Physics – Uspekhi **49** (1) 89 – 102 (2006)

CONFERENCES AND SYMPOSIA

PACS number: 01.10.Fv

Joint scientific session of the Physical Sciences Division of the Russian Academy of Sciences and the Joint Physical Society of the Russian Federation "Nonlinear acoustic diagnostics" (28 September 2005)

A joint scientific session of the Physical Sciences Division of the Russian Academy of Sciences (RAS) and the Joint Physical Society of the Russian Federation was held on September 28, 2005 in the Conference Hall of the Lebedev Physics Institute, RAS under the name "Nonlinear acoustic diagnostics." The following reports were presented at the session:

(1) **Rudenko O V** (Lomonosov Moscow State University) "Giant nonlinearities in structurally inhomogeneous media and the fundamentals of nonlinear acoustic diagnostics methods";

(2) **Zaitsev V Yu, Nazarov V E, Talanov V I** (Institute of Applied Physics, RAS, Nizhny Novgorod) "'Nonclassical' manifestations of microstructure-induced nonlinearities: new prospects for acoustic diagnostics";

(3) **Esipov I B, Rybak S A, Serebryanyi A N** (Andreev Acoustics Institute, RAS) "Nonlinear acoustic diagnostics of the ocean and rock";

(4) **Preobrazhenskii V L** (Research Center for Wave Studies, Prokhorov Institute of General Physics, RAS, European Laboratory in Nonlinear Magneto-acoustics (LEMAC)) "Parametrically phase-conjugate waves: applications in nonlinear acoustic imaging and diagnostics."

An expanded version of the report by Rudenko is published in the 'Physics of our days' section of this issue. An abridged version of reports 2 – 4 is given below.

PACS numbers: **43.25.+y**, 62.20.Mk, **62.65.+k**
DOI: 10.1070/PU2006v049n01ABEH005877

'Nonclassical' manifestations of microstructure-induced nonlinearities: new prospects for acoustic diagnostics

V Yu Zaitsev, V E Nazarov, V I Talanov

1. 'Classical' lattice nonlinearity and microstructure-induced acoustic nonlinearity

In solid-state physics, deviations from linear elasticity are traditionally attributed to a weak unharmonicity of the interatomic potential [1]. To describe such nonlinearity, it is usually sufficient to amend the linear term in Hooke's law by terms proportional to the square and cube of the deformation tensor, $\sigma = E\,(\varepsilon + \gamma^{(2)}\varepsilon^2 + \gamma^{(3)}\varepsilon^3 + \ldots)$, where σ is the elastic tension, ε is the deformation tensor, and E is the elastic modulus. For simplification, we limit our analysis to longitudinal deformations. For homogeneous amorphous materials and monocrystals, the dimensionless quadratic and cubic nonlinearity coefficients $\gamma^{(2)}$ and $\gamma^{(3)}$ are typically of the order of unity, whereas the deformations are usually quite small (for instance, $\varepsilon < 10^{-5}$ even for intense acoustic loads and $\varepsilon < 10^{-3}$ for 'usual' mechanical loads). Therefore, nonlinear corrections are usually very small in comparison with the linear term, but just these corrections are responsible for the well-known effects of thermal expansion and the dependence of the velocity of elastic waves on mechanical load and temperature. Based on such effects, the estimated nonlinearity coefficients are in good agreement with the conventionally adopted shape of the interatomic potential characterized by weak unharmonicity [1]. But the discrepancy of 2 to 4 orders of magnitude between the theoretical estimates of the breakdown threshold loads and the experimental data for the same potential was already found in the 1920s. The search for the reasons for this discrepancy has led to the understanding of the importance of microstructure imperfections and has significantly contributed to the formation of an important research field, the physics of dislocations.

Breakdown is an extreme form of materials' nonlinear behavior. Even though it is extremely sensitive to microstructure imperfections, it obviously cannot be used for diagnostic purposes. The amplitudes of deformations in the acoustic range are smaller than the breakdown deformations by several orders of magnitude; however, they already reveal microdefects by modified nonlinear properties of the medium. In the 1970s, the changes in metal microstructure due to accumulated fatigue-caused damage were first experimentally observed to result in a severalfold higher acoustic nonlinearity manifesting itself, for instance, in a significantly larger amplitude of higher harmonics of the probing acoustic wave [2 – 4]. Later, it was found that in contrast to the manifestations of the 'classical' power-law elastic nonlinearities, qualitative modifications of nonlinear effects may often occur, including functional modifications in amplitude dependences, the appearance of new nonlinear dissipative properties, and hysteresis. Data of this kind are available for many very different media, such as metals with fatigue- or heat-caused damage, rocks, many man-made materials (including composites) with crack-like imperfections, delaminations and inter-granular contacts [2 – 12], metal nanocrystals [13], and granular media [14].

