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Millimeter-wave response in the magnetoconductivity of highly perfect two-dimensional electron systems

S I Dorozhkin

1. State of the art

This talk reviews the state of the art research on photoresponse in the magnetoconductivity of 2D electron systems, with emphasis on the experimental aspects of the phenomenon and including the latest results from the cond-mat preprint archive; and presents a photoresponse model involving a radiation-induced nonequilibrium in the electron distribution function. It also puts forward the hypothesis that

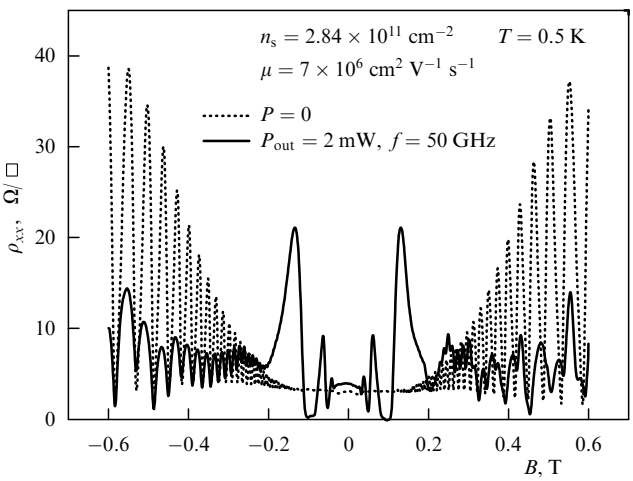

Figure 1. Magnetoresistivity ρ_{xx} of a 2D electronic system near a single GaAs/AlGaAs heterojunction in the absence of radiation (dotted curve) and in the presence of 50 GHz radiation (solid curve). Experimental parameters are shown in the figure. Here and hereafter the generator output power is indicated as radiation power (2 mW).

