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Radiation of superluminal sources in empty space

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Abstract. **We consider the theoretical prerequisites for and experimental papers on the coherent radiation of electromagnetic waves by moving sources whose speed exceeds the speed of light in empty space. Examples of this kind are provided by a light spot or a charged filament incident on an interface between two media.**

1. Introduction

It has been a rather long time since it was realized that a uniformly moving source that produces perturbation fields in a medium can radiate directed waves when the source speed exceeds the velocity of wave propagation in the medium through which the source is traveling. This effect was supposedly considered for the first time in hydrodynamics by the example of waves diverging from a moving ship. More recently, Ernst Mach considered the sound waves generated by a shell projected with a supersonic speed. Mach showed that these waves propagate in the direction that makes an angle θ with the shell velocity, with $\cos\theta = u/v$, where u is the speed of sound and v is the shell speed. Mach even succeeded in photographing the conical wave emerging in this case. These works were highly valued by physicists of that time, which is evident, in particular, from Einstein's paper in commemoration of Mach [1].

Based on the analogy, one might assume that similar effects take place in electrodynamics, namely, a charged body traveling with a speed that exceeds the speed of light waves becomes a radiation source of directed electromagnetic waves. However, this fact was realized much later. Mention can be made of several apparent reasons. One reason is that the speed of light in both empty space and a refracting medium is rather high, and it was hard to imagine a material body with a speed higher than that of light. It may well be that this is the reason why Heaviside's work of 1888 [2], in which he considered this possibility, did not attract the attention of physicists. Another reason is that the speed of a material body cannot, according to the relativity theory, exceed the speed of light. That is why the very possibility of a superluminal speed was called into question. But, in reality, the relativity theory forbids the motion of material bodies with a speed exceeding the speed of light in the vacuum, which is approximately equal to 300000 km s^{-1}. In a refracting medium, the speed of light turns out to be substantially lower. For instance, in glass with the refractive index 1.5, the speed of light is approximately equal to 200000 km s^{-1}. When the particle energy is high enough, its speed may exceed the speed of light in the medium and nevertheless remain below the speed of light in empty space. This was realized with perfect clarity after the work of Tamm and Frank [3], which explained the results of the Vavilov – Cherenkov experiments [4, 5].

At present, the theory of Vavilov – Cherenkov radiation has been adequately elaborated, probably more amply than the theories of similar effects in other areas of physics (for instance, the theory of Mach waves). This is attributable both to the practical significance of this phenomenon in high-energy physics and to the circumstance that the treatment of similar effects in hydrodynamics and acoustics turned out to be substantially more intricate due to the strong influence of nonlinear processes.

In 1904, Sommerfeld [6] considered the electromagnetic field of a particle traveling through empty space at a speed exceeding the speed of light. He showed that this gives rise to directional radiation. The emitted waves propagate at an angle θ to the direction of charged particle velocity, with

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$\cos\theta = c/v$, where c is the speed of light in empty space and v is the particle speed. The next year saw the final formulation of the special theory of relativity. The motion of material objects with a superluminal speed turned out to be prohibited and therefore Sommerfeld's work was forgotten for a long time. Nevertheless, the question of whether there may exist radiation sources whose speed exceeds that of light in free space proved to be by no means a simple one. It is likely that the first model of such a source was the model considered by Heaviside in his monograph 'Electromagnetic Theory' published in 1912 [7].

2. Heaviside's model

Shown in Fig. 1 is an illustration from Heaviside's book [7]. The horizontal line, in which points P, A, and Q are located, depicts a perfectly conducting planar surface. A plane wave pulse, shown at the right of the drawing, is incident on this surface. The incident pulse contacts the plane on the segment PAQ. The direction of propagation of the incident pulse is defined by the vector $\mathbf{X}$ and makes an angle θ with the normal to the plane. The PAQ domain is not immobile but shifts rightward along the perfectly conducting plane. It is easily seen that the PAQ domain moves with the speed v defined by the equality

$$v = \frac{c}{\sin\theta}\,, \tag{1}$$

where c is the speed of light in free space.

Because $\sin\theta \leqslant 1$, $v \geqslant c$, i.e., the speed of motion of the PAQ domain along the plane is equal to or exceeds the speed of light in free space. At the same time, currents and charges in the PAQ domain are excited under the action of the incident wave. The PAQ domain travels along the surface with a speed exceeding that of light in free space, and this domain should therefore become a source of Vavilov – Cherenkov radiation. The emitted wave is shown in the left part of the drawing. The propagation direction of this wave is given by a vector $\mathbf{Y}$. It is easily verified that the vector $\mathbf{Y}$ makes the same angle θ with the normal to the interface as the vector $\mathbf{X}$, which characterizes the incident wave.

Therefore, the wave radiated by the spot (the PAQ domain) traveling along the surface is a reflected wave. It can consequently be treated as the Vavilov – Cherenkov radiation of the traveling domain with currents and charges, which is induced at the interface under the action of the incident wave.

More recently, Frank [8] considered a more general case of a source traveling with a superluminal speed taking into account not only the incident and reflected waves but also the refracted wave. We emphasize that both Heaviside and Frank considered the currents and charges induced at the interface by the incident wave. But the incident wave can also induce currents and charges in the domain adjacent to the spot PAQ and traveling along the interface. These bulk currents and charges also constitute a superluminal source. Below, we give Frank's example [8].

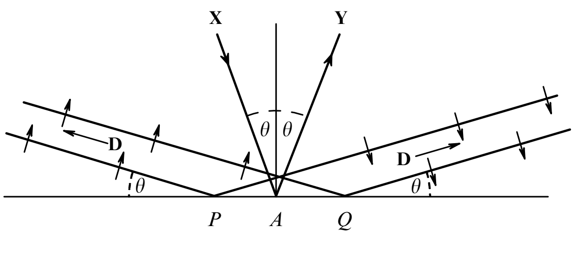

Figure 1. Reflection of a plane wave incident on a surface.

3. Frank's model

We consider a pulse made up of plane electromagnetic waves. We assume that the electromagnetic field is nonzero between two parallel planes and vanishes in the remaining space. The pulse propagates through a medium with a permittivity ε_1 and is incident on the boundary with a medium with a permittivity ε_2. The geometry of the problem is shown in Fig. 2. The incident wave front is labeled *1* in the drawing. The angle of incidence on the interface is denoted by θ_0. The pulse speed in the first medium is $v_1 = c/\sqrt{\varepsilon_1}$. It is easily seen that the pulse – interface intersection domain (the spot) moves along the interface with the speed

$$v = \frac{v_1}{\sin\theta_0} = \frac{c}{\sqrt{\varepsilon_1}\sin\theta_0}\,. \tag{2}$$

This speed exceeds the speed of light in the medium, and the spot velocity exceeds the speed of light in a vacuum when $\varepsilon_1 = 1$. There is no contradiction with the special relativity theory whatsoever, because the spot on the interface is not a material body: at each time instant, the spot is produced by different portions of the pulse front. This pulse nevertheless induces real currents and charges in the spot area, and this domain of induced currents and charges travels along the interface together with the spot. Therefore, a traveling radiation source results whose speed always exceeds the speed of light in the first medium. Such a source should produce Vavilov – Cherenkov radiation. The front of the Vavilov – Cherenkov wave propagating through the first medium is labeled *2* in Fig. 2. The angle $\theta_{1(\mathrm{V-Ch})}$ between the propagation direction of wave *2* and the spot velocity is defined by the formula

$$\cos\theta_{1(\mathrm{V-Ch})} = \frac{c}{\sqrt{\varepsilon_1}\,v}\,. \tag{3}$$

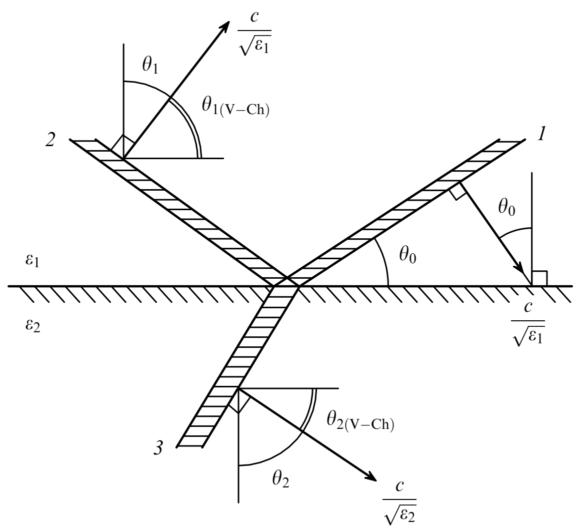

Figure 2. Reflection and refraction of waves as the Vavilov – Cherenkov effect. Geometry of the problem. *1* — incident wave, *2* — reflected wave, *3* — refracted wave.

