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A generalized adiabatic principle for electron dynamics in curved nanostructures

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1. Introduction

Progress in nanotechnology has made possible the production of extended thin quasi-one-dimensional and quasi-two-dimensional structures of complex geometries, namely, nanotubes and nanofilms. In our model, these structures are the areas of the thin bent 'twisted-edge' cylinder type or of the thin curved film type. Beyond these areas, the wave function $\Psi(\mathbf{r}, t)$ of a quantum particle either drops exponentially (a soft wall model) or equals zero (a hard wall model). As in Refs [1, 2], we assume that the electron (three-dimensional) quantum dynamics in nanostructures in a magnetic field is given by the so-called Rashba Hamiltonian

$$\widehat{\mathcal{H}} = \frac{\widehat{\mathbf{P}}^2}{2m} + v_{\text{int}}(\mathbf{r}) + v_{\text{ext}}(\mathbf{r}, t) - \frac{e\hbar}{2mc}\,\langle \boldsymbol{\sigma}, \mathbf{H}\rangle + \widehat{\mathcal{H}}_{\text{so}}\,. \quad (1)$$

Here, $\mathbf{r}$ is the radius vector of a point in three-dimensional space; $\widehat{\mathbf{P}} = -\mathrm{i}\hbar\nabla - (e/c)\mathbf{A}(\mathbf{r}, t)$; e is the electron charge; m is the effective mass of a quasiparticle; $v_{\text{int}}(\mathbf{r})$ is the confinement potential; v_{ext}, $\mathbf{A}$ are the external field potentials; $\mathbf{H}(t) = \operatorname{rot}\mathbf{A}(\mathbf{r}, t)$ defines a homogeneous magnetic field; $\boldsymbol{\sigma} = \{\sigma_1, \sigma_2, \sigma_3\}$ are the Pauli matrices, and $\widehat{\mathcal{H}}_{\text{so}} = \alpha\langle\boldsymbol{\sigma}, [\nabla v_{\text{int}}, \widehat{\mathbf{P}}]\rangle$ is the operator for the interaction of the electron spin with the crystal electric field, with α constant being dependent on the given crystal type [3]. In the case of $v_{\text{int}}(\mathbf{r}) = 0$ and a zero wave function at the tube or film boundaries, we get 'empty structure' models. d and l_0 stand for the characteristic thickness and linear dimensions (e.g., tube length), respectively, of a tube or a film. In extended thin nanostructures, the scale difference may be conveniently characterized by a small 'adiabatic' parameter $\mu = d/l_0 \ll 1$. We restrict ourselves to the case of a weak enough magnetic field, thus considering the Larmor frequency $\omega_H = e|\mathbf{H}|/(mc) \sim \hbar/(mdl_0)$, and magnetic length $l_H \sim \sqrt{dl_0}$.

The Schrödinger nonstationary equation for electron quantum states $\Psi(\mathbf{r}, t)$ (including stationary ones) acquires the form

$$\mathrm{i}\hbar\Psi_t = \widehat{\mathcal{H}}\,\Psi\,. \quad (2)$$

We limit our consideration to a quantum particle (electron) in such tubes and films with slowly changing geometric characteristics, whose small segments with linear scales of order close to their thicknesses d may be quite accurately considered a right cylinder and a flat layer, respectively. It is clear that the effective electron dynamics in such structures have to be one- or two-dimensional and be represented by the equation with the effective Hamiltonian $\hat{L}^{\nu}$ for the wave function ψ^{ν} on the tube axis or the film surface:

$$\mathrm{i}\hbar\psi^{\nu}_t = \hat{L}^{\nu}\psi^{\nu}\,, \quad (3)$$

where ν is the number of a 'dimensional quantization subband'. To go over from Eqn (2) to Eqn (3), we use the procedure in which the function Ψ for the lower subbands with $\nu \ll \mu^{-1}$ is retrieved for ψ^{ν} by the action of 'intertwining' operator on ψ^{ν} (see Section 3.1). In stationary problems, the derivative $\mathrm{i}\hbar\partial/\partial t$ is substituted by the energy E.

Different characteristic dimensions and the presence of free carriers make possible the consideration of nanostructures as quantum waveguides or confined quantum systems [6 – 15]. Similar problems related to waveguides emerge in electrodynamics, acoustics, the theory of elasticity, marine physics, and so forth. Confinement elimination leading to lower system dimensionality is usually done by applying the adiabatic approximation equivalent to asymptotic partitioning of oscillations into longitudinal and transverse modes. For the Helmholtz equation, this partitioning was done in Ref. [16]. In that work, equation (3) for a longitudinal mode was given and it was shown that single-mode bound-state resonators may be made by varying the waveguide curvature [17]. Analogous equations for quantum-mechanical problems have been subsequently deduced in Refs [1, 2, 7 – 15]. It should be emphasized that the waveguide problems are similar to those of molecular physics, with the role of confinement potential being played by the Coulomb potential with 'frozen' coordinates of heavy nuclei. In mathematical literature, equations emerging in problems with different scales are called the operator-valued symbol equations [19].

The longitudinal-state wave functions ψ^{ν} can be: (1) delocalized and significantly changing on the scale of order l_0; (2) delocalized and rapidly oscillating, i.e., changing on the scale of $\lambda_{\parallel} \ll l_0$, and (3) asymptotically localized in the small sections with scales $\ll l_0$. We characterize the rate of the wave function change by a 'quasiclassical' parameter $h = \lambda_{\parallel}/l_0$, where $\lambda_{\parallel} = \max|\partial\psi^{\nu}/\partial x|^{-1}$ is the wavelength characteristic of ψ^{ν}. The final expressions for Ψ are significantly dependent on the relationships among $\lambda_{\parallel}$, d, and l_0 or, equivalently, on the relationship between the parameters μ and h.

In this report, we introduce the effective equations for 'dimensional quantizing subbands' (3), which are accurately derived and can be utilized to describe all the above-mentioned longitudinal states. The class of these states proves to be considerably wider than that described in Refs [7–11]. Bound states and traps due to the variable tube thickness, the spin effect on the classical one-dimensional dynamics in the tubes placed in a magnetic field (see Section 4), the possibility of spin flipping in curved tubes, and so forth are derived from the equations given.

We limit our consideration to nanotubes. The authors' results on nanofilms and optical planar waveguides can be found in Refs [15, 17]. The works [18, 19] are also devoted to nanofilms.

