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# Neutrino oscillations and the CPT

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## Contents



**Abstract. The experimental discovery of neutrino mixing is now a well-established fact. Its phenomenological interpretation in the theory of neutrino oscillations gives convincing evidence for a nonzero neutrino mass. Precise measurements of the differences of the neutrino mass squared and mixing angles are in progress. Dedicated experiments to search for CP- and T-odd effects will start soon. At the same time, the consequences of hypothetical CPT violation in the leptonic sector of the Standard Model, in neutrino physics in particular, are being widely discussed. This paper represents a brief review of published articles on the manifestations of CPT violation in the processes of neutrino oscillations and resonant changes of their flavor.**

## 1. Introduction

Recent reviews by Bilenky [1] and Akhmedov [2] in *Physics Uspekhi*, review talks at the conferences 'Neutrino-2002' [3] and 'ICHEP-2002' [4] convened in Munich and Amsterdam in 2002, as well as a number of reviews that appeared later, e.g., prior to the 'Neutrino-2004' conference, focused on the pressing problems of neutrino oscillations; these problems have been a subject of discussion in the literature in the last three or four years. However, there has been insufficient emphasis on the problem of possible CPT violation in neutrino physics; this problem has attracted attention recently, also in connection with the idea of the hypothetical mass difference between the neutrino and the antineutrino ($m_{\bar{\nu}} \neq m_{\nu}$). The Munich talks by Akhmedov [5] and Kayser [6] and the plenary talk by Gonzalez-Garcia [7] in Amsterdam contained brief remarks on $m_{\bar{\nu}} \neq m_{\nu}$. Papers by Kostelecky and Mewes [8] specially considered the problem of the CPT symmetry in neutrino physics. A talk by Kostelecky [9] and two review papers by Mavromatos (talk [10] and lectures [11]) were published in 2004; a relevant section is also included in the analytical review by Bahcall, Gonzalez-Garcia, and Peña-Garay [12]. The bibliography of publications on this topic covers much more than a hundred papers that study the general consequences of CPT violation in neutrino-involving processes and analyze experimental data on the oscillations of solar and atmospheric neutrinos and antineutrinos, as well as the well-known LSND anomaly [13].[1]

This review was written in order to briefly outline the available information on CPT violation in neutrino oscillations and on the process of resonance change of neutrino

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[1] See the discussion of this anomaly in Sections 6.1 and 9 of review [1], in Section 7 of review [2], and in the present review (in the middle of Section 5.1 and in 6.3 and 7).

flavors.[2] Following the presentation style of the summarizing papers listed above, we here reproduce only the concluding statements of papers chosen for discussion, without going into the details of the relevant experiments; these can be found in the original publications cited in review papers that have appeared in the last two years [22 – 33].

Several remarks are due on the structure of the paper and the notation used. Publications devoted to general theoretical and phenomenological approaches to the main topic are given in Sections 2 – 4. We need to note here that while the approaches presented in Sections 4.1 and 4.2 are a natural generalization of the Standard-Model formalism, the 'unconventional' ones discussed in Section 4.4 are typically hypothetical.

Those publications that mainly focused on comparing experimental and observational data with phenomenological expectations in the framework of the relevant theoretical approaches are discussed in Sections 5.1 – 5.3, 6.1 – 6.4, and 7; we followed the principle of maximum correspondence of the contents to the section title. Because this arrangement of the material cannot be fully unambiguous, sections containing additional information are cited where necessary.

We use the following main acronyms: CPT, NO, MSW, LI, EP, and DR; these stand for the CPT symmetry, neutrino oscillations, the Mikheyev – Smirnov – Wolfenstein resonance solution in medium, the Lorentz invariance, the equivalence principle, and the dispersion relations (interpreted as relations between the energy, 3-momentum, and mass of a particle). The abbreviated names of experiments are those used throughout the literature; brief descriptions are provided where necessary.

Numerous remarks and references to publications added to the main body of the review are given as footnotes.

## 2. The CPT theorem (quotations)

Even though the Pauli – Lüders – Schwinger CPT theorem is directly related to developments in theories of the weak interaction and in neutrino physics, these general aspects are not covered in this review. Consequently, the fundamental conclusions of the theory concerning the CPT problem is only represented by quotations from familiar monographs and collected volumes (see also Section 3).

• ...Unter sehr allgemeinen und wohlbergündeten Voraussetzungen, zu denen die für die spezielle Relativitätstheorie characteristische Lorentz-Invarianz gehört, gilt nämlich das sogenannte CPT-Theorem. Dieses sagt aus, dass aus diesen allgemeinen Voraussetzungen — wir verweisen für Einzelheiten hier auf die Literatur [34 – 36] — die Invarianz der Theorie für die Zusammensetzung (Produkt) aller drei Operationen C, P und T (in irgend einer Reihenfolge) bereits folgt.

Dieses hat unter anderem zur Folge, dass die Massen von Teilchen und Antiteilchen (allgemeiner die Energiewerte eines Systems von Teilchen und die der zu ihnen *C*-konjugierten Teilchen einander gleich sein müssen. (W Pauli [39, § 1].)

[...Very general and well-founded assumptions, including the requirement of Lorentz invariance in special relativity, imply the so-called CPT theorem. The theorem states that these general assumptions (for details see [34 – 36][3]) immediately imply the invariance of the theory relative to the union (product) of all three operations C, P, T (in an arbitrary order).

This in turn implies, among other things, that the masses of particles and antiparticles (in the general case — energy levels of a system of particles and a system of charge-conjugated particles) must be identical. (W Pauli [39, § 1].)]

• ...We assume further for the sake of simplicity the *local* character of the field equation, which means that all field quantities are spinors or tensors of finite rank and that the interaction part of the Lagrangian (or the Hamiltonian) contains only derivatives of finite order of these field quantities... (W Pauli [36, § 1].)

• Unabhängig von Schwinger [35] kam Lüders [34] zu dem sehr nahverwandten Resultat, dass unter sehr weiten Voraussetzungen eine P invariante Theorie, in welcher die normalen Vertauschungsrelationen bestehen, automatisch CT invariant ist.

Die endgültide und allgemeine Formulierung des hier zuständigen Theorems aber stammt wiederum von Pauli [36] und lautet CTP *Theorem*: Eine Bezüglich der eigentlichen Lorentzgruppe invariante Feldtheorie mit normalen Vertauschungsrelationen ist auch CTP invariant.

Der Fortschritt der neuen Fassung besteht darin, dass (natürlich vor der Entdeckung der Paritätsverletzung) nur die Invarianz bezüglich der eigentlichen Lorentzgruppe vorausgesetzt wird. Ausserdem wird das Theorem fur beliebigen Spin bewiesen, während Lüders sich auf die wichtigsten Spinwerte 0, 1/2 und 1 beschränkt... (R Jost [40, Abschn. 1, § 3].)

[Lüders, independently of Schwinger [35], obtained a very similar result [34], namely that under not very restricting assumptions, a P-invariant theory with conventional commutation relations is automatically invariant under CT.

However, the final and general formulation of this theorem is again Pauli's [36]; the CPT *theorem* states: a field theory with conventional commutation relations, invariant under the Lorentz eigengroup, is also CPT-invariant.

The advantage of the new formulation is the fact that only invariance under the Lorentz eigengroup is assumed (obviously, prior to the discovery of parity nonconservation). Furthermore, the theorem is proven for an arbitrary spin while Lüders only considered the more important spin values 0, 1/2 and 1... . (R Jost [40, Ch. 1, § 3].)]

• Normal commutation relations are defined as follows: tensor fields (belonging to one-valued representations of $L_+^\uparrow$) commute with themselves and with the spinor fields (belonging to two-valued representations of $L_+^\uparrow$) at space-like separation; spinor fields anticommute at space-like separation...

If we anticipate the results of the last chapter, where particles are introduced into Wightman field theory, then the above results imply the law of connection between spin and statistics: particles with integer spin obey Bose – Einstein statistics, particles with half integer spin obey Fermi – Dirac statistics. (R Jost [41, Ch. V, § 3].)

[2] The general situation with the origin of the neutrino mass and neutrino oscillations was discussed in reviews by Gershtein, Kuznetsov, and Ryabov [14] and Kozlov, Martem'yanov, and Mukhin [15] published in 1997 in *Physics Uspekhi*, and also in a review by Bettini [16] [*Physics Uspekhi,* (2001)]; for the latest data on oscillations, see Wark's plenary talk [17] in Amsterdam, as well as *Review of Particle Physics*, 2004 (the general review by Kayser [18], reviews by Vogel and Piepke [19], Groom [20], and Nakamura [21]) and recent reviews [22 – 33].

[3] For nonlocal theories Jost [37] produced a condition equivalent to the CPT theorem that holds identically for local theories. For further applications see [38]. (*Pauli's comment* [39, § 1].)

• Let us next consider the restrictions imposed by the requirement that the theory be invariant under (Wigner) time inversion. An important theorem by Pauli [36] and Lüders [34] (this discovery was essentially anticipated by Shell (1948) and by Schwinger [42]), and currently known as the *TCP* theorem, asserts that within the framework of relativistically invariant *local* field theories, assuming the usual connection between spin and statistics, invariance under time reversal is equivalent to invariance under $U_{\mathrm{P}}U_{\mathrm{C}}$, i.e., the combined operation of charge conjugation ($U_{\mathrm{C}}$) and space inversion ($U_{\mathrm{P}}$). In a Lagrangian formulation, the TCP theorem is a result of the assumed invariance under the proper Lorentz transformation, the Lagrangian hermiticity, the locality of the theory, and the assumption that particles of internal spin (bosons) must obey Bose–Einstein statistics and those of half-integral spin (fermions) must obey Fermi–Dirac statistics, i.e., the particles obey the usual connection with statistics. ([43, Part 3, 10b, p. 264].)

• ...Hence it follows that the Lagrangian (14.16) from which we have demanded only that it must be Hermitian and invariant under the proper Lorentz transformations, is also invariant under PCT (CPT, TCP, and so on). This is the essence of the Lüders–Pauli CPT theorem (for further details see Pauli [36] and Grawert, Lüders and Rollnik [44]). The above discussion can readily be generalized to the case of an arbitrary Hermitian Lagrangian, written in the form of a polynomial of finite degree in the fields and their derivatives (of finite order) which transform in accordance with the irreducible representations of the proper Lorentz group.

The requirement that the interaction is local has played an essential role in the above discussion. In the axiomatic formulation of quantum field theory, this requirement can be made less stringent. The proof of the CPT theorem in axiomatic approach has been given by Jost [41], Streater and Wightman [45], and Bogolyubov, Logunov, and Todorov [46]. In this approach, it is also assumed that the Lagrangian is written in the form of the normal product and there is a connection between spin and statistics: fields with integer spin commute with one another and with other fields, whereas fields with half-integer spin anticommute with one another but commute with integer-spin fields. ([47, Ch. II, § 14].)

