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# Development of quantum measurement methods (Methodological notes on part of Einstein's scientific legacy)

V B Braginsky

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## Contents



**Abstract. Historical development of indirect quantum measurements is briefly reviewed. Using several examples, considerable resources are shown to exist for increasing the sensitivity of various types of quantum measurements.**

## 1. Historical foreword

This year (2005) marks the centenary of the first publications by Albert Einstein, two of them on the special relativity theory and the third one on the photoelectric effect. This anniversary is celebrated as a natural recognition of the outstanding contribution the great scientist made to modern physics. The development of the special and general relativity (GR) theories is usually considered the most important part of Einstein's scientific legacy. This point of view tends to overshadow his pioneering work on the photoelectric effect [1]. The judgment of the Nobel Committee, "the Prize of 1921 to Professor A Einstein, Berlin, for his services to Theoretical Physics, and especially for his discovery of the law of the photoelectric effect," emphasizes the importance of this work. In my opinion, this work is the first description, or prediction, of a quantum measurement.

In Paragraph 8 of paper [1], Einstein formulates his heuristic standpoint as follows:

"Energy quanta penetrate the surface layer of the body, and their energy is converted, at least in part, into kinetic energy of electrons. The simplest case occurs if a light quantum gives up all its energy to a single electron; we shall assume that this happens. In addition, we assume that each electron, in leaving the body, has to do an amount of work $A$ (which is characteristic of the body).

... The kinetic energy of such an electron is

$$\frac{m_e v_e^2}{2} = \hbar\omega - A\,.\text{"} \qquad (1)$$

Equation (1) is presented here in the same form as it has been written in textbooks for more than 60 years. A university physics student will reasonably say that it gives the energy conservation law for a photon wave function collapsing near the surface of a solid due to the interaction of the photon with the electron. With the help of Eqn (1), by measuring the electron velocity $v_e$, an experimentalist can find (estimate) the energy $\hbar\omega$ of the photon before its death (collapse). The same equation predicts the red boundary of the photoeffect. Such boundaries were indeed discovered for various solids almost immediately after paper [1] had been published.

Development of the quantum picture of the world, which was started by Planck and Einstein, has been going on for more than a hundred years and, in my opinion, is still far from being accomplished (if it can be accomplished at all). It is interesting to see how slowly, in hindsight, the physics community approached the understanding of what happens in the simplest quantum interactions (measurements) involving a photon and, for instance, an electron. In 1922, i.e., seventeen years after the publication of Einstein's paper [1], Compton discovered [2] that a photon can 'survive' after such an interaction, losing part of its energy, and that the interaction preserves both the sum of the energies and the vector sum of momenta of the photon and the electron. This discovery was made nine years after Bohr had published his paper [3], where he explained the basic features of a hydrogen-like atom emission spectrum. In 1958, i.e., thirty years after the theoretical foundations of the quantum theory had been developed by Heisenberg, Schrödinger, and Dirac, Mössbauer discovered that emission or absorption of a photon by a nucleus in a solid can involve momentum exchange with many neighboring nuclei.

† We remind the readers that, as it has been mentioned in the January issue (Vol. 175, p. 40), throughout 2005 the Editorial Board of *Physics-Uspekhi* plans to reflect the celebration of the World Year of Physics on its pages. The corresponding materials will come under the heading of Annus Mirabilis.

**V B Braginsky** M V Lomonosov Moscow State University, Department of Physics,
119992 Moscow, Vorob'evy gory, Russian Federation
Tel. (7-095) 939 55 65. Fax (7-095) 932 88 20
E-mail: brag@hbar.phys.msu.ru

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In the middle of the 1950s, a research field that was quite important for quantum measurements appeared. This research was (and is) aimed at understanding the properties of groups (ensembles) of photons and preparing such groups experimentally. Two years before Mössbauer's discovery, experimentalists Hanbury Brown and Twiss [4, 5] found out that photons emitted by a single source can be correlated. At the same time, Basov, Prokhorov, and Townes created a principally new clock standard, a microwave oscillator using ammonia molecules (maser), which was based on the effect of stimulated emission of photons. Five years later, Maiman created a similar oscillator in the optical range — a laser. In masers and lasers, large groups of photons are in quantum coherent states. Nowadays, lasers and masers of various types are widely used in many fields of physics research, as well as in technology. It must be noted that both lasers and masers are based on the effect predicted by Einstein in 1916 [6], namely, on the fact that a photon created by an atom via stimulated emission is an exact copy of the initial ('stimulating') photon.

