

REVIEWS OF TOPICAL PROBLEMS

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Particle self-action effects in a gravitational field

N R Khusnutdinov

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Abstract. **The particle self-energy and self-force in a gravitational field are considered. The particle self-action phenomena in a gravitational field of black holes, in spaces of both the infinitely thin and finite-thickness cosmic strings, as well as of a point global monopole are discussed. Some effects related to particle self-action are covered.**

1. Introduction

For the case of flat Minkowski space – time,[1] the phenomenon of self-action for electromagnetic particles has been thoroughly studied and is described in great detail in reviews and monographs (see, for instance, the books [1, 2] and the reviews [3, 4]). The origin of the particle self-force is related to the inertial properties of the electromagnetic field: radiation emitted by an accelerated particle carries away momentum, thus slowing down the particle. In other words, the self-force is the reaction of radiation. The covariant 4-force of the self-action of a particle with charge e and a 4-velocity u^{μ} in the Minkowski space – time is described by the well-known Dirac – Lorentz formula

$$F^{\mu}_{\rm DL} = \frac{2e^2}{3}\,\frac{D^2 u^{\nu}}{{\rm d}s^2}\,(\delta^{\mu}_{\nu} + u^{\mu}u_{\nu})\,, \tag{1}$$

where δ^{μ}_{ν} is the Kronecker delta. This force, which has been thoroughly studied in the relativistic and nonrelativistic regions [1, 2], may give rise to substantial effects. For instance, if a charged particle moves in an electromagnetic field, the self-force has a profound effect on its law of motion. This leads to a situation in which the particle energy, after the particle has flown through a region filled by an electromagnetic field, cannot exceed a certain threshold value determined by the electromagnetic field [2, § 76].

Here are the main features of the Dirac – Lorentz force (1) in flat space – time. First, the force is proportional to the derivative of the particle acceleration and is, therefore, zero for a particle at rest, moving uniformly, or having a constant acceleration. Second, the formula for the self-force (1) is valid for any trajectory followed by the particle and does not depend on the electromagnetic field.

The description of the self-action effect in general relativity becomes much more complicated, since in this theory any type of energy (including fields) generates a gravitational field. This makes it impossible to obtain in explicit form the Green function of an electromagnetic field in an arbitrary external gravitational field, and it becomes necessary to calculate the Green function for each configuration of the gravitational field. The local expansion of the Green function of fields in a gravitational field [5] shows that in addition to the standard terms there appear an infinite number of additional terms that depend on the external gravitational field. In space – time with an even dimension, the structure of the singularities of the Green function also changes: in addition to the standard pole part there appears a logarithmic divergence. Eventually this leads to a violation of the Huygens principle in the sense that a plane or spherical light wave propagating in curved space – time loses its shape and acquires 'tails' [6]. According to the principle of equivalence, a free particle as a local object tends to move along a geodesic line. The electromagnetic field linked to the particle is a nonlocal, extended object, for which a gravitational field is the media in which the field propagates and scatters. For this reason, in addition to the Dirac – Lorentz self-force (1) there appears an additional gravity-induced self-force which is related both to the electromagnetic field and to the gravitational field of the particle, since gravitational radiation also produces a reaction. The first to analyze electromagnetic self-force for a charged particle in an arbitrary gravitational field were DeWitt and Brehme [6], and later Hobbs [7, 8] added his own ideas to their analysis. The result was the following equation of particle motion with

[1] The system of units adopted in this review and the main quantities are defined in the Appendix.

N R Khusnutdinov Physics Department, Kazan' State Pedagogical University, ul. Mezhlauka 1, 420021 Kazan', Russian Federation
Tel. (7-8432) 92 53 73
Fax (7-8432) 92 42 69
E-mail: nail@theory.kazan-spu.ru

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allowance for the self-force:

$$m\,\frac{\mathrm{D}u^\mu}{\mathrm{d}s} = eF^{\mu\nu}_{\mathrm{ext}}u_\nu + \frac{2e^2}{3}\,\frac{\mathrm{D}^2u^\nu}{\mathrm{d}s^2}\,(\delta^\mu_\nu + u^\mu u_\nu) + \frac{e^2}{3}\,R^\nu_\beta u^\beta(\delta^\mu_\nu + u^\mu u_\nu) + e^2u^\alpha\int_{-\infty}^{s} f^{\mu}_{\cdot\alpha\beta'}u^{\beta'}(s')\,\mathrm{d}s'\,. \quad (2)$$

The first term on the right-hand side of equation (2) describes the electromagnetic Lorentz force with which the external field F_{ext} acts on the particle. The second term is the covariant-generalized expression for the Dirac – Lorentz force (1). The third term, introduced by Hobbs [7, 8], appears because of the local distribution of matter – energy. Finally, the last term depends on the entire prehistory of the charge's motion. The origin of this term is related to the scattering of the electromagnetic radiation by the gravitational field (on the curvature) and, as noted earlier, its emergence eventually leads to violation of the Huygens principle. The integrand $f^{\mu}_{\cdot\alpha\beta'}$, which cannot be obtained in general form, depends on the space – time curvature. Since, as we will shortly see, the function $f^{\mu}_{\cdot\alpha\beta'}$ satisfies the identity $u_\mu f^{\mu}_{\cdot\alpha\beta'} = 0$, expression (2) for the self-force retains the requirement that particle velocity be perpendicular to particle acceleration.

There are substantial differences between the gravity-induced self-force (2) and the Dirac – Lorentz self-force (1). First, the gravity-induced part in equation (2) generally contains not only a local term but also a nonlocal term that depends on the entire prehistory of the charge's motion. In other words, the gravity-induced self-force depends on both local and global properties of space – time. Second, it depends not on the derivative of acceleration but on velocity and, therefore, may be nonzero, even for a particle at rest.

Let us now briefly discuss the main ideas in deriving formula (2). The procedure is based on a method first proposed by Dirac [9] in deriving the particle self-force in flat Minkowski space – time. The initial equation is that of the balance of energy between the particle and the field. In the absence of external fields of nongravitational origin, the particle – field energy balance equation takes the form of a covariant law of conservation of the total energy – momentum tensor for a particle and a field:

$$T^{\mu\nu}_{;\nu} = 0\,, \quad (3a)$$

where the energy – momentum tensor $T^{\mu\nu} = T^{\mu\nu}_{\mathrm{mech}} + T^{\mu\nu}_{\mathrm{el}}$ has two components, namely, the mechanical

$$T^{\mu\nu}_{\mathrm{mech}}(x) = m_0\int g^{\mu}_{\cdot\alpha}\big(x, x(s)\big)\,g^{\nu}_{\cdot\beta}\big(x, x(s)\big) \times u^\alpha(s)u^\beta(s)\delta^{(4)}\big(x - x(s)\big)\,\frac{\mathrm{d}s}{\sqrt{-g}} \quad (3b)$$

and the electromagnetic

$$T^{\mu\nu}_{\mathrm{el}}(x) = \frac{1}{4\pi}\left(F^{\mu\alpha}F^{\nu}_{\ \ \alpha} - \frac{1}{4}\,g^{\mu\nu}F^{\alpha\beta}F_{\alpha\beta}\right). \quad (3c)$$

Here, $g^{\mu}_{\cdot\alpha'}(x, x')$ is the bivector of parallel transport along the geodesic line connecting the points x' and x.

Then this equation integrates along a world tube of radius ε and length $\mathrm{d}s$ surrounding the world line of the particle, and at the end of the calculations the tube's radius is turned to zero. Since the left-hand side of equation (3a) is a vector at point x, we must first transport in parallel the vector to a point lying on the world line inside the interval $\mathrm{d}s$, and only after that can we integrate the resulting biscalar. This procedure leads to the following equation of motion:

$$m_0\,\frac{\mathrm{D}u^\mu}{\mathrm{d}s}\,\mathrm{d}s = -\lim_{\varepsilon\to 0}\int g^{\mu\cdot}_{\cdot\alpha'}(x, x')T^{\alpha'\beta'}\,\mathrm{d}\Sigma_{\beta'}\,. \quad (4)$$

The left-hand side of the last relationship is connected with the mechanical part of the energy – momentum tensor (3b), while the electromagnetic part (3c) contributes to the right-hand side.

The first term in the expansion of the right-hand side of Eqn (4) in powers of ε diverges as $1/\varepsilon$. Since the structure of this term coincides with that of the left-hand side, this divergence can be removed by defining the 'observed' mass m of a particle as

$$m = m_0 + \lim_{\varepsilon\to 0}\frac{e^2}{2\varepsilon}\,. \quad (5)$$

The contribution that tends to infinity actually stands for the contribution from the infinite electromagnetic particle self-energy. The next terms, which are finite in the limit $\varepsilon \to 0$, lead to equation (2) in which the function

$$f_{\mu\nu\alpha'} = v_{\mu\alpha';\nu} - v_{\nu\alpha';\mu} \quad (6)$$

is expressed in terms of the bivector $v_{\mu\nu'}$ determined in turn by the Feynman vector Green function in the Hadamard form

$$G^{\mathrm{F}}_{\mu\nu'}(x, x') = \frac{1}{(2\pi)^2}\left\{\frac{\Delta^{1/2}}{\sigma + \mathrm{i}0}\,g_{\mu\nu'} + v_{\mu\nu'}\ln(\sigma + \mathrm{i}0) + w_{\mu\nu'}\right\}. \quad (7)$$

Here, $\sigma(x, x') = s^2/2$, s is the interval between the points x and x' along the shortest geodesic line connecting these points, $g_{\mu\nu'}$ is the bivector of parallel translation along this geodesic, and

$$\Delta(x, x') = \frac{\det\big(\sigma_{;\mu\nu'}(x, x')\big)}{\det\big(g_{\mu\nu'}(x, x')\big)}$$

is the Van Vleck – Morret determinant. The quantities $v_{\mu\nu'}$ and $w_{\mu\nu'}$ have no singularities in the coincidence limit as $\sigma \to 0$. We can directly express $f_{\mu\nu\alpha'}$ in terms of the retarded vector Green function in the following way:

$$f_{\mu\nu\alpha'} = 4\pi(G^{\mathrm{ret}}_{\nu\alpha';\mu} - G^{\mathrm{ret}}_{\mu\alpha';\nu})\,.$$

Such a representation is more preferable [10] because expression (7) for the Green function in the Hadamard form is reasonable only if defined locally. The origin of the nonlocal term in equation (2) is related to the fact that the retarded Green function obtained from formula (7) and needed in the calculation of the right-hand side of Eqn (4), namely

$$G^{\mathrm{ret}}_{\mu\nu'}(x, x') = \frac{\theta(x, x')}{4\pi}\left\{\Delta^{1/2}g_{\mu\nu'}\delta(\sigma) - v_{\mu\nu'}\theta(-\sigma)\right\},$$

contains both local and nonlocal contributions. Here, the function $\theta(x, x')$ is defined in such a way that it is equal to unity if the event x resides in the causal future of the event x', and is equal to zero otherwise; $\theta(x)$ is the ordinary step function.

A more thorough investigation of the self-action effect in a local frame of reference has been carried out by Hobbs [7], who removed some of the inaccuracies in Ref. [6] and obtained expression (2) with allowance for a local term proportional to the Ricci tensor. If one employs Einstein equations, this term can be expressed via the matter – energy local distribution.

