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Nonlinear acoustic diagnostics of the ocean and rock

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1. Introduction

In the 1940s and 1950s, Russian studies on nonlinear acoustics were stimulated by the pioneering work of N N Andreev. His article 'On some second-order values in acoustics' was published in the first volume of *Akusticheskii Zhurnal* issued in 1955 [1]. In the 1960s, it was followed by the technological and medical applications of powerful ultrasound introduced by L D Rosenberg [2]. It was then that L M Brekhovskikh formulated the problem of the interaction between acoustic and oceanic waves [3]. Many Russian research groups worked in these areas. Recently, experts in propagation and interactions of nonlinear waves have started to address waves in granular media and continental shelf internal waves because both are essentially nonlinear. In granular media, the nonlinearity parameter is 3 – 4 orders of magnitude greater than that in homogeneous media, and the 'giant nonlinearity' term has been coined [4, 5]. Due to the high amplitudes and slow velocities, internal waves undergo essential nonlinear transformations.

In this report, the results of experimental observations of new nonlinear wave processes in the ocean and granular rocks are given.

2. Granular media

Acoustic waves are known to propagate in granular and in solid media differently. The mechanical properties of granular media are largely determined by the inter-granular contacts. This property allows assigning granular media to a broad class of media with nonlinear structural elasticity. Whereas the nonlinear acoustic properties of monocrystals, homogeneous fluids, and other solid media are determined by the molecular nature of their strain, the properties of granular media are determined by their structure. In this sense, the properties of granular media are seen at the mesoscale, that is, the scale of the granules [4]. This results in considerable qualitative and quantitative differences, including differences in the equations of state for the media. For a solid medium, the relative strain Δ is assumed to be proportional to the stress applied, $\Delta \approx P$, but the relation for the spherical granule is $\Delta \approx P^{2/3}$ [6]. In granular media, the velocity of acoustic waves is therefore $c = (\partial P/\partial \rho)^{1/2} \approx P^{1/6}$, which is a nonlinear function of the stress P applied. In agreement with this, the nonlinearity parameter $\alpha = \rho_0 \partial c^2/\partial P \approx P^{-5/6}$ is also dependent on the stress applied. (Here, ρ is the medium density and ρ_0 is its value at equilibrium.) The nonlinear properties of granular media prove noticeable even at quite moderate strain. In rocks, for instance, nonlinear distortions are already observed at a strain equal to $\Delta \approx 10^{-9}$ [4]. This strain range is typical even for quite moderate acoustic perturbations.

The behavior of granular media is currently being studied at the single-granule level [7 – 11]. Under low stress, the stress – strain relations are noticed to measurably deviate from regular relations. As a rule, the latter work only asymptotically under high enough stress, when the granular medium may be considered well-packed. Such media usually respond to repeated stress with stress – strain hysteresis. This property of granular media leads to a nonlinear distortion of acoustic waves with the second harmonic proportional to the squared signal amplitude and possibly exceeding the third harmonic. Oscillations of a single granule in a constant acoustic field are shown to slowly fluctuate [7].

We discuss the results of the experimental analysis of slow fluctuations of the nonlinear oscillations of the granule in a medium where the acoustic field is propagating. One-two Granite bits of the size 1 to 2 cm served as the granular medium in our experiments. The sound was produced by a piezoceramic plate and the detection was done by accelerometers mounted among the granules. The experimental setup is detailed in [12]. We note that the accelerometers were of the size scale of the granules.

The detected signal versus the intensity of sound produced is plotted in Fig. 1. In this and subsequent experiments, the frequency of the produced sound was 5.6 kHz. The signal was detected by two accelerometers positioned equidistantly from the sound source and separately from each other. The common character of response measured by both detectors can be seen. A linear relation between the sound intensity and the signal holds only on average, for a large range of signal amplitude variations. The response is different for the different detectors, pointing to the independent propagation

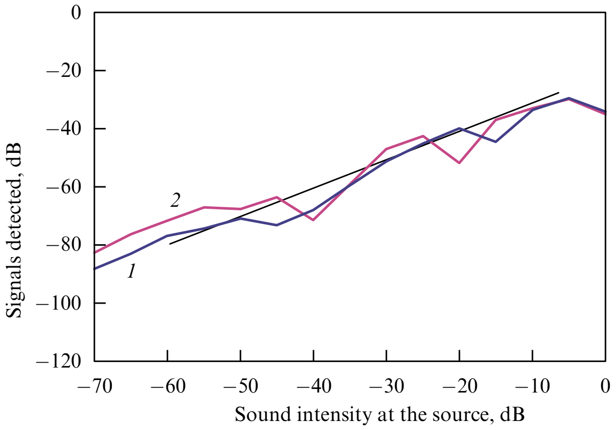

Figure 1. Signals detected in the granular medium versus sound intensity. The straight line corresponds to the linear response. Curves *1* and *2* are from different detectors.

