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Microwave-induced negative conductivity and zero-resistance states in two-dimensional electronic systems: history and current status

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1. Introduction

The history of the effect of absolute negative conductivity in semiconductor structures and its current studies are briefly reviewed here. We focus primarily on this effect in two-dimensional electron systems in a magnetic field under microwave radiation in the context of the so-called zero-resistance, as well as zero-conductance, states observed in recent experiments.

The possibility of realizing negative dc conductivity in a non-equilibrium electron system, i.e., the situation when the dc current \mathbf{J} flows in the direction opposite to the direction of the electric field \mathbf{E} , was qualitatively discussed for the first time by Krömer in the late 1950s [1] in connection with his proposal of negative electron mass. In such a situation, the usual dc conductivity $\sigma_D = \mathbf{J}\mathbf{E}/E^2 < 0$, so that one can say that we have to deal with the effect of absolute negative conductivity (ANC). This effect should be distinguished from

the effect of negative differential conductivity (NDC) which occurs in many semiconductor structures, in particular in Gunn diodes. Different realistic mechanisms of ANC in two- and three-dimensional electron systems (2DESs and 3DESs, respectively) in which a substantial deviation from equilibrium is associated with intraband or interband absorption of optical radiation were considered more than three decades ago [2–7] (see also Ref. [8]). A mechanism of ANC in a 2DES subjected to a magnetic field and irradiated with microwaves was first proposed by the author [9]. This mechanism is related to 2D electron scattering by impurities accompanied by the absorption of microwave photons. It was shown that the dissipative dc conductivity (the diagonal component of the conductivity tensor) is an oscillating function of the ratio of the microwave frequency Ω to the electron cyclotron frequency $\Omega_c = eH/mc$, where $e = |e|$ and m are the electron charge and effective mass, H is the magnetic field strength, and c is the speed of light. At Ω somewhat exceeding Ω_c or a multiple of Ω_c , the photon-assisted impurity scattering of 2D electrons with their transitions between the Landau levels (LLs) results in a contribution to the dissipative current flowing opposite to the electric field. At sufficiently strong microwave radiation, this scattering mechanism can dominate, leading to ANC when $\Omega \geq A\Omega_c$, where $A = 1, 2, 3, \dots$. Possible transformation of the current – voltage characteristic of a 2DES with increasing microwave power P is schematically shown in Fig. 1.

There were several early but unsuccessful attempts to observe experimentally the effect of ANC associated with the mechanisms in question, although some features of the transport phenomena in 3DESs and 2DESs studied at that time can be attributed to these mechanisms. Early theoretical and experimental studies related to ANC have been eclipsed by the discovery of the integral and fractional quantum Hall effects and very extensive related activities.

Recently, Mani et al. [10] and Zudov et al. [11] have observed experimentally the effect of vanishing electrical resistance (in the Hall bar configuration) in high-quality 2DESs (with electron mobility on the order of $\mu \sim 10^7 \text{ cm}^2 \text{ V}^{-1} \text{ s}^{-1}$) at low temperatures ($T \sim 0.1 \text{ K}$) in rather weak magnetic fields ($H \sim 0.1 \text{ T}$) irradiated with microwaves (with frequencies $f = \Omega/2\pi$ in the range of several tens of GHz). The obtained experimental magnetic-field dependences of the longitudinal resistance R_{xx} are shown in Fig. 2 [10, 11]. As shown, in the presence of sufficiently strong microwave radiation, R_{xx} vanishes in some ranges of the magnetic field. Due to an association

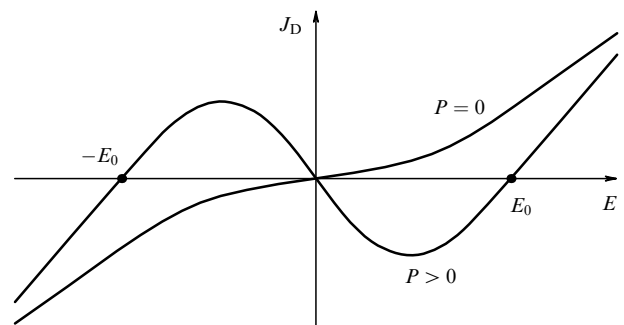


Figure 1. Schematic view of current – voltage characteristics without ($P = 0$) and with ($P > 0$) microwave radiation.