During the last three decades of the last century, physicists working in quantum optics and quantum electronics predicted and discovered several types of photon group quantum states that are essentially different from coherent states. In particular, these are the so-called squeezed states (for instance, phase-squeezed, energy-squeezed, and quadrature-squeezed states). In such states, one of the observables is prepared with an uncertainty smaller than that of a coherent state, while the conjugate observable has a larger uncertainty than that of a coherent state. The energy-squeezed states are often called Fock states. In addition to these, other states have been discovered, for instance, states with bunching or antibunching of photons in space, as well as some 'exotic' states, e.g., with frequency-anticorrelated photons in an ensemble. Most of such states are described in modern textbooks (see, e.g., [7, 8]).

In these notes, the author does not aim to review all possible types of photon-based quantum measurements (for example, those where photons interact with an atom or a molecule) or quantum meters (starting from the Geiger counter or Wilson's chamber) that enabled the discovery of the world of elementary particles. The aim of these notes is actually quite limited — to describe the advances and some perspectives of quantum measurements where the object is a macroscopic body and the task is to measure one or several observables of this body by means of an indirect quantum measurement involving groups of photons. The existence of considerable sensitivity resources in various quantum measurements has already been demonstrated, using several examples, in a recent short review [9]. The present notes are offered to the reader as an extension of that review.

## 2. Quantum nondemolition measurements and the Bohr – Einstein dispute

A simple calculation [10] made in 1967 revealed the existence of quantum sensitivity limits for the measurement of observables of a macroscopic oscillator. In the example considered, the oscillator mass (or part of the oscillator) was one of the two mirrors of an optical Fabry – Perot (FP) cavity, the other mirror being fixed. If the cavity is coherently pumped at a near-resonance frequency, this optical meter can detect small variations of the mass coordinate by registering small changes in the power of light passing through the cavity. Calculations showed [10] that if one has to measure a variable force $F(t) = F_0 \sin \omega_m t$ acting, during a time $\tau > 2\pi\omega_m^{-1}$, on the oscillator at the frequency $\omega_m$ equal to its resonance frequency, the smallest possible value of $F_0$ that can be detected is

$$(F_0)_{\min} \simeq \frac{2}{\tau} \sqrt{\hbar m \omega_m}\,, \tag{2}$$

where $m$ is the oscillator mass.

The existence of limit (2) originates from the fact that a photon leaving the FP cavity and 'taking' the information about the mass coordinate is at the same time passing a momentum to the mass. Calculations showed that limit (2) can be achieved only for an optimal pump power $W_{\mathrm{opt}}$, which does not depend on $\hbar$ but depends on $\tau$, $m$, and the optical parameters of the FP cavity.

Similar calculations [11] showed that using the same method, one cannot measure the momentum of a free mass $m$ with an accuracy better than

$$\Delta P \geqslant \sqrt{\frac{\hbar m}{2\tau}}\,. \tag{3}$$

The uncertainty of the coordinate $x$ of the mass is then not less than

$$\Delta x \geqslant \sqrt{\frac{\hbar \tau}{2m}}\,. \tag{4}$$

Simple analysis [11] also shows that sensitivity limits (2) – (4) can be alternatively derived from the Heisenberg uncertainty relations if one takes into account that the measurement time $\tau$ is finite and the meter measures the coordinate. Following the suggestion of Thorne, such limits are called the standard quantum limits (SQLs).

Similar limits also exist for an electromagnetic cavity if the meter registers the coordinate (the electric field strength). Hence, there exists a 'family' of SQLs.

The recipe for designing a quantum meter whose sensitivity could overcome the SQLs was found rather soon after the publication of Ref. [10]. Several suggested versions included mechanical systems and electromagnetic cavities [12 – 5] for measuring observables corresponding to the integrals of motion [16], i.e., not coordinates. The simplest example is the measurement of the energy contained in an electromagnetic cavity. This energy can be measured without the demolition (destruction) of the photons inside the resonator, by registering the displacement of one of the cavity walls (which must be movable) due to the ponderomotive force. This displacement is proportional to the energy [12]. In practice, it is convenient to use two cavities, a signal one and a measurement one, whose electromagnetic fields have a partial overlap in a dielectric medium with nonzero cubic nonlinearity [17]. In this case, the eigenfrequency shift of the measurement cavity is proportional to the energy of the signal one. Such measurements are conventionally called quantum nondemolition measurements (QNDMs).

The suggested schemes for QNDMs [12 – 15, 17] were noticed by physicists working in quantum optics, and several groups demonstrated QNDMs in the optical frequency range (see review [18]). The resolution achieved in these experiments was several times better than the SQLs. Even more impressive was the result obtained by Haroche and his colleagues: they have built a counter of single (!) microwave photons in which the photons are not destroyed, as they are in the photoelectric effect [19]. QNDMs with macroscopic quantum objects still remain a challenge, although quite a number of various experimental schemes have been analyzed in the literature. They are reviewed in more detail in Section 3.