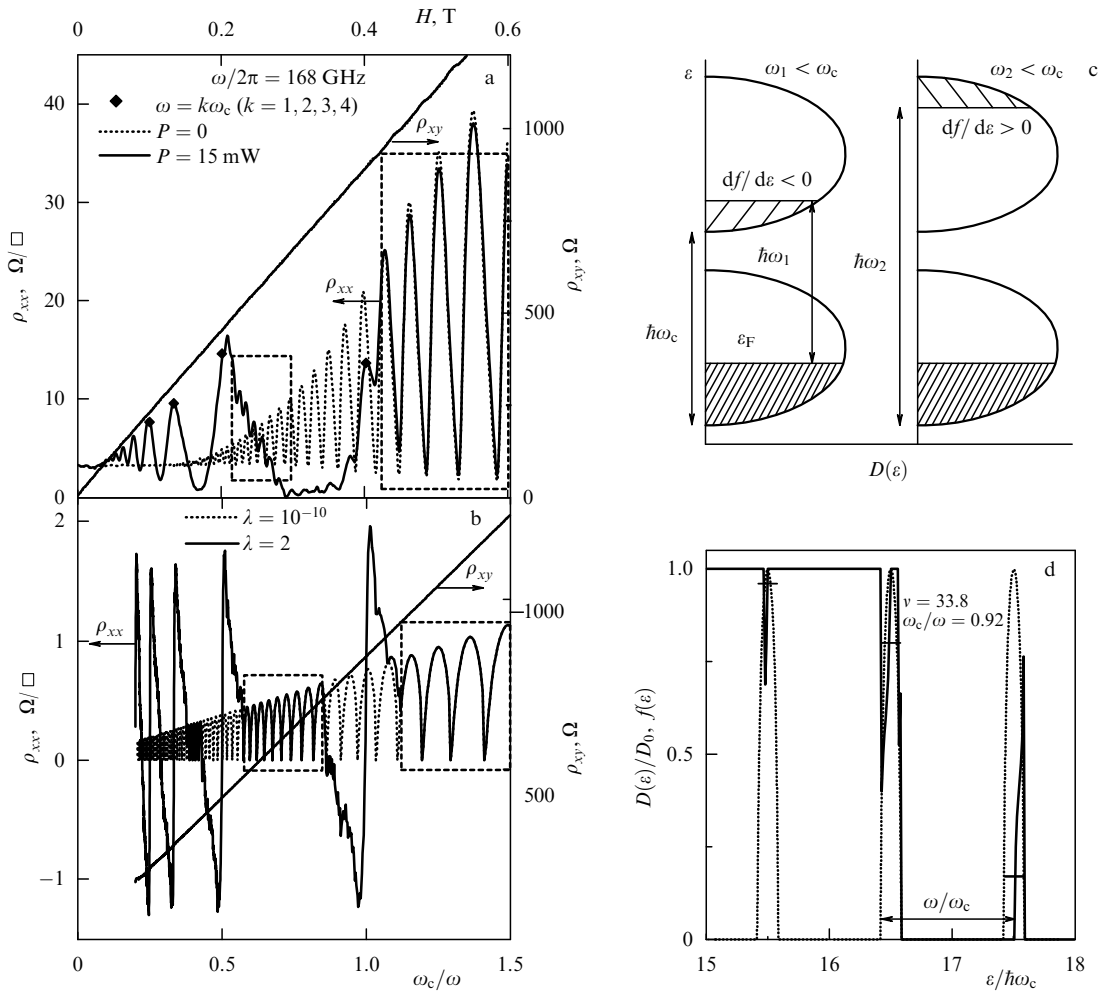

Figure 2. Measured (a) and calculated (b) magnetic field dependence of magnetoresistivity ρ_{xx} and Hall resistivity ρ_{xy}. Theoretical curves in Fig. b are given as a function of the ratio ω_c/ω, which is proportional to the magnetic field. Dotted curves in both figures correspond to the case of no radiation. Dotted Hall resistivity curves are practically indistinguishable from solid ones in both figures. Solid curves in Fig. a are measured under 168-GHz 15-mW radiation. The dashed boxes in both figures are regions where the Shubnikov – de Haas amplitudes are weakly sensitive to radiation. Solid lines in Fig. b were calculated from equations of Ref. [20] for long-period potential fluctuations using the following parameter values: $\lambda = 2$, $\hbar\omega \approx 6.6 \times 10^{-2}\epsilon_F$, $\Gamma_n \approx 4.9 \times 10^{-3}\,\epsilon_F \approx 7.5 \times 10^{-2}\hbar\omega$, $\Gamma_n^{xx} \approx 0.15(\omega/\omega_c)^{1/2}\Gamma_n$. Fig. c is a schematic of the semi-elliptic-shaped density of states as calculated for broadened Landau levels within the self-consistent Born approximation. The wide hatched areas are the regions of the partly occupied states produced by radiation. The states below the Fermi level which are fully occupied in the absence of radiation are shown as narrow hatched. The arrows indicate 1) transitions to the edges of the regions of partly occupied states for the cases $\omega_1 < \omega_c$ and $\omega_2 > \omega_c$; and 2) the cyclotron transition energy $\hbar\omega_c$. In Fig. d is shown the density of states (normalized to its maximum value $D_0 = 4N_0/\pi\Gamma_n$) for three Landau levels closest to the Fermi level and the nonequilibrium distribution function $f(\epsilon)$ calculated for the parameters of the curves in Fig. b using the value $\omega_c/\omega \approx 0.918$ which corresponds to $\nu = 33.8$ and $\epsilon_F/\hbar\omega_c \approx 16.56$. The magnitude of ω/ω_c is shown by an arrow. The distribution function under radiation differs from the equilibrium one only for the three levels shown in the figure. The bold horizontal segments represent, within the Landau levels, the hypothetical distribution function which provides zero magnetoresistivity.

near-zero magnetoresistance states might be due to an energy-inverted electron distribution rapidly relaxing within a single broadened Landau level.

