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Fundamental physical constants: their role in physics and metrology and recommended values

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Abstract. **A brief review on recent CODATA recommended values of the fundamental physical constants, based on data obtained by the end of 2002, is presented. Preceding the review is an analysis of the role that fundamental constants play in fundamental physics and practical measurements. In view of the increasing role of fundamental constants for the realization of the standards of units of physical quantities, some questions related to the International System of Units (SI) are also discussed.**

1. Introduction

Recently, new "CODATA recommended values of the fundamental physical constants" obtained as a result of the adjustment of the values of basic physical constants have been presented to the scientific community [1]. The results may be of interest to the broader public. At the same time, a thorough exposition of the numerous technical details concerning the procedure of adjusting a diverse set of experimental and theoretical data on fundamental physical constants [1] would be of interest only to specialists. With all this in mind, I have compiled a review to explain the goals and strategy of such adjustment. Listed below are the new recommended values. I will also discuss the various aspects of the problem of fundamental constants.

Physics is a practical science and is based on measuring various physical quantities. Most of these quantities are dimensional, and we should agree on the units of physical quantities if we want to measure anything. At present, the majority of units of physical quantities are fixed by a special agreement known as the *International System of Units* (SI). This system has undergone several modifications, and its modern version is presented in Ref. [2].[1] There are also practical recommendations by a number of international committees on realizing the SI units, for instance, recommendations on realizing the meter [4], the ohm [5], and the volt [6, 7] (see also Ref. [8]).

In addition to SI units there are other, non-SI (or conventional) units, and only some of these, e.g., the unified atomic mass unit (with a unit symbol 'u'), are accepted for use with the SI.

To describe nature quantitatively is the goal of physics. Experimental data are used to create theories containing a small number of fundamental physical laws. Knowing the laws that describe natural phenomena is not enough — to arrive at quantitative results we must know the numerical values of the parameters that these laws contain. Fundamental constants play a key role among physical quantities

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[1] An earlier version of the *International System of Units* can be found in Ref. [3].

(both dimensional and dimensionless) that are of experimental or theoretical interest.

Fundamental constants enter equations from various areas of physics, demonstrating in this way their universal nature. In view of this, these constants form the main instrument that makes it possible to compare theory and experiment. The uncertainty of values of fundamental physical quantities determines the limits of theoretical calculations. A perfect illustration of this fact is provided by the highly accurate calculations in quantum electrodynamics (QED) — their capacity for prediction is limited by the accuracy to which the Rydberg constant R_∞, the fine-structure constant α, the electron – muon mass ratio m_e/m_μ, etc. are known.

In the process of perfecting our understanding of nature we realize that the theoretical descriptions of real physical phenomena are approximate. The extent to which the values of physical quantities are self-consistent points to the accuracy of theoretical concepts as a whole.

Refining the values of fundamental physical quantities is extremely important, not only to physics but also to metrology. The metrological importance is determined both by the role that fundamental constants play for the standards, and by the role that the standards play in determining the values of the constants. The speed of light c and the electric constant ε_0, which directly enter into the definitions of the SI base units, may serve as an illustration of the first point, while various determinations of the fine-structure constant and the Planck constant by 'electrical methods', whose realization in making electrical measurements requires high-standard accuracy, may serve as an illustration of the second point.

Several general remarks about metrology must be made when we take into account the metrological aspects of the problem. Very often it is erroneously believed that metrology is the science concerned with the measurements. Actually, metrology is not a science but a practical area of human activity that solves a great diversity of problems, scientific and legislative in particular.

Generally speaking, the standards of units are not the best 'devices' for reproducing the units. Standards (and calibration services, which make it possible to compare the standards with less accurate devices) are the elements of a legislatively adopted system of official measurements: certification, attestation, licensing, etc. From the scientific standpoint, the documents regulating the parameters of standards do not carry, and cannot carry, any special power, but legally these documents are more important than real characteristics. It may happen that a standard works better than official documents, but the opposite is also possible.

From the scientific viewpoint, traceability of measurements is needed so that we can compare the experimental data obtained in different laboratories, by different methods, etc., and in scientific work we have the right to error. Legally, a system of standards exists so that we can delegate responsibility for claiming that a particular product has the announced characteristics. In this case, if the application of an official document (say, the recommendations on reproduction of SI units) or the use of legally accepted standards leads to erroneous results, this may result in lawsuits and subsequent financial losses.

The Metric Convention and the International System of Units are characteristic products of the metrological community and, hence, amount not only to a system of units. They also incorporate many acts of law aimed at the implementation and domination of this system. For instance, the use of the competing centimeter – gram – second (CGS) system is not allowed in teaching. [2] At the same time, discussions about the merits and drawbacks of SI and CGS units are still raging, and there are still some consistent advocates of the CGS units (see Refs [9 – 11]). For a number of reasons, both practical and stemming partly from fundamental physics, I am a strong advocate of the SI units being the main system of units (see Section 4.2).

It must be noted, however, that the idea of a statutory ban on certain methods of teaching physics is unacceptable. Moreover, the SI units cannot be a success when we teach physics proper, since the system of units of physical quantities (in view of its practical importance) is adopted not only by physicists and not solely for physicists. Scientific applications play a marginal role in the usage of the SI units. In this context, the work of D V Sivukhin [10] published by a decision of the Bureau of the Division of General Physics and Astronomy of the USSR Academy of Sciences is characteristic. Some aspects related to the standard usage of the SI units are discussed in Section 6.

Fundamental physical quantities are of a universal nature and enter into a wide diversity of equations of physics and in interrelated combinations. Establishing the values of these constants is a nontrivial problem and is regularly done by the CODATA (Committee on Data for Science and Technology) Task Group on Fundamental Constants. CODATA is a committee of the International Council of Science, ICSU. The procedure for determining the most accurate self-consistent set of values of constants is called the least-squares adjustment of values of fundamental physical constants.

The essence of the work in adjustment is not the statistical processing of the available data but a critical understanding of the highly accurate results as the input data for the adjustment of constants. And all physical theories are approximate, while any experimental method is also based on approximations. The adjustment of values of fundamental constants is probably the only attempt to verify the meaningfulness and consistency of approximations as a whole.

Comparison of the accuracy of the modern values of fundamental constants helps in answering the question of what actually is measured in practice. In view of the practical nature of physics, while the questions of measurability and accuracy of measurements are in the shadow of the basic equations (such as, for example, the Maxwell equations), they still play a role that is no less important than the equations proper, since it is experiments that make physics what it is.

The present paper is devoted to a discussion of the most recent set of recommended values of fundamental constants [1]. These results are compared to their respective predecessors issued by CODATA in 1986 and 1998 [13, 14]. (Some details of earlier adjustments can be found in Refs [15, 16].) In view of the key role that some constants play in determining the size of the units, their recommended values also appear in publications of the corresponding Consultative Committees of the International Committee for Weights and Measures, CIPM. The CIPM recommendations are compared to the results of CODATA least-squares adjustment.

[2] Using the SI units in teaching is regulated not so much by legal documents pertaining to this system as by documents dealing with the educational system. The degree of independence of the educational system varies from country to country much more strongly than legislation concerning the usage of the SI units.

The various aspects of the problem of fundamental physical constants are also discussed in this paper. Among these are the origin of the constants, their role in physics and metrology, the extent to which they are fundamental and so forth. In particular, it examines the role of research in the field of fundamental constants for low-energy tests of QED and in creating a system of standards of the basic electrical quantities.

2. Fundamental constants in modern physics

2.1 Fundamental constants and various physical phenomena

The first truly universal parameters appeared in physics several centuries ago. It should be noted, however, that the idea of 'universality' has different meanings depending on whether we are interested in fundamental physics or in applied physics. From the standpoint of fundamental physics, one should speak about the basic laws of nature; from the practical standpoint more significant is the role that universal parameters plays in the most important measurements. For instance, in classical mechanics there were two types of problems: the mechanics of bodies on the Earth's surface, and the mechanics of heavenly bodies. In the first case, the acceleration of free fall g was an important 'constant'. In the second, the product of the Sun's mass by the Newtonian constant of gravitation G was probably more important than this constant proper, as were the ratios of masses of various nebular bodies (the Sun, the planets, and their moons).

The parameters of water (density, specific heat capacity, and boiling and freezing points) and the parameters of the Earth's atmosphere ('normal' atmospheric pressure) were also important constants. Constants describing an 'average' human being, the size of the Earth, the rotation of the Earth, and the Earth revolution around the Sun also emerged. They made it possible to introduce various 'natural' units (foot, meter, stadion, calorie, day, year, etc.) as well as the Celsius and Fahrenheit temperature scales.

In modern physics, the parameters of atoms act as constants of nature, and these can be used to define units and create standards. In classical physics, a similar idea, i.e., taking a certain class of objects as constants of nature, has a limited application. All characteristics for which metrological applications can be found emerge as the average properties of equivalent objects or as the properties of a unique object of artificial (the kilogram prototype) or natural (the Earth) origin.

Generally speaking, classical physics does not assume that all the atoms of an element are the same: they are the same on the average, and the differences can be ignored. (For instance, in some problems the presence of isotopes can be ignored.) Classical physics has no way of verifying that particles are 'identical'. However, quantum physics provides ways of doing this, namely, by using the interference between two or more particles. Here, we are not talking about a special experiment in which the interference between the particles is measured. Fermi statistics with its Pauli exclusion principle makes the Periodic Table of Elements one of the strongest pieces of evidence in favor of the fact that different electrons are identical, while the shell structure of nuclei is proof that protons are identical (and so are neutrons). Bose condensation observed in various systems convinces us of the identity of simple (and compound) Bose particles.

Thus, nonrelativistic quantum mechanics makes it possible to verify through experiments the identity of electrons, protons, and neutrons (within each class). If all matter consists of only three types of 'building blocks', atoms and molecules of one type are also identical. In turn, relativistic quantum mechanics (quantum field theory) presupposes the building of an exhaustive theory from a fairly small number of fundamental fields and answers the question of why all the building blocks are the same.

The thermodynamic properties of matter play an intermediate role. Ideally, they are governed by quantum physics but manifest themselves in the classical macroscopic world. Being the properties of macroscopic objects, thermodynamic parameters depend on the details of the sample: its size, shape, etc. Theoretically, this dependence can be limited or removed. However, here there is a more important problem, a technological one, having to do with impurities and defects which can be understood only within atomic theory.

Quantum physics is a more fundamental science than classical physics, with the result that it introduces units and constants that are more fundamental. The majority of classical units and constants can, at least theoretically, be derived from more fundamental units and constants. For instance, the mass of the prototype of the kilogram can be defined as the sum of the masses of the constituent particles (with a correction for the computable binding energy), and all these particles can be counted. Note that over the last decades such a project is actually being realized as a determination of the Avogadro constant, but the accessible accuracy is much lower than that corresponding to the artifact adopted as the prototype of the kilogram.

The Newtonian constant of gravitation G and the speed of light in vacuum c emerged as constants of classical physics, but their role goes beyond the limits of classical physics. Quantum physics adds the Planck constant h to the truly fundamental classical constants G and c and the properties of elementary objects (electrons, protons, and neutrons) and compound objects. (The best-known of these are atoms of caesium-133 and carbon-12, and the water molecule.)

From the fundamental standpoint, the properties of particles and space–time (G, h, and c) are, of course, fundamental to different degrees. However, the overwhelming number of measurements deal with common (atomic) substance around us, and so from the practical standpoint the properties of the simplest constituents of common substance are no less important (and maybe even more important) for physics than the constants G, h, and c. The electron and the proton (in ions) are agents that carry electric charge. The proton and the neutron are the main carriers of mass, and the properties of the electron fully determine the characteristics of most spectral lines.

The properties of the proton and the neutron, which are the building blocks of nuclei, also play their role in the case of hyperfine splitting, isotopic effects, and the rotational and vibrational spectra of molecules. The properties of the electron determine in the final analysis the characteristic binding energies and, hence, the most important thermodynamic properties (such as the temperatures of phase transitions). Understandably, in 'practical' physics some parameters of the three most important particles (the proton charge[3] e, the proton mass $m_{\rm p}$, the neutron mass $m_{\rm n}$, the

[3] Hereinafter, the elementary charge e is defined as the proton charge and, hence, is positive.

electron mass m_e, and the Rydberg constant R_∞) are extremely important, since they are involved in the various measurements of frequencies, wavelengths, electrical quantities, and masses.

While new principles of measurements were developed in the first half of the 20th century, in the second half progress largely amounted to increasing the accuracy and range of measurements. In particular, the development of QED, the first successful attempt to create a quantum field theory, was made possible by perfecting the measurements. The problem that confronted the builders of QED was that calculations on a perturbation theory led to divergent quantities. To overcome this difficulty, the renormalization procedure was introduced. The idea was to express observables (say, the electron energy levels in an atom) in terms of other observables, say the observed electron charge and mass. But in this case we find that the observed mass m_e and charge e of the electron are not the quantities m_0 and e_0 that enter into a fundamental equation (on the Planck scale) but are the results of the perturbation. If we want to find these fundamental quantities, we are forced to introduce a regularization procedure, i.e., we must redefine the theory at small distances in such a way that the 'naively' divergent results become finite.

The fact that the results of calculations for observables do not depend on the details of regularization is the most important achievement of renormalization, which guarantees the viability of this idea. The 'restored' initial fundamental parameters m_0 and e_0 depend on these details. We do not know how physics works at small distances and so do not know the 'truly fundamental' parameters of the electron. The parameters that we are forced to deal with are in a certain sense nothing more than effective parameters.

2.2 Fundamental constants and systems of units

Measurements amount to comparing two quantities of the same dimensions. Nevertheless, we must distinguish between relative and absolute measurements. In *relative* measurements, the compared quantities are in a certain sense contiguous. For instance, they may be two frequencies within a certain frequency range. In *absolute* measurements, one of the measured quantities must be related in one way or another to a universal agreement (e.g., the SI units). For instance, a relative measurement of the magnetic moment of a nucleus can be realized by comparing this moment to another nuclear magnetic moment. In an absolute measurement of such a moment, we must calculate or measure the magnetic field intensity in SI units, which, of course, is much more complicated and is substantially less accurate than relative measurements. The magnitudes with which we do the comparing are SI units. However, there exist a vast number of natural and practical units, and often the measurements are done in several stages: comparison of various practical units to SI units, and the measurement of the quantities of interest in the most appropriate practical units, which is close to relative measurements.

Initially, units emerged from the need to describe the world that surrounded us. This is the world of the macroscopic phenomena of classical physics and, as noted earlier, its characteristic natural constants are determined by the standard acceleration of gravity g, the properties of water, the 'average' human being, and the Earth as a planet. Meanwhile, fundamental phenomena occur on quite a different physical scale, the scale of atomic and nuclear physics and particle physics.

Preserving continuity, the SI does not change the magnitude of the units significantly, but it employs the phenomena of quantum physics to reproduce these units more successfully. Hence, one could expect that because the units emerged as macroscopic classical nonrelativistic units and their magnitudes have not since changed significantly, the numerical values of all the dimensional fundamental constants are much larger or much smaller than unity. This is true, however, only for mechanical units. There is one constant on the order of unity, the Rydberg constant (more exactly, the corresponding energy in electron-volts):

$$hcR_\infty \simeq 13.6\ \text{eV}\,. \tag{1}$$

Of course, the electron-volt is a non-SI unit, but the given numerical value corresponds to the ionization potential of the hydrogen atom:

$$I_\text{H} \simeq \frac{hcR_\infty}{e}\,\frac{1}{1+m_e/m_p} \simeq 13.6\ \text{V}\,, \tag{2}$$

which is measured in volts (an SI unit) and relates to the category of fundamental parameters describing the hydrogen atom no worse and no better than the Rydberg constant. Such a natural value (by order of magnitude) of the volt appeared because initially potential differences were generated by chemical reactions (or were compared to potentials needed for breaking bonds in atoms and molecules or exciting atoms and molecules).

The selection of practical units and the structure of building a system of standards are determined by expediency. Generally speaking, they may differ from each other or even not be obvious from the fundamental standpoint. Of course, they are also determined by our inertness and history, but the fact that physics began from classical and nonrelativistic physics is not only a historical fact (from the standpoint of fundamental physics) but also a clear indication of the practical importance of the corresponding type of measurement. For instance, the use of modern navigation systems on the Earth's surface, at sea, and in outer space clearly demonstrates the importance of precise measurements of common distances and time intervals at present as well. The shear volume of measurements of 'classical macroscopic nonrelativistic' quantities (with the wide use of quantum methods and with allowance, if necessary, for relativistic effects) exceeds many times over the volume of measurements of essentially quantum or essentially relativistic objects.