2. The effective Hamiltonian for a dimensional quantizing subzone of a nanotube

2.1 Hamiltonian for a ν-th subband of a bent tube

Let us define a suitable coordinate system in the vicinity of the tube axis. We will assume that the tube axis is a curve γ given by the equation $\mathbf{r} = \mathbf{R}(x)$, $\mathbf{r} \in \mathbb{R}^3$, $x \in \mathbb{R}$, where $\mathbf{R}(x)$ is a smooth vector function, x is a natural parameter in the curve γ, i.e., the tube length measured from a fixed point x^*, $|\partial_x \mathbf{R}(x)| = 1$, and $\partial_x = \partial/\partial x$. If the axis curvature $k(x) = |\partial_x^2 \mathbf{R}| \neq 0$, the Frenet trihedron is defined as $\{\partial_x \mathbf{R}, \mathbf{n} = \partial_x^2 \mathbf{R}/|\partial_x^2 \mathbf{R}|, \mathbf{b} = [\partial_x \mathbf{R}, \mathbf{n}]\}$ and the axis twisting $\varkappa(x)$ is governed by the equation $\partial_x \mathbf{n} = -\varkappa \mathbf{b} - k\partial_x \mathbf{R}$, $\partial_x \mathbf{b} = \varkappa \mathbf{n}$. Rotating $\mathbf{n}(x)$, $\mathbf{b}(x)$ through the angle $\theta(x) = \int_{x^*}^{x} \varkappa(x)\,\mathrm{d}x$, we get the vectors $\mathbf{n}_1(x)$, $\mathbf{n}_2(x)$. Then, the curvilinear coordinates (x, y_1, y_2) introduced by the relationship $\mathbf{r} = \mathbf{R}(x) + \mathbf{y}(x, y_1, y_2)$, $\mathbf{y}(x, y_1, y_2) = y_1 \mathbf{n}_1(x) + y_2 \mathbf{n}_2(x)$ will be orthogonal in the vicinity of the tube axis.

In this report, closed tubes and tubes with ends satisfying the Born–Karman periodicity condition for the function Ψ are called periodic tubes. The tube period is represented by l_0. In passage through l_0, the vectors $\mathbf{n}_1(x)$, $\mathbf{n}_2(x)$ go over into $\Pi(\theta_0)\,\mathbf{n}_1(x)$, $\Pi(\theta_0)\mathbf{n}_2(x)$, where $\Pi(\theta_0)$ is the rotation matrix through the angle $\theta_0 = \int_{x^*}^{x^*+l_0} \varkappa(x)\,\mathrm{d}x$. The coordinates (x, y_1, y_2) are therefore not global — that is, the same point in a three-dimensional space corresponds to coordinates $\big(x + nl_0,\, \Pi(-n\theta_0)\, y\big)$, $n = 0, \pm 1, \pm 2, \ldots$, where $y = \{y_1, y_2\}^{\mathrm{T}}$ is a two-component vector column, and the periodicity condition takes the form $\Psi(x, y) = \Psi\big(x + l_0, \Pi(-\theta_0)\, y\big)$.

To simplify expressions, we limit our further consideration to a class of model potentials represented by the formula $v_{\mathrm{int}} = v_{\mathrm{int}}(x^*, D(x)^{-1}\Pi(\Phi)^{-1}\mathbf{y})$, where $D(x) > 0$, $v_{\mathrm{int}}(x^*, y_1, y_2)$ is a smooth function, $\Pi(\Phi)$ is the rotation matrix through the angle $\Phi(x)$ of 'inner' twisting, and x^* is some fixed point on the tube axis.

To apply the adiabatic approximation, the 'prompt transverse' functions should be defined. They are represented by the formula $\exp(\mathrm{i}\langle \mathbf{y}, \mathbf{A}(\mathbf{R})\rangle)\, w_j^{\nu}$, $j = 1, \ldots, r$, where w_j^{ν} are the problem eigenfunctions normalized to unity with respect to y:

$$\left[-\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial y_1^2} + \frac{\partial^2}{\partial y_2^2}\right) + v_{\mathrm{int}}(x, y)\right] w_j^{\nu} = \varepsilon_{\perp}^{\nu}(x)\, w_j^{\nu}\,. \qquad (4)$$

The appearance of a number j of functions w_j^{ν} is related to the possible degeneracy of $\varepsilon_{\perp}^{\nu}(x)$; we will assume that *the degeneracy order r does not depend on the longitudinal coordinate x*. The functions $w_j^{\nu}(x, y)$ and $\varepsilon_{\perp}^{\nu}(x)$ are expressed through $w_j^{\nu}(x^*, y)$, $\varepsilon_{\perp}(x^*)$:

$$w_j^{\nu}(x, y) = D^{-1}(x)\, w_j^{\nu}\left(x^*, D^{-1}(x)\,\Pi^{-1}(x)\, y\right) \times \exp\left[\mathrm{i}\beta_j(x - x^*)\right],$$

$$\varepsilon_{\perp}^{\nu}(x) = \frac{\varepsilon_{\perp}^{\nu}(x^*)\, D^2(x^*)}{D^2(x)}\,.$$

For a nonperiodic tube, $w_j^{\nu}(x^*, y)$ and β_j are chosen in ambiguous manner; β_j may be set equal to zero. For a periodic tube, it is appropriate to choose such a β_j that $w_j^{\nu}(x + l_0,\, \Pi(-\theta_0)\, y) = w_j^{\nu}(x, y)$. Then, the effective quantum matrix Hamiltonian that is one-dimensional with respect to x will be defined as

$$\begin{aligned}\hat{L}^{\nu} &= \frac{\hat{p}^2}{2m} + v_{\mathrm{ext}}(x) + \varepsilon_{\perp}^{\nu}(x) - \frac{\hbar^2 k^2(x)}{8m} \\ &\quad + \frac{e}{c}\int_0^x \left\langle \partial_x \mathbf{R}(x'), \frac{\partial \mathbf{A}(\mathbf{R}(x'), t)}{\partial t}\right\rangle \mathrm{d}x' \\ &\quad + \hbar B \otimes E_2\, \frac{\hat{p}}{m} + \hat{L}_y \otimes E_2 - \frac{e\hbar}{2mc}\, E_r \otimes \langle \boldsymbol{\sigma}, \mathbf{H}\rangle + \hat{L}_{\mathrm{sy}}, \qquad (5)\end{aligned}$$

