \frac{mc}{(32\pi G A_{\mathrm{r}})^{1/2}} = \left(\frac{m^2c^2}{32\pi G\rho_{\max}}\right)^{1/6}. \tag{105}$$

The presence of the minimum scale factor (105) implies that the process of time dilation by the gravitational field during the contraction of the Universe stops. Therefore, the gravitational field cannot act so as to stop the flow of time.

Thus, *the nonzero graviton mass and hence the appearance of effective forces related to the change in the flow of time eliminate the cosmological singularity* and the expansion of the Universe starts with the finite scale factor in (105). The surprising feature of the gravitational field to create repulsion forces that stop the contraction of the Universe and initiate its later accelerated expansion is fully manifested here.

We emphasize that the commonly used terms 'gravitational forces of attraction' and 'gravitational forces of repulsion' mean that the increase and decrease in density and pressure of the Universe is caused not by pressure gradients, which are absent here, but by the change in the flow of time and the volume occupied by a given mass under the action of the gravitational field changing in time.

Based on (101) and (105), we can determine the initial acceleration that was the primordial '*kick*' initiating the expansion of the Universe:

$$\frac{1}{a}\frac{\mathrm{d}^2a}{\mathrm{d}\tau^2}\bigg|_{\tau=0} = \frac{8\pi G}{3}\rho_{\max}\,.$$

Hence, according to RTG, at the radiation-dominated stage *during the period of accelerated expansion preceding* the Friedmann stage, the scalar curvature is nonzero and

$$R = -\frac{16\pi G}{c^2}\rho_{\max}$$

at $\tau = 0$, while it vanishes in GR. When the scale factor increases to

$$a^2(\tau) = \frac{3}{2}a^2_{\min}\,,$$

the Hubble constant reaches a maximum,

$$H_{\max} = 3^{-2}(32\pi G\rho_{\max})^{1/2}\,,$$

the scalar curvature is

$$R = -\left(\frac{2}{3}\right)^3 \frac{16\pi G\rho_{\max}}{c^2}\,,$$

and the invariant is

$$R_{\rho\lambda\mu\nu}R^{\rho\lambda\mu\nu} = 8\times 3^{-7}\left(\frac{32\pi G}{c^2}\rho_{\max}\right)^2.$$

Because the scalar curvature R and the invariant $R_{\rho\lambda\mu\nu}R^{\rho\lambda\mu\nu}$ depend on $\rho_{\max}$, multiple production of gravitons at the radiation-dominated stage can be expected. A nonthermal relativistic primordial gravitational wave background can originate in this way.

6.3 The impossibility of an unlimited 'expansion of the Universe'

In considering the gravitational field $\phi^{\mu\nu}$ as a physical field in the Minkowski space, the causality principle must be imposed. This means that the light cone in the effective Riemannian space must lie inside the light cone of the Minkowski space, i.e., for $\mathrm{d}s^2 = 0$, the condition $\mathrm{d}\sigma^2 \geqslant 0$ must be satisfied. Writing $\mathrm{d}\sigma^2$ in spherical coordinates as

$$\mathrm{d}\sigma^2 = c^2\,\mathrm{d}t^2 - (\mathrm{d}r^2 + r^2\,\mathrm{d}\theta^2 + r^2\sin^2\theta\,\mathrm{d}\phi^2) \tag{106}$$

and deriving the spatial part of the interval from the condition $\mathrm{d}s^2 = 0$, we obtain

$$\mathrm{d}\sigma^2 = c^2\,\mathrm{d}t^2\left(1 - \frac{a^4}{\beta^4}\right) \geqslant 0\,,$$

i.e.,

$$(a^4 - \beta^4) \leqslant 0\,. \tag{107}$$

Thus, the scale factor a is limited by the condition $a \leqslant \beta$, and it is therefore natural to accept its maximum value as

$$a_{\max} = \beta\,.$$

Choosing $a_{\max}$ in such a way provides the flow of time $\mathrm{d}\tau_{\mathrm{g}}$ at the stopping point of the expansion equal to that of the inertial time t in the Minkowski space, although the second derivative $\ddot{a}$ and, hence, the scalar curvature R are nonzero. From this point, attraction forces make the Universe contract and the time dilation $\mathrm{d}\tau_{\mathrm{g}}$ continues until the contraction stops and the reverse process of the acceleration of time $\mathrm{d}\tau_{\mathrm{g}}$ up to the flow of the inertial time t in the Minkowski space begins. *These physical consequences require that the condition $a_{\max} = \beta$ be satisfied.* As we see in what follows (see Section 6.7), the value of β is determined by an integral of motion.

Condition (107) does not allow an unlimited increase in the scale factor with time, i.e., prohibits the unbounded 'expansion' of the Universe (in the sense discussed above). We note that the Universe itself is then infinite because the radial coordinate is defined everywhere in the range $0 \leqslant r \leqslant \infty$.