It is widely known that for many years Bohr and Einstein argued about the accuracy achievable in quantum measurements. Mostly, the dispute concerned the well-known relation between the error $\Delta\mathcal{E}$ in the energy measurement and the time interval $\tau$ of the measurement:

$$\Delta\mathcal{E}\,\tau \geqslant \frac{\hbar}{2}\,. \tag{5}$$

Historians of physics usually conclude that in this dispute, Bohr always found arguments in favor of his standpoint. In 1949, Bohr published quite a detailed paper [20] describing a gedankenexperiment (!) where relation (5) always holds. He suggested to weigh an electromagnetic cavity by means of scales (placed on the earth) and, during a finite time interval $\tau$ when the photons are inside the cavity, to measure the momentum $P_y = \mathcal{E} g\tau c^{-2}$ that the scales acquire in the vertical direction due to the weight of the photon energy $\mathcal{E}$. (Here, $g$ is the gravity acceleration on the earth.) The error in the measurement of the energy is then $\Delta\mathcal{E} = c^2(g\tau)^{-1}\Delta P_y$. But the measurement of $\Delta\mathcal{E} = c^2(g\tau)^{-1}\Delta P_y$ inevitably leads to the uncertainty $\Delta y = \hbar(2\Delta P_y)^{-1}$ in the vertical coordinate measurement. And because the frequency of the photons (oscillations in the electromagnetic cavity) in a gravitational field depends on the gravitational potential (a prediction of Einstein's GR theory!), one can easily see that relation (5) holds (see also [21]). The last point, most probably, provided an additional argument for Bohr.

It is worth noting that measurement of the electromagnetic energy of a cavity by determining the change in the cavity weight is actually a QNDM of the energy. Bohr probably paid no attention to this fact.

There is, however, a more important point. Namely, one can propose a different measurement procedure, also based on QNDM, which, at the same time, does not lead to formula (5) [22]. It follows from (3) that if the energy $\mathcal{E}$ of a free mass $m$ is measured from its momentum $P$ (i.e., the kinetic energy is measured) using a coordinate meter, the accuracy $\Delta\mathcal{E}$ is given by

$$\Delta\mathcal{E}_{\mathrm{SQL}} = \frac{(\Delta P)^2}{2m} = \frac{\hbar m}{2\tau}\,\frac{1}{m} = \frac{\hbar}{2\tau}\,, \tag{6}$$

in other words, we obtain the result in (5).

However, if $P$ is measured in a quantum nondemolition way, the resolution

$$\Delta P_{\mathrm{QNDM}} = \xi\Delta P_{\mathrm{SQL}} = \xi\sqrt{\frac{\hbar m}{2\tau}} \tag{7}$$

can be reached, where $\xi$ can be much less than unity.

Correspondingly, the kinetic energy $\mathcal{E}_{\mathrm{kin}}$ of the free mass is determined with the accuracy

$$\Delta\mathcal{E}_{\mathrm{QNDM}} = \xi^2\,\frac{\hbar}{2\tau}\,. \tag{8}$$

In other words, this second measurement procedure violates Bohr's rule (5). As a 'payoff' for this sensitivity benefit, one faces additional uncertainty of the mass coordinate, which is the larger, the smaller the chosen $\xi$:

$$\Delta x = \frac{1}{\xi}\sqrt{\frac{\hbar\tau}{2m}}\,. \tag{9}$$

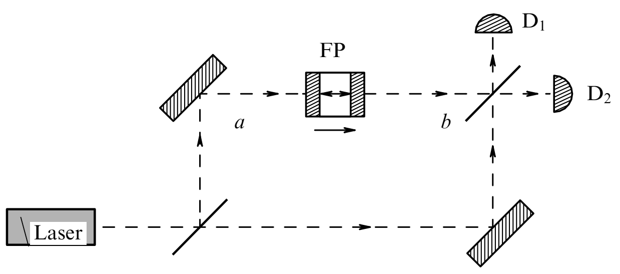


**Figure 1.** A scheme for the measurement of a free mass horizontal velocity.

Figure 1 shows a scheme for measuring the horizontal velocity component $v$ of a free mass $m$, which is a small optical FP cavity. The resonance frequency of the cavity is exactly equal to the frequency of the pump laser. We note that in the nonrelativistic case, both $v$ and $P$ are quantum nondemolition observables. The cavity is placed in one of the arms of a Mach–Zehnder interferometer. Clearly, at $v = 0$, the interferometer does not sense the position of the FP cavity within the interval $ab$. (The counting rates of the detectors $D_1$ and $D_2$ are independent of this position.) But if the mass (the FP cavity) is moving with a velocity $v$, then a phase shift