where $\hat{p} = -\mathrm{i}\hbar\partial/\partial x$, and

$$\begin{aligned}\hat{L}_y &= \Lambda\left(\frac{\hbar}{m}\,(\partial_x \Phi)\,\hat{p} - \frac{e}{2mc}\langle \partial_x \mathbf{R}, \mathbf{H}\rangle\right) - \frac{e}{mc}\langle \mathbf{Y}_{\perp}, \mathbf{H}\rangle\,\hat{p} \\ &\quad + \left\langle \mathbf{Y}, \nabla v_{\mathrm{ext}} + \frac{e}{c}\,\frac{\partial \mathbf{A}(\mathbf{R}, t)}{\partial t} + k\mathbf{n}\,\frac{\hat{p}^2}{2m}\right\rangle,\end{aligned}$$

$$\begin{aligned}\hat{L}_{\mathrm{sy}} &= \alpha\big(M^0 \otimes \langle \boldsymbol{\sigma}, \partial_x \mathbf{R}\rangle + M^1 \otimes \langle \boldsymbol{\sigma}, \mathbf{n}_1\rangle\,\hat{p} \\ &\quad + M^2 \otimes \langle \boldsymbol{\sigma}, \mathbf{n}_2\rangle\,\hat{p}\big)\,.\end{aligned}$$

The following designations were introduced above: E_n is a unit $n \times n$ matrix; B is a diagonal $r \times r$ matrix having elements $B_{jj'} = \beta_j \delta_{jj'}$; $\Lambda(x)$ is an $r \times r$ momentum matrix having elements $\Lambda_{jj'} = \langle w_j^{\nu}, \hat{l} w_{j'}^{\nu}\rangle_y$, $\hat{l} = -\mathrm{i}\hbar(y_1\partial/\partial y_2 - y_2\partial/\partial y_1)$; $M^k(x)$ are the $r \times r$ matrices ($k = 0, 1, 2$) such that

$$(M^0)_{jj'} = -\mathrm{i}\hbar\left\langle w_j^{\nu}, \big((\partial_1 v_{\mathrm{int}})\,\partial_2 - (\partial_2 v_{\mathrm{int}})\,\partial_1\big)\, w_{j'}^{\nu}\right\rangle_y,$$

$$(M^1)_{jj'} = \left\langle w_j^{\nu}, (\partial_2 v_{\mathrm{int}})\, w_{j'}^{\nu}\right\rangle_y,$$

$$(M^2)_{jj'} = -\left\langle w_j^{\nu}, (\partial_1 v_{\mathrm{int}})\, w_{j'}^{\nu}\right\rangle_y;$$

$\partial_i = \partial/\partial y_i$; $\otimes$ denotes the tensor product of matrices; $\mathbf{Y}(x) = Y_1\mathbf{n}_1 + Y_2\mathbf{n}_2$ and $\mathbf{Y}_{\perp}(x) = Y_2\mathbf{n}_1 - Y_1\mathbf{n}_2$ are the three-dimensional 'vectors' with components that are 'dipole' 2×2 matrices $(Y_i)_{jj'}(x) = \langle w_j^{\nu}, y_i\, w_{j'}^{\nu}\rangle_y$, $i = 1, 2$, and, finally, $\langle .,. \rangle_y$ designates integrating over y variables.

For the $\lambda_{\parallel} \gg d$ case, formula (5) type Hamiltonian had been obtained in Refs [1, 2]. The 'geometric potential' $-\hbar^2 k(x)^2/(8m)$ originally introduced in Ref. [16] should be taken into consideration in the long-wave limit ($\lambda_{\parallel} \sim l_0$). Namely this potential generates bound states in an empty waveguide [16] by producing effective attraction to the maximum curvature points on an axis.

If the function ψ^{ν} is the effective solution of equation (3), the function ψ^{ν} is retrieved according to the relationship

$$\Psi^{\nu}(x, y, t) \approx G(x, y)^{-1/4}\big(\chi_0^{\nu}(x, y, t) + \hat{\chi}_1^{\nu}\big) \times \exp\left(\frac{\mathrm{i}e}{\hbar c}\int_{x^*}^{x} \big\langle \partial_x \mathbf{R}, \mathbf{A}(\mathbf{R})\big\rangle\,\mathrm{d}x\right)\psi^{\nu}(x, t)\,,$$

where $G(x,y) = (1 - k\langle \mathbf{y}, \mathbf{n}\rangle)^2$, and

$$\chi_0^\nu = \exp\Big(\mathrm{i}\langle \mathbf{y}, \mathbf{A}(\mathbf{R})\rangle\Big) \times \left\| \begin{matrix} w_1^\nu(x,y) & 0 & \cdots & w_{\nu+1}^\nu(x,y) & 0 \\ 0 & w_1^\nu(x,y) & \cdots & 0 & w_{\nu+1}^\nu(x,y) \end{matrix} \right\|, \tag{6}$$

with $\hat{\chi}_1^\nu = \chi_0^\nu(x, \hat{p}, y, t)$ being the corrective differential operator whose explicit definition is not important for further consideration.

2.2 Geometric phases and boundary conditions

In an open tube, it is natural to consider the problems of wave packet scattering and evolution for equations (3). In a periodic tube, the condition for Ψ is followed by the Bloch condition for a vector function ψ^ν:

$$\psi^\nu(x + l_0, t) = \exp\left(\frac{\mathrm{i}e}{\hbar c}\int_0^{l_0} \langle \partial_x \mathbf{R}, \mathbf{A}(\mathbf{R})\rangle \,\mathrm{d}x\right) \psi^\nu(x,t)\,.$$

The phase for a closed tube is given by

$$\frac{e}{\hbar c}\int_0^{l_0} \langle \partial_x \mathbf{R}, \mathbf{A}(\mathbf{R})\rangle \,\mathrm{d}x = \frac{2\pi\Phi}{\Phi_0}\,,$$

where Φ is the magnetic flux through the area enclosed by the tube axis:

$$\Phi = \int_0^{l_0} \langle \partial_x \mathbf{R}, \mathbf{A}(\mathbf{R})\rangle \,\mathrm{d}x\,,$$

and $\Phi_0 = 2\pi\hbar c/e$ is a magnetic flux quantum. This equality is an *Aharonov – Bohm effect* manifestation [20].