The great interest in the magnetoresistivity behavior of highly perfect two-dimensional electron systems in a high-frequency electromagnetic field has been driven by the discovery in 2002 [1] of radiation-stimulated giant magnetoresistance oscillations (GMOs) with close-to-zero resistance in the fundamental minima, as exemplified in Fig. 1. The position of oscillations in a magnetic field approximately corresponds to that of cyclotron resonance harmonics, $\omega = n\omega_c^{(n)}$. Here, ω is the radiation frequency, $n = 1, 2, 3, \ldots$, and $\omega_c^{(n)} = eH^{(n)}/m^*c$ is the cyclotron frequency in the field $H^{(n)}$ (m^* is the electron effective mass). Thus, the giant magnetoresistance oscillations turn out to be periodic in an inverse magnetic field. Superficially, what distinguishes GMOs from Shubnikov – de Haas oscillations and the quantum Hall effect is the absence of, respectively, oscillations and quantized plateaus in the Hall resistance which remains virtually unchanged by radiation (Fig. 2a). The observation of close-to-zero magnetoresistance states led the authors of Ref. [1] to suggest that superconductivity might have occurred under the conditions of their experiment. Giant oscillations and states with close-to-zero magnetoresistance were almost simultaneously observed in Ref. [2] and shortly afterwards reproduced elsewhere [3]. It should be noted that GMO-like oscillations of photocurrent were predicted many years ago for 2D systems in a quantized magnetic field in a regime nonlinear in the electric field [4], and that this prediction was later extended to magnetoresistance [5] (for more details see the talk by V I Ryzhiĭ in this issue) — so that it was generally clear immediately after the publication of Ref. [1] that there was no need to invoke the idea of superconductivity to account for these experiments. Soon

thereafter, the diagonal ρ_{xx} and Hall ρ_{xy} components of the magnetoresistivity tensor and the dissipative component σ_{xx} of the magnetoconductivity tensor were simultaneously measured [6] and shown to be related between themselves by the usual relation $\sigma_{xx} = \rho_{xx}/(\rho_{xx}^2 + \rho_{xy}^2)$, with the implication that at the GMO minima, under the experimental conditions used ($\rho_{xx} \ll \rho_{xy}$), the magnetoresistivity ρ_{xx} and magnetoconductivity σ_{xx} simultaneously go to zero. As a result, when the Corbino samples used in Ref. [6] (the term referring to a ring made of a 2D system and having Ohmic contacts on its inner and outer sides) were in a state with near-zero ρ_{xx}, their resistance, on the contrary, sharply increased (because the resistance of a Corbino sample is proportional to σ_{xx}^{-1}) — something which is clearly inconsistent with the idea of superconductivity.

The proposed theoretical explanations of GMOs [7–9] relied mainly on the idea [4] of indirect optical transitions involving a change in electron momentum due to scattering by an impurity. Simultaneously, it was suggested by the present author [3] that the non-equilibrium occupation of disorder-broadened Landau levels might be an alternative explanation of GMOs. The authors of Refs [10, 11] developed this idea further by including the energy relaxation effect — which left the results qualitatively unchanged, though (in Ref. [12], the role of non-equilibrium electron distribution was briefly discussed by some of these authors). However, while all these approaches were adequate for describing the positions of GMO features (such as the minima, maxima, and zeroes of photoresponse), they failed to account for the near-zero values of magnetoresistance at the minima. Instead, all the above theories yielded negative values of magnetoresistance for the fundamental GMO minima at intense enough radiation (see Fig. 2b, for example). Today, this contradiction is usually resolved in the following way: as is well known [13], uniform states with negative resistance are unstable, so a uniform system is likely to break into domains in such a way that, given that the $I-V$ curve of the uniform state is an S-type, the domain-structured sample will have a near-zero resistance (see, the review paper [14], for example). For 2D electron systems irradiated under conditions when the Hall conductivity greatly exceeds the dissipative one, the formation of domain structure was first investigated in Ref. [15]. What the above discussion implies then is that one of the key problems facing experimenters today is testing the domain formation idea. The present author's own search for such domain structures has met with no success so far, no are there reports on such structures from other research groups who presumably must be working along similar lines. At the end of this paper, a scenario other than the domain structure will be suggested for the appearance of states with near-zero magnetoresistance.

GMOs located near cyclotron resonance harmonics nicely fit the picture of a one-particle energy spectrum in a magnetic field but are a completely unexpected feature for collective modes in limited-size samples (i.e., for bulk and boundary magnetoplasmons) because, given the typical parameters of the samples used in GMO studies, microwave radiation is expected (and observed) to be absorbed at frequencies which are quite far from the cyclotron resonance and its harmonics (see, for example, Ref. [16] and references therein). To the author's knowledge, there is currently no answer to the question of why effects related to the excitation of collective modes are dominated in magnetotransport measurements by giant magnetoresistance oscillations.