6.4 Evolution of the early Universe

At the radiation-dominated stage of the Universe ($\rho = \rho_{\mathrm{r}}$) for $a \ll 1$, Eqns (101) and (102) take the form

$$\left(\frac{1}{\xi}\frac{\mathrm{d}\xi}{\mathrm{d}\tau}\right)^2 = \frac{1}{\tau_{\mathrm{r}}^2}\left(1 - \frac{1}{\xi^2}\right)\frac{1}{\xi^4}\,, \tag{108}$$

$$\frac{1}{\xi}\frac{\mathrm{d}^2\xi}{\mathrm{d}\tau^2} = \frac{1}{\tau_{\mathrm{r}}^2}\left(\frac{2}{\xi^2} - 1\right)\frac{1}{\xi^4}\,, \tag{109}$$

where

$$\xi = \frac{a(\tau)}{a_{\min}}\,, \qquad \tau_{\mathrm{r}} = \left(\frac{3}{8\pi G\rho_{\max}}\right)^{1/2}.$$

The solution of Eqn (108) is given by

$$\frac{\tau}{\tau_{\mathrm{r}}} = \frac{1}{2}\left\{\xi(\xi^2 - 1)^{1/2} + \ln\left[\xi + (\xi^2 - 1)^{1/2}\right]\right\}. \tag{110}$$

For $\xi - 1 \ll 1$ ($\tau \ll \tau_{\mathrm{r}}$),

$$a \simeq a_{\min}\left[1 + \frac{1}{2}\left(\frac{\tau}{\tau_{\mathrm{r}}}\right)^2 - \frac{7}{24}\left(\frac{\tau}{\tau_{\mathrm{r}}}\right)^4\right].$$

Adding Eqns (108) and (109) yields

$$\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2 = \frac{(mc)^2}{12a^6}\,,$$

where $\dot{a} = \mathrm{d}a/\mathrm{d}\tau$.

In GR, the left-hand side of this equation at the radiation-dominated stage is exactly zero, and hence the Friedmann stage with the scale factor $a(\tau)$ changing as $\tau^{1/2}$ is realized. In RTG, according to this equation, there is a 'pre-Friedmann' stage at the radiation-dominated epoch with the scalar curvature

$$R = -\frac{1}{2}\,(mc)^2\,\frac{1}{a^6}\,.$$

The particle horizon is equal to

$$R_{\mathrm{part}}(\tau) = a(\tau)\int_0^{\tau}\frac{c\,\mathrm{d}\tau'}{a(\tau')} \simeq c\tau\left(1 + \frac{1}{3}\frac{\tau^2}{\tau_{\mathrm{r}}^2}\right).$$

Accelerated expansion occurs, according to (109), until $\xi = \sqrt{2}$ (i.e., $a = \sqrt{2}\,a_{\min}$) during the time

$$\tau_{\mathrm{in}} = \tau_{\mathrm{r}}\,\frac{1}{2}\left[\sqrt{2} + \ln(1 + \sqrt{2})\right] \simeq 1.15\tau_{\mathrm{r}}\,.$$

The quantity $\dot{a}/a$ reaches its maximum $(\dot{a}/a)_{\max} = 2/3\sqrt{3}\tau_{\mathrm{r}}$ somewhat earlier: at $a/a_{\min} = \sqrt{3/2}$ and at $\tau \sim 0.762\,\tau_{\mathrm{r}}$. *Large acceleration when the scale factor grows from its minimum value $(\ddot{a}/a)_0 = 1/\tau_{\mathrm{r}}^2$ is established by the effective forces that appear due to the difference in the flow of time t and τ (see Eqn* (94)) *caused by gravity*. These forces are due to the term m^2/a^6 in Eqns (101) and (102). At $\tau > \tau_{\mathrm{in}}$, the deceleration follows the acceleration. When $\xi \gg 1$, the expansion (110) reaches the Friedmann regime corresponding to the radiation-dominated stage,

$$a(\tau) = a_{\min}\,\xi \simeq a_{\min}\left(\frac{2\tau}{\tau_{\mathrm{r}}}\right)^{1/2}$$

with the well-known dependence

$$\rho \simeq \rho_{\mathrm{r}}(\tau) = \frac{3}{32\pi G\tau^2}\,, \qquad \tau \gg \tau_{\mathrm{r}}\,. \tag{111}$$

For the primordial nucleosynthesis conditions to be satisfied during the first seconds after the expansion starts, it suffices to have $\tau_{\mathrm{r}} \lesssim 10^{-2}$ s. The corresponding constraint for $\rho_{\max}$ is fairly weak:

$$\rho_{\max} > 2 \times 10^{10}\ \mathrm{g\ cm^{-3}}\,.$$

The value of $\rho_{\max}$ at the electroweak energy scale $kT \simeq 1$ TeV, with all the degrees of freedom of leptons, quarks, etc. taken into account is

$$\rho_{\max} \simeq 10^{31}\ \mathrm{g\ cm^{-3}}\,,$$

and at the Grand Unification scale $kT \simeq 10^{15}$ GeV,

$$\rho_{\max} \simeq 10^{79}\ \mathrm{g\ cm^{-3}}\,.$$

Consequently, because the scale factor a cannot be zero according to RTG, no *Big Bang* could have taken place in the Universe in this theory. In the past, everywhere in the Universe, *matter* was in the gravitational field and had a high temperature and density, and then evolved as described above.