3. The generalized adiabatic principle

3.1 The 'operator' separation of variables

Here are the basic ideas of the 'generalized adiabatic principle' resulting in Eqns (3) and (5). The wave functions $\Psi = \Psi^\nu(x,y,t)$ for the lower subbands of transverse quantizing are given as the result of an action of $x, \hat{p} = -\mathrm{i}\hbar\partial/\partial x$, t, y-dependent 'intertwining' operator $\hat{\chi}^\nu = = \chi^\nu(x, \hat{p}, y, t)$ on a function $\psi^\nu(x,t)$ independent of y:

$$\Psi^\nu(x,y,t) = \chi^\nu(x, \hat{p}, y, t)\,\psi^\nu(x,t)\,. \tag{7}$$

Since the operators x and $\hat{p}$ do not commute, an agreement should be made about the order of their actions. For the sake of definiteness, we will assume that the operator $\hat{p}$ acts first, and the operator x acts second (see Refs [22, 23]). In addition to formula (7), ψ^ν is required to satisfy equation (3). As in eigenfunction and eigenvalue problems, the operators $\hat{\chi}^\nu$ and $\hat{L}^\nu$ are defined simultaneously. Calculating the operator $\hat{L}^\nu$ generalizes the *Peierls substitution and dispersion relation quantization* (cf. Ref. [24]). Considering the possible degeneracy of $\varepsilon_\perp^\nu(x)$ in problem (4), $\psi^\nu(x,t)$ should be viewed as a $2r$-dimensional vector function, and $\hat{\chi}^\nu$ should be viewed as a $2 \times 2r$-matrix function of the operators $\hat{p}$ and x.

The representation of Ψ^ν according to formula (7) is a natural generalization of the Born – Oppenheimer method in which the function χ^ν does not depend on $\hat{p}$, and expressions [21] for WKB (Wentzel – Kramers – Brillouin) functions ψ^ν, including the turning point and focusing situations. Formula (7) realizes the quantum averaging idea [23 – 26] that some block of the diagonalizing operator $\hat{U}$ is asymptotically (μ-wise) represented in the form $\hat{U} = \exp(\mathrm{i}\hat{S}) \approx \sum_\nu |\hat{\chi}^\nu\rangle\langle\hat{\chi}^\nu|$, $\langle \hat{a} \,|\, \hat{b}\rangle = = \int \mathrm{d}y\,(\hat{a}^+\hat{b})$, and $\langle\hat{\chi}^\nu|\hat{\chi}^{\nu'}\rangle = \delta_{\nu\nu'}$. After the 'twisting', equation (2) takes the form

$$\hat{U}^{-1}\left(-\mathrm{i}\hbar\frac{\partial}{\partial t} + \hat{\mathcal{H}}\right)\hat{U} = \sum_\nu |\hat{\chi}^\nu\rangle\left(-\mathrm{i}\hbar\frac{\partial}{\partial t} + \hat{L}^\nu\right)\langle\hat{\chi}^\nu| = 0\,.$$

For the function $\Psi^\nu = \hat{\chi}^\nu\psi^\nu$, we arrive at the equation

$$\left(-\mathrm{i}\hbar\frac{\partial}{\partial t} + \hat{\mathcal{H}}\right)\Psi^\nu = |\hat{\chi}^\nu\rangle\left(-\mathrm{i}\hbar\frac{\partial}{\partial t} + \hat{L}^\nu\right)\psi^\nu = 0$$

equivalent to Eqn (3).

Representation of $\hat{\chi}^\nu$ as a function of x and $\hat{p}$ operators is fundamental. In mathematical literature, a function of x, $\hat{p}$ is called a *pseudo-differential operator* whose master function is called a *symbol*. A passage from operators $\hat{\chi}^\nu$ and $\hat{L}^\nu$ to symbols χ^ν and L^ν allows us not to be concerned whether operators x and $\hat{p}$ commute. Symbols χ^ν and L^ν cannot usually be exactly computed, and one can only calculate the coefficients of the asymptotic expansion of χ^ν and L^ν into a series in the adiabatic parameter μ:

$$\chi^\nu = \chi_0^\nu(x,p,y) + \chi_1^\nu(x,p,y) + \chi_2^\nu(x,p,y) + \ldots, \chi_j^\nu \sim \mu^j\,,$$

$$L^\nu = L_0^\nu(x,p) + L_1^\nu(x,p) + L_2^\nu(x,p) + \ldots, L_j^\nu \sim \frac{\mu^j\hbar^2}{md^2}\,.$$

General expressions for χ_k^ν and L_k^ν are given in Ref. [15], including the case of a degenerate effective Hamiltonian. To produce them, some basic concepts and formulae of operator algebra [22] known to junior or senior students majoring in physics and mathematics are needed. On the other hand, deriving the 'twisting' operator in quantum averaging is a quite complex problem. A realization of the adiabatic approximation as in Eqns (7) and (3) is therefore thought to be its most pragmatic and simple form.

In many problems, χ_j^ν and L_j^ν are power series in p, therefore, $\hat{\chi}^\nu$ and $\hat{L}^\nu$ are the differential operators. The function χ_0^ν does not usually depend on momentum, and in this case $\hat{\chi}_0^\nu$ is a regular function of variables x, y. But, as a rule, χ_1^ν depends on p, thus restricting the possibility of using the adiabatic approximation (see Section 3.3).

3.2 Energy renormalization and quasiclassical asymptotics

The operator (5) spectrum strongly depends on the pattern of the effective potential change, i.e., on the effective force applied to a particle in the direction of a tube axis.

As an example, consider a tube in the absence of an external field. Then, the effective 'longitudinal' potential (an analog of the Morse potential in molecular physics) is mainly determined by 'fluctuations' in the cross section dimensions and, as a consequence, by the 'prompt' transverse energy $\varepsilon_\perp^\nu(x)$. Let us consider the following cases with a periodic tube: (1) $\varepsilon_\perp^\nu = \varepsilon_\perp^\nu(x_0) = \text{const}$, and (2) $\varepsilon_\perp^\nu(x)$ has a single point of minimum x_0 over the period. In case (1), the operator (5) spectrum is taken as $E^{\nu n} = \varepsilon_\perp^\nu + \varepsilon_\parallel^{\nu n}$. The part of the spectrum where $\varepsilon_\parallel^{\nu n} \ll \varepsilon_\perp^\nu$ may be significantly influenced by the axis curvature (bound states [9, 16] may be organized) and by spin-related terms [2]. In case (2), the point x_0 of the minimum produces a spectral series of asymptotically localized eigenfunctions ('trapped' states analogous to the

lower states of nuclei in molecular physics) and eigenvalues $E^{\nu n} = \varepsilon_\perp^\nu(x_0) + \varepsilon_\parallel^{\nu n}$, with geometric potential practically not playing a role and possibly considered by the perturbation theory. The longitudinal wavelength $\lambda_\parallel$ is related to $\varepsilon_\parallel^{\nu n}$ through the estimate $\varepsilon_\parallel^{\nu n} \sim \hbar^2/(2m\lambda_\parallel^2)$.