$$\Delta\varphi = \frac{v}{c}\,Q_{\mathrm{FP}} \tag{10}$$

appears between the interferometer arms, $Q_{\mathrm{FP}}$ being the quality factor of the FP cavity. This shift can be detected by measuring the difference in the counting rates of detectors $D_1$ and $D_2$. An accurate calculation for this scheme [23] gives the following value for the laser power $W_{\mathrm{SQL}}$ required for achieving the SQL:

$$W_{\mathrm{SQL}} = \frac{mc^2\omega_{\mathrm{opt}}}{16\pi Q_{\mathrm{FP}}^2} \simeq 4\times10^{12}\ \mathrm{erg\,s^{-1}} \times \frac{m}{1\ \mathrm{g}} \times \left(\frac{\omega_{\mathrm{opt}}}{2\times10^{15}\ \mathrm{s^{-1}}}\right) \times \left(\frac{Q_{\mathrm{FP}}}{10^{11}}\right)^{-2}. \tag{11}$$

This numerical estimate is based on the assumption that the size of $m$ is about 1 cm and the FP cavity mirrors have the best reflectivity achieved by now: $(1-R) = 10^{-6}$. It must be noted that the scheme requires a small 'addition': due to the passage of photons through the FP cavity, the mass acquires a small momentum, and hence a small velocity. This tiny velocity change should be taken into account or compensated.

If $v_{\mathrm{QNDM}} \simeq 0.1\,v_{\mathrm{SQL}}$ is to be measured with the help of this scheme, then either $W$ should be increased two orders of magnitude or mirrors with $(1-R) = 10^{-7}$ should be available. In addition to these very strict conditions, there are requirements, also very strict, for the quality factors of the internal mechanical modes of the FP cavity oscillations. These requirements are necessary for suppressing the phase fluctuations of the optical beam caused by thermal Brownian vibrations of the mirrors; see also [24].

Summarizing the discussion of this scheme, we must admit that with the present state of the art, this measurement procedure can only be viewed as a gedankenexperiment. In Section 3, much more practical schemes are considered.

As a conclusion, we can say that the Bohr–Einstein dispute ended in a draw: rule (5) indeed holds in many measurement procedures, but there nevertheless exist other procedures where it is violated.

## 3. Potential sensitivity resources in quantum nondemolition measurements of observable macroscopic objects

It is known that Einstein published the GR theory in 1916 [25]. Two years later, he discovered one of the GR solutions, the gravitational waves, which are emitted by nonuniformly accelerated masses [26]. Under earth-based laboratory conditions, this emission is rather weak. In the second half of the 1950s, Weber [27] formulated a task: in an earth-based laboratory, to create an antenna that would be capable of detecting gravitational-wave bursts coming from extremely strong astrophysical catastrophes. (In such catastrophes, a few percent of the total energy $mc^2$ of a star or several stars turn into a burst of gravitational radiation.) Such catastrophes are not frequent even in a single galaxy. Therefore, their signals can be expected to reach earth quite seldom, about once a month from distances larger than dozens of megaparsecs. A gravitational wave, i.e., a wave of the transverse acceleration gradient caused by a perturbation of the metric with the amplitude $h$, leads to a variable force $F_{\rm grav}$ acting on a test mass $m$ from another test mass placed at a distance $L$,

$$F_{\rm grav} = \frac{1}{2}\, hLm\omega_{\rm grav}^2\,, \tag{12}$$

where $\omega_{\rm grav}$ is the gravitational wave frequency.

In 1962, Gertsenshtein and Pustovoit [28] suggested using two freely suspended mirrors of an FP cavity as the test masses of such an antenna, such that small oscillations of one mirror with respect to the other due to the force $F_{\rm grav}$ can be registered by measuring the transmission of the cavity. The amplitude of these oscillations is

$$\Delta L = \frac{1}{2}\, hL\,. \tag{13}$$

This is the basic principle of several so-called laser gravitational-wave antennas (LIGO, VIRGO, Geo-600, TAMA). Preliminary laboratory experiments with models of antennas were started in the middle of the 1970s. At present, the sensitivity $h \simeq 10^{-21}$ has been achieved for one of the two LIGO antennas ($m = 2\times10^4$ g, $L = 4\times10^5$ cm, for $\omega_{\rm grav} \simeq 10^3$ rad s$^{-1}$). This sensitivity is sufficient for detecting a burst of gravitational radiation from two neutron stars merging at a distance about 10 Mpc from our galaxy (see recent reviews [29, 30]).