We substitute the value of v from formula (2) in formula (3) to obtain

$$\cos\theta_{1(\mathrm{V-Ch})} = \frac{c}{\sqrt{\varepsilon_1}}\,\frac{\sqrt{\varepsilon_1}\sin\theta_0}{c} = \sin\theta_0\,. \tag{4}$$

It is easily seen that wave *2* recedes from the interface, its propagation direction making the angle $\theta_1 = \theta_0$ with the normal to the interface. Therefore, wave *2* is precisely the reflected wave if pulse *1* is the incident wave.

We now consider the field behind the opposite side of the interface, i.e., in the medium with the permittivity ε_2. A spot traveling along the interface with the speed defined by formula (2) may become a source of radiation in the second medium only when the spot speed exceeds the speed of light in the second medium, i.e., when the inequality

$$v = \frac{c}{\sqrt{\varepsilon_1}\sin\theta_0} > \frac{c}{\sqrt{\varepsilon_2}} \tag{5}$$

is satisfied. In this case, a Vavilov–Cherenkov wave, numbered *3* in Fig. 2, emerges in the second medium. The angle between the propagation direction of this wave and the spot velocity is defined by the formula

$$\cos\theta_{2(\mathrm{V-Ch})} = \frac{c}{\sqrt{\varepsilon_2}\,v} = \sqrt{\frac{\varepsilon_1}{\varepsilon_2}}\sin\theta_0\,. \tag{6}$$

We let θ_2 denote the refraction angle $\pi/2 - \theta_{2(\mathrm{V-Ch})}$ and thus arrive at the well-known relation

$$\sqrt{\varepsilon_1}\sin\theta_0 = \sqrt{\varepsilon_2}\sin\theta_2\,. \tag{7}$$

This is nothing more nor less than Snell's law. Therefore, the Vavilov–Cherenkov radiation from the spot gives rise to the refracted wave in the second medium. When the spot speed is lower than the speed of light in the second medium, i.e.,

$$v = \frac{c}{\sqrt{\varepsilon_1}\sin\theta_0} < \frac{c}{\sqrt{\varepsilon_2}}\,, \tag{8}$$

the Vavilov–Cherenkov wave is not generated in the second medium and the incident pulse *1* does not propagate through the second medium. Evidently, inequality (8) is equivalent to the condition for total internal reflection

$$\sin\theta_0 > \sqrt{\frac{\varepsilon_2}{\varepsilon_1}}\,. \tag{9}$$

Therefore, the reflection and refraction of waves at an interface may be considered as the Vavilov–Cherenkov radiation of the charges and currents induced by the incident wave on the interface.

4. Light spot

It is pertinent to note that a light spot traveling along an interface can also be produced by other means. For instance, when a light beam reflected from a mirror is directed to an interface between two media, a light spot also forms at the interface. When the mirror rotates, this spot travels along the interface and may become a source of directional radiation under certain conditions. We consider the properties of the field generated by a rotating light source [9, 10].

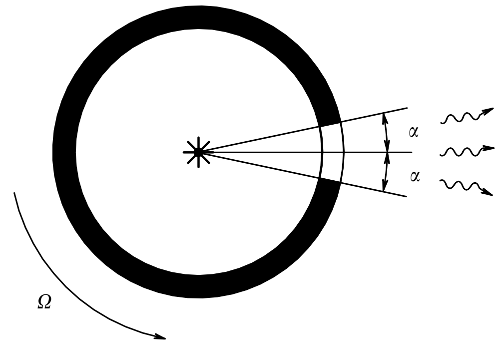

Figure 3. Rotating beacon.

We introduce a cylindrical coordinate system r, φ, z, where z is the axis of the cylindrical coordinate system, r is the radial distance of a point, and φ is the azimuthal angle. We assume that the z axis is the axis of a hollow cylinder of radius a. Let the cylinder shell be transparent for angles φ satisfying the inequality $\varphi \leqslant |\alpha|$ and the remaining cylindrical shell surface be opaque. Here, α is some given angle. We assume that a light source is located along the z axis. The geometry of the problem under discussion is shown in Fig. 3 in the plane perpendicular to the z axis. Let the cylinder rotate about the z axis with an angular velocity Ω. Clearly, the entire structure resembles the radiating device of a lighthouse. In this case, the transparent part of the cylinder plays the role of the source aperture. The frequency of light from the source located on the z axis is denoted by ω. Then, on the cylinder surface $r = a$, the electric field E as a function of time t and the polar angle φ is described by the relation

$$E_{r=a} = \begin{cases} E_0\exp[-\mathrm{i}\omega t] & \text{for} \quad |\varphi + \Omega t| \leqslant \alpha\,, \\ 0 & \text{for} \quad |\varphi + \Omega t| > \alpha\,. \end{cases} \tag{10}$$

We expand $E_{r=a}$ into the Fourier series in the azimuthal angle φ. The expansion is of the form

$$E_{r=a} = \sum_n E_n \exp\left[-\mathrm{i}\omega t - \mathrm{i}n(\varphi + \Omega t)\right], \tag{11}$$

where E_n are some constants. In view of the cylindrical symmetry of the problem, far away from the z axis, the field is given by

$$E = \sum_n E_n \exp\left[-\mathrm{i}n\varphi - \mathrm{i}(\omega + n\Omega)\,t\right] \times \frac{1}{\sqrt{r}}\exp\left[\mathrm{i}\left(\frac{\omega + n\Omega}{c}\right)r\right]. \tag{12}$$

We see from formula (12) that the field consists of diverging waves whose frequency is determined by both the initial wave frequency ω and the cylinder rotation frequency Ω. The constant-phase surface for these waves is of the form

$$-n\varphi - \omega_n t + \frac{\omega_n}{c}\,r = \mathrm{const}\,, \tag{13}$$

where $\omega_n = \omega + n\Omega$.

At a given time instant t, Eqn (13) describes the spiral of Archimedes. The distance between two successive turns of the spiral is $n\lambda_n$, where $\lambda_n = 2\pi c/\omega_n$; λ_n is the wavelength corresponding to the harmonic with the number n.

We consider a cylindrical surface at a distance R well away from the source (Fig. 4). We select a portion of this surface

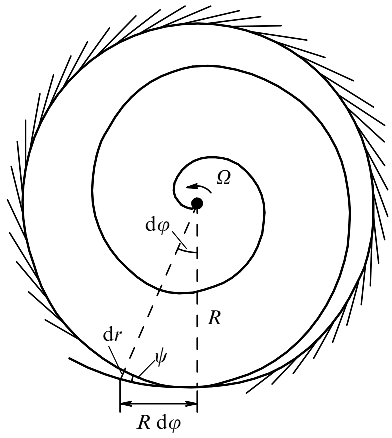

Figure 4. Cylindrical surface of a wave front at a distance R from the source.

corresponding to an interval $\mathrm{d}\varphi$ of azimuthal angles. It is evident from the drawing that the angle between the surface $r = R$ and the constant-phase surface is given by

$$\tan\psi \simeq \psi = \frac{\mathrm{d}r}{R\,\mathrm{d}\varphi} = \frac{cn}{R\omega_n}\,, \tag{14}$$

where the derivative $\mathrm{d}r/\mathrm{d}\varphi$ was obtained from Eqn (13) for the constant-phase surface. Clearly, ψ is the angle of wave incidence on the surface $r = R$. Well away from the source, the curvature may be neglected for sufficiently small portions of the surface $r = R$, and therefore the physical picture of the effect turns out to be essentially the same as for a plane wave incident on a plane interface (see Fig. 2).

