

CONFERENCES AND SYMPOSIA

PACS number: 01.10.Fv

Scientific session of the Physical Sciences Division of the Russian Academy of Sciences (19 April 2006)

A scientific session of the Physical Sciences Division of the Russian Academy of Sciences (RAS) was held in the Conference Hall of the P N Lebedev Physics Institute, RAS on 19 April 2006. The following reports were presented at the session:

(1) **Kagan M Yu, Klaptsov A V, Brodsky I V** (P L Kapitza Institute for Physical Problems, RAS, Moscow), **Combescot R, Leyronas X** (Ecole Normale Supériure, Paris, France) "Composite fermions and bosons in ultracold gases and in high-temperature superconductors";

(2) **Andriyash A V, Loboda P A, Lykov V A, Politov V Yu, Chizhkov M N** (Russian Federal Nuclear Center 'E I Zababakhin All-Russian Research Institute of Technical Physics', Snezhinsk, Chelyabinsk region) "Lasers and high energy density physics at the All-Russian Research Institute of Technical Physics";

(3) **Murtazaev A K** (Institute of Physics, Dagestan Scientific Center, RAS, Makhachkala) "Monte Carlo studies of critical phenomena in spin lattice systems";

(4) **Cherepenin V A** (Institute of Radioengineering and Electronics, RAS, Moscow) "Relativistic multiwave oscillators and their possible applications".

An abridge version of the reports is given below.

PACS numbers: 03.75.Mn, **21.45.+v**, 32.80.Pj
DOI: 10.1070/PU2006v049n10ABEH006097

Composite fermions and bosons in ultracold gases and in high-temperature superconductors

M Yu Kagan, A V Klaptsov, I V Brodsky,
R Combescot, X Leyronas

1. Introduction

The physics of ultracold quantum gases has recently — and deservedly — received a great deal of attention from both theorists and experimentalists, working at the intersection of condensed matter and atomic physics [1].

Indeed, ultracold Fermi – Bose gases and Fermi – Bose mixtures in magnetic and dipole traps provide an excellent testing ground for the theory of strongly correlated electron systems in its various forms now in existence. Furthermore, the experimental possibility of varying in a controlled way the density and interaction parameters of ultracold gases — and hence the possibility of proceeding from the weak-coupling regime to the more realistic strong-coupling regime — makes magnetic traps and optical lattices ideal tools for checking the currently most popular scenarios of high-temperature superconductivity (HTSC). The recent flurry of experimental and theoretical work on ultracold Fermi – Bose gases was largely stimulated by the experimental realization of the Feshbach effect in this class of systems [2]. With the help of this effect it is possible, by applying an external magnetic field, to change abruptly the sign and magnitude of the scattering amplitude in the quasiresonant case (if there is a real or virtual shallow level in the interaction potential of two particles). As the magnetic field passes through the resonant value B_0, the interaction in the system abruptly reverses sign, so that for $B > B_0$ we proceed from the positive scattering length $a > 0$ (corresponding to the formation of real molecules) to a negative one, $a < 0$ (corresponding to the absence of a real bound state). Moreover, as $B \to B_0$ the effective scattering length can reach an absolute value of 2000 to 3000 Å, whereas the bare quasiresonant scattering length is typically around 15 – 20 Å in the absence of a magnetic field. Figure 1 presents a schematic illustration of the Feshbach resonance. The magnetic field dependence of the scattering length can be expressed analytically by the well-known formula [2]

$$a = a_{\mathrm{bg}}\left(1 + \frac{\Delta}{B - B_0}\right), \qquad (1)$$

where a_{bg} is the bare scattering length, and Δ is the Feshbach resonance width measured in gausses. As $B \to B_0$, the

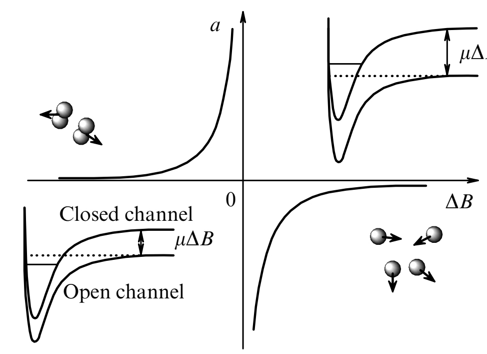

Figure 1. Schematic illustration of the Feshbach effect. Applying a magnetic field causes two energy terms to cross in the open and closed channels, changing the system from one with a bound state for $a > 0$ to one with a virtual state for $a < 0$. $\Delta B = B - B_0$ is the deviation of the magnetic field B from the Feshbach resonance field B_0, and μ is the magnetic moment of the atom.