Depending on the relation between $\varepsilon_\parallel^{\nu n}$ and $\varepsilon_\perp^\nu(x_0)$, the following classification of operator (5) eigenvalues is worth proposing:

(a) long-wave states with

$$\lambda_\parallel \sim l_0\,, \quad \varepsilon_\parallel^{\nu n} \sim \frac{d^2}{l_0^2}\,\varepsilon_\perp^\nu(x_0)\,, \quad h \sim 1\,;$$

(b) medium-wave states with

$$\lambda_\parallel \sim \sqrt{d l_0}\,, \quad \varepsilon_\parallel^{\nu n} \sim \frac{d}{l_0}\,\varepsilon_\perp^\nu(x_0)\,, \quad h \sim \sqrt{\mu}\,;$$

(c) short-wave states with

$$\lambda_\parallel \sim d\,, \quad \varepsilon_\parallel^{\nu n} \sim \varepsilon_\perp^\nu(x_0)\,, \quad h \sim \mu\,;$$

(d) ultra-short-wave states with

$$\lambda_\parallel \sim \frac{d^{3/2}}{l_0^{1/2}}\,, \quad \varepsilon_\parallel^{\nu n} \sim \frac{l_0}{d}\,\varepsilon_\perp^\nu(x_0)\,, \quad h \sim \mu^{3/2}\,.$$

Given the longitudinal wavelength

$$\lambda_\parallel \sim \frac{d^2}{l_0}\,, \quad \varepsilon_\parallel^{\nu n} \sim \frac{l_0^2}{d^2}\,\varepsilon_\perp^\nu(x_0)\,, \quad h \sim \mu^2\,,$$

the adiabatic approximation is no longer valid. A similar classification for an abstract problem was made in Ref. [27]. In case (a), equation (3) should be solved exactly; in cases (b) through (d), the quasiclassical approximation may be used, with spin possibly influencing classical dynamics in case (b); finally, in case (d), the quasiclassical approximation coincides with the Born approximation (see examples in Section 4).

3.3 On the accuracy of equation derivation in dimensional quantizing subbands

Using the asymptotic procedure [15], operator (5) can be obtained with any given accuracy. Its derivation, however, faces some technical difficulties. In the case where $\hat{\mathcal{H}}$ is not time-dependent, the operator $\hat{L}^\nu$ is reasonably built with such a degree of accuracy that its spectrum $E^{\nu n}$ is an approximation to the spectrum $\varepsilon^{\nu n}$ of the operator $\hat{\mathcal{H}}$. To do this, the condition $|E^{\nu n} - \varepsilon^{\nu n}| \ll |E^{\nu n} - E^{\nu(n+1)}|$ must be satisfied. Three terms of the operator $\hat{L}^\nu$ expansion are usually sufficient. To find the excited eigenfunctions of operator (5), two first terms are enough.

In the case (d) (see Section 3.2), a characteristic period of transverse oscillations $md^2/\hbar$ becomes comparable with the time $m\lambda_\parallel l_0/\hbar$ that it takes a particle to pass through a tube, therefore the instantaneous partition of oscillations into longitudinal and transverse ones does not make sense and the adiabatic approximation no longer works. In formula (7), the result of operator $\hat{\chi}_1^\nu$ action on function ψ^ν cannot be considered a 'correction' because $\hat{\chi}_1^\nu \psi^\nu \sim 1$. The part of the operator $\hat{L}^\nu$ spectrum, corresponding to those states, does not approximate any eigenvalues of $\hat{\mathcal{H}}$. This situation can sometimes be analyzed by using the complex WKB method [28]. The accuracy of asymptotics dependent on the relationship between μ and h parameters was analyzed in Refs [14, 15].

4. Some properties of circular cross-section tubes

Let us select $v_{\rm int} = m\Omega^2(x)(y_1^2 + y_2^2)/2$ as a confinement potential. Then, $\varepsilon_\perp^\nu(x) = \Omega(x)(\nu+1)$, the problem (4) degree of degeneracy is $\nu+1$, and the matrix Hamiltonian $\hat{L}^\nu$ has the $2(\nu+1)\times 2(\nu+1)$ dimensionality. For w_j^ν, we will select the eigenfunctions of the momentum operator

$$\hat{l} = -\mathrm{i}\hbar\left(y_1\,\frac{\partial}{\partial y_2} - y_2\,\frac{\partial}{\partial y_1}\right) = -\mathrm{i}\hbar\,\frac{\partial}{\partial\varphi}\,,$$

which may be expressed as $\exp(\mathrm{i}l\varphi)\,u^{|l|k}(r)$, where $l = 0, \pm 1, \ldots$; $k = 0, 1, \ldots$; $y_1 = r\cos\varphi$, and $y_2 = r\sin\varphi$. On the basis $\{w_j^\nu\}$, the Hamiltonian $\hat{L}^\nu$ is a diagonal block matrix with $\hat{L}^{\nu l}$ 2×2 blocks corresponding to the momentum projections onto the tube axis, which are equal to l:

$$\hat{L}^{\nu l} = -\frac{\hbar^2}{2m}\,\frac{\partial^2}{\partial x^2} + \hbar\Omega(x)\,(\nu+1) - \frac{\hbar^2 k^2}{8m} - \frac{\hbar e}{2mc}\,\langle\partial_x\mathbf{R}, \mathbf{H}\rangle\, l - \frac{\hbar}{2}\langle\boldsymbol{\sigma}\,, \mathbf{a}^l(x)\rangle\,. \tag{8}$$

Here, $\mathbf{a}^l(x) = e(mc)^{-1}\mathbf{H} - 2\alpha m l\Omega(x)^2\partial_x\mathbf{R}$, and $k(x)$ is the tube axis curvature at the point x. Thus, the vector $\psi^\nu(x,t)$ is composed of two-component vector functions $\psi^{\nu l}(x,t)$ satisfying the equations $\mathrm{i}\hbar\psi_t^{\nu l} = \hat{L}^{\nu l}\psi^{\nu l}$. Below are some precise and some asymptotic solutions of this equation.