The general conclusions concerning the structure of the gravity-induced self-force and the nonlocal term in it can be drawn for conformally flat spaces [8] and in the approximation of a weak gravitational field and the nonrelativistic motion of particles [11]. For conformally flat spaces, the bivector $v_{\mu\nu'}$ is the gradient of the scalar function: $v_{\mu\nu'} = \Phi_{,\mu\nu'}$. For this reason $f_{\mu\nu\alpha'} = 0$, and the nonlocal term vanishes, too. What remains is the local part of the particle self-force [8]. Calculations that rely on the approximation of a weak gravitational field and nonrelativistic particles were done by the DeWitts [11]. In such an approximation, the nonlocal equations of motion (2) become local. The reason is that retardation effects, which lead to nonlocality, in the case of nonrelativistic motion become unessential [11]. In this case, in addition to being subjected to the Dirac – Lorentz self-force (a dot over indicates a time derivative $\dot{\mathbf{v}} = \mathrm{d}\mathbf{v}/\mathrm{d}t$)

$$\mathbf{f}_{\mathrm{DL}} = \frac{2}{3}\, e^2 \ddot{\mathbf{v}}$$

[the nonrelativistic form of expression (1)], a charge is subjected to the additional gravity-induced self-force

$$\mathbf{f}_G = e^2 \int \frac{\mathbf{x} - \mathbf{x}'}{|\mathbf{x} - \mathbf{x}'|^4}\, \varrho(\mathbf{x}')\, \mathrm{d}^3 x'\,,$$

where $\varrho(\mathbf{x}')$ is the density of matter that generates the weak gravitational field. The complete equation of motion in this case has the form

$$m(\dot{\mathbf{v}} + \nabla U) = \mathbf{f}_{\mathrm{DL}} + \mathbf{f}_G\,,$$

where

$$U(\mathbf{x}) = -\int \frac{\varrho(\mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|}\, \mathrm{d}^3 x'$$

is the gravitational potential of matter.

2. Self-force and self-energy in the spaces of black holes

Further investigations dealt with specific spaces and trajectories of particles. A lot of work went into the study of self-action phenomenon in the Schwarzschild space – time:

$$\mathrm{d}s^2 = -\left(1 - \frac{2M}{r}\right)\mathrm{d}t^2 + \left(1 - \frac{2M}{r}\right)^{-1}\mathrm{d}r^2 + r^2(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\varphi^2)\,, \tag{8}$$

the Reissner – Nordstrom space – time:

$$\mathrm{d}s^2 = -\left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right)\mathrm{d}t^2 + \left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right)^{-1}\mathrm{d}r^2 + r^2(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\varphi^2)\,, \tag{9}$$

and the Kerr space – time:

$$\mathrm{d}s^2 = -\left(1 - \frac{2Mr}{\Sigma}\right)\mathrm{d}t^2 + \frac{\Sigma}{\Delta}\,\mathrm{d}r^2 + \Sigma\,\mathrm{d}\theta^2 - \frac{4JMr\sin^2\theta}{\Sigma}\,\mathrm{d}t\,\mathrm{d}\varphi + \sin^2\theta\left(r^2 + J^2 + \frac{2J^2Mr\sin^2\theta}{\Sigma}\right)\mathrm{d}\varphi^2\,. \tag{10}$$

Here, $\Sigma = r^2 + J^2\cos^2\theta$ and $\Delta = r^2 - 2Mr + J^2$, with M, Q, and J being the mass, charge, and angular momentum of the black hole, respectively. The element of length (8) describes a static massive (mass M) black hole formed as a result of the collapse of a massive body; formula (9) describes a massive charged (charge Q) black hole, and, finally, Eqn (10) describes a massive, stationary, rotating black hole with angular momentum J. The event horizon of the black hole (8) is the Schwarzschild radius $r_{\mathrm{S}} = 2M$, that of the Reissner – Nordstrom black hole is the quantity $r_Q = M + \sqrt{M^2 - Q^2}$, and the Kerr black hole possesses an event horizon $r_J = M + \sqrt{M^2 - J^2}$. In these spaces, the results of the DeWitts [11] cannot be applied directly, since the gravitational field is not weak everywhere.

Smith and Will [12] studied the self-force for a charge particle at rest in the Schwarzschild field. Initially, this problem was studied by Vilenkin [13] for particles far from a black hole. In this case, all the terms on the right-hand side of Eqn (2), except the last one, vanish; the particle is repelled from the black hole in the radial direction, and the self-force for a particle at a point with the coordinate r has the form

$$F^r_{\mathrm{em}} = \frac{r_{\mathrm{S}} e^2}{2r^3}\sqrt{1 - \frac{r_{\mathrm{S}}}{r}}\,, \qquad |\mathbf{F}_{\mathrm{em}}| = \frac{r_{\mathrm{S}} e^2}{2r^3}\,, \tag{11}$$

where $r_{\mathrm{S}} = 2M$ is the Schwarzschild radius of the black hole. This formula has been obtained both in the local and global approaches.

The local approach, similar to that used by DeWitt and Brehme [6], is based on calculating the density of the external force $\mathcal{F}^\mu$ needed to keep the particles in the Schwarzschild field in equilibrium. This density has the form of a 4-divergence:

$$\mathcal{F}^\mu = T^{\mu\nu}{}_{;\nu}\,, \tag{12}$$

where the energy – momentum tensor comprises two components: the mechanical (3b), and the electromagnetic (3c). Expression (12) for the force density is then integrated in the frame of reference of a freely falling observer across a sphere of radius ε surrounding the particle; at the end of calculations, the radius of the sphere is turned to zero.

When calculating the self-force, we must determine in explicit form the electromagnetic 4-potential of a particle in the Schwarzschild field, the potential depending on the particle's trajectory. Copson [14] found the electromagnetic potential for a particle at rest at point x_p [14]. The zeroth component of the vector potential has the following form

$$\begin{aligned} A_0^{\mathrm{part}}(\mathbf{x}, \mathbf{x}_p) &= \frac{e}{r_p r}\,\frac{(r - M)(r_p - M) - M^2\cos\chi}{R}\,, \\ R^2 &= (r - M)^2 + (r_p - M)^2 - 2(r - M)(r_p - M)\cos\chi - M^2\sin^2\chi\,, \end{aligned} \tag{13}$$

where $\cos\chi = \cos\theta\cos\theta_p + \sin\theta\sin\theta_p\cos(\varphi-\varphi_p)$. This potential behaves 'incorrectly' at infinity:

$$A_0^{\text{part}}\Big|_{r\to\infty} \approx \frac{e}{r}\,\frac{r_p - M}{r_p}\,.$$

At infinite distances from a black hole and a particle, space – time becomes plain and the electromagnetic potential of the particle must be expressed in the Coulomb form. Potential (13) is of the Coulomb type only if the particle is far from the black hole in an asymptotically plain region.

To solve this problem, Linet [15] proposed adding to expression (13) the solution of the homogeneous equation:

$$A_0^{\text{hom}} = \frac{eM}{rr_p}\,. \tag{14}$$

In this case, the total potential $A_0 = A_0^{\text{part}} + A_0^{\text{hom}}$, or

$$A_0(\mathbf{x},\mathbf{x}_p) = \frac{e}{r_p r}\,\frac{(r-M)(r_p-M) - M^2\cos\chi}{R} + \frac{eM}{rr_p} \tag{15}$$

takes the Coulomb form at infinity:

$$A_0\big|_{r\to\infty} \approx \frac{e}{r}\,, \tag{16}$$

while on the horizon of the black hole, with $r = 2M$, it is a constant quantity

$$A_0\big|_{r=2M} = \frac{e}{r_p}\,. \tag{17}$$

A charge located on the horizon of a black hole generates an electric Coulomb field

$$A_0\big|_{r_p=2M} = \frac{e}{r}\,. \tag{18}$$

Cohen and Wald [16], and Hanni and Ruffini [17] also obtained an expression for the potential A_0 of a charged particle at rest in the Schwarzschild field, in the form of an expansion in spherical harmonics and with allowance for the additional term (14).

To make our exposition complete, it may be beneficial to present appropriate formulas for other black holes. An expression for the electrostatic potential of a charged particle in the space – time of a Reissner – Nordstrom black hole (9) was derived by Leaute and Linet [18] (see also Ref. [19]):

$$A_0(\mathbf{x},\mathbf{x}_p) = \frac{e}{r_p r}\,\frac{(r-M)(r_p-M) - (M^2-Q^2)\cos\chi}{R_Q} + \frac{eM}{rr_p}\,,$$

$$R_Q^2 = (r-M)^2 + (r_p-M)^2 - 2(r-M)(r_p-M)\cos\chi - (M^2-Q^2)\sin^2\chi\,. \tag{19}$$

Clearly, this potential meets the same conditions (16) – (18) in which for the Schwarzschild radius we must take $r_Q = M + \sqrt{M^2-Q^2}$.

An expression for the electromagnetic potential of a particle located on the symmetry axis of a Kerr black hole (10) was derived by Léauté [20]:

$$\begin{aligned}
A_0(\mathbf{x},\mathbf{x}_p) &= \frac{e}{(r_p^2+J^2)\Sigma}\Bigg[(r_p r + J^2\cos\theta) \\
&\quad\times\left(M + \frac{(r-M)(r_p-M) - (M^2-J^2)\cos\theta}{R_J}\right) \\
&\quad + J^2(r - r_p\cos\theta)\,\frac{(r-M) - (r_p-M)\cos\theta}{R_J}\Bigg]\,,\\
A_\varphi(\mathbf{x},\mathbf{x}_p) &= -\frac{eJ}{(r_p^2+J^2)}\Bigg\{\frac{\sin^2\theta}{\Sigma}\Bigg[(r_p r + J^2\cos\theta) \\
&\quad\times\left(M + \frac{(r-M)(r_p-M) - (M^2-J^2)\cos\theta}{R_J}\right) \\
&\quad + J^2(r - r_p\cos\theta)\,\frac{(r-M) - (r_p-M)\cos\theta}{R_J}\Bigg] \\
&\quad - R_J + (r - r_p\cos\theta)\,\frac{(r-M) - (r_p-M)\cos\theta}{R_J} \\
&\quad - M(1-\cos\theta)\Bigg\}\,,\\
R_J^2 &= (r-M)^2 + (r_p-M)^2 \\
&\quad - 2(r-M)(r_p-M)\cos\theta - (M^2-J^2)\sin^2\theta\,.
\end{aligned}$$

If a charge is placed on the horizon of a Kerr black hole, i.e., if we put $r_p = r_J = M + \sqrt{M^2-J^2}$, the potential of the charge will have, in complete agreement with the electromagnetic potential outside a Kerr – Newman black hole (see, e.g., Ref. [21]), the following nonzero components

$$A_0 = \frac{er}{\Sigma}\,, \qquad A_\varphi = -\frac{eJr\sin\theta}{\Sigma}\,.$$

The use of the potential A_0 in the form (15) in equation (12) leads to a particle self-force (11). In deriving the expression for this force, we must perform an infinite renormalization of the particle mass, according to which the observed mass is given by the expression

$$m = m_{\text{bare}} + \lim_{\varepsilon\to 0}\frac{e^2}{2\varepsilon}\,.$$

The infinite term emerges from the electromagnetic part of the energy – momentum tensor.

The global approach is based on using the law of energy conservation: the work that a force does during virtual radial displacement of a charge is equal to the product of force by displacement. This energy is transferred to infinity, and an infinitely remote observer measures the change δM in the mass of the system, which can be calculated by Carter's formula [22] through the variation δA of the surface area of the horizon and the black hole's surface gravity κ:

$$\delta M = \frac{\kappa}{8\pi}\,\delta A - \frac{1}{8\pi}\,\delta\int G_0^0\sqrt{-g}\,\mathrm{d}^3x + \frac{1}{16\pi}\int G^{\mu\nu}\delta g_{\mu\nu}\sqrt{-g}\,\mathrm{d}^3x\,,$$

where $G_{\mu\nu}$ is the Einstein tensor. If the particle moves rather slowly, the energy is not carried over the horizon and $\delta A = 0$. The last term is also zero, since we are considering the motion of a *test* particle against the background of the Schwarzschild metric. Thus, there remains only one term which, using the

Einstein equations, we can rewrite as follows:

$$\delta M = -\delta \int T_0^0 \sqrt{-g}\, d^3 x\,.$$

Both approaches give the same result: the total force needed to keep the particle in the Schwarzschild field in equilibrium takes the form

$$F^r_{\text{em}} = \frac{r_S m}{2r^2} - \frac{r_S e^2}{2r^3}\sqrt{1-\frac{r_S}{r}}\,.$$

The ratio of the particle self-force (the second term) to the gravitational attractive force (the first term), namely

$$\delta = \frac{\lambda}{r}\sqrt{1-\frac{r_S}{r}}\,,$$

reaches its maximum at a distance $r_* = (3/2) r_S$ and has the maximum value

$$\delta = \frac{2}{3\sqrt{3}}\frac{\lambda}{r_S}\,,$$

where $\lambda = e^2/m$. When the Schwarzschild radius is critical, $r_S = 2\lambda/(3\sqrt{3})$, the self-force balances the gravitational force of attraction to the black hole.