From formula (14), it is clear that the value of the angle ψ at large distances from the source can become arbitrarily small. At the same time, ψ is nothing but the angle of wave incidence on the surface; to be more precise, ψ is the angle of incidence of the portion of the constant-phase surface that is close to the interface. Therefore, if we want to determine the speed of the point of intersection of the constant-phase surface with the surface of the screen $r = R$, we can resort to expression (2) to obtain

$$v \simeq \frac{c}{\psi} = \frac{R\omega_n}{n}\,. \tag{15}$$

Hence, it follows that the speed of a spot produced by a rotating source is proportional to its distance from the screen and can exceed the speed of light when the distance is long enough. Such a rotating spot may become a radiation source. It is noteworthy that this radiation source at each time instant is excited by different segments of the wave front, as is the case when a plane wave is incident on a plane interface.

The model of a superluminal light spot was realized in the investigation of the ionosphere with a high-power beam of radio waves [11]. The beam of radio waves from a radio station located on the terrestrial surface is directed upwards. At the lower ionospheric boundary, this beam produces a region with induced currents and charges (a light spot), which can be set in motion along the lower edge of the ionosphere, with the capability to select this translation velocity. When it exceeds the speed of light, the light spot becomes a source of Vavilov – Cherenkov radiation. When the light spot is produced by a modulated wave, its radiation is frequency-shifted relative to the modulation frequency by the formulas for the Doppler effect.

5. Coherent transition radiation as a source of Vavilov – Cherenkov radiation and the Doppler effect

We consider one more model of a superluminal radiator, wherein the peculiarity of the effect is manifested with exceptional clarity [12]. The peculiarity consists in the fact that the radiation region travels with a superluminal speed but each radiation pulse is caused by a new particle.

Let there be a perfectly conducting plane. We introduce a rectangular coordinate system x, y, z such that the z axis is perpendicular to the plane (Fig. 5). Let a charged particle arrive at a point $x = 0$ at $t = 0$. In this case, a flash of transition radiation occurs. The transition radiation field $\mathbf{E}$ at a long distance $\mathbf{r}$ from the point $x = 0$ can be written as

$$\mathbf{E}_1 = \mathbf{E}(\mathbf{k}) \exp\left[\mathrm{i}(\mathbf{kr} - \omega t)\right], \tag{16}$$

where $\mathbf{k}$ is the wave vector. The field $\mathbf{E}(\mathbf{k})$ is commonly defined as a function of the angle θ between the wave vector $\mathbf{k}$ and the normal to the surface (the z axis). We assume that a second charged particle with the same speed arrives at a point $x = L$ in a time T. The second flash of transition radiation then occurs, and its field well away from the point $x = L$ is given by

$$\mathbf{E}_2 = \mathbf{E}(\mathbf{k}) \exp\left[\mathrm{i}\mathbf{k}(\mathbf{r} - \mathbf{L}) - \mathrm{i}\omega(t - T)\right]. \tag{17}$$

Let this process be periodically repeated, i.e., the nth particle intersects the plane at a point $x_n = (n-1)L$ at the instant $t_n = (n-1)T$. The resultant radiation is defined by the sum of the fields $\mathbf{E}_1, \mathbf{E}_2, \ldots, \mathbf{E}_n$. When the particles are infinite in number, summing up these fields gives

$$\begin{aligned} \mathbf{E} &= \sum_n \mathbf{E}_n(\mathbf{k}) = \mathbf{E}(\mathbf{k}) \exp\left[\mathrm{i}(\mathbf{kr} - \omega t)\right] \\ &\quad \times \left\{1 + \exp\left[\mathrm{i}(\mathbf{kL} - \omega T)\right] + \exp\left[2\mathrm{i}(\mathbf{kL} - \omega T)\right] + \ldots\right\} \\ &= 2\pi\mathbf{E}(\mathbf{k}) \exp\left[\mathrm{i}(\mathbf{kr} - \omega t)\right] \sum_m \delta(\mathbf{kL} - \omega T - 2\pi m), \end{aligned} \tag{18}$$

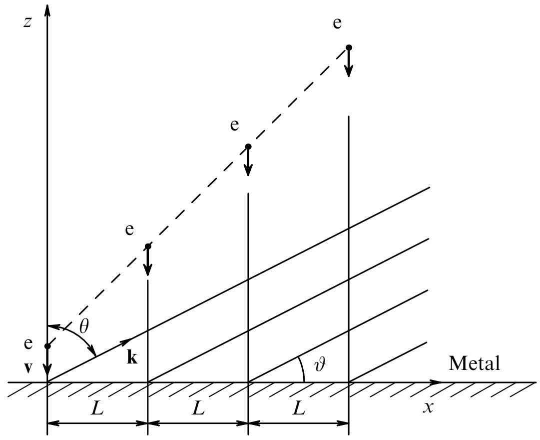

Figure 5. Model of a traveling source.

where $\delta(x)$ is the Dirac delta function and m is an arbitrary integer.

Therefore, the field of total transition radiation is different from the radiation field produced in the incidence of a single charge by the factor $\sum \delta(\mathbf{kL} - \omega T - 2\pi m)$, which imposes an additional condition on the field radiated. The condition reduces to the vanishing of the argument of the delta function, i.e., only the waves with $\omega T - \mathbf{kL} = 2\pi m$ remain in the transition radiation. Taking the geometry of the problem unto account, we rewrite this condition as

$$\omega = \frac{2\pi m}{T[1 - (L/cT)\cos\theta]}\,, \tag{19}$$

where θ is the angle between the x axis and the radiation direction.

The resultant radiation may be regarded as the radiation of a blinking source that travels along the x axis with the speed $v = L/T$ and flashes with the period T. When $m = 0$ in expression (19), the radiation with a frequency ω may be nonzero only if

$$1 - \frac{L}{cT}\cos\theta = 0\,. \tag{20}$$

In this case, the radiation is nonzero on the cone of the angle θ, with $\cos\theta = c/v$. This is the condition for the emergence of Vavilov–Cherenkov radiation in a vacuum. We emphasize that the quantity $v = L/T$, unlike the speed of material bodies, is essentially the speed of a spot and not of a material body and may exceed the speed of light. When $m \neq 0$, condition (19) defines the set of frequencies characteristic of the Doppler effect. Therefore, the field of transition radiation in the problem under discussion contains factors characteristic of Vavilov–Cherenkov radiation and the Doppler effect.

When the number of particles incident on the surface is finite, the sum in expression (18) contains a finite number of terms and yields the factor

$$S_N = \frac{1 - \exp(\mathrm{i}N\Delta)}{1 - \exp[\mathrm{i}\,\Delta]} = \exp\left[\mathrm{i}\,\frac{(N-1)\,\Delta}{2}\right]\left(\frac{\sin(N\Delta/2)}{\sin(\Delta/2)}\right), \tag{21}$$

where N is the number of incident particles and $\Delta = \mathbf{kL} - \omega T$. We see that apart from the principal maxima, subsidiary maxima also appear in this case.

6. Vavilov–Cherenkov radiation in a waveguide. Gyrocon

Using a sequence of incident particles considered above, it is possible to produce a radiation source in a waveguide. The phase velocity of waves in a waveguide exceeds the speed of light in a vacuum, and therefore a single charged particle moving through a waveguide cannot be a source of Vavilov–Cherenkov radiation. However, it is possible to realize the following method of wave excitation. For simplicity, we consider a rectangular waveguide. Let a charged particle whose velocity is directed perpendicular to the longitudinal waveguide axis intersect the waveguide in some lateral section. In a time T, another charged particle traveling with the same velocity traverses another lateral section of the waveguide spaced at a distance L from the first one. At the instant $2T$, a third charged particle traverses the lateral section of the waveguide spaced at the distance $2L$ from the first one, and so on. Every intersection is accompanied by a flash of transition radiation in the waveguide. We can say that a radiation source that travels along the longitudinal axis with the speed $v = L/T$ appears in the waveguide. When the magnitude of v coincides with the phase velocity of some natural wave in the waveguide, field generation occurs at the corresponding harmonic. In this case, as in the problem considered in Section 5, it is possible to model both the Vavilov–Cherenkov radiation and the Doppler effect.