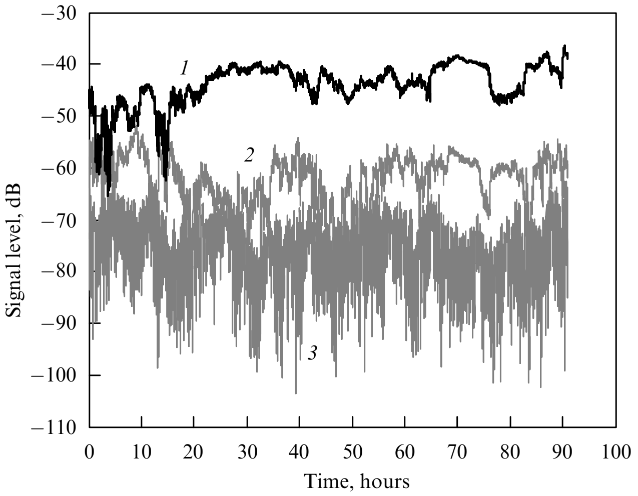

Figure 2. Time dependences of the harmonics of the signal detected in a granular medium: *1* — frequency 5.6 kHz, *2* — the second harmonic at 11.2 kHz, *3* — the subharmonic at 2.8 kHz.

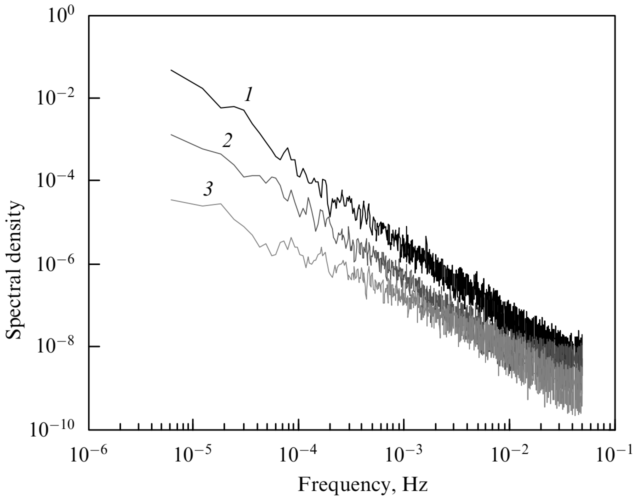

Figure 3. Fluctuation spectrum of spectral components of the signal in a granular medium. The signal level is 10 dB, *1* — frequency 5.6 kHz, *2* — frequency 11.2 kHz, *3* — frequency 2.8 kHz.

of the signal from the source to each detector. The maximum signal received corresponds to the source piezoceramic plate vibration with the acceleration 0.6 m c^{-2} and the amplitude equal to a mere 5 Å. These parameters correspond to a -10 dB sound level. The detected level of the granules' oscillations proved to be approximately 10dB lower. The nonmonotonic response of single granules to the stress applied to the medium is considered the result of percolate development of bonding of granules involved in transduction of the elastic signal from the source to the detector [13]. Such a percolate chain of inter-granule contacts is very sensitive to the stress parameters. A rise in stress leads to the chain restructuring and results in changes to its effective elasticity.

The results of the measurements of the acoustic field in the medium over time is given below. The signal at one of the detectors is given in Fig. 2 as a function of time, together with levels of the harmonics of the signal. First of all, we note that a subharmonic has appeared in the signal. The fluctuations of harmonics of the signals from both detectors are similar. The signal with the carrier frequency 5.6 kHz randomly changes by more than 5–6 dB. At the same time, the harmonics fluctuate much stronger, indicating the giant fluctuations of acoustic signals in granular media. The subharmonics fluctuates most intensely, as much as several tenfold.