During the next five years, it is planned to replace all key elements of LIGO (the mirrors and their suspensions and systems of insulation from internal and external noises, and the pump laser), with the aim to reach $h \simeq 10^{-22}$. This value is close to the SQL for $h$:

$$h_{\rm SQL} = \frac{1}{L}\sqrt{\frac{8\hbar}{m\omega_{\rm grav}^2\tau}} \simeq 2\times10^{-23}\times\left(\frac{m}{10^4\ {\rm g}}\right)^{-1/2}\times \left(\frac{\omega_{\rm grav}}{10^3\ {\rm s}^{-1}}\right)^{-1}\times\left(\frac{L}{4\times10^5\ {\rm sm}}\right)^{-1}\times\left(\frac{\tau}{10^{-2}\ {\rm s}}\right)^{-1/2}. \tag{14}$$

In this section, we present some considerations showing that the threshold for detectable $h$ can be reduced and, in particular, $h_{\rm SQL}$ can be overcome. As we see in what follows, this requires a substantial reduction of losses in both the mechanical parts of the antenna (mirrors and their suspensions) and the optical elements of the meter.

The initial (simplest) example is the requirement that the center of mass of a mirror should be very well insulated from the external thermostat. Indeed, according to the fluctuation–dissipation theorem, the thermostat gives rise to the fluctuation force $F_{\rm therm}$, which is proportional to the friction coefficient. In other words, friction in the suspension should be sufficiently small to satisfy the condition $F_{\rm grav} > F_{\rm therm}$. If a mirror is considered as a point mass, then achieving $F_{\rm grav} = F_{\rm SQL}$ requires that

$$\frac{2kT}{\omega_{\rm grav}^2\tau_{\rm M}^*} < \hbar\,, \tag{15}$$

where $\tau_{\rm M}^* = m\,k_{\rm fr}^{-1}$ is the time of mechanical relaxation and $k_{\rm fr}$ is the friction coefficient.

About ten years ago, my colleagues Mitrofanov and Tokmakov succeeded in reaching a value of more than 5 years for $\tau_{\rm M}^*$. For the temperature $T = 300$ K (!) and $\omega_{\rm grav} \simeq 10^3$ s$^{-1}$, this value of $\tau_{\rm M}^*$ is sufficient to satisfy condition (15) and to overcome $h_{\rm SQL}$ by at least a factor of two [see Eqn (14)]. A detailed description of this research can be found in review [29] and the references therein.

From the same origin come the requirements for the quality factors $Q_{\rm M}$ of the internal mechanical degrees of freedom of a mirror. (Here, the mirror itself is considered a thermostat.) Calculations, which are omitted here, show that for achieving a sensitivity better than $h_{\rm SQL}$, it is necessary that $Q_{\rm M}$ exceeds $10^8$. This condition was recently satisfied by a group of researchers [31] who made models of mirrors using high-purity fused silica. It is noteworthy that this result was obtained with 'no consultancy' from the dissipation theory of solids. The only known fundamental 'obstacle' for increasing $Q_{\rm M}$ of mechanical oscillators is described in the model suggested by Barton [32], in which dissipation is caused by zero-point vacuum fluctuations. Still, the predicted limitation for $Q_{\rm M}$ in this model is of the order $10^{60}$.

The measurement system used in the two LIGO antennas operating now is in fact the usual optical FP cavity, which provides detection of small relative oscillations of the coordinates of the mirrors (with the amplitudes $\Delta L \simeq 2\times10^{-16}$ cm at frequencies within the range 50–1000 Hz). In other words, this measurement system is a coordinate meter, and hence its sensitivity cannot exceed $h_{\rm SQL}$. In the new version of LIGO, which has been mentioned above and which will be 'commissioned' in five years, no principal changes are planned, except for increasing the laser power and the main cavity relaxation time. Thus, the sensitivity will become ten times better than the present one but will still remain worse than $h_{\rm SQL}$.

At present, several research groups have developed various schemes of quantum 'readout' meters for the third stage of the LIGO project. These meters will provide sensitivities better than $h_{\rm SQL}$. In one such scheme, developed and analyzed by Thorne and my colleagues from MSU [33], measuring the velocity of one of the mirrors in the quantum nondemolition way is suggested. This will be done using two coupled microwave cavities based on ring 'whispering gallery' resonators made of $Al_2O_3$ monocrystals. The setup requires cooling to the temperature $T = 1$ K, which will provide the quality factor about $Q_{\text{microwave}} \simeq 5 \times 10^9$. In this case, calculation shows that $h \simeq h_{\text{SQL}}/3$ can be achieved. It is important that such a high quality factor (low dissipation) allow one to substantially reduce the pump power and hence to use cryogenic technology. We recall that the realization of this or a similar scheme will not only increase the sensitivities of gravitational-wave antennas but also allow performing the experiment mentioned in Section 2 in connection with the Bohr – Einstein dispute.