The radiation source in a waveguide can also be produced with the aid of a continuous beam of charged particles if the beam traverses the waveguide such that the intersection point moves along the waveguide [13, 14]. When the synchronism condition is satisfied, this beam generates the corresponding proper wave. An example of such a generator is a gyrocon [15], which is schematically represented in Fig. 6. An accelerating device 1 forms an electron beam. The particle flow passes through a deflecting resonator *3* with a rotating electromagnetic field. Upon exiting the resonator, the beam trajectory is an unwinding helix lying on the surface of a cone of angle α. The beam then traverses an electrostatic deflecting system *5*, following which the particle beam moves parallel to the direction of the initial motion, i.e., the beam trajectory is a helix. This beam enters a circular resonator *6*. That part of the beam that is inside the resonator moves in a circle. In this case, the sweep frequency is selected such that the speed of beam displacement in the resonator exceeds the speed of light. When the displacement speed coincides with the phase velocity of one of the proper harmonics of the circular resonator, the beam begins to generate this harmonic. The resonator *6* is essentially a waveguide bent into a ring, which has an opening in the upper wall to inject the beam. That is

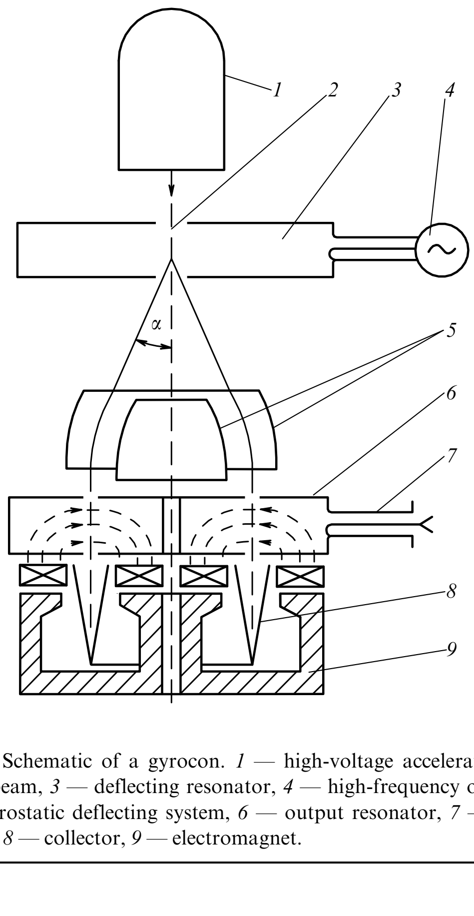

Figure 6. Schematic of a gyrocon. *1* — high-voltage accelerator, *2* — electron beam, *3* — deflecting resonator, *4* — high-frequency oscillator, *5* — electrostatic deflecting system, *6* — output resonator, *7* — energy extractor, *8* — collector, *9* — electromagnet.

why the phase velocities of the proper harmonics exceed the speed of light. The authors of Ref. [15] indicate, in particular, that a harmonic with a phase velocity 1.84 times the speed of light was adopted for the generation. Operating in a continuous mode, the gyrocon produced radiation in the wavelength range from 30 cm to 1.1 m. The output radiation power amounted to 5 MW and the beam-to-radiation energy conversion efficiency amounted to 80%.

7. Other possible realizations of superluminal sources

With the aid of a collimated bunch of charged particles, it is possible to produce a radiation source following an arbitrary path with any speed. Indeed, let a well-collimated beam of charged particles be incident on a conducting plane. The point at which the beam crosses the plane is a source of transition radiation. Using a deflecting system, it is possible to make this point (or spot) shift in the plane, and the displacement speed may exceed the speed of light. It is possible, for instance, to create a condition whereby the spot uniformly follows a circular path. In this case, the generated radiation has much in common with synchrotron radiation [16, 17]. In particular, the emission spectrum is made up of frequencies that are multiples of the revolution frequency. However, there are significant distinctions as well. Specifically, the speed of spot motion in the plane may exceed the speed of light, while such a motion is impossible for a material particle. It is noteworthy that a light spot can also follow an arbitrary path, which can also be treated as a radiation source.

Other superluminal source models have also been proposed [18, 19]. In particular, Carron and Longmire [19] considered a situation where a light spot traveling along a surface is generated by X-ray instead of optical radiation. If the angle of wave incidence is denoted by θ, the wave produces excitation on the metal surface, which travels with the speed $v = c/\sin\theta$ along the surface. Unlike optical radiation, the X-ray wave knocks electrons out of the metal surface to give rise to photoelectron current, which serves as the source of secondary radiation. Because the photocurrent is synchronously related to its exciting X-ray radiation wave, the photoelectron emission excitation domain travels along the surface with the same speed as the wave incident on the surface. The emission current therefore becomes a source of Vavilov – Cherenkov radiation. This possibility was experimentally investigated in the Russian Federal Nuclear Center (All-Russian Scientific Research Institute of Experimental Physics), where the generation of Vavilov – Cherenkov radiation was accomplished by an emission current excited by an X-ray wave incident on a metal surface [20, 21]. The experimental facility is schematically diagrammed in Fig. 7. For an X-ray source, use was made of plasma produced in the focusing of subnanosecond laser radiation. The X-ray radiation wave consisted of photons with energies of several hundred electron-volts. In these experiments, it was shown that the incidence of such a wave on a planar metal surface gives rise to secondary electromagnetic radiation that has directivity characteristic of the Vavilov – Cherenkov radiation from a source traveling through a vacuum with the speed $v = c/\sin\theta$ (i.e., with the speed with which the excitation generated by the X-ray wave travels along the metal surface).

Ardavan et al. [22] performed an experimental investigation of a superluminal radiation source made up of a chain of

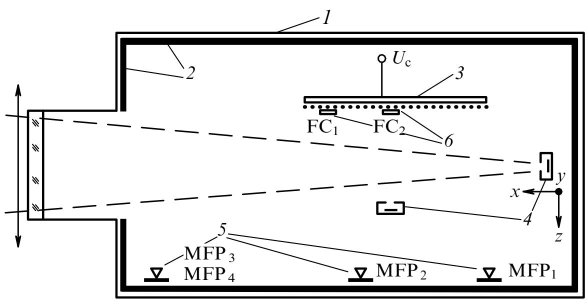

Figure 7. Installation diagram. *1* — vacuum chamber, *2* — absorbing coating, *3* — diode (aluminum plate and mesh), *4* — screened target, *5* — magnetic field pickups, *6* — Faraday cups.

dielectric-filled capacitors. Upon application of voltage to an individual capacitor, there appeared polarization in the dielectric located between the plates. The voltage was applied to individual capacitors such that a polarization wave propagated along the chain of capacitors. The principle of source operation is illustrated in Figs 8a and 8b. The capacitors that make up the chain may be arranged such that the polarization wave propagates along some curve rather than a straight line. This radiator is diagrammed in Fig. 8c. As is seen from the diagram, one plate is common for all capacitors. This design feature is of no fundamental significance. The radiation properties in this scheme were theoretically considered in Refs [23, 24].

It is pertinent to note, by the way, that in radiophysics and, in particular, in radio astronomy, use is made of receiving and transmitting multielement antennas in which the radiation source is a wave traveling along a chain of antennas.

We consider antennas arranged on the x axis at points $x = nL$, $n = 1, 2, \ldots, N$, where N is the total number of radiators. Let the first antenna emit a short pulse at the initial time instant $t = 0$. The second antenna emits the same pulse in a time interval T. The third antenna emits the same pulse in a time $2T$, etc. We can say that the phase of emitted signal travels along the antenna chain with the speed $v = L/T$. The field radiated by this system is determined by formulas (16) – (21), where $\mathbf{E}(\mathbf{k})$ is this time the field radiated by a single antenna. If the quantity $v = L/T$ exceeds the speed of light, directed radiation is emitted at the angle θ to the x axis with $\cos\theta = c/v = cT/L$.