• The TCP-theorem is remarkable because a discrete symmetry is shown to exist in theories which, to begin with, are only assumed to be invariant under connected continuous groups... . (R Jost [41, Ch. V, § 2].)

• ...A very important consequence concerns the equality [48] of masses and total lifetimes of particle and antiparticle, a result which is true irrespective of the particle conjugation non-invariance of the weak decay interactions... . ([49, Ch. 3, § 5].)

## 3. The theoretical and experimental status of CPT

The general principles of quantum field theory that lie at the foundation of the CPT theorem and were formulated in the mid-20th century connect any violation of the CPT invariance with far-reaching changes in fundamental concepts of the theory such as the causality principle (locality of the Lagrangian) and the relation between spin and statistics (see, e.g., Ref. [50]). Hence, a critical discussion of modern unconventional (and also Lorentz-noninvariant) theories involving CPT violation and their experimental testing are necessary elements in the progress of physics. Further theoretical scrutiny of the current status of the CPT and of the conditions of validity of the CPT theorem are no less important.

Does the Lorentz invariance (LI) still hold in the theory when the CPT symmetry breaks down, the way this occurs in models with unequal masses of particles and antiparticles ($\bar{m} \neq m$)? As follows from Greenberg's paper [51], the answer is negative: the general Greenberg theorem states that the interacting fields that break the CPT symmetry inevitably break the LI as well. The CPT invariance here is necessary but not sufficient for the LI. Theories that break the CPT symmetry as a consequence of mass difference between particles and antiparticles must be nonlocal. Greenberg then discusses what the property of locality may mean in quantum field theory.

The starting points of Greenberg's work are as follows [51]. Quantum field theory is Lorentz-covariant on the mass shell if vacuum matrix elements of unordered products of the fields $\phi(x_n)$ (Wightman functions $W^{(n)}$ [45]) are covariant. The Lorentz covariance (in fact, the Poincaré covariance) on the mass shell is assumed from the beginning. Quantum field theory is covariant off the mass shell if the vacuum matrix elements of time-ordered products of the fields ($\tau$ functions) are covariant. For the LI to hold, quantum field theory must be covariant both on and off the mass shell.

Greenberg's proof employs Jost's axiomatic approach. Jost formulated the fundamental theorem [37] stating that the necessary and sufficient condition of the CPT symmetry is that the so-called weak local commutativity hold at Jost points in the form

$$W^{(n)}(x_1, x_2, \ldots, x_n) = W^{(n)}(x_n, x_{n-1}, \ldots, x_1)\,, \tag{1}$$

where $W^{(n)}$ are defined by the equalities

$$W^{(n)}(x_1, x_2, \ldots, x_n) = \langle 0|\phi(x_1)\phi(x_2)\ldots\phi(x_n)|0\rangle\,. \tag{2}$$

Because the $\tau$ functions can be expressed in terms of the properly arranged sum of Wightman functions, it follows that the invariance condition is a constraint on $W^{(n)}$ given by relations (1), i.e., by the condition of weak local commutativity; this immediately implies the CPT symmetry. Consequently, any violation of the CPT invariance in any of the Wightman functions signifies noncovariance of the corresponding $\tau$ function and, hence, breaking of the LI of the theory. Greenberg also noted that there was no reason to deny the possibility of CPT violation in scattering and other physical processes even if particles and antiparticles had equal masses.

In a later paper, Greenberg [52] gave a critical analysis of an attempt to justify the model with CPT violation caused by $\bar{m} \neq m$ [53] by introducing free hybrid ('homeotic') fields that are, in the case of appropriate normalization, linear combinations of positive- and negative-frequency components of Dirac fields with the masses $m$ and $-m$. It was shown that even though such free fields could satisfy the Lorentz covariance condition on the mass shell, the interacting hybrid fields inevitably violate the Lorentz covariance in accordance with the Greenberg theorem [51]. The model proposed in Ref. [53] cannot serve as an example of a theory with CPT violation. When discussing the fundamental nature of the CPT symmetry in quantum field theory as compared to other discrete symmetries or their combinations, Greenberg emphasized that for LI to result in CPT, it is necessary and sufficient to have a certain weakened form

of space – time commutativity (or anticommutativity) — the so-called weak local commutativity (WLC).[4] This remark explains why free fields with $\bar{m} \neq m$ can satisfy LI on the mass shell but at the same time violate the CPT symmetry.

Summarizing his investigation of the relation of the LI of a theory to the CPT symmetry, Greenberg [54] returned to the question of what was lacking for the CPT symmetry to hold (in the presence of LI). "A free or generalized free field can be Lorentz covariant but not obey CPT invariance if the particle and antiparticle masses are different [51]. What fails in that case is that WLC does not hold at Jost points.... Note that although the fields in these examples transform covariantly their time-ordered products are not covariant. Thus if we require that time-ordered products be covariant as part of Lorentz covariance of a theory then, as shown in [51], free fields that violate CPT are not covariant. See [52] for a detailed analysis of hybrid Dirac fields ('homeotic' fields [53]) which can be covariant only when they are non-interacting but even in the free case have time-ordered products that are not covariant [54]."

The information given above on the general theoretical status of the CPT problem are directly connected with the experimental study of CPT conservation (in elementary particle physics[5] and also in neutrino physics) — from the standpoint of both standard field-theory approaches (mostly discussed in Sections 4.1 – 4.3, 5.1, 6.1 and partially in Section 6.2) and more contemporary models (see Sections 4.4, 5.2, 5.3, 6.2 – 6.4, and 7) in which the sources of possible CPT violation[6] are typically tied to theories of extended objects and to quantum gravity.

We also need to remind the reader that the exceptional importance of testing the CPT experimentally was first understood in connection with the discovery of the violation of P-, C- and CP invariance (see review talk [63] and review [64]).

The constraint that is usually quoted is the rigid constraint on CPT violation due to the difference $\Delta(\mathrm{K}^0, \bar{\mathrm{K}}^0)$ of the masses of $\mathrm{K}^0$- and $\bar{\mathrm{K}}^0$-mesons: $|m_{\mathrm{K}^0} - m_{\bar{\mathrm{K}}^0}|/\langle m_{\mathrm{K}}\rangle < 10^{-18}$ [65, p. 73]. However, because this difference caused by the transition $\mathrm{K}^0 \to \bar{\mathrm{K}}^0$ is small from the very beginning, this constraint is not exclusively characteristic of the CPT-odd interaction: the true parameters of CPT nonconservation in the $\mathrm{K}^0 - \bar{\mathrm{K}}^0$ [56] system can only be bounded at the level $10^{-3} - 10^{-4}$ (see review [66] and also [67]).[7] The general constraints imposed by analyticity and discrete symmetries P, C, CP, TCP on the description of binary systems of neutral mesons of the type $(\mathrm{K}^0, \bar{\mathrm{K}}^0)$ were obtained in the framework of quantum field theory in [68].

The best bound on the CPT violation in the lepton sector is dictated by the relative difference between the $g$-factors of the electron and the positron [65]:

$$\frac{g_{\mathrm{e}^+} - g_{\mathrm{e}^-}}{\langle g_{\mathrm{e}}\rangle} = (-0.5 \pm 2.1) \times 10^{-12}\,.$$

The current status of the CPT symmetry was also presented in recent reviews [69 – 73]:[8] the first four discuss theoretical sources and experimental constraints on CPT violation. The talk [73] dealing with the classification of the effects of breakdown of all discrete symmetries also describes the relation between CPT violation and the Hermitian nature of the Lagrangian of the theory. The processes discussed are those that have not yet been studied experimentally. These are the circular polarization of $\gamma$ quanta in the $\pi^0 \to 2\gamma$ and $\eta^0 \to 2\gamma$ decays (and also the longitudinal polarization of muons in the decay $\eta^0 \to \mu^+\mu^-$) and circular polarization of photons in the decay of parapositronium.[9] It is emphasized that in contrast to the case where $\bar{m} \neq m$, the above examples of CPT-odd polarizations can be formulated in a Lorentz-invariant manner.

## 4. General consequences of hypothetical CPT violation in neutrino physics

The history of studying the relation between possible CPT violation and neutrino physics and neutrino oscillations (NO) covers two decades. The first attempt was the paper by Bigi in 1982 [74]. Starting with the speculations in the literature on possible violation of LI outside the Standard Model that were detectable in the lepton sector of the theory, Bigi investigated not the possibility of interpreting the NO data but a more general problem of expanding the range of phenomena and experiments whose analysis could promise sufficient progress in improving the sensitivity of results to CPT violation. Bigi pointed out that at least in principle, the effects of CP and CPT nonconservation could be separated: CPT conservation signifies the equality of the probabilities $P(\nu_\alpha \to \nu_\beta) = P(\bar{\nu}_\beta \to \bar{\nu}_\alpha)$, while CP conservation results in the equality $P(\nu_\alpha \to \nu_\beta) = P(\bar{\nu}_\alpha \to \bar{\nu}_\beta)$; here and throughout the text, the indices $\alpha$, $\beta$ of the neutrino $\nu$ denote its flavor: $\alpha, \beta = \mathrm{e}, \mu, \tau$. The subsequent description of neutrino oscillations for $m_{\bar{\nu}} \neq m_\nu$ in the case of CPT violations was achieved by introducing a double set of parameters without writing the Lagrangian (see also monograph [69]) and without introducing an explicit definition of the masses of $\nu$ and $\bar{\nu}$. Obviously, this description corresponds at the same time to the violation of the LI (see remarks on the discussion in the first part of Section 3) with all the consequences implied for the theory. As a result, the neutrino oscillation models with $m_\nu \neq m_{\bar{\nu}}$ mentioned in Section 7 are theoretically unfounded and in fact incorrect.

We also note that the discussion in [75] of the relation of the CP, T, and CPT symmetries to NO made it quite clear for

[4] We note that only the normal spin – statistics relationship is possible in the axiomatic approach to quantum field theory as discussed here, because selecting incorrect commutation relations for the field results in the field vanishing identically (see, e.g., Ref. [43, Ch. 17, § 1]).

[5] See, e.g., reviews on CPT conservation [55, 56] (Review of Particle Physics, 2004).

[6] Two approaches are typically mentioned in connection with the mechanisms that could produce spontaneous CPT violations in string theories: one phenomenological [57] and the other based on decoherence due to quantum gravity effects [58] (see also [59, 60] and review talks [61, 62]).