When the electromagnetic field is replaced by the massive vector Proca field, the self-force only changes sign [13] but remains the same in value (11), and the particle will be attracted to the black hole. The reason is that the requirement that the energy–momentum tensor be finite on the horizon of the black hole leads to different boundary conditions for the 4-potentials of the massless electromagnetic field and the massive field [23, 24]. The energy–momentum tensor (3c) of the electromagnetic field contains only derivatives of the 4-potential, while the energy–momentum tensor of the massive Proca field, namely

$$T^{\mu\nu}_{\text{Pr}} = \frac{1}{4\pi}\left(H^{\mu\alpha}H^{\nu}_{\ \alpha} - \frac{1}{4}g^{\mu\nu}H^{\alpha\beta}H_{\alpha\beta} - \frac{m_B^2}{\hbar^2}\left[B^{\mu}B^{\nu} - \frac{1}{2}g^{\mu\nu}B_{\alpha}B^{\alpha}\right]\right),$$

has terms proportional to the square m_B^2 of the field mass and containing the 4-potential in explicit form, without derivatives. (Here, $H_{\mu\nu} = B_{\mu;\nu} - B_{\nu;\mu}$.) This implies that physically meaningful solutions of the Proca equations must, first, satisfy the boundary condition $B_0|_{r=2M} = 0$ on the horizon of the black hole [13, 23, 24] and, second, the invariant $B_{\mu}B^{\mu}$ must be finite on the horizon of the black hole. To satisfy these conditions, we must add the appropriate solutions of the homogeneous equation, which do not depend on the field's mass and which, eventually, change the sign of the particle self-force. Although the massless limit in the energy–momentum tensor has no singularities, this is not the case with the boundary conditions. For small values of the mass of the vector field, $(m_B/\hbar)^{-1} \gg M$, and at distances $M \ll r$ and $r_p \ll (m_B/\hbar)^{-1}$, the zeroth component B_0 of the vector potential of the Proca field assumes the following form [13]

$$B_0 \approx A_0 - \frac{2Me}{rr_p} = A_0^{\text{part}} + \frac{Me}{rr_p}\,,$$

where A_0 is the zeroth component of the electromagnetic potential discussed earlier [see equation (15)]. Thus, on the horizon of the black hole the potential obtained is zero: $B_0|_{r=2M} = 0$, and, in accordance with the 'Black Hole No Hair' theorem, a charge on the horizon of a black hole generates no field outside the hole, or $B_0|_{r_p=2M} = 0$. The above formula shows that the term that is the addition to Copson's solution is independent of the field's mass m_B and, in contrast to the electromagnetic case, has a different sign. At large distances from the black hole, the Proca field is described by the superposition of the fields of a point charge e and a charge $e' = -2Me/r_p$ beneath the horizon of the black hole:

$$B_0|_{r\to\infty} \approx \frac{e^2}{r} - \frac{2Me}{rr_p}\,.$$

The reason for such a difference between massive and massless vector fields lies in the gauge invariance of the latter. The potential of the electromagnetic field is unobservable because of the gauge invariance of the theory. The observables here are the electric and magnetic fields. On the other hand, the massive vector Proca field is not gauge invariant, so that the potential of this field is an observable. Using the gauge invariance of the electromagnetic field, we can always guarantee the 'correct' behavior of the potential at infinity and make the field energy density regular on the horizon. The requirement that the Proca field energy density be finite on the horizon together with the absence of gauge invariance of the theory leads to different boundary conditions at infinity, being independent of the field's mass. Such differences are closely related to the 'Black Hole No Hair' theorem. The electromagnetic potential generated by a particle residing on the horizon of a black hole is nonzero and is of exactly Coulomb form

$$A_0|_{r_p=2M} = \frac{e}{r}\,,$$

and, therefore, a charge crossing the horizon of a black hole leaves an electric 'hair' outside the hole. The situation is different in the case of a massive Proca field. A charge crossing the horizon does not leave 'hairs' in the outer region of the field:

$$B_0|_{r_p=2M} = 0\,.$$

In the case of a scalar particle with a scalar charge q and a minimally coupled massless field, the self-force for the particle at rest is zero [19, 25–28] both in a Schwarzschild field and in a Reissner–Nordstrom field. This is not the case, however, for a scalar particle moving in a Schwarzschild field [26] or at rest in a Kerr field [29, 30]. The equations of motion of a scalar particle are similar to the equation (2) of motion in electrodynamics [31]. The one thing that we must do is to replace e^2 with $q^2/2$ and

$$e^2 u^{\alpha} f^{\mu}_{\ .\,\alpha\beta'} u^{\beta'} \to q^2 (g^{\mu\nu} + u^{\mu}u^{\nu}) G_{;\nu}\,,$$

where G is the scalar Green function.

The scalar potential generated by a scalar charge q placed at the point x_p has the form

$$V^{\text{part}}(\mathbf{x}, \mathbf{x}_p) = -\frac{q}{R}\sqrt{\frac{r_p - 2M}{r_p}}\,. \qquad (20)$$

At large distances from a black hole ($r \gg r_p$), one finds

$$V^{\mathrm{part}}\big|_{r\to\infty} \approx -\frac{q}{r}\sqrt{\frac{r_p-2M}{r_p}}\,.$$

A scalar charge located on the horizon of a black hole does not generate a scalar field outside the hole: $V^{\mathrm{part}}|_{r_p=2M}=0$, in accordance with the 'Black Hole No Hair' theorem. This is due to the structure of scalar current [26]. The thing is that for a scalar charge (spin 0) at rest, the current density has one component proportional to $q/u^t = q\sqrt{g_{tt}}$, while the t-component of the electromagnetic current density (spin 1) is proportional to $eu^t/u^t = e$, and for the tt-component of the energy density of a tensor (rank 2) field we have $m(u^t)^2/u^t = m/\sqrt{g_{tt}}$. Here, g_{tt} is the tt-component of the metric.

The potential induced by the charge on the horizon of a black hole, namely

$$V^{\mathrm{part}}\big|_{r=2M} = -\sqrt{\frac{r_p-2M}{r_p}}\,\frac{q}{r_p-M-M\cos\chi}\,,$$

is not constant, as it is in the case of an electromagnetic field, but depends on the position of the charge. Eventually, the potential (20) leads to a zero self-force for a scalar particle at rest in a Schwarzschild field. If the scalar field is not minimally coupled, the self-force for a particle at rest is no longer zero [19, 28]. In this case, there is only the radial component of the force

$$F^r_{\mathrm{sc}} = \xi\,\frac{q^2 r_{\mathrm{S}}}{r^3}\sqrt{1-\frac{r_{\mathrm{S}}}{r}}\,,\qquad |\mathbf{F}_{\mathrm{sc}}| = \xi\,\frac{q^2 r_{\mathrm{S}}}{r^3}\,,$$

proportional to the nonminimal coupling constant ξ. Positive values of $\xi > 0$ correspond to the case where the particle is repelled from the black hole.

Parker [32, 33] calculated the self-force of an atom freely falling in the Schwarzschild field and found that the force is zero for an electroneutral atom. The explanation goes as follows. Earlier we stated that for the 'correct' behavior of the charge potential at infinity, we must add to Copson's solution A_0^{part} (13) the solution A_0^{hom} (14) of the homogeneous equation. Then the local expansion of the potential near the charge contains, besides the Coulomb part which can be obtained from expression (13), an additional term of the form $-eK_i\,\Delta x^i$, whose origin is related to the fact that the solution (14) of the homogeneous equation is added. Here, $K_r = 1/r_p^2$, while the other components vanish. This term is responsible not only for the emergence of the particle self-force but also for supplementary interaction with the nucleus. The self-force (repulsion) acting on the electron of the atom has the form $f_i = e^2K_i$, while the force of attraction to the nucleus with charge Z is $f'_i = -Ze^2K_i$. Thus, the total force acting on the charge equals $F_i = (1-Z)e^2K_i$ or, if the atom has Z' electrons, $F_i = (Z'-Z)e^2K_i$, and it is zero for an electroneutral atom. Thus, in an electroneutral atom the self-force (repulsion) acting on the electron clouds is balanced by the additional attraction to the nucleus.

Furthermore, Parker [33] found that the other components A_k of the vector potential also contain additional terms which near the charge have the form $-eL_{kj}\,\Delta x^j$, where the L_{kj} quantities form an antisymmetric matrix with constant coefficients. These terms lead to a situation in which at the point where the charge is located there emerges a magnetic field $B_i = -e\epsilon_{ijk}L_{kj}$ acting on the charge and determining a force that can be called self-torque, $f^t_j = -2e\mu_k L_{kj}$, where μ_k is the magnetic dipole moment of an electron. By analogy with the case examined earlier, we conclude that for an electroneutral atom the self-torque force acting on the electron clouds is balanced due to interaction with the nucleus by a force of the same magnitude. Thus, we conclude that no additional self-forces act on a freely falling electroneutral atom. It should be emphasized that the K_i and L_{kj} depend only on the global properties of space – time.

Recently, Linet [34 – 37] derived an expression for the self-action potential in spherically symmetric space – time with an element of length

$$\mathrm{d}s^2 = -N^2(r)\,\mathrm{d}t^2 + B^2(r)(\mathrm{d}r^2 + r^2\,\mathrm{d}\theta^2 + r^2\sin^2\theta\,\mathrm{d}\varphi^2)\,. \tag{21}$$

The electrostatic self-action potential of a particle at rest at point r_p in the space of a black hole with the surface gravity $\kappa = N'(r_{\mathrm{h}})/B(r_{\mathrm{h}})$ assumes the form

$$U^{\mathrm{em}}(r_p) = \frac{1}{2}\,e^2 s\big[a(r_p)\big]^2\,,\qquad s = \frac{1}{a(r_{\mathrm{h}})}\left(1-\frac{\kappa}{a(r_{\mathrm{h}})}\right), \tag{22}$$

where

$$a(r) = \int_r^\infty \frac{N(r)}{r^2B(r)}\,\mathrm{d}r$$

is the potential of a unit charge residing beneath the horizon, and r_{h} is the horizon radius of the black hole. The approach amounts to the following. The electrostatic potential generated by the charged particle that is outside the horizon of the black hole (21) is written down as the sum of the solution in the Hadamard form [in the case of a Schwarzschild black hole this is Eqn (13), and in the case of a Reissner – Nordstrom black hole, Eqn (19)] and the solution of the homogeneous Maxwell equation (equation (14) for a Schwarzschild black hole). The potential represents a symmetrical function in relation to the location of the charge and the point from which the field is observed. The solution of the homogeneous Maxwell equation is represented in the form $esa(r)a(r_p)$, where $a(r) = A_0/e$ is the potential generated by a unit charge beneath the horizon of the black hole (outside the black hole this is simply the centrally symmetric solution of homogeneous equations). The proportionality factor s can be found from Gauss's theorem stating that the flux of a field through a closed surface gathering round the charge equals $4\pi e$. Computations of the energy for such a configuration with allowance for the infinite renormalization of mass lead to expression (22).

Calculations of this type have been done in connection with estimation of the upper limit on the entropy of a black hole [37 – 40], with Zaslavskii being the first to make such estimates [41]. The thing is that when a charged particle crosses the horizon of a black hole, the self-energy is also absorbed by the hole, which should lead to a shift in the upper limit on the black hole's entropy. The following restriction on the entropy of a black hole has been obtained:

$$S \leqslant 2\pi ml + \frac{\pi e^2}{\kappa}\big(sa(r_{\mathrm{h}})-1\big)\,a(r_{\mathrm{h}})\,, \tag{23}$$

which with allowance for formulas (22) yields

$$S \leqslant 2\pi\left(ml - \frac{1}{2}\,e^2\right), \tag{24}$$

where m, l, and e are the mass, radius, and charge of the object, respectively. It is interesting that the known estimate (24), according to which the upper limit on the entropy is independent of the parameters of the black hole, is a corollary of the more profound formula (23) in which the upper limit depends both on the parameters of the black hole and on the self-energy.