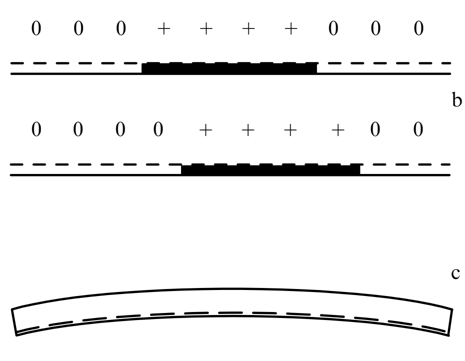

Figure 8. Schematic of a superluminal source. A positively charged plate is marked with a sign (+) and an uncharged plate with a sign (0). (a) charge distribution at a time instant t_0, (b) distribution at an instant $t_0 + \delta t$, (c) schematic of the experimental facility.

Instead of radiating antennas, it is possible to consider a system of receiving antennas arranged in the same manner. By selecting the phase shift between the neighboring antennas, the system can be tuned to receive radio waves with a specific direction.

8. Amplification of electromagnetic waves by reflection from a moving medium

Until now, we considered the radiation effects in a medium at rest. In this case, the radiation of a moving object is a threshold effect, the threshold speed being the speed of light in the medium or in free space. When the source speed exceeds the threshold, radiation emerges. In this connection, it is interesting to note that the motion of the medium can also result in the emission of electromagnetic waves, with a threshold speed occurring in this case as well. Below, we consider a simple example to illustrate this possibility — the scattering of electromagnetic waves from a rotating cylinder with a nonzero conductivity. This problem was first considered by Zel'dovich [25, 26].

We consider a circular cylinder of radius $r = a$ whose axis coincides with the z axis of a cylindrical coordinate system r, φ, z (Fig. 9). The cylinder is uniformly rotating with an angular velocity Ω. A plane electromagnetic wave is incident on the cylinder from free space. We assume that the electric field of this wave is directed parallel to the cylinder axis and the coordinate dependence of the field is given by

$$E_z = E_0 \exp\left[\mathrm{i}(kr\cos\varphi - \omega t)\right], \tag{22}$$

where E_0 is the field amplitude. This formula describes a plane wave whose wave vector is perpendicular to the cylinder axis and aligned with the y axis (the y axis is aligned with the ray $\varphi = 0$). We use the equality

$$\exp(\mathrm{i}kr\cos\varphi) = \sum_{m=-\infty}^{m=+\infty} \mathrm{i}^m J_m(kr)\exp(\mathrm{i}m\varphi), \tag{23}$$

where $J_m(x)$ is the mth-order Bessel function. This formula represents the field as the sum of harmonics of the form $\exp(\mathrm{i}m\varphi)$ with all possible values of m. As is well known, the number m determines the angular momentum of the electromagnetic field described by the corresponding harmonic. We select some value of m and consider the field with the characteristic dependence $\exp(\mathrm{i}m\varphi)$ on the azimuthal angle φ. The electric field inside the cylinder is written as

$$E^m_{z\omega} = E_0 \exp\left[\mathrm{i}\,(m\varphi - \omega t)\right] f(r), \tag{24}$$

where the function $f(r)$ is a solution of the Bessel equation with the boundary conditions at the cylinder surface. We do not specify the form of this function.

The medium of which the cylinder is made up is described by the permittivity ε,

$$\varepsilon = \varepsilon_0 + \frac{4\pi\sigma\mathrm{i}}{\omega}, \tag{25}$$

where ε_0 is the real part of the permittivity and σ is the conductivity of the medium.

The electric field excites currents inside the cylinder and produces work on these currents. If the cylinder does not rotate, the electric current excited by the field E is

$$j = \sigma E^m_{z\omega}. \tag{26}$$

If the cylinder rotates, the field–current relation is defined, with the medium motion taken into account, as

$$j = \sigma\left[E^m_{z\omega} + \left(\frac{v_\varphi}{c}\,B^m_{r\omega}\right)\right], \tag{27}$$

where $v_\varphi = \Omega r$ is the linear speed of motion of the medium at a distance r from the cylinder axis and $B^m_{r\omega}$ is the magnetic field.

We use relations (24) and (27) for the field E to obtain the expression for the magnetic field

$$B^m_{r\omega} = \frac{c}{\mathrm{i}\omega r}\,\frac{\partial E^m_{z\omega}}{\partial\varphi} = \frac{\Omega m}{\omega}\,E^m_{z\omega}. \tag{28}$$

In view of (28), we rewrite formula (27) as

$$j = \sigma\left(\frac{\omega - m\Omega}{\omega}\right)E^m_{z\omega}. \tag{29}$$

The work W_ω of the field on the current j is given by

$$W_\omega = \int_{V_\tau} (jE)\,\mathrm{d}V = \sigma\left(\frac{m\Omega - \omega}{\omega}\right)\int_{V_\tau} |E^m_{z\omega}|^2\,\mathrm{d}V, \tag{30}$$

where V_τ is the cylinder volume. The integral in this expression is a positive quantity and the sign of the entire expression (30) is determined by the sign of $(\omega - m\Omega)$. For $\omega < m\Omega$, the work of the field on the charge is positive, i.e., the field energy is absorbed inside the cylinder. Conversely, the field energy increases if $\omega > m\Omega$. It is evident that this occurs at the expense of the kinetic energy of the rotating cylinder. If the quantity W_ω is negative, the energy of the mth harmonic increases, as does the amplitude of the corresponding wave.

The condition for the enhancement and attenuation of harmonics has a simple physical interpretation. The expression for the wave incident on the cylinder is given by (24), and the phase is determined by the exponent

$$\phi = m\varphi - \omega t. \tag{31}$$

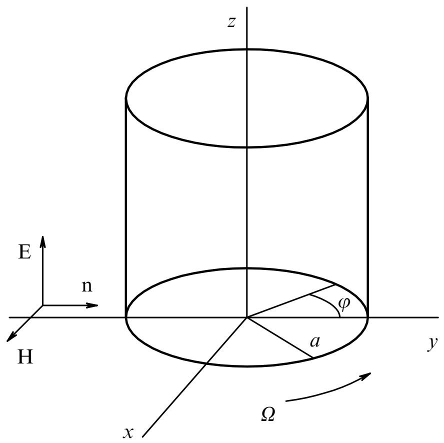

Figure 9. Amplification of electromagnetic waves in the reflection from a rotating cylinder.

The point of constant phase is defined by the equation

$$m\varphi - \omega t = \text{const}\,, \tag{32}$$

and hence

$$\varphi = \text{const} + \frac{\omega}{m}\, t\,. \tag{33}$$

The point of constant phase rotates with the angular velocity

$$\frac{\mathrm{d}\varphi}{\mathrm{d}t} = \frac{\omega}{m}\,. \tag{34}$$

At the cylinder surface (for $r = a$), the corresponding point moves with a linear (phase) velocity

$$v_\varphi = a\,\frac{\omega}{m}\,, \tag{35}$$

and the velocity of the medium on the cylinder surface is

$$v = a\Omega\,. \tag{36}$$

A comparison with formula (30) shows that the work of the field on currents is proportional to the difference between the medium velocity on the boundary and the phase velocity of the wave at the cylinder surface.

In the case of a plane interface, the amplification of the incident wave may also occur at reflection, but the amplification condition is not necessarily satisfiable. For instance, when a plane wave is incident from a vacuum on the boundary of a moving medium, the speed of the light spot exceeds the speed of light, and the medium cannot move faster than the light spot. For a cylinder, it is always possible to select a value of the multipole number m such that the amplification condition is satisfied.

Wave amplification in wave scattering by a rotating cylinder was also considered in Refs [27–30].

9. Extended electron bunch. Superluminal expansion of the radiating domain

It is noteworthy that the radiation characteristics depend not only on the law of spot motion but also on the variations of the dimension and shape of the spot itself. A consideration of the transition radiation of an extended bunch can provide some idea of the special features of the radiation in this case [12].