6.5 The total relative density of matter and the graviton mass

Let a_0 be the present value of the scale factor and ρ_{c}^0 be the critical density determined by the present values of the Hubble constant

$$H = \left(\frac{1}{a}\frac{\mathrm{d}a}{\mathrm{d}t}\right)_0$$

by the relation

$$H^2 = \frac{8\pi G}{3}\,\rho_c^0\,.$$

Introducing the variable

$$x = \frac{a}{a_0}$$

and the density ratios

$$\Omega_r^0 = \frac{\rho_r^0}{\rho_c^0}\,, \qquad \Omega_m^0 = \frac{\rho_m^0}{\rho_c^0}\,, \qquad \Omega_q^0 = \frac{\rho_q^0}{\rho_c^0}\,,$$

and taking formula (104) into account, we can rewrite Eqns (101) and (102) as

$$\left(\frac{1}{x}\frac{\mathrm{d}x}{\mathrm{d}\tau}\right)^2 = H^2\left[\frac{\Omega_r^0}{x^4} + \frac{\Omega_m^0}{x^3} + \frac{\Omega_q^0}{x^{3\nu}} - \frac{f^2}{6}\left(1 - \frac{3}{2\beta^4 a^2} + \frac{1}{2a^6}\right)\right], \tag{112}$$

$$\frac{1}{x}\frac{\mathrm{d}^2x}{\mathrm{d}\tau^2} = -\frac{H^2}{2}\left[\frac{2\Omega_r^0}{x^4} + \frac{\Omega_m^0}{x^3} - 2\left(1 - \frac{3\nu}{2}\right)\frac{\Omega_q^0}{x^{3\nu}} + \right.$$
$$\left. + \frac{f^2}{3}\left(1 - \frac{1}{a^6}\right)\right], \tag{113}$$

where

$$f = \frac{mc}{H} = \frac{m_g c^2}{\hbar H}\,. \tag{114}$$

For the present value of the quantities at $a_0 \gg 1$, Eqn (112) yields

$$1 = \Omega_{tot}^0 - \frac{f^2}{6}\,,$$

i.e., the total relative density is

$$\Omega_{tot}^0 = \frac{\rho_{tot}^0}{\rho_c^0} = \Omega_r^0 + \Omega_m^0 + \Omega_q^0 = 1 + \frac{f^2}{6}\,. \tag{115}$$

Thus, the Universe with the Euclidean space geometry (according to RTG) must have $\Omega_{tot}^0 > 1$, while in theories with primordial inflationary expansion leading to the flat geometry of space, the condition $\Omega_{tot}^0 = 1$ must be satisfied with high accuracy ($\sim 10^{-3} - 10^{-5}$). Equation (115) allows the graviton mass to be determined from the modern measurements of Ω_{tot}^0 and H.

6.6 The upper limit on the graviton mass

The determination of cosmological parameters from measurements of the angular inhomogeneity of the cosmic microwave background (CMB) systematically leads to the mean value $\Omega_{tot}^0 > 1$. This follows from the first quantitative experiments COBE [13], Maxima-1 [14], and Boomerang-98 [15], whose joint analysis [16] yields $\Omega_{tot}^0 = 1.11 \pm 0.07$, and from the superb data of the WMAP experiment [17–19], which alone (without invoking measurements of distant type-Ia supernovae [20, 21] or processing galaxy catalogs like 2dFGRS [22] and SDSS [23]) suggest (depending on the parameters assumed)

$$\Omega_{tot}^0 = 1.095^{+0.094}_{-0.144} \quad \text{and} \quad \Omega_{tot}^0 = 1.086^{+0.057}_{-0.128}\,.$$

Within measurement errors, these values, of course, do not contradict the flat Universe $\Omega_{tot}^0 = 1$, as predicted by inflationary cosmology; however, they can also indicate a nonzero graviton mass in accordance with Eqns (114) and (115). In any case, taking the value $\Omega_{tot}^0 = 1.3$, which is more than 2σ higher than the mean value Ω_{tot}^0, we obtain a 95%-probability upper limit on the mass of the graviton from (114) and (115). The quantity f in (114) can be conveniently expressed as the ratio of the graviton mass to the value

$$m_H = \frac{\hbar H}{c^2} = 3.80 \times 10^{-66}\,h\,,$$

which could be named the 'Hubble mass.' For $f^2/6 = 0.3$, the upper limit on the graviton mass is [24]

$$m_g \leqslant 1.34\,m_H \approx 5.1 \times 10^{-66}\,h\,,$$

or with $h = 0.70$,

$$m_g < 3.6 \times 10^{-66}\ \mathrm{g}\,. \tag{116}$$

The Compton length of a graviton turns out to be comparable to the Hubble radius of the Universe c/H:

$$\frac{\hbar}{m_g c} \leqslant 0.75\,\frac{c}{H}\,.$$

Estimates of the upper limit on the graviton mass obtained in earlier works were based on the gravitational potential of the Yukawa form for massive gravitons. In papers [22, 26], from the analysis of the dynamics of galaxy clusters and conservative estimates of distances ($\sim$ 580 kpc) where the gravitational connection between galaxies in clusters still exists, the upper limit on the graviton mass was obtained as

$$m_g < 2 \times 10^{-62}\ \mathrm{g}\,.$$

Our estimate (116) is more than 5000 times stronger than this limit. This is due to our systematic treatment of the gravitational field in the Minkowski space. Our consideration includes both the equation showing that the potential of a weak gravitational field has the Yukawa form and the general equations (1) and (2), which are consistent with all gravitational phenomena observed in the Solar system and applicable to the entire Universe at a scale of the order $c/H \simeq 10^{28}$ cm, i.e., 5000 times as high as the distance between gravitationally bound galaxies in clusters.

6.7 The evolution integral of the Universe and the present value of the scale factor

Using Eqn (103) to eliminate the pressure p from Eqn (101), we obtain

$$\frac{1}{a}\frac{\mathrm{d}^2a}{\mathrm{d}\tau^2} = \frac{4\pi G}{3}\left(a\frac{\mathrm{d}\rho}{\mathrm{d}a} + 2\rho\right) - \frac{1}{6}(mc)^2\left(1 - \frac{1}{a^6}\right),$$

and then rewrite this equation as

$$\frac{\mathrm{d}^2a}{\mathrm{d}\tau^2} + \frac{\mathrm{d}V}{\mathrm{d}a} = 0\,, \tag{117}$$

where

$$V = -\frac{4\pi G}{3}\,a^2\rho + \frac{(mc)^2}{12}\left(a^2 + \frac{1}{2a^4}\right). \tag{118}$$

Multiplying both parts of Eqn (117) by $\mathrm{d}a/\mathrm{d}\tau$, we obtain

$$\frac{\mathrm{d}}{\mathrm{d}\tau}\left[\frac{1}{2}\left(\frac{\mathrm{d}a}{\mathrm{d}\tau}\right)^2 + V\right] = 0\,,$$

or

$$\frac{1}{2}\left(\frac{\mathrm{d}a}{\mathrm{d}\tau}\right)^2 + V = E = \mathrm{const}\,. \tag{119}$$

Expression (119) resembles the energy of a unit mass. If the quantity a had had the dimension of length, the first term in (119) would have been the kinetic energy and the second term the potential energy. The quantity

$$-\frac{4\pi G}{3}\,a^2\rho$$

in (118) corresponds to the gravitational potential at the boundary of a ball with the radius a filled with matter with the constant density ρ, and additional terms in (118), proportional to m^2, correspond to the effective forces emerging due to the gravitational influence on the flow of time, as we discussed above.

The quantity E is an integral of the evolution of the Universe. It is extremely small but nonzero if $m \neq 0$. Expressing $(\mathrm{d}a/\mathrm{d}\tau)^2$ from Eqn (102) and substituting it in Eqn (119), we find

$$E = \frac{(mc)^2}{8\beta^4}\,. \tag{120}$$

Therefore, the constant β [see (107)], which enters the expression for interval (95) and according to (107) bounds the growth of the scale factor a, can be expressed through the integral of motion E.

In what follows, we need the present value of the scale factor a_0. It can be estimated from the following considerations. Assuming the evolution of the Universe to begin at the radiation-dominated epoch, we express $a_0/a_{\min}$ as

$$\frac{a_0}{a_{\min}} = \left(\frac{\rho_{\max}}{\rho_{\mathrm{r}}^0}\right)^{1/4},$$

where ρ_{r}^0 is the present-day radiation energy density. In turn, ρ_{r}^0 can be expressed through the relative density Ω_{r}^0 and the critical density ρ_{c}^0:

$$\rho_{\mathrm{r}}^0 = \Omega_{\mathrm{r}}^0\rho_{\mathrm{c}}^0 = \Omega_{\mathrm{r}}^0\,\frac{3H^2}{8\pi G}\,.$$

Therefore,

$$\frac{a_0}{a_{\min}} = \left(\frac{8\pi}{3}\,\frac{G\rho_{\max}}{H^2\Omega_{\mathrm{r}}^0}\right)^{1/4} \approx 1.34\times 10^{10}(G\rho_{\max})^{1/4}\,,$$

where $G\rho_{\max}$ is expressed in s^{-2}. (The numerical coefficient in this expression was calculated using the standard values $H = h/3.0857\times 10^{17}c$ and $\Omega_{\mathrm{r}}^0 = \Omega_{\gamma}^0 = 2.471\times 10^{-5}/h^2$.)