In the planned version of LIGO, the reflection coefficients of the mirrors will be $(1-R)=10^{-5}$. In the presence of additional mirrors, the corresponding relaxation time of the basic optical mode will be $\tau^* \simeq 1$ s. Under these conditions, the total energy accumulated in the basic mode will be $\mathcal{E} \simeq 4 \times 10^8$ erg (i.e., about $N \simeq 2 \times 10^{20}$ photons). Such a large number of photons are required because the expected sensitivity $h \simeq 10^{-22}$ corresponds to the phase shift $4 \times 10^{-10}$ rad; at the same time, the phase uncertainty for a coherent state, which is the state of the ensemble of photons in the cavity, is $\Delta\varphi \simeq (N)^{-1/2}$. It is worth noting that the measurement procedure destroys (in the same way as in Einstein's photoelectric effect) quite a small part of the total number of photons, because the interferometer is kept close to the 'dark field' regime. Therefore, almost all $2 \times 10^{20}$ photons pass through the cavity while only about $10^{10}$ are used (destroyed).

Clearly, it is not reasonable to further increase $\mathcal{E}$ and $N$ such that $h_{\text{SQL}}$ is overcome to a better extent. One of the obstacles here is laser breakdown: for the given value of $\mathcal{E}$ and the chosen size of the mode caustic, the electric field is $E \simeq 4 \times 10^3$ V cm$^{-1}$, which is not very far from the typical laser damage threshold $10^6$ V cm$^{-1}$.

There exists yet another approach to solving this problem, which is to combine QNDMs with the phase squeezing of the optical field. In this case, the maximum squeezing corresponds to the phase uncertainty $\Delta\varphi = (2N)^{-1}$ [34 – 37]. Hence, instead of $N \simeq 2 \times 10^{20}$ photons, only $N \simeq 10^9$ photons are required, i.e., the optical energy 'charges' can be reduced eleven orders of magnitude (!).

However, in order to obtain this energy benefit, one has to solve a certain problem. Namely, in a maximally phase-squeezed quantum state, the loss of a single photon leads to a phase perturbation of the state of the remaining photons by approximately $(2N)^{-1}$. It follows that if the measurement time for the gravitational-wave antenna is $\tau \simeq 10^{-2}$ s, then the relaxation time $\tau^*_{\text{FP}}$ of the FP cavity must be

$$\tau^*_{\text{FP}} \simeq \tau N \simeq 10^{-2}\ \text{s} \times 10^9 \simeq 10^7\ \text{s}\,. \tag{16}$$

For the best multilayer dielectric-coated mirrors available to experimentalists, $(1-R) \leqslant 10^{-6}$. This value corresponds to $\tau^*_{\text{FP}} \simeq 10^1$ s for LIGO. According to Macowsky, one of the manufacturers of such dielectric coatings, we can expect that mirrors with $(1-R) \simeq 10^{-9}$ will appear in the nearest future. This figure should provide $\tau^*_{\text{FP}} \simeq 10^4$ s. In other words, we are only three orders of magnitude 'apart' from the maximal phase squeezing. But even less phase-squeezed states will provide an energy benefit of several orders of magnitude.

In my view, a much more important problem for the experimentalists is to develop a method for preparing a maximally phase-squeezed state of electromagnetic radiation inside an FP cavity. As an example, one can consider the way phase squeezing is obtained for a mechanical oscillator [34]. The corresponding scheme (see Fig. 2) is based on the principle of 'zero detection'. If the coordinate of the oscillator mass differs from $x = 0$ by a value $\Delta x_{\text{YES}}$ that is much smaller than $\Delta x_{\text{SQL}}$, the detector should give the signal 'YES'. For all other values, the detector should give the signal