It has recently been established [17, 18] that the giant suppression of magnetoresistivity due to irradiation can be observed not only at GMO minima near the cyclotron resonance harmonics but (for relatively low-frequency radiation) also in a wide magnetic field range for which $\omega \ll \omega_c$ (Fig. 3). In this region no single-photon transitions occur between the Landau levels. As before, no change in the Hall resistance accompanies the giant suppression of magnetoresistance (Fig. 3a). The essential point is that irradiation can reduce magnetoresistance by more than one order of magnitude, to well below its values at the minima of 'dark' Shubnikov – de Haas oscillations. It turned out that the calculation of Ref. [17] using a non-equilibrium distribution function was adequate for this effect as well (Figs 3b, d). Another experimental finding of Ref. [17] was that under certain circumstances, radiation of low intensity has no effect on the Shubnikov – de Haas amplitudes. This effect arises in a threshold manner with respect to the irradiation frequency near the second subharmonic $\omega_0 = \omega_c/2$ of the cyclotron resonance at the threshold frequency ω_0. This result can be explained quantitatively [17] as a change from radiation-induced transitions between Landau levels to those within one broadened Landau level. Below the threshold frequency and at a relatively high power of radiation, an additional minimum of GMOs is observed near the second subharmonic [1–3] — which, it seems, can naturally be associated with two-photon processes.

2. GMO characteristics

To understand the GMO effect and to be able to use it to measure radiation frequencies, a crucial point is to know exactly where GMO features (minima, maxima, etc.) are located. Most theories predict the existence of only one type of features with well-defined positions — namely, where photoresponse is absent (i.e., magnetoresistance is unchanged by irradiation). Theoretically, these features are shown to correspond to the cyclotron resonance and its harmonics. The best experimental work on the subject seems to have been performed in Ref. [19], where the magnetic fields corresponding to GMO features were measured to high precision. These data confirmed that the most characteristic feature of the oscillations — and one practically independent of radiation power — is a point near the position of the cyclotron resonance at which radiation has no influence on magnetoresistance (see Figs 1 and 2). According to Ref. [19], this point is shifted by about 2% toward lower magnetic fields from the cyclotron resonance position as calculated from the known electron effective mass $m^* = 0.067m_0$ for GaAs. The amount by which similar features shift from the positions of the corresponding cyclotron resonance harmonics for higher-number GMOs increases with increasing oscillation number. The disagreement with theoretical predictions is most likely due to different Landau levels being nonequivalent in terms of conductivity (because, for example, the width of the level depends on its number [20]). The positions in magnetic field of GMO minima and maxima are shifted respectively towards weaker and stronger fields relative to the position of the cyclotron resonance and its harmonics $\omega = n\omega_c$. Increasing the radiation power leads to the maxima and minima being shifted toward the positions of the harmonics (see, for example, Ref. [3]).

Two points of interest here are the shape of the $I-V$ curve and the absolute values of magnetoresistance at GMO minima. As is well established (see, for example, Ref. [1]),