The self-energy and self-force of an electromagnetic particle and a scalar particle in the space of a charged Reissner – Nordstrom black hole (9) were studied by Zel'nikov and Frolov [19]. They found that the force has only a radial component and derived expressions of the self-forces of an electromagnetic particle:

$$F^r_{\rm em} = \frac{r_{\rm S} e^2}{2r^3}\sqrt{1 - \frac{r_{\rm S}}{r} + \frac{Q^2}{r^2}}\,, \qquad |\mathbf{F}_{\rm em}| = \frac{r_{\rm S} e^2}{2r^3}\,,$$

and a scalar particle:

$$F^r_{\rm sc} = \frac{2\xi q^2}{r^3}\sqrt{M^2 - Q^2}\sqrt{1 - \frac{r_{\rm S}}{r} + \frac{Q^2}{r^2}}\,,$$
$$|\mathbf{F}_{\rm sc}| = \frac{2\xi q^2}{r^3}\sqrt{M^2 - Q^2}\,.$$

The total energy of a particle at rest in the field of such a black hole, namely

$$E_{\rm em} = m_{\rm em}\sqrt{1 - \frac{r_{\rm S}}{r} + \frac{Q^2}{r^2}} + \frac{e^2 r_{\rm S}}{4r^2}\,,$$

is the sum of the rest energy of the particle and its self-energy. The infinite electromagnetic self-energy of the particle is removed by the procedure of classical renormalization of mass:

$$m_{\rm em} = m_{\rm bare} + \frac{e^2}{2\varepsilon}\,.$$

For a scalar particle we obtain in analogous fashion:

$$E_{\rm sc} = m_{\rm sc}\sqrt{1 - \frac{r_{\rm S}}{r} + \frac{Q^2}{r^2}} - \frac{\xi q^2\sqrt{M^2 - Q^2}}{r^2}\,,$$

where

$$m_{\rm sc} = m_{\rm bare} - \frac{q^2}{2\varepsilon}\,.$$

Gal'tsov [29, 42] calculated the particle self-force in the field of a rotating black hole with the Kerr metric for massless fields of spin 0, 1, and 2, while Leaute and Linet [30] calculated the same in the particular case of an electromagnetic field and a particle resided on the symmetry axis. Gal'tsov's computing method in Ref. [29] is based on using the radiative Green function (the half-difference of the retarded and advanced Green functions), which is the part of the Green function describing radiation. He showed that the self-force acting on a particle at rest in the Kerr field has azimuthal components

$$F^{\rm sc}_\varphi = -\frac{1}{3}\,Jq^2M^2\,\frac{\sin^2\theta}{r^4}\,,$$
$$F^{\rm el}_\varphi = -\frac{2}{3}\,Je^2M^2\,\frac{\sin^2\theta}{r^4}\,,$$
$$F^{\rm gr}_\varphi = -\frac{8}{5}\,Jm^2M^4\,\frac{\sin^2\theta}{r^6}\left(1 + \frac{3J^2}{4M^2}(5\sin^2\theta - 1)\right)$$

for a scalar particle, an electromagnetic particle, and a massive particle ($r \gg M$), respectively. Here, J is the angular momentum of the black hole, and q, e, and m are, respectively, the scalar and electromagnetic charges and the mass of a particle. A rotating black hole tends to move the particle in the direction of its rotation. Hence, the rotation of the black hole slows down due to the total angular momentum conservation law. This phenomenon, discussed earlier by Hawking and Hartle [43], became known as tidal friction.

A great many works based on different methods have been written about the gravitational self-force which emerges because of the reaction of gravitational radiation. An approach similar to the one proposed by DeWitt and Brehme was utilized by Mino et al. [44] in the first approximation in the test-particle mass, i.e., in the approximation of a weak gravitational field. The researchers obtained the following equation of motion of a particle with allowance for the self-force:

$$m\,\frac{{\rm D}u^\alpha}{{\rm d}s} = -m\bigg(\frac{1}{2}\,u^\alpha u^\beta u^\gamma u^\delta + g^{\alpha\beta}u^\gamma u^\delta - \frac{1}{2}\,g^{\alpha\delta}u^\beta u^\gamma - \frac{1}{4}\,u^\alpha g^{\beta\gamma}u^\delta - \frac{1}{4}\,g^{\alpha\delta}g^{\beta\gamma}\bigg)\psi_{\beta\gamma;\delta}\big(x(s)\big)\,, \qquad (25)$$

where

$$\psi_{\mu\nu}\big(x(s)\big) = \mp 2m\int_{\mp\infty}^{s_z}{\rm d}s'\,v_{\mu\nu\alpha'\beta'}\big(x(s);x(s')\big)\,u^{\alpha'}(s')\,u^{\beta'}(s')\,, \qquad (26)$$

with the upper and lower signs corresponding to the retarded and advanced boundary conditions, respectively. The quantity $\psi_{\mu\nu}$ describes the 'tail' part of the perturbation of the metric $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$:

$$\psi_{\mu\nu}(x) = h_{\mu\nu}(x) - \frac{1}{2}\,g_{\mu\nu}(x)h(x)\,,$$

the perturbation induced by a point particle moving along a trajectory $z^\alpha(s)$. The complete solution is the sum of the 'tail' part $\psi_{(v)\,\mu\nu}$ and the local part $\psi_{(u)\,\mu\nu}$. As in the case of the electromagnetic self-force (2), the 'tail' part of the perturbation of the metric is determined by the logarithmic part of the Green function

$$G_F^{\mu\nu\alpha'\beta'}(x,x') = \frac{1}{(2\pi)^2}\left(\frac{u^{\mu\nu\alpha'\beta'}}{\sigma + {\rm i}0} + v^{\mu\nu\alpha'\beta'}\ln(\sigma + {\rm i}0) + w^{\mu\nu\alpha'\beta'}\right).$$

For the Schwarzschild space – time, there is also a different method of calculating self-forces, developed in Refs [45 – 51]. The method is based on renormalization of each term in the orbital angular momentum expansion of the potential. The renormalization amounts to subtracting the necessary number of terms in the asymptotic expansion at high orbital angular momenta.

Recently, Quinn and Wald [10] and Quinn [31] developed an axiomatic approach. The basic 'axiom of comparison' [10] can be formulated as follows. The difference in the self-forces of two particles carrying equal charges e and having the same accelerations is the ordinary Lorentz force generated by the (appropriately defined) difference in the electromagnetic fields of the particles. Actually, such an axiom is needed to avoid divergence, commonly removed by classical renormalization of mass. The results obtained by such an approach agree with those of DeWitt and Brehme [6], and Mino et al. [44]. Note, however, that the methods used in Refs [6, 10, 31, 44] make it possible to obtain only the general structure of the particle self-force, which nevertheless is important for understanding the nature of self-action phenomenon.

3. Self-force and self-energy in topological defect spaces

In this section, we discuss the self-action effect in spaces of topological defects. Most fully topological defects, their emergence, evolution, and interaction are described in the monograph [53] and the reviews [53, 54]. The aspects that have been studied so far are those of self-action in the space of an infinitely thin cosmic string [53, 55], in the space of a cosmic string with a finite cross section [56, 57], and in the space of a point global monopole [58].

We begin with the space – time of an infinitely thin cosmic string [53, 55] with the metric

$$ds^2 = -dt^2 + dr^2 + \frac{r^2}{\nu^2}\, d\varphi^2 + dz^2\,, \qquad 0 \leqslant \varphi \leqslant 2\pi\,. \quad (27)$$

Such space – time is described by a singular curvature tensor [54]. The parameter ν is related to the linear mass μ of the string by the formula $\nu^{-1} = 1 - 4\mu$ [53]. This situation is of interest in that the space – time is locally flat and in consequence the emerging particle self-force is only determined by the singular structure of the manifold. Linet [59, 60] and Smith [61] were the first to examine the self-force in the space – time of an infinitely thin cosmic string, and both used the following approach.

Let us consider a particle that carries a charge e and is at rest at a distance r from the string. In this case, the equation for the zeroth component of the vector potential has the form

$$\Delta A^0 = -\frac{4\pi e}{\sqrt{g^{(3)}}}\, \delta(r - r')\, \delta(\varphi - \varphi')\, \delta(z - z')\,,$$

where (x', φ', z') are the coordinates of the particle, $g^{(3)} = r^2/\nu^2$ is the determinant of the three-dimensional spatial part of the string's metric, and $\Delta = g^{ik}\nabla_i\nabla_k$ is the three-dimensional Beltrami operator. Thus, the component A^0 is proportional to the scalar Green function of a three-dimensional Laplace operator in a conical space:

$$A^0(x) = 4\pi e G_\nu(x, x')\,.$$

The potential, energy, and force of self-action are determined through the limit of coincidence for the renormalized Green function as follows:

$$\Phi(x) = 4\pi e G_\nu^{\mathrm{ren}}(x, x)\,, \quad (28a)$$

$$U(x) = \frac{1}{2}\, e\Phi(x)\,, \quad (28b)$$

$$\mathbf{F}(x) = -\boldsymbol{\nabla} U(x)\,. \quad (28c)$$

For renormalization and removal of divergences it is enough to subtract from the exact Green function the Green function $G_{\mathrm{M}}(x, x')$ in the Minkowski space – time, which can be obtained from the Green function in the conical space by passage to the limit $\nu \to 1$:

$$G_{\mathrm{M}}(x, x') = G_{\nu=1}(x, x')\,.$$

Thus, one obtains

$$G_\nu^{\mathrm{ren}}(x, x') = G_\nu(x, x') - G_{\nu=1}(x, x')\,.$$

Since in our case space – time is flat everywhere, the Feynman Green function can be expressed in explicit form as [62]:

$$G_\nu(x; x') = \frac{\mathrm{i}\nu^2}{8\pi^2 r r'}\, \frac{\sinh \nu\eta}{\sinh \eta \left[\cosh \nu\eta - \cos(\Delta\varphi)\right]}\,, \quad (29)$$

where

$$\cosh \eta = 1 + \frac{1}{2rr'}\,(-\Delta t^2 + \Delta z^2 + \Delta r^2)\,,$$

and $\Delta x = x - x'$.

The square of interval (27) can be reduced by a simple transformation of the angular variable $\varphi = \nu\phi$ to the appropriate expression for the Minkowski space – time. Although formally the element of length in this case appears to be the element of length of the Minkowski space – time, all information about the presence of a string is contained in the boundary conditions: the Green functions are periodic in the angular variable ϕ with a period of $2\pi/\nu$. If we now go back to the variable φ, the Green functions are periodic in the variable φ with a period of 2π, but then the metric coefficients differ from such coefficients for the Minkowski space – time. For this reason, the resulting Green function differs from that in the Minkowski space – time.