Uspekhi Fizicheskikh Nauk **176** (10) 1105 – 1130 (2006)
Translated by E N Ragozin, E G Strel'chenko, E Yankovsky;
edited by A Radzig

scattering length $a \to \infty$, taking the system to the so-called unitary limit for quantum gases. Notice that for an ultracold Fermi gas the case $a > 0$ corresponds to the formation of a molecule (a composite boson) consisting of two elementary fermions $f_\uparrow f_\downarrow$. The energy of this bound state is given by

$$E_b = -\frac{1}{ma^2}\,. \qquad (2)$$

As applied to a Fermi – Bose mixture with a fermion resonantly interacting with a boson, the case $a > 0$ for the fermion – boson scattering amplitude corresponds to the formation of a molecule (a composite fermion) consisting of an elementary boson b and an elementary fermion f [3]. If the fermion and the boson are of equal mass, $m_B = m_F$, then their binding energy is again given by $E_b = -1/(ma^2)$. Finally, in a Bose gas with two sorts of bosons, the case $a > 0$ for the scattering amplitude of a boson of one sort from a boson of another sort corresponds to a composite boson ($b_1 b_2$ molecule) being formed [4]. It should be noted that both composite bosons $f_\uparrow f_\downarrow$ ($^6Li_2$, $^{40}K_2$) and composite fermions fb ($^{40}K + {}^{87}Rb$) have recently been observed in optical dipole traps in Feshbach resonance experiments by the groups of Jin [5], Ketterle [6], Grimm [7], and Salomon [8].

In this article we consider composite fb fermions and composite $f_\uparrow f_\downarrow$, $b_1 b_2$ bosons in Fermi – Bose gases and Fermi – Bose mixtures with a resonantly large scattering amplitude ($a \gg r_0$, where r_0 is the range of the potential). We will determine exactly the scattering amplitude of an elementary fermion (boson) from a molecule (a composite boson or fermion) and the scattering amplitude of a molecule from a molecule in resonant 3D and 2D Fermi – Bose gases, and we will investigate the role of these amplitudes in constructing phase diagrams for this class of systems. We will also determine exactly all the bound state energies of three- and four-member complexes of resonantly interacting particles in the 2D case where we will show that the number of bound states is finite. We will conclude by briefly discussing the probability that five or more resonantly interacting particles can form a complex in a 2D Fermi – Bose gas and a Fermi – Bose mixture. Finally, we will touch briefly on a possible HTSC scenario in which two composite holes (two spin polarons or two strings) form a local pair in the d-wave channel. What we mean by a composite hole is in fact a bound state between a spinon $f_{i\sigma}$ and a holon b_i in a confining string potential that arises as a hole moves in an antiferromagnetic (AFM) background in the 2D and 3D cases [9, 10].

2. Scattering of a molecule by an atom

Constructing phase diagrams for resonant gases requires the knowledge of the scattering amplitudes for three or four particles, namely, a_{2-1}, the scattering amplitude of a molecule from an atom, and a_{2-2}, the scattering amplitude of a molecule from a molecule.

If there is a strong attraction between the particles, then the binding energies for trios, E_3, for quartets, E_4, and for large droplets containing $N > 4$ particles also need to be calculated. Notice that in the bare (Born) approximation for three resonantly interacting $f_\uparrow f_\downarrow$ and $f_{\uparrow,\downarrow}$ fermions, the sign of the scattering amplitude a_{2-1} corresponds to attraction. Similarly, the fermion $f_{\uparrow,\downarrow}$ repels itself from the molecule fb composed of a fermion and a boson [3]. On the other hand, the Born approximation predicts the boson b to be attracted to the molecule bb. Similarly, the boson b is attracted to the molecule bf [3]. The only factor causing the bare interaction to differ in sign is the Pauli principle (i.e., the statistics of the interacting particles).

3. Skorniakov – Ter-Martirosian equations

With the signs of the bare interaction now known, we are in a position to exactly determine the scattering amplitudes a_{2-1} for three-particle complexes. This is done by exactly solving the so-called Skorniakov – Ter-Martirosian integral equations [11], whose graphic representation is given in Fig. 2. In the case of three resonantly interacting particles, Skorniakov – Ter-Martirosian equations can be written analytically as [11, 12]

$$T_3(p_1, p_2, P) = \mp G(P - p_1 - p_2)$$
$$\mp \mathrm{i} \int \frac{d^4 q}{(2\pi)^4}\, G(P - p_1 - q) G(q) T_2(P - q) T_3(q, p_2, P)\,, \qquad (3)$$

where, for three fermions, p_1, p_2 are the initial and final four-momentum vectors of an elementary fermion; $P - p_1$ and $P - p_2$ are the initial and final four-momenta for the molecule $f_\uparrow f_\downarrow$; P is the total four-momentum; q is the intermediate four-momentum of an elementary fermion; $d^4 q = d^3\mathbf{q}\, d\Omega_q$ is the measure of integration over an intermediate four-momentum vector, the '$-$' sign stands for the bare repulsion of the fermion $f_{\uparrow,\downarrow}$ by the molecules $f_\uparrow f_\downarrow$ and fb, and the '$+$' sign for the bare attraction of a boson by the molecules bb and bf. To evaluate the scattering amplitudes a_{2-1} of a fermion from the molecule $f_\uparrow f_\downarrow$ we can, without loss of generality, assume that $P = \{E, \mathbf{P}\} = \{-|E_b|, 0\}$ and $p_2 = 0$ in Eqn (3).