An example of expediency in building a system of units is the mole, the unit of amount of matter. From the standpoint of fundamental physics, there is no need to introduce a special dimension — we need only count the atoms. However, weighing atoms is simpler (for not very precise common applications) and more accurate (for single precision measurements). Hence, instead of speaking of the number of atoms and their macrovalue as a quantity that incorporates a fixed number of atoms, it is more advisable to speak of a definite mass of atoms (say, 0.012 kg) of a certain element (say, carbon-12).

Comparison of atomic (molecular) masses can be done fairly simply and reliably, as also can the weighing of macroscopic amounts of matter. The Avogadro constant which determines the number of atoms in one mole of matter becomes measurable and dimensional. (By fixing it we could measure the amount of matter in, say, teraatoms and the Avogadro constant would be dimensionless, just as the conversion factor between a fermi and a meter is.) Note that here we are speaking of weighing rather than of measuring mass: comparison of weights under the assumption that the acceleration of free fall is constant and, if necessary, the 'absolute weighing' of mass (a combination of measuring the weight and the acceleration of free fall g) are still the most exact and simplest way of determining the mass of a body (although, of course, we know of the dependence of g on time and place).

Strictly speaking, we must distinguish between the dimensional constant, the Avogadro constant N_A, which has the dimension of a number of particles per mole, and its numerical value in the SI. Both quantities have physical meaning: the dimensional quantity is the molar particle number density, while the numerical value corresponds to the mass of a carbon atom: [4]

$$m(^{12}\mathrm{C}) = \frac{12\ \mathrm{g}}{\{N_A\}}\ . \tag{3}$$

From the practical standpoint, it is not advisable to reduce temperature to mechanical units (as we did not with amount of matter). From the standpoint of fundamental physics, in principle we can even say that measurements of temperature and the amount of matter in special independent units that cannot be reduced to units of length, time, and mass implement the idea of relative measurements under special conditions. However, the world community considers these conditions so important that such units employed in measurements have been legalized as the SI base units!

The situation is even more intricate in the case of electrical units; there is even a trace of the 'atavistic' notion of ether, which is inadmissible, of course. For instance, the constants ε_0 and μ_0 not so long ago were called the permittivity and permeability of vacuum, respectively. At present, these quantities are called the electric constant and the magnetic constant (of vacuum), which really does not alter the essence of the issue.

At the same time, there are strong reasons for introducing $\varepsilon_0 \neq 1$ (in addition to historical reasons). In the case of amount of matter and temperature we spoke of quantities that could, at least principally, be reduced to mechanical quantities (number of atoms, energy, etc.) measurable in already known units. It goes without saying that when we are dealing with electric charge, we mean a certain non-mechanical quantity that can be measured in terms of mechanical units. Hence, the question is modified: Is there any sense in measuring very different quantities in the same units? We will revert to this question below.

As noted above, historically the scale of the magnitudes of the SI units was determined on the classical grounds for macroscopic phenomena (at the end of the 18th century but on the basis of the evolution of units over the course of several centuries!), but this did not stop scientists from realizing (in the 20th century) that only quantum phenomena can provide really universal and unchangeable quantities that can be applied to build the units of measure. Actually, quantum phenomena provide some natural units and the number of these units is fairly large.

[4] From here on, the expressions in curly brackets stand for the numerical values of the dimensional quantities in the SI if another system of units is not indicated explicitly.

The tendency of the development of the SI units is the adoption of new definitions based on quantum phenomena, where the corresponding quantum units define the SI units but are not equal to them, since the metrological community is conservative and we, of course, do not want to change the size of the units. The definition is done by fixing the values of certain natural constants (fundamental to this or that degree) such as, for instance, the ground-state hyperfine splitting interval in caesium-133 [2]:

$$\nu_{\mathrm{HFS}}(^{133}\mathrm{Cs}) = 9\,192\,631\,770\ \mathrm{Hz} \tag{4}$$

and the speed of light in vacuum:

$$c = 299\,792\,458\ \mathrm{m\ s^{-1}}\ . \tag{5}$$

There is a whole range of natural units that are of different status in the SI. If we take the reciprocal interval of hyperfine splitting in caesium, $\nu^{-1}_{\mathrm{HFS}}(^{133}\mathrm{Cs})$, as such a unit, then the second in SI is a derivative of this unit:

$$1\ \mathrm{s} = 9\,192\,631\,770\ \frac{1}{\nu_{\mathrm{HFS}}(^{133}\mathrm{Cs})}\ . \tag{6}$$

The speed of light in vacuum c is a natural unit of speed, so that the SI unit is its derivative:

$$1\ \mathrm{m\ s^{-1}} = \frac{1}{299\,792\,458}\ c\ . \tag{7}$$

The unified atomic mass unit

$$1\ \mathrm{u} = \frac{1}{12}\ m(^{12}\mathrm{C}) \tag{8}$$

is a non-SI unit actually defining the mole:

$$\frac{1\ \mathrm{g}}{1\ \mathrm{u}} = \{N_A\}\ . \tag{9}$$

In addition to non-SI units there are unofficial units whose role is often played by fundamental constants. For instance, nuclear magnetic moments in many handbooks are often given in units of the nuclear magneton

$$\mu_N = \frac{e\hbar}{2m_p}\ . \tag{10}$$

Sometimes, to avoid writing the unit explicitly, specially normalized quantities are introduced, such as the atomic number A (the number of nucleons, which in some problems approximates fairly well the mass of the atom or nucleus in atomic mass units), the nuclear g-factor (to within a simple factor depending on the nucleus spin this is the magnetic moment μ_{Nucl} of the nucleus represented in terms of nuclear magnetons μ_N), and the quantum number corresponding to the total angular momentum or its part, say spin (coinciding with the corresponding angular momentum divided by $\hbar$). Often, the constant c is used as the unit of speed. For instance, in astronomy distance may be measured in light years, while in accelerator physics the momenta are measured in MeV$/c$. The list can be continued.

Natural units emerge as an interpretation of dimensional fundamental constants [17] (Table 1). Some of them may be considered conversion factors between different units as well. Dimensionless and some dimensional constants may also be interpreted as conversion factors between different units (and

Table 1. Some natural units and their recommended values in SI units and others [1].

Natural unit	Symbol	Value	Relative standard uncertainty u_r
Unified atomic mass unit (u)	$\frac{1}{12}\,m\,(^{12}\mathrm{C})$	$1.660\,538\,86(28)\times10^{-27}$ kg $931.494\,043(80)$ MeV c^{-2}	1.7×10^{-7} 8.6×10^{-8}
Proton mass	m_p	$1.672\,621\,71(29)\times10^{-27}$ kg $938.272\,029(80)$ MeV c^{-2}	1.7×10^{-7} 8.6×10^{-8}
Electron mass	m_e	$9.109\,382\,6(16)\times10^{-31}$ kg $0.510\,998\,918(44)$ MeV c^{-2}	1.7×10^{-7} 8.6×10^{-8}
Bohr magneton	$\mu_\mathrm{B}=\frac{e\hbar}{2m_\mathrm{e}}$	$9.274\,009\,49(80)\times10^{-24}$ J T^{-1} $1\,836.152\,672\,61(85)\,\mu_\mathrm{N}$	8.6×10^{-8} 4.6×10^{-10}
Nuclear magneton	$\mu_\mathrm{N}=\frac{e\hbar}{2m_\mathrm{p}}$	$5.050\,783\,43(43)\times10^{-27}$ J T^{-1} $2.179\,872\,09(37)\times10^{-18}$ J	8.6×10^{-8} 1.7×10^{-7}
Rydberg energy	hcR_∞	$13.605\,692\,3(12)$ eV	8.5×10^{-8}
Proton charge	e	$1.602\,176\,53(14)\times10^{-19}$ C	8.5×10^{-8}
Planck constant	h	$6.626\,069\,3(11)\times10^{-34}$ J s	1.7×10^{-7}
	$\hbar=\frac{h}{2\pi}$	$1.054\,571\,68(18)\times10^{-34}$ J s	1.7×10^{-7}
Planck mass	$M_\mathrm{Pl}=\left(\frac{\hbar c}{G}\right)^{1/2}$	$2.176\,45(16)\times10^{-8}$ kg $1.220\,90(9)\times10^{19}$ GeV c^{-2}	7.5×10^{-5} 7.5×10^{-5}
Planck length	$l_\mathrm{Pl}=\frac{\hbar}{M_\mathrm{Pl}c}$	$1.616\,24(12)\times10^{-35}$ m	7.5×10^{-5}
Planck time	$t_\mathrm{Pl}=\frac{l_\mathrm{Pl}}{c}$	$5.391\,21(40)\times10^{-44}$ s	7.5×10^{-5}
Planck temperature	$T_\mathrm{Pl}=\frac{M_\mathrm{Pl}c^2}{k}$	$1.416\,79(11)\times10^{32}$ K	7.5×10^{-5}
von Klitzing constant	$R_\mathrm{K}=\frac{h}{e^2}$	$25\,812.807\,449(86)$ Ω	3.3×10^{-9}

their interpretation is not exhausted by this!). Below we consider all possibilities.

(1) Dimensionless constants that are conversion factors between 'same-type' units for a 'same' quantity. For instance, the various ratios formed including the mass hR_∞/c (corresponding to the Rydberg energy), electron mass m_e, proton mass m_p, the mass of a carbon atom, and the Planck mass M_Pl may be considered units. Since these units are natural and fundamental (to different degrees), their ratios are also fundamental constants. The numerical values of dimensionless constants do not depend on the choice of units and are the most interesting from the standpoint of physics.

(2) Dimensional constants that are conversion factors between different units for a 'single' quantity. For instance, the Boltzmann constant k links the units of temperature (average energy per degree of freedom) and the standard energy units. As noted earlier, a special dimensional constant is the frequency of hyperfine splitting of the ground state in caesium-133, the numerical value of which is fixed by definition: it relates the frequency of hyperfine splitting in caesium as a natural unit with the SI unit. A constant that emerges from the SI may be a constant of nature with a fairly nontrivial meaning. For instance, the numerical value of the Boltzmann constant k characterizes the triple boiling point of water. Such a constant can also be 'only' a conversion factor characterizing the agreement on units, just as the numerical value of the hyperfine interval $\nu_\mathrm{HFS}(^{133}\mathrm{Cs})$ is a factor.

There is no distinct difference between dimensionless and dimensional constants that serve as conversion factors. The reason is that the SI units and some non-SI units (such as the electron-volt or the atomic mass unit) contain 'built-in' dimensional constants with fixed numerical values. For example, the frequency of the two-photon $(1s-2s)$-transition in the hydrogen atom [20]:

$$\nu_\mathrm{H}(1s{-}2s)=2\,466\,061\,413\,187\,103(46)\ \mathrm{Hz} \tag{11}$$

characterizes a natural hydrogen unit closely related to the hartree

$$E_\mathrm{H}=2h\,c\,R_\infty\,.$$

On the other hand, the same result in dimensionless terms:

$$\frac{\nu_\mathrm{H}(1s{-}2s)}{\nu_\mathrm{HFS}(^{133}\mathrm{Cs})}=268\,265.005\,592\,310\,7(50) \tag{12}$$

can be interpreted without the loss of accuracy as the conversion factor between the hydrogen and the caesium units. Obviously, here we are speaking about the same physical quantity, i.e., equations (11) and (12) carry the same information, but their dimensions and numerical values are different.

(3) Dimensional constants that are conversion factors between different units for 'different' (but related) quanti-

ties. Here are examples: distances and time intervals; mass, energy, and momentum; frequency and energy, as well as charge and a combination of units emerging from the Coulomb law at $\varepsilon_0 = 1$ [$\mathrm{m}^{3/2}\,\mathrm{kg}^{1/2}\,\mathrm{s}^{-1}$ is a unit similar to the unit of charge in the CGS system]. These constants can be fixed. Two of them, the speed of light in vacuum c [see equation (5)] and the electric constant ε_0, are fixed in the SI:

$$\varepsilon_0 = \frac{1}{c^2 \mu_0}\,, \qquad \mu_0 = 4\pi \times 10^{-7}\ \mathrm{H\,m^{-1}}\,. \tag{13}$$

The constants c and ε_0 can be made dimensionless. But will we benefit from this? The answer depends on whether we consider these quantities (measured by 'identified' units) the same quantities. If we assume that mass, momentum, and energy are, in essence, the same thing (different components of a single 4-vector), we will benefit from simplifying the equations. But if we assume that these quantities are really different, measuring them by the same units will lead only to a superficial simplification, which will hide the important physical factors and may lead to misunderstanding.

There can be no absolutely obvious answer to the above question because these quantities are closely related but are not identical. They are 'almost' identical, and the term 'almost' is a sly one mathematically, since it cannot be expressed numerically and has no comparative degree.

As for relativistic vectors (4-coordinate or 4-momentum), their components often have an independent physical meaning, partially dictated by history and nonrelativistic limits. While we can assign a different meaning to each component of a 4-vector and measure them, either explicitly or implicitly, in different units, the situation with tensors of higher rank becomes more complicated and unpleasant and the desire to simplify may prevail. However, 4-tensors, with rare exceptions (e.g., the electromagnetic field-strength tensor $F_{\mu\nu}$ which incorporates both the electric and magnetic fields), are less important for applications than various vectors.

Relativistic physics tends to measure different quantities in the same units and forces us to introduce (in addition to time and energy) zero components of the corresponding 4-vectors:

$$x_0 = c\,t\,, \qquad p_0 = \frac{E}{c}$$

with dimensions corresponding to distance and momentum, respectively. A consistent passage from t and E to x_0 and p_0 actually means that $c = 1$, since the speed of light in vacuum drops out from the equations. Thus, instead of the dimensional speed $v_i = \partial x_i/\partial t$ in equations there naturally emerges its dimensionless analog $\beta_i = \partial x_i/\partial x_0$.

At the same time, quantum (and statistical) mechanics insist on different interpretations of time and space coordinates, and energy and momentum. We are interested in the time evolution of a system occupying a certain volume in space. Quantum mechanics (nonrelativistic as well as relativistic) use the language of stationary (quasi-stationary) states and discrete energy spectra. In other words, a clearly noncovariant approach to space and time proves convenient and useful in many cases.

We also note that in nonrelativistic physics with $c = 1$ certain difficulties emerge in applying the dimensional method, with the result that the theory becomes less clear and transparent. Of course, we can speak of the limit of small distances ($\Delta x \ll \Delta t$) and of series expansions in terms of the parameter $\Delta x/\Delta t$. Then, quantities of the same order infinitesimal with $c = 1$ will correspond to the same dimension in the case of the common interpretation of the speed of light in vacuum as a dimensional quantity. However, there may be many characteristic times and lengths and, hence, small parameters. Moreover, reasoning consistently, we would logically want $\hbar$ to be set to unity. Such a system of units would simplify the equations of relativistic quantum mechanics, but it is very inconvenient for nonrelativistic classical physics which occupies an important place in measurements.

It must be understood that setting c to unity is more a question of jargon [17]. Actually, we are choosing natural units of speed in which the numerical value of the speed of light in vacuum equals unity. Unfortunately, when employing relativistic or some other special units, physicists do not all have clear knowledge where they are dealing with questions of jargon.

A standard compromise is to consider covariant and noncovariant notations on an equal basis. Covariant notation is used in relativistic problems, and noncovariant notation in nonrelativistic problems. Noncovariant notation is also applied, if necessary, for directly measured quantities, since in real measurements the frame of reference is always fixed. The forced condition $c = 1$, one of the merits of which is the simplification of equations in relativistic physics, would narrow the advantages of the noncovariant approach when it is asked for.

Irrespective of the choice of the system of units, we know that light travels and, hence, its speed c is an objective quantity. The very fact that in a certain system of units it can assume a trivial value does not mean that the constant disappears. We have a certain experience of measuring different quantities in the same units, for instance, weight (i.e., a particular case of force) in kilograms. The problem of the 'disappearance' of constants is a misunderstanding that emerges in the process.

Suppose the speed of light is not isotropic or depends on time and position. Comparing its values, we may not even notice this in a system of units where the numerical value of c is not fixed, since we must monitor the isotropy and constancy of the units proper. However, in a flat world with a varying speed of light we are forced to cope with the problems of the geometric interpretation of data that rely on the propagation of light, and so geometric results may prove to be not unambiguously defined and not flat.

Note that the above reasoning is not an abstract problem but the reality of global positioning systems which use satellites. Of course, in this case the problems of the speed of light do not center around the fundamental non-Lorentzian nature of our world but around the presence of the Earth's atmosphere. Nevertheless, this situation is a simple illustration of the fact that studies covering the speed of light cannot be reduced to direct measurements of the speed of light propagation and can be expressed in terms not directly related to the details of the system of units.