Then, using definition (114), we can express the value $a_{\min}$ from (105) as

$$a_{\min} = \left(\frac{f^2}{6}\right)^{1/6}\left(\frac{3}{16\pi}\,\frac{H^2}{G\rho_{\max}}\right)^{1/6} = 8.21\times 10^{-7}\left(\frac{f^2}{6}\right)^{1/6}\frac{1}{(G\rho_{\max})^{1/6}}\,,$$

where

$$\frac{f^2}{6} = \Omega_{\mathrm{tot}}^0 - 1$$

in accordance with (115). On the electroweak scale,

$$a_{\min} \simeq 5\times 10^{-11}\,,$$

and on the GUT scale,

$$a_{\min} \simeq 5\times 10^{-19}\,.$$

From the relation $a_0/a_{\min}$ for a_0, we have[4]

$$a_0 = \left(\frac{f^2}{6}\right)^{1/6}\left(\frac{2\pi}{3}\,\frac{G\rho_{\max}}{H^2}\right)^{1/12}\frac{1}{(\Omega_{\mathrm{r}}^0)^{1/4}} \simeq 1.1\times 10^4\left(\frac{f^2}{6}\right)^{1/6}(G\rho_{\max})^{1/12}\,. \tag{121}$$

For $\rho_{\max}$ taken on the electroweak scale, we have

$$a_0 \simeq 5\times 10^5\,,$$

and on the GUT scale,

$$a_0 \simeq 5.5\times 10^9\,.$$

In RTG, the absolute value of the scale factor acquires meaning (see Section 6.1). For the mean value $\Omega_{\mathrm{tot}} = 1.02$ (i.e., $f^2/6 = 0.02$) and $\rho_{\max} \gtrsim 10^{10}\ \mathrm{g\,cm^{-3}}$, the value $a_0 \gg 1$. This justifies approximations made in deriving Eqn (115).

6.8 Incompatibility of RTG with the nonzero cosmological term (ΛCDM-cosmology). The need for a quintessence with $\nu > 0$

As noted above, in considering the gravitational field as a physical field in the Minkowski space, the causality principle must be satisfied. This requirement, when applied to the evolution of the Universe, leads to inequality (107), according to which the scale factor is bounded by the inequality $a \leqslant a_{\max} = \beta$. In other words, according to RTG, unlimited expansion of the Universe is impossible. Mathematical equations of RTG automatically ensure this condition *when the matter density decreases as the scale factor increases*. Indeed, the structure of the *term* proportional to m_{g}^2 in Eqn (102) is such that the positive definite left-hand side of the equation implies that the third term in parentheses ensures the absence of the cosmological singularity at $a \ll 1$ and the first term restricts the minimum value of the matter density

[4] For numerical estimates, the relative density of relativistic particles Ω_{r}^0 is taken to be equal to the CMB energy density Ω_{γ}^0, because neutrino oscillations suggest that at least two types of neutrino are nonrelativistic at present. When extrapolating to the early Universe, one should of course take into account that the CMB temperature increases due to $\mathrm{e}^+\mathrm{e}^-$ annihilations. Before that, it was equal to the temperature of the neutrino gas, which at that time consisted of relativistic neutrinos and contributed to the total density of relativistic particles. Similarly, when extrapolating to the early Universe, the density of the relativistic gas increases due to the creation of other relativistic particles. But because the quantity Ω_{r}^0 enters (121) as $(\Omega_{\mathrm{r}}^0)^{1/4}$, the numerical coefficient in (121) changes less than three times (even if we assume the number of the degrees of freedom in the relativistic gas to be as high as 100).

from below (and hence, the scale factor from above) for $a \gg 1$. The condition

$$\frac{8\pi G}{3}\,\rho - \frac{(mc)^2}{6} = 0\,,$$

written as

$$\frac{H^2}{\rho_c^0}\,\rho - \frac{(mc)^2}{6} = 0\,,$$

where H is the present value of the Hubble constant, leads to the equality

$$\rho_{\min} = \frac{(mc)^2}{6H^2}\,\rho_c^0\,,$$

or, in another form,

$$\frac{\rho_{\min}}{\rho_c^0} = \frac{f^2}{6} = \Omega_{\text{tot}}^0 - 1\,. \tag{122}$$

The field theory of gravity turns out to be inconsistent with the existence of a constant cosmological term leading to unlimited expansion of the Universe. Indeed, when $a \gg 1$, Eqn (112) and the causality principle imply

$$\Omega_\Lambda^0 < \frac{f^2}{6}\,.$$

But this inequality is inconsistent with the condition

$$\Omega_\Lambda^0 > \frac{f^2}{6}\,,$$

which is required for accelerated expansion to be realized at the present time in accordance with Eqn (113).

Thus, the only possibility in the framework of RTG to explain the accelerated expansion of the Universe observed presently is to assume the existence of a quintessence with $v > 0$ or another substance whose density decreases with the increase in the scale factor (but slower than const$/a^2$*). RTG excludes the possible existence of both the cosmological constant term ($v = 0$) and 'phantom' expansion with ($v < 0$)* [27].