[7] The reason for this is that, as was pointed out in Ref. [66], it is more logical to compare the magnitude of $\Delta(\mathrm{K}^0, \bar{\mathrm{K}}^0)$ not with $\langle m_{\mathrm{K}}\rangle$ but with the CP- and CPT-even difference $\Delta(\mathrm{K_L}, \mathrm{K_S})$.

[8] It must be noted that neither monograph [69] nor paper [74] used by the authors of Ref. [69] contain a theoretical justification of the validity of the relations in the case of unequal masses of the neutrino and the antineutrino. What is hiding behind this fact is in all likelihood an internal inconsistency in these relations.

[9] The decays listed above preserve the C parity, while the magnitude of the PT-odd effect of the **sk**-type correlation of the photon spin and momentum is controlled by the difference $\beta = g^* h - g h^*$, where $g$ and $h$ are the coefficients in the effective Lagrangian with scalar and pseudoscalar terms. Therefore, an experimental observation of the effect would indicate that $\beta \neq 0$, i.e., that the CPT symmetry is violated because the Lagrangian is non-Hermitian.

some time (see also review [72]) that if the CPT symmetry is preserved, the effects of CP and T violation could only occur in experiments that would monitor an excess of neutrinos with the initial flavor. At the same time, CPT violation may also manifest itself (in contrast to the CP- or T-noninvariance) in experiments measuring the deficit of initial-flavor neutrinos.

### 4.1 Extension of the Standard Model: spontaneous Lorentz invariance and CPT violation

Regardless of paper [74] mentioned above but also in connection with searching for new more stringent constraints on the presence of Lorentz-noninvariant terms in the Lagrangian of the Standard Model, other perturbation-theory approaches to describing CPT-odd effects have been formulated. For instance, in order to construct a CPT-noninvariant generalization of the Standard Model in the framework of an effective low-energy theory, an approach was developed [76] for treating spontaneous CPT and LI violation in quantum field theory and in relativistic quantum mechanics. In this case, the neutrino component of the Lagrangian contains only left-handed neutrinos ($L_a$) and has the form [8]

$$\mathscr{L} = \frac{1}{2}\,\mathrm{i}\bar{L}_a\gamma^\mu \overleftrightarrow{\mathrm{D}}_\mu L_a - (a_\mathrm{L})_{\mu ab}\bar{L}_a\gamma^\mu L_b + \frac{1}{2}\,\mathrm{i}(c_\mathrm{L})_{\mu\nu ab}\bar{L}_a\gamma^\mu \overleftrightarrow{\mathrm{D}}^\nu L_b\,, \tag{3}$$

where $\mu, \nu = 1, 2, 3, 4$, $a, b = \mathrm{e}, \mu, \tau$, the first term is the standard kinetic term, the second and third terms correspond to LI violation, and the term with $a_\mathrm{L}$ corresponds to CPT violation. This extension of the Standard Model was carefully investigated in [77] taking gravity into account.[10]

A detailed general analysis of a possible violation of the LI and CPT symmetry in the neutrino sector, not using the assumption of space isotropy,[11] was given in Refs [8, 9]. The authors gave a clear scheme for estimating the sensitivity of various neutrino experiments relative to the value of three parameters — $a_\mathrm{L}$ and $c_\mathrm{L}$ involved in Lagrangian (3), and the difference between the squared masses of neutrino eigenstates $\Delta m^2$, which determines the NO. It was shown that even in the framework of the simplest scheme (with a nonzero element $c_\mathrm{L}$ in the case of an isotropic effective Hamiltonian for the transitions $\nu_\mathrm{e} \leftrightarrow \nu_\mathrm{e}$ and with equal nonzero real elements $a_\mathrm{L}$ in the case of the preferred direction along the rotation axis of the earth for the transitions $\nu_\mathrm{e} \leftrightarrow \nu_\mu$ and $\nu_\mathrm{e} \leftrightarrow \nu_\tau$), it is still possible to reproduce the main features of the experimental behavior of the probabilities of the corresponding NO. The simplified model with two free parameters analyzed by the authors (instead of the usual four in the case of standard oscillations), with $\Delta m^2 = 0$ and no $\nu \leftrightarrow \bar{\nu}$ mixing, predicts, among other things, that LI violation should cause a considerable azimuthal dependence for the number of atmospheric neutrinos and a large decrease in the half-annual variation in the flux of solar neutrinos during some weeks before and after the equinox. The authors of Ref. [8] emphasized that the model serves to illustrate certain key effects caused by LI violation and demonstrates how the presence of Lorentz noninvariance and CPT nonconservation on the $M_\mathrm{Pl}$ scale can be identified using a certain signal in NO.

The results of analyzing the consequences of the Standard Model extension for neutrino physics [8] were summarized in a recent talk [9], which gave an exhaustive description of the theoretical investigation of LI and CPT violation in NO. This work is based on considering the standard equations of motion for the Dirac and Majorana neutrinos, in which matrices in the spinor space are rewritten in a more general form:

$$(\mathrm{i}\Gamma^\nu_{AB}\partial_\nu - M_{AB})\nu_B = 0\,, \tag{4}$$

$$\Gamma^\nu_{AB} \equiv \gamma^\nu\delta_{AB} + c^{\mu\nu}_{AB}\gamma_\mu + d^{\mu\nu}_{AB}\gamma_5\gamma_\mu + e^\nu_{AB} + \mathrm{i}f^\nu_{AB}\gamma_5 + \frac{g^{\lambda\mu\nu}_{AB}\sigma_{\lambda\mu}}{2}\,,$$

$$M_{AB} \equiv m_{AB} + \mathrm{i}m_{5AB}\gamma_5 + a^\mu_{AB}\gamma_\mu + b^\mu_{AB}\gamma_5\gamma_\mu + \frac{H^{\mu\nu}_{AB}\sigma_{\mu\nu}}{2}\,. \tag{5}$$

Here, all neutrino fields (including the C-conjugate ones) are collected into a single spinor $\nu_B$; $A, B = 1, 2, \ldots, 2N$, where $N$ is the number of neutrino types; $\lambda, \mu, \nu = 1, 2, 3, 4$; $m$ and $m_5$ are the mass terms, and the other coefficients in (4) correspond to LI violation, with $a$, $b$, $e$, $f$, $g$ determining CPT violation. If the coefficients of the type $g$ and $H$ are nonzero, a $\nu \leftrightarrow \bar{\nu}$ mixing arises. In the framework of the scheme described here, the terms with LI violation are characterized by dimensionless combinations of $a^\mu L$, $b^\mu L$, $H^{\mu\nu}L$ and $c^{\mu\nu}LE$, $d^{\mu\nu}LE$, $g^{\lambda\mu\nu}LE$ and can be reduced to direction-dependent effects in oscillations.

### 4.2 Additional perturbation-theory terms with violation of Lorentz invariance, CPT, and the equivalence principle

Similarly to the approach in [76], outlined in Section 4.1, Coleman and Glashow [85] developed a general scheme aimed at introducing Lorentz-noninvariant and CPT-nonsymmetric perturbation-theory terms into the theory. Coleman and Glashow [85] started with the specific problem of testing special relativity theory in connection with cosmic rays and NO [86–89]. A renormalizable and gauge-invariant CPT-odd LI-violating additional term in the Standard Model Lagrangian results in the emergence of the maximum attainable velocity (MAV) of particles and, in addition, is invariant under rotations in a certain preferred reference frame. Each particle type $a$ is put in correspondence with not only the mass $m_a$ that characterizes it but also the MAV value in a vacuum denoted by $c_a$, with $c_a \neq c_\gamma$. It has been found that this assumption is sufficient [85, 86] for oscillations to appear even with massless neutrinos,[12] which are typically described in terms of the differences $\Delta c_\nu \equiv c_{\nu_i} - c_{\nu_j}$ and angles of the corresponding mixing matrix for MAV eigenstates. In the most general form, the neutrino eigenstates with a given momentum $p$ are characterized in the ultrarelativistic case by the following sum of three Hermitian $3 \times 3$-matrices [85, 89]:

$$\hat{c}p + \frac{\hat{m}^2}{2p} + \hat{b}\,. \tag{6}$$

[10] For a discussion of the problem of calculation of the NO phase in curved space–time see, e.g., Refs [78, Pt. II; 79, 80] and the references therein.

[11] The isotropy of space was tested relatively recently in measurements of the direction independence of the gravitational constant $G$, in experiments with light propagation using the theory and practical methods of wave front inversion [81], as well as in experiments measuring the amplitude $A(t)$ in the angular dependence $1 + A(t)\cos\theta$ of the emission of $\mathrm{e}^-$ in β-decay of $^{90}$Sr, where $\theta$ is the angle relative to the South–North axis. It was found that $\Delta G/G$ does not exceed the level $10^{-10}$ (see, e.g., the analysis in Ref. [82]) and that the speed of light in air and refraction index in glass are independent of direction, at least to within $5 \times 10^{-8}$ [83]; the upper bound on $A$ was found to be $1.4 \times 10^{-5}$ (90% CL) [84].

[12] "Massless neutrinos cannot oscillate if special relativity is unbroken. However, they can oscillate if different neutrinos travel at slightly different speeds *in vacua*." [86].

Here, $\hat{c}$ is the matrix of MAV values for the neutrino, $\hat{m}^2$ is the diagonal matrix of squared Majorana masses, $m^2 = mm^\dagger$, and $\hat{b}$ is the matrix related to the CPT-noninvariant additional term $\bar{\nu}_\alpha b_\mu^{\alpha\beta}\gamma_\mu \nu_\beta$ in the Lagrangian (the case of timelike $b_\mu \sim (b, \mathbf{b})$ for $\mathbf{b} = 0$ is considered). The matrices $\hat{c}$ and $\hat{m}^2$ determine the energy eigenstates as MAV states in the high-energy limit and as mass states in the low-energy limit, respectively. In the case of oscillations of the neutrino of two flavors, the expression for the probability of a diagonal transition on the baseline $L$ is given by

$$P(\nu_\alpha \to \nu_\alpha) = 1 - \sin^2 2\Theta \sin^2 \frac{L\Phi}{4}\,. \tag{7}$$

The generalized mixing angle $\Theta$ and the phase factor $\Phi$ are written explicitly in terms of eight parameters, three mixing angles $\theta_m$, $\theta_b$, and $\theta_c$, three differences $\Delta m^2$, $\Delta b$, and $\Delta c$ corresponding to the matrices $\hat{m}^2$, $\hat{b}$, and $\hat{c}$, and two complex phases $\eta$ and $\eta'$:

$$\begin{aligned} \Phi \sin 2\Theta &= \left|\Delta m^2 E^{-1} \sin 2\theta_m + 2\exp(\mathrm{i}\eta)\,\Delta b \sin 2\theta_b \right. \\ &\quad \left. + 2\exp(\mathrm{i}\eta')\,\Delta c E \sin 2\theta_c \right|, \\ \Phi \cos 2\Theta &= \left|\Delta m^2 E^{-1} \cos 2\theta_m + 2\exp(\mathrm{i}\eta)\,\Delta b \cos 2\theta_b \right. \\ &\quad \left. + 2\exp(\mathrm{i}\eta')\,\Delta c E \cos 2\theta_c \right|. \end{aligned} \tag{8}$$

Clearly, the type of possible LI and CPT violation can be found from the essentially different dependences of the terms containing $\Delta m^2$, $\Delta b$, and $\Delta c$ on $E$.