Thus, we can say that a system of units (and a system of standards) presupposes that certain laws of physics are true. If reality contradicts those laws of physics (for instance, the definition of the meter presupposes that the speed of light in vacuum is a universal constant indepen-

Table 2. 'Energy' and 'frequency' equivalents of various units (recommended values [1]). Here, $E_{\rm H} = \alpha^2 mc^2 = 2c\,h\,R_\infty$ is the atomic energy unit (hartree), and $\nu' = 1/\lambda$ is the wavenumber.

Unit	Relation to energy	Numerical value of equivalent		
		1 J	1 eV	1 Hz
1 J		1	$6.241\,509\,47(53) \times 10^{18}$	$1.590\,190\,37(26) \times 10^{33}$
1 eV	$E = eV$	$1.602\,176\,53(14) \times 10^{-19}$	1	$2.417\,989\,40(21) \times 10^{14}$
1 Hz	$E = h\nu$	$6.626\,069\,3(11) \times 10^{-34}$	$4.135\,667\,43(35) \times 10^{-15}$	1
1 m^{-1}	$E = ch\nu'$	$1.986\,445\,61(34) \times 10^{-25}$	$1.239\,841\,91(11) \times 10^{-6}$	299 792 458
1 K	$E = kT$	$1.380\,650\,5(24) \times 10^{-23}$	$8.617\,343(15) \times 10^{-5}$	$2.083\,664\,4(36) \times 10^{10}$
1 kg	$E = mc^2$	$8.987\,551\,787\ldots \times 10^{16}$	$5.609\,588\,96(48) \times 10^{35}$	$1.356\,392\,66(23) \times 10^{50}$
1 u	$E = mc^2$	$1.492\,417\,90(26) \times 10^{-10}$	$931.494\,043(80) \times 10^{6}$	$2.252\,342\,718(15) \times 10^{23}$
1 $E_{\rm H}$	$E_{\rm H} = \alpha^2 mc^2$	$4.359\,744\,17(75) \times 10^{-18}$	$27.211\,384\,5(23)$	$6.579\,683\,920\,721(44) \times 10^{15}$

dent of any factor, the definition of the ampere presupposes that Ampere's law is valid, while its realization depends on the validity of the Maxwell equations), there are always ways of detecting such contradictions and interpreting them.

It should be emphasized that instead of the dimensionless unit $c = 1$ we can explicitly adopt a dimensional equality. However, this palliative does not fully use the merits of the lingo approach with $c = 1$ and partially preserves the problems of different dimensions. For instance, time t (in years) and distance x (in light years) are still measured in different units.

Various quantities are often measured in 'energy' units (which is convenient from the standpoint of fundamental physics) and in frequency units (which is convenient from the practical standpoint, since frequency measurements are the most precise ones). The conversion factors are the values of such fundamental constants as h, e, c, k, etc. The 'energy' and 'frequency' equivalents of some of these units are listed in Table 2.

2.3 Physical quantities, units, and the standards of units of physical quantities

The units of physical quantities are fixed by a certain official agreement, the SI units, but the system of standards may not correspond to the system of units. For instance, the SI presupposes a single base electromagnetic unit introduced by fixing the magnetic constant μ_0. Formally, this is done on the basis of Ampere's law. Actually, from the standpoint of the SI, it does not matter which electromagnetic unit is fixed and which law formed the basis for its definition (Coulomb's law, Ampere's law, or some other laws) — all are equivalent.

Two factors are important in choosing the principal standards for the system of standards under construction: these should be optimum standards from both the point of view of the precision of the base unit and the point of view of the realization of the sizes of the derived units. From the standpoint of practice, it is convenient to divide the standards into groups: length – frequency – time, mass, electromagnetic quantities, etc. In the case of electromagnetic quantities, it is convenient to select two standards, say, the volt and the ohm. This creates a redundant system. Hence, in addition to determining the magnitudes of the other electrical quantities, we must solve the problem of matching the units of mechanical power (expressed in terms of the base units as $1\ {\rm W} = 1\ {\rm kg\ m^2\ s^{-3}}$) and electric power ($1\ {\rm W} = 1\ {\rm V^2\ \Omega^{-1}}$). We will discuss this problem in greater detail in Section 4.2.

The above discussion shows that there are three related, and yet very different, systems:

- the system of physical quantities;
- the system of physical units, and
- the system of standards of physical units.

These systems may have different structures. For practical reasons, the same physical quantity can, in certain conditions, be measured in different units (e.g., in kelvins and in joules). On the other hand, different quantities (say, weight and mass) can be measured in the same units. The hierarchy of standards does not follow the hierarchy of units.

The situation with standards is even more complicated because we are speaking of reproducing the units and of maintaining the size of the units. In the case of maintaining the size of the units, a standard is a device delivering a certain unit that does not coincide with a unit belonging to the SI and whose magnitude (in SI units) is, strictly speaking, unknown. Such a unit makes it possible to do 'local' measurements. If this unit is based on quantum phenomena (which is important if we are doing frequency – length – time, voltage, or resistance measurements), it is natural and, hence, universal in the entire world and is time-independent (in contrast to classical national units based on artifacts).

To obtain (reproduce) an SI unit, the maintained unit must be correctly calibrated. If the maintained units are based on numerical values of fundamental constants, the independent measurements of their values transforms maintenance into reproduction. Thus, there emerges the possibility of realizing the same SI units in different laboratories, i.e., to ensure traceability of measurements without direct comparison of standards. We will discuss the problem of reproduction and maintenance of units by turning to the example of electromagnetic quantities in Section 4.2.

In the practical realization of the SI units, the electromagnetic units occupy a special place. We could, for example, note that from the formal (mathematical) viewpoint the unit of length is a derived unit, while the base units are those of time (caesium clock) and speed (in terms of the speed of light in vacuum). However, there is no way in which the unit of speed can be considered a base unit, since it is impossible to realize a standard for this unit. On the other hand, the unit of length has no such drawback and can be considered a base unit.

All the SI base units except the ampere correspond to the main standards according to which standards of derived units are built as needed. There is practically no alternative to any of the base units, since these units are the most convenient ones (each in its own area). The situation with electromagnetic units is qualitatively different. The choice of the main standards is dictated by expediency, and its results may differ depending on the development of technology for a given period. Furthermore, the ampere is clearly defined as a unit of *electrodynamics*, which describes the physics of fields, forces, interactions, etc. However, the most important (and most accurate) applications are in *electrical engineering* which studies electric circuits.

Actually, the definition of the ampere is not a definition of a unit that can be used for reproduction but the definition of the value of the magnetic constant μ_0, which can be used to reproduce various units in very different ways. A substantial fraction of units from electrical engineering (current, voltage, resistance, capacitance, and inductance) can be maintained fairly easily and compared to each other. In electrical engineering, only the released thermal power is directly related to mechanical units. However, thermal phenomena do not present the best possibility for determining the units of energy, power, etc. Hence, the national standards of electromagnetic units were built in practice in a certain sense independently from the definitions of the SI units (as artifacts). Then, in the course of special experiments, cross-comparisons were conducted between

- units with the same dimensions (say, between the units of the volt or the ohm in different countries);
- units with different dimensions (say, between the farad and the ohm), and
- national and international units and the SI units.

This tendency remains even today with the only correction that the comparison of national standards of different countries has lost its significance in view of the use of universal quantum phenomena. The actual independence in the determination of electric charge forces us to introduce the constants ε_0 and μ_0. One cannot exclude the possibility that in the future natural magnitudes of the base electrical units will be adopted on the basis of macroscopic quantum phenomena (the Josephson effect and the quantum Hall effect). In this case, electric charge will be measured in units of the electron charge (with a fixed normalization factor), so that the appearance of a proportionality factor in Coulomb's law is inevitable. In a system of units with a fixed value of the electron charge e, the proportionality factor in Coulomb's law must be determined by measurements and cannot be fixed (see Section 4.2).

The above example is not the only case where the requirements of transparency of physical equations ($\varepsilon_0 = 1$) and accuracy of measurements clash. Another example is the measurements of the frequency, energy, and wavelength of gamma radiation. Traditionally, the results are given in energy units, although what is measured directly is the emission wavelength. The binding energy of the deuteron has a clear physical meaning, however, what is measured directly is the other quantity — minimum frequency of a photon that leads to photoionization. The conversion of this frequency into the non-SI unit of energy, the electron-volt, increases the error.

A similar situation emerges in the precision physics of simple atoms (hydrogen, muonium, and positronium), where the transition frequencies are measured but the energy levels are calculated. The problem is solved in favor of accuracy: analytical expressions are derived for the energy, while numerical values are given for the corresponding frequency (cf. Refs [18, 19]). This must be done not only for the sake of comparing theoretical results with experimental data, but also because the corresponding normalization factors (such as the electron mass m_e) are known with higher accuracy not in units of mass or energy ($m_e c^2$) but in units of frequency ($m_e c^2/h$).

2.4 Fundamental constants in modern physics: origin and fundamental nature

Let us briefly discuss the origins of physical constants and the extent to which these constants are considered fundamental in modern physics. These problems are also of practical importance, since their solution may lead to an understanding of how some constants can be derived from other constants, and broadly speaking, whether or not constants can be calculated from first principles, if the constants can (or must) change in time, etc.

The question of whether some constants can be derived from other constants belongs to the problem of 'secondary' objects. In the case of macroscopic quantum effects and atomic energy levels, this problem amounts to estimating the accuracy of calculations and, to a certain extent, to definitions. For instance, in the case of the Josephson effect and the quantum Hall effect (see Section 4.2), the corresponding universal proportionality factors are practically defined as the Josephson constant and the von Klitzing constant:

$$K_J = \frac{2e}{h}\,, \qquad R_K = \frac{h}{e^2}\,. \tag{14}$$

Hence, the accuracy with which these constants are expressed in terms of the electron charge and so forth amounts to the accuracy of the respective theories. At present, no corrections to the above expressions, either theoretical or experimental, are known.

Let us now turn to the hydrogen atom. Here, the name 'Rydberg constant' was used not for an observable quantity but for a special combination of the electron charge and mass:

$$R_\infty = \frac{\alpha^2 m_e c}{2h}\,, \tag{15}$$

where the fine-structure constant is introduced as follows:

$$\alpha = \frac{e^2}{4\pi\varepsilon_0 \hbar c}\,. \tag{16}$$

The Rydberg constant determines the energy levels in the hydrogen atom only if one ignores the radiative and relativistic effects as well as the effects of finite mass and structure of the nucleus. Hence, the constant R_∞ is expressed exactly in terms of more fundamental constants. However, the accuracy with which the observed transition frequencies and the ionization potential in the hydrogen atom are related to the Rydberg constant is determined by the accuracy of the theory. To what extent we are able to calculate the mass of the nuclei (to calculate or measure the correction for their binding energy), their magnetic moments, etc. is also debatable. Although the accuracy of such calculations is often not very high, there is no doubt about the 'secondary' nature of the above objects.

A more intricate question is whether we can calculate the fundamental constants that refer to elementary objects. The ability to do calculations from first principles is closely related to the fundamental nature of these objects. First, it should be noted that the constants associated with the properties of particles are not, strictly speaking, fundamental. Earlier, we discussed the problem of renormalization. A correct understanding of the role of renormalization in the fundamental nature of constants and, hence, in the computability of constants shows that all speculation concerning the exact prediction of the value of the fine-structure constant α, based on general considerations, cannot be correct and has no meaning. Simple expressions may correspond only to unrenormalized quantities defined on the Planck scale, and we are still unable to derive the observable value of the fine-structure constant.

The four constants in formulas (14)–(16) incorporate the elementary charge e, which is set equal to the positron or proton charge. Strictly speaking, at present there are no theoretical arguments corroborated by experiments in favor of the equality of these two charges. The equality finds its explanation within various theories that unify the strong, weak, and electromagnetic interactions. Such theories naturally follow from our ideas about fundamental interactions, but so far they find no experimental confirmation. Thus, from the practical viewpoint, we simply assume that the positron and proton charges 'coincide', and this assumption is corroborated with high accuracy. The difference between the positron and proton charges is smaller by many orders of magnitude than the errors with which these charges are determined (e.g., see Ref. [25]), so that in practical matters we can speak of a single constant, the elementary charge.

Today, only a limited number of fundamental constants can be calculated from first principles. In some cases, advances are achieved only for unrenormalized constants, such as the electron and muon g-factors. Their unperturbed values can easily be found, but to be able to compare them with the real values we must take into account the QED effects, and the accuracy with which these effects can be calculated is limited.

Another example is the sine of the Weinberg angle, $\sin^2\theta_W$, in grand unified models (i.e., the unification of the electromagnetic, weak, and strong interactions) [12]. After the minimal model based on the SU(5) group proved incapable of explaining the much too long lifetime of the proton, various modifications of this model came into play. Further experimental (i.e., selection of the correct modification) and theoretical (i.e., calculation of corrections) progress may lead to computation of this constant.

Speaking of the fundamental nature of constants, we must bear in mind two additional aspects of the problem. They are related to the simple fact that our Universe (in which we exist) is a unique object, and it is still unclear what in it is governed by laws and what by probability. The building of a modern theory of electroweak interaction, the various attempts of grand unification [12], and the inflationary-universe model [21] assume that the effects of spontaneous symmetry breaking play an important role in our world. Hence (and possibly for other reasons), some constants might assume different values, and only in our Universe or in our version of the evolution of the Universe are they such as we see them. Perhaps some constants cannot be calculated in principle.

At the same time, there are certain macroparameters that describe the Universe as a whole: the Hubble constant, the average density of energy and matter, the ratio of the number of baryons to the number of photons, etc. Nor do we know how fundamental these constants are: perhaps they are just 'facts of the particular biography' of our Universe. Possibly, within the *anthropic cosmological principle* these two aspects are simply two sides of the same question. The idea of the anthropic principle is that a Universe could have very different properties (and different constants), but only their certain fine-tuned combination can produce a Universe that contains observers capable of studying it. Note that here we are not speaking of life or organic matter. For instance, for certain ratios of the constants nucleosynthesis could proceed differently and no stars would be born (e.g., see Refs [22, 23]). Maybe the expectation that the properties of the Universe are universal will prove to be something similar to the adoption of the idea that such 'earthly' constants as the acceleration of free fall and the density of water are important.

Another aspect of the fundamental nature of constants is their constancy. Models can exist in which the 'truly' fundamental constants may change, but it is much easier to construct a scenario in which the observed 'effective' parameters change. For instance, in the inflationary-universe model [21] there are phase transitions in which the electron suddenly acquires mass. This is accompanied by the sudden emergence of a finite vacuum average of some fields, but the parameters of the fundamental Hamiltonian do not change in the process. This example is an indication that sometime in the past the fundamental constants probably varied in time or, possibly, vary even today, but of course not so noticeably. A discussion of the current situation with the search for possible variations of the values of fundamental constants with time can be found in Ref. [24].

As we approach the end of this section, I would like to note that we must distinguish between the dimensional constants proper and their numerical values. The numerical values can be manipulated, including a change in dimensions and the fixing of their value. But no matter how we change the system of units, the speed of light c and the ground-state hyperfine splitting interval in caesium, $\nu_{\rm HFS}(^{133}{\rm Cs})$, exist objectively. What is interesting is that manipulations can strongly affect the extent to which the numerical values are fundamental. For example, the Rydberg constant (15) is a combination of the fundamental properties of the electron and 'knows' nothing about the magnetic moment of caesium-133.

On the other hand, the numerical value of the Rydberg constant R_∞ can, in principle, be calculated if we know the magnetic moment of caesium (in Bohr magnetons) and the fine-structure constant α. The value of the electron mass plays a very small role in such calculations (through the effects of the finiteness of the nucleus mass). Actually, the numerical value of the Rydberg constant in SI units is (to within normalization) the reciprocal frequency of hyperfine splitting in caesium-133 measured in natural (atomic) units. It characterizes not the Rydberg constant but the hyperfine splitting in caesium and the SI.

3. Adjustment of the values of fundamental physical constants

3.1 The structure of adjustment and the recommended values of the basic fundamental physical constants (according to results published before 2002)

Adjustment of the values of fundamental physical constants is not a simple problem and differs dramatically from simple combined processing of data by the least-squares method. One of the problems stems from the close interrelation of the data. To find, say, the Rydberg constant R_∞, we must know the fine-structure constant α. On the other hand, for some ways of determining the fine-structure constant we must know the Rydberg constant. In the given case, the interrelation between the data can almost be ignored. The thing is that the Rydberg constant is known with an accuracy that is several orders of magnitude greater that with which the fine-structure constant is known, with the result that in determining α we can ignore all errors in R_∞.