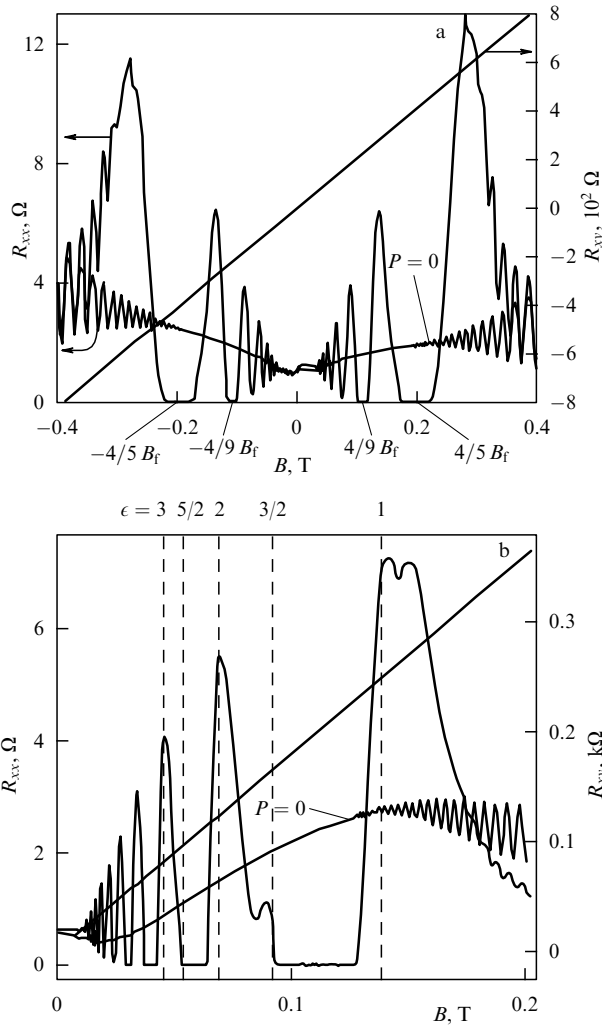


Figure 2. Magnetic-field dependences of 2DES resistance without ($P = 0$) and with microwave radiation vs. magnetic field: (a) observed by Mani et al. [10] and Zudov et al. [11]. Smooth nearly linear dependence corresponds to the Hall resistance.

between the longitudinal resistivity ρ_{xx} and longitudinal conductivity σ_{xx} (i.e., the dissipative conductivity σ_D), the zeros of $R_{xx} \propto \rho_{xx}$ should correspond to the zeros of σ_{xx} . As a result, the magnetic-field dependence of the conductance of a 2DES in the Corbino disk configuration should exhibit zero-conductance ranges. Soon after the first experiments on zero-resistance (ZR) states, the zero-conductance (ZC) states were observed by Yang et al. [12] (Fig. 3).

These experimental findings have been confirmed by other experimental groups. As noted by Zakharov [13] and discussed in early papers on ANC in 3DESs [14] and 2DESs [7, 15], homogeneous states of systems with ANC are unstable because the current–voltage characteristic with ANC, like that shown in Fig. 1, corresponds to NDC in some range of the electric fields around $E = 0$. This instability usually results in the formation of the electric-field domain structures and the stratification of current flow (see, for example, the review by Volkov and Kogan [16]). In accordance with this, Andreev et al. [17] and Volkov with co-workers [18] suggested that the appearance of ZR-states (and ZC-states) can be attributed to some mechanism of ANC, so that the regions of ZR and ZC correspond to inhomogeneous