The frequency spectra of the fluctuations of the signal harmonics are given in Fig. 3. The measurements were averaged over 32 realizations. The spectral analysis shows that random oscillations of low frequencies are the strongest fluctuations. Their frequencies are in the range $f = 10^{-5} - 10^{-1}$ Hz and monotonically drop with the frequency as $S(f) \approx Af^{-n}$. The exponent n proves to be different for different harmonics. It varies from $n = 2$ for the fundamental frequency 5.6 kHz to $n = 1.3$ for the subharmonics and always remains greater than one. Interestingly, at low frequencies, the exponent for the relation describing the spectrum of fluctuation frequencies happened to be near the values $n = 1.7 - 2.2$ measured in an experiment involving passing an acoustic signal through a medium composed of identical glass beads [7]. This points to a common mechanism of the low-frequency modulation of sound passing through media composed of granules of different shapes and sizes. The spectrum of fluctuations of the sound recorded must obviously reach a maximum or at least saturation at low frequencies when $n > 1$, because the intensity of fluctuations must remain finite. Our several-day-long experiments could not confirm this statement.

The experimental data involving registration of acoustic waves in granular media by granule-sized detectors point to the statistical nature of the process. In these conditions, a detector has a limited number of contacts with the adjacent granules and can be regarded as one of the elements of the medium. Even low constant amplitudes of sound in a medium result in great acoustic field fluctuations and in harmonics and subharmonics produced in it. The nonmonotonic dependence of the field in the medium on the sound amplitude points to a considerable role played by the inter-granular contacts in the formation of the acoustic field in a granular medium. In this case, acoustic perturbations are transmitted between granules via contacts only, which occupy a very small part of the granules and cannot therefore stabilize the medium. Thus, a granular medium may be considered to be in a metastable state and even a low-amplitude acoustic field may change the structure of its contacts. Such contacts form a structure of chains transmitting signals from the source to a detector. The density of such elastic chains is determined by the granule sizes and the number of inter-granular contacts. Acoustic oscillations shift inter-granular contacts slightly, possibly radically changing the structure of elastic chains and, therefore, the effective elastic properties of the medium. The acoustic impedance of the medium then also changes. During the contraction of a medium, the number of inter-granular contacts grows, resulting in a higher density of the elastic chains that transmit acoustic perturbations. The elastic properties of the medium then tend to saturation.

A possible mechanism of slow changes in the structure of contacts discussed in [7] is the thermal strain at the contact points of granules where acoustic energy is concentrated. The stable frequency response over the range $10^{-6} - 10^{-1}$ Hz may be due to the fractal structure of the elastic chains transmitting signals from the source to the detector.

To induce the subharmonics of an acoustic signal, quite intense acoustic fields are required. This phenomenon is usually observed during phase transitions. For a granular medium with elasticity determined by inter-granular contacts, the threshold amplitude for the phase transition is the specific gravity g. This value determines when a consolidated medium becomes nonconsolidated. The subharmonics appearing even at quite low signal amplitudes of 0.5 m c^{-2} point to the localization of the elastic energy of acoustic oscillations at separate granules of a granular medium.

Thus, the data obtained point to the complex character of the propagation of acoustic signals in granular media. The analysis of an acoustic field on the scale of a single granule has revealed the statistical nature of the process and its strong nonlinearity even at moderate amplitudes of the signal. The obtained experimental data allow developing a model for slow fluctuations of the acoustic field in granular media.

The acoustic diagnostics of granular media also have other features. The granular and other heterogeneous media belong to the class of acoustically dispersed nonlinear media. The combination of the nonlinearity and the dispersion allows obtaining soliton solutions for acoustic waves in such media. In particular, we derived the following equations for waves with resonance dispersion (RDE):

$$\left(\frac{\partial^2}{\partial t^2}+\omega_0^2\right)\left(\Delta U-\frac{1}{c^2}\,\frac{\partial^2}{\partial t^2}\,U+\alpha\Delta(U^2)+\Gamma\Delta\,\frac{\partial}{\partial t}\,U\right) - \sigma^2\,\frac{\partial^2}{\partial t^2}\,U=0\,,$$

where α is the media nonlinearity, Γ is the dissipation, and σ^2 is the dispersion, which is proportional to the concentration of the oscillators in the medium [14, 15]. The solutions of this equation shown in Fig. 4 are specially shaped solitary solitons with vertical fronts. These solutions are quite sensitive to changes to the parameters of the medium such as dispersion, nonlinearity, and absorbance. Thus, following the shapes of such wave perturbations allows monitoring the state of media with resonance dispersion.