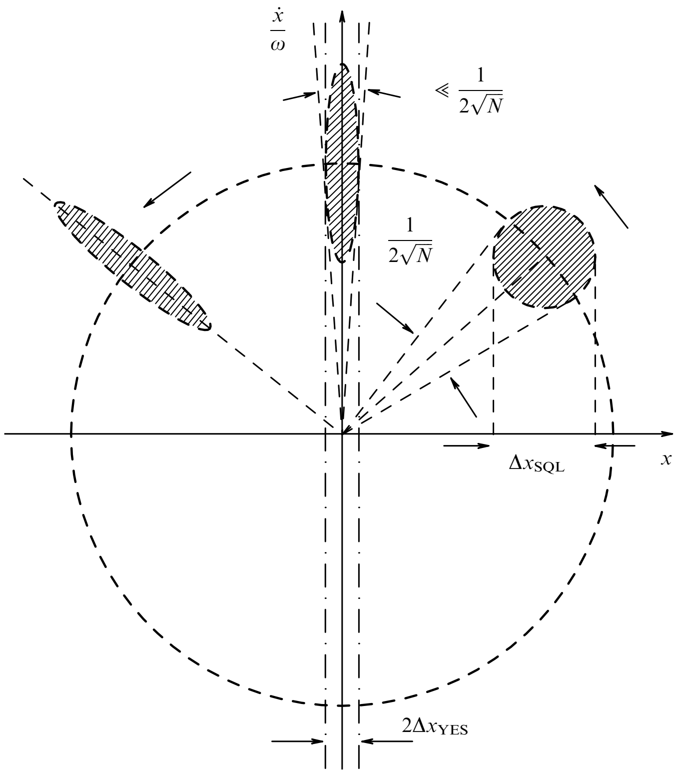


**Figure 2.** A scheme illustrating how phase squeezing can be obtained for a mechanical oscillator.

'NO'. This can be realized using an FP cavity with a small value of $(1 - R)$ and a nanogram mass $m$ of the mirror,

$$\Delta x_{\rm SQL} = \sqrt{\frac{\hbar}{2m\omega_{\rm M}}} = \sqrt{\frac{10^{-27}}{2 \times 10^{-9} \times 1}}\ {\rm cm}\ \simeq 7 \times 10^{-10}\ {\rm cm}\,,$$

$$\Delta x_{\rm YES} = \frac{\lambda(1-R)}{2\pi} = \frac{6 \times 10^{-5} \times 10^{-6}}{2\pi}\ {\rm cm}\ \simeq 1 \times 10^{-11}\ {\rm cm}\,. \tag{17}$$

For the given values of $\lambda$, $m$, and $\omega_{\rm M}$, a phase squeezing coefficient of 70 can be achieved. Figure 2 shows how a quantum state that is initially coherent becomes a phase-squeezed state. It is clear that the dynamical range for the coordinate $x$ is $\lambda/4 \simeq 1.5 \times 10^{-5}$ cm. Therefore, 'the only problem' for experimentalists is to find how this principle of preparing a phase-squeezed state can be transferred from mechanics to optics.

The examples given above do not exhaust the list of various suggestions aimed at overcoming the 'threshold' of $h_{\rm SQL}$. We mention an elegant principle proposed by Khalili [39]. It turns out that adding a relatively light mirror to the basic mirrors of the antenna and using the mechanical rigidity caused by optical fields (ponderomotive rigidity) allows obtaining an 'optical lever': the response of the small mirror has an amplitude larger than $hL/2$ (for more details, see [39]).

## 4. Conclusion

In fact, this paper has just given a brief review of developments in only one area of quantum measurements, a field started by Einstein's work [1]. The 'ultimate goal' of this area is to detect bursts of gravitational radiation, also predicted by Einstein [26]. Achieving this goal will mean opening a new information channel in astrophysics, which will inevitably lead to the discovery of novel elements of physical reality (see, e.g., the review by Thorne [40]).

It is worth mentioning that quantum nondemolition measurements are not the only way to overcome the SQL. Recently, Vyatchanin proposed a new principle of quantum variation measurements [41], which can be another tool for overcoming the SQL.

Indirect quantum measurements, where groups of photons are used as a 'probing instrument', are not the only possible way for quantum measurements. Indeed, in rather recent scattering experiments, Karlsson and Lovesey successfully prepared entangled states of neutrons with hydrogen isotopes with lifetimes $10^{-15} - 10^{-16}$ s [42] (see also [43]).

Quite recently, Krauss and colleagues [44] from the Max-Planck-Institute for Quantum Optics, Garching, developed a technique that allows optical pulses to be compressed to several femtoseconds (1 fs $= 10^{-15}$ s). In such pulses, even for a modest total energy $\simeq 10^5$ erg, the peak electric field amplitude exceeds the interatomic field. Under the action of such a pulse, an atomic beam generates short (attosecond, $10^{-18}$ s) pulses of X-ray radiation. We note that such optical pulses have allowed building a table-top proton accelerator with the energy 100 MeV.

The gap between the achieved time resolution $10^{-18}$ s and the Planck time interval $\tau_{\rm Pl} = \sqrt{\hbar G/c^5} = 5 \times 10^{-44}$ s can probably serve as a measure of the 'space' open for researchers of future generations with a keen interest in the problem of quantum measurements.