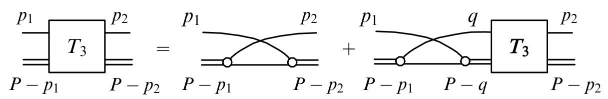

Figure 2. Skorniakov – Ter-Martirosian equation for three-particle scattering represented graphically. T_3 is the exact three-particle T matrix, while the double line denotes the exact two-particle T-matrix T_2.

It should be noted that in Eqn (3) G is the single-particle Green's function [in a vacuum $G = 1/(\omega - p^2/(2m) + \mathrm{i}0))$], the free term corresponds to the Born approximation, T_2 is the two-particle T matrix, and T_3 is the three-particle T matrix.

In the 3D case, the required T-matrix for two particles of equal mass takes the form [12]

$$T_2(\omega, \mathbf{p}) = \gamma\, \frac{4\pi}{m^{3/2}}\, \frac{\sqrt{|E_b|} + \sqrt{p^2/(4m) - \omega}}{\omega - p^2/(4m) + |E_b|}\,, \qquad (4)$$

where $|E_b| = 1/(ma^2)$ is the absolute value of the two-particle binding energy: $\gamma = 2$ for indistinguishable particles, and $\gamma = 1$ for two different particles (e.g., a fermion and a boson, or two bosons of different sorts).

Similarly, in the 2D case one finds

$$T_2(\omega, \mathbf{p}) = \gamma\, \frac{4\pi}{m}\, \frac{1}{\ln \dfrac{p^2/(4m) - \omega}{|E_b|}}\,, \qquad (5)$$

where, as in the case of three dimensions, $\gamma = \{1, 2\}$.

The three-particle T-matrix T_3 is related to $a_{2-1}(\mathbf{k})$, the scattering amplitude of an elementary particle from a

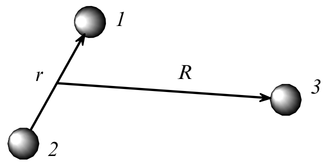

Figure 3. The coordinate R is the distance from the elementary boson *3* to the center of gravity of the molecule composed of two bosons *1* and *2*.

molecule, by the relationship

$$\frac{8\pi}{m^2 a}\, T_3\left(\left\{\frac{k^2}{2m}, \mathbf{k}\right\}, 0, -|E_b|\right) = \frac{3\pi}{m}\, a_{2-1}(\mathbf{k})\,. \tag{6}$$

Substituting $a_{2-1}(\mathbf{k})$ and $T_2(\omega, \mathbf{p})$ into Eqn (3) and integrating over frequencies one arrives at the relationship for three fermions in the 3D case:

$$\frac{(3/4)a_{2-1}(\mathbf{k})}{\sqrt{m|E_b|} + \sqrt{3k^2/4 + m|E_b|}} = \frac{1}{k^2 + m|E_b|} - 4\pi \int \frac{a_{2-1}(q)}{q^2\left(k^2 + q^2 + \mathbf{kq} + m|E_b|\right)} \frac{d^3\mathbf{q}}{(2\pi)^3}\,. \tag{7}$$

Solving this equation we find that the s-wave scattering length of a fermion by a molecule equals

$$a_{2-1}(0) = 1.18a\,, \tag{8}$$

consistent with the classical Skorniakov–Ter-Martirosian result.

Note that in the case of bare attraction between an elementary particle and a molecule, the only thing which is needed in fact to determine the binding energy of a three-particle complex reduces to solving the homogeneous Skorniakov–Ter-Martirosian equation and to determining the pole of the three-particle vertex of T_3 from it. There is, however, an important point to make here. A kernel analysis of Eqn (3) for three bosons bbb or two bosons and a fermion, fbb, shows that in the 3D case the homogeneous Skorniakov–Ter-Martirosian equation has a solution independent of how large the absolute values of the negative energy are chosen to be [13]. In reality, however, $|E_3| < 1/(mr_0^2)$, where $r_0 \ll a$ is the range of potential, and there exist $N \sim (1/\pi)\ln(a/r_0)$ three-particle levels [14] on the energy interval $1/(ma^2) < |E_3| < 1/(mr_0^2)$. This phenomenon — known as the Efimov effect [15] — is related to the presence of the attractive potential $V_{\text{eff}} \sim -\alpha/R^2 < 0$ (see Fig. 3) in the three-particle Schrödinger equation for three bosons in the 3D case.

So what we come up with in a three-particle system is the phenomenon of falling to the center and the possibility of appearing the arbitrarily strongly bound states in the 3D case. Analysis shows that the Efimov effect occurs in space dimensions $2.3 < D < 3.8$ [16]. In the 2D case, the Efimov effect is absent [16]. The three-particle Schrödinger equation now contains a repulsive term $V_{\text{eff}} \sim \beta/R^2 > 0$, making negative-energy three-particle levels $|E_3|$ finite in number in the 2D case.