From the standpoint of the possible use of nonlinear acoustic effects in diagnostics, it is especially important that

Uspekhi Fizicheskikh Nauk **176** (1) 97 – 112 (2006)
Translated by D V Belyaev; edited by A M Semikhatov

the high 'structural sensitivity' of the acoustic nonlinearity (concerning both its quantitative level and qualitative features) is often observed at the very first stage of damage of a material when its linear elastic moduli remain practically unchanged. Traditional linear methods, e.g., based on registering variations in the velocities of propagation of elastic waves or the equivalently changing fundamental frequencies cannot yet produce a definitive result at this stage of material damage.

2. Why is the variability of the nonlinear acoustic properties so great?

Due to the diversity of types of microstructural imperfections and to the broad range of media demonstrating anomalous acoustic nonlinearity both qualitatively and quantitatively, the mechanisms of its origin may also seem to be quite diverse, and specific physical models of nonlinear properties should therefore have predictive force only for rather limited classes of materials. On the other hand, alternative purely phenomenological, experiment-based stress – strain relations are more general, but do not allow relating the nonlinear properties of the medium to its microstructural features. Therefore, the models needed should reasonably compromise between the above-mentioned extreme cases and be useful for predictions.

The analysis of copious data by various groups on microstructure-induced acoustic nonlinearity (disregarding the exotic interactions of the acoustic mode with the other nonacoustic strongly nonlinear mode, which is coupled, for example, with the electron or spin subsystem) allows making the following fairly general statement. In most cases of microinhomogeneous media, a strong increase in acoustic nonlinearity of a material results from the presence of some structural features with strongly decreased rigidity, the sizes of these soft imperfections being small on the scale of the elastic wavelength, and their concentration being also small (in the sense specified below). Cracks are characteristic examples of such relatively soft imperfections. Various known models agree that a crack may be completely closed by the compressing load that produces a strain in the surrounding medium approximately equal to the ratio of the crack opening d to its diameter L. This actually means that a crack is approximately L/d times softer than the surrounding medium. The values of the d/L ratio are often quite low, $10^{-3}-10^{-5}$. Another characteristic example is intergrain contacts, which, due to their geometry (small contact area), are much more compressible than the grain bulk material. Aggregates of dislocations at grain boundaries in polycrystals are also significantly more compliant, first and foremost, to tangential loads, than the surrounding areas consisting of more homogeneous material.

Without specifying the types of imperfections, the features noted above can be described by a rather simple rheological model of a microinhomogeneous medium with soft contrasting imperfections [15, 16]. Even a one-dimensional variant of the model allows drawing some nontrivial conclusions. The model is based on the evident statement that nonlinearity (that is, the maximum deviations from Hooke's law) and increased dissipation are localized in the imperfections due to their high compressibility and, therefore, due to locally increased strain and the strain rate. The homogeneous medium that surrounds imperfections can be viewed here as a linearly elastic material that obeys Hooke's law, $\sigma = E\varepsilon$.

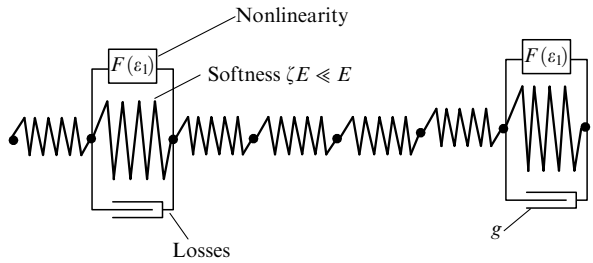

Figure 1. A rheological model of a medium with embedded contrasting soft defect inclusions at which nonlinearity and dissipation are localized.

This model of a microinhomogeneous medium is schematically given in Fig. 1. In this model, the most important parameters of imperfections are their compliance parameter (relative to the elastic modulus E of the matrix medium) given by a small parameter $\zeta \ll 1$, the concentration of soft imperfections ν, that is, in the one-dimensional case, the linear concentration of soft imperfections, or, in the three-dimensional case, their relative volume. We consider the imperfection size to be much smaller than the elastic wave length, and the imperfections are considered to be viscoelastic and weakly nonlinear when the strain of the imperfections is measured on their own scale (we note that this local strain must be distinguished from the mean macroscopic strain). For each imperfection with the compliance parameter ζ, the equation of state taking the above properties into account is

$$\sigma = \zeta E\left[\varepsilon_1 + F(\varepsilon_1)\right] + g\,\frac{\mathrm{d}\varepsilon_1}{\mathrm{d}t}\,. \tag{1}$$