We assume that a uniformly charged bunch of spherical shape is moving along the z axis with a speed v. The bunch radius is denoted by r_0. The moment the bunch touches an interface is taken to be the initial time instant. We divide the bunch into thin layers parallel to the interface. These layers sequentially cross the interface during the bunch motion and each of them generates transition radiation. The interference of the radiation from the layers determines the transition radiation of the entire bunch. Each layer is a circle with a uniform charge density. The radius $a(t)$ of the circle intersecting the interface varies with time. At the initial time instant, the circle radius is equal to zero. At an instant t, the radius is defined by the formula

$$a(t) = \sqrt{r_0^2 - (r_0 - vt)^2}\,. \tag{37}$$

The time variation of the circle radius determines the evolution of the radiating domain. When the spherical bunch passes through a plane, the circle radius varies from zero to the bunch radius and then decreases to zero. We consider the rate of change of the radiating domain radius. From formula (37), we obtain

$$\frac{\mathrm{d}a}{\mathrm{d}t} = v\,\frac{r_0 - vt}{\sqrt{2r_0 vt - v^2t^2}}\,. \tag{38}$$

We see from equality (38) that the radius of the radiating domain early in its development increases with a rate exceeding the speed of light. For $t = 0$, the rate $\mathrm{d}a/\mathrm{d}t$ is infinite. Then, as the bunch traverses the interface, the rate of the radiating domain broadening decreases, but the domain continues to broaden until the center of the bunch crosses the interface. At this instant, the broadening rate is zero, then the radius begins to decrease, vanishing at $t = 2r_0/v$. In the vicinity of this time instant, the radiating domain collapses to the center with a rate exceeding the speed of light.

The radiation of the spherical bunch intersecting an interface can be represented as the radiation of some domain located at the interface; as the bunch traverses the interface, this domain emerges, broadens (at some time instant, it broadens with a superluminal speed), and then decreases and vanishes. If the bunch speed is close to the speed of light, in the time interval from $t = 0$ to $t \simeq 0.3r_0/c$, the speed of domain expansion exceeds the speed of light and in the time interval from $t \simeq 1.7r_0/c$ to $t = 2r_0/c$, the velocity with which the domain boundary travels to the center also exceeds the speed of light.

We consider the contribution to spherical bunch radiation made by the radiation of a circle of radius $a(t)$. Figure 10 shows the geometry of the problem. The xy plane coincides with the interface. The circle of radius $a(t)$ represents the layer of the spherical bunch that is located at the interface at a time t. Each element of the circle is a radiation source. We consider the radiation field at a point D located at a long distance R_0 from the center of the circle ($R_0 \gg a$). The radius vector drawn from the center to the point D forms an angle θ with the normal to the interface (the z axis). The direction from the center of the circle to the point D is characterized by a unit vector $\mathbf{n}$. Without loss of generality, it may be assumed that the point D lies in the xz plane. We select a small element $\mathrm{d}S = r\,\mathrm{d}r\,\mathrm{d}\varphi$ in the radiating domain located at a distance r from the center and determined by an angle φ. The distance of the radiating element from the point D is denoted by R. The transition radiation field from the element $\mathrm{d}S$ at the point D is

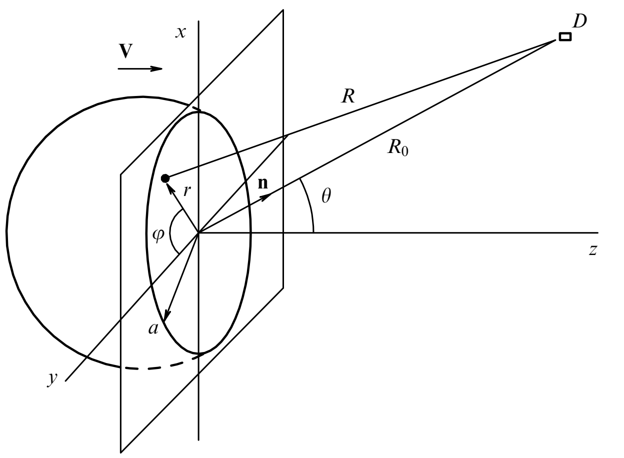

Figure 10. Geometry of the problem of transition radiation generated by a spherical bunch.

given by

$$E(\omega) = \frac{\sigma}{\pi c R}\left(\frac{\beta \sin\theta}{1-\beta^2\cos^2\theta}\right)\exp\left(\mathrm{i}\frac{\omega}{c}R\right)\mathrm{d}S\,, \tag{39}$$

where σ is the surface charge density and $\beta = v/c$ is the relative charge speed. The total field is given by the integral over the area of the circle of radius a

$$E(\omega) = \frac{\sigma}{\pi c}\int_0^a r\,\mathrm{d}r\int_0^{2\pi}\left(\frac{\beta\sin\theta}{1-\beta^2\cos^2\theta}\right)\frac{1}{R}\exp\left(\mathrm{i}\frac{\omega}{c}R\right)\mathrm{d}\varphi\,. \tag{40}$$

In our case, $R_0 \gg a$, and we can represent the distance R as

$$R = R_0 - \mathbf{nr} = R_0 - r\sin\theta\sin\varphi\,.$$

With this relation, we integrate (40) to obtain

$$E(\omega) = \frac{\sigma}{\pi}\left(\frac{\beta\sin\theta}{1-\beta^2\cos^2\theta}\right)\frac{a(t)\,J_1[(\omega/c)\,a\,(t)\sin\theta]}{\omega\sin\theta}\,. \tag{41}$$

The angular distribution is determined by two factors. The first one (the expression in parentheses) describes the transition radiation of a unit point charge and the second factor takes the interference of radiation from different segments of the circular layer into account.

When the spherical bunch passes through the interface, the radius of the radiating domain changes, and the total radiation is defined by the integral over the period during which the entire bunch passes through the interface (or, equivalently, by the integral over all layers). The ultimate expression for the field is

$$E(\omega) = \frac{\sigma}{\pi}\left(\frac{\beta\sin\theta}{1-\beta^2\cos^2\theta}\right)\int_0^{2r_0/v}\sqrt{2r_0vt - v^2t^2}$$
$$\times\frac{J_1\left[(\omega/c)\sin\theta\sqrt{2r_0vt-v^2t^2}\,\right]}{\omega\sin\theta}\exp\left(\mathrm{i}\frac{\omega}{c}\,t\right)\mathrm{d}t\,. \tag{42}$$

It is easily shown that the integral in (42) is the Fourier component of the charge density distribution in the spherical bunch.

Plotted in Fig. 11 are transition radiation intensities in a unit solid angle $\mathrm{d}I(\omega)/\mathrm{d}\Omega \sim E^2(\omega)$ as functions of the radiation angle θ. The intensity values are normalized to the peak intensity. The dependences plotted in Fig. 11 were obtained according to expression (42) for different ratios between the radiation wavelength λ and the sphere diameter $2r_0$. We note that a sphere is the simplest body characterized by a single dimension. Curve *1* in Fig. 11 refers to the case where the wavelength exceeds the diameter of the sphere ($\lambda/2r_0 = 1.5$). The dashed line shows the value of the field for a point charge. We see that the distribution of the bunch radiation in this case differs little from the radiation of a point charge, although the difference in the field magnitude becomes appreciable for large values of the angle. Calculations showed that the difference becomes smaller as the ratio $\lambda/2r_0$ increases. Curve *2* applies to the case where $\lambda/2r_0 = 0.73$. Apart from the narrow peak directed at an angle $\theta \simeq 1/\gamma$, as is evident from Fig. 11, additional peaks appear in the radiation distribution, making the radiation of a lengthy bunch at large angles significantly different from the radiation of a point charge. As the ratio $\lambda/2r_0$ decreases, the radiation at low angles is suppressed and the radiation at large angles becomes significant. Curve *3* shows the distribution for $\lambda/2r_0 = 0.7$. Calculations showed that a further decrease in the ratio $\lambda/2r_0$ entails an increase in the number of peaks.

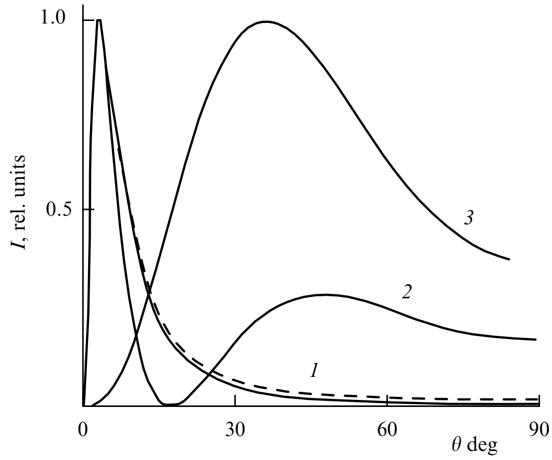

Figure 11. Angular dependence of the intensity of transition radiation. *1* — $\lambda/2r_0 = 1.5$; *2* — $\lambda/2r_0 = 0.73$; *3* — $\lambda/2r_0 = 0.7$.