The exact solution of the Skorniakov–Ter-Martirosian equation in the 2D case shows that for three resonantly interacting bosons there are only two levels with binding energies [17]

$$E_3^{(1)} = 16.4E_b\,, \qquad E_3^{(2)} = 1.3E_b\,. \tag{9}$$

Similarly, for a boson b scattered by a composite fermion fb or for a boson of one kind b_1 scattered by a molecule $b_1 b_2$ (composed of two unlike bosons) there is only one bound level [12]

$$E_3 = 2.4E_b\,, \tag{10}$$

for the fermion mass equal to that of the boson, $m_B = m_F$. It should be emphasized that the binding energies of the three-particle complexes (9) and (10) are functions of the two-particle binding energy $|E_b|$ alone in the 2D case.

4. Scattering of a molecule by a molecule

We next consider four resonantly interacting particles in 3D and 2D cases and calculate the scattering amplitude a_{2-2} of a molecule from a molecule and the bound state energies E_4. In the bare (Born) approximation, it can be shown — again based on the Pauli principle (i.e., the particle statistics) alone — that the two molecules $f_\uparrow f_\downarrow$ and $f_\uparrow f_\downarrow$ repel each other. At the same time, the two other molecules (for example, bb and bb) attract each other.

We have been able to derive exact integral equations for the quartets [12] — the analogue of the Skorniakov–Ter-Martirosian equation for the trios. A graphical representation of these equations is given in Fig. 4.

When represented algebraically, these equations take the following form [12]:

$$\begin{aligned}\Phi(q_1, q_2, p_2, P) &= -G(P - q_1 + p_2)G(P - q_2 - p_2) \\ &- \mathrm{i}\int \frac{d^4k}{(2\pi)^4}\, G(k)G(2P - q_1 - q_2 - k) \\ &\times T_2(2P - q_1 - k)\Phi(q_1, k, p_2, P) \\ &+ \frac{1}{2}\iint \frac{d^4Q}{(2\pi)^4}\frac{d^4k}{(2\pi)^4}\, G(Q - q_1)G(2P - Q - q_2)T_2(2P - Q) \\ &\times T_2(Q)G(k)G(Q - k)\Phi(k, Q - k, p_2, P) + (q_1 \leftrightarrow q_2),\end{aligned} \tag{11}$$

$$T_4(p_1, p_2, P) = \frac{\mathrm{i}}{2}\sum_{\alpha\beta}\int \frac{d^4k}{(2\pi)^4}\, \chi(\alpha, \beta)G(P + p_1 - k)G(k) \times \chi(\beta, \alpha)\, \Phi(P + p_1 - k, k, p_2, P)\,, \tag{12}$$

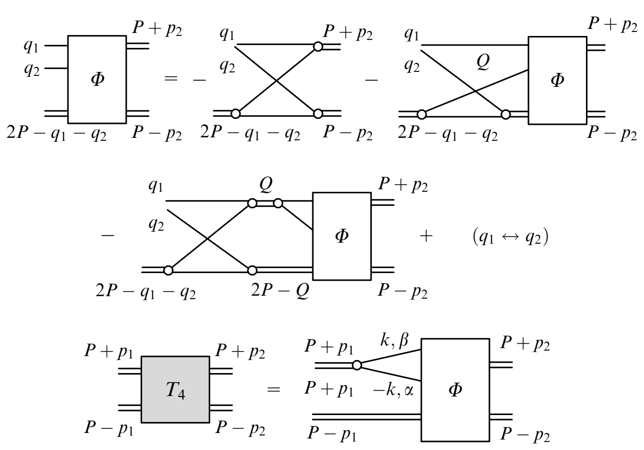

Figure 4. Exact integral equations for four resonantly interacting particles. The double line represents the exact two-particle T-matrix T_2.

where Eqn (12) relates the canonical four-particle T-matrix T_4 and the auxiliary vertex Φ which we introduced into Eqn (11). Importantly, the skew-symmetric vertex Φ is the end point for two lines representing elementary particles with four-momenta q_1 and q_2, and for one double line representing a molecule. At the same time, Φ is the starting point for two molecular lines. Also note two molecular double lines entering and leaving the canonical T-matrix T_4. In Eqn (12), $\chi(\alpha, \beta)$ is the spin factor, and P is the total four-momentum. The minus sign in front of the free term in Eqn (11) corresponds to the Born repulsion between the two $f_\uparrow f_\downarrow$ molecules. To find the s-wave scattering amplitude a_{2-2} of a molecule from the molecule, we can assume without loss of generality that $p_2 = 0$, $P = \{0; -|E_b|\}$ and use the following relationship between a_{2-2} and T_4:

$$\left(\frac{8\pi}{m^2 a}\right)^2 T_4\big(0, 0, -2|E_b|, 0\big) = \frac{2\pi(2a_{2-2})}{m} . \qquad (13)$$