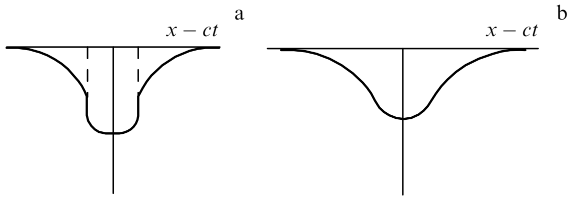

Figure. 4. A soliton of the wave equation with resonance dispersion: (a) medium without damping, (b) medium with damping.

3. Nonlinear acoustic processes in the ocean

We consider the capabilities of acoustic diagnostics of the ocean with the observations of internal waves taken as an example. Internal gravitational waves are frequent in the ocean and in the atmosphere. They exist because of the vertically stable stratification of the density of the medium. Any perturbation of such a medium leads to waves in it because the particles of the vertical column shifted from the equilibrium tend to return to their initial position under the action of the Archimedean force. A number of natural phenomena can produce this 'first push.' The most effective sources include tides, atmospheric perturbations, and ocean currents. The numerous sources of internal waves are so effective that, according to the well-known oceanologist W Munk, "no researcher of internal waves has yet reported calm at the depths of the ocean" [16]. Thus, internal waves are ubiquitous in the ocean; they create vertical water movements and facilitate the internal mixing necessary for life in the ocean. The characteristic parameters of internal waves are as follows: the periods vary from several days to several minutes and the wave speeds vary from several decimeters to several meters per second. The distribution of frequencies of the ocean internal waves drops proportionally to the frequency squared and has two peaks corresponding to long and short waves. The peak at long waves is from inertial and tidal waves, and the peak at short waves corresponds to the buoyancy frequency [17, 18]. The internal waves may be as high as several meters. In some regions of the ocean, giant internal waves higher than 100 m have been registered [19–22]. The internal waves come in series in space and time and propagate in a specific direction. In addition, solitary waves are observed. Internal-wave solitons are a widespread phenomena. Internal waves are typical for both the deep sea and the shelf. The shelf is where the internal waves are produced and simultaneously broken down with the energy of long waves transferred to short waves and further into turbulence. Here, the nonlinearity of the internal waves is seen most clearly [23, 24].

In recent years, we have analyzed internal waves of the Sea of Japan shelf by a multi-beam pulse sonar. We used the Acoustic Doppler Current Profiler (ADCP) Rio Grande 600 kHz from RD Instruments. Besides measuring a vertical and two horizontal components of the current, this instrument provides the data on reflection of the signal and locates the water density jump interfaces (pycnoclines) along which the propagating internal waves are seen. For the measurements, a small research boat was anchored for 24 hours or tacking at sea.

The research conducted has revealed the general picture of the internal wave dynamics and allowed more accurate measurements of their main parameters. During the measurements, a number of nonlinear effects fundamental for internal waves were confirmed. These effects were previously independently observed at other continental shelves by contact methods. Such effects include, first of all, the 'vertical' and the 'horizontal' asymmetry of the internal waves profile and the 'effect of changing the polarity of internal waves.'

The vertical asymmetry of crests and troughs. Intense internal waves propagating at a pycnocline (a layer with a density jump) that is closer to the bottom than to the sea surface have a characteristic profile with smooth feet and sharp crests and are therefore waves of elevation. In the case where the internal waves propagate at the thermocline that is closer to the surface than to the bottom, the wave profiles are characterized by smooth crests and sharp troughs and are therefore waves of depression. We often saw waves of elevation and waves of depression. Given in Fig. 5a is the record of a backscattered signal reflected from the deep water that was recorded in October of 2003 during passage through the Peter the Great Gulf in the Sea of Japan in the direction transverse to the coastline. Seen at the cross section obtained is the area of the subsurface pycnocline from the range 15–25 m, over which the solitary 5–10 m high waves of depression propagate. The internal waves propagate in the direction of the coast toward more shallow water. Given in Fig. 5b is the similar record taken in September of 2005 at the inner shelf when the density jump was near the bottom. In this case, a series of the internal waves of elevation with rank-ordered amplitudes up to 7 m propagates along the pycnocline toward the coast.