The decrease in forward-directed radiation may be attributed to the fact that a lengthy bunch comprises volume elements radiating in counterphase. This decrease becomes especially significant when the bunch length is comparable to or longer than the wavelength.

The enhancement of radiation emitted at large angles is attributable to the fact that the expansion of the radiating domain occurring with a superluminal rate gives rise to directional coherent radiation. This radiation may be considered an analogue of the Vavilov–Cherenkov radiation. This radiation was called the quasi-Cherenkov radiation by Nagornyi and Potylitsyn [31], who considered similar effects. In Section 10, we discuss an experiment in which the radiating domain as a whole moves along the interface.

10. Extended electron bunch. Superluminal motion of the radiating domain. Experimental results

As is well known [32, 33], the energy emitted by a particle that escapes from a conductor perpendicular to its surface is equal to zero in the direction of particle velocity and peaks at the angle $\theta = \theta_m$ to the velocity direction, where $\theta_m \simeq \gamma^{-1}$, γ is the relative particle energy. The radiation intensity smoothly decreases with a further increase in the angle θ. The power emitted along the conductor surface (i.e., at an angle $\theta \simeq 90°$) is approximately γ^2 times lower than in the direction $\theta_m \simeq \gamma^{-1}$.

When an extended charged particle bunch traverses an interface, the transition radiation of the bunch is a result of the interference of radiation from a large number of particles and may markedly differ from the radiation of a single charge, as already noted in Section 9. This difference is most pronounced when the radiation is recorded at a wavelength comparable with the bunch dimension. In this connection, we mention that the transition radiation from bunches of finite dimensions was theoretically investigated in several papers (see monograph Ref. [33]). As a rule, the authors were concerned with conditions whereby the radiation of a bunch containing N particles is the same as the radiation of a point particle of charge eN. Such treatment results in certain requirements on the bunch dimensions. However, the problem of the effect of bunch dimensions and the particle distribution in the bunch on the angular or spectral distribution of transition radiation is practically important. In this case, perfect radiation coherence is not necessarily attained, i.e., the bunch radiation intensity is not N^2 times the radiation intensity of a single particle.

Serov et al. [34] measured the angular distribution of the radiation emitted when a bunch of particles accelerated in a microtron traverses a metal foil. It turned out that the resultant radiation exhibited the properties of both transition and Vavilov – Cherenkov radiation.

The experimental setup is schematically represented in Fig. 12. The electron source is a microtron operating by the type-I acceleration mode. The particles had the energy 7.4 MeV and were extracted from the microtron using a magnetic channel with the internal diameter 8 mm. Then, the beam traveled through a 1.5 m long drift region and went into the atmosphere through a 100 μm thick foil *1* located on the microtron flange. The extracted beam traversed a foil *2* in its further motion. The radiation generated by the bunch at the intersection of the second foil was recorded by a detector *D*. The detector was arranged in the microtron orbital plane (in the xz plane in Fig. 12) at different distances x_0 from the beam axis and was displaced parallel to this axis.

The dimensions of the second foil and its position relative to the beam could be varied. To shield the detector *D* from the radiation generated by the beam at the foil *1*, the second foil was bent at a right angle and located in front of the flange, as shown in Fig. 12. The distance *d* from the beam axis to the bent section of the foil was equal to 85 mm.

In this experiment, the reduced energy was $\gamma \simeq 15$, and the angle for which the transition radiation intensity reached its peak was $\theta_m = \gamma^{-1} \simeq 3.5°$. The angles θ at which the radiation was recorded were much larger than θ_m and lay in the 45° – 90° range. According to the theory, the radiation intensity is proportional to $\sin^2\theta/(1-\beta^2\cos^2\theta)^2$, where $\beta = v/c$ is the ratio between the charge speed v and the speed of light c. As the observation angle is changed from 45° to 90°, the radiation intensity should therefore lower smoothly by a factor of two.

Figure 13 shows the experimentally measured radiation intensities as functions of the longitudinal detector coordinate z. The curves refer to different values of the transverse detector coordinate x_0. The measurements were carried out at a wavelength $\lambda \simeq 8$ mm comparable with the bunch dimensions. As Fig. 13 suggests, the distribution is markedly different from the distribution predicted by the theory for a single charge. In the experiment, the transition radiation of the bunch exhibits sharp peaks at large angles. For instance, the clearly defined peak in curve *1* is attained for $z \simeq 20$ mm and $x_0 = 95$ mm, which corresponds to the emission angle $\theta \simeq 70°$.

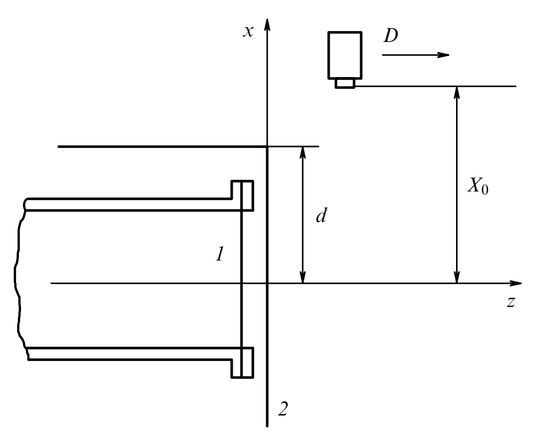

Figure 12. Schematic of experimental setup.

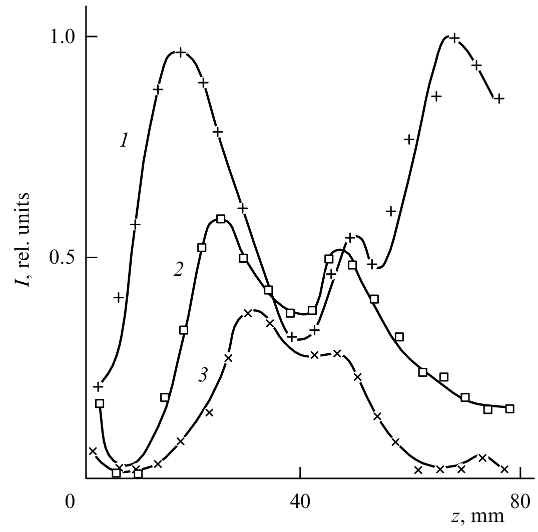

Figure 13. Radiation intensity I_ω as a function of the longitudinal coordinate z. $\gamma = 15$, $\lambda \simeq 8$ mm. *1* — $x_0 = 95$ mm, *2* — $x_0 = 135$ mm, *3* — $x_0 = 165$ mm.

The measurements showed that there is yet another significant distinction between the transition radiation of a bunch and that of an electron, specifically, the angular radiation distribution at large angles is asymmetric with respect to the z axis. If we select two points for a given z value whose x coordinates are equal in magnitude and opposite in sign, the radiation intensity at these two points is different. In the experiments, for the same z value, the intensity of emission to the domain with positive values of the transverse detector coordinate x_0 was higher (by a factor of 5 – 8) than to the negative-x_0 domain.

The reason why the measured angular distribution is different from the theoretically predicted one lies with the coherent nature of the radiation. In the experiment, the sensor records the radiation not of a single electron but the result of the interference of the waves emitted by the electrons of the bunch. This interference has the effect that the angular distributions of transition radiation for the bunch and for an individual electron differ widely. In this case, the governing factor that affects the angular distribution is the mutual arrangement of the radiating particles.

To account for the experimental data, numerical calculations were performed for the charge distribution in a bunch accelerated in the microtron and for the angular distribution of transition radiation generated by this bunch.