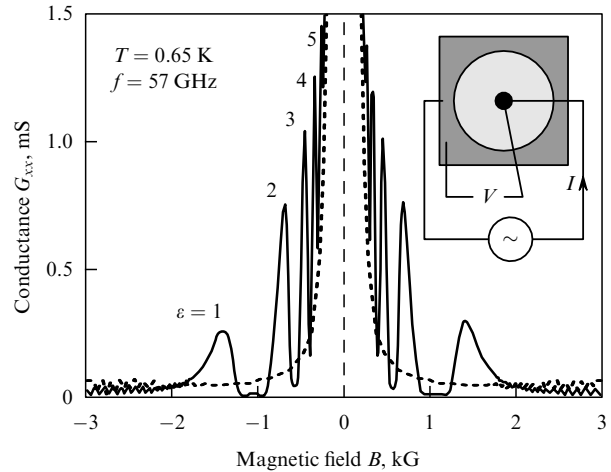


Figure 3. Conductance of a 2DES in the Corbino disk configuration without (dotted curve) and with (solid curve) microwave radiation vs. magnetic field [12].

distributions of the electric field and the current density, formed due to an instability of homogeneous states associated with ANC. The occurrence of ANC in the experiments under consideration was attributed to the contribution to the conductivity of photon-assisted impurity scattering of 2D electrons, i.e., to the mechanism put forward in [9, 19].

The structure of the electric-field distributions and the distributions of the current density corresponding to ZR- and ZC-states formed as a result of the instability is determined by the shape of the current–voltage characteristic (in particular, by the value of E_0) and the features of the diffusion processes. Examples of such domain structures are schematically shown in Fig. 4 [20]. Recent experimental findings [10–12] have stimulated a surge of experimental (for example, [21–25]) and theoretical (for example, [26–34]) papers. In particular, the results of early theoretical studies of ANC caused by photon-assisted impurity scattering were generalized by the inclusions of LL broadening and a high microwave power effects [28–30] (see also [19, 35]). A quasi-classical model which is valid at

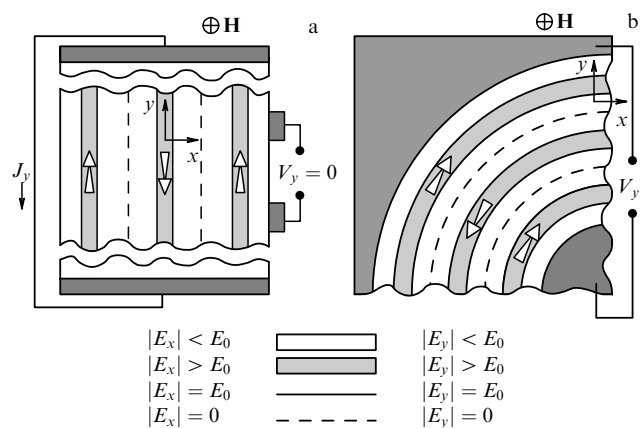


Figure 4. Schematic view of possible domain structures in a 2DES [20] corresponding to (a) ZR states in the Hall bar configuration (J_y is the input current and V_y is the measured voltage) and (b) ZC states in the Corbino geometry (V_y is the applied voltage). Arrows show directions of the Hall current.

large filling factors and a sufficiently strong electric field (or when a long-range disorder determines the dissipative current) was developed by Vavilov and Aleiner [31]. The possible role of photon-assisted acoustic phonon scattering was discussed in [32–34].

This paper is a supplement to some preliminary attempts to restore the whole picture of the effects in question relying on early and recent theoretical papers and the results of experimental observations which have been recently published [36–40] and discussed at several conferences.

2. Magnetotransport and Shubnikov–de Haas oscillations

The Hamiltonian of noninteracting electrons in crossed homogeneous dc electric and magnetic fields $\mathbf{E} \perp \mathbf{H}$ can be presented (disregarding spin) as follows:

$$\mathcal{H}_0 = -\frac{\hbar^2}{2m} \left[\frac{\partial^2}{\partial \rho^2} + \left(\frac{\partial}{\partial \xi} + i \frac{e}{c\hbar} H \rho \right)^2 \right] + eE\rho. \quad (1)$$

Here, \hbar is the Planck constant, and the axes ρ and ξ are in the 2DES plane; they are directed along the direction of the net dc electric field \mathbf{E} , which includes the applied and Hall components, and perpendicular to both \mathbf{E} and \mathbf{H} , respectively, i.e., in the direction of the Hall drift. The energy spectrum of 2D electrons with the Hamiltonian (1) is determined by the quantum numbers N and ρ_k (or N and k) and is given (disregarding Zeeman splitting) by the following formula:

$$\varepsilon_{N, \rho_k} = \left(N + \frac{1}{2} \right) \hbar\Omega_c + eE\rho_k. \quad (2)$$