As already noted, the first term on the right-hand side of equation (11) for Φ corresponds to $-GG$, i.e., to the Born approximation [18]. Substituting the Born-approximated Φ into Eqn (12) we arrive at T_4 proportional to the integral of four Green's functions G^4 and, furthermore, $a_{2-2} = 2a$, in accordance with the Pauli principle — as previously shown in the well-known work by Hausmann [18]. The third term on the right-hand side of equation (11) for Φ corresponds to the ladder approximation. It was studied by Pieri and Strinati [19] and is responsible for an infinite number of rescatterings of one molecule by another molecule, occurring without either of them losing their identity. When the first and third right-hand terms of expression (11) for Φ are substituted into equation (12) for T_4, one obtains $a_{2-2} = 0.75a$ [19]. Finally, the second term on the right-hand side of the expression for Φ accounts for the molecule – molecule scattering dynamics, describing both the virtual decay of a single molecule into virtual trios and units, and — provided the crossing effect is taken into consideration — the virtual decays of both molecules with elementary particle exchange between them. Such processes of molecule – molecule scattering were previously analyzed by Petrov, Salomon, and Shlyapnikov [20] using a properly chosen ansatz for the solution of the four-particle Schrödinger equation. Substituting all three terms of the expression for Φ into Eqn (12) for T_4 yields $a_{2-2} = 0.6a > 0$, consistent with the results of Ref. [20]. Similarly, one finds in the 2D case that [21]

$$f_{2-2}(\varepsilon) = \frac{1}{\ln\big(1.6\,|E_b|/\varepsilon\big)} > 0 . \qquad (14)$$

Finally, let us consider again the bound state of four particles attracting each other resonantly. In the 3D case, we again arrive at the analogue of the Efimov effect, and in the event of two interacting molecules (bb and bb, $f_\uparrow$b and $f_\downarrow$b, fb and bb) the homogeneous equations corresponding to integral equations (11) and (12) have a solution at arbitrarily large absolute values of the binding energy $|E_4|$. For a real situation, again, one has $1/(ma^2) < |E_4| < 1/(mr_0^2)$.

In the 2D case, there is again no analogue to the four-particle Efimov effect and the number of bound states is finite as before. For four interacting bbbb bosons there exist two bound states with the energies

$$E_4^{(1)} = 194\,E_b , \qquad E_4^{(2)} = 24\,E_b . \qquad (15)$$

The energies of these levels were first determined by Bruch and Tjon [22]. For the two-boson two-fermion complex $f_\uparrow b f_\downarrow b$ or for the complex $b_1 b_2 b_1 b_2$ involving different types of bosons there are again two bound states with the energies [12]

$$E_4^{(1)} = 10.7\,E_b , \qquad E_4^{(2)} = 2.9\,E_b . \qquad (16)$$

Finally, for the complex fbbb with the fermion and boson masses equal, $m_B = m_F$, only one energy level

$$E_4 = 4.1\,E_b \qquad (17)$$

exists [12]. In the last case it should be noted that the second ('ladder') term in the expression for Φ is absent altogether [12].

5. Phase diagram of a resonant Fermi gas

The results of Sections 3 and 4 for the scattering amplitudes a_{2-1} and, especially, for a_{2-2}, as well as for E_3 and E_4, are of key importance in constructing phase diagrams for 3D and 2D Fermi – Bose gases, individual or mixed.

The phase diagram of a resonant Fermi gas in the 3D case is shown qualitatively in Fig. 5 taken from Ref. [23], where the plot of the dimensionless temperature T/ε_F versus the inverse gas parameter $1/ap_F$ portrays the superfluid BCS region which corresponds to the formation and simultaneous Bose condensation of extended Cooper pairs, and also shows a BEC region where local pairs (molecules) form and then also undergo Bose condensation. The BCS region corresponds to a negative two-particle scattering length $a < 0$, and a positive chemical potential $\mu > 0$. In the weak-coupling region $1/(ap_F) \to -\infty$, Cooper pairing in a Fermi gas occurs near the Fermi surface, so that at $1/(ap_F) \to -\infty$ one finds $\mu \approx \varepsilon_F$.

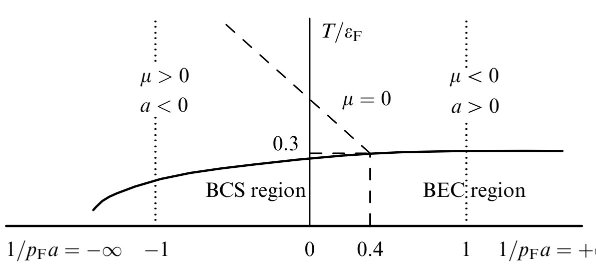

Figure 5. Phase diagram of a resonant Fermi gas in the 3D case. Regions labelled BCS and BEC correspond to Copper pairing and to Bose condensation of local pairs, respectively.