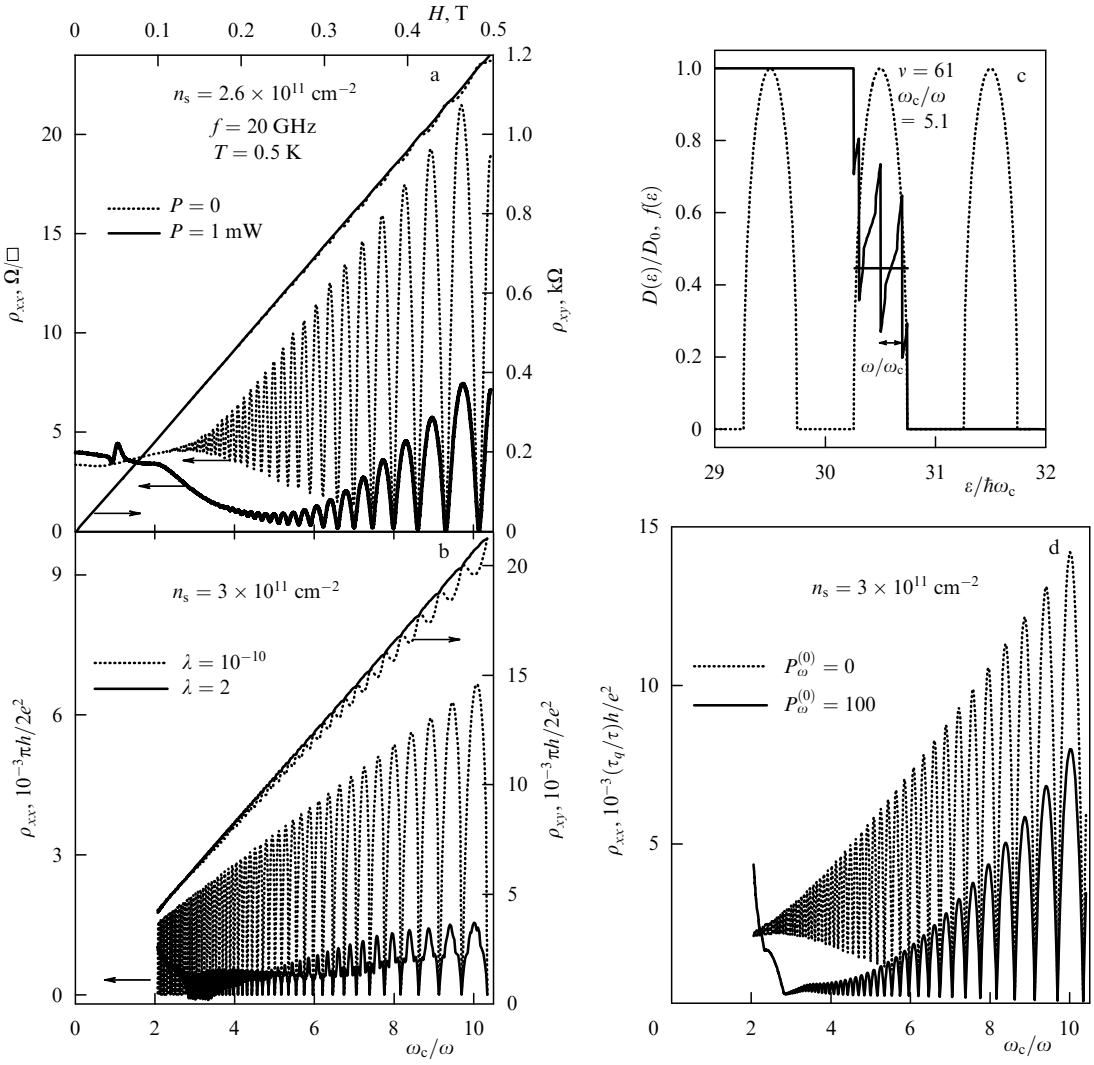

Figure 3. The basic notation in this figure is the same as in Fig. 2. (a) Experimental dependences $\rho_{xx}(H)$ and $\rho_{xy}(H)$ are obtained for 20 GHz irradiation. (b) $\rho_{xx}(\omega_c/\omega)$ and $\rho_{xy}(\omega_c/\omega)$ calculated from the model of Ref. [3] for short-period potential fluctuations using the following parameter values: $\hbar\omega \approx 6.6 \times 10^{-3}\epsilon_F$, $\Gamma_n/\hbar\omega_c \approx 0.55(\omega_c/\omega)^{-1/2}$, $\Gamma_n^{xx} = (n + 1/2)^{1/2}\Gamma_n$. (c) Density of states and nonequilibrium distribution function for intra-Landau level transitions for $\omega_c/\omega \approx 5.1$, which corresponds to $\nu = 61$ and $\epsilon_F/\hbar\omega_c \approx 30.5$. (d) Calculations of Ref. [17] for finite temperature $T \approx 4.0 \times 10^{-3}\epsilon_F$ for the same frequency and energy spectrum parameters as used in Fig. b.