Table 3. Values of the fundamental physical constants adopted by definition and, therefore, known with absolute accuracy (exactly).

Fundamental constant	Symbol	Value
Speed of light in vacuum	c, c_0	299 792 458 m s^{-1}
Hyperfine interval in ground state of caesium	$\nu_{\rm HFS}\,(^{133}\rm Cs)$	9 192 631 770 Hz
Magnetic constant	$\mu_0 = 4\pi \times 10^{-7}$ H m^{-1}	$1.256\,637\,061\ldots \times 10^{-6}$ H m^{-1}
Electric constant	$\varepsilon_0 = \dfrac{1}{c^2\mu_0}$	$8.854\,187\,817\ldots \times 10^{-12}$ F m^{-1}
Atomic mass of carbon-12	$m\,(^{12}\rm C)$	12 u

Table 4. Recommended values of the fundamental physical constants [1] that are known with high accuracy and play an auxiliary role in the adjustment procedure.

Fundamental constant	Symbol	Value (2002)	Relative standard uncertainty $u_{\rm r}$
Rydberg constant	R_∞ cR_∞	10 973 731.568 525(73) m^{-1} $3.289\,841\,960\,360(22) \times 10^{15}$ Hz	6.6×10^{-12} 6.6×10^{-12}
Electron mass	$m_{\rm e}$	$5.485\,799\,094\,5(24) \times 10^{-4}$ u	4.4×10^{-10}
Proton mass	$m_{\rm p}$	1.007 276 466 88(13) u	1.3×10^{-10}
Proton – electron mass ratio	$\dfrac{m_{\rm p}}{m_{\rm e}}$	1 836.152 672 61(85)	4.6×10^{-10}
Proton magnetic moment	$\mu_{\rm p}$	$1.521\,032\,206(15) \times 10^{-3}\,\mu_{\rm B}$	1.0×10^{-8}
Proton g-factor	$g_{\rm p} = \dfrac{2\mu_{\rm p}}{\mu_{\rm N}}$	5.585 694 701(56)	1.0×10^{-8}
Neutron mass	$m_{\rm n}$	1.008 664 915 60(55) u	5.5×10^{-10}
Deuteron mass	$m_{\rm d}$	2.013 553 212 70(35) u	1.7×10^{-10}
Deuteron magnetic moment	$\mu_{\rm d}$	$0.466\,975\,456\,7(50) \times 10^{-3}\,\mu_{\rm B}$	1.1×10^{-8}
Deuteron g-factor	$g_{\rm d} = \dfrac{\mu_{\rm d}}{\mu_{\rm N}}$	0.857 438 232 9(92)	1.1×10^{-8}
Helion (helium-3 nucleus) mass	$m_{\rm h}$	3.014 932 243 4(58) u	1.9×10^{-9}
Alpha particle mass	m_α	4.001 506 179 149(56) u	1.4×10^{-11}
Fermi coupling constant *	$\dfrac{G_{\rm F}}{(\hbar c)^3}$	$1.166\,39(1) \times 10^{-5}$ GeV^{-2}	8.6×10^{-6}
Weak mixing angle *	$\sin^2\theta_{\rm W}$	0.222 15(76)	3.4×10^{-3}

At the same time, in determining the Rydberg constant R_∞, the fine-structure constant α enters only into the additive corrections to the relative order of α^2, and one can easily see that the error in determining the fine-structure constant is also insignificant. However, the situation is not always so good, and we are forced to process some blocks of data simultaneously and minimize the errors. This procedure is called *adjustment* of the values of the fundamental physical constants. During the last decade, the structure of adjustment has remained unchanged.

(1) Several constants are 'known' with absolute accuracy, since their values are fixed by the determinations of SI units or non-SI units, e.g., the unified atomic mass unit. Of course, adjustment does not alter these constants (Table 3). The errors associated with these constants do not disappear but are transferred from their numerical values to the definitions of the dimensional units in terms of which they are expressed, and in this way to the errors associated with other quantities.

(2) The values of some constants are known with such high accuracy that the corresponding errors have no effect on the determination of other fundamental constants. These are, say, the Rydberg constant R_∞, the ratio $m_{\rm e}/m_{\rm p}$ of electron mass to proton mass, and some other constants (Table 4). Constants of this kind act as auxiliary constants in the adjustment, since they are found prior to the main adjustment procedure and do not change as a result of adjustment.

Note that the absolute value of the relative uncertainty is not very important. What is important is whether the given value enters into the principal contribution as a factor (as a rule, the Rydberg constant enters in exactly this manner) or as small corrections whose accuracy may be low enough. For instance, the contribution of weak interactions to various quantities (the anomalous magnetic moments of the electron and the muon, and the energy levels of hydrogen, deuterium, helium, and muonium) is so small that the errors associated with it play no role. Meanwhile, the Rydberg constant is known with the highest accuracy among measurable constants, while the accuracy with which the Fermi coupling constant $G_{\rm F}$ is known is much lower than the accuracy of most constants that enter into adjustment. The accuracy of the weak mixing angle is also low. [5]

[5] The numerical values [1] of the weak-interaction constants labeled by asterisks in Table 4 are listed in accordance with Ref. [25]. In the 2004 edition [26] they were slightly changed: $G_{\rm F}/(\hbar c)^3 = 1.166\,37(1)\times 10^{-5}$ GeV^{-2} [9×10^{-6}], and $\sin^2\theta_{\rm W} = 0.222\,12(37)$ [1.7×10^{-3}], which has no effect on the results of adjustment (the relative standard uncertainties are given by the figures in square brackets).

Table 5. Atomic masses measured with the highest accuracy [27].

Isotope	Atomic mass, u	Relative standard uncertainty
^{1}H	1.007 825 032 07(10)	1.0×10^{-10}
^{2}H	2.014 101 777 85(36)	1.8×10^{-10}
^{3}H	3.016 049 277 7(25)	8.3×10^{-10}
^{3}He	3.016 029 319 1(26)	8.6×10^{-10}
^{4}He	4.002 603 254 15(6)	0.15×10^{-10}
^{12}C	12	0
^{13}C	13.003 354 837 8(10)	0.75×10^{-10}
^{14}C	14.003 241 988 7(41)	2.9×10^{-10}
^{14}N	14.003 074 004 78(62)	0.44×10^{-10}
^{15}N	15.000 108 898 23(75)	0.50×10^{-10}
^{16}O	15.994 914 619 56(16)	0.10×10^{-10}
^{20}Ne	19.992 440 175 4(19)	1.0×10^{-10}
^{22}Ne	21.991 385 114(19)	8.6×10^{-10}
^{23}Na	22.989 769 280 9(29)	1.3×10^{-10}
^{24}Mg	23.985 041 700(14)	5.8×10^{-10}
^{28}Si	27.976 926 532 5(19)	0.7×10^{-10}
^{29}Si	28.976 494 700(22)	7.5×10^{-10}
^{30}Si	29.973 770 17(3)	1.0×10^{-10}
^{36}Ar	35.967 545 106(29)	8.2×10^{-10}
^{40}Ar	39.962 383 122 5(29)	0.75×10^{-10}
^{85}Rb	84.911 789 738(12)	1.4×10^{-10}
^{87}Rb	86.909 180 527(13)	1.5×10^{-10}
^{133}Cs	132.905 451 933(24)	1.8×10^{-10}
^{134}Cs	133.906 718 475(28)	2.1×10^{-10}

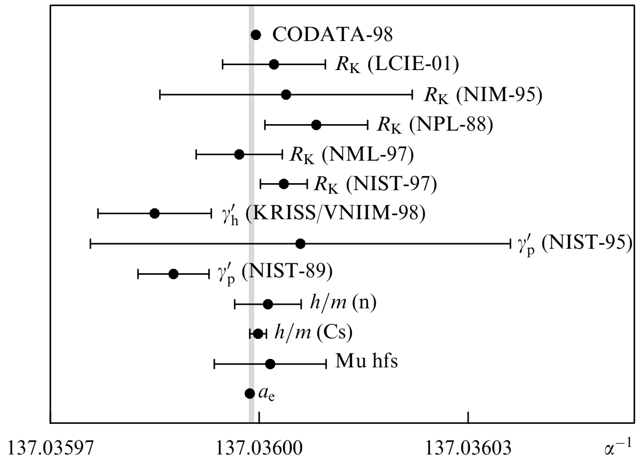

Figure 1. Various values of the reciprocal fine-structure constant α^{-1}, obtained from the data in an adjustment of constants. The vertical shaded belt corresponds to the CODATA recommended value [1]. The indices indicate the directly measured values. For explanations and references to experimental works, see Ref. [1].

There is also a specific class of auxiliary constants, namely, atomic masses expressed in unified atomic mass units. Some of these are known to a very high accuracy. Recent results of the processing of the data on atomic masses has been published in Ref. [27]. The most exact findings are listed in Table 5. The values of the masses utilized in Ref. [1] may differ somewhat (by only a small fraction of the error) from those listed in the table.

(3) There are two large blocks of data. The block with the data that have been measured with the highest accuracy includes the fine-structure constant α, the von Klitzing constant R_K, and the values of h/m with atomic masses (caesium-133) or particle masses (the neutron). Since the most important mass ratios (of atoms and the neutron to the proton, and of the proton to the electron) are known better than α, by measuring h/m and combining it with the mass ratio m/m_e and the Rydberg constant R_∞ we can find the value of the fine-structure constant

$$\alpha = \left(\frac{2\,R_\infty}{c}\,\frac{m}{m_e}\,\frac{h}{m} \right)^{1/2} . \tag{17}$$

We can get an idea about the input data from Fig. 1 which shows the different values of the fine-structure constant α found from input data. These results are of illustrative value only.

The results of the adjustment for this block are listed in Table 6. Note the presence of a such a 'nonfundamental' quantity as the magnetic moment (gyromagnetic ratio) of the shielded helion (the nucleus of helium-3), measured for the ground state of the neutral atom. Of course, this quantity differs somewhat from the unperturbed magnetic moment of the isolated nucleus. However, the accuracy of the measurements exceeds the precision of numerical evaluations for shielding effects. Since so far there have been no direct measurements of the magnetic moment of the nucleus proper and all results refer to the atom, this constant is more important from the standpoint of experimental physics.

(4) The less exact block of data is related to the elementary charge e, the Planck constant h, the Josephson constant K_J, and the Avogadro constant N_A. The interrelation of the first three constants is obvious (as the fine-structure constant α is known with higher accuracy). As for the Avogadro constant, it must be noted that there are two microscopic types of mass measurement with high accuracy: in unified atomic mass units (i.e., $m\{N_A\}$ in grams), and in frequency units (i.e., measurements of mc^2/h). Their comparison leads to a precise definition of the value of the molar Planck constant hN_A, with some of the results shown in Fig. 2.

Two competing approaches in the microscopic determination of mass correspond also to two possibilities of 'natural' definitions of the kilogram as the mass of a fixed number of carbon atoms and as the mass corresponding to rest energy equal to the energy of photons with a fixed sum of frequencies. (Here, we are speaking rather about a 'large' number of 'soft' photons than about a single photon with a frequency exceeding the Planck scale, or about an indirect measurement.) The factor hN_A is, to within the well-known factor $1/c^2$, the conversion factor between these 'natural' definitions of the unit of (macroscopic) mass.

The results for the Planck constant, obtained from the data used in adjustment, are shown in Fig. 3 and are of illustrative value only. The characteristic accuracies of the data in the α-block and in the h-block (the results of adjustment for this block are listed in Table 7) differ by a factor of about 10, so that the processing of the data in the blocks is done practically independently.

(5) Some constants are not included in any blocks simply because the experiments are fully isolated as, for instance, in the case of the Newtonian constant of gravitation G (Table 8). Adjustment in this case is reduced to common averaging.

(6) There are also constants that could have been included in the blocks but are known from measurements with an accuracy that is too low, so that the respective experimental data are not included in the adjustment procedure. The related results are derived from the adjusted values of other constants. Some constants of this type are listed in Table 9.

A major portion of the quantities listed in Tables 6 and 7 (the Bohr radius, the electron Compton wavelength, the

Table 6. Recommended values of the fundamental physical constants [1] related in the course of adjustment to the fine-structure constant α. The gyromagnetic ratio of the shielded ^{3}He nucleus corresponds to neutral helium-3. There are several somewhat different agreements concerning the sign of g-factors. In contrast to Ref. [1], here the sign of the g-factor of elementary particles is defined relativistically; hence, it is positive for the elementary particles and coincides with that for the electron and the positron. Negatively charges particles, such as the electron and the muon, have a negative magnetic moment and a positive g-factor. For compound objects such as nuclei, the g-factor may be either positive or negative, depending on the directions of spin and magnetic moment.

Fundamental constant	Symbol	Value (2002)	Relative standard uncertainty
Fine-structure constant	$\alpha = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{\hbar c}$	$7.297\,352\,568(24) \times 10^{-3}$	3.3×10^{-9}
	α^{-1}	$137.035\,999\,11(46)$	3.3×10^{-9}
Electron magnetic moment anomaly	$a_e = \frac{g_e - 2}{2}$	$1.159\,652\,185\,9(38) \times 10^{-3}$	3.2×10^{-9}
Bohr radius	$a_0 = \frac{\hbar}{\alpha m_e c}$	$5.291\,772\,108(18) \times 10^{-11}$ m	3.3×10^{-9}
Muon – electron mass ratio	$\frac{m_\mu}{m_e}$	$206.768\,283\,8(54)$	2.6×10^{-8}
Muon – proton magnetic moment ratio	$\frac{\mu_\mu}{\mu_p}$	$-3.183\,345\,118(89)$	2.6×10^{-8}
Muon magnetic moment anomaly	a_μ	$1.165\,919\,81(62) \times 10^{-3}$	5.3×10^{-7}
Electron Compton wavelength	$\lambda\!\!\!^{-}_C = \frac{\hbar}{m_e c}$	$386.159\,267\,8(26) \times 10^{-15}$ m	6.7×10^{-9}
von Klitzing constant	$R_K = \frac{h}{e^2}$	$25\,812.807\,449(86)\ \Omega$	3.3×10^{-9}
Quantum of circulation	$\frac{h}{2m_e}$	$3.636\,947\,550(24) \times 10^{-4}$ m^2 s^{-1}	6.7×10^{-9}
Electron gyromagnetic ratio	$\gamma_e = \frac{2\mu_e}{\hbar}$	$1.760\,859\,74(15) \times 10^{11}$ s^{-1} T^{-1}	8.6×10^{-8}
Proton gyromagnetic ratio	$\gamma_p = \frac{2\mu_p}{\hbar}$	$2.675\,222\,05(23) \times 10^{8}$ s^{-1} T^{-1}	8.6×10^{-8}
Shielded helion magnetic moment	μ'_h	$-2.127\,497\,723(25)\ \mu_N$	1.2×10^{-8}
Shielded helion gyromagnetic ratio	$\gamma'_h = \frac{2\mu'_h}{\hbar}$	$2.037\,894\,70(18) \times 10^{8}$ s^{-1} T^{-1}	8.6×10^{-8}
Molar Planck constant	hN_A	$3.990\,312\,716(27) \times 10^{-10}$ J s mol^{-1}	6.7×10^{-9}

quantum of circulation, the electron gyromagnetic ratio, the molar Planck constant, particle masses expressed in SI units, and the nuclear magneton and the Bohr magneton expressed in SI units, to name just some) also belong to this category of data.

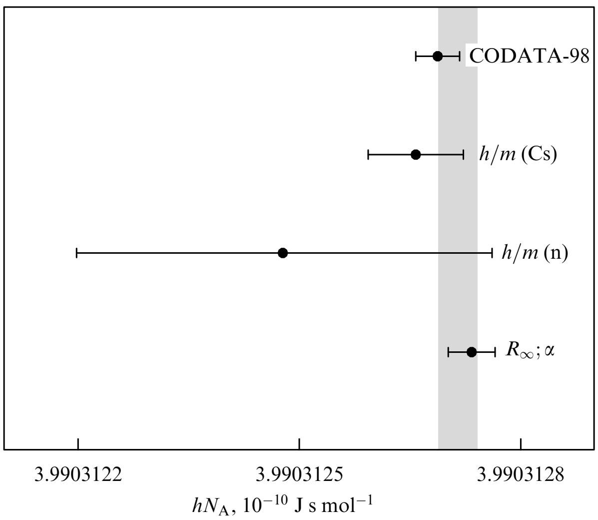

Figure 2. Determining the molar Planck constant hN_A on the basis of the input data for an adjustment of constants. In calculations with the Rydberg constant and the fine-structure constant, the value of R_∞ which is a result of the adjustment and the value of α stemming from the anomalous magnetic moment of an electron were used. References can be found in Ref. [1].