4.1 Explicitly solved model of a helical tube

Let us consider a helical tube of a cylindrically symmetric cross-section and the axis

$$\mathbf{R}(x) = \left(\rho_1 \sin\frac{x}{\rho}\,,\ -\rho_1\cos\frac{x}{\rho}\,,\ \rho_2\frac{x}{\rho}\right), \quad \rho = \sqrt{\rho_1^2 + \rho_2^2}\,.$$

Its axial curvature is defined as $k(x) = \rho_1/\rho^2 = \text{const}$, and the axial twisting is $\varkappa(x) = \rho_2/\rho^2 = \text{const}$.

In the case of the constant tube cross-section $\Omega(x) = \Omega = \text{const}$, and the magnetic field parallel to the helical axis, $\mathbf{H} = (0, 0, H)$, the Hamiltonian (8) is unitary equivalent to the operator with the constant coefficients:

$$U^{-1}\hat{L}^{\nu l}U = -\frac{\hbar^2}{2m}\,\frac{\partial^2}{\partial x^2} - \frac{\hbar}{2m\rho}\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}\left(-\mathrm{i}\hbar\,\frac{\partial}{\partial x}\right) + \hbar\Omega(\nu+1) - \frac{l\rho_2}{2\rho}\,\hbar\omega_H + \frac{\hbar^2\rho_2^2}{8m\rho^4} - \frac{\hbar\omega_H}{2}\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix} + \alpha m\Omega^2\,\frac{\hbar l}{\rho}\begin{pmatrix}\rho_2 & \rho_1\\ \rho_1 & -\rho_2\end{pmatrix},$$

$$U(x) = \begin{pmatrix}\exp\left[-\dfrac{\mathrm{i}x}{2\rho}\right] & 0\\ 0 & \exp\left[\dfrac{\mathrm{i}x}{2\rho}\right]\end{pmatrix},$$

where $\omega_H = eH/(mc)$ is the Larmor frequency. The spectrum of the operator $\hat{L}^{\nu l}$ (8) in an infinite helical tube is therefore continuous and is given by

$$E^{\nu l}(q) = \frac{q^2}{2m} + \hbar\Omega(\nu+1) - \frac{\rho_2\omega_H l}{2\rho} + \frac{\hbar^2\rho_2^2}{8m\rho^4} - \sigma_{\uparrow\downarrow}\,\frac{\hbar}{m\rho}\sqrt{\left(\frac{q + m\omega_H\rho}{2} - \alpha m^2\Omega^2 l\rho_2\right)^2 + (\alpha m^2\Omega^2 l\rho_1)^2}\,,$$

where $\sigma_{\uparrow\downarrow} = \pm 1$.

Let wave function $\Psi^{\nu}(x, y)$ be periodical with the period equal to N turns. This condition results in the Bloch condition for a function $\psi^{\nu l}$ with the quasimomentum $I/(2\pi\hbar\rho N)$, $I = -I_{\mathrm{AB}} + I_{\mathrm{B}}^{l}$, where

$$I_{\mathrm{AB}} = \frac{e}{c}\int_0^{2\pi\rho N} \left\langle \partial_x \mathbf{R}, \mathbf{A}(\mathbf{R})\right\rangle \mathrm{d}x = \frac{e}{c}\, N\Phi\,,$$

$$I_{\mathrm{B}}^{l} = \hbar l \int_0^{2\pi\rho N} \varkappa\, \mathrm{d}x\,.$$

Thus, the periodicity condition is written down as

$$N\left(\frac{2\pi\rho q^{ln}}{\hbar} - \pi - 2\pi\rho\varkappa l + \frac{2\pi\Phi}{\Phi_0}\right) = 2\pi n\,.$$

From this it follows that under the given Bloch conditions, the operator $\hat{L}^{\nu l}$ spectrum is discrete: $E^{\nu ln} = E^{\nu l}(q^{ln})$, where the notation was used:

$$q^{ln} = \frac{\hbar}{\rho}\left(\frac{n}{N} + \frac{l\rho_2}{\rho} + \frac{1}{2}\right) - \frac{\Phi}{\rho\Phi_0}\,,$$

and $\Phi = \pi\rho_1^2 H$ is the magnetic flux through the helix projection area. The term I_{AB} is a manifestation of the Aharonov – Bohm effect, and I_{B}^{l} are the Berry phases. Note that $I_{\mathrm{B}} = 0$ when $l = 0$. It follows from Ref. [14] that in the case of an undegenerate term $I_{\mathrm{B}} = 0$ for any cross-section of a tube. At $\rho_2 = 0$, $E^{\nu ln}$ determine the spectrum of a closed toroidal tube.

In Sections 4.2 – 4.6, we will consider situations where the quasiclassical approximation may be applied to equation (3).

4.2 Spin dynamics for a short-wave mode

Let us consider a tube of variable thickness with $\Omega(x) \neq \mathrm{const}$ and the nonstationary equation $\mathrm{i}\hbar\psi_t^{\nu l} = \hat{L}^{\nu l}\psi^{\nu l}$. We will analyze the evolution of wave packets $\psi^{\nu l}(x, t)$ given by the condition $\psi^{\nu l}(x, 0) = \exp(\mathrm{i}S_0^{\nu}(x)/\hbar)\, A_0^{\nu l}(x)$, where $A^{\nu l}$ is a two-dimensional vector, in a 'short-wave' mode [14]. In this case, the functions $\psi^{\nu l}$ are quasiclassical and are retrieved through:

(1) the trajectories $x = X(x_0, t)$ of the Newton equation

$$m\ddot{x} = -\hbar\Omega'(x)(\nu + 1),\ x|_{t=0} = x_0\,,$$

$$m\dot{x}|_{t=0} = \frac{\partial S_0^{\nu}}{\partial x}(x_0)\,;$$

(2) the solutions $A^{\nu l}(x_0, t)$ of the $A^{\nu l}$-spinor equation

$$\frac{\mathrm{d}A^{\nu l}}{\mathrm{d}t} - \frac{i}{2}\left\langle \boldsymbol{\sigma}, \mathbf{a}^{l}\big(x(t)\big)\right\rangle A^{\nu l} = 0\,, \quad A^{\nu l}|_{t=0} = A_0^{\nu l}(x_0)\,. \tag{9}$$