The parameter g describes the effective viscosity of imperfections and the function $F(\varepsilon_1)$ describes their nonlinearity. For instance, $F(\varepsilon_1) = \gamma\varepsilon_1^2$ for quadratically nonlinear defects. The parameter γ here characterizes the imperfection nonlinearity on the scale of its own deformation ε_1, the magnitude of γ being in the standard range of several units. Using this model of a microinhomogeneous medium, for a low concentration of identical defects, we obtain [16] the following relation between the macroscopic material strain and stress:

$$\begin{aligned}\sigma(\varepsilon) = {} & E\varepsilon - E\nu\Omega\int_{-\infty}^{t}\varepsilon(\tau)\,\exp\left[-\zeta\Omega(t-\tau)\right]\mathrm{d}\tau \\ & + \nu E\Omega\zeta\int_{-\infty}^{t}\exp\left[-\zeta\Omega(t-\tau)\right] \\ & \times F\left\{\Omega\int_{-\infty}^{\tau}\varepsilon(\tau')\exp\left[-\zeta\Omega(\tau-\tau')\right]\mathrm{d}\tau'\right\}\mathrm{d}\tau\,.\end{aligned} \tag{2}$$

Here, the notation $\Omega = E/g$ is used, with $\zeta\Omega$ denoting the relaxation frequency of the defects. In the case of different defects, averaging over their properties, that is, over the distribution $\nu(\zeta, g)$, must also be included in this equation. The first term on the right-hand side of (2) comes from the linear matrix medium, the second term is from the dissipation and linear reduction of the elastic modulus due to imperfections, and the third nonlinear term accounts for the combined effects of nonlinear and relaxational properties of imperfections. This term demonstrates the essence of the 'contrast structural mechanism' of the nonlinearity increase due to locally strongly increased strain at the soft defects. This is especially clear in the quasistatic limit. In this case, for instance, for the defects characterizes by a power-type

nonlinearity of the nth order, with $F(\varepsilon_1) = \gamma^{(n)}\varepsilon_1^{(n)}$, the macroscopic stress – strain relation takes the simple form

$$\sigma = \varepsilon E_{\text{eff}} \left(1 + \varepsilon^{n-1}\gamma_{\text{eff}}^{(n)}\right),$$

which is valid for $0 \leqslant v \leqslant 1$ [15], where

$$\frac{E_{\text{eff}}}{E} = \frac{1}{1 - v + v/\zeta}, \quad \frac{\gamma_{\text{eff}}^{(n)}}{\gamma^{(n)}} = \frac{1 - v + v/\zeta^n}{(1 - v + v/\zeta)^n}. \tag{3}$$

These relations imply (see Fig. 2) that the locally increased deformation at the soft defects (characterized by the compliance parameter $\zeta \ll 1$) predominantly manifests itself in the growth of the nonlinear terms in the equation of state rather than in the drop in the linear elastic modulus. For a given value of the compliance parameter ζ, there is a range of imperfection concentrations $v < \zeta \ll 1$ where the linear elastic modulus remains practically unchanged, whereas the material's nonlinear parameter already grows severalfold (see Fig. 2). In this range of low concentrations, linear methods are not sensitive enough to produce a definite result and the use of nonlinear effects is attractive for early detection of 'weak features,' primarily, the incipient of cracks in material. Interestingly, the dependence of the nonlinearity coefficients $\gamma_{\text{eff}}^{(n)}/\gamma^{(n)}$ of different orders on the concentration of imperfections v has a clear maximum $\gamma_{\text{eff}}^{(n)}/\gamma^{(n)} = [(n-1)/\zeta]^{n-1}/n^n \gg 1$. With the higher nonlinearity order, the maximum is reached at the lower imperfection concentration $v_{\text{opt}} \approx \zeta/(n-1) \ll 1$ and the maximum value is greater. This nonmonotonic nonlinearity growth is caused by the interplay between the locally increased deformations at the soft defects and their concentration (volume content). The analogous phenomenon is known in acoustics of gas – liquid mixtures exhibiting strongly increased nonlinearity due to the structural 'contrast mechanism,' whereas pure liquids and gases are classical weakly nonlinear media [17].

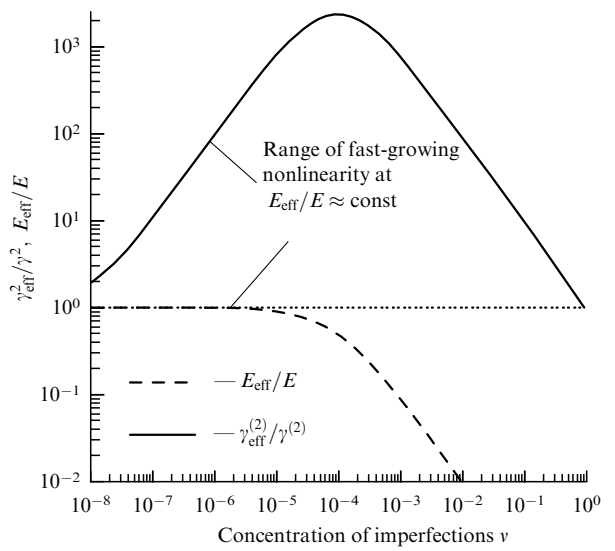

Figure 2. Complementary relative changes in the elastic modulus E_{eff}/E and in the quadratic nonlinearity parameter $\gamma_{\text{eff}}^{(2)}/\gamma^{(2)}$ for a microinhomogeneous medium containing defects with the relative compliance parameter $\zeta = 10^{-4}$, which is typical of cracks.