magnetoresistance at minima falls off rapidly (approximately as an activation law) with lowering temperature; however, it is practically universally reported that at low temperatures magnetoresistance goes to a finite value. Low temperature observations showed radiation-induced reductions of up to two orders of magnitude in magnetoresistance. Resistance at minima turns out to be Ohmic over a wide range of measuring currents (approximately to 80 μA in Ref. [21]). One further point to mention is the negative voltage observed on potential contacts at GMO minima [22]. This effect is not reproduced when the magnetic field is reversed in sign and is presumably due to radiation being detected on the contacts, where boundary magnetoplasmons may be involved [23]. A significant asymmetry with respect to the sign of the magnetic field is observed, in particular, also in Fig. 1 at $|H| > 0.2$ T.

3. Conductivity in quantizing magnetic fields and the role of disorder in GMOs

To observe GMOs, it is essential that the 2D electrons have a high mobility — of more than 3×10^6 cm^2 V^{-1} s^{-1}, according to all available data. The observation of GMOs was in fact a follow-up of the studies of Refs [24, 25], which had used less perfect samples and therefore showed smaller-amplitude oscillations and no states with near-zero magnetoresistance. Still further back, experiments [26] on samples with a mobility of 1.2×10^6 cm^2 V^{-1} s^{-1} had demonstrated only solitary magnetoresistance features, which were located near the anticipated positions of magnetoplasmon resonances — positions which depended on the density of 2D electrons and the size of the sample and differed substantially from the position of the cyclotron resonance or from that of any of its harmonics. So the described evolution of photoresponse with changing electron mobility seems to suggest that well resolved Landau levels are a condition for GMOs to be observed.

On the other hand, the observation of giant oscillations clearly requires that disorder to be present in the system of 2D electrons. There are two fundamental reasons for this. First, the fact that GMOs are close to cyclotron resonance harmonics indicates that radiation-stimulated transitions between Landau levels whose numbers differ by more than

one are important. The selection rules which forbid such transitions in ideal systems cease to apply if a system is disordered on a spatial scale less than the cyclotron radius. It seems important to discuss the nature of random potential in the samples that were studied. The above mobility requirements for the observation of GMOs can be fulfilled only in selectively doped GaAs/AlGaAs heterostructures, in which a donor impurity supplying electrons to the 2D system is separated from the 2D layer by a nominally undoped barrier spacer typically about 800 Å thick. The widely held view is that the dominant scattering potential in such structures arises from fluctuations in the concentration of charged donors behind the spacer — with the result that the minimum spatial dimension, $\lambda_{\min}$, of potential fluctuations in the plane of the 2D electron system turns out to be approximately equal to the spacer thickness d_s. The upper limit for the spatial scale of the fluctuations is on the order of the sample size. Because the cyclotron radius r_c decreases as the magnetic field increases, one and the same random potential has a short-period component ($r_c > \lambda_{\min}$) in weak magnetic fields, which vanishes when the cyclotron radius becomes less than the spacer thickness. In this connection, it is useful to estimate the cyclotron radius $r_c = \sqrt{2\pi n_s}(\hbar c/eH)$ corresponding to the Landau level closest to the Fermi level. Under typical experimental conditions ($n_s \sim 3 \times 10^{11}$ cm^{-2}), the radius $r_c \approx d_s = 800$ Å in a field of about 1 T. Hence, in fields weaker than that a short-period component of the fluctuation potential is present in a sample, which lifts the selection rules and allows any photon-absorbing transitions that conserve energy. In stronger fields, the observation of GMOs becomes impossible, and only cyclotron resonance can be seen.