The phase of neutrino oscillations caused by the violation of the equivalence principle (EP) of general relativity (first treated in Refs [91, 92]) depends on $E$ in the same way as the term with $\Delta c$. We should expect that a natural corollary of EP violation in gravity theories discussed currently in the literature[13] would also be LI and CPT nonconservation. As mentioned in Ref. [87], the phenomenological equivalence of NO under EP or LI violation makes it possible to calculate the constraints on the parameters $\Delta c$ and $\theta_c$ from the range of values of $|\phi\Delta f|$ and $\sin 2\theta_G$ found in the former case ($\phi$ is the dimensionless gravitation potential, $\Delta f$ characterizes the degree of EP violation, and $\theta_G$ is the corresponding mixing angle). In addition to references to previous publications on the relation of NO to the effects of EP violation,[14] paper [87] offers an important general statement that the experimental observation of neutrino oscillations in itself is insufficient for a decisive conclusion on the nonzero mass of at least one of the neutrinos, because the oscillations may be caused by a very small violation of LI and/or EP.

### 4.3 A remark on the type of neutrino mass and the lepton number conservation

We discussed the initial introduction of LI violation in the lepton sector in Sections 4.1 and 4.2 for Majorana masses. In general, the situation with regard to extending the Standard Model appears to be more complicated [99]. First of all, we do not know whether neutrino processes violate the conservation of the lepton number L and whether neutrinos are identical to their own antiparticles. In itself, introduction of the Dirac neutrino mass into the model keeps L conserved. We note that any nonconservation of L would imply the presence of Majorana mass terms that transform the neutrino into an antineutrino. With CPT conserved and in the presence of Majorana masses, the mass eigenvalues are of the Majorana type, i.e., the neutrino is its own antiparticle. If the CPT symmetry is conserved but the theory has no Majorana mass terms, then mass states are of the Dirac type, L is conserved, and the neutrinoless double β-decay is forbidden. Barenboim et al. [99] discuss CPT violations using a simple example of a theory with a single neutrino ν interacting with the electron, and its CPT-conjugate antiparticle $\bar{\nu}$ coupled to the positron. The neutrino mass matrix $M_\nu$ in the selected direction of spin has the form

$$\begin{pmatrix} \mu + \Delta & y^* \\ y & \mu - \Delta \end{pmatrix}, \tag{9}$$

where the upper row corresponds to the neutrino and the lower to the antineutrino. For stable neutrinos, the matrix $M_\nu$ is Hermitian and hence the parameters $\mu$ and $\Delta$ (Dirac masses) are real; $\Delta \neq 0$ indicates CPT violation and $y \neq 0$ (Majorana mass) indicates nonconservation of L. An analysis shows that the neutrino mass eigenstate for $\Delta \neq 0$ cannot correspond to a Majorana particle any more. Nevertheless, if $y \neq 0$, we have mixing of ν with $\bar{\nu}$, the lepton number L is not conserved, and the neutrinoless double β-decay is allowed.

### 4.4 'Nonstandard' mechanisms of violating Lorentz invariance, CPT, and the equivalence principle (decoherence, modification of dispersion relations)

'Nonstandard' sources of LI or EP violation and novel NO mechanisms are usually connected with certain properties of the vacuum on Planckian (or even considerably larger) scales. These aspects were treated in recent review talks [10, 100], which offered arguments in favor of the inherent sensitivity of the NO to CPT violation in comparison with experimental data involving other particles. The mechanism that could explain the loss of unitarity in quantum gravity [58, 59] — which would result in LI and CPT violation in one form or another — is so far illustrated only by a hypothetical, although visually clear picture of the manifestation of the space – time structure (the 'foam') at the quantum level; this is caused by the creation and annihilation of black holes and large metric fluctuations accompanied by the generation of virtual horizons. In these talks, Mavromatos starts with the idea (see, e.g., Ref. [101]) that when a particle crosses such horizons, the information on its state may be partly lost.[15] Correspondingly, it is suggested that the density matrix with $\rho_{\text{out}} = \$\rho_{\text{in}}$, $\$ \neq SS^\dagger$, where $S$ is the conventional scattering matrix and $\$$ is the noninvertible matrix introduced by Hawking, be considered instead of a pure quantum-mechanical state. One consequence is the loss of unitarity in the effective low-energy theory and CPT violation in accordance with Wald's theorem [102], which states that if $\$ \neq SS^\dagger$, then the CPT theorem is violated, at least in its rigorous form, because the CPT operator is ill-defined. In this connection, Mavromatos [10] discussed the problem of the relation

[13] Halprin and Leung [93] considered oscillations (even for massless mass-degenerate neutrinos) caused by EP violation due to string theory effects (see also Ref. [94]) that contribute to macroscopic gravity, itself caused by the scalar dilaton partner of the graviton [95].

[14] See in addition papers [96 – 98], which discuss experiments carried out by the time of publication on solar, accelerator (including LSND), and atmospheric neutrinos.

[15] Hawking stated in his talk at the GR (*17th Intern. Conf. on General Relativity and Gravitation, Dublin, July 2004*) that information is not lost when black holes are formed or evaporate because in all likelihood the true (not the apparent) horizon is never generated.

between this scenario of CPT symmetry breaking and the LI (see Refs [103, 104]). A range of aspects of this problem are considered in Mavromatos's other publications, in which he also discusses other possibilities, e.g., the ill-defined definition of antiparticle [105], as well as the idea of direct violation of CPT [100] caused by a nonzero $\Lambda$-term that accelerates the expansion of the universe and results in the formation of a cosmological horizon.

In addition to a brief review of theoretical ideas concerning CPT violation at lengths of the order of $M_{\rm QG}^{-1}$ that are characteristic of quantum gravity, Mavromatos [10] considered a wide range of aspects of phenomenological tests of the CPT symmetry in various neutrino processes, including astrophysical and cosmological manifestations. He also reviewed a number of papers that assume that the above picture is valid and investigated whether it is possible to use the available data, including NO data, for the evaluation of parameters that characterize, first, the openness of the system that results in quantum-mechanical decoherence[16] (see Refs [58–60]) in accordance with the right-hand side of the Liouville equation

$$\dot{\rho} - \mathrm{i}[\rho, H] = \delta H \rho\,, \tag{10}$$

and second, the distortion of standard dispersion relations (DR) (see Refs [107–109]) via the addition of new terms (which are represented in the general case by a model-dependent function $F$),

$$E^2 = p^2 + m^2 - F(E, \mathbf{p}, M_{\rm QG})\,, \tag{11}$$

resulting in CPT and LI violation.[17]

In contrast to the analysis of renormalizable Lorentz-noninvariant terms in Refs [76, 85, 86] that we discussed in Section 4.2, the general discussion of the possible LI violation on Planck scales that would affect NO was focused on studying renormalizable effects [113] that result in the energy dependence of the oscillation length of the type $L_{\rm osc} \propto E^{-n}$ with $n = 2$. A dependence with $n \neq -1$ is essential evidence of LI and CPT violation [114]: the cases with $n = 0$ published in the literature [79, 86, 115],[18] $n = 1$ [85, 91], $n = 2$ [117–119],[19] and $n = -3$ [121][20] belong to this group. We note that all possible terms in the effective action that are renormalizable and invariant under rotations correspond to $n = 0, \pm 1$.

Lambiase [79] used $n = 0$ and considered the consequences of EP violation in noninertial reference frames; if $\Delta m^2 = 0$ and linear acceleration is zero, $L_{\rm osc}^{-1} \propto \omega \cos\beta$, where $\omega$ is the angular momentum of the system (in the case of the earth, $\omega \sim 7 \times 10^{-5}$ rad s$^{-1}$) and $\beta$ is the angle between the rotation axis and the momentum of the neutrino. Lambiase also points to the fact (already discussed in the literature) that the choice of metric affects the estimates of EP violation from NO data.

Adunas et al. [121], setting the dependence on energy to correspond to $n = -3$, started with the assumption that the standard commutation relation has to be modified on Planck scales to the form $[\mathbf{x}, \mathbf{p}] = \mathrm{i}\hbar(1 + L_{\rm Pl}^2 \mathbf{p}^2/\hbar^2)$, where $L_{\rm Pl} = \sqrt{\hbar G/c^3} \sim 10^{-33}$ cm. This quantum violation of EP is considered because it can be tested in the next generation of atomic interference experiments and because it may provide an explanation of solar neutrino experiments; the anticipated expression for the NO length is $L_{\rm osc} \propto E^3/(\Delta m^2)^2$.

The general case of a phenomenological description of oscillations of neutrinos with two flavors treated as an open system was analyzed in detail by Benatti and Floreanini [123]. Dissipation effects on the right-hand side of (10) were treated in the approximation in which quantum gravity results in linear decoherence (with a linear dependence on the density matrix);[21] they are simply parameterized by six real variables. These quantities are related via a number of inequalities that correspond to the property of 'total positivity' required to ensure that the density matrix, which describes the states of the extended system that includes not only neutrinos but also their microenvironment with a characteristic scale length, is positive. Three additional parameters are then introduced into the effective Hamiltonian; they correspond to the interaction with the surrounding part of the system (for simplicity, the authors kept only the parameter that is additive to the conventional parameter $\Delta m^2/2E$).

Benatti and Floreanini [123] obtained and analyzed formulas for the NO probability in the case of a general dependence of decoherence effects on all parameters; they emphasized that these effects manifest themselves even with massless neutrinos and depend on the CP-odd phase, which is present in the mixing matrix for the Majorana neutrino. In principle, this feature may serve to distinguish this case from that of the Dirac neutrino.

## 5. Experimental and observational consequences of a hypothetical CPT violation in neutrino physics

In this section, we give information on publications dealing with those specific models of CPT, LI, and EP violation in various neutrino processes where flavor changes and for which the estimates of parameters that characterize the appropriate violation were obtained by comparison with measurement data. Information on this is also given at the end of Section 6.1 and in Sections 6.2 and 6.4.