3. Qualitative 'nonclassical' features of the microstructure-induced nonlinearity

In the case of solids, the soft defects that we consider have some features, in addition to compliance itself, leading to unusual consequences. For example, microcontact-like and crack-like imperfections have both relaxational and specific nonlinear properties. In particular, the Hertz contacts have the fractional nonlinearity exponent 3/2 with respect to compression load and at the same time behave as diods, because they do not 'hold' a tensile load [14]. In addition, due to friction and adhesion, the same imperfections may result in stress – strain hysteresis of the material [5 – 7].

These features combined provide a wide diversity of 'nonclassical' manifestations of structure-induced nonlinearity. For example, even simplified Eqn (2) shows that for the quite common quadratic elastic nonlinearity $F(\varepsilon_1) = \gamma\varepsilon_1^2$ of the defects, their relaxational properties result in a pronounced frequency dependence of the effective nonlinear parameters, which is not typical of the classical lattice nonlinearity. Actually, a traditional intuitive approach, which accounts for nonlinearity and relaxation via additive terms, does not work for a microinhomogeneous medium. The additive approach becomes inapplicable because both nonlinear and relaxational properties of a microinhomogeneous medium are mainly localized in the same places, the soft imperfections. Thus, the relaxational 'freezing' of the response of imperfections to acoustic waves with frequencies greatly exceeding the relaxational frequency $\zeta\Omega$ simultaneously weakens their nonlinear response. For example, for the conventionally studied frequency mixing effect, the effective values of the quadratic nonlinear parameter may be considerably lower for the sum harmonics than for the difference-frequency harmonics.

Equation (2) also predicts another nontrivial manifestation of this contrast soft – rigid microheterogeneity of the medium: at moderate static and dynamic strains (e.g., $\varepsilon \sim 10^{-5}$, typical of acoustic waves), there is a possibility of a severalfold change in the acoustic dissipation in microinhomogeneous materials due to the combined action of the linear relaxational dissipation and the purely elastic nonlinearity $F(\varepsilon_1)$ [18]. It is important that at such strong changes in the dissipation, the complementary changes in the linear elasticity may still be very small, about 1%. Indeed, almost all dissipation in microinhomogeneous media is localized at soft defects such as cracks, which only weakly affect the macroscopic elastic modulus at small concentration. Therefore, even the complete closing of the cracks under compressing load may change the material elasticity merely insignificantly, whereas the *relative* change of dissipation may be arbitrarily large (from a finite value down to nearly zero). In a homogeneous medium, when the Kelvin – Voight or similar classical rheological visco-elastic models are applicable, such a drastic difference in variations of elasticity and dissipation is impossible.

The lack of space does not allow us to discuss these and other interesting consequences of the rheological model in more detail [15, 16]. We note that its implications are supported by the direct analysis of nonlinear and thermoelastic effects at Hertzian microcontacts in cracks [11, 12]. First, this analysis predicts a highly increased level of thermoelastic losses, up to 4 – 6 orders of magnitude larger than the estimates of the thermoelastic contribution to dissipation conventionally discussed in geophysics. Second,

it explains the high sensitivity of these losses to quite moderate average deformation of the medium. Indeed, the internal dissipating contacts may be considerably disturbed at typical acoustic strains $\varepsilon \sim 10^{-7} - 10^{-5}$, which are much smaller than the mean strains $\varepsilon \sim 10^{-4} - 10^{-3}$ required for the complete closing/opening of cracks.

4. Experimental examples

One of the first impressive demonstrations of the high sensitivity of acoustic nonlinearity to microstructural imperfections was the application of this phenomenon to quality control of the adhesion bonding of the thermal-insulation tiles for the Soviet space shuttle 'Buran.' The method exploited the enhanced level of the second harmonic at the local vibrational excitation of the debonded area [19]. The standard ultrasonic defectoscopy cannot be applied there because of the too high absorption of ultrasound by the tile material.

In many cases, however, the modulation of a probe wave by another independently generated perturbation (including impact perturbations) is more convenient than the registration of higher harmonics. An example of using the modulation spectra to detect a single crack of several millimeters in size in a railroad wheel axle is given in Fig. 3. The probe ultrasonic wave was modulated by the impact-generated oscillations at eigenfrequencies of the samples.