Note that comparison of the accuracy of different data plays an important role in their breakdown into the blocks, but this is not the decisive criterion. Adjustment represents a procedure similar to finding weighted mean values, the only difference being that data are diverse and are characterized by multiple correlations. A weighted mean value is calculated with weights inversely proportional to the error squared. If on

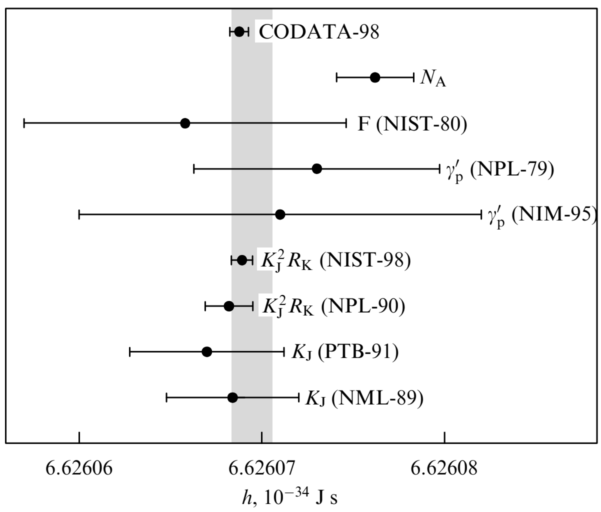

Figure 3. Values of the Planck constant h, obtained on the basis of the input data for an adjustment of constants. The vertical shaded belt corresponds to the value which is a result of the adjustment. For references, see Ref. [1].

Table 7. Recommended values of the fundamental physical constants [1] related in the course of adjustment to the Planck constant h and the elementary charge e.

Fundamental constant	Symbol	Value (2002)	Relative standard uncertainty
Planck constant	h	$6.626\,069\,3(11) \times 10^{-34}$ J s	1.7×10^{-7}
	$\hbar = \dfrac{h}{2\pi}$	$1.054\,571\,68(18) \times 10^{-34}$ J s	1.7×10^{-7}
Elementary charge	e	$1.602\,176\,53(14) \times 10^{-19}$ C	1.7×10^{-7}
Avogadro constant	$N_{\rm A}$	$6.022\,141\,5(10) \times 10^{23}$ mol^{-1}	1.7×10^{-7}
Faraday constant	$F = eN_{\rm A}$	$96\,485.338\,3(83)$ C mol^{-1}	8.6×10^{-8}
Josephson constant	$K_{\rm J} = \dfrac{2e}{h}$	$483\,597.879(41) \times 10^{9}$ Hz V^{-1}	8.5×10^{-8}
Electron mass	$m_{\rm e}$	$9.109\,382\,6(16) \times 10^{-31}$ kg	1.7×10^{-7}
Proton mass	$m_{\rm p}$	$1.672\,621\,71(29) \times 10^{-27}$ kg	1.7×10^{-7}
Neutron mass	$m_{\rm n}$	$1.674\,927\,28(29) \times 10^{-27}$ kg	1.7×10^{-7}
Bohr magneton	$\mu_{\rm B} = \dfrac{e\hbar}{2m_{\rm e}}$	$927.400\,949(80) \times 10^{-26}$ J T^{-1}	8.6×10^{-8}
Nuclear magneton	$\mu_{\rm N} = \dfrac{e\hbar}{2m_{\rm p}}$	$0.505\,078\,343(43) \times 10^{-26}$ J T^{-1}	8.6×10^{-8}
Proton magnetic moment	$\mu_{\rm p}$	$1.410\,606\,71(12) \times 10^{-26}$ J T^{-1}	8.7×10^{-8}

Table 8. Recommended values of the fundamental physical constants [1], the results for which are in no way related to other constants.

Fundamental constant	Symbol	Value (2002)	Relative standard uncertainty
Newtonian constant of gravitation	G	$6.674\,2(10) \times 10^{-11}$ m^3 kg^{-1} s^{-2}	1.5×10^{-4}
Molar gas constant	$R = kN_{\rm A}$	$8.314\,472(15)$ J mol^{-1} K^{-1}	1.7×10^{-6}
Neutron magnetic moment	$\mu_{\rm n}$	$-1.041\,875\,63(25) \times 10^{-3}\,\mu_{\rm B}$	2.4×10^{-7}
		$-1.913\,042\,73(45)\,\mu_{\rm N}$	2.4×10^{-7}

Table 9. Recommended values of the fundamental physical quantities [1] that are directly accessible from experiments with a fairly low accuracy and, therefore, can be derived from auxiliary constants (Table 4) and more exact constants (Tables 6 – 8).

Fundamental constant	Symbol	Value (2002)	Relative standard uncertainty
Boltzmann constant	$k = \dfrac{R}{N_{\rm A}}$	$1.380\,650\,5(24) \times 10^{-23}$ J K^{-1}	1.8×10^{-6}
Stefan – Boltzmann constant	$\sigma = \dfrac{\pi^2}{60}\,\dfrac{k^4}{\hbar^3 c^2}$	$5.670\,400(40) \times 10^{-8}$ W m^{-2} K^{-4}	7.0×10^{-6}

the whole the data agree with each other, the most accurate values dominate, and the data whose accuracy is lower by a factor of approximately five play practically no role. But if an essentially less accurate value differs from the more accurate one by several standard deviations, it affects the mean value.

Furthermore, in the absence of consistency of data, we are confronted with questions of confidence in the input data and of meaningful estimates of the errors in the final results. In the optimum case (in the absence of like problems) the correlation between blocks plays a marginal role, but nevertheless such correlations exist (e.g., they shift some values by several fractions of their errors).

The results of adjustment are published in the form of tables of recommended values of fundamental physical constants [1] (some of these values are listed in Tables 1, 2, 4, and 6 – 9). What is the field of application of the recommended values? The answer is somewhat discouraging. In cases where high accuracy is not needed very much, these tables are convenient and useful, and their use ensures unity of the quantities employed. But when high precision is really important, everything depends on the type of measurements (calculations). If we are speaking of relative measurements (say, of the nuclear magnetic moment in units of the proton magnetic moment or of a certain electrical resistance in units of resistance of a Hall sample determined by the von Klitzing constant $R_{\rm K}$) with the results subsequently converted into SI units, the CODATA recommended values are, unquestionably, the best choice. However, the values obtained in SI units play rather an illustrative role.

High-precision values of fundamental constants are indeed important when we have *to compare* different results. However, the situation changes dramatically when the comparison of the results of calculations and/or measurements is critical to the choice of the values of fundamental physical constants. Usually, it happens that a new experiment, or theoretical calculation, is a development of an earlier one already included in the adjustment scheme, and there is a certain correlation between the two. Hence, in the case of precision investigations one must consider the recommended value as an illustration, while the adjustment procedure should be considered a detailed and critical compilation of the original data with which the new precise findings should be compared.

One should clearly understand that the real objective of adjustment is not to compile tables of the most accurate values of fundamental physical constants. Adjustment makes it possible to evaluate the extent to which precise theoretical and experimental methods taken from various fields of

Table 10. Progress in refining the fundamental constants according to the results of the 1998 and 2002 adjustments. The change Δ is defined as the shift of a 2002 result [1] in relation to the 1998 result [14], σ refers to the combined standard deviation (with allowance for the correlation of new and old data), and $\varkappa = \sigma_{1998}/\sigma_{2002}$. All data except those in the last two columns are directly taken from Refs [1, 14].

Constant and unit	Value (1998)	Relative standard uncertainty	Value (2002)	Relative standard uncertainty	$\varkappa$	Δ/σ
h, J s	$6.626\,068\,76(52)\times10^{-34}$	7.8×10^{-8}	$6.626\,069\,3(11)\times10^{-34}$	1.7×10^{-7}	2.1	0.5
G, $m^3\ kg^{-1}\ s^{-2}$	$6.673(10)\times10^{-11}$	1.5×10^{-3}	$6.674\,2(10)\times10^{-11}$	1.5×10^{-4}	0.1	0.1
e, C	$1.602\,176\,462(63)\times10^{-19}$	3.9×10^{-8}	$1.602\,176\,53(14)\times10^{-19}$	8.5×10^{-8}	2.2	0.5
α^{-1}	$137.035\,997\,76(50)$	3.7×10^{-9}	$137.035\,999\,11(46)$	3.3×10^{-9}	0.9	−1.3
m_e, kg	$9.109\,381\,88(72)\times10^{-31}$	7.9×10^{-8}	$9.109\,382\,6(16)\times10^{-31}$	1.7×10^{-7}	2.2	0.5
R_∞, m^{-1}	$10\,973\,731.568\,549(83)$	7.6×10^{-12}	$10\,973\,731.568\,525(73)$	6.6×10^{-12}	0.9	−0.3
μ_B, J T^{-1}	$9.274\,008\,99(37)\times10^{-24}$	4.0×10^{-8}	$9.274\,009\,49(80)\times10^{-24}$	8.6×10^{-8}	2.1	0.6
a_e	$1.159\,652\,186\,9(41)\times10^{-3}$	3.5×10^{-9}	$1.159\,652\,185\,9(38)\times10^{-3}$	3.2×10^{-9}	0.9	−0.24
m_p/m_e	$1\,836.152\,667\,5(39)$	2.1×10^{-9}	$1\,836.152\,672\,61(85)$	4.6×10^{-10}	0.22	1.3
μ_p/μ_B	$1.521\,032\,203(15)\times10^{-3}$	1.0×10^{-8}	$1.521\,032\,206(15)\times10^{-3}$	1.0×10^{-8}	1.0	0.2
g_p	$5.585\,694\,675(57)$	1.0×10^{-8}	$5.585\,694\,701(56)$	1.0×10^{-8}	1.0	0.46
m_μ/m_e	$206.768\,265\,7(63)$	3.0×10^{-8}	$206.768\,283\,8(54)$	2.6×10^{-8}	0.85	2.9
a_μ	$1.165\,916\,02(64)\times10^{-3}$	5.5×10^{-7}	$1.165\,919\,81(62)\times10^{-3}$	5.3×10^{-7}	1.0	4.3
μ_μ/μ_p	$-3.183\,345\,39(10)$	3.2×10^{-8}	$-3.183\,345\,118(89)$	2.6×10^{-8}	0.9	2.7
R_K, Ω	$25\,812.807\,572(95)$	3.7×10^{-9}	$25\,812.807\,449(86)$	3.3×10^{-9}	0.9	−1.4
K_J, Hz V^{-1}	$483\,597.898(19)\times10^{9}$	3.9×10^{-8}	$483\,597.879(41)\times10^{9}$	8.5×10^{-8}	2.2	−0.5
N_A, mol^{-1}	$6.022\,141\,99(47)\times10^{23}$	7.9×10^{-8}	$6.022\,141\,5(10)\times10^{23}$	1.7×10^{-7}	2.1	−0.5
k, J K^{-1}	$1.380\,650\,3(24)\times10^{-23}$	1.7×10^{-6}	$1.380\,650\,5(24)\times10^{-23}$	1.8×10^{-6}	1.0	0.1
hN_A, J s mol^{-1}	$3.990\,312\,689(30)\times10^{-10}$	7.6×10^{-9}	$3.990\,312\,716(27)\times10^{-10}$	6.7×10^{-9}	0.9	0.9
R, J mol^{-1} K^{-1}	$8.314\,472(15)$	1.7×10^{-6}	$8.314\,472(15)$	1.7×10^{-6}	1.0	0.0

measurement and theory agree with each other. A major portion of the experiments included in the adjustment scheme deal with measurements involving the direct use of modern standards (and sometimes prototypes of future standards, which has actually happened to the quantum Hall effect and the Josephson effect) and thus make it possible to test their viability. The most advanced methods of measurements and calculations are often used in the fierce competition for higher precision, and to a certain extent the reliability of these methods is not obvious and requires tests (see Refs [18, 19] and Section 4.1).

Thus, from the standpoint of fundamental physics, specific verification of QED is one of the results of adjustment. The special role of QED objects (both free particles and simple atoms) stems from the fact that such objects represent a set of nonfundamental constants that can be directly expressed in terms of more fundamental constants, and these constants cannot only be evaluated but also be measured with high precision.

The special feature of such verification is the focus on the most accurate measurements. [6] Such measurements can verify QED only if we know the proper value of the fine-structure constant α. The comparison of QED quantities with quantities obtained outside QED is one of the most effective verifications.

3.2 Progress in determining the values of fundamental constants

The latest recommended values [1] are compared to the results of the previous adjustment [14] in Table 10. For a smaller set of constants (R_∞, m_p/m_e, α, h, N_A, and some others), Fig. 4 demonstrates the outcome of comparing the respective accuracies in several adjustments [1, 13, 14, 28]. The CODATA Task Group on Fundamental Constants was established in 1969, and all four adjustments represented in Fig. 4 have been conducted by this group.

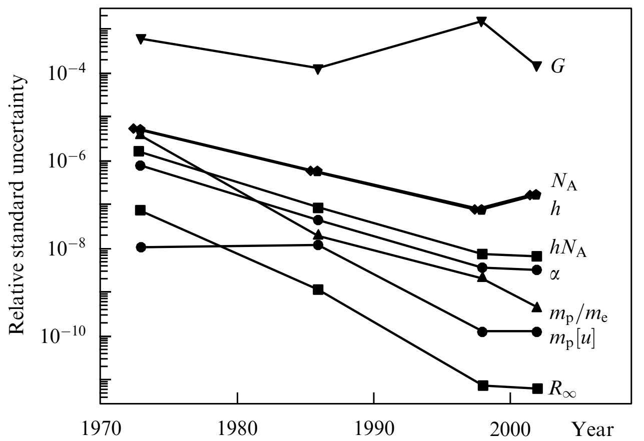

Figure 4. Progress in raising the accuracy of determining the values of fundamental physical constants (the period from 1969 to 2002).

The main changes in the values of fundamental constants that occurred in four years and are presented in Table 4 amount to the following:

- the shift of the value of the fine-structure constant α caused by improvements in the theory of the anomalous magnetic moment of the electron (a correction to the numerical value was found [29] that is larger than its standard deviation), and one more highly accurate value of α was obtained by the Raman spectroscopy method applied to caesium [30];
- an essential refinement of the value of the proton–electron mass ratio obtained by an entirely new method [31] (a certain shift of the result and a reduction of the uncertainty);
- slight shifts in the value of the Planck constant h and the related constants (e, K_J, N_A, and μ_B) and an increase of its uncertainty caused by the scatter in the data (see Ref. [1] for a discussion of Fig. 3 and references);

[6] It should be noted that it is not an only possible approach to test QED: positronium can be used for studying recoil effects no less efficiently than muonium and other atomic systems, but with a relatively lower degree of accuracy; the situation with the hyperfine splitting of the 2s-level in a helium-3 ion is similar.

• a shift in the muon – electron mass ratio (in connection with the earlier overly optimistic estimate of the error in the theory [7]);
• a shift in the value of the anomalous magnetic moment of the muon, a_μ, determined in Ref. [14] on a theoretical basis, and in Ref. [1] on the basis of an improved experiment [33] and a theory in which hadronic contributions have been recalculated (both their values [34] and the errors [35]), and
• considerable progress in determining the value of the Newtonian constant of gravitation G (the error in G in both adjustments is determined not by the accuracy of some of the data but by their scatter (Fig. 5); in 2002, the situation with the consistency of the data improved (see a discussion and references in Ref. [1]).

It should be emphasized that the errors do not decrease in time for all results. In some cases, the new data contradict the earlier data or new sources of errors, which reduce the accuracy, are discovered. Some constants vary noticeably from adjustment to adjustment, which is now carried out once every four years (for the 2000 adjustment of the 1998 results see Ref. [14], and for the 2004 adjustment of the 2002 results see Ref. [1]). However, an earlier adjustment deals with the 1986 results [13].

Why the recommended values were not published more frequently? The fact that the properties of elementary particles are published once every two years, and the volume of these publications (e.g., see Ref. [25]) noticeably exceeds the volume of CODATA publications [1], would seem a good enough example. Publication [25] is the result of a large collaboration effort which involved more than 100 scientists, and the greater parts of the sections are devoted to compilations of experimental data, done by separate groups independently. The main volume of papers in this area has to do with theoretical work, but compilations of theoretical results are rare.

The situation with fundamental constants is quite different. First, there is no real need to publish compilations of experimental data, since they refer to different quantities. Second, theory is critically involved in the processing of the data. Third, often we cannot limit ourselves to experiments in which the quantity we are interested in is measured directly. Here is an example involving the muon mass (actually, the muon – electron mass ratio) that explains the situation. There are several kinds of experiments in which this quantity can be obtained.