### 5.1 Evaluations of parameters of LI and CPT violation (not violating the equivalence principle)

In this section,[22] we review papers that consider constraints on the parameters of LI and CPT violation (with the EP not violated); these parameters are predicted or expected on the basis of analyzing NO manifestations.

[16] Nonunitary evolution of a quantum system in which a pure state is transformed into a mixed one was discussed by Marinov [106], who used equations of a type similar to (10).

[17] Mavromatos [10] also discussed a nonlinear modification of LI, both in connection with unitary inequivalence of Fock's flavor and mass spaces in the NO description in quantum field theory [110], and in view of the natural requirement of invariant definition of the scale of Planck lengths (energies) [111, 112].

[18] Cases of energy-independent NO at $m_\nu = 0$ are treated in [116]: by analogy with solid state physics, LI and CPT violations are introduced in the fermion vacuum of quantum field theory.

[19] In the case considered of $m = 0$, we have $L_{\rm osc} \propto M_{\rm QG}^2/E^2$ in a $q$-deformed noncommutative theory [120] if it is assumed that LI is violated as a result of unequal values of MAV due to recoil effects in neutrino scattering by virtual D-branes [108].

[20] EP violation in effective Schwarzschild geometry modified by the hypothetical presence of the maximum acceleration $\mathcal{A}_{\rm m} = 2mc^3/\hbar$ in the chosen gravity model corresponds to $L_{\rm osc} \propto \Delta m^2/E^3$ [122] (see also Ref. [80]).

[21] This linear approximation may not comply with the comprehensive theory [124] (see also Ref. [104]).

[22] See also the end of Section 6.2.

A comparison of expressions (5) – (7) with neutrino data of the 1990s showed that Lorentz-noninvariant terms are found to be too small and do not affect the interpretation of the available NO results (except for CPT-odd effects on very long baselines). At the same time, further investigation of oscillations of solar and accelerator neutrinos with $E \sim 10^3$ GeV and oscillations on the $L \sim 10^3$ km baseline may detect LI violation when $\Delta c \sim 10^{-25}$ [85, 89]. A recent analysis in [125] showed that a more stringent constraint than earlier ones may be obtained from the Super-Kamiokande (S-K) and MACRO experimental results for atmospheric neutrinos at $E \sim 100$ GeV and $L \sim 10^4$ km: $\Delta c < 10^{-25}$.

Barger et al. [126] treated the cases of the manifestation of CPT-odd effects caused by the interference of terms with $\Delta m^2$ and $\Delta b$ in (7) when resonance amplification of the NO amplitude becomes possible at $\sin^2 2\Theta = 1$, by analogy to the MSW-resonance (observed when neutrinos pass through a sufficiently dense medium).[23] For instance, the resonance occurs when the denominator of the generalized mixing angle $\Theta$

$$\tan 2\Theta = \frac{\kappa^2 \sin 2\theta}{\kappa^2 \cos 2\theta - 2\sqrt{2}\, G_{\mathrm{F}} E N_{\mathrm{e}}} \tag{12}$$

vanishes for $\alpha = \mathrm{e}$ in (6) for a medium with the number density $N_{\mathrm{e}}$ in the simplified situation where $\theta_m = \theta_b \equiv \theta$ and $\eta = 0$; here, $\kappa^2 = \Delta m^2 + 2E\Delta b$ and $G_{\mathrm{F}}$ is the Fermi constant. Analyzing the CPT violation in atmospheric neutrinos, Barger et al. [126] assume that, in principle, it is possible to achieve estimates as low as $\Delta b \sim 5 \times 10^{-14}$ eV. Also, CPT violations at the $3\sigma$ level in neutrino factories could be detectable for $\Delta b \approx (1-3) \times 10^{-14}$ eV depending on the baseline length $L$ (for the energy of stored muons 29 GeV).

At the same time, new analyses of the data on the absence of $\nu_{\mathrm{e},\mu} \to \nu_\tau$ oscillations in the latest accelerator short-baseline experiments CHORUS and NOMAD are expected to furnish the limiting values at the levels $\Delta b < 10^{-9}$ eV and $\Delta c < 10^{-20}$ [8].

Preprint [127] gives the results of studying LI violations in short-baseline NO experiments in the framework of the general formalism of the Standard Model extension that we described in Section 4.1 [see formulas (4) and (5)]. The use of the general form of parameterization of this effect [8, 9] allowed Kostelecky and Mewes [127] to consistently combine the descriptions of the results of the accelerator and reactor LSND experiments,[24] CHOOZ and KARMEN. The two-flavor analysis of LSND data that covered a large number of parameters [the terms $a^\mu L$ and $c^{\mu\nu} LE$ in (5) correspond to taking 41 degrees of freedom into account, including the dependence on direction] yielded a nonzero value for a combination of coefficients that give the value of the LI violation. It is found to be $(3 \pm 1) \times 10^{-19}$ GeV, which is characteristic of effects on the Planck scale of energies and is based on the probability $P(\bar{\nu}_\mu \to \bar{\nu}_{\mathrm{e}}) \simeq 0.26 \pm 0.08\%$ measured in the LSND experiment.

As for the value of $\Delta b$, from a joint analysis [128] of the data on the solar neutrino and the expected sensitivity of the KamLAND reactor experiment, the obtained upper bound is at the level $10^{-11} - 10^{-12}$ eV. The result of the latest (June 2004) global fitting of solar and reactor data point to $\Delta b < 0.6\,(1.5) \times 10^{-11}$ eV at $1\sigma\,(3\sigma)$, respectively [12].

A still more stringent restriction can be derived by taking the neutrino and the charged lepton sectors of the theory into account. An estimate for spatial components of the quantity $b$ ($b_i < 10^{-17}$ eV) in the sector of left-handed neutrinos was obtained for models with heavy right-handed Majorana neutrinos [130] on the basis of the available (see [129]) very stringent restriction on the axial term $\bar{e} b_\mu \gamma_\mu \gamma_5 e$ defined by the expression $|\mathbf{b}_{\mathrm{electron}}| \lesssim 10^{-19}$ eV.[25] This estimate is weakened by four orders of magnitude ($b_i \sim 10^{-4} b_0$) by accounting for the motion of the solar system relative to the galactic halo and that of the earth around the sun, such that the selection of a reference frame for $b_\mu$ brings the obtained constraint down to the level $10^{-13}$ eV — still much more stringent than is anticipated for direct neutrino experiments.

To make the manifestations of the possible LI and CPT violations in oscillations accessible for realistic observations, the neutrino sector should be 'shielded' [131] from the sector of charged leptons. Choubey and King [131] connected the implementation of this idea with a unique operator $h^{\mu\nu}_{\alpha\beta} (\overline{\nu^{\mathrm{C}}_{\mathrm{L}}})_\alpha \sigma_{\mu\nu} (\nu_{\mathrm{L}})_\beta$ that emerges in the light left-handed neutrino sector via a seesaw-type mechanism through the introduction of the appropriate LI violation for the heavy Majorana neutrino characterized by the constants $H_{\alpha\beta}$. This approach results in nonconservation of the lepton number L ($\Delta \mathrm{L} = 2$), while the LI violation (with CPT conserved) valid for conventional neutrinos does not cover charged leptons via the radiation corrections in all orders of the perturbation theory. The appropriate oscillation length is found to be independent of energy (as it is in the case of flavor transitions, by virtue of the magnetic moment of the neutrino) and is determined only by the constants $H_{\alpha\beta}$. A comparison of this approach with experimental data provided the authors with a justification for generating the following constraints: $H_{\mu\tau} \lesssim 10^{-11}$ eV (relative to atmospheric $\nu_\mu$), $H_{\mu\beta} \lesssim 10^{-13}$ eV (relative to accelerator $\nu_\mu$ on a long baseline), $H_{\mu\beta} \lesssim 10^{-14}$ eV (relative to $\nu_\mu$ in neutrino factories), $H_{\mathrm{e}\beta} \lesssim 10^{-10}$ eV (relative to $\nu_{\mathrm{e}}$ of the CHOOZ and Palo Verde reactors); the results of the KamLAND experiment with reactor neutrinos are described in this case for $H_{\mathrm{e}\beta} \lesssim 7.2 \times 10^{-13}$ eV.

Finally, Datta et al. [132] calculated the ratios of the expected numbers of $\nu_\mu$ and $\bar{\nu}_\mu$ events in connection with new multikiloton magnetized iron calorimeter projects for studying atmospheric neutrinos in laboratories at Gran Sasso (Italy) and INO (India); the authors compared the predictions with those obtained with the CPT and LI violation approach. The obtained dependence of this ratio on $L$, $L/E$, and $LE$ for a number of values of $\Delta b$ and $\Delta c$ confirmed the possibility of detecting these violations for $\Delta b > 3 \times 10^{-14}$ eV and $\Delta c > 7 \times 10^{-25}$. These constraints are more stringent than is expected in future neutrino factory projects.

Additional information on constraints for the quantities $c$ and $b$ in (8) that follow from NO data was presented in talk [72]. Similar information on constraining the parameters of the possible LI and CPT violation that follow from experiments with atomic systems and muons was reported in review talks [133]. Constraints on LI violation parameters on Planck scales are also obtained by analyzing the high-energy parts of

[23] The phenomenon of a resonance change of flavor of the MSW-transition type was reported earlier in [90 – 92] and in other papers on EP violation (see, e.g., the references in [87]).

[24] Attempts to explain the LSND anomaly are discussed in Sections 6.3 and 7; see also footnote 14 in Section 4.2.

[25] This constraint was obtained in a precision experiment with torsional balance in which the probe weight possessed certain remanent magnetization caused by the spin dipole moment (originating with the polarization of electrons).

the cosmic ray spectrum; these indicate that there is no effect of Cherenkov radiation in the vacuum for p, e, μ, and ν [134].

### 5.2 Violation of the equivalence principle in neutrino oscillations and in neutrino astrophysics

Review talk [135], which outlined the fundamentals of EP and LI violation in NO when neutrinos interact with the background gravitational field, presented the corresponding results of the analysis of solar and atmospheric neutrino data available at the end of 1990s, as well as references to earlier work.[26] The best constraint (the safest estimate) on the parameters $\Delta c$ and $|\phi\Delta f|$ for atmospheric neutrinos was $6\times10^{-24}$ and $3\times10^{-24}$, respectively, at 90% CL, regardless of the value of the mixing angle [138]. The result for solar neutrinos was found to fall to a similar level but was affected by the choice of assumptions. A detailed analysis of the atmospheric neutrino data available at the time with arbitrary values of the parameter $n$ in the general expression for the oscillation length as a function of neutrino energy, $L_{\rm osc}^{-1}\propto E^n$, resulted in the constraint [138] $n=-0.9\pm0.4$ for 90% CL ($n=-1$ corresponds to ordinary oscillations of massive neutrinos). Still earlier results of experiments with atmospheric neutrinos failed to provide an opportunity to exclude any of the existing scenarios of EP violation in spin-$J$ field exchange [139]: scalar with $J=0$ and $n=-1$ (dilaton), vector with $J=1$ and $n=0$ (twisting in the Einstein–Cartan theory), tensor with $J=2$ and $n=+1$ (graviton).