Unlike the absorption in homogeneous media, the absorption in microinhomogeneous media is pronouncedly amplitude-dependent. Together with the conventional modulation technique, this amplitude-dependent dissipation allows using the effect of amplitude modulation transfer from a carrier wave ('pump') to another initially nonmodulated probe wave for the purposes of diagnostics. This acoustic effect is a direct analog of the so-called Luxemburg – Gorky effect of cross-modulation of radio waves in ionosphere. Figure 4 shows an example of using the acoustical cross-modulation technique to monitor structural rearrangements in a grainy medium subjected to weak artificial 'seismic events' in laboratory conditions [21]. In the experiment, the amplitude modulation (30 Hz) was transferred from the carrier 'pump' wave (7 kHz) to a probe wave (10 kHz).

Successful field observations of the probe seismic wave modulation under the action of another deformation field [22, 23] and seismic-wave self-action [24] at amplitudes that are typical of the existing seismic sources [22 – 25] suggest that the

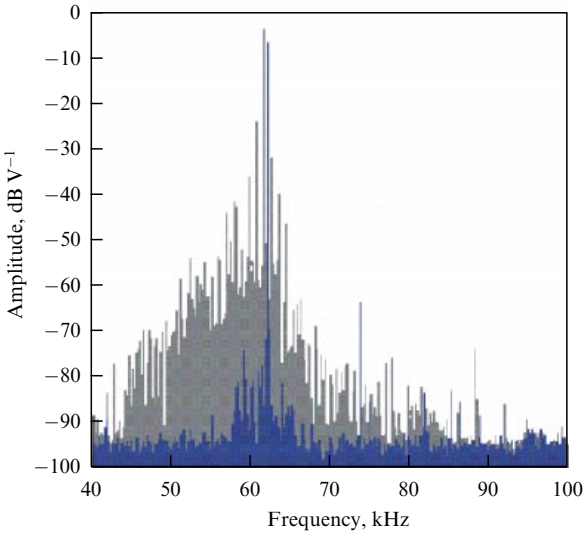

Figure 3. The diagnostics of microcracks in railroad wheel axles by nonlinear modulation. The modulation spectra of a defect-free axle and an axle with a single crack are drawn by the darker and by the lighter color, respectively. The modulation contrast is over 45 dB.

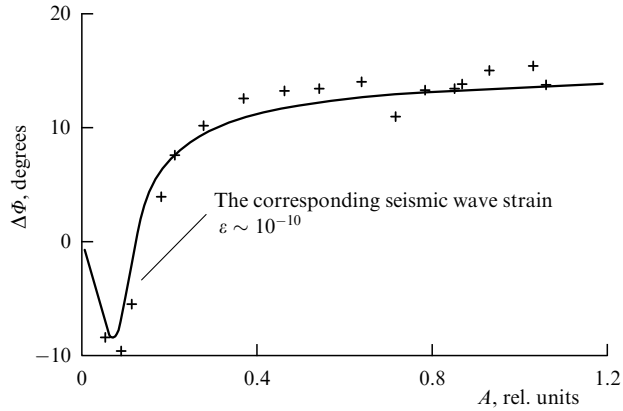

Figure 5. Nonlinear phase shift versus amplitude for a seismic wave of an extremely low amplitude radiated by a highly coherent source with the frequency 230 Hz. Crosses are the experimental data and the curve is theoretical simulation [24].

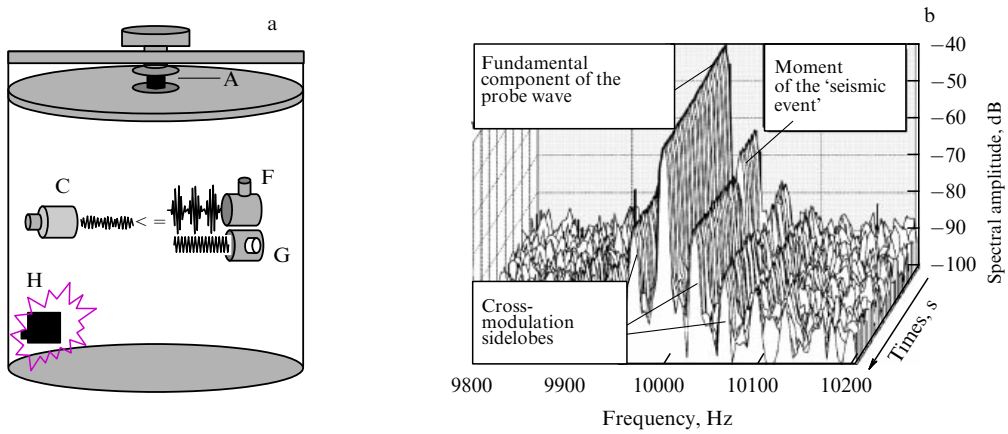

Figure 4. (a) Sketch of the experimental setup for observation of the acoustic analog of the Luxemburg – Gorky effect: A — force cell, C — receiver, F and G — sources of the pump and probe waves, H — shaker. (b) An example of strong (10 – 15 dB) variations of pump-induced cross-modulation sidelobes of the probe wave in an artificial granular medium. The variations in the probe wave amplitude at the fundamental frequency were hardly noticeable [21].