Here $N = 0, 1, 2, \dots$ is the LL index, $\rho_k = -L^2k + F/m\Omega_c^2$ is the coordinate of the electron orbit center in the direction of the net dc electric field, $F = -eE$, and $L = (c\hbar/eH)^{1/2}$ is the quantum Larmor radius. The first term on the right-hand side of Eqn (2) is the electron kinetic energy, while the second one can be attributed to its potential energy in the net electric field. The wave function of a 2D electron is presented as

$$\psi_{N, \rho_k}(\rho, \xi) \propto \exp(ik\xi) \varphi_N\left(\frac{\rho - \rho_k}{L}\right), \quad (3)$$

where $\varphi_N(x/L)$ is the oscillator wave function. Equations (2) and (3) are valid when the effective mass approximation is applicable to the 2DES under consideration, i.e., when the ratio E/H is not too large, $E/H \ll \sqrt{\varepsilon_g/2mc^2}$ [41], where ε_g is the energy gap.

The dissipative electron transport in the direction parallel to the electric field and perpendicular to the magnetic field is due to hops of the electron Larmor orbit centers caused by scattering processes. These hops result in a change in the electron potential energy $\delta\varepsilon = -F\delta\rho$, where $\delta\rho = \rho_k - \rho'_k$ is the displacement of the electron orbit center. When electron scattering by impurities dominates and the LL broadening is insignificant, one can find the displacement of the electron orbit center caused by electron scattering with its transition from the state N, ρ_k to the state N', ρ'_k using the energy conservation law (disregarding spin-flip effects):

$$F\delta\rho = A\hbar\Omega_c, \quad (4)$$

where $A = N' - N$. The matrix elements of the electron transitions due to impurity scattering \mathcal{M}_i and, consequently,

the probability of the scattering processes in question are determined by the overlap of the electron wave functions (3) corresponding to two spatially separated states (i.e., with different coordinates of the electron orbit center ρ_k). In the case of electron transitions between low LLs, the pertinent matrix element decreases exponentially when $\delta\rho > L$ because $|\mathcal{M}_i|^2 \propto \exp(-\delta\rho^2/2L^2)$. Due to this, the contribution of each act of inter-LL electron scattering by impurities to the dissipative current is

$$\delta J_D^{\text{inter}} \propto -e\delta\rho \exp\left(-\frac{\delta\rho^2}{2L^2}\right). \quad (5)$$

In real situations, $|E| < E_c = \hbar\Omega_c/eL$, so that the transitions between only the neighboring LLs ($A = 1$) are efficient. In this case, considering Eqns (4) and (5) and taking into account the electron–impurity interaction in the Born approximation, one can arrive at the following formula obtained by Tavger and Erukhimov [42] for the dissipative current due to inter-LL electron transitions in a nondegenerate 2DES in which only low LLs are essential:

$$J_D^{\text{inter}} \propto \frac{\Gamma_i \Omega_c}{e^2 |E| EL^2} \exp\left[-\frac{1}{2} \left(\frac{\hbar\Omega_c}{eEL}\right)^2\right] \propto \exp\left[-\frac{1}{2} \left(\frac{E_c}{E}\right)^2\right]. \quad (6)$$

Here, Γ_i is the frequency characterizing the electron–impurity collisions. The exponential electric-field dependence given by Eqn (6) is due to inter-LL transitions which become crucial under a sufficiently strong electric field. Such inter-LL transitions can be called the Zener tunneling transitions between LLs (see, for example, [43]). If Zener tunneling between LLs is due to a resonant transition via impurity level [44], the calculated dissipative current–voltage characteristic remains exponential. The inter-LL transitions in a strong electric field considered above were recently invoked to explain new experimental observations of specific oscillations of the magnetoresistance in relatively weak magnetic fields [43]. It is instructive to note that Eqn (6) yields a non-analytic electric-field dependence which corresponds to the dissipative conductivity $\sigma_D = J_D/E$ tending to zero in the limit of weak electric fields and rapidly increasing when the electric field becomes comparable with E_c [42]. In degenerate 2DESs with the Fermi energy ε_F markedly exceeding the LL separation $\hbar\Omega_c$, in which many LLs are occupied, the electron inter-LL transitions with $\delta\rho \sim L_F = L\sqrt{2N_F + 1}$ can provide the main contribution to the dissipative current. Here, $N_F = \varepsilon_F/\hbar\Omega_c$ is the so-called filling number. In this case, a substantial increase in the dissipative conductivity occurs at $E \geq E_c^{(F)} = \hbar\Omega_c/L_F \approx E_c/\sqrt{2N_F}$.