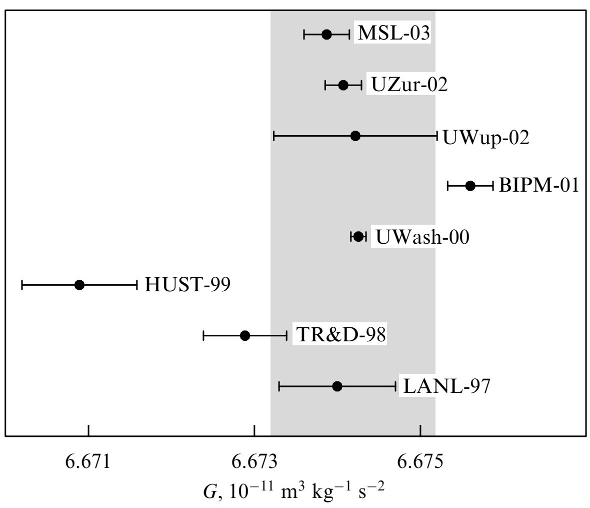

Figure 5. Determining the Newtonian constant of gravitation G. The shaded area corresponds to the recommended value. References can be found elsewhere [1].

[7] To our mind, this error has been understated in the modern value, too (a discussion of the realistic error can be found in Ref. [32]).

(1) When the *precession of the muon spin is measured* or *the positions of the Breit – Rabi levels in a magnetic field are determined*, relatively small corrections to the magnetic moments appear, but what is important is that the muon magnetic moment can be measured only in relative units. The proton magnetic moment is used as such a unit. To convert to mass, we should employ the equation

$$\frac{m_\mu}{m_e} = (1 + a_\mu)\,\frac{\mu_B}{\mu_p}\left|\frac{\mu_p}{\mu_\mu}\right|, \tag{18}$$

where to find the mass ratio we must determine, via separate investigations, the muon magnetic moment anomaly a_μ, and the proton magnetic moment in units of the Bohr magneton.

(2) The *mass ratio* m_μ/m_e can be measured directly. Such a ratio appears when we compare the frequencies of the $(1s-2s)$-transitions in hydrogen and muonium. In this case, the QED theory plays an important role.

(3) The numerical value that dominates in adjustment scheme [1] emerges from measurements and calculations of the ground-state *hyperfine* interval in the *muonium* atom. Here, the QED theory plays the key role. After all corrections have been taken into account, we can move from the real observed splitting to its naive magnitude, known as the Fermi energy, which corresponds to the nonrelativistic interaction of the electron and the muon magnetic moments, and then we can find μ_μ/μ_B.

Thus, different constants are measured in various experiments, and to interpret the constants considerable theoretical efforts and additional measurements are needed.

Sometimes the involvement of the data from other experiments in precision metrology takes on forms that appear very strange to a physicist. Slightly exaggerating, we can say that in some cases the constants that are really measured are not those that are claimed to be measured. Here is an example. To determine the fine-structure constant α, we can determine the ratio h/m and make use of formula (17). Such an experiment was carried out for the neutron [36], but its interpretation proved much more complicated than one would expect. The thing is that a crystal lattice was utilized to determine precisely the neutron de Broglie wavelength, with the result that the researchers measured $h/m_n v_n d$ (where d is the lattice spacing), while the neutron speed v_n was determined separately in 'macroscopic' units (i.e., irrespective of the lattice).

The research teams in some international centers have a lot of experience in taking relative measurements (comparisons) of lattice spacings of various crystals, and this is done fairly simply and reliably. The crystal lattice employed in the neutron experiment was compared to a lattice whose spacing had been measured in SI units (meters). At the same time, this absolutely calibrated lattice was utilized in measurements of the Avogadro constant and, in particular, to determine the lattice spacing of a perfect crystal. Of course, there can be no such thing as a perfect crystal, but one can grow a special crystal and estimate the corrections to its parameters (caused by the nonideality of the crystal).

It was found that perfect crystals grown and studied in different laboratories produce unlike results for the lattice spacing. After comparing the lattices proper rather than the measurement data, it was found that the lattice spacings of the crystals are consistent. This means that there were several contradicting experiments on absolute measurements of the lattice spacing, while the comparison of the lattice spacings was done correctly (see Ref. [1] for a detailed discussion and references). However, the problem of the lattice spacing and a perfect crystal goes far beyond the experiment in which the neutron Compton wavelength is measured.

Another example is the measurement of the gyromagnetic ratio of the proton and the helion (the nucleus of helium-3 atom) and the Faraday constant (see Figs 1 and 2). In all precision electrical experiments, measurements are actually taken not in SI units but in practical units, in other words, by means of the Josephson effect and the quantum Hall effect. As a result, in addition to the nominally measured constant there appear characteristic factors of the form

$$K_{\rm J}^n\, R_{\rm K}^m\,,$$

which, of course, changes the interpretation of the experiment.

Another important fact that sets the work on the adjustment of constants [1] apart from the compilation of data [25] is that all data in Ref. [1] have been correlated. As noted earlier, the data are divided into blocks, with the correlations between them being weak (but, nevertheless, they exist). What is more, this must be verified. In the event of incompatible accuracies, correlations may emerge when less accurate data do not agree with more accurate data and there occur common systematic effects. For instance, the accuracies with which the anomalous magnetic moments of the electron and the muon may be determined are not competitive, while the theories are largely similar. If there are doubts about the reliability of QED calculations for one constant, the other constant must also be considered unreliable. Another example is presented by the data for measuring the Avogadro constant. The error in determining this constant is determined by the errors in measuring the density, while the lattice constant is known with a much higher accuracy. Nevertheless, systematic effects may be caused by the same crystal defects.

When the various data disagree, the question arises of how to take them into account in a meaningful manner. Here, one often has in mind not so much the statistical evaluation of the data but rather the re-examination of their errors and the search for systematic effects. A similar problem arises when accuracy is improved substantially, by using a new method, and in some other cases. A critical study of the input data constitutes an important part of the work on the adjustment of constants, which sets adjustment apart from simple compilation of original results and their combined processing with the least-squares method.

4. Applications of fundamental physical constants

4.1 Quantum electrodynamics and fundamental physical constants

As noted earlier, quantum electrodynamics (QED) plays a special role in determining the values of fundamental physical constants. By studying free particles and simple atoms one can determine with high precision the values of the Rydberg constant R_∞, the fine-structure constant α, the electron–proton mass ratio $m_{\rm e}/m_{\rm p}$, the electron–proton magnetic moment ratio $\mu_{\rm e}/\mu_{\rm p}$, the electron and the muon g-factors, the mass and magnetic moment of the muon in natural units (say, the $m_\mu/m_{\rm e}$ and $\mu_\mu/\mu_{\rm p}$ ratios), and some other quantities. All these values can be obtained either by various methods that incorporate QED (thus verifying the self-consistency of QED) or by methods that do not incorporate QED (thus verifying it absolutely or testing its consistency with our understanding of certain phenomena from other areas of physics). Due to the limited space allotted to this review, I am unable to analyze in full the application of QED to the problem of determining fundamental constants (for a fuller account see, e.g., Ref. [19]). Here, I will briefly discuss the main QED phenomena that are important for these constants.

First, note that this area of physics is very close to *precision low-energy tests of QED*, and four types of errors may be present:

(1) the experimental error of the measured quantity proper (say, the hyperfine splitting in muonium or the frequency of the $(1s-2s)$-transition in hydrogen);

(2) the theoretical error related to the QED calculations proper (numerical evaluations, estimates of unknown contributions, etc.);

(3) the computational error related to the accuracy of the fundamental constants that enter into the calculations (of the Rydberg constant, the fine-structure constant, the muon mass, and others), and

(4) the error that emerges in calculating non-QED contributions (say, allowing for the contributions of the hadronic polarization of a vacuum to the muon magnetic moment anomaly, and the effects of nucleus structure for hyperfine splitting and the $(1s-2s)$-transition in hydrogen atom requires that additional experiments be done, and the accuracy of the data obtained in such experiments is not always satisfactory).

Today, QED calculations for free particles and two-body atoms (hydrogen, deuterium, muonium, and hydrogen-like ions) that have metrological applications place no limitations on the accuracy of comparing theory and experiment in any case [19]. The limiting factors in various cases could be the structure of the nucleus and the accuracy of calculations of hadronic contributions, experimental errors, the accuracy with which the values of the necessary constants are determined, but not the QED theory proper. In the case of three-particle atoms, such as neutral helium, and in more complicated systems, the accuracy of QED calculations may not be so high. It is also important to realize that from the standpoint of the problems being solved and the problems that emerge in the process, there are two quite different areas of precision application of QED.

(1) In the theory of free particles, problems emerge because of the large number of complex multiloop diagrams consisting of simple blocks. In the case of the purely electron fourth-order contribution to the anomalous magnetic moment of the electron, one has to deal with a 891 four-loop diagram consisting of free electron and photon propagators.

(2) In the theory of bound states, the most complicated problem is to find an adequate approximation and build an effective perturbation theory covering bound particles. The number of diagrams is not very large and they are fairly simple, but the blocks that form them take into account coupling effects and differ significantly from free propagators. The emergence of bound states is by itself a nonpertur-

bative effect and, hence, corresponds to the sum of an infinite number of free diagrams, in view of which the perturbation theory needs to be modified and becomes more complex. The current state of the precision physics of simple atomic systems is discussed in Refs [37, 38].

Let us now briefly analyze the main QED phenomena that have been studied with high accuracy.

Precision QED calculations for free particles are exhausted by the theory of anomalous magnetic moments of the electron and the muon, a_e and a_μ. The results of highly accurate measurements and calculation of the a_e value serve as a main source for finding the value of the fine-structure constant α in Refs [1, 14]. On the whole, the other values of α in Ref. [1] are less accurate, but they agree with $\alpha(a_e)$ (see Fig. 1). In the case of the muon, the magnetic moment a_μ is of certain interest as an isolated constant. What, possibly, is more important is that with the current methods of QED calculations there are substantial correlations between the computations of a_e and a_μ, but it is difficult to express them quantitatively.

The Rydberg constant is found on the basis of precision spectroscopy of hydrogen and deuterium. In doing this, at least two transitions must be measured so as to separate the contribution of the Rydberg constant and the QED contribution (the Lamb shift). The very process of finding R_∞ is a way to test on QED, since the number of exactly measured transitions exceeds the required minimum. After the Rydberg constant and the Lamb shift have been determined, we can proceed to a more substantial test of QED: a verification of the theory of the Lamb shift, which is one of the most important tests of QED methods in the theory of bound states. This test is most sensitive to effects corresponding to the behavior of an electron in an external field (the Coulomb field of an infinitely heavy nucleus).

For some time, the hyperfine splitting in muonium served as a means for finding the fine-structure constant. With the current level of precision, this effect is highly interesting if we wish to find the ratios m_μ/m_e and μ_μ/μ_p. The theory of hyperfine splitting differs dramatically from the theory of the Lamb shift, since recoil effects in the first case play a much more important role. Hence, the study of hyperfine splitting in muonium is a way to test another sector of QED for bound states. In the event of hyperfine splitting in hydrogen, the effects of finite size and structure of the nucleus happens to be very large compared to the accuracy of QED calculations. Therefore, muonium, which is free from these effects, is very attractive as a probe QED atom.

The magnetic moments of bound particles in two-body atoms differ from those of free particles, but they can be measured relatively easily and the corrections for the binding effects are calculated fairly easily as well. The measurements are done by placing the atoms in a uniform magnetic field. Research of this type plays an important role in determining a variety of constants:

- research involving muonium makes it possible to find μ_μ/μ_p;
- measurements in hydrogen and deuterium lead to the most accurate results for μ_p/μ_e and μ_d/μ_e, and
- experiments with hydrogen-like carbon and oxygen (comparison of the precession frequency of the ion spin and the ion cyclotron frequency) make it possible to determine m_e/m_p with the highest accuracy.

It is expected that investigations into the Lamb shift in muonic hydrogen will make it possible to substantially refine the charge radius of the proton and thus to improve the accuracy and reliability of theoretical calculations for the Lamb shift in ordinary hydrogen, which in turn will affect the way in which the Rydberg constant is determined.

Recent progress in theory and experiment in the fine structure of helium suggests that in the near future helium studies will lead to a new method of determining α.

There are also some other quantities that are of interest from the standpoint of verifying the efficiency of modern QED approaches to the problem of bound states but which have no applications to fundamental constants: various transition frequencies in the spectrum of positronium and its annihilation widths, hyperfine splitting of the excited $2s$-state in hydrogen, deuterium, and helium-3 ion (for more details see Ref. [19]), and some spectral characteristics of multiply charged ions and muonic atoms [37, 38].

4.2 Macroscopic quantum phenomena and units of electrical quantities

As noted earlier, a system of electromagnetic units demonstrates its salient features. Several scenarios for realizing such a system are possible. The base units may be the ampere, the volt, and the ohm, where the unit of resistance can, in principle, be replaced by the unit of capacitance or inductance. These three base units may be 'independent'. In this case, for them to coincide with SI units, the following three conditions must be satisfied:

$$\mu_0 = 4\pi \times 10^{-7}\ \mathrm{H}_*\,\mathrm{A}_*^{-2}\,, \tag{19}$$

$$\mathrm{W}_* = \mathrm{A}_*\,\mathrm{V}_*\,, \tag{20}$$

$$\mathrm{V}_* = \mathrm{A}_*\,\Omega_*\,. \tag{21}$$

If the units are 'independent', this means that measurements are done in certain maintained units, which are not realized on the basis of the SI, and their correspondence to the SI units constitutes a separate problem, so that some of the above conditions may be violated. But if the units are realized from the start as SI units, all three conditions (19)–(21) are met automatically. To distinguish between 'independent' maintained units and SI units, we label the former with asterisks.

If condition (21) is not satisfied, the system of electrical units is not self-consistent and an additional factor appears in it, in contradiction to Ohm's law. The other two conditions (19) and (20) do not play any special role since the 'electrical' watt may be represented in different ways: $\mathrm{A}_*\,\mathrm{V}_*$, $\mathrm{A}_*^2\,\Omega$, or $\mathrm{V}_*^2\,\Omega_*^{-1}$. If Ohm's law is violated, all three units of power are different. A similar situation occurs in interpreting condition (19).

If condition (21) is met, then, on the contrary, feasibility of condition (20) becomes meaningful: it controls the consistency of mechanical and electrical units. If all three units are matched, we must check condition (19) which makes it possible to answer the question of whether we are dealing with the SI ampere or with a slightly different ampere. For instance, if we define the unit of charge in terms of the electron charge and do the same with other electrical quantities, conditions (20) and (21) are satisfied and the system of electrical units is self-consistent and matches the mechanical units, but the ampere defined in this way differs from the SI ampere.

The different scenarios for realizing electrical units depend on how many units are realized as 'independent'. Here there are four possibilities of a fundamental nature.

(1) We reproduce all electrical units strictly in accordance with SI definitions, and then there are no 'independent' units.

(2) We realize a system with one 'independent' electrical unit, but then condition (19) is violated.

(3) From the practical viewpoint, two is the smallest number of 'independent' units. This makes it possible to realize a self-consistent system of highly stable standards of units of all electrotechnical quantities. But conditions (19) and (20) are violated in this case, which is of great importance only for a limited number of measurements.

(4) A system based on two 'independent' units can be set up on the basis of macroscopic quantum phenomena. To compare maintained units with SI units, we can employ the respective values of fundamental physical quantities, but the accuracy of measurements in SI units will be lower than that of measurements done in maintained 'independent' units. One of the problems that emerges here is whether our ideas about macroscopic quantum phenomena are meaningful. The system of standards with three 'independent' units partially solves this problem. If our understanding of macroscopic quantum phenomena (and the standards based on these phenomena) is correct, Ohm's law [condition (21)] is valid. But if there are unaccounted theoretical errors in describing the macroscopic quantum phenomena or systematic errors in realizing the standards, Ohm's law is violated.

At present, there exists a system of standards based on the 'independent' standards of ohm and volt. A system based on an 'independent' ampere standard is currently being developed.

The elegance of macroscopic quantum phenomena in metrology lies in the rare combination of the 'quantum nature' and 'macroscopic nature' of such phenomena. Thus, most quantum phenomena manifest themselves on the atomic scale (or even over shorter distances of nuclear physics and particle physics), and it is difficult to compare the characteristics of these phenomena with macroscopic objects. But the standards of units must be macroscopic devices. Macroscopic quantum phenomena make it possible to build macroscopic devices fairly easily, and such devices partly have quantum characteristics that, by their very nature, can be reduced to natural constants.