Substituting Green function (29) into equations (28), we arrive at the following expressions for the particle self-energy and self-force:

$$U^{\mathrm{em}}(r) = \frac{e^2}{2r}\, L_0(\nu)\,, \qquad F_r^{\mathrm{em}}(r) = \frac{e^2}{2r^2}\, L_0(\nu)\,, \quad (30)$$

where

$$L_0(\nu) = \frac{1}{\pi}\int_0^\infty \frac{\nu \coth(\nu x) - \coth(x)}{\sinh(x)}\, dx\,. \quad (31)$$

Numerical calculations done by Smith [61] exhibit an almost perfect linear dependence of $L_0(\nu)$ on ν. For a small angle deficit, $|\nu - 1| \ll 1$, this dependence takes the form

$$L_0(\nu) \approx \frac{\pi}{8}\,(\nu - 1)\,, \quad (32)$$

with the result that the charged particle is repelled from the string in the transverse direction by a force

$$F_{\mathrm{r}}^{\mathrm{em}} \approx \frac{\pi e^2}{16 r^2}\,(\nu - 1)\,. \quad (33)$$

The electromagnetic self-force in the space of an infinitely thin string for an arbitrary particle trajectory has been studied in Refs [63 – 65]. The following approach was utilized in calculating the self-force. Consider a particle carrying a charge e and moving along a trajectory $x^\mu(\tau')$ with a 4-velocity $u^\mu(\tau')$. First, we derive an expression for the 4-potential A^μ generated by the particle at the point of observation x^μ (Fig. 1a). Next, we place a certain fictitious particle with charge e and velocity u^μ at this point and calculate the Lorentz 4-force $F^\mu = eF^{\mu\nu}u_\nu$ acting on it (Fig. 1b). Then we place the fictitious charge on the particle's

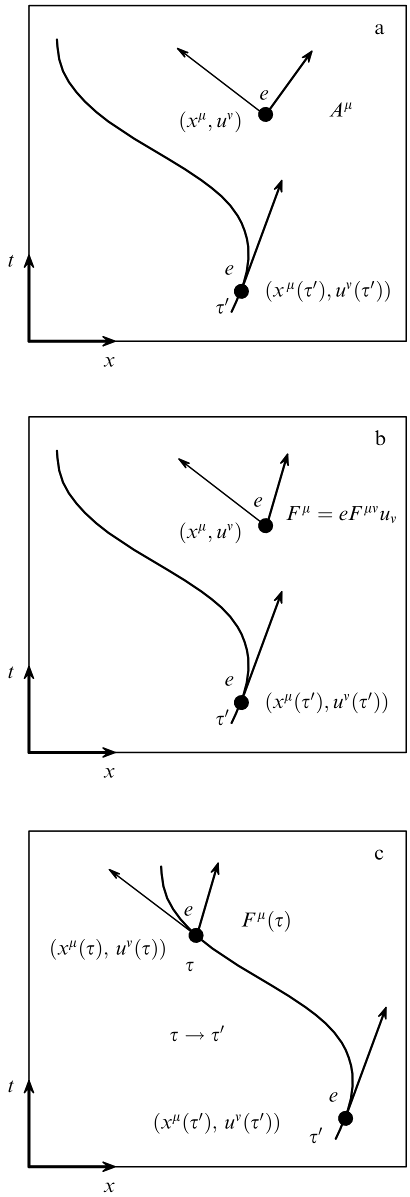

Figure 1. A schematic of the procedure for calculating the particle self-force. (a) First, we find the electromagnetic potential at the point where the test charge is located. (b) Then, we calculate the Lorentz force acting on the test charge. (c) Finally, we place a test particle on the mechanical trajectory, i.e., assume it is the initial particle at a later moment in time, and direct the two particles toward each other.

trajectory: $x^\mu \to x^\mu(\tau)$ and $u^\mu \to u^\mu(\tau)$. Thus, the fictitious charge constitutes simply the initial particle at a later moment τ in time. To find the self-force, we direct the two particles toward each other, i.e., we pass to the limit $\tau \to \tau'$ (Fig. 1c). A well-known fact of the general theory of Green's functions in curved space–time (e.g., see Ref. [5, p. 170 of the Russian edition]) is that the retarded Green function has two components: the local, and the nonlocal. The local component proportional to the delta-function $\delta(\sigma)$ of the square of the interval between the points $x^\mu(\tau)$ and $x^\mu(\tau')$ determines (after renormalization of mass) the local part of the self-force, i.e., the Dirac–Lorentz force and the matter–energy contribution [see Eqn (2)]. The nonlocal part of the Green function, which is proportional to $\theta(-\sigma)$, yields the nonlocal part of the self-force.

In the particular case of a particle at rest, it was found that the increase in the parameter $L_0(\nu)$ with the angle deficit ν is related to the increase in the number N of closed geodesics on the cone, whose number is the integer part of $\nu/2$:

$$N = \left[\frac{\nu}{2}\right]. \tag{34}$$

For the case of a supermassive string ($\nu \to \infty$, and $\mu \to 1/4$), the following estimate was given [65]:

$$L_0 \approx \frac{\nu}{\pi} \ln \frac{2\nu}{\pi}.$$

In this limit, the exact field equations lead to the metric of cylindrical space–time [66]. The case of supermassive cosmic strings has also been discussed in the context of topological inflation [67–73].

Linet [60] studied a more general situation and calculated the self-force for both an electrically charged particle and particles that are charges of massive scalar or vector fields. In such cases, the self-energy has the form

$$U(r) = \varepsilon \frac{q^2}{2r} L(\nu, mr),$$

where

$$L(\nu, mr) = \frac{\nu \sin(\pi\nu)}{\pi} \int_0^\infty \frac{\exp(-2mr\cosh x)}{\cos(\pi\nu) - \cosh(2\nu x)} \frac{\mathrm{d}x}{\cosh(x)},$$

q is the charge of the particle related to the scalar and vector massive or massless fields, and ε equals -1 for a scalar field, and $+1$ for a vector field.

Thus, a particle interacting with a scalar field is attracted to the string, while a particle interacting with a vector field (massive or massless) is repelled from the string. The difference is due to the form of the interaction term in the scalar field Lagrangian [74]. The particle self-energy in this situation is defined as

$$U(x) = \varepsilon \frac{q}{2} \Phi(x).$$

In the case of an electromagnetic field ($q = e$, $m = 0$, and $\varepsilon = +1$) we arrive at formula (30). Note that in deriving formulas (30) we must take into account an identity that holds for $\nu < 2$:

$$\int_0^\infty \frac{\nu \sin(\pi\nu)}{\cos(\pi\nu) - \cosh(2\nu x)} \frac{\mathrm{d}x}{\cosh(x)} = \int_0^\infty \frac{\nu \coth(\nu x) - \coth(x)}{\sinh(x)} \,\mathrm{d}x.$$

The particle self-energy for the case of massless fields is inversely proportional to the distance to the string [the first formula in Eqn (30)], while in the massive case [60] it is represented by the following exponential dependence

$$U(x) \approx \varepsilon \frac{m\nu q^2}{4\sqrt{\pi}} \cot\left(\frac{\pi\nu}{2}\right) \frac{\exp(-2mr)}{(mr)^{3/2}}, \qquad mr \gg 1.$$

The gravitational self-force for an uncharged test particle of mass m has been calculated by Smith [61] and Gal'tsov [75] in an approximation linear in the Newtonian constant of gravitation G. The researchers found that the particle is attracted to the string with a force

$$F_r^{\rm gr}(r) = -\frac{m^2}{2r^2}\,L_0(\nu)\,. \tag{35}$$

The opposite signs in formula (35) and in the expression for the self-force in the electromagnetic case [the second formula in Eqn (30)] can be explained by the fact that gravitational charges (masses) are always attracted, while electromagnetic like charges are always repelled.

The electromagnetic and gravitational self-forces for a particle at rest in the space–time of a cosmic dispiration (dislocation plus disclination) [76–78] were studied by De Lorenci and Moreira [79]. This space–time can be obtained from the Minkowski space–time by identifying the cylindrical coordinates according to the following rule

$$(t, r, \phi, Z) = \left(t, r, \phi + \frac{2\pi}{\nu}\,,\ Z + 2\pi\kappa\right).$$

The parameters ν and κ describe a disclination and dislocation, respectively. In the new coordinates $\varphi = \phi\nu$, $z = Z - \kappa\nu\phi$, the element of length takes the form

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + \mathrm{d}r^2 + \frac{r^2}{\nu^2}\,\mathrm{d}\varphi^2 + (\mathrm{d}z + \kappa\,\mathrm{d}\varphi)^2\,,$$

$$z \in (-\infty, +\infty)\,, \qquad 0 \leqslant \varphi \leqslant 2\pi\,.$$

The researchers arrived at the following expressions for self-energy:

$$U^{\rm em} = \frac{e^2}{2r}\,L(\nu,\kappa)\,, \qquad U^{\rm gr} = -\frac{m^2}{2r}\,L(\nu,\kappa)\,,$$

where the function $L(\nu,\kappa)$ is expressed in terms of the coincidence limit of the renormalized scalar Green function:

$$L(\nu,\kappa) = -\frac{\ln 2}{\pi} - 2\sum_{n=1}^{\infty}\int_0^{\infty}\frac{x^2 - \pi^2(4n^2/\nu^2 - 1)}{\left[x^2 + \pi^2(2n/\nu + 1)^2\right]\left[x^2 + \pi^2(2n/\nu - 1)^2\right]} \times \frac{\mathrm{d}x}{\sqrt{\cosh^2(x/2) + (\pi n\kappa/r)^2}}\,.$$

It should be emphasized that $L(\nu,\kappa)$ can be either positive or negative [79], i.e., the particle self-force may have different signs at different distances from the string. The natural scale of distance in this space is the dislocation parameter κ. As expected, in the case of an infinitely small disclination (or at great distances from the string), $\kappa/r \ll 1$, the above expression for self-energy is reduced to expression (31) for the space of an infinitely thin string: $L(\nu,\kappa) \to L_0(\nu)$. In the opposite situation where the disclination prevails (or at small distances from the string), $\kappa/r \gg 1$, we get

$$L(\nu,\kappa) \approx -\frac{\ln 2}{\pi}\,.$$

Thus, when disclination prevails or when the distances from the string are small, the self-energy is independent of the parameters of the string. The situation is exactly the opposite for an infinitely thin string; precisely, the electromagnetic self-force attracts the particle, while the gravitational self-force repels it.

Although the space–time of string is locally flat, the self-force can still be obtained by the method used by the DeWitts [11] in the case of weak gravitational fields. In our situation, the small angle deficit acts as the parameter characterizing the weakness of the gravitational field. Boisseau et al. [80] did such calculations and obtained the following formula (in the first approximation in the Newtonian constant of gravitation G):

$$U^{\rm em}(x) = \frac{\pi e^2}{4}\int\frac{2T_0^0 - T_1^1 - T_2^2 - T_3^3}{\rho(x,x')}\,\mathrm{d}^2x'\,, \tag{36}$$

where $\rho(x,x')$ is the Euclidean distance in the plane perpendicular to the string, and T_ν^μ is the string's energy–momentum tensor. For an infinitely thin string, when

$$T_0^0 = T_1^1 = \mu\delta^{(2)}(x')\,, \qquad T_2^2 = T_3^3 = 0\,,$$

equation (36) yields the well-known expression for the electromagnetic self-energy [see formula (33)]:

$$U^{\rm em} = \frac{\pi e^2\mu}{4r} = \frac{\pi e^2(\nu - 1)}{16r}\,.$$

The electromagnetic self-force in the space of many cosmic strings has been studied in Refs [81–87]. The results are valid for three-dimensional spaces with a metric $(a,b,c = 1,2)$

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + \gamma_{ab}(x^c)\,\mathrm{d}x^a\,\mathrm{d}x^b\,,$$

and for four-dimensional spaces with the interval

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + \mathrm{d}z^2 + \gamma_{ab}(x^c)\,\mathrm{d}x^a\,\mathrm{d}x^b\,.$$

The spatial part can be effectively reduced to two dimensions by a special choice of current density

$$J^\mu(x^c) = \left(J^t(x^c), 0, 0, J^z(x^c)\right).$$

Since two-dimensional Riemannian surfaces are conformally flat, we can introduce coordinates in terms of which one obtains

$$\gamma_{ab} = \exp\left[-\Omega(x^c)\right]\delta_{ab}\,.$$

Bezerra de Mello et al. [81] calculated the linear energy density $\mathcal{U}$ and the linear self-force density $\vec{\mathcal{F}}$ for a steady current flowing along the string through the point (x_1^1, x_1^2):

$$J^\mu(x^c) = (J^t, 0, 0, J^z)\,\frac{\delta(x^1 - x_1^1)\,\delta(x^2 - x_1^2)}{\sqrt{\gamma(x)}}\,.$$

The researchers also found that

$$\mathcal{U} = -\frac{1}{2}\,\Omega(J^{t2} + J^{z2})\,,$$

$$\vec{\mathcal{F}} = \frac{1}{2}\exp\left(\frac{\Omega}{2}\right)\boldsymbol{\nabla}\Omega(J^{t2} - J^{z2})\,.$$

For the multiconical space formed by N parallel strings, Staruszkiewicz [88], Letelier [89], and Deser et al. [90] arrived at the following expressions with the conformal factor

$$\Omega = \sum_{k=1}^{N} 2(1 - \nu_k^{-1})\ln|\mathbf{x} - \mathbf{x}_k|$$

for the particle self-energy and self-force:

$$\mathcal{U} = -(J^{t2} + J^{z2}) \sum_{k=1}^{N} (1 - \nu_k^{-1}) \ln |\mathbf{x} - \mathbf{x}_k| \,, \tag{37a}$$

$$\vec{\mathcal{F}} = \exp\left(\frac{\Omega}{2}\right)(J^{t2} - J^{z2}) \sum_{k=1}^{N} (1 - \nu_k^{-1}) \frac{\mathbf{n}_k}{|\mathbf{x} - \mathbf{x}_k|} \,, \tag{37b}$$

where

$$\mathbf{n}_k = \frac{\mathbf{x} - \mathbf{x}_k}{|\mathbf{x} - \mathbf{x}_k|} \,.$$

Thus, the self-energy (37a) is an additive quantity, while the self-force (37b) is not additive due to its dependence on the conformal factor.