However, in relatively low electric fields $E < E_b = \hbar\Gamma/eL$ (or $E < E_b^{(F)} = \hbar\Gamma/eL_F$), where Γ is the LL broadening, the contribution of intra-LL electron scattering is important, so that the dissipative current $J = J^{\text{intra}} + J^{\text{inter}}$. Moreover, in weak electric fields, intra-LL electron transitions can provide the main contributions to the dissipative current, so that the LL broadening becomes crucial. The dissipative conductivity of 2DESs in a magnetic field associated with impurity scattering was considered by the author [45] (see also Ref. [15]). It was assumed in Ref. [45] that the LL broadening is mainly due to electron–electron interactions ($\Gamma \approx \Gamma_{ee} > \Gamma_i$). This can be justified, in particular, in the

2DESs in which the electron sheet concentration is on the same order of magnitude as the sheet concentration of remote donors (separated from the 2DES by a sufficiently thick spacer). Using the method of Green functions, in which the interaction of electrons with impurities was considered as a perturbation whereas the dc electric field was taken into account strictly using the wave functions (3), the following formula for the dissipative current associated with intra-LL electron transitions in the case $\Gamma_i < \Gamma \ll \Omega_c$ at $E < E_b$ was obtained [45]:

$$J_D^{\text{intra}} \propto E \Gamma_i \sum_N b_N^{(i)} \left(-\frac{\partial f_N}{\partial \zeta_F} \right). \quad (7)$$

Here, $f_N = [\exp(N\hbar\Omega_c/T - \zeta_F) + 1]^{-1}$ is the Fermi distribution function, $\zeta_F = \varepsilon_F/T$ is Fermi energy normalized to the temperature T , and $b_N^{(i)}$ are coefficients depending on the matrix elements of the impurity potential.

When $E \geq E_c \gg E_b$, the models used in Ref. [42] and in Refs [15, 45] lead to the same formula for the dissipative current. Thus, formula (7) corresponds to the impurity scattering accompanied with the transitions within the LLs, i.e., the intra-LL transitions (see also Ref. [46]). In weak electric fields ($E \ll E_c$, $E_c^{(F)}$), the inter-LL transitions require the hops of the electron Larmor orbit centers with fairly large $\delta\rho$. Such hops are characterized by an exponentially small probability because of a small overlap of the electron wave functions in the initial and final states. This implies that although electron–electron interaction does not change the total momentum of the electron system and, hence, does not lead to dissipative conductivity, it may mediate the momentum transfer to the scatterers and, therefore, strongly affect the electron transport phenomena [47, 48]. Invoking the mechanism of intra-LL impurity scattering mediated by an electron–electron collision, one can understand the association of the dissipative current with the Joule heating. Indeed, in the case of the mechanism in question, the change in the electron potential energy associated with the electron orbit center displacements in the direction of the electric force is compensated by an increase in the kinetic energy (heating) of all participating electrons. This problem, to the best of my knowledge, is not resolved in the framework of the model that solely considers impurity scattering (even beyond the Born approximation). There is no such problem in the case of a 3DES in a magnetic field because the potential energy acquired by an electron from the electric field due to the displacement of its Larmor orbit center associated with impurity scattering goes to an increase in the kinetic energy of the electron motion along the magnetic field.

When $\Omega_c \gg \Gamma_i$, the values of characteristic fields E_c and E_b are quite different ($E_b \ll E_c$). For example, for a AlGaAs/GaAs 2DES, assuming $H = 0.2$ T and $\Gamma = 10^{10} \text{ s}^{-1}$, one can obtain $E_b \approx 1 \text{ V cm}^{-1}$ and $E_c \approx 60 \text{ V cm}^{-1}$. If $N_F = 50$, one obtains $E_b^{(F)} \approx 0.1 \text{ V cm}^{-1}$ and $E_c^{(F)} \approx 6 \text{ V cm}^{-1}$.