There are three effects that are extremely attractive from the standpoint of building standards for the units of the main electrical quantities:

- the Josephson effect, in which the potential difference $U_n^{(\mathrm{JE})}$ may take only special fixed values;
- the quantum Hall effect, in which the resistance $R_n^{(\mathrm{QHE})}$ assumes quantized values, and
- single-electron tunneling, in which the current $I^{(\mathrm{SET})}$ is quantized.

The corresponding equations take the form

$$U_n^{(\mathrm{JE})} = \frac{n\nu}{K_{\mathrm{J}}} , \tag{22}$$

$$R_n^{(\mathrm{QHE})} = n\, R_{\mathrm{K}} , \tag{23}$$

$$I^{(\mathrm{SET})} = Q_{\mathrm{e}}\, \nu , \tag{24}$$

where

$$K_{\mathrm{J}} = \frac{2e}{h} , \tag{25}$$

$$R_{\mathrm{K}} = \frac{h}{e^2} , \tag{26}$$

$$Q_{\mathrm{e}} = e , \tag{27}$$

n is a certain integer, and ν is the measured frequency. The notation Q_{e} is not common and often equation (24) incorporates e. This is not done here for the following reason.

If the theory of the above three macroscopic quantum effects and the realization of standards agree with the above equations, then

$$K_{\mathrm{J}} R_{\mathrm{K}} Q_{\mathrm{e}} = 2 . \tag{28}$$

But if there are errors, condition (28) is not satisfied and then the units based on formulas (22)–(27) violate Ohm's law. Hence, when checking condition (28), it is important to distinguish between Q_{e} and e.

At present, the Josephson effect and the quantum Hall effect are successfully employed in standards, while the single-electron tunneling effect has formed a promising area of research but so far has not achieved the precision required by standards. In 1988, the International Committee on Weights and Measures (CIPM) adopted practical recommendations for reproducing and maintaining the standards of ohm and volt [5–7]. The recommendations took effect on January 1, 1990, and the corresponding units are called ohm-1990 (Ω_{90}) and volt-1990 (V_{90}). These units are defined on the basis of equations (23) and (22) and fixed values of the fundamental constants in these equations and their errors. The corresponding values are listed in Table 11.

The CIPM recommendations read in two ways:

- using the values of constants without errors [5, 6], we can operate with the practical units (Ω_{90} and V_{90}) which are universally defined the world over but which violate conditions (19) and (20), and
- adopting the values of constants with errors [5, 7], we can conduct measurements in the SI units but with a loss of accuracy.

When discussing the *recommendations on practical realization of the definitions of units* (e.g., see Refs [4–8]), one should bear in mind the special nature of metrology (see Section 1). The approach to accuracy in official metrological documents differs substantially from the purely scientific approach in favor of higher reliability (in view of possible legal consequences). Papers [4–8] and other publications presenting the recommendations are often simply reprints or expositions of the official documents prepared by various Consultative Committees and approved by CIPM. The uncertainties used in these documents differ from those of the same quantities given in scientific compilations, such as the CODATA materials [1].

The CODATA recommended values [1] are the result of the work of a Task Group on Fundamental Constants and *contain no* recommendations for reproducing the units, which

Table 11. CIPM recommended values of the Josephson [5] and von Klitzing [6, 7] constants.

Constant	Symbol	Value in practical units	Value in SI units	Relative standard uncertainty (in SI units)
Josephson constant	K_{J}	483 597.9 GHz V_{90}^{-1}	483 597.9(2) GHz V^{-1}	4×10^{-7}
von Klitzing constant	R_{K}	25 812.807 Ω_{90}	25 812.807 0(25) Ω	1×10^{-7}

Table 12. Results of direct measurements of the Josephson constant K_J included in the adjustment procedure [1].

Josephson constant K_J	Relative standard uncertainty	References
483 597.91(13) GHz V^{-1}	2.7×10^{-7}	[39]
483 597.96(15) GHz V^{-1}	3.1×10^{-7}	[40]

Table 13. Results of direct measurements of the von Klitzing constant R_K included in the adjustment procedure [1].

von Klitzing constant R_K	Relative standard uncertainty	References
25 812.803 31(62) Ω	2.4×10^{-8}	[41]
25 812.807 1(11) Ω	4.4×10^{-8}	[42]
25 812.809 2(14) Ω	5.4×10^{-8}	[43]
25 812.808 4(34) Ω	1.3×10^{-7}	[44]
25 812.808 1(14) Ω	5.3×10^{-8}	[45]

makes it possible to solve the problem of the most exact consistent values of fundamental constants. Progress in the field of electrical standards is illustrated by the data listed in Tables 12 and 13, which give some results of *direct* measurements of the K_J and R_K constants.

It should be emphasized that *indirect* determinations of the constants K_J and R_K dominate in the results of adjustment, which agrees with the modern strategy of reproducing units. *Direct* measurements are close to the traditional way of reproducing units: the K_J and R_K constants are determined by comparing quantum standards (which maintain practical units) with classical standards (which reproduce SI units) and to a great extent encounter the same difficulties. A similar role is played by the direct measurements of the quantity [46, 47]

$$K_J^2 R_K = \frac{4}{h} \tag{29}$$

which realizes the 'electrical' definition of the Planck constant.

In the absence of a better term, the formal status of the ohm and the volt in the modern system of units can be characterized as *adjustable units*. We note the important fact that drawing-up a legal document (recommendations) is a much more complicated and time-consuming procedure, with the result that the official recommendations are published less frequently and with a time lag. For instance, since 1988 no new CIPM recommendations on the practical realization of the magnitudes of the volt and, hence, the Josephson constant have been published. In Ref. [7], the accuracy of determining the von Klitzing constant is doubled, but even today the accuracy of two constants is much lower than that of the CODATA recommended values (Figs 6 and 7). In view of this, we propose a differentiated approach to the use of official recommendations. In doing scientific research it is advisable to employ the CODATA recommended values [1] of the von Klitzing and Josephson constants or even original results, while in building standards or commercial devices one should strictly follow the CIPM recommendations.

5. Concluding remarks

Fundamental physical constants play an ever increasing role in modern standards. They make the units more natural and universal. This, in turn, makes the system of standards more 'democratic'. A fairly wide segment of users maintains the sizes of units based on quantum phenomena and employs

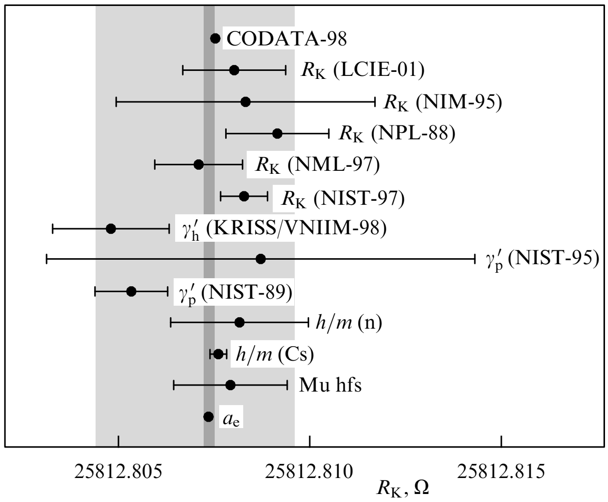

Figure 6. Determining the von Klitzing constant R_K. The indices correspond to the directly measured values. The wide shaded belt conforms to the CIPM recommended value [7], while the narrow shaded belt complies with the CODATA recommended value [1]. Explanations and references to experimental work can be found in Ref. [1].

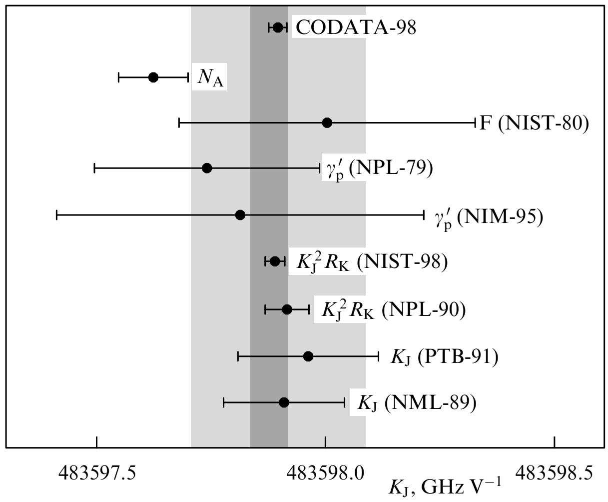

Figure 7. Determining the Josephson constant K_J. The indices correspond to the directly measured values. The wide shaded belt conforms to the CIPM recommended value [5], while the narrow shaded belt complies with the CODATA recommended value [1]. Explanations and references to experimental work can be found in Ref. [1].

them in various measurements. A narrow segment of specialists determines the values of natural parameters needed so that the maintained units coincide with the SI units. While precision measurements of the values of constants involve extremely complicated experiments comparable to classical standards, the realization of standards maintaining natural units may be accessible for nearly ordinary user.

What is important is that a direct 'instrumental' connection between two groups of researchers (using standards and determining the true sizes of units) is reduced to a minimum (control tests and certification) or is simply absent. Let us just recall the situation with the standard of the second. In the classical setting, time was determined by astronomical measurements, and then, after several stages, clocks of ordinary users were compared with the principal clock calibrated by the Earth's motion. Today caesium clocks can simply be bought. In contrast to other quantum standards, there is no need here to measure any constant: it is built into the SI by definition. One of the most fundamental constants, the speed of light in vacuum c, directly enters into the definition of the SI units. Recommendations on the reproduction of the SI ohm and volt are also related to fundamental constants, but their values are obtained through measurement.

At the same time, a correct understanding of research in the field of fundamental constants and natural units makes for a better understanding of nature. One of the areas where modern standards intersect with fundamental physics is the search for possible changes in the values of fundamental physical constants with the passage of time.

What is needed now is a better understanding of physics at small distances, which could dramatically change our approach to fundamental constants. An important step must be taken from effective observed parameters, such as the electron charge, the electron mass, and the proton mass, with which we deal in practical life, to more fundamental quantities. However, our understanding (on the quantitative level) largely has to do with rejecting unreasonable suppositions (such as Dirac's hypothesis for large numbers, which assumes that some constants change linearly with time) rather than with constructive theories with verifiable predictions. It is very possible that the next step toward understanding small-distance physics and truly fundamental constants will not be taken soon.

Nevertheless, 'effective' constants, which we constantly encounter in experiments and whose values are given in the present review, while being not quite fundamental constants from the standpoint of modern theory, play a fundamental role in modern physics and determine our capability of doing precision measurements and realizing a system of units that most fully meets modern requirements of both fundamental physics and many applications. For a science based on experiment this is not such a small achievement.

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6. Appendix. On the *Système International d'Unités* (SI)

6.1 SI base units

The International System of Units proposes seven base units (Table 14). These have been defined in very different ways to design most precise and stable standards and to ensure that the most accurate measurements in terms of these units can be made. Thus, the second is defined in terms of a natural unit of time (frequency), the hyperfine splitting interval in caesium, while the kilogram is the last representative of units defined on the basis of artifacts.

Table 14. Base units of the International System (SI).

Physical quantity	Name	Symbol
Length	meter	m
Mass	kilogram	kg
Time	second	s
Electric current	ampere	A
Thermodynamic temperature	kelvin	K
Amount of substance	mole	mol
Luminous intensity	candela	cd

Actually, only three units are truly independent: the unit of mass, the unit of time, and the unit of length. In the latest version of the SI [2], the unit of length (the meter) is defined as the distance that light travels in a certain time. Of course, such a definition is highly impractical, and in reality the reproduction of the meter is related to the wavelength of electromagnetic radiation of a known frequency. From the standpoint of physics, these two approaches are equivalent, but the adopted definition is accessible to the public, which is basically important.

One might suspect that with the adoption of such a definition the unit of length can be derived from the unit of time, but this is not the case. The unit of length is a derived unit of time and the natural unit of speed (which is the speed of light in vacuum). Thus, the number of independent units remains unchanged. For certain reasons it is convenient to retain the status of the meter as a base unit.

The units of thermodynamic temperature and the amount of matter are not quite independent, but neither can they be considered derived units, since the conversion factors are, generally speaking, unknown and are the subject of measurements. The definition of the unit of electric current (ampere) appears to be a definition of a derived unit, and in the CGS system this unit was a derived one. The question of the status of the ampere is not an indisputable one, and there exist various viewpoints concerning this question. My position is explained in Section 6.2. I believe that the SI units are not without faults and the ampere is not the only unit that needs commenting on.

In addition to the above-mentioned base units, there are 'geometric' units (the radian and the steradian) which had the status of *supplementary units*.[8] The problems of units is closely related to the problem of dimensions. Dimensionless quantities need no units, but often have one to deal with specific quantities, i.e., quantities related to one atom (one molecule) or a certain number of atoms (molecules). Formally, the number of atoms is a dimensionless quantity, while mass related to a single atom has the same dimension as a 'simple' mass. However, in practical life this quantity (the number of atoms) is occasionally considered dimensional.

Without going into detail of the SI units, we nevertheless emphasize the special status of the unit of luminous intensity, the *candela*. This unit is defined as "*the luminous flux in a given direction emitted at 540×10^{12} Hz in a solid angle and equal to 1/1.683 watt per steradian*". This definition appears to be

[8] These units have been included into the derived units in 1995.

rigorous enough. At the same time, in Appendix 2 of the official description of the SI we read [2]: "*The definition of the candela given in page 98 is expressed in strictly physical terms. The objective of photometry, however, is to measure light in such a way that the result of the measurement correlates closely with the visual sensation experienced by a human observer of the same radiation*." This differs substantially from W sr^{-2}, while the respective quantity (luminous flux) corresponds not to a physical quantity but to the action of a certain physical factor (light) on the human eye.

In our opinion, such a unit is superfluous in a system of physical units. It is a well-established fact that certain physical phenomena (light, sound, radiation, etc.) highly specifically affect the human body and are detected by us very subjectively. The action or detection effect depends both on the absorbed energy (or power) proportional to the energy flux and on the sensitivity depending on frequency and, possibly, other factors. Fluxes, doses, etc. can be measured by physical instruments in ordinary units (in particular, in W sr^{-2} instead of the candela), while the sensitivity of the human body must be taken into account by methods adopted by the respective metrological organizations that deal with questions of physics as well as biology and medicine. Here, the units may have special names, such as the 'roentgen equivalent man' (rem), which clearly indicates a physical unit adopted as a basis and takes into account additional nonphysical factors. Such units must in no way be adopted as SI units, since SI is a system of units of physical quantities.

Clearly, the choice is very limited in such a situation. By agreement we can adopt a corresponding function that takes into account the effect of light on the human eye. In this case, we are speaking of modeling a subjective nonphysical quantity with a certain physical quantity, and so we are confronted with the problem as to what extent the model is conventional and how the model is related to SI agreements. Or we can measure the real function corresponding to the sensitivity of the human eye. Then, possibly, we must introduce a special unit, but the quantity becomes more biological or medical than physical, since it requires that two factors be rigorously defined:

- the 'average' human being, and
- the 'same effect' or the 'same action'.

In the case of the luminous flux there are several questions that must be answered. We can take a brief look at two parts of a room and decide which part is exposed to more light. But if we take a long look, the conclusion may be different. Finally, instead of making subjective conclusions concerning the perception of illuminance, we can do various tests that are critical in illuminance (speed of reaction or quality of work). It may very well be that all such tests will produce results that show different sensitivities of the human eye as functions of frequency. These questions have to do not with the accuracy of measurements but with the extent to which the definitions of the measured quantities are subjective.

Let us discuss in greater detail how a similar problem has been solved in studies of radiation. Roentgen is a non-SI unit. In the SI, radiation and its effect are described somewhat differently. The radiation effect is related to ionization. The ionizing power of radiation is characterized by the kinetic energy of ions and the electrons liberated in ionization. Their energy per unit mass is measured in grays (1 Gr = 1 J kg^{-1}).

Following the recommendations of the International Committee on Radiological Protection (ICRP) (e.g., see Ref. [48]), we break down these quantities into three groups:

- 'physical quantities', which characterize ionizing radiation proper and do not depend on the human body (say, the absorbed radiation dose measured in grays);
- 'normalized quantities', which determine the measure of damage inflicted by ionizing radiation, i.e., nonphysical quantities that fully and meaningfully describe the effect of physical factors on the human body, such as the effective equivalent dose measured in sieverts (Sv), and
- 'operational quantities', which are rigorously defined in terms of the physical characteristics of the radiation field and are intended for estimating normalized quantities such as the ambient equivalent dose of radiation absorbed by the human body, which is also measured in sieverts.

The sievert is a subjective unit and is defined as the effect of a given type of radiation of a dose of 1 Sv on a given human organ. This unit coincides with the effect of 1 Gr of radiation dose with a small linear energy transfer (no higher than 10 keV μm^{-1}), which corresponds, for instance, to X-ray radiation.