In the description of particle motion in a microtron, advantage is conventionally taken of numerical computational techniques specifically elaborated for this purpose [35]. Experimental investigations of acceleration modes in the microtron showed that these calculations describe the electron capture and bunch formation with adequate accuracy. As is well known [36], the microtron operation depends on several parameters: the dimension and shape of the accelerating cavity, the emitter location, and the amplitude

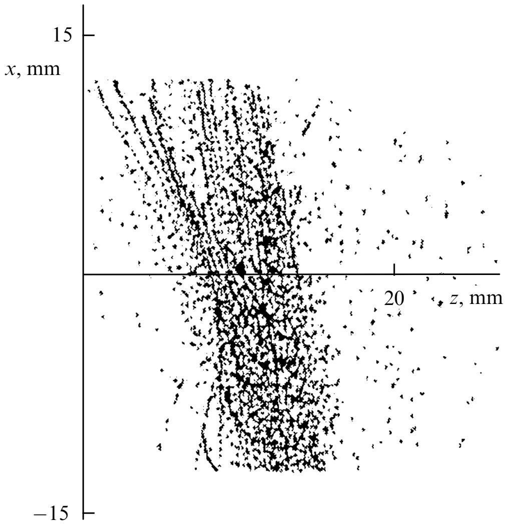

Figure 14. Spatial distribution of bunch particles prior to traversing the foil.

of the accelerating high-frequency field and the guiding magnetic field. In the numerical calculation, it is therefore necessary to take the geometrical dimensions of the cavity and the operating mode of a specific microtron into account. The results of particle dynamics calculations in this microtron are outlined in Ref. [37].

Figure 14 shows the spatial distribution of the particles prior to their passage through the foil. The distribution was obtained by numerical simulations of electron dynamics in the acceleration mode that was used in the experiment. We see from Fig. 14 that the bunch has a rather sharp boundary. In the transverse direction, the boundaries are determined by the diameter of the extraction channel and the length of the drift region; in the longitudinal direction, the boundaries are determined by the features of phase particle motion in the microtron. In the microtron operating mode involved upstream of the foil, the bunch dimensions were $\simeq 9$ mm longitudinally and $\simeq 28$ mm and 4 mm in the horizontal and vertical directions, respectively.

The transition radiation of the bunch was calculated based on the spatial particle distribution derived from the simulations. When a charged particle intersects a plane's metal boundary to find itself in a vacuum, the radiation field at a given frequency ω is described by

$$E_{\omega} = \frac{q}{\pi c R}\left(\frac{\beta \sin\theta}{1-\beta^2\cos^2\theta}\right)\exp\left(\mathrm{i}\,\frac{\omega}{c}\,R - \mathrm{i}\omega t_s\right), \qquad (43)$$

where q is the charge, R is the distance between the transition and observation points, and t_s is the time instant at which the particle escapes from the metal. The bunch radiation field is equal to the sum of the fields generated by individual particles.

Calculations were made of the transition radiation intensity $I_{\omega} \sim E_{\omega}^2$ as a function of the longitudinal z coordinate for a given value of x_0. The calculations were performed for different wavelengths. Figure 15 shows the calculated data. Curve *1* refers to the case where the wavelength is $\lambda = 8$ mm. This is precisely the wavelength at which the radiation intensity I was measured as a function of the longitudinal z coordinate (see Fig. 13, curve *1*). We see

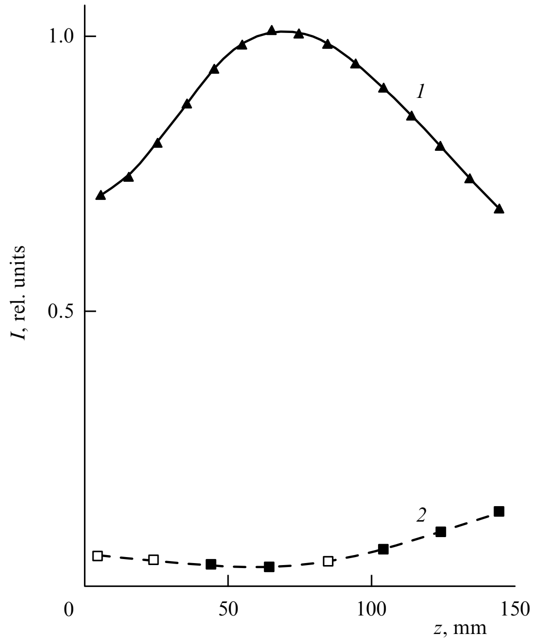

Figure 15. Radiation intensity vs the longitudinal z coordinate. $\lambda = 8$ mm, *1* — $x_0 = 95$ mm, *2* — $x_0 = -95$ mm.

that the calculated data are in qualitative agreement with the experiment: for $x_0 = 95$ mm, the dependence $I_{\omega}(z)$ has a clearly defined peak and its amplitude is many times higher than for $x_0 = -95$ mm. Quantitatively, however, the calculated data are somewhat different from the experimental ones: according to the calculations, the radiation peaks at an angle $\theta \simeq 60°$, while in the experiment, this angle is $\theta \simeq 70°$. The discrepancy is supposedly attributable to the fact that certain factors affecting the spatial electron distribution in the bunch are neglected in the calculation of electron motion. At the same time, even small alterations in the spatial distribution of bunch particles may have a substantial effect on the radiation characteristics.

Figure 16 shows the angular distribution of the radiation intensity $I_{\omega}(\theta)$ in the xz plane. In these calculations, the distance between the bunch transition point and the radiation detector was taken to be equal to 100 mm. It is evident from Fig. 16 that the transition radiation of the bunch is strongly asymmetric at large angles to the direction of particle motion. In addition to the peaks at the angles $\theta = \pm 1/\gamma$, the radiation has a peak at an angle $\theta \simeq 60°$.

Of importance in accounting for the results obtained is the fact that the bunch front is tilted relative to the velocity, i.e., the normal to the front is not aligned with the velocity. The angle between them is equal to about 10°. Different sections of the front therefore intersects the foil at different time instants, the line of intersection moving along the foil with a speed exceeding the speed of light. Therefore, the dimensions, as well as the location, of the foil region from which the transition radiation emanates vary with time. It is easily shown that the boundaries of the radiating region move with a superluminal speed. The superluminal motion of the boundaries (and of the whole radiating region) has the effect that the resultant radiation has the properties of transition radiation as well as of Vavilov–Cherenkov radiation. In this case, it is as if the source of transition radiation were traveling along the normal to the foil and the source of Vavilov–

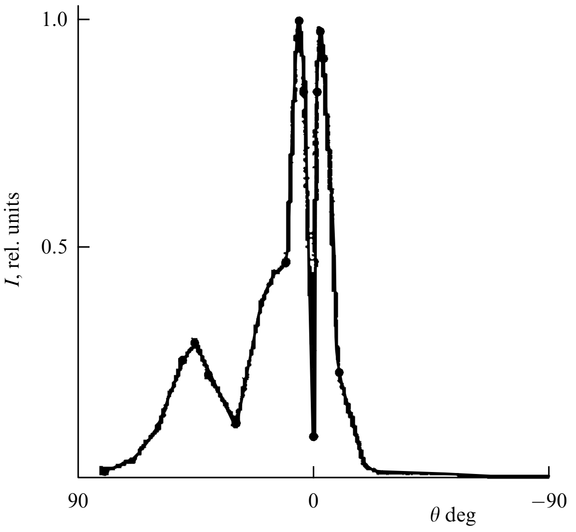

Figure 16. Angular radiation intensity distribution. $\lambda = 8$ mm, $R = 100$ mm.

Cherenkov radiation were moving in the plane of the foil. It should be borne in mind that every individual particle of the bunch produces only the transition radiation in transit through the foil, but the source of transition radiation moves with a superluminal speed along the interface. Due to the interference of the waves emitted by separate particles, a directivity characteristic of the Vavilov – Cherenkov radiation appears.

It is pertinent to note that such features of coherent radiation may be observed in experiments with relativistic beams produced in the majority of linear accelerators. The emission peaks at large angles will be most pronounced in the cases where the transverse dimension of the bunch exceeds its longitudinal dimension, as, for instance, in the Nuclear Physics Laboratory accelerator of Tohoku University in Sendai, Japan [38].

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