Second, disorder is a crucial factor in determining the dissipative conduction of 2D electrons in a magnetic field where, as is well known, there is no current along the electric field if the electrons are not scattered. In this case, the classical motion of a 2D electron consists of two components, a cyclotron rotation in the plane of the electron system with (cyclotron) frequency $\omega_c = eH_z/m^*c$ (where H_z is the magnetic field component perpendicular to the 2D electron system); and a straight line motion with velocity cE/H_z in the direction perpendicular to the electric field E lying in the plane of the system. The resulting cycloid motion of the electron corresponds to the zero values of the diagonal (dissipative) components of the magnetoconductivity and magnetoresistance tensors. A current along the electric field appears only due to electron scattering and the associated shift of the line of drift in this direction. At low temperatures the predominant electron scattering is usually elastic scattering by impurities. The line of drift can shift either along or opposite to the electric field direction as a result of scattering, respectively contributing negatively or positively to the dissipative conductance. In a classical treatment, the dissipative conduction is positive because electrons are predominantly scattered opposite to the electric field, due to the fact that during its cycloid motion an electron moves along the drift most of the time. Quantum-mechanically, the preferential shift of the line of drift (i.e., of the center of the Landau orbital) is determined by the electron energy distribution function. As a result, the dissipative conductivity has the form

$$\sigma_{xx} \sim \int \sigma_{xx}(\epsilon)\left(-\frac{\mathrm{d}f}{\mathrm{d}\epsilon}\right)\mathrm{d}\epsilon$$

(for equilibrium distribution functions, a similar formula was first obtained in Ref. [27]; for non-equilibrium functions, the reader may be referred to Refs [28, 29]). Clearly, in the equilibrium case we always have $(-\mathrm{d}f/\mathrm{d}\epsilon) > 0$ and $\sigma_{xx} > 0$. The presence of inverted population ($-\mathrm{d}f/\mathrm{d}\epsilon < 0$) regions in the momentum-symmetric part of the non-equilibrium function leads to a negative contribution to the dissipative conductivity. In 2D electron systems the energy spectrum consists of broadened Landau levels. As Fig. 2c demonstrates, for this spectrum regions of inverted population may be expected to appear when the photon energy either somewhat exceeds a multiple of the cyclotron energy or is less than the Landau level width (in the latter case the appearance of such regions depends on the position of the Fermi level within the broadened Landau level). Whether an inverted distribution will appear undoubtedly depends on how fast the electron system comes to equilibrium and how fast it loses its energy — the processes which, for systems with a Landau spectrum (and especially for disordered ones), are so complex that actually no analyses of them have been made.

4. GMO model using a non-equilibrium distribution function

In this Section we demonstrate how GMOs and the giant suppression of magnetoresistance at $\omega < \omega_c$ can potentially be described in terms of a non-equilibrium distribution function. For this purpose, we will look in more detail at the results of Ref. [3], where the non-equilibrium distribution function was calculated for a model in which stimulated and spontaneous transitions were only allowed to occur between levels differing in energy by $\hbar\omega$. For the case of the giant suppression of magnetoresistance, we will also present the numerical results of Ref. [17], which were obtained using the distribution function equation (2) of Ref. [10] — an equation which uses the relaxation time approximation to describe relaxation in energy. In both cases, the conductivity formulas used were those obtained in the framework of the self-consistent Born approximation for the case of non-overlapping Landau levels. Whether this approximation and the formulas we borrowed from Refs [20, 10] are applicable to the experiment under discussion will be left 'off-screen' in our discussion, and instead our purpose will be to see how a non-equilibrium electron energy distribution affects the conductivity.

In Ref. [3] the density of states $D(\epsilon)$ and the conductivity tensor components were given by the following formulas [20]:

$$D(\epsilon) = \sum_{n=0}^{\infty} \frac{2N_0}{\pi\Gamma_n}\left[1-\left(\frac{\epsilon-\epsilon_n}{\Gamma_n}\right)^2\right]^{1/2} \equiv \sum_{n=0}^{\infty} \frac{2N_0}{\pi\Gamma_n} Z_n^{1/2}(\epsilon), \quad (1)$$

$$\sigma_{xx} = \frac{e^2}{\pi^2\hbar}\sum_{n=0}^{\infty}\left(\frac{\Gamma_n^{xx}}{\Gamma_n}\right)^2 \int_{\epsilon_n-\Gamma_n}^{\epsilon_n+\Gamma_n}\left(-\frac{\mathrm{d}f}{\mathrm{d}\epsilon}\right) Z_n(\epsilon)\,\mathrm{d}\epsilon\,, \quad (2)$$

$$\sigma_{xy} = -\frac{n_s ec}{H} + \frac{e^2}{\pi^2\hbar}\sum_{n=0}^{\infty}\frac{(\Gamma_n^{xy})^4}{\Gamma_n^3\hbar\omega_c}\int_{\epsilon_n-\Gamma_n}^{\epsilon_n+\Gamma_n}\left(-\frac{\mathrm{d}f}{\mathrm{d}\epsilon}\right) Z_n^{3/2}(\epsilon)\,\mathrm{d}\epsilon. \quad (3)$$