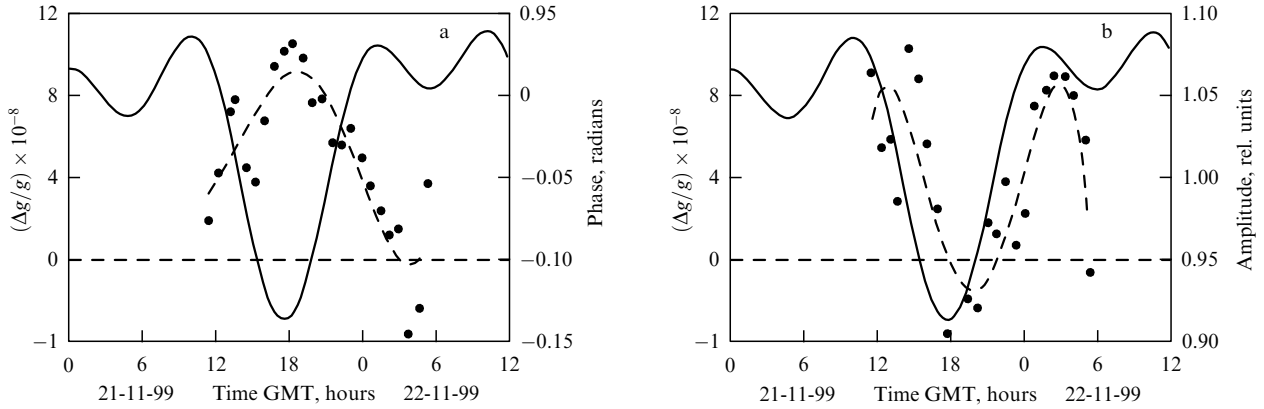

Figure 6. Tidal variations of the amplitude (a) and the phase (b) of a seismic wave with the frequency 167 Hz propagating in gas- and oil-saturated sandstone over the distance 360 m. The dashed lines fit the experimental points and the solid lines represent tidal variations in the local gravitational acceleration [23].

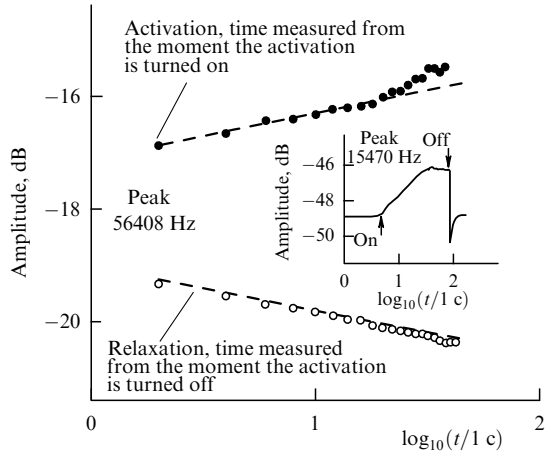

Figure 7. An example of slow dynamics of amplitudes (quality factors) of two probe-wave resonances in a steel sample with a single crack during acoustic 'loading' of it by another wave and during the relaxation after it is turned off. The solid circles and white circles represent the activation and the relaxation stages with the time measured from the moments the loading wave turned on and off, respectively. In the inset, the time scale is continuous, and the time points when the loading wave is turned on and off are shown by arrows.

nonlinear technique can be useful not only in nondestructive testing but also in seismic monitoring (at least at seismic-engineering scales of several hundred meters). Figure 5 presents an example of the seismic wave phase shift as a function of the excitation amplitude for the propagation distance 120 m in sandy sediments [24]. Another example of a field seismic experiment [23] is given in Fig. 6. Here, a seismic wave from a powerful down-hole source [25] exhibits amplitude and phase modulation by the field of tidal deformations.

Other examples of 'nonclassical' manifestations of microstructure-induced nonlinearity include a time-reversible effect of slow thermoelastic dynamics recently found to occur for cracks, combined slow and 'instant' nonlinear effects, the use of nonlinear acoustics for evaluation of the distribution of contact forces in granular media in the previously inaccessible range of very weak forces much below the average value, and other effects [11, 12, 14, 21, 26]. We limit ourselves here to an example (Fig. 7) of the logarithmic-in-time, reversible, and velocity-symmetric slow thermoelastic dynamics observed for a single crack [12, 26].