In the particular case of a single string ($N = 1$), an expression for the self-force of the current flowing parallel to the string was obtained by Bezerra de Mello et al. [91]:

$$\vec{\mathcal{F}} = (J^{t2} - J^{z2})(\nu - 1) \frac{\boldsymbol{\rho}}{\rho^2} \,,$$

where $\rho = \nu |\mathbf{x}|^{1/\nu}$. Formally, this force coincides with the interaction force of two currents in the Minkowski space–time: J^μ, and the 'induced' current $J^\mu_{\text{ind}} = (\nu - 1) J^\mu / 2$. The sign of the force depends on the sign of the square of the 4-current: a spacelike current is attracted, while a timelike current is repelled.

In the case of a nontrivial inner structure of the current (the presence of nonzero moments), in addition to the induced current there will be induced moments [82, 83]. The self-energy of an electric dipole moment coincides with the interaction energy of the dipole $\mathbf{d}$ and the induced dipole moment

$$\mathbf{d}_{\text{ind}} = -\frac{1}{24} (\nu^2 - 1) \mathbf{d} \,.$$

For a magnetic moment $\boldsymbol{\mu}$ and a quadrupole moment D^{ab}, we have, respectively

$$\boldsymbol{\mu}_{\text{ind}} = -\frac{1}{24} (\nu^2 - 1) \boldsymbol{\mu} \,, \qquad D^{ab}_{\text{ind}} = \frac{1}{1440} (11 + \nu^2)(\nu^2 - 1) D^{ab} \,.$$

The inner structure of the current also leads to the emergence of a moment of self-force [83].

The above expression for the self-force diverges as the particle approaches the string, and the reason lies in the adopted cosmic string model. One can expect that the presence of a nontrivial inner structure will lead to a substantial modification of the picture in hand. Such calculations were done in Ref. [92] for a cosmic string with a constant substance density $\mathcal{E}$ inside it [56, 57]. The metric of the space–time of such a string is determined by the following relations

$$\mathrm{d}s^2_{\text{in}} = -\mathrm{d}t^2 + \mathrm{d}\rho^2 + \frac{\rho_\text{o}^2}{\epsilon^2} \sin^2\left(\frac{\epsilon\rho}{\rho_\text{o}}\right) \mathrm{d}\varphi^2 + \mathrm{d}z^2 \tag{38a}$$

in the inner ($\rho \leqslant \rho_\text{o}$) region of the string, and

$$\mathrm{d}s^2_{\text{out}} = -\mathrm{d}t^2 + \mathrm{d}r^2 + \frac{r^2}{\nu^2} \mathrm{d}\varphi^2 + \mathrm{d}z^2 \tag{38b}$$

in the outer ($r \geqslant r_\text{o}$) region, where ρ_o and r_o are the radii of the string in the respective coordinates.

The condition for the C^1-continuity of the metric coefficients on the string's surface leads to the following relationships

$$\frac{r_\text{o}}{\rho_\text{o}} = \frac{\tan\epsilon}{\epsilon} \,, \qquad \cos\epsilon = \frac{1}{\nu} \,.$$

The parameter ϵ is determined in terms of the 'energy' radius of the string, $\rho_* = 1/\sqrt{8\pi\mathcal{E}}$:

$$\frac{\rho_\text{o}}{\rho_*} = \epsilon \,.$$

Since in the absence of an angle deficit the particle self-energy and self-force disappear, it is convenient to single out the factor

$$\frac{e^2}{2r_\text{o}} \frac{\nu^2 - 1}{\nu} \,,$$

i.e., write down the self-energy U and the height $U_{\max}$ of the energy barrier of a charged particle in the following form:

$$U = \frac{e^2}{2r_\text{o}} \frac{\nu^2 - 1}{\nu} \mathcal{U}(\nu, R) \,, \tag{39a}$$

$$U_{\max} = \frac{e^2}{2r_\text{o}} \frac{\nu^2 - 1}{\nu} \mathcal{U}_{\max}(\nu) \,. \tag{39b}$$

The height of the energy barrier is determined by the value of self-energy at the center of the string, i.e., where this energy is at its maximum. The results of a numerical analysis of $\mathcal{U}(\nu, R)$ as a function of $R = r/r_\text{o}$ are shown in Fig. 2, and $\mathcal{U}_{\max}(\nu)$ as a function of ϵ in Fig. 3.

Figure 4 depicts the dependences of the self-energy of a charged particle on the proper distance D to the string, with the distance measured in units of the string radius r_o, for spaces formed by an infinitely thin string and a string with a finite cross section. At a distance equal to two string diameters, the energy values almost coincide. Only inside the string do these values differ significantly.

The self-force repelling the particle from the string has only a radial component. The quantity $\mathcal{F}_r$ as a function of R, defined by the expression

$$F_r = \frac{e^2}{2r_\text{o}^2} \frac{\nu^2 - 1}{\nu} \mathcal{F}_r(\nu, R) \,, \tag{40}$$

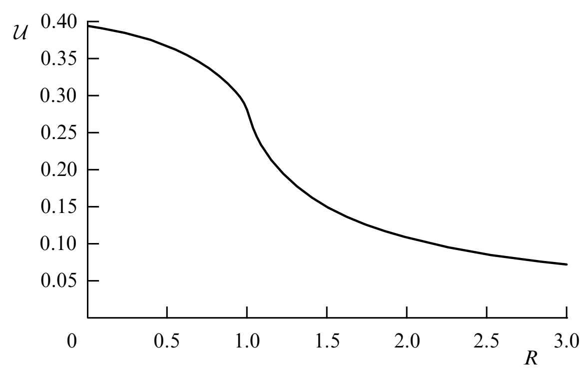

Figure 2. Dependence of the self-energy $\mathcal{U}$ on the particle position $R = r/r_\text{o}$. The cone parameter $\epsilon = 0.1$.

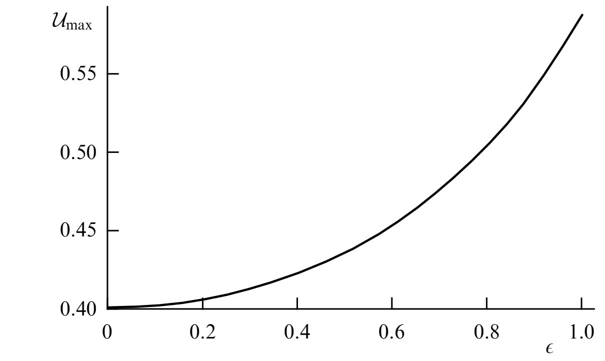

Figure 3. Dependence of the maximum self-energy $\mathcal{U}_{\max}$ (barrier height) on ϵ. For $\epsilon \leqslant 0.1$, the function $\mathcal{U}_{\max} \approx 0.39$.

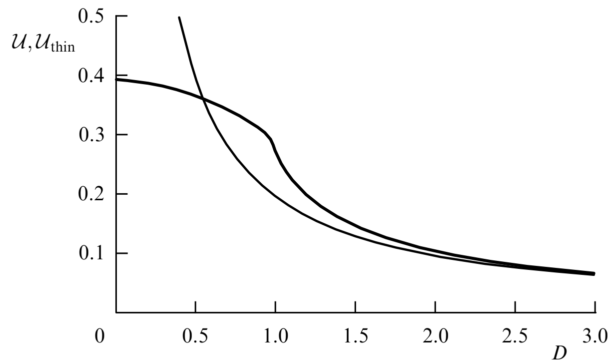

Figure 4. Self-energies of a particle in the space – time of a 'thick' cosmic string, $\mathcal{U}$ (heavy curve), and of a particle in the space – time of an infinitely thin string, $\mathcal{U}_{\text{thin}}$ (light curve), as functions of the proper distance D. The cone parameter $\epsilon = 0.1$.

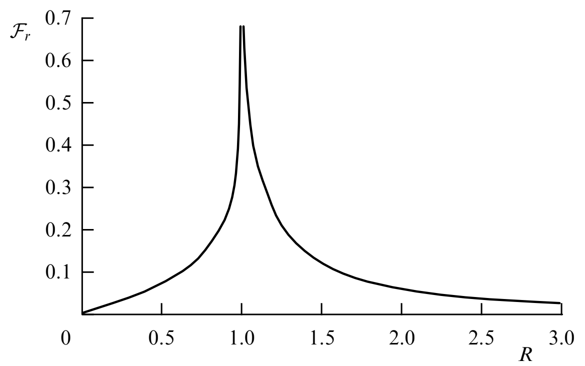

Figure 5. Self-force $\mathcal{F}_r$ of a charged particle in the space – time of a Gott – Hiscock string as a function of the particle position $R = r_p/r_{\text{o}}$. The cone parameter $\epsilon = 0.1$.

is depicted in Fig. 5. Near the string's surface, at $R \approx 1$, the force diverges logarithmically according to the law

$$F_r \approx -\frac{e^2}{2r_{\text{o}}^2}\,\frac{v^2-1}{8}\,\ln|R-1|\,, \qquad (41)$$

but the work against self-forces remains finite and is equal to the barrier height $U_{\max}$. Such divergence is probably caused by the chosen string model: the metric shows up as C^1-smooth, and the curvature experiences a discontinuity at the string's surface (inside the string, the space – time has constant curvature, while outside the string it is flat). The barrier height for a string with the Grand Unification parameters is $U_{\max} = 2.8 \times 10^5$ GeV.

Examining the problem of calculating the particle self-energy in the space – time of a string with a finite cross section is important in the context of the string catalysis of baryonic decay [93 – 99]. The point is that the self-force prevents the penetration of the inner region of the string by particles, and only in that region may baryonic decay take place. Perkins and Davis [98] studied this problem from the qualitative angle. A detailed description of a study of the electromagnetic self-force in the space – time of a string with a finite cross section can be found in Ref. [92]. The situation can be expected to be the same (qualitatively) with a string possessing a different inner structure.

The electromagnetic and gravitational self-action potentials for a charged particle in the field of a point global monopole with an element of length ($0 \leqslant \theta \leqslant \pi$ and $0 \leqslant \varphi \leqslant 2\pi$)

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + \alpha^{-2}\,\mathrm{d}r^2 + r^2(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\varphi^2) \qquad (42)$$

were studied by Bezerra de Mello and Furtado [100]. The parameter α is related to the scale η characterizing spontaneous symmetry breaking by the formula $\alpha^2 = 1 - 8\pi\eta^2$ [58]. This space is curved, in contrast to the space formed by an infinitely thin string. The approach used in the calculations was the same as in Refs [59 – 61]. The Green function was renormalized by subtracting the Green function of the Minkowski space – time. The resulting electromagnetic and gravitational self-action potentials have the following form:

$$U^{\text{em}}(r) = \frac{e^2}{2r}\,S(\alpha)\,, \qquad U^{\text{gr}}(r) = -\frac{m^2}{2r}\,S(\alpha)\,, \qquad (43)$$

where

$$S(\alpha) = \sum_{l=0}^{\infty}\left[\frac{2l+1}{\sqrt{\alpha^2 + 4l(l+1)}} - 1\right].$$

When the solid-angle deficit is small, $|1-\alpha| \ll 1$, the last expression yields

$$S(\alpha) \approx \frac{\pi}{8}(1-\alpha)\,.$$

Thus, as in the case of an infinitely thin string, the electromagnetic self-force repels the particle, while the gravitational self-force attracts it.