If for $t < t^*$ the Jacobian $J^{\nu l}(x_0, t) = |\partial X/\partial x_0| \neq 0$, then

$$\psi^{\nu l}(x, t) \approx \exp\left[\frac{\mathrm{i}S^{\nu}\big(x_0(x, t), t\big)}{\hbar} + \mathrm{i}\vartheta\big(x_0(x, t), t\big)\right] \times \frac{A^{\nu l}\big(x_0(x, t), t\big)}{\sqrt{J^{\nu l}\big(x_0(x, t), t\big)}}\,,$$

where

$$S^{\nu}(x_0, t) = S_0^{\nu}(x_0) + \int_0^t \left[\frac{m}{2}\,\dot{X}(x_0, t)^2 - \hbar\Omega\big(X(x_0, t)\big)(\nu + 1)\right] \mathrm{d}t\,,$$

$$\vartheta(x_0, t) = \frac{l}{2}\int_0^t \left\langle \partial_x \mathbf{R}\big(X(x_0, t)\big), \mathbf{H}\right\rangle \mathrm{d}t\,,$$

$x_0(x, t)$ is the solution to the equation $X(x_0, t) = x$ in x_0. For $t > t^*$, after focal points appear (for instance, at the points of the narrowing of the tube and near its end), the asymptotics are determined by the canonical operator [29].

4.3 Spectral asymptotics in a short-wave mode

In a variable thickness tube, let the functions Ψ^{ν} satisfy the periodicity condition at every spiral turn and let the frequency $\Omega(x)$ with the same period have a single minimum point x_0 per turn. Then, functions with energies $E^{\nu ln} < \max \varepsilon_{\perp}^{\nu}(x)$ are localized in a classically accessible region; functions with energies $E^{\nu ln} > \max \varepsilon_{\perp}^{\nu}(x)$ are delocalized and specify 'ballistic' states [30].

Localized states may be sorted into two groups:

(1) lower states corresponding to the oscillatory approximation [Born – Oppenheimer approximation for equation (2)] and having the spectrum

$$E^{\nu nl} \approx \hbar\Omega(x_0)(\nu + 1) + \hbar\sqrt{\frac{\hbar\Omega''(x_0)(\nu + 1)}{m}}\left(n + \frac{1}{2}\right) - \frac{\mu\rho_1}{2\rho}\, H_2 l \sin\left(\frac{x_0}{\rho}\right) - \frac{\mu\rho_2}{2\rho}\, H_3 l + \hbar\sigma_{\uparrow\downarrow}\left|\frac{e}{c}\,\mathbf{H} - 2\alpha m l\Omega^2(x_0)\,\partial_x\mathbf{R}(x_0)\right|,$$

where $\sigma_{\uparrow\downarrow} = \pm 1$ corresponds to spins directed along the vector $(e/c)\mathbf{H} - 2\alpha m l\Omega(x_0)^2\partial_x\mathbf{R}(x_0)$;

(2) excited states corresponding to rapidly oscillating WKB-solutions having the spectrum

$$E_{\mathrm{s}}^{\nu ln} = E_0^{\nu n} - \hbar\omega_H^{\nu ln} - \hbar\omega_{\mathrm{s}}^{\nu ln} + O(\mu^2)\,,$$

where $E_0^{\nu n}$ is found from the Bohr – Zommerfeld quantization condition

$$\frac{1}{\pi}\int_{x_1}^{x_2} \mathcal{P}\,\mathrm{d}x = \hbar\left(n + \frac{1}{2}\right), \quad \mathcal{P} = \sqrt{2m\big(E_0^{\nu n} - \hbar\Omega(x)(\nu + 1)\big)}\,.$$

Here, $x_1 < x_2$ are the solutions to the equation $\mathcal{P} = 0$, with

$$\omega_H^{\nu nl} = el(2mcT)^{-1}\int_0^T \left\langle \partial_x\mathbf{R}\,(X(x_0, t)), \mathbf{H}\right\rangle \mathrm{d}t = el(mcT)^{-1}\int_{x_1}^{x_2} \left\langle \partial_x\mathbf{R}, \mathbf{H}\right\rangle m\,\mathcal{P}^{-1}\,\mathrm{d}x\,,$$

$$T = 2\int_{x_1}^{x_2} m\,\mathcal{P}^{-1}\,\mathrm{d}x$$

being the period of a closed trajectory at the energy level $E_0^{\nu n}$, $\omega_{\mathrm{s}}^{\nu nl}$ are the Floquet indices of the monodromy matrix of system (9) [that are real due to the $\left\langle \boldsymbol{\sigma}, \mathbf{a}^{l}(x(t))\right\rangle$ matrix self-conjugacy and are such that $-\pi/T < \omega_{\mathrm{s}}^{\nu nl} \leqslant \pi/T$]. For localized states, the phases are $I_{\mathrm{AB}} = 0$, $I_{\mathrm{B}}^{l} = 0$.

The spectrum of *ballistic states* takes the form

$$E = E_0^{\nu n} - \hbar\omega_H^{\nu nl} - \hbar\omega_{\mathrm{s}}^{\nu nl} + \hbar\omega_{\mathrm{B}}^{l} + O(\mu^2)\,,$$

where $E_0^{\nu n}$ is determined by the condition

$$\int_0^{2\pi\rho N} \mathcal{P}\,\mathrm{d}x = 2\pi\hbar n - I_{\mathrm{AB}}\,,$$

$$\omega_H^{\nu nl} = el(cT)^{-1}\int_0^{2\pi\rho N} \left\langle \partial_x\mathbf{R}, \mathbf{H}\right\rangle \mathcal{P}^{-1}\,\mathrm{d}x\,,$$

$$T = \int_0^{2\pi\rho N} m\,\mathcal{P}^{-1}\,\mathrm{d}x\,,$$

$\omega_{\mathrm{s}}^{vnl}$ are the Floquet indices of the system (9), and $\omega_{\mathrm{B}}^{l} = (\hbar T)^{-1} I_{\mathrm{B}}^{l}$.