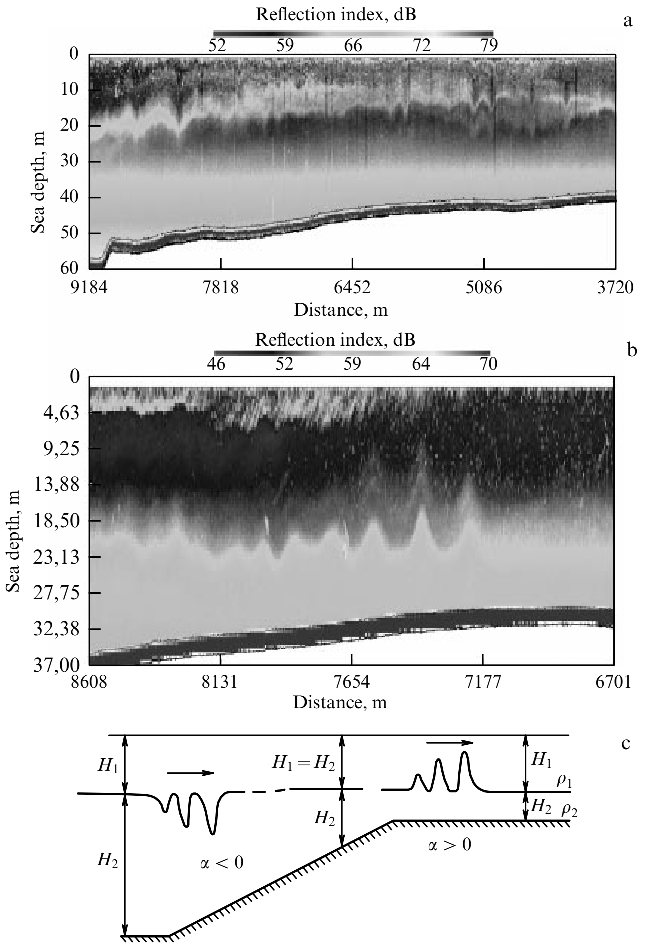

Figure 5. The vertical asymmetry of internal waves at a shelf. At the ocean cross sections of the refraction indices produced by the ADCP, the waves of depression (a) and the waves of elevation (b) are seen. (c) The explanation of the effect of changing the polarity of internal waves at a shelf; H_1 is the depth of the pycnocline, and H_2 is the depth from the pycnocline to the bottom.

The properties of vertically asymmetric internal waves at the shelf are close to those of solitons [24], and we can say that soliton-like waves of different polarities propagate at the shelf. This feature of internal waves at shelves results in an interesting phenomenon that we call the 'effect of changing the polarity of internal waves.' During the summer, the pycnocline at the near-coast part of the shelf is usually near the sea bottom, while at the deep sea part of the shelf, it is closer to the sea surface. Because soliton-like internal waves form at the shelf mainly from long tidal waves coming from the open sea and propagate toward the coast, they must switch from the near-surface pycnocline to the near-bottom pycnocline. At this point, the negative solitons (with negative nonlinearity) switch to the zone with the positive nonlinearity, and the waves of depression break down and become waves of elevation (Fig. 5c). At this point, the pycnocline is in mid-depth, and we call it the 'overturning point.' The first experimental proof of this effect came from observations at the Sea of Japan shelf [25, 23, 27] and was later confirmed by results from the Mediterranean Sea [26]. Recently, the best experiment under natural conditions was conducted in the South China Sea [28]. A high-frequency multi-beam acoustic sonar was used there as well. The transformation of the negative solitons of the internal waves at the switching point was numerically modeled [29]. The results of a special experiment studying the effect of changing the polarity of internal waves recently conducted by us at the Sea of Japan shelf are being prepared for publication.

The horizontal asymmetry of internal waves. The vertical asymmetry of internal waves discussed above is the main indicator of their nonlinearity. However, many intense internal waves are not only vertically but also horizontally asymmetric due to different inclinations of the front and the back slopes of the internal wave. This difference is a feature of a nonstationary wave, that is, a wave that is being destroyed. We repeatedly saw the horizontal asymmetry of internal waves during the measurements by the contact volume temperature sensors. The profiles of two internal waves of elevation propagating toward the coast are shown in Fig. 6. The recording in Fig. 6a was taken on October 1, 1982 at a sea depth of 30 m by the line temperature sensors and in Fig. 6b is the ADCP recording done on September 17, 2004 at a sea depth of 43 m. Both waves are about 10 m high, but their front slopes are considerably steeper than the back slopes. The recording from the contact volume temperature sensors is shown for two sensors situated at different distances from the coast: as the wave approaches the coast, it becomes steeper. The horizontal symmetry is very common for both elevation and depression intense internal waves at the shelves [30].