Earlier this method was employed in calculating the self-force of a particle in the field of a strong plane Bondi – Pirani – Robinson gravitational wave [101], which in group coordinates is described by the interval

$$\mathrm{d}s^2 = -2\,\mathrm{d}u\,\mathrm{d}v + A(u)(\mathrm{d}x^2)^2 + B(u)(\mathrm{d}x^3)^2 - C(u)\,\mathrm{d}x^2\,\mathrm{d}x^3\,.$$

The wave propagates with the speed of light along the x^1 axis; $u = (t - x^1)/\sqrt{2}$ and $v = (t + x^1)/\sqrt{2}$ are, respectively, the 'retarded' and 'advanced' times. Despite the fact that the Green function has a nonzero nonlocal component and the space – time is not conformally flat, the resulting self-force proves to be completely local for any particle's trajectory [102]. This can be explained by the structure of the nonlocal part of the Green function. The quantity $v_{\mu\nu'}$ in expression (6) depends only on one coordinate u and has the following structure: $v_{\mu\alpha'} \sim \delta^u_\mu \delta^{u'}_{\alpha'} v(u)$. Clearly, in this case $f_{\mu\nu\alpha'} = 0$ and, indeed, the nonlocal part of the self-force vanishes.

The gravity-induced self-force may be the cause of several interesting effects. Since in the string's space – time it is nonzero for a particle at rest and takes the form of the Coulomb interaction of the charge e with a fictitious charge $e' = eL_0(v)/2$ [see Eqn (30)] located on the string, particles are scattered by the string, with the scattering cross section being proportional to the Rutherford cross section (here, ε is the energy of the colliding particle):

$$d\sigma_{\text{self}} = \frac{1}{4} L_0^2(v)\, d\sigma_{\text{R}} = \frac{1}{4} L_0^2(v) \left(\frac{e^2}{2\varepsilon}\right)^2 \frac{\cos(\theta/2)}{\sin^3(\theta/2)}\, d\theta\,. \quad (44)$$

A similar effect occurs in the space – time of a global monopole, where in calculating the cross section we need only replace $L_0(v)$ with $S(\alpha)$ from formulas (43). Other types of cross sections have also been obtained (see Refs [95, 103, 104]).

Everett [103] calculated the scalar-particle scattering cross section per unit length of a string with a finite cross section r_{o}:

$$\frac{d\sigma}{d\theta\, dl} = \frac{\pi\hbar}{2p_\perp} \frac{1}{\ln^2(p_\perp r_{\text{o}}/\hbar)}\,. \quad (45)$$

Here, $p_\perp$ is the component of particle momentum that is transverse in relation to the string. This cross section emerges when the particles directly interact with a string having a nonzero cross section and is independent of the coupling constant. Everett [103] considered a model interaction of scalar particles with a scalar field inside the string. Perkins et al. [97] obtained a similar expression for the scattering of charged fermions interacting with the magnetic field inside the string. Using these results, Vilenkin [53] arrived at an expression for energy losses of a string moving with a velocity v through a medium:

$$\frac{dE}{dl\, dt} \approx \frac{\hbar n v^2}{\ln^2(p_\perp r_{\text{o}}/\hbar)}\,,$$

where n is the particle number density.

The cross section for particle scattering by a string, calculated by Alford and Wilczek [95] and de Sousa Gerbert and Jackiw [104] and found to be identical to the Aharonov – Bohm cross section [105]

$$\frac{d\sigma}{d\theta\, dl} = \frac{\hbar}{2\pi p_\perp} \frac{\sin^2(\pi\alpha)}{\sin^2(\theta/2)}\,, \quad (46)$$

leads to the following formula for energy losses:

$$\frac{dE}{dl\, dt} = \frac{2\hbar n v^2}{\sqrt{1-(v^2/c^2)}}\,,$$

where $\alpha = e\Phi/2\pi$ is the field flux in units of magnetic flux quantum.

The reason for such interaction consists in the following. As shown by Alford and Wilczek [95], the strings that appear in field models as a result of spontaneous symmetry breaking contain a magnetic field inside them, while in the outer region of the string the field is transferred into pure gauge without a magnetic field. Thus, the configuration of the fields is identical to that of a solenoid [105], which leads to such a formula for the scattering cross section.

Neither scattering cross section (45) or (46) is related to the conical structure of space – time; rather, both are caused either by the specific distribution of the fields generating the string or by the particle – field interaction inside the string. In the calculations of the above-mentioned cross sections, it was assumed that the space – time is of the Minkowski type. On the other hand, the origin of the scattering cross section (44) is directly related to the conical structure of space – time.

Acceleration caused by the particle self-force results, as is known, in emission of electromagnetic waves. Indeed, the self-energy of a particle in the field of a monopole can be expressed in the form of Coulomb interaction of a particle carrying a charge $e_1 = e$ with a fictitious particle carrying a charge $e_2 = eS(\alpha)/2$ located at the monopole center (43). For an infinitely thin string (30), the fictitious charge is $e_2 = eL_0(v)/2$. Coulomb-type interaction gives rise to bremsstrahlung [2]. For the ultrarelativistic case [2, § 74], we have the following expression for the energy emitted in the course of motion:

$$E = \frac{\pi e_1^4 e_2^2 \gamma^2}{4m^2\rho^3}\,,$$

where γ and ρ are the relativistic factor and the impact parameter, respectively.

At the same time, in the space – time of topological defects there exists another process of emission of electromagnetic waves, which is related to self-forces rather than to acceleration. As shown by a number of researchers (e.g., see Refs [106 – 112]), particles in the space – time of a cosmic string emit radiation even when they move along geodesic lines. Such a process is forbidden in Minkowski space – time but is allowed in the cosmic-string space – time. The researchers thoroughly studied these phenomena against the background of the space – time of an infinitely thin cosmic string. In Ref. [113], similar phenomena were studied for a charged particle in the field of a different topological defect, namely, a point global monopole described by the metric (42). For the ultrarelativistically moving particles, the energy emitted because of self-force is much lower than the energy emitted as a particle moves along a geodesic line [113].

Let us turn to the quantum phenomena associated with self-action effects. Several general remarks must be made if we are dealing with quantum phenomena in a gravitational field. A satisfactory theory combining the theory of relativity and quantum theory has yet to be created. Certain progress has been made in the theory of strings (e.g., see the review article by Marshakov [114]), where it is shown that in the low-energy limit we can arrive at Einstein's theory of relativity. For this reason, a semiclassical theory of gravitation is under intense development. Within this theory, all fields except gravitational ones are quantized fields (see the monographs [5, 42, 115 – 117]). The semiclassical approximation breaks down when the characteristic scale of a gravitational field,[2] l_{gr}, becomes comparable to the Planck length $l_{\text{P}} = \sqrt{G\hbar/c^3} \approx 10^{-33}$ cm [118]. On such scales, the full quantum theory of gravitation is required.

All the ideas expressed below concern the semiclassical theory of gravitation. From the viewpoint of quantum field theory, all matter fields must be quantized, i.e., representable

[2] The characteristic scale of a gravitational field is defined in such a way that the characteristic components of the curvature tensor obey the relationship $|R^\alpha{}_{\beta\mu\nu}| \approx l_{\text{gr}}^{-2}$.

by the assemblage of elementary excitations. The dynamics of these excitations is described by single-particle equations. The characteristic scale in these equations is the Compton wavelength of the particles, $l_C = \hbar/mc$. On scales larger than this length, we can speak of excitations as of particles. The requirement that the theory of relativity be generally covariant leads to a situation in which all the equations must be invariant under coordinate transformations belonging to the Poincaré group. By replacing partial derivatives with covariant derivatives, we can easily obtain covariant generalizations of the equations for the scalar and vector particles, while the Fock – Iwanenko coefficients make it possible to covariantly generalize the Dirac equation of a spin-1/2 particle. When the equations are covariantly generalized, we are still free to add terms that vanish as the gravitational field disappears. For instance, the requirement that the equations for a scalar particle be conformally invariant leads to a situation in which a term proportional to the scalar curvature R must be added to the equation [119].

If the characteristic scale on which the gravitational field varies is comparable to the Compton wavelength, so that $l_{\rm gr} \sim l_C$, the effect of pair production from a vacuum is made possible. In these conditions, the tidal interaction 'tears apart' a virtual pair which becomes a real pair. The best-known effect in the semiclassical theory of gravitation is the Hawking effect [120] which became known as 'evaporation of black holes'. After collapse has finished, there appears a uniform flux of particles with an effective temperature

$$T = \frac{\hbar\kappa}{2\pi c k_{\rm B}} ,$$

where $k_{\rm B}$ is the Boltzmann constant, and κ is the surface gravity. In the static case, one has

$$T = \frac{\hbar c^3}{8\pi G M k_{\rm B}} ,$$

where M is the mass of the black hole. Thus, a black hole is not really black — it emits radiation with a Planck spectrum.

When the radius of curvature of the gravitational field is large compared to the Compton wavelength of the particles, particle production can be ignored. A gravitational field manifests itself in that the vacuum averages of fluctuations of quantum fields become nonvanishing. This leads to a manifestation of the vacuum polarization effect [5, 42, 115 – 117]. Within the semiclassical theory of gravitation, all these phenomena must be taken into account by the Einstein equations. Instead of the classical energy – momentum tensor we must take the energy – momentum tensor with quantum corrections. The corresponding equations are known as the semiclassical Einstein equations. Effect of particle production becomes important when the gravitational field is strong, i.e., when the characteristic size of variation of the gravitational field becomes comparable to the Planck length. It must also be noted that the semiclassical theory of gravitation, built in this manner, is unrenormalizable. The counterterms needed in order to remove the emerging divergences have a structure that is quadratic in curvature, which does not agree with the structure of the initial Lagrangian. For this reason, the renormalized one-loop Einstein equations contain higher derivatives of the metric. A more profound treatment of these aspects can be found in the monographs [5, 21, 42, 115 – 117].

The gravity-induced self-force leads to various quantum phenomena. Since a particle carries additional self-energy even when at rest, we must take into account the effect of this energy on the particle's state. The additional self-energy may influence not only the states of scattered particles — it may even lead to bound states. The question of the presence of bound states in the spectrum of nonrelativistic zero-spin particles has been examined in Ref. [121], and for relativistic particles with spin 0 or 1/2 in Ref. [122].

Since the electromagnetic (33) and gravitational (35) self-forces have the same structure, we can examine their combined effect on a quantum particle. The presence or absence of bound states is determined by the sign of the quantity $Z = (e^2 - m^2)/\hbar$. For positive Z (when repulsive forces prevail over attractive forces), there are no bound states. Conversely, for negative Z (when attractive forces prevail over repulsive forces), say in the case of a massive uncharged particle, bound states exist. For a spin-1/2 particle, the spectrum determined by the localized solutions of the Dirac equation ($U = Zc\hbar/r$)

$$\left[\gamma^\mu(x)\widetilde{\nabla}_\mu + \frac{m}{\hbar} + U\right]\Psi = 0$$

takes the form

$$E_{N,M} = \pm\sqrt{m^2 + p_3^2}\left[1 - \frac{m^2}{m^2 + p_3^2}\,\frac{Z^2}{(N + \sqrt{\nu^2 M^2 + Z^2})^2}\right]^{1/2} , \tag{47}$$

where N is the principal quantum number, $N = 0, 1, \ldots$, and M is the total orbital angular momentum (half-integral), $M = n + 1/2$, $n = 0, \pm 1, \ldots$. The quantity p_3 is the longitudinal (along the string) component of the momentum. In the case of a spin-0 relativistic scalar particle, whose wave function satisfies the Klein – Gordon equation

$$\left\{\Box - (m + U)^2\right\}\Psi = 0 ,$$

the spectrum has a similar structure, but N must be replaced with $N + 1/2$, and M with n, $n = 0, \pm 1, \ldots$ [total orbital angular momentum (integral)]:

$$E_{N,n} = \pm\sqrt{m^2 + p_3^2} \times\left[1 - \frac{m^2}{m^2 + p_3^2}\,\frac{Z^2}{\left(N + 1/2 + \sqrt{\nu^2 n^2 + Z^2}\right)^2}\right]^{1/2} . \tag{48}$$

In the nonrelativistic limit $c \to \infty$ ($Z \to \infty$ and $m \to \infty$), we arrive at the following value for the energy of bound states (without the rest energy):

$$E_{N,n} = -\frac{m(e^2 - Gm^2)^2}{2\hbar^2\left(N + 1/2 + \nu|n|\right)^2} ,$$

in complete agreement with the results of Ref. [121].