4.4 Spin-orbit splitting and the perturbation theory

Let $\mathbf{H} = 0$, $\alpha \ll (m\Omega)^{-1}$. Then, the correction $\omega_{\mathrm{s}}^{vnl}$ may be calculated with perturbation theory [2]. Indeed, the monodromy matrix of equation (9) is given by

$$\mathcal{M} = E + 2\alpha ml \int_0^T \Omega\big(X(x_0,t)\big)^2 \big\langle \boldsymbol{\sigma}, \partial_x \mathbf{R}\big(X(x_0,t)\big)\big\rangle\,\mathrm{d}t + O(\alpha^2)$$

and the Floquet indices with an accuracy of $O(\alpha^2)$ are the eigenvalues of the matrix

$$2\alpha mlT^{-1} \int_0^T \Omega\big(X(x_0,t)\big)^2 \big\langle \boldsymbol{\sigma}, \partial_x \mathbf{R}\big(X(x_0,t)\big)\big\rangle\,\mathrm{d}t$$
$$= 2\alpha mlT^{-1} \int_0^{2\pi\rho N} \Omega(x) \big\langle \boldsymbol{\sigma}, \partial_x \mathbf{R}(x)\big\rangle\, m\,\mathcal{P}^{-1}\,\mathrm{d}x\,.$$

It should be emphasized that the perturbation theory is valid only at $\alpha \ll (m\Omega)^{-1}$.

4.5 The impact of spin on the classical dynamics of weakly excited states, depending on the direction of H

Let $|\mathbf{a}^l(x)| \neq 0$ (terms are not overlapping) and $\Omega(x) = \Omega = \text{const}$. Then, the set of equations $\mathrm{i}\hbar\psi_t^{vl} = \hat{L}^{vl}\psi^{vl}$ has the fast-oscillating WKB-solutions which are retrieved through the trajectories of two classical systems $m\ddot{x} = -\nabla v_{\uparrow\downarrow}^{l}$ [14], where

$$v_{\uparrow\downarrow}^{l} = -\frac{e\hbar}{(2mc)} \big\langle \partial_x \mathbf{R}, \mathbf{H}\big\rangle\, l - \frac{\sigma_{\uparrow\downarrow}}{2}\,\hbar\big|\mathbf{a}^l(x)\big|\,,$$
$$\big|\mathbf{a}^l(x)\big| = \sqrt{\omega_H^2 + (2\alpha m\Omega^2 l)^2 - 2\,\frac{e}{c}\,\alpha l\Omega^2 \big\langle \partial_x \mathbf{R}, \mathbf{H}\big\rangle}\,.$$

The $v_{\uparrow\downarrow}^{l}$ extrema are found from the condition

$$\big(|\mathbf{a}^l(x)| - \sigma_{\uparrow\downarrow}\alpha m\Omega^2\big)\,\partial_x\bigg(-\frac{1}{2}\big\langle \partial_x \mathbf{R}, \mathbf{H}\big\rangle\, l\bigg) = 0\,.$$

The $v_{\uparrow\downarrow}^{l}$ minima correspond to 'traps'. For the $\sigma_{\downarrow}$ direction, the minima and maxima of the potential $v_{\uparrow\downarrow}^{l}$ correspond to minima and maxima of $-(1/2)\langle \partial_x \mathbf{R}, \mathbf{H}\rangle\, l$, i.e., the phase portrait of a spin particle qualitatively coincides with that of an analogous zero-spin particle. For the direction $\sigma_{\uparrow}$, a similar situation takes place when $\alpha m\Omega^2 < |\mathbf{a}^l(x^*)|$, where x^* corresponds to the points of minimum and maximum of $-(1/2)\langle \partial_x \mathbf{R}, \mathbf{H}\rangle l$. For the direction $\sigma_{\downarrow}$, the potential maximum grows by $(\hbar/2)|\mathbf{a}^l(x^*)|$ in comparison with that for a spin-zero particle, and for $\sigma_{\uparrow}$ it drops by the same value. An interesting possibility of *separation of spin particles* having a given momentum projection l results: in the energy range

$$\hbar\Omega(v+1) - \frac{\hbar e}{2mc}\big\langle \partial_x \mathbf{R}(x^*), \mathbf{H}\big\rangle\, l - \frac{\hbar}{2}\big|\mathbf{a}^l(x^*)\big|$$
$$< E < \hbar\Omega(v+1) - \frac{\hbar e}{2mc}\big\langle \partial_x \mathbf{R}(x^*), \mathbf{H}\big\rangle\, l + \frac{\hbar}{2}\big|\mathbf{a}^l(x^*)\big|\,,$$

a particle with spin $\sigma_{\uparrow}$ will pass through the tube, whereas a particle with spin $\sigma_{\downarrow}$ will be deflected from it.

If $\alpha m\Omega^2 > |\mathbf{a}^l(x^*)|$, then for the direction $\sigma_{\uparrow}$ in the center of a 'magnetic' trap (the point of a minimum of $-(1/2)\langle \partial_x \mathbf{R}, \mathbf{H}\rangle\, l$) a barrier appears and the trap breaks up into two traps; at the points of the maximum of $-(1/2)\langle \partial_x \mathbf{R}, \mathbf{H}\rangle l$, additional points of the minimum appear.

4.6 Spin flip

Let us consider the case $e/(mc)\mathbf{H} = 2\alpha ml\Omega^2 \partial_x \mathbf{R}(x^*)$. Then, x^* is a point of minimum of $-(1/2)\langle \partial_x \mathbf{R}, \mathbf{H}\rangle\, l$, and $|\mathbf{a}^l(x^*)| = 0$. In this case, the term overlapping occurs at points x^*, corresponding to spin flip.

As an example, take a circular arc-shaped axis $\mathbf{R}(x) = \rho(\cos(x/\rho), \sin(x/\rho), 0)$ and the field $e/(mc)\mathbf{H} = 2\alpha ml\Omega^2(-1,0,0)$. Then the potential is given by

$$v_{\uparrow\downarrow}^{l} = -\hbar\,|l|\,\omega_H \sin\left(\frac{x}{\rho}\right) - \sigma_{\uparrow\downarrow}\hbar\omega_H \left|\sin\left(\frac{\pi}{4} - \frac{x}{2\rho}\right)\right|,$$

and $x^* = \pi\rho/2$, with the principal term of the WKB-solution of Eqn (3) determined by the equations $m\ddot{x} = -(v_{\pm}^{l})'$, where

$$v_{\pm}^{l} = -\hbar|l|\omega_H \sin\left(\frac{x}{\rho}\right) \mp \hbar\omega_H \sin\left(\frac{\pi}{4} - \frac{x}{2\rho}\right)$$

[31, 32]. The potential $v_{\pm}^{l}$ does not coincide with the potential $v_{\uparrow\downarrow}^{l}$.

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