The results given here and similar data by other authors suggest a wide diversity of applications of nonlinear acoustic effects in material science, nondestructive testing, and seismic monitoring, which explains the worldwide growing attention to these studies.

In conclusion, we note that this brief review of the research conducted at the Institute of Applied Physics, RAS was to a large extent motivated by the pioneering works on nonlinear acoustics done since the 1960s – 1970s under the direction of V A Zverev and L A Ostrovskii.

The results presented were obtained with the partial support of the Russian Foundation for Basic Research (grants No. 05-05-64941 and 05-02-17355), the Ministry of Industry and Science, the program "Coherent acoustic fields and signals" of the Physics Division of RAS (grant No. NSh — 1641.2003.02), and the joint program of the Russian Academy of Sciences and Centre National de la Recherche Scientifique, France (project PECO — NEI No. 16366).

References

1. Kittel Ch *Introduction to Solid State Physics* (New York: Wiley, 1976) [Translated into Russian (Moscow: Nauka, 1978)]
2. Gedroits A A, Zarembo L K, Krasil'nikov V A *Dokl. Akad. Nauk SSSR* **150** 515 (1963) [*Sov. Phys. Dokl.* **8** 478 (1963)]
3. Gits I D, Gushchin V V, Konyukhov B A *Akust. Zh.* **19** 335 (1973)
4. Buck R O, Morris W L, Richardson J M *Appl. Phys. Lett.* **33** 371 (1978)
5. Nazarov V E et al. *Phys. Earth Planet. Inter.* **50** 65 (1988)
6. Nazarov V E *Akust Zh.* **37** 150 (1991) [*Sov. Phys. Acoust.* **37** 75 (1991)]
7. Guyer R A, Johnson P A *Phys. Today* **52** (4) 30 (1999)
8. Nazarov V E, Kolpakov A B *J. Acoust. Soc. Am.* **107** 1915 (2000)
9. Van Den Abeele K E-A *NDT&E Int.* **34** 239 (2001)
10. Ostrovsky L A, Johnson P A *Riv. Nuovo Cimento* **24** (7) 1 (2001)
11. Zaitsev V, Gusev V, Castagnede B *Phys. Rev. Lett.* **89** 105502 (2002)
12. Zaitsev V, Gusev V, Castagnede B *Phys. Rev. Lett.* **90** 075501 (2003)
13. Lang M J et al. *Nanostruct. Mater.* **12** 811 (1999)
14. Tournat V et al. *Phys. Rev. Lett.* **92** 085502 (2004)
15. Zaitsev V Yu *Acoust. Lett.* **19** 171 (1996)
16. Zaĭtsev V Yu, Nazarov V E, Belyaeva I Yu *Akust. Zh.* **47** 220 (2001) [*Acoust. Phys.* **47** 178 (2001)]
17. Rudenko O V, Soluyan S I *Teoreticheskie Osnovy Nelineinoi Akustiki* (Theoretical Foundations of Nonlinear Acoustics) (Mos-

cow: Nauka, 1975) [Translated into English (New York: Consultants Bureau, 1977)]
18. Zaitsev V, Sas P *Acta Acust. Acust.* **86** 429 (2000)
19. Antonets V A, Donskoi D M, Sutin A M *Mekhanika Komposit. Mater.* (5) 934 (1986)
20. Matveev A L et al. *V Mire Nerazrush. Kontr.* (4(26)) 65 (2004)
21. Zaitsev V Yu et al. *Europhys. Lett.* **70** 607 (2005)
22. Bagmet A L et al. *Dokl. Ross. Akad. Nauk* **346** 390 (1996) [*Dokl. Earth Sci.* **346** 135 (1996)]
23. Bogolyubov B N et al. *Geol. Geofiz.* **45** 1045 (2004)
24. Zaitsev V Yu, Nazarov V E, Talanov V I *Akust. Zh.* **45** 799 (1999) [*Acoust Phys.* **45** 720 (1999)]
25. Averbakh V S et al. *Akust. Zh.* **51** (dop. vyp.) 31 (2005) [*Acoust Phys.* **51** (Suppl.) 23 (2005)]
26. Zaitsev V, Nazarov V, Gusev V, Castagnede B *NDT&E Int.* (2005) (in press)

PACS numbers:: **43.25.+y**, 43.30.Pc, 83.80.Fg, **92.10.–c**
DOI: 10.1070/PU2006v049n01ABEH005878