The critical temperature in the BCS region is determined from the well-known formula by Gor'kov and Melik-Barkhudarov [24]:

$$T_c \approx 0.28\varepsilon_F \exp\left(-\frac{\pi}{2p_F|a|}\right) . \qquad (18)$$

As the point $1/(ap_F) \to -0$ is approached, we pass to the so-called unitary limit. In this limit there is no small parameter of the theory and all the quantities involved, including the total energy of the system, its chemical potential μ, and T_c, are expressed in terms of the Fermi energy alone [25]. From the Monte Carlo calculations by Pieri, Pisani, and Strinati [26] (2005) and by Astrakharchik et al. [27] (2004), it follows that $\mu = 0.44\varepsilon_F > 0$, i.e., the sign of the chemical potential corresponds to the BCS region. On the other hand, a Monte Carlo calculation of T_c by Burovski et al. [28] (2006) yields $T_c/\varepsilon_F = 0.15$. The chemical potential goes to zero at the point T_c only at $ap_F \approx 2.5$ [23], i.e., in the range of positive a values. Notice that for $1 \lesssim ap_F \lesssim 3$ we have $na^3 = p_F^3 a^3/3\pi^2 \lesssim 1$, so that local pairs still do not strongly overlap but only touch one another slightly.

In Fig. 5, the boundary between the BCS and BEC regions is shown by the dashed line. Along this line $\mu(T) = 0$. For $ap_F < 2.5$ $\big($or $1/(ap_F) > 0.4\big)$, we go over into the BEC region where in the limit of weak coupling $\big(1/(ap_F) \gg 1\big)$ the critical temperature is given by [29]

$$T_c = 0.2\varepsilon_F[1 + 1.3\, a_{2-2}\, n^{1/3}]\,. \tag{19}$$

Corrections to the Einstein formula in Eqn (19), which are linear in the scattering length a_{2-2} of a molecule from a molecule, were determined by Kashurnikov, Prokof'ev, and Svistunov [29] using the Monte Carlo method. A point to note is the existence in the BEC region of yet another characteristic temperature which is given by the Saha formula [29]

$$T_* = \frac{|E_b|}{(3/2)\ln\big(|E_b|/\varepsilon_F\big)} \gg T_c\,. \tag{20}$$

This temperature governs a smooth crossover and is found from the condition of thermodynamic equilibrium between the unpaired fermions and molecules:

$$n_F(T_*) = 2n_M(T_*) = \frac{n}{2}\,, \tag{21}$$

where n is the total density. For $T_c \ll T \ll T_*$, we find ourselves in the region of a normal Bose gas of molecules with the mass $2m$ and density $n/2$ [30].

Note also that the scattering length a_{2-2} of a molecule from a molecule determines the speed of sound in the superfluid state of a resonant Fermi gas in the BEC region for $T \ll T_c$ [31, 32]:

$$c^2 = \frac{n_M}{m_M}\frac{d\mu_M}{dn_M}\,, \tag{22}$$

where

$$\mu_M = \frac{4\pi a_{2-2}}{2m}\frac{n}{2} > 0 \tag{23}$$

is the chemical potential of a weakly nonideal Bose gas of molecules with the mass $m_M = 2m$ and density $n_M = n/2$.

Analogously, given the knowledge of the scattering amplitude f_{2-2} in a 2D Fermi gas and the three- and four-particle binding energies in a 2D Fermi–Bose mixture with resonant boson–fermion attraction, it is possible to determine the characteristic Saha temperatures and the superfluid transition temperature for these systems, thus completing construction of phase diagrams for them.

It should be noted that in a 2D Bose gas or Fermi–Bose mixture with bosons outnumbering fermions ($n_B > n_F$), large complexes (droplets) involving $N > 4$ particles can, in principle, form. The binding energy of such a droplet again will be limited only by the presence of the attractive core: $|E_N| < 1/(mr_0^2)$. For multiparticle complexes, the closed system of Skorniakov–Ter-Martirosian type integral equations is extremely difficult to construct and solve. A more promising approach here is the variational method, proposed by Hammer and Son [34], among others. In this method, the energy of a droplet in a 2D Bose gas grows exponentially with N (the number of particles in the droplet), so that $|E_N| \sim |E_b| \exp(2N)$, provided that $N < N_{max} = 0.9\ln(a/r_0)$. Such large droplets may have been observed in experiments conducted by Roati et al. [35] and Modugno et al. [36] on the collapse of a Bose gas in Fermi–Bose mixtures for $n_B > n_F$.

6. A spinon–holon mixture in high-temperature superconductors

Finally, a few words on the mixture of spinons and holons in weakly doped HTSC systems are in order. For HTSC systems we suggest, starting from the Hamiltonian for a strongly interacting Fermi–Bose mixture of spinons $f_{i\sigma}$ and holons b_i, to derive the effective one-band Hamiltonian for weakly interacting composite holes or spin polarons:

$$h_{i\sigma} = f_{i\sigma}\, b_i\,. \tag{24}$$

Lying behind this idea is the well-known string solution for a composite hole, which was obtained by Bulaevskii, Nagaev, and Khomskii [9] and also by Brinkman and Rice [10], and which predicts for 3D and 2D cases that a hole produces a linear track (or string) of frustrated spins in its wake as it moves in an AFM background. As this takes place, the spinon–holon binding energy in a confining string potential is given by

$$E_b \sim (zJS^2)^{2/3}\, t^{1/3}\,, \tag{25}$$

where $m_b \sim 1/t$ is the holon mass, $m_f \sim 1/J$ is the spinon mass, t is the hopping integral, J is the exchange integral, and $J \ll t$ for real HTSC systems. Allowing for quantum fluctuations [the term $J(S_i^+ S_j^- + S_i^- S_j^+)$ in the $t-J$-model], the composite hole acquires a large but finite mass $M \sim 1/J$, leading to the expression

$$E_h = E_b + J(\cos k_x + \cos k_y)^2 \tag{26}$$

for the composite hole spectrum on a 2D square lattice [39].