## References

1. Einstein A *Ann. Phys.* (Leipzig) **17** 132 (1905)
2. Compton A H *Phys. Rev.* **21** 483 (1923)
3. Bohr N *Philos. Mag.* **26** 1 (1913)
4. Hanbury Brown R, Twiss R Q *Nature* **177** 27 (1956)
5. Hanbury Brown R, Twiss R Q *Nature* **178** 1046 (1956)
6. Einstein A *Verhandl. Deutsch. Phys. Ges.* **18** 318 (1916)
7. Auletta G *Foundations and Interpretation of Quantum Mechanics* (Singapore: World Scientific, 2000)
8. Mandel L, Wolf E *Optical Coherence and Quantum Optics* (Cambridge: Cambridge Univ. Press, 1995)
9. Braginsky V B *Usp. Fiz. Nauk* **173** 89 (2003) [*Phys. Usp.* **46** 81 (2003)]
10. Braginsky V B *Zh. Eksp. Teor. Fiz.* **53** 1434 (1967) [*Sov. Phys. JETP* **26** 831 (1968)]
11. Braginsky V B, Vorontsov Yu I *Usp. Fiz. Nauk* **114** 41 (1974) [*Sov. Phys. Usp.* **17** 644 (1975)]
12. Braginsky V B, Vorontsov Yu I, Khalili F Ya *Zh. Eksp. Teor. Fiz.* **73** 1340 (1977) [*Sov. Phys. JETP* **46** 705 (1977)]
13. Thorne K S et al. *Phys. Rev. Lett.* **40** 667 (1978)
14. Braginsky V B, Vorontsov Yu I, Khalili F Ya *Pis'ma Zh. Eksp. Teor. Fiz.* **27** 296 (1978) [*JETP Lett.* **27** 277 (1978)]
15. Unruh W G *Phys. Rev. D* **18** 1764 (1978)
16. Golubtsova G A, Mensky M B *Int. J. Mod. Phys. A* **4** 2733 (1989)
17. Braginsky V B, Vyatchanin S P *Dokl. Acad. Nauk SSSR* **259** 570 (1981) [*Sov. Phys. Dokl.* **26** 686 (1981)]
18. Grangier P, Levenson J A, Poizat J-P *Nature* **396** 537 (1998)
19. Nogues G et al. *Nature* **400** 239 (1999)
20. Bohr N, in *Albert Einstein, Philosopher-Scientist* (The Library of Living Philosophers, Vol. 7, Ed. P A Schilpp) (Evanston, Ill.: Library of Living Philosophers, 1949); see also: Wheeler J A, Zurek W H (Eds) *Quantum Theory and Measurement* (Princeton, NJ: Princeton Univ. Press, 1983)
21. Braginsky V B, Khalili F Ya *Quantum Measurement* (Cambridge: Cambridge Univ. Press, 1992)
22. Braginsky V B *Proc. SPIE* **3516** 911 (1999)
23. Khalili F Ya, Levin Yu *Phys. Rev. D* **54** 4735 (1996)
24. Khalili F Ya "Quantum speedmeter and laser interferometric gravitational-wave antennae", gr-qc/0211088
25. Einstein A *Ann. Phys.* (Leipzig) **49** 769 (1916)
26. Einstein A *Sitzungsber. Preuss. Akad. Wiss.* **1** 154 (1918)
27. Weber J *Phys. Rev. Lett.* **21** 395 (1968)
28. Gertsenshtein M E, Pustovoit V I *Zh. Eksp. Teor. Fiz.* **43** 605 (1962) [*Sov. Phys. JETP* **16** 433 (1963)]
29. Braginsky V B *Usp. Fiz. Nauk* **170** 743 (2000) [*Phys. Usp.* **43** 691 (2000)]
30. Barish B C, Weiss R *Phys. Today* **52** (10) 44 (1999)
31. Ageev A et al. *Class. Quantum Grav.* **21** 3887 (2004)
32. Barton G *J. Phys. A: Math. Gen.* **24** 991 (1991)
33. Braginsky V B et al. *Phys. Rev. D* **61** 044002 (2000)
34. Braginsky V B, Khalili F Ya, Kulaga A A *Phys. Lett. A* **202** 1 (1995)
35. Collett M J *Phys. Scripta* **T48** 124 (1993)
36. "Quantum phase and phase dependent measurements: Special issue" (Eds W P Schleich, S M Barnett) *Phys. Scripta* **T48** (1993)
37. Freyberger M, Schleich W *Phys. Rev. A* **49** 5056 (1994)
38. Macowsky J M, private communication
39. Khalili F Ya *Phys. Lett. A* **298** 308 (2002)
40. Thorne K S, in *Three Hundred Years of Gravitation* (Eds S W Hawking, W Israel) (Cambridge: Cambridge Univ. Press, 1987)
41. Vyatchanin S P, Lavrenov A Yu *Phys. Lett. A* **231** 38 (1997)
42. Karlsson E B, Lovesey S W *Phys. Rev. A* **61** 062714 (2000)
43. Karlsson E B *Solid State Phenomena: as Seen by Muons, Protons, and Excited Nuclei* (Oxford: Clarendon Press, 1995)
44. Kienberger R et al. *Science* **297** 1144 (2002)