At $p_3 = 0$, $\nu = 1$, and $Z = -Gmm_0$, the spectrum obtained coincides with the spectrum of the bound states of an electron with mass m placed in a Newtonian gravitational field of mass m_0 [123, 124]. Note the dramatic difference in the nature of spectrum (47) and that obtained in Refs [123, 124]. In the case of flat space, we have only attraction between the electron and the mass m_0, which leads to the bound states discussed in Refs [123, 124]. For this reason,

Z is always negative, $Z = -Gmm_0$. In the space–time of a cosmic string, as is well known (see Ref. [53]), there is no Newtonian interaction between the string and the particle. The origin of spectra (47) and (48) is related to a specific particle–string interaction, namely, self-action. Both the mass and the charge lead to such interaction. There are no bound states if the electromagnetic (repulsive) self-force prevails over gravitational (attractive) particle self-force. In the opposite case, bound states appear, since the total self-force becomes attractive.

Here are some numerical estimates. The energy scale of the spectrum obtained, which characterizes the distance between energy levels, is the quantity

$$E_{\rm scale} = Z^2 mc^2 .$$

For an electrically neutral particle, one finds

$$E_{\rm scale} = L_0^2 \left(\frac{Gm^2}{\hbar c}\right)^2 mc^2 = L_0^2 \left(\frac{m}{m_{\rm P}}\right)^4 mc^2$$
$$= 0.2 \times 10^{-96} \left(\frac{L_0}{L_0^{\rm GUT}}\right)^2 \left(\frac{m}{m_{\rm e}}\right)^5 \ [\text{eV}] .$$

Here, $L_0^{\rm GUT} \approx (\pi/8) \times 10^{-6}$ is the characteristic value of the parameter L_0 in the Grand Unification Theory, $m_{\rm P} = \sqrt{\hbar c/G}$ is the Planck mass, and $m_{\rm e}$ is the electron mass.

Thus, the spectrum obtained is practically continuous for elementary particles. One should expect a distinctive effect for very massive particles. For instance, the energy scale $E_{\rm scale}$ is 1 eV for a particle with the mass $m = 60 m_{\rm P} \approx 10^{-3}$ g.

4. Conclusions

Since the phenomenon of self-action of a particle in a gravitational field has been covered fairly thoroughly in this review, several quantitative aspects should be mentioned. The field of a point particle is a nonlocal object which 'feels' not only local properties of space–time but also its global properties. For this reason, the phenomenon of self-action of a particle in a gravitational field has several distinctive features. Below we list only the main features.

Self-force in general depends on the throughout history of the particle's motion. In some situations, the particle self-force is local, say in the case of a weak gravitational field, where retardation can be ignored, or on certain particle trajectories. It is noteworthy that the self-force may be finite even for a particle at rest. Qualitatively, this is quite understandable. If the gravitational field is inhomogeneous, then even for a particle at rest the effect of self-field on the particle is inhomogeneous, too. In the absence of a gravitational field, the origin of self-action is different: self-action is caused by the reaction of the radiation emitted by the particle. For such an effect to occur, the particle must be accelerated.

Particle self-force also emerges when the gravitational field is localized within an area (even of infinitesimal size), such as an infinitely thin cosmic string. It is interesting that here the Newtonian potential of the string is zero: the particle does not 'feel' the string, but still there occurs specific interaction carried by the nonlocal object, the particle's field.

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5. Appendix. Definitions of the main quantities and concepts

In the present review we adopted the so-called geometric system of units, in which the Newtonian constant of gravitation and the speed of light are set equal to unity: $G = c = 1$. Where it was necessary, we introduced these constants explicitly. The mass of a black hole is denoted as M, while m, e, and q stand for the mass, electric charge, and scalar charge of a particle, respectively.

The ***surface gravity*** κ of a black hole characterizes the strength of the gravitational field near the surface of the black hole. Here is the exact definition of κ [125]:

$$\kappa^2 = -\frac{1}{2} (\nabla^\mu \chi^\nu)(\nabla_\mu \chi_\nu) ,$$

where all quantities are calculated on the hole's horizon, and χ^μ is the Killing field normal to the horizon of the black hole. If we introduce the acceleration a^μ of the orbit χ^ν, namely

$$a^\mu = \frac{\chi^\nu \nabla_\nu \chi^\mu}{-\chi^\alpha \chi_\alpha} ,$$

we can rewrite the above definition of the surface gravity as follows:

$$\kappa = \lim\,(Va) ,$$

where $a = (a^\mu a_\mu)^{1/2}$, $V = (-\chi^\mu \chi_\mu)^{1/2}$, and lim stands for movement toward the horizon of the black hole. For the static case, the surface gravity can be shown to represent the force needed in order to keep a unit mass at rest on the horizon of the black hole. (Obviously, a local force a becomes infinite.) This explains the name chosen for κ. For the charged Kerr–Newman black hole, i.e., a black hole of mass M, charge Q, and angular momentum J, the surface gravity has the form

$$\kappa = \frac{(M^2 - J^2 - Q^2)^{1/2}}{2M\left[M + (M^2 - J^2 - Q^2)^{1/2}\right] - Q^2} .$$

Space–time is described by an element of length

$$\mathrm{d}s^2 = g_{\mu\nu}\, \mathrm{d}x^\mu\, \mathrm{d}x^\nu ,$$

where $g_{\mu\nu}$ is the space–time metric. The metric signature corresponds to the following choice of signs in the Minkowski metric: $\eta_{\mu\nu} = \mathrm{diag}\,(-1, +1, +1, +1)$. The indices denoted by the Greek letters $\alpha, \beta, \ldots$, incorporate time, in contrast to those denoted by Latin letters $i, j, \ldots$. The covariant derivative of the vector v^μ, denoted by a semicolon, takes the form

$$v^\mu_{;\nu} = \partial_\nu v^\mu + \Gamma^\mu_{\alpha\nu} v^\alpha ,$$
$$\frac{\mathrm{D}v^\mu}{\mathrm{d}s} = u^\nu \nabla_\nu v^\mu .$$

Here, $u^\nu = \mathrm{d}x^\nu/\mathrm{d}s$ is a tangent vector. A partial derivative is denoted by a comma: $\partial_\nu v^\mu = v^\mu_{,\nu}$.

A bivector of parallel translation $g^{\mu}_{.\alpha'}(x, x')$ is a two-point object, i.e., a vector defined both at the point x and at the point x' by the relationship

$$v^\mu(x) = g^{\mu}_{.\alpha'}(x, x')\, v^{\alpha'}(x') ,$$

where the vector v^{μ} at the point x is obtained from the vector $v^{\alpha'}$ at the point x' through parallel transport along a geodesic. The interested reader will find a detailed discussion of this object in the monographs [5, 116]. The need to use a parallel transport bivector in expression (3b) can be explained by the fact that only scalars can be integrated, while $g^{\mu}_{\cdot\alpha'}(x,x')\,u^{\alpha'}(x')$ is a scalar at the point x', and a vector at the point x.

The Riemann, Ricci, and Einstein tensors as well as the scalar curvature are defined in the standard manner:

$$R^{\alpha}_{\cdot\,\beta\gamma\delta} = \partial_{\gamma}\Gamma^{\alpha}_{\beta\delta} - \partial_{\delta}\Gamma^{\alpha}_{\beta\gamma} + \Gamma^{\alpha}_{\gamma\sigma}\Gamma^{\sigma}_{\beta\delta} - \Gamma^{\alpha}_{\delta\sigma}\Gamma^{\sigma}_{\beta\gamma}\,,$$

$$R_{\beta\delta} = R^{\alpha}_{\cdot\,\beta\alpha\delta}\,,\quad G_{\beta\delta} = R_{\beta\delta} - \frac{1}{2}\,g_{\beta\delta}R\,,\quad R = R^{\beta}_{\beta}\,.$$

A ***scalar field*** ϕ in the theory of relativity obeys the covariantly generalized Klein – Gordon – Fock equation

$$\left[\Box - \frac{m^2}{\hbar^2} - \xi R\right]\phi = 0\,,$$

where $\Box = g^{\mu\nu}\nabla_{\mu}\nabla_{\nu}$ is the four-dimensional Beltrami operator, and ξ is the nonminimal coupling constant. At $\xi = 1/6$, the field equation becomes conformally invariant, while at $\xi = 0$ the field is called minimally coupled.

If a massless scalar field has a source characterized by a charge q, it obeys the equation

$$[\Box - \xi R]\phi = -4\pi j\,,$$

where the scalar current has the form

$$j = q\int\delta^{(4)}\big(x - x(s)\big)\,\frac{\mathrm{d}s}{\sqrt{-g}}\,.$$

The 'Black Hole No Hair' theorem. What this theorem means is that after the collapse stage has been finalized, an external observer can measure only four quantities characterizing the black hole: mass M, electric charge Q, magnetic charge Q_{m}, and angular momentum J. Each of these quantities J A Wheeler aptly named 'hair'.

A massive body falling on a black hole disturbs the spherically symmetric Schwarzschild space – time, and the further evolution of the black hole is related to the emission of radiation modes. In Refs [126, 127] and [128, 129], it was shown that for electromagnetic and gravitational fields respectively, all radiation modes with orbital angular momenta higher than the field's spin are emitted. After all radiation modes have been emitted, the space – time again becomes the Schwarzschild space – time, but with a different mass. Later, this result was obtained in a different way in Refs [23, 24], and a detailed description of this problem can be found in the monograph [21]. At present, it is proved that the theorem is true not for all fields (e.g., see the review [130]). For instance, a self-consistent treatment of Yang – Mills fields and the Einstein equations has shown that a spherically symmetric solution of the equations is not only described by conserved quantities such as mass, charge, and angular momentum, but also characterized by the presence of a short-range external non-Abelian field.

Topological defects appear as a result of phase transitions in various field models. There are four types of defects: monopoles, cosmic strings, domain walls, and textures, as well as their hybrid compounds. Here, we will mention only the first two. Because of the unusual equation of state of matter inside these objects, space – time also acquires extraordinary properties. In the purest form this manifests itself when one considers structureless topological defects, i.e., a point monopole and an infinitely thin string. The space – time of a straight, infinitely thin cosmic string with the metric (27) is everywhere locally flat except for the string itself, where the curvature becomes delta-like:

$$R^{r\varphi}_{\cdot\cdot\,r\varphi} = R^{r}_{\cdot r} = R^{\varphi}_{\cdot\varphi} = (\nu - 1)\,\frac{\delta(r)}{r}\,.$$

Globally, space – time has a plane-angle deficit. The section $t = \mathrm{const}$ and $z = \mathrm{const}$ is a conical space. The strings that appear in the Grand Unification Theory, where all the known interactions are united into one interaction, have the following parameters: $r_{\mathrm{o}} \approx 10^{-29}$ cm, and $\nu - 1 \approx 10^{-6}$.

In contrast to the space – time of a cosmic string, the space – time of a point global monopole with the metric (42) is curved:

$$R^{\theta\varphi}_{\cdot\cdot\,\theta\varphi} = R^{\theta}_{\cdot\theta} = R^{\varphi}_{\cdot\varphi} = \frac{1 - \alpha^2}{\alpha^2 r}\,,$$

and has a solid-angle deficit. A detailed description of the theory of topological defects can be found in the monograph [52] and the reviews [53, 54].

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