As shown in Ref. [140], we can also expect that estimates of EP violation can be obtained in a wide range of values of parameters by recording changes of neutrino flavor in ordinary NO manifestations in muon storage rings.

A detailed study of possible constraints on the EP and LI violation parameter in neutrino factories [141] led to the conclusion that measuring the T-odd probability difference $P(\nu_\alpha\to\nu_\beta)-P(\nu_\beta\to\nu_\alpha)$ provides the most sensitive evaluation of this violation. A limiting value $|\phi\Delta f|\lesssim10^{-26}$ [141], comparable to the maximum constraint in the $(\nu_\mu,\nu_\tau)$ sector and obtained in Ref. [138] for atmospheric neutrinos $\nu_{\rm atm}$ using the S-K results, can be achieved for the sectors $(\nu_e,\nu_\mu)$ and $(\nu_e,\nu_\tau)$ with a suitable baseline of several thousand kilometers.

Guzzo et al. [142] gave a brief review of the literature on the consequences of EP violation in various neutrino processes, including neutrino astrophysics and primary nucleosynthesis.[27] It was shown that the absence of any significant indications of NO coming from the supernova SN1987A (i.e., the low probability of the $\bar\nu_e\to\bar\nu_{\mu,\tau}$ transitions) points to a very strong constraint on the corresponding parameters[28] for massless or mass-degenerate neutrinos: $|\Delta f|\lesssim O(10^{-31})$ and $\tan^2\theta_G\ll10^{-4}$. However, these constraints become invalid or weakened if the effect due to the mass is present. We note that in this case, an analysis of consequences of EP violation in NO, which may be detected in future observations of super-high-energy neutrinos arriving from cosmologically remote active nuclei of galaxies, is likely to yield an even stronger constraint, at the level $|\Delta f|\sim10^{-41}$ [144].

EP violation may be directly related to the formation of neutrino stars, namely to the fact that pulsars at the moment of birth acquire considerable peculiar velocities. First evaluations of EP violation in resonance flavor transitions (for the maximum efficiency case of $J=2$) that can sustain the required velocities in the anisotropic ejection of neutrinos from a presupernova (provided there is a magnetic field $>10^{15}$ G) yielded the value $|\Delta f|\simeq10^{-10}-10^{-9}$ [145]. The translational and rotational motion of pulsars caused by directionality of neutrino ejection can be interpreted even in zero magnetic field [146] if resonance transitions are assumed to be caused by anisotropy effects [137] in the post-Newtonian approach to gravitational neutrino interactions.

Finally, Denisov et al. [147] recently attracted attention to the importance of simultaneous neutrino and optical monitoring of type-II presupernovas. The data on the time of recording and the characteristics of both signals, as well as the observation of the frequency difference in the spectra of atoms on the surface of the star before and after the neutrino ejection pulse, provide information both on the gravitational potential of the neutrino flux and the neutrino mass and on the possible EP violation.

Constraints on the EP violation parameter are also discussed in Section 6.3.

### 5.3 Manifestations of decoherence and modification of dispersion relations

This section mostly deals with neutrino processes involving flavor change owing to one of the three effects (see Section 4.4), two of which are caused by the quantum-mechanical decoherence. The analysis is based on Eqn (10) in the linear formalism mentioned at the end of Section 4.4, in correspondence with modified DR (11) in the Einstein, as well as in loop, quantum gravity; at the end of this section, we also mention several more exotic models with possible LI violation.[29]

Several papers reported an analysis of NO data in the two-flavor approximation based on the possible decoherence effect that is described by the right-hand side of (10) and parameterized by six variables (as mentioned at the end of Section 4.4). Earlier attempts [148] of explaining the deficit of solar neutrinos, as well as data on atmospheric neutrinos, ignored the requirement of 'total positivity' that relates these parameters to one another. Then stringent constraints for one of them, $\gamma=\gamma_0E^k$ $(k=-1,0,1,2)$, were found in the simplest single-parameter case (in the limit of the neutrinos weakly influenced by the surrounding medium); this parameter characterizes the suppression of the conventional oscillation term with $\Delta m^2$ through the additional factor $\exp(-2\gamma L)$. Strong constraints for the parameter $\gamma_0$ were obtained from the atmospheric neutrino $\nu_{\rm atm}$ data (for 90% CL, assuming $k=0$ and $k=2$), equal to $3.5\times10^{-23}$ GeV and $0.9\times10^{-27}$ GeV$^{-1}$, respectively [149]. A detailed analysis of a more realistic case of $k=-1$ [150] on the basis of the result of S-K and K2K experiments in the channel $\nu_\mu\to\nu_\tau$ failed to detect evidence of the decoherence effect; however, it equally failed to eliminate its presence at $\Delta m^2=0$.

Earlier fitting of short-baseline reactor and accelerator experiments (CHOOZ, CHORUS, E776, CCFR) established upper bounds $\gamma_0$ for all values of $k$ for 99% CL [151];

[26] Also see talk [114], paper [136], and the references therein, and also paper [79] mentioned earlier (in the second part of Section 4.4) and [137], where the gravitational interaction with the neutrino in the post-Newtonian approach takes into account, in addition to the potential $\phi$, next-generation terms that describe new anisotropy effects.

[27] Also see Ref. [143] on NO in wormhole-type objects.

[28] Evaluations of EP violation expressed in terms of constraints on the parameter $|\phi\,\Delta f|$ typically assume that the quantity $\phi=\text{const}\sim3\times10^{-5}$ is determined by the mass of the local galaxy supercluster.

[29] These topics are presented in detail in lectures [11].

constraints in the $\nu_\mu \to \nu_\tau$ channel were found to be considerably weaker than those obtained from solar neutrino data $\nu_{sol}$. By order of magnitude, they were $10^{-22}$ GeV$^2$, $5 \times 10^{-22}$ GeV, $5 \times 10^{-24}$, and $10^{-26}$ GeV$^{-1}$, respectively, in the channel $\nu_\mu \to \nu_e$ for $k = -1, 0, 1, 2$ (it appears that the last two constraints will unlikely be improved using the data for $\nu_{sol}$); the limits in the channel $\nu_e \to \nu_\tau$ are such that the results are more stringent than those obtained for the $\nu_{sol}$ data only if $k = 2$: $\gamma_0 \lesssim 10^{-24}$ GeV$^{-1}$.

The same authors conducted a quantitative analysis [152] of the potential uses of long-baseline accelerator experiments — K2K, MINOS, OPERA — and of a neutrino factory in order to discriminate between ordinary NO and NO due to pure decoherence effects in atmospheric transitions $\nu_\mu \to \nu_\tau$.

The next publication by the same authors [153] extended the initial formalism in [123] to the general approach that is independent of specifics of the model of decoherence interaction between the neutrino system and the surrounding matter to the three-flavor system and obtained explicit formulas for NO probabilities in this case; this work also studied the correspondence of the above two-flavor decoherence analysis of data for $\nu_{atm}$. Two qualitative scenarios were investigated: flavor change in NO due to decoherence only and as a result of the cumulative effect of this mechanism and the conventional one. It was shown that with a simplifying assumption of diagonality of dissipation matrices on the right-hand side of (10), both versions of taking decoherence into account fail to comply with experimental data if mixing in the channel $\nu_e \to \nu_\mu$ or $\nu_e \to \nu_\tau$ is included into the model.

We need to mention in this context that there is an extremely strong astrophysical constraint on the decoherence effect: $\gamma_0 \lesssim 10^{-40}$ GeV at $k = 0$ [154]. It is based on the published estimate of the limiting probability of recording the fact of NO in the flux from the supernova SN1987A, $P(\nu_e \to \nu_{\mu,\tau}) < 0.2$ [155]; in all likelihood, this constraint imposes a very considerable limitation on the expectation of observing the effect in other experiments, even though the data on NO in connection with active galactic nuclei may amplify it by many orders of magnitude [154].

Another source of CPT nonconservation is the interaction of fermion spin with the spin connection of the external gravitational field of Einstein's theory provided its sign is not reversed under the CPT transformation. The contributions of this interaction to the energy for the Dirac neutrino and antineutrino are of opposite signs. Scattering on primordial black holes [156] in the early universe and in axially symmetric cosmological solutions [156, 157] and, in today's epoch, scattering on rotating black holes [156, 158, 159], result in unequal number densities of $\nu$ and $\bar{\nu}$ and also in energy- and mass-independent oscillations $\nu \leftrightarrow \bar{\nu}$ [160].

Another group of papers in which Alfaro et al. [161] start from one of the models of gravity (loop quantum gravity) treats a modified DR of form (11). Among other things, the theory assumes the existence of an intermediate scale $\mathcal{L} \gg L_{Pl}$ that separates the lengths $d \ll \mathcal{L}$ on which the loop structure of space manifests itself and the lengths $d \gtrsim \mathcal{L}$ on which flat classical geometry is reconstructed. Detailed investigation showed [118, 161] that in the framework of this approach, the function $F$ in (11) for a neutrino in a vacuum is in general parameterized by nine constants of different degrees of suppression (compared to unity) through a factor $(L_{Pl}/\mathcal{L})^{3\Upsilon+2}$ ($\Upsilon \geqslant 0$ is an additional phenomenological parameter that is possibly a function of energy), with the scale $\mathcal{L}$ found for two scenarios ($\mathcal{L} \sim 1/E$ and $\mathcal{L} \sim$ const); the term linear in $p$ involved in $F$ includes the symbol '$\pm$' for the possible helicity. Alfaro et al. [161] analyzed in detail the possibility of extracting information (also evaluating the restrictions on the parameter $\Upsilon$ based on the data for $\nu_{atm}$) on two characteristics of observable (in principle) effects of cosmic gamma ray bursts as they are accompanied with powerful emissions of massive neutrinos with $E \sim 10^5 - 10^{10}$ GeV: (1) by signal delay time for various neutrino states in comparison with the light signal, found to be of the order of $(EL_{Pl})L/c \approx 10^4$ s;[30] (2) by the energy dependence $L_{osc}^{-1} \propto E^2 L_{Pl}$, which differs from that discussed in Ref. [138] (see the beginning of Section 5.2).