Nonlinear acoustic diagnostics of the ocean and rock

I B Esipov, S A Rybak, A N Serebryanyi

1. Introduction

In the 1940s and 1950s, Russian studies on nonlinear acoustics were stimulated by the pioneering work of N N Andreev. His article 'On some second-order values in acoustics' was published in the first volume of *Akusticheskii Zhurnal* issued in 1955 [1]. In the 1960s, it was followed by the technological and medical applications of powerful ultrasound introduced by L D Rosenberg [2]. It was then that L M Brekhovskikh formulated the problem of the interaction between acoustic and oceanic waves [3]. Many Russian research groups worked in these areas. Recently, experts in propagation and interactions of nonlinear waves have started to address waves in granular media and continental shelf internal waves because both are essentially nonlinear. In granular media, the nonlinearity parameter is 3 – 4 orders of magnitude greater than that in homogeneous media, and the 'giant nonlinearity' term has been coined [4, 5]. Due to the high amplitudes and slow velocities, internal waves undergo essential nonlinear transformations.

In this report, the results of experimental observations of new nonlinear wave processes in the ocean and granular rocks are given.

2. Granular media

Acoustic waves are known to propagate in granular and in solid media differently. The mechanical properties of granular media are largely determined by the inter-granular contacts. This property allows assigning granular media to a broad class of media with nonlinear structural elasticity. Whereas the nonlinear acoustic properties of monocrystals, homogeneous fluids, and other solid media are determined by the molecular nature of their strain, the properties of granular media are determined by their structure. In this sense, the properties of granular media are seen at the mesoscale, that is, the scale of the granules [4]. This results in considerable qualitative and quantitative differences, including differences in the equations of state for the media. For a solid medium, the relative strain Δ is assumed to be proportional to the stress applied, $\Delta \approx P$, but the relation for the spherical granule is $\Delta \approx P^{2/3}$ [6]. In granular media, the velocity of acoustic waves is therefore $c = (\partial P/\partial \rho)^{1/2} \approx P^{1/6}$, which is a nonlinear function of the stress P applied. In agreement with this, the nonlinearity parameter $\alpha = \rho_0 \partial c^2/\partial P \approx P^{-5/6}$ is also dependent on the stress applied. (Here, ρ is the medium density and ρ_0 is its value at equilibrium.) The nonlinear properties of granular media prove noticeable even at quite moderate strain. In rocks, for instance, nonlinear distortions are already observed at a strain equal to $\Delta \approx 10^{-9}$ [4]. This strain range is typical even for quite moderate acoustic perturbations.

The behavior of granular media is currently being studied at the single-granule level [7 – 11]. Under low stress, the stress – strain relations are noticed to measurably deviate from regular relations. As a rule, the latter work only asymptotically under high enough stress, when the granular medium may be considered well-packed. Such media usually respond to repeated stress with stress – strain hysteresis. This property of granular media leads to a nonlinear distortion of acoustic waves with the second harmonic proportional to the squared signal amplitude and possibly exceeding the third harmonic. Oscillations of a single granule in a constant acoustic field are shown to slowly fluctuate [7].

We discuss the results of the experimental analysis of slow fluctuations of the nonlinear oscillations of the granule in a medium where the acoustic field is propagating. One-two Granite bits of the size 1 to 2 cm served as the granular medium in our experiments. The sound was produced by a piezoceramic plate and the detection was done by accelerometers mounted among the granules. The experimental setup is detailed in [12]. We note that the accelerometers were of the size scale of the granules.

The detected signal versus the intensity of sound produced is plotted in Fig. 1. In this and subsequent experiments, the frequency of the produced sound was 5.6 kHz. The signal was detected by two accelerometers positioned equidistantly from the sound source and separately from each other. The common character of response measured by both detectors can be seen. A linear relation between the sound intensity and the signal holds only on average, for a large range of signal amplitude variations. The response is different for the different detectors, pointing to the independent propagation

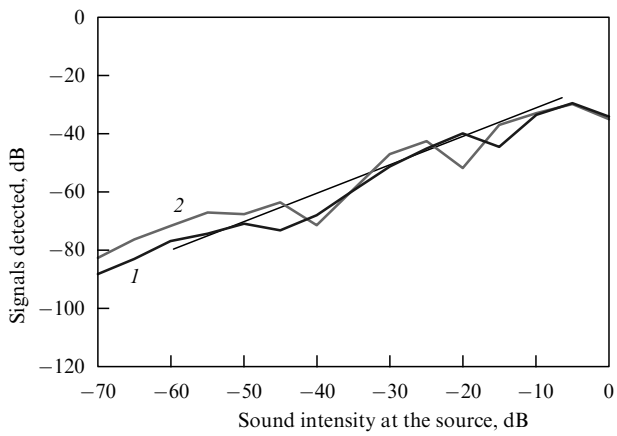

Figure 1. Signals detected in the granular medium versus sound intensity. The straight line corresponds to the linear response. Curves *1* and *2* are from different detectors.