6.9 The beginning and the end of the present accelerated expansion

The strongest constraints on the value $\Omega_{\text{tot}}^0 = 1.018^{+0.013}_{-0.022}$ are inferred from the WMAP experiment [17–19] using the ΛCDM-model and the SDSS galaxy catalog in combination with the SNIa data. These results allow $\Omega_{\text{tot}}^0 = 1.03$ within 1σ uncertainty. The difference of this value from unity, according to RTG relations (114) and (115), *determines the graviton mass*

$$m_g = 0.424\,m_H = 1.6 \times 10^{-66}\,h\,.$$

For definiteness, we use this value of the graviton mass in the estimates below. Because $\Omega_r \ll \Omega_m$ and $a \gg 1$ before the beginning of the present accelerated expansion, it follows that the beginning and the end of the accelerated expansion is determined in accordance with (113) by roots $x_1 < x < x_2$ of the equation $F(x) = 0$, where

$$F(x) = \frac{\Omega_m^0}{x^3} - 2\left(1 - \frac{3v}{2}\right)\frac{\Omega_q^0}{x^{3v}} + \frac{f^2}{3}\,.$$

The accelerating stage is possible if $v < 2/3$. The value of the first root x_1 is related to the redshift Z_1 corresponding to the start of the acceleration stage:

$$\frac{1}{x_1} = \frac{a_0}{a_1} = Z_1 + 1\,. \tag{123}$$

The time since the beginning of expansion until the present acceleration stage can be found from Eqn (112). If we neglect the duration of the radiation-dominated stage and the value of the scale factor at its end, we obtain

$$\tau_1 \approx \frac{1}{H}\int_0^{x_1} \frac{\mathrm{d}x}{x\big(\Phi(x)\big)^{1/2}} = \frac{1}{H}\int_{Z_1+1}^{\infty} \frac{\mathrm{d}y}{y(\Omega_m^0 y^3 + \Omega_q^0 y^{3v} - f^2/6)^{1/2}}\,,$$

where

$$\Phi(x) = \frac{\Omega_m^0}{x^3} + \frac{\Omega_q^0}{x^{3v}} - \frac{f^2}{6}$$

and we assumed $\Omega_m^0 = 0.27$, $\Omega_q^0 = 0.73$ following Refs [17–19].

Accordingly, the time of the end of the accelerated expansion and the transition to deceleration is

$$\tau_2 = \frac{1}{H}\int_0^{x_2} \frac{\mathrm{d}x}{x\big(\Phi(x)\big)^{1/2}}\,,$$

and the present age of the Universe is

$$\tau_0 = \frac{1}{H}\int_0^{1} \frac{\mathrm{d}x}{x\big(\Phi(x)\big)^{1/2}}\,.$$

The physical distance traveled by light (the particle horizon) up to the present time is determined by the relation

$$D_{\text{part}}(\tau_0) = \frac{c}{H}\int_1^{a_0/a_{\min}} \mathrm{d}y\,\left[\Omega_r^0 y^4 + \Omega_m^0 y^3 + \Omega_q^0 y^{3v} - \frac{f^2}{6}\left(1 + \frac{y^6}{2a_0^6}\right)\right]^{-1/2} \simeq \frac{2}{\sqrt{\Omega_m^0}}\,\frac{c}{H}\,.$$

This value determines the size of the presently observable Universe. Qualitatively (using arbitrary scales), the time dependence of the scale factor and its derivatives $\dot{a}$ and $\ddot{a}$ are shown in Fig. 1.

6.10 The maximum value of the scale factor and the evolution integral of the Universe

The time corresponding to the end of the accelerated expansion and the transition to deceleration stopping the expansion strongly depends on the parameter v (see Table 1).

Table 1. The time of the beginning of accelerated expansion of the Universe τ_1, the time of its end τ_2, and the duration of the maximum expansion (the oscillation half-period) $\tau_{\max}$, in billions of years.

v	τ_1	τ_2	$\tau_{\max}$
0.05	7.0–8.2	980–1080	1220–1360
0.10	7.0–8.2	440–485	620–685
0.15	7.1–8.3	275–295	430–460
0.20	7.1–8.3	190–205	325–347
0.25	7.2–8.5	142–149	263–280
0.30	7.5–8.7	109–113	227–235