By using working parameterization constants and comparing theoretical results with NO data and with the spectrum of cosmic rays of extragalactic origin on the basis of the above formalism, it was also possible to analyze the energy dependence of the NO length [163], constraints on the intermediate scale ($\mathcal{L} \gtrsim 10^{-18}$ eV$^{-1}$ [163, 164]) and the working constant [164], and a novel mechanism [165] for generating the primary cosmological asymmetry of the universe originating with the density difference of neutrinos and antineutrinos caused by the above difference in signs for the linear-in-momentum contribution to the function $F$ in (11).

Finally, a separate problem discussed in the literature stems from attempts at describing the closeness in scale of the observed neutrino masses and the energy scale of the so-called dark energy, which determines the acceleration of the cosmological expansion of the universe. Barenboim and Mavromatos, extending and specifying the general line of their earlier work [104], developed a new interpretation of all NO data mentioned in Section 6.3 and also continued the study [166] of possible model approaches to interpreting the cosmological $\nu - \bar{\nu}$ asymmetry and to obtaining a meaningful evaluation of the vacuum energy (the so-called dark energy, also known as the cosmological term, caused by mixing of neutrinos through decoherence).[31] The starting point in Ref. [166] is the assumption that decoherence effects [caused by the interaction of the neutrino with the foam structure of the vacuum (see Section 4.4)] contribute to the terms of the Hamiltonian in Eqn (10) for the evolution of the density matrix and result in the emergence of effective mass shifts, by analogy to the MSW effect in the medium.

We note that the application of the idea of a single origin of the neutrino mass and dark energy in one form or another is typical of a considerable number of publications.[32] Among earlier papers in this field, we mention [169], where EP

[30] More specific estimates of delay time can be found in earlier publications cited in [161], as well as in preprint [162], where time dependence of the Hubble constant and the problem of dark energy are additionally taken into account.

[31] The authors assume that the vacua of flavor and mass states of the neutrino are not equivalent; this results in a nontrivial contribution to the cosmological term owing to the mixing effect [167].

[32] Fardon et al. [168] (see also the references therein) considered a scenario with dark energy treated as the total energy of the liquid at negative pressure, composed of the neutrino field and some scalar field whose magnitude is dictated by the cosmological neutrino density. The neutrino mass then depends on the magnitude of this field such that the lower the number density of neutrinos, the heavier they are. A very weak interaction between the scalar field and ordinary matter arising in this scenario could change the magnitude of the field relative to its vacuum value, resulting in a neutrino mass that is affected by the medium and in turn affects the NO.

violation is discussed in the framework of cosmological 'quintessence'.

Completing the discussion of the general consequences of the relation of the CPT problem to neutrino physics, we need to remind the reader that the CPT noninvariance of the theory results first and foremost in the independence of the CP- and T-symmetry violation effects.

## 6. Interpretations of neutrino oscillations based on CPT violation

In what follows, we outline various (different in principle) manifestations of CPT violation in terrestrial NO experiments. We begin (Section 6.1) with publications in which no true (fundamental) violation of CPT invariance is assumed in the theory but which treat oscillations in the conventional (typically, CPT-nonsymmetric) medium. Section 6.2 presents attempts to interpret NO not by deriving them from the fact that they are massive but on the basis of LI and CPT symmetry violations in the theory (similar to those we discussed in Section 4, including those connected with possible EP violations in gravity theories). Some hypothetical ways of explaining the LSND anomaly are presented in Section 6.3. Section 6.4 discusses papers aimed at generating constraints on the parameters of the fundamental CPT violation that originates with the introduction of a doubled set of oscillation parameters (or at least $m_{\bar{\nu}} \neq m_{\nu}$); we note, however, that the authors of these papers assume without any justification that LI is preserved (see the discussion of this problem at the beginning of Section 4).

### 6.1 False CPT-odd effects in medium

A number of papers [170–172] analyzed spurious CPT-odd effects caused by NO in the medium while preserving the CPT invariance of the theory; these effects result, first of all, in a nonzero asymmetry of the probabilities of diagonal transitions $\Delta P_{\alpha\alpha}^{\mathrm{CPT}} \equiv P_{\nu_\alpha \to \nu_\alpha} - P_{\bar{\nu}_\alpha \to \bar{\nu}_\alpha}$, i.e., the difference of survival probabilities of the neutrino and the antineutrino of a given flavor, and in similar asymmetries $\Delta P_{\alpha\beta}^{\mathrm{CPT}} \equiv P_{\nu_\alpha \to \nu_\beta} - P_{\bar{\nu}_\beta \to \bar{\nu}_\alpha}$ for nondiagonal transitions. For instance, the dependence of $\Delta P_{\mu\mu}^{\mathrm{CPT}}$ on energy and the $\theta_{13}$ angle was demonstrated [171] in very-long-baseline $L$ experiments for terrestrial and atmospheric neutrinos, i.e., in a dependence on a small matrix element $U_{e3}$ that characterizes the NO in atmospheric and solar ranges of $\Delta m^2$. If $L \gtrsim 7000$ km, the familiar resonance effect manifests itself clearly in atmospheric neutrinos through the interactions of $\nu$ and $\bar{\nu}$ in the earth's mantle and crust; the measurement of the CPT-odd asymmetry will provide a means of extracting information on $\theta_{13}$ and on the sign of the corresponding difference of squared masses. A calculation of $\Delta P_{\alpha\alpha}^{\mathrm{CPT}}$ for reactor-based oscillation experiments on long baselines from 730 to 3200 km was also given in Ref. [172].

A detailed list of approximate analytic formulas for $\Delta P_{\alpha\alpha}^{\mathrm{CPT}}$ and $\Delta P_{\alpha\beta}^{\mathrm{CPT}}$ in a medium with an arbitrary density distribution is given in [173]. Furthermore, particular cases of constant density and stepwise density distribution are considered, the latter corresponding to NO in long-baseline accelerator and reactor experiments, as well as in future neutrino factories. Estimates were obtained (numerically and on the basis of the perturbation theory and low-energy approximation) for about a dozen experiments — current and in preparation — for evaluating the indicated CPT-odd differences. Also shown graphically is the effect as a function of the energy $E$ and the baseline $L$ for three more efficient accelerator experiments, KamLAND, BNL NWG, and NuMI, for which numerical values of $\Delta P_{ee}^{\mathrm{CPT}}$ and $\Delta P_{\mu e}^{\mathrm{CPT}}$ are $-0.033$, 0.032, and 0.026, respectively.

### 6.2 Violation of Lorentz invariance and the equivalence principle

The last part of talk [9] cited in Section 4.1 contains, in addition to a discussion of the application of the extended Standard Model to neutrino physics, a description of qualitative features of the simplified two-parameter model and an analysis of its compatibility with the data on atmospheric and solar neutrinos. It is noted that it is currently difficult and would most likely be wrong to exclude the possibility of describing the observed oscillations in terms of LI and CPT violations instead of assigning mass to the neutrino.

In addition to the general evaluation (see Section 4) of the possibility of explaining NO exclusively in terms of CPT- and LI-noninvariant models, detailed analyses of the results of available experiments were carried out recently.

Gago et al. [174] gave a description of the global fitting of all solar neutrino data obtained before the publication of the results of the experiment SNO 2002 with solar neutrinos, and also provided information on interpretations of oscillation data, including those based on EP violation. [33] The purpose of this work was to obtain a numerical comparison of the quality of experimental results in terms of different flavor-varying mechanisms for $\nu_e$. It was shown that in addition to the known strong-mixing LMA(MSW) solution, oscillations can be explained at the same confidence level ($\gtrsim 60\%$) by EP violation, by neutrino flavor variation through interaction of its magnetic moment with the external magnetic field, and by nonstandard neutrino interaction (NSNI) parameterized by two constants, one of which characterizes the contribution of flavor-changing interactions and the second determines the ordinary neutrino–medium interaction and plays a role similar to that of $\Delta m^2$ in the MSW resonance. It is also emphasized that experimental data do not warrant obtaining stringent constraints for the existence of solutions based on the NSNI or on EP violation. [34] The best description of solar neutrino oscillations, $\nu_e$, reported in Ref. [174] corresponds, in the case where they are caused by EP violation, to $|\phi\,\Delta f| \simeq 1.6 \times 10^{-24}$ and $\sin 2\theta_G = 1$, and predicates the vacuum solution; the MSW-type resonance solution [178] requires values of $|\phi\,\Delta f|$ that would be incompatible with the CCFR data.

The fitting of atmospheric neutrino data on the basis of EP violation or in the presence of NSNI provides very poor results (see talks [179] and the references therein); no interpretation of these data on the basis of NSNI only is acceptable for 99% CL [180], mostly due to the energy-independence of the NSNI mechanism.

[33] See also talk [175] and papers [176, 177].

[34] Many papers have appeared recently in the literature that interpret neutrino oscillations data in terms of NSNI mechanisms. Their phenomenological manifestations are typically in the same group because the effect is independent of neutrino energy, which is also characteristic of the contribution of the parameter $\Delta b$ in (8). We saw in the discussion in Section 4.2 that this is also the case for the scalar version of EP violation. Information on the limiting values of $\Delta b$ and $|\phi\,\Delta f|$ is likely to be extractable from the data that yielded constraints on the parameters of NSNI.

The latest fitting [181] of atmospheric S-K data and the K2K experimental results showed that resorting to LI and CPT violation as an additional mechanism of NO hardly affects the standard parameters. The following restrictions [giving eightfold improvement on the results in [138] (see Section 5.2)] were obtained at the level 90% CL:

$$|\Delta c| \leqslant 8.1 \times 10^{-25}\,, \qquad |\phi\,\Delta f| \leqslant 4.0 \times 10^{-25}\,,$$

$$|\Delta b| \leqslant 3.2 \times 10^{-14}\ \text{eV}, \qquad |\Delta\delta_0| \leqslant 4.0 \times 10^{-14}\ \text{eV}.$$

This is a constraint on the CPT-odd interaction with a vector-type twisted field $(J = 1)$ for $n = 0$; the corresponding $3\sigma$ limits were also obtained and found to be greater by a factor of 1.5 – 2.

Another difficult and still unsolved problem is the nonstandard interpretation of experimental results with solar and atmospheric neutrinos in the three-flavor analysis [182]; it is made still more complicated if mutual influences of these two sectors of experimental data are taken into account.

### 6.3 CPT-noninvariant 'ether', decoherence, and the LSND anomaly

Another idea tested for the interpretation of neutrino experiments was the CPT-noninvariant 'ether' [183] acting as a dense medium responsible for interaction potentials of opposite signs for the neutrino and antineutrino. It was shown in [184] that this model involving Lorentz-noninvariant effective operators cannot solve the problems of solar neutrino deficit or the anomalous result of LSND.