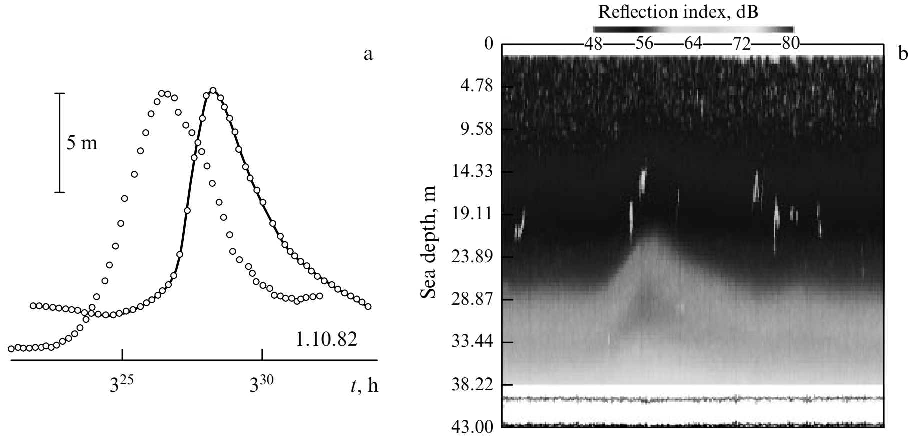

Figure 6. The horizontal asymmetry of internal waves measured by line temperature sensors (a) and by ADCP (b).

4. Conclusion

The considered examples of the acoustic diagnostics of nonlinear waves demonstrate new physical phenomena in granular rocks and the ocean. The metastable state of the granular media is an important condition for studies of acoustic effects on percolation in rocks. The nonlinear interaction of internal waves determines the transformation of the tidal energy into oceanic turbulence. The acoustic diagnostics prove to be effective for studying the dynamics of processes at the continental shelf.

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Parametrically phase-conjugate waves: applications in nonlinear acoustic imaging and diagnostics

V L Preobrazhenskii

1. Introduction

Wave phase conjugation (WPC) is typically understood as a wave process whose time development is the reverse of an arbitrarily specified incident wave. The interest in acoustic WPC stems from the ability of phase-conjugate waves (PCWs) to automatically focus onto objects that scatter the incident wave and to compensate the phase distortions of wave propagation in heterogeneous refractive media. Various applications of these properties for ultrasonic diagnostics, therapy, surgery, nondestructive evaluation, and underwater communications have recently been intensely discussed.

Comprehensive research of physical principles and mechanisms underlying the ultrasonic WPC began more than 20 years ago and was stimulated considerably by the results in the nonlinear optics for WPC of light. Since the early 1980s, the mainstream objective of several years of research has been to find adequate approaches to the problem of generation of ultrasonic PCWs in various condensed media. The systematized results of this research are reviewed in [1, 2].

The most promising techniques of acoustic PCWs, including phase conjugation of ultrasonic waves in parametrically active solid media such as piezoelectrics and magnetics [3 – 6] and time reversal of the acoustic signals in receiving and transmitting multichannel electronics [7, 8], appeared in the late 1980s – early 1990s. WPC techniques based on time reversal in waveguides and cavities with multiple reflection are being intensely developed [9, 10]. The supercritical parametric phase conjugation of magnetoelastic waves in magnetostrictive ceramics [2, 4, 11] is an effective way to produce PCWs with frequencies from several megahertz to several dozen megahertz for applications in medicine and defectoscopy. The fairly strong coupling between elastic and magnetic subsystems of spinel ferrite-based ceramics allows an effective modulation of the speed of sound by a high-frequency magnetic field. The modulation depth may then considerably exceed the threshold of parametric instability for the longwave phonons at room temperature. Under the conditions of supercritical pumping, a parametrically active element works as a source of stimulated phase-conjugate phonon pairs providing the giant (over 80 dB) amplification of the PCW compared with the incident wave. The ceramic technology allows manufacturing active elements of a wide variety of shapes and sizes according to the requirements of specific applications.

A numerically modeled animation of supercritical amplification in an active medium taken from [12] is available in Supplement 1 to the on-line version of this report.

The possible uses of magnetoacoustic PCW methods to autofocus ultrasound in liquid and solid matter and to autotarget ultrasonic beams to sound-scattering objects in liquids have been demonstrated in a set of experiments (reviewed in [2]). A stroboscopic visualization of autotargeting of