To achieve superconductivity in the system, we need to create a pair of composite holes $h_{i\sigma}h_{j-\sigma}$, which is in fact a quartet $f_{i\sigma}b_i\, f_{j-\sigma}\, b_j$ containing two spinons and two holons located at the i and j sites of the square lattice.

Whether or not a bound state of two composite holes forms depends on the nature of the residual interaction between them. The residual interaction of two holes at low ($x \ll 1$) concentrations is of a dipole–dipole nature, as revealed by Shraiman and Siggia [40] in 1990, and has the form $V(r) \sim \lambda/r^2$ in the 2D case.

As shown by Belinicher and co-workers [41, 42] (1997, 1995), for holes interacting in this way on a lattice, a shallow bound state of two composite holes can form in the $d_{x^2-y^2}$-wave channel in the limit of small hole concentrations.

We note that this result was obtained by applying the spin-polaron approximation to the $t-J$-model with only the nearest neighbor hopping terms t and the next nearest hopping terms t' and t'' being neglected. In the opposite limit of high hole concentrations, the d-wave pairing (of Cooper type this time) was obtained in the framework of the $t-J$-model by Kagan and Rice [43]. Finally, a recent result by Plakida and co-workers [44] — an exact expression for T_c

in the d-wave channel for the $t-J$-model taking into account the preexponential factor — opens up an interesting possibility for studying the T_c-vs-x curve for HTSC systems as a BCS – BEC crossover for the pairing of two composite holes in the d-wave channel.

7. Conclusion

Concluding we briefly outline the main results presented in this report. Based on the resonance ($a \gg r_0$) approximation, we derived and exactly solved integral equations for trios and quartets in the 3D and 2D cases. We calculated the scattering amplitude of a molecule from a molecule in a 3D and 2D resonant Fermi gas, and we determined bound state energies for all the possible three- or four-particle complexes in the 2D case. As a result, we were able to construct the phase diagrams for a resonant Fermi gas and a resonant Fermi – Bose mixture. We also proposed a new superconductivity scenario for HTSC systems, in which two composite holes, each containing a spinon and a holon, form a superconducting pair in the d-wave channel.

Acknowledgments. The authors acknowledge fruitful discussions with A F Andreev, Yu Kagan, L V Keldysh, F Nosieres, P Woelfle, D Vollhardt, I A Fomin, G V Shlyapnikov, P Fulde, L P Pitaevskii, and C Salomon. The work was supported by the Russian Foundation for Basic Research (grant No. 06-02-16449).