However, in principle, the increased number of fitting parameters and their nonstandard dependence on energy provide a possibility of describing NO data, including LSND. In the middle of Section 5.1, we quoted the hypothesis of the possibility of interpreting experiments on a short baseline only in terms of the LI violation effect in the extended Standard Model [127]. This approach, incorporating the assumption in [104] on nonidentical decoherence parameters for the neutrino and antineutrino because of the strong CPT violation but with $m_\nu = m_{\bar\nu}$, has allowed Barenboim and Mavromatos to successfully fit all the available results. The application of the three-flavor analysis, of simplifying assumptions on parameterization of the decoherence effect (which is valid in this particular model only in the antineutrino sector and is described by two quantities that are directly and inversely proportional to $E$), and of the conventional NO mechanism in terms of $\Delta m^2$ made it possible to explain the LSND anomaly [104, 166].

This experiment deserves some additional remarks. We know (see recent reviews [1, 2, 5, 6, 22 – 33]) that the data on the deficit of solar and atmospheric neutrinos (confirmed in a number of experiments) were successfully explained in the model with three flavors mixed; this model operates with only two independent differences of squared masses $\Delta m^2_{ij}$ $(i, j = 1, 2, 3)$. Therefore, the indication in favor of a third value of $\Delta m^2$ that was obtained in the LSND experiment required a modification of the model through incorporation of sterile neutrinos (i.e., by adding $i, j > 3$) or through a radical increase in the number of its free parameters (see the first paragraphs of Section 4).

The LSND experiment [13] searched for $\bar\nu_e$-events originating in decays of positive muons that were created in the decays of stopped pions formed in interactions of protons from the linear accelerator of the meson factory LAMPF. An analysis of the data led Aguilar et al. [13] to the conclusion that a nondiagonal transition $\bar\nu_\mu \to \bar\nu_e$ at $\Delta m^2_{\text{LSND}} \sim 1\ \text{eV}^2$ was present. This result was only obtained in a single experiment and was never confirmed by similar measurements at KARMEN2 [185] (see also the conclusions in [186] on a joint analysis of the data [13] and of [185] and the results of the search for oscillations $\bar\nu_\mu \to \bar\nu_e$ in the accelerator experiment NuTeV [187]).

The upcoming accelerator experiment Mini-BooNE [188] (FNAL) is aimed at testing the LSND result. The refutation of this result would mean that there is no need to introduce sterile neutrinos or a hypothetical inequality of the neutrino and antineutrino masses. A confirmation of the LSND-anomaly may attract additional attention to using the simplest, even though theoretically unfounded, model for interpreting oscillations in terms of $m_\nu \neq m_{\bar\nu}$ (see Section 7).

### 6.4 Constraints on the values of $m_{\bar\nu} \neq m_\nu$ derived from NO data

As for the generation of constraints in the case of $m_\nu \neq m_{\bar\nu}$, it was shown, for example, that sensitivity at the level $|\bar m_3 - m_3| \lesssim 1.9 \times 10^{-4}$ eV can be achieved in neutrino factory experiments [189].

Strumia [190] carried out data fitting in S-K and K2K experiments at $m_\nu \neq m_{\bar\nu}$ in the range of squared mass differences typical of atmospheric neutrinos. The first parameter on the $\Delta m^2_\nu$ vs $\Delta m^2_{\bar\nu}$ diagram proved to be constrained in the same way as with CPT conserved while the second, by contrast, showed allowed values greater by about an order of magnitude.

NO experimental data were also analyzed in order to evaluate $\delta = \Delta m^2_\nu - \Delta m^2_{\bar\nu}$ [191]. Murayama [191] cites the result $(-7.5 \times 10^{-3}\ \text{eV}^2 < \delta < 5.5 \times 10^{-3}\ \text{eV}^2)$ presented by the S-K collaboration at the conference ICHEP-2002 [4]; the result was based on studying the flux of atmospheric neutrinos and pointed to its dependence on the assumption of mixing being maximum and identical for $\nu$ and $\bar\nu$. However, a more detailed analysis of the most recent SNO data on solar neutrinos [193] using the mechanism with MSW transitions inside the sun and of the information on the deficit of reactor antineutrinos in the KamLAND experiment [194] yielded a better constraint for $\delta$ in the form of the inequality $|\Delta m^2_\nu - \Delta m^2_{\bar\nu}| < 1.3 \times 10^{-3}\ \text{eV}^2$ (90% CL) [191]; Murayama compares it with the known upper bound on the mass difference between $K^0$- and $\bar K^0$-mesons $(|m_{K^0} - m_{\bar K^0}|/\langle m_K\rangle < 0.25\ \text{eV}^2)$ and considers this the most stringent experimental bound on the possible violation of CPT invariance.

## 7. Attempts to interpret oscillation data for $m_{\bar\nu} \neq m_\nu$

In connection with the difficulties involved in interpreting the entire gamut of experimental results on NO, even resorting to 'marginal' solutions with the sterile neutrino (see, e.g., Refs [190, 192]),[35] an extended set of squared mass differences was used (by using an independent value of $\Delta\bar m^2_{ij}$ for the antineutrino); to justify this, reference was made to a

[35] Recent SNO experiments (see Refs [193, 195]) eliminate the need to consider sterile neutrinos; for the latest estimates, see review [22], paper [196], and talk [197].

hypothetical violation of the CPT invariance in the neutrino sector of the theory. We emphasize that all models involved [53, 183, 198, 200] lack theoretical foundation: in fact, the difference between the neutrino and antineutrino masses is introduced into them 'by hand'; indeed, as we stressed in Section 3, CPT violation is impossible in a Lorentz-invariant theory (see Refs [51, 52]).[36]

Murayama and Yanagida [198] were the first to try this approach to interpreting NO experimental results (see also [199]). Their paper proposed a scheme of neutrino and antineutrino masses that is compatible with all NO data and the LSND anomaly without adding the sterile neutrino; in addition, it does not contradict neutrino events from the SN1987A supernova. We note that the LSND result is interpreted as a consequence of the large squared mass difference of the antineutrino $\Delta m^2_{\bar{\nu}}$. In view of this, the authors considered it essential to begin the forthcoming MiniBooNE experiment with an antineutrino beam. By analyzing the energies of the indicated SN1987A events in the Kamiokande and IMB experiments, these authors also obtained arguments against the preferred values of $\Delta m^2_{\rm LSND} \approx 0.1-1$ eV$^2$ suggested earlier.

The current situation with interpretation of NO data in terms of a CPT-noninvariant neutrino mass spectrum is outlined in [201] and in review talk [202]. An analysis of all results, with the LSND experiment either taken or not taken into account, was carried out in terms of the three-flavor approach. It was shown that no areas allowed for the LSND or for all other experiments on the $\Delta m^2$ vs $\sin^2 2\theta$ plane overlap at the $3\sigma$ level, while the values of these parameters that correspond to the best fitting result are practically identical both in scenarios with CPT conservation and in those with violation of the CPT.

Finally, Barger et al. [203] used still more exotic scenarios to interpret LSND data in conjunction with all other NO results: with the fourth sterile-type neutrino and imposed equality of $\nu$ and $\bar{\nu}$ parameters (also for $m_\nu \neq m_{\bar{\nu}}$). It was shown that while mass spectra of the types $3+1$ and $2+2$ were possible for $\bar{\nu}$, only the $3+1$ scenario is possible for $\nu$.[37]

## 8. Conclusion

The information given in Sections 3 to 6 characterizes the current status of the old problem of theoretical and experimental investigation of the hypothetical violation of CPT invariance within the lepton sector of the Standard Model. It gives an indication that neutrino oscillations constitute a novel area in which promising possibilities are opened for the search for effects of CPT symmetry violation. One should not forget, however, the status of the CPT invariance as one of the fundamental concepts underpinning the theory. This is a proper place to quote the following remarks that are undoubtedly shared by most of the researchers: "Of course, whatever could be measured should be measured and whatever could be tested should be tested. There should be no reservations: such a fundamental symmetry as CPT should be tested. However one should keep in mind that unlike breaking of C, P, T, CP, PT, and TC, the breaking of CPT is non-compatible with the standard quantum field theory, the only basis for a self-consistent phenomenological description of any process, which we know up to now. Therefore the chances that CPT breaking would be discovered are vanishingly small." [73].

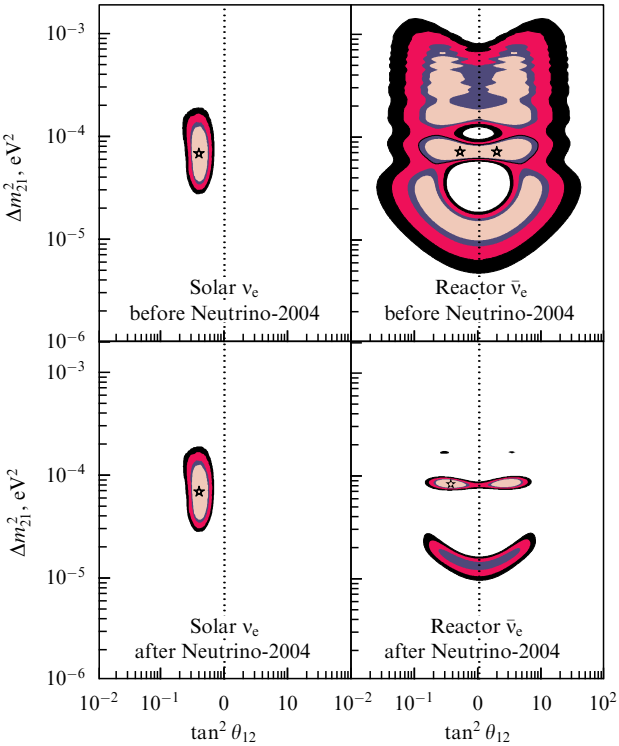


**Figure 1.** Allowed areas for the parameters of neutrino oscillations [12].

Nevertheless, a discussion of the LI and CPT symmetry problem in NO was represented at the recent Neutrino-2004 XXI International Conference on Neutrino Physics and Astrophysics (14–19 June 2004, Paris) with a special talk by A de Gouvêa. The latest measurements of CPT nonconservation in the neutrino sector are also summarized in review [12]; the review graphically characterizes the situation before and after the Neutrino-2004 conference by comparing the admissible values of NO parameters (see Fig. 1) for solar $\nu_e$ and reactor $\bar{\nu}_e$ (including the most recent KamLI results presented to this conference [204]). This comparison demonstrates the complete agreement of totally independent neutrino and antineutrino data, confirming the CPT symmetry in the electron neutrino sector.

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[36] An obvious assumption is made of the correct relation between spin and statistics.

[37] For mass spectra taking the sterile neutrino into account, see, e.g., review [2].

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