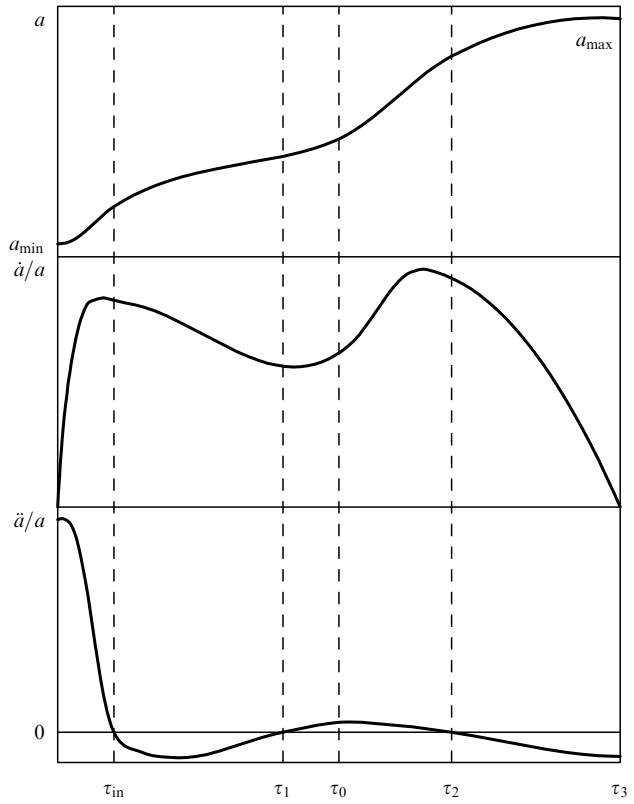

Figure 1. The qualitative time dependences of the scale factor a, the expansion velocity $\dot{a}/a$, and the acceleration $\ddot{a}/a$. Here, $\tau_{\text{in}} = 1.15\tau_{\text{r}}$ and τ_0 is the present time. Initially, the scale factor increases from some minimum value $a_{\min}$ with a very large acceleration that vanishes over a quite short period of time τ_{in}. During this period, the expansion velocity increases from zero to a maximum value, while the scale factor does not increase significantly: $a(\tau_{\text{in}}) = \sqrt{2}a_{\min}$. Then the expansion occurs with a negative acceleration that vanishes at some instant τ_1. The expansion velocity decreases and at an instant slightly later than τ_1 reaches some minimum. The scale factor continues to increase here (the expansion continues). The motion with positive acceleration proceeds up to some instant τ_2. The expansion velocity and the scale factor increase. For $\tau > \tau_2$, the motion with negative acceleration resumes and proceeds up to the instant τ_3, when it stops. The scale factor reaches its maximum value, the half-cycle ends, and everything repeats in the reverse order: contraction alternates with expansion. The first maximum of $\dot{a}/a$ is reached at $a = \sqrt{3/2}\,a_{\min}$ ($\tau \sim 0.76\,\tau_{\text{r}}$), somewhat earlier than τ_{in}; similarly, the second maximum occurs earlier than τ_2. The minimum of $\dot{a}/a$, in contrast, is reached later than τ_1. This follows from the negative value of $(\mathrm{d}/\mathrm{d}\tau)(\dot{a}/a) = \ddot{a}/a - \dot{a}^2/a^2$ at $\ddot{a} = 0$.

The scale factor corresponding to the end of expansion is determined by the root of Eqn (112) and for small v is, to high accuracy,

$$x_{\max} \simeq \left(\frac{6\Omega_{\text{q}}^0}{f^2}\right)^{1/3v} = \left(\frac{\Omega_{\text{q}}^0}{\Omega_{\text{tot}}^0 - 1}\right)^{1/3v}. \tag{124}$$

After substituting a_0 from Eqn (121) in (124), we find

$$a_{\max}^4 = \frac{1}{\Omega_{\text{r}}^0}\left(\frac{f^2}{6}\right)^{2/3}\left(\frac{2\pi}{3}\,\frac{G\rho_{\max}}{H^2}\right)^{1/3}\left(\frac{\Omega_{\text{q}}^0}{\Omega_{\text{tot}}^0 - 1}\right)^{4/3v}.$$

Taking this equality into account and considering the integral of motion

$$E = \frac{(mc)^2}{8a_{\max}^4},$$

we obtain

$$E = \frac{(mc)^2}{8}\,\Omega_{\text{r}}^0\left(\frac{6}{f^2}\right)^{2/3}\left(\frac{3}{2\pi}\,\frac{H^2}{G\rho_{\max}}\right)^{1/3}\left(\frac{\Omega_{\text{tot}}^0 - 1}{\Omega_{\text{q}}^0}\right)^{4/3v}.$$

This shows that the integral of motion for the evolution of the Universe is extremely small. Using the expression for $x_{\max}$, it is easy to determine the relative attraction acceleration at the end of expansion

$$\frac{\ddot{a}}{a} \sim -\frac{v}{4}\left(\frac{m_{\text{g}}c^2}{\hbar}\right)^2,$$

and hence the scalar curvature

$$R = \frac{3v}{2c^2}\left(\frac{m_{\text{g}}c^2}{\hbar}\right)^2.$$

It is essential that the relative minimum value of the density $\rho_{\min}/\rho_{\text{c}}^0$ corresponding to the expansion maximum depend only on the difference $\Omega_{\text{tot}}^0 - 1$, i.e., on the graviton mass [see (115) and (116)]. For $\Omega_{\text{tot}}^0 = 1.02$, the value of $\rho_{\min}$ is quite large and even strongly exceeds the *present radiation density*. In paper [24], the authors used the present age of the Universe $(13.7 \pm 0.2) \times 10^9$ years from [17, 19], which is inferred using the ΛCDM-*model*. It is very important that the new observations of SN1a [28, 29] at $Z \gtrsim 1$ provide *direct information* on the beginning of the present acceleration. According to the data obtained in the seminal paper [29], the present acceleration alternates with deceleration at the red-shift

$$Z = 0.46 \pm 0.13\,.$$

This result is consistent with the evolutionary picture considered here. It allows the direct determination of x_1 [see (123)] and precise measurement of cosmological parameters.[5]