References

1. Pitaevskii L, Stringari S *Bose – Einstein Condensation* (Oxford: Clarendon Press, 2003); Dalfovo F, Giorgini S, Pitaevskii L P, Stringari S *Rev. Mod. Phys.* **71** 463 (1999)
2. Inouye S et al. *Nature* **392** 151 (1998)
3. Kagan M Yu et al. *Phys. Rev. A* **70** 023607 (2004)
4. Kagan M Yu, Efremov D V *Phys. Rev. B* **65** 195103 (2002)
5. Greiner M, Regal C A, Jin D S *Nature* **426** 537 (2003); Inouye S et al. *Phys. Rev. Lett.* **93** 183201 (2004)
6. Zwierlein M W et al. *Phys. Rev. Lett.* **91** 250401 (2003)
7. Jochim S et al. *Science* **302** 2101 (2003)
8. Bourdel T et al. *Phys. Rev. Lett.* **93** 050401 (2004)
9. Bulaevskii L N, Nagaev E L, Khomskii D I *Zh. Eksp. Teor. Fiz.* **54** 1562 (1968) [*Sov. Phys. JETP* **27** 836 (1968)]
10. Brinkman W F, Rice T M *Phys. Rev. B* **2** 1324 (1970)
11. Skorniakov G V, Ter-Martirosian K A *Zh. Eksp. Teor. Fiz.* **31** 775 (1956) [*Sov. Phys. JETP* **4** 648 (1957)]
12. Brodsky I V et al. *Pis'ma Zh. Eksp. Teor. Fiz.* **82** 306 (2005) [*JETP Lett.* **82** 273 (2005)]; *Phys. Rev. A* **73** 032724 (2006)
13. Danilov G S *Zh. Eksp. Teor. Fiz.* **40** 498 (1961) [*Sov. Phys. JETP* **13** 349 (1961)]
14. Minlos R A, Faddeev L D *Zh. Eksp. Teor. Fiz.* **41** 1850 (1961) [*Sov. Phys. JETP* **14** 1315 (1962)]
15. Efimov V N *Yad. Fiz.* **12** 1080 (1970); *Phys. Rev. C* **44** 2303 (1991)
16. Jensen A S et al. *Rev. Mod. Phys.* **76** 215 (2004)
17. Nielsen E et al. *Phys. Rep.* **347** 373 (2001); Nielsen E, Fedorov D V, Jensen A S *Few-Body Syst.* **27** 15 (1999)
18. Hausmann R *Z. Phys. B* **91** 291 (1993)
19. Pieri P, Strinati G C *Phys. Rev. B* **61** 15370 (2000)
20. Petrov D S, Salomon C, Shlyapnikov G V *Phys. Rev. Lett.* **93** 090404 (2004)
21. Petrov D S, Baranov M A, Shlyapnikov G V *Phys. Rev. A* **67** 031601(R) (2003)
22. Bruch L W, Tjon J A *Phys. Rev. A* **19** 425 (1979)
23. Combescot R, Leyronas X, Kagan M Yu *Phys. Rev. A* **73** 023618 (2006)
24. Gor'kov L P, Melik-Barkhudarov T K *Zh. Eksp. Teor. Fiz.* **40** 1452 (1961) [*Sov. Phys. JETP* **13** 1018 (1961)]
25. Heiselberg H *Phys. Rev. A* **63** 043606 (2001)
26. Pieri P, Pisani L, Strinati G C *Phys. Rev. B* **72** 012506 (2005)
27. Astrakharchik G E et al. *Phys. Rev. Lett.* **93** 200404 (2004)
28. Burovski E et al. *Phys. Rev. Lett.* **96** 160402 (2006)
29. Kashurnikov V A, Prokof'ev N V, Svistunov B V *Phys. Rev. Lett.* **87** 120402 (2001)
30. Landau L D, Lifshitz E M *Statisticheskaya Fizika* (Statistical Physics) Pt. 1 (Moscow: Nauka, 1976) [Translated into English (Oxford: Pergamon Press, 1980)]
31. Kagan M Yu et al. *Phys. Rev. B* **57** 5995 (1998)
32. Bartenstein M et al. *Phys. Rev. Lett.* **92** 203201 (2004)
33. Combescot R, Kagan M Yu, Stringari S *Phys. Rev. A* **74** 042717 (2006); cond-mat/0607493
34. Hammer H-W, Son D T *Phys. Rev. Lett.* **93** 250408 (2004); Platter L, Hammer H-W, Meißner U-G *Few-Body Syst.* **35** 169 (2004)
35. Roati G et al. *Phys. Rev. Lett.* **89** 150403 (2002)
36. Modugno G et al. *Science* **297** 2240 (2002)
37. Anderson P W *Science* **235** 1196 (1987)
38. Laughlin R B *Phys. Rev. Lett.* **60** 2677 (1988); Fetter A L, Hanna C B, Laughlin R B *Phys. Rev. B* **39** 9679 (1989)
39. Fulde P *Electron Correlations in Molecules and Solids* 2nd ed. (Berlin: Springer-Verlag, 1993)
40. Shraiman B I, Siggia E D *Phys. Rev. B* **42** 2485 (1990)
41. Belinicher V I, Chernyshev A L, Shubin V A *Phys. Rev. B* **56** 3381 (1997)
42. Belinicher V I et al. *Phys. Rev. B* **51** 6076 (1995)
43. Kagan M Yu, Rice T M *J. Phys.: Condens. Matter* **6** 3771 (1994)
44. Plakida N M et al. *Zh. Eksp. Teor. Fiz.* **124** 367 (2003) [*JETP* **97** 331 (2003)]

PACS numbers: **42.62.–b**, **52.38.–r**, 52.50.Jm
DOI: 10.1070/PU2006v049n10ABEH006098

Lasers and high energy density physics at the All-Russian Research Institute of Technical Physics (VNIITF)

A V Andriyash, P A Loboda, V A Lykov, V Yu Politov, M N Chizhkov

1. Introduction

The build-up of high energy density physics (HEDP) as an independent area of research was caused by the development of nuclear weaponry. By the end of the 1980s and the beginning of the 1990s, the activity of research in this area sharply increased. The reason was the growth in importance, in the atmosphere of the nuclear test ban, of laboratory studies aimed at confirming the reliability and security of nuclear stockpiles. To this end, the high-power laser facility projects of National Ignition Facility (NIF) and Megajoule laser facility (LMJ — Laser Mégajoule) with a total energy of laser radiation approaching 1.8 MJ are being pursued by Lawrence Livermore National Laboratory (LLNL, USA) and Commissariat a l'Energie Atomique (CEA, France), respectively [1, 2]. In addition to research concerned with nuclear stockpile stewardship, there are plans to utilize these facilities to demonstrate the possibility of employing fusion ignition to solve energy problems: the goal is to implement fusion ignition of microtargets with an energy yield higher than 20 MJ and more than 10^{19} 14-MeV neutrons per flash. At the Russian Federal Nuclear Center (RFNC) 'All-Russian Research Institute of Experimental Physics' (VNIIEF in Russ. abbr.), it is planed to build a solid-state-laser ISKRA-6 facility with an energy of up to 300 kJ in a nanosecond pulse [3]. Another factor determining the increase in the pace of research in HEDP is the astonishing