The term 'normalized quantities' is not fully adequate. The desire to 'normalize' the real effect of radiation by various legal documents is quite understandable, but a real strategy amounts, of course, to the study of normalized quantities in order to develop their description by operational quantities in the most meaningful way (in other words, to develop an appropriate model) and to employ thereafter the operational quantities in legal documents.

The division of quantities into physical and nonphysical and the use of different units for these two types of quantities is very indicative and may serve as an example for photometry and other similar areas. An important element in describing the effect of radiation on the human body is the introduction of operational quantities that play an intermediate role.

The reader will recall that rejection of units related to the 'average' human being was predetermined not only by the rise in measurement accuracy but also by the fact that the very notion of an 'average' human being has an ambiguous physical status. For instance, a considerable number of human beings live in appalling conditions, which determine their level of health. Do different sensitivities depend on the 'average' level of health? If they do, they must change with time. Moreover, in view of various social, historical, and climatic factors, some sensitivities may vary from region to region. It is also natural to expect different sensitivities of the eyes of human beings who spend most of their life in conditions of artificial lighting and under natural light.

Such problems should be the subject of scientific research. Hence, the fixation of sensitivity as a function of frequency may be convenient from the viewpoint of lawmaking in the fields of safety and ecology, but it does not satisfy a rigorous definition of luminous intensity, which is related to the real properties of the human eye.

Thus, a rigorous definition of luminous intensity (on the basis of the real properties of the 'average' human eye) contradicts the definition of the unit of luminous intensity, which presupposes a certain fixed function of eye sensitivity. If sensitivity is measurable and, hence, a function that is not well known, the unit of measurement is actually ambiguous. We believe that incorporating such poorly defined quantities into a physical system of units is a step backward.

From the standpoint of building the SI units, it is also important to note that the unit of luminous flux (candela) is incorporated into the SI as a base unit. Meanwhile, neither the candela nor the luminous flux is an essential element of standard courses of college physics. At the same time, the other SI base units (second, meter, kilogram, ampere, kelvin, and mole) are vital to high-school physics courses. The introduction of a special (subjective) and not quite independent unit as a base unit cannot but be surprising.

6.2 The SI and the laws of electromagnetism in a vacuum

Critics of the SI units (e.g., see Refs [9–11]) of the way the system is applied to electromagnetic phenomena remark on the unjustified complications in the expressions for the basic laws of electromagnetism and a number of misunderstandings related to the presence of four nonindependent fields of different dimensions: **E**, **B**, **D**, and **H**. Note that here two quite different concepts are confused: the system of units and the system of physical quantities. The two systems emerged and developed in parallel, but their history and the practice of their use are not the same. Most criticized the system of quantities utilized in describing electromagnetic phenomena. The SI can be kept intact, but the way the quantities are used to describe electrodynamics in a vacuum can be changed. Taking into consideration their redundancy, this can be done easily.

Suppose that we only use

- the electric constant ε_0, expressing the magnetic constant μ_0 in terms of ε_0 and c, and avoid using μ_0 explicitly, and
- the electric field strength **E** and the magnetic flux density **B**, and avoid using the electric displacement **D** and the magnetic field strength **H**.

If we adopt these two ideas as recommendations for education and science, we can reduce to a minimum the drawbacks of the SI while retaining its advantages.

For instance, the Maxwell equations become

$$\begin{aligned} &\operatorname{div} \mathbf{E} = \frac{1}{\varepsilon_0}\,\rho\,, \\ &\operatorname{curl} \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}\,, \\ &\operatorname{div} \mathbf{B} = 0\,, \\ &\operatorname{curl} \mathbf{B} = \frac{1}{c^2}\left[\frac{\partial \mathbf{E}}{\partial t} + \frac{1}{\varepsilon_0}\,\mathbf{j}\right]. \end{aligned} \tag{30}$$

The equations differ from those in the centimeter–gram–second system by the presence of the factor ε_0 at the sources of the fields (charge density and current density). Common practice, of course, stems from the ether concept, where μ_0, **D**, and **H** had independent physical meaning. But these three quantities have lost their independency, and so there is no need to apply them in the equations for a vacuum.

The speed of light in vacuum c also enters into equations (30) somewhat differently than it does in the appropriate CGS equations, while the dimensions of the electric field (**E**) and the magnetic field (**B**) differ, but it is questionable whether this is a drawback. If we do not want to set $c = 1$, we must remember that the magnetic field corresponds to the purely spatial components $F_{\mu\nu}$ of the 4-tensor, while for the electric field one index is temporal. Thus, the magnetic field contains the term $\partial A_i/\partial x_j$, while the electric field contains the term $\partial A_i/\partial t$. In noncovariant form, it is natural to take the derivative with respect to time t rather than with respect to the zero component $x_0 = ct$ of the 4-vector. Thus, the dimensions of **E** and **B** must be different, i.e., just as $x_0 = ct$ and $\rho = (1/c)\,j_0$, it is natural to put $\phi = (1/c)A_0$ and define

$$F_{\mu\nu} = \left(\frac{1}{c}\,\mathbf{E},\ \mathbf{B}\right). \tag{31}$$

What are the advantages of the SI units? The answer is not the convenient practical magnitudes of the ohm, ampere, volt, etc. The problem is much deeper. Electrical quantities can be compared one to the other much more easily and accurately than to mechanical quantities. This is characteristic of relative measurements. In such a case, we first adopt a special practical unit (or even several units), then all the results are expressed in terms of this unit (units).

Most units of this type are either non-SI (as the atomic mass unit) or unofficial (as the nuclear magneton). However, in some special cases these units are incorporated into the system of units as base units. (For instance, the mole and the kelvin are, actually, special units of the number of particles and the specific energy of matter.) This is done because of the particular importance of the physical quantities measured in these units. Bearing in mind the importance of electromagnetic measurements, it is advisable to do the same with electrical units, for which it is enough to adopt a special definition of one of these units. We can fix its relation to mechanical units (as is done at present) or, to cite an example, we can express charge in terms of the electron charge (the consistent application of macroscopic quantum effects in building the standards will lead to this).

Why does the fixation of the electrical unit in terms of mechanical units require that $\varepsilon_0 \neq 1$? The answer lies in the practice. Decades ago the primary standards which directly realized the definitions were based on classical physics. (We are speaking of various electrical balances and capacitors with calculable capacitances.) However, such a standard is cumbersome and impractical. An alternative could be artifact standards similar to the kilogram prototype. In this case, the electric constant $\varepsilon_0 \neq 1$ would be a measurable quantity. Clearly, the normal cells and the standard resistors are subject to many more hazards than standard weights, so that within such an approach it is impossible to ensure high reliability and long-term stability.

Experience has shown that the greatest error in an artifact standard is the systematic one. The standard may give reproducible results and be used in measurements. Taking into account the inevitable rather than the possible slow drift of parameters (related, in particular, to the ageing of the elements of the standard), we can monitor the reproducibility of results over relatively short periods of time, while it is extremely difficult to test the long-term reproducibility. Together with the use of the standard for measurements and realization of derived units, the standard must be checked for drifts of parameters and for unaccounted systematic effects. The latter was possible, since it usually consisted of artifacts of a single type (say, normal cells), so that drifts could be estimated by studying the relative drifts and so on.

The strategy consisted in combining two types of standards: primary, based on SI definitions, and artifacts, which were calibrated with the primary standards. The nature of the errors was such that it was found to be convenient to adopt the 'intermediate' results as the practical units (e.g., ampere-1969) that could be used up to the point in time when more precise determinations would be made. No 'final' results were possible in principle, since the evolution of the cumbersome precision macroscopic devices would never end, and this was true of primary standards, too, due to their cumbersomeness and the systematic errors that were sure to appear. The magnitudes of the electrical units 'lived their own lives', and fixing ε_0 as a dimensionless constant would simply be an illusion.

Later the strategy changed. All the base units of electrical quantities are reproduced on the basis of macroscopic quantum phenomena, and their size is determined by the values of fundamental physical constants (the Josephson constant, and the von Klitzing constant) which are measured separately. The relation of the maintained units to fundamental constants removes the problem of drifts. The next step that can be taken in the future is to fix the constants by definition. The fixation of one of them means that electric charge will be measured in units of electron charge (the electric constant ε_0 will remain dimensional but will be measurable), while the fixation of two constants will immediately lead to a new definition of the kilogram, based on the fixed value of the Planck constant, which together with the frequency standard will make it possible to reproduce the units of energy and mass. The problems associated with the realization of standards of electrical units are discussed in more detail in Section 4.2.

Strange as it might seem, from the standpoint of fundamental physics it is also advisable to introduce the parameter $\varepsilon_0 \neq 1$. The modern approach to problems of measurement units and standards is that the most natural standards are the ones based on fundamental constants, and one should strive for such a system of units and standards. The units of time and length are defined in terms of natural constants. When certain progress has been achieved, we can express the unit of charge in terms of the fixed value of the electron charge e, and the unit of mass in terms of the fixed value of the Planck constant h. But there is also a special dimensionless combination of the constants that incorporates the constants e and h, namely, the fine-structure constant

$$\alpha = \frac{e^2}{4\pi\varepsilon_0 \hbar c} \,.$$

The value of α is fixed by nature and can be known only to a finite accuracy. If we want to fix the values of the dimensional constants e and h exactly (note that the speed of light in vacuum c is already fixed) and not arrive at contradictions, we need the constant $\varepsilon_0 \neq 1$. In the above scenario this constant becomes measurable and actually gives an answer to the question that such a constant in a fundamental theory must give: how is the fundamental charge e related to another, possibly more fundamental, analog of charge, $(\hbar c)^{1/2}$.

In other words, if we ignore the fact that the numerical values of the natural SI units are expressed in terms of the appropriate natural units by use of specially selected coefficients (which differ from unity and conserve the illusion that the SI units remain unchanged), the constant ε_0 will turn out to be a dimensional analog of α. This is especially evident in a system of units in which frequency, energy, mass, and potential difference are measured in the same units and, hence, $h = c = e = 1$. In such a system of units, one has

$$\varepsilon_0 = \frac{1}{2\alpha} \,.$$

Note that the Avogadro constant, the Boltzmann constant, and other dimensional constants (used for conversion of 'different' units of a 'single' physical quantity) are in a certain sense dimensional analogs of dimensionless numbers. A constant acquires dimensions because of inconsistencies in the definition of the dimensions of units, which makes it impossible to reduce some units to multiples of other units with an absolute accuracy. Sometimes, such inconsistency is of historic nature and can be removed, but sometimes (as in the above scenario with the constants ε_0 and α) the inconsistency is of a physical nature.

One must also bear in mind that the Maxwell equations for the three-dimensional electric and magnetic fields in a vacuum have two natural generalizations: the relativistic equations with 4-tensors for electrodynamics in a vacuum, and the equations for electromagnetic effects in media. In the latter case, for the majority of problems it is only natural to immediately select a reference frame in which the medium is at rest. Hence, the requirements that the three-dimensional equation in a vacuum be natural, graphic, transparent, etc. are not really so obvious: they depend on what generalization is considered more important. For instance, from the standpoint of the physics of fields in a vacuum, the description of two fields by four vectors is redundant, while from the standpoint of the electrodynamics of continuous media such a description is natural as an important preparatory step.

The problem of the redundancy in the description of electrodynamics in a vacuum appeared not in connection with the SI units but historically: as the problem of ether and of the noncovariance of the approach. For instance, neither in SI nor in CGS can the terminology be considered satisfactory: one of the 4-tensors is formed by the *electric field strength* **E** and the *magnetic induction* (the magnetic flux density) **B**, while the other is formed by the *electric displacement* (earlier, *electric induction*) **D** and the *magnetic field strength* **H**. The reason is that the phenomenologies of electrostatics and magnetostatics were built asymmetrically and without proper understanding of the interrelation and analogy between them. Relativistically, it would be more natural to call **B** the magnetic field strength, not **H**.

The redundancy in the description of electrodynamics in a vacuum is also due to education. The traditional school and college education in Russia gravitates largely to the logic of the electrodynamics of media. First, electrostatics in a vacuum is studied, then in media, next comes magnetostatics (in a vacuum and in media), and only then are the physics of variable fields and the Maxwell equations studied. In such an approach, vacuum is considered a particular limiting case of matter rather than the general fundamental case.

Vacuum as 'electromagnetically inert' matter with unit relative permittivity and permeability and a naturally fixed reference frame is not a very big step away from the ether concept. The introduction of four vectors even in the case of a vacuum becomes natural. More natural, in my opinion, is the approach of the Berkeley Physics Course (Electricity and Magnetism) [49], where the first eight chapters are devoted to electrical and magnetic phenomena in a vacuum, while Chapters 9 and 10 are devoted to electric and magnetic fields in media.

Returning to equations (30), we note that in contrast to fixing the units (which requires certain procedures), the selection of reproducible quantities characterizing the fields requires no special agreements, provided that there is redundancy. The situation with electrodynamics in media is more complex, since, by analogy with the CGS system, it is natural to use instead of **D** and **H** units with the same dimensions as those of **E** and **B**, which is impossible without reassessment of the respective agreements.

Nevertheless, the introduction of new notation such as, say, the '*normalized*' electric displacement

$$\mathbf{D}' = \frac{\mathbf{D}}{\varepsilon_0} \,,$$

in a separate book or paper also requires no international agreements. In any case, the problem of units (and, hence, of the requirement that $\varepsilon_0 \neq 1$ or, in other words, the form of Coulomb's law) must be separated from the decision of what physical quantities must characterize the electromagnetic field.

Sooner or later the logic of the development of the standards of electrical units will lead to a situation in which the elementary charge e will be chosen as the natural unit of charge, and the constant μ_0 and, hence, the constant ε_0 will become measurable. With the value of ε_0 fixed legally, one can discuss the merits and drawbacks of SI and CGS as competing systems of units, but when the constant ε_0 is measurable, there can be no alternative to the SI.

Our arguments in favor of the introduction of a special dimension for charge and of the inevitability of selecting a system of units with $\varepsilon_0 \neq 1$ do not mean that the CGS system or any other system with $\varepsilon_0 = 1$ is not needed. On the contrary, a practical system of units, i.e., a system determined by the requirements of experiment, is not necessarily the most suitable one for teaching and theoretical calculations. Various details of a practical system may seem unnatural from the theoretical viewpoint.

It is obvious that introducing special units for the amount of matter (instead of the number of particles) and temperature (instead of the mean energy) is illogical. To natural units belong, say, the atomic, relativistic, and Planck systems of units. Natural units are closely related to specially selected numerical values of fundamental constants. Some of the units are usually set equal to unity, and one often hears of constants being equal to unity, e.g., $\hbar = c = 1$. Statements of this kind are more jargon than truth [17].

In natural units, equations are simplified and some estimates become trivial. However, usually the 'natural' nature of units and the advantages of using such units refer only to a narrow area of physics. It is clear that the Planck units are as unnatural in atomic phenomena as atomic units are for quantum gravitation.

Irrespective of our viewpoint on the CGS system of units proper, we believe it is highly important that physicists in teaching physics and in (theoretical) research feel free to use any system of units they choose.

In a certain sense, a system of units and the accompanying recommendations concerning the nomenclature and notation constitute a language and the experience of teaching in both schools and universities can be used here, i.e., nonlanguage subjects can be taught in foreign languages, but qualifying examinations must be held in the official language. This is similar to the experience in the use of special units (atomic, relativistic, or Planck units) that considerably simplify the teaching of special subjects and do not present any difficulties in expressing the final results in practical units (SI units) needed for measurements. It must also be noted that the qualification of physicists (as users of systems of units) is much higher than the average qualification, so that the transfer from one system of units to another should not (and does not) constitute an essential problem for them.

It is necessary to regulate by law only what the holder of a degree must know and use in practice, and in the case of nonphysicists this knowledge and these skills must be expressed in the language of a practical system of measurement units, or the SI units. But the way this knowledge is acquired must not be restricted to one system. As for future physicists, it is crucial to know how to teach them physics in the SI [so far as the degree of a researcher in physics supposes (in Russia) the right to teach physics in schools and colleges], then proper research can be done, if necessary, in other systems of units, say, the CGS system.

Finally, it must be noted that an excessively rigorous system of education is detrimental to the SI units proper. There is no such thing as a once-and-for-all rigorously fixed International System of Units (SI). The agreements in this system are being constantly modified. Some additions (say, the inclusion of new units) are adopted only after they have demonstrated their viability. However, prior to their adoption they are not part of the SI, although they may be in wide use. In a rigorous and consistently regulated system of education we should be forced to avoid such common units, which would be foolish.

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