The expansion to the maximum scale factor and the subsequent contraction lead to the oscillating evolution of the Universe. The concept of the oscillating Universe has been argued earlier by many authors, primarily from philosophical considerations (see, e.g., Refs [30–32]). This regime, in principle, could be expected in the closed Friedmann model with $\Omega_{\text{tot}} > 1$. But the need to pass through the cosmological singularity in this model and the entropy increase from cycle to cycle [32] seam to be unsurmountable obstacles.

We emphasize that in the framework of the Hilbert–Einstein equations, a flat Universe cannot be oscillating.[6] In RTG, an infinite Universe does not encounter these

[5] We note that the distance to supernovae (D_L) derived from the relation $F = L/4\pi D_L^2$ (where L is the luminosity of the standard SN1a and F is the flux obtained) is expressed through the cosmological parameters in RTG as

$$D_L = \frac{c}{H}(Z+1)\int_1^{1+Z}\left[\Omega_{\text{m}}^0 y^3 + \Omega_{\text{q}}^0 y^{3v} - \frac{f^2}{6}\right]^{-1/2}\mathrm{d}y\,.$$

[6] Paper [33] on the oscillating evolution of the Universe is erroneous because the 'solution' given there is in fact not a solution of the original Hilbert–Einstein equations, as can be checked by direct substitution. Also erroneous is paper [34] because the system of equations (3), (17), and (18) in that paper is inconsistent. We also note that Eqn (21) in paper [34] directly contradicts the system of equations (17) and (18), because (21) implies that $\dot{R}$ is a discontinuous function, while (17) and (18) show that $\dot{R}$ is continuous.

difficulties. *Because there is no singularity in RTG, the Universe could have existed for an infinite period of time during which its different regions interacted, which ultimately led to homogeneity and isotropy of the Universe* with some structural inhomogeneity that we neglected in our study for simplicity.

In this approximation, the scale factor x_2 corresponding to the end of the accelerated expansion is related to $x_{\max}$ as

$$x_2 = \left(1 - \frac{3}{2}\,v\right)^{1/3v} x_{\max} \approx \frac{1}{\sqrt{\mathrm{e}}}\, x_{\max}\,.$$

The time corresponding to the stopping of the expansion (the oscillation half-period) for the graviton mass $m_{\mathrm{g}} = 0.49\, m_{\mathrm{H}}$ chosen in [24] is about 1300×10^9 years at $v = 0.05$, 650×10^9 years at $v = 0.10$, and around 270×10^9 years at $v = 0.25$.

The attractiveness of the oscillating Universe was noted in the recent paper [35]. The oscillating regime in that model is due to introducing a scalar φ field interacting with matter and using the concept of extra dimensions. Important ideas have been put forward that the stage of accelerated expansion helps to preserve entropy in the repeating evolutionary cycles. *In RTG, the oscillating evolution of the Universe is entirely due to treating the gravitational field as a physical field generated by the total energy–momentum tensor in the Minkowski space.*

7. Conclusion

The above results suggest that the field approach, in which the gravitational field is considered a classical physical field in the Minkowski space, leads to the conclusion that the gravitational field can both slow down the flow of time and stop doing so, and, hence, stop the gravitational collapse of matter. This is in full agreement with the general statement: *if the gravitational field in a physical theory can slow down time, this theory must stop slowing down the flow of time, otherwise the flow of time would be fully stopped by the gravitational field, which is physically unacceptable.*

This is revealed in the property of 'self-limitation' of the gravitational field precluding the unlimited compression of matter. In an isotropic and homogeneous Friedmann Universe, this property is responsible for the cyclic evolution from the maximum matter density to the minimum value, etc. This model solves the well-known cosmological paradoxes [such as singularity, causality (horizon), flatness (Euclidean space)].

The theory implies the existence of a large hidden mass ('dark matter') in the Universe. This conclusion was made in 1984 in paper [36]. More accurate observational measurements of the total relative density of matter in the Universe Ω_{tot} could serve as a critical test for our theory. The 'self-limitation' property of the gravitational field precludes the existence of 'black holes' (objects without material boundaries and 'isolated' from the surrounding world).

According to our theory, the spherically symmetric accretion of matter onto a massive body at the final evolutionary stage (after the nuclear resources have been exhausted) must be accompanied by an appreciable energy release, while such an accretion onto a 'black hole' proceeds with a small energy release because the infalling matter brings energy inside the 'black hole.' Searches for and observations of such objects could shed light on the fate of massive stars at the final evolutionary stages after their nuclear resources have been exhausted.

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