Here, $\epsilon_n = \hbar\omega_c(n+1/2)$ is the energy of the nth spin-degenerate Landau level of width Γ_n, and the total number of states at this level is $N_0 = 2eH/hc$ (per unit area). The contributions from this level to σ_{xx} and σ_{xy} are characterized by the parameters Γ_n^{xx} and Γ_n^{xy}, respectively. The only modification we made to Eqns (1) — (3) was to use the non-

equilibrium distribution function $f(\epsilon)$ which is calculated for the case of zero temperature under the approximation mentioned above (spontaneous and induced transitions causing an energy change by $\hbar\omega$). Under these assumptions, and using the stationarity condition, it is easy to write the recurrence relation for the distribution function,

$$f(\epsilon) = \frac{\lambda f(\epsilon - \hbar\omega)}{\lambda + 1 - f(\epsilon - \hbar\omega)} \,. \tag{4}$$

Here, the parameter λ is proportional to the radiation power (the number of photons). The above relation is valid for $D(\epsilon) \neq 0$ and $D(\epsilon - \hbar\omega) \neq 0$. We assume that $f(\epsilon) = f_0(\epsilon)$ for $D(\epsilon - \hbar\omega) = 0$ and $\epsilon > \epsilon_{\mathrm{F}}$; and also for $D(\epsilon + \hbar\omega) = 0$ and $\epsilon < \epsilon_{\mathrm{F}}$ (where $f_0(\epsilon)$ is the Fermi distribution function at zero temperature). Within the approximation used, the condition of electron number conservation has the form

$$\sum_{n=-\infty}^{\infty} \left[f(\epsilon + n\hbar\omega) - f_0(\epsilon + n\hbar\omega) \right] D(\epsilon + n\hbar\omega) = 0 \,, \tag{5}$$

which is valid for any value of energy ϵ. The solutions of Eqns (4) and (5), which were obtained numerically, determine the non-equilibrium distribution function. An example of such a function is shown in Fig. 2d for parameter values corresponding to Fig. 2b. Figure 2b shows magnetoresistivity tensor components calculated for a long-period fluctuation potential. These results correctly describe such things as the position of the GMOs and their asymmetric shape; the absence of the effect in the Hall resistivity; the weak magnetic field dependence of the GMO amplitude; and the magnetic field regions where radiation has little effect on the Shubnikov–de Haas amplitudes. As already noted, where experiment and theory disagree the most is in the values of magnetoresistance at the minima of the giant oscillations (which are negative in theory and close to zero in experiment).

The results of similar calculations for the case of short-period potential fluctuations are shown in Fig. 3, which represents the region of relatively strong magnetic fields ($\omega < \omega_{\mathrm{c}}$). In Fig. 3b, this region reveals a significant suppression of the Shubnikov–de Haas oscillation maxima — an effect which leads to a considerable reduction in the average values of magnetoresistance and therefore provides a qualitative explanation of the giant suppression of magnetoresistance as demonstrated by the experimental results in Fig. 3a. Predictions and experiment are brought much closer together (see Fig. 3d) by using the distribution function equation (2) of Ref. [10], which is applicable at non-zero temperatures and includes relaxation processes. As can be seen from the distribution function of Fig. 3c, in the region of the giant suppression of magnetoresistance radiation-induced transitions occur within the broadened Landau levels.

In conclusion, we would like to discuss a scenario in which the states with close-to-zero magnetoresistance arise by a mechanism which does not involve (as of yet undiscovered) domains. This scenario consists in the distribution function rapidly levelling off within one individual Landau level in situations in which, in the absence of relaxation, a distribution function with inverted-population regions must appear. An example of such a hypothetical distribution function is shown in Figs 2d and 3c. Clearly, for such a distribution function the magnetoconductivity of a uniform state would be zero, in correspondence with experimental observations.

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