The dissipative conductivity in a low electric field when the inter-LL transitions (Zener inter-LL tunneling) are insignificant can also be associated with the electron intra-LL scattering processes involving acoustic phonons [20, 49–51]. The main result of these papers is that the intra-LL acoustic scattering of electrons is ‘switched on’ only when the net electric field exceeds the threshold value $E_a = sH/c$, where s and c are, respectively, the speeds of sound and light. This condition corresponds to the electron Hall drift velocity $v_H = eE/H$ faster than s . At $E > E_a$, the intra-LL acoustic

scattering can be essential, providing a marked increase in the dissipative conductivity.

Using Eqn (7) and taking into account that

$$\frac{\partial f_N}{\partial \zeta_F} = \frac{\exp(N\hbar\Omega_c/T - \zeta_F)}{[1 + \exp(N\hbar\Omega_c/T - \zeta_F)]^2},$$

the low-field dissipative conductivity can be presented as

$$\sigma_D \propto \Gamma_i \sum_N \frac{b_N^{(i)} \exp(N\hbar\Omega_c/T - \zeta_F)}{[1 + \exp(N\hbar\Omega_c/T - \zeta_F)]^2}. \quad (8)$$

As follows from Eqn (8), at $T < \hbar\Omega_c$, the dissipative conductivity is a strongly oscillating function of Ω_c and ζ_F . These are the well-known Shubnikov–de Haas oscillations in a 2DES observed experimentally in the 1960s by Fowler et al. [52] (early experimental papers were discussed in detail in Ref. [46]). One can see from Eqn (8) that σ_D reaches a maximum when $\varepsilon_F = N\hbar\Omega_c$, whereas the minimum of σ_D corresponds to $\varepsilon_F = (N + 1/2)\hbar\Omega_c$. At the minimum,

$$\sigma_D \propto \exp\left(-\frac{\hbar\Omega_c}{2T}\right). \quad (9)$$

Thus, despite the fact that the dissipative conductivity in the case of the low electric field under consideration is determined by the electron transitions within LLs, it exhibits an activation behavior with activation energy $\varepsilon_A = \hbar\Omega_c/2$, which is determined by the separation between LLs. At sufficiently low temperatures, the dissipative conductivity exhibits giant Shubnikov–de Haas oscillations with its exponentially small values under the condition of quantum Hall effect $\varepsilon_F \approx (N + 1/2)\hbar\Omega_c$.

3. ANC due to photon-assisted electron scattering mechanisms

Microwave radiation with frequency Ω comparable with cyclotron frequency Ω_c or its harmonics can substantially affect the dissipative conductivity in 2DESs. Since the dissipative conductivity depends on electron energy distribution, which can often be characterized by an effective electron temperature, the absorption of microwave photons can markedly change this conductivity. However, as has been predicted [9], microwave radiation can rather effectively influence the electron scattering processes, in particular, on impurity scattering. Moreover, such scattering processes in which an electron absorbs or emits a certain number of photons (which is called ‘photon-assisted impurity scattering’) can change not only the value of the dissipative conductivity but also its sign, i.e., can lead to ANC if the microwave power is sufficiently large. This can be clarified using the diagrams for the electron inter-LL transitions shown in Fig. 5 (left-hand side) and considered in some papers [19, 26, 40]. According to the diagrams of Fig. 5, the effect of ANC in a 2DES system in a magnetic field under microwave irradiation is associated with the following [9, 19]. As explained above, if the electron orbit center displaces in the direction of the electric force ($\delta\rho > 0$ and $\delta\varepsilon < 0$), the electron potential energy decreases. At equilibrium, the electron orbit center hops in this direction are dominant, so the dissipative electron current flows in the direction of the net dc electric field. However, in some cases, the displacements of the electron orbit centers in the direction opposite to the electric force (with $\delta\rho < 0$ and, hence, $\delta\varepsilon > 0$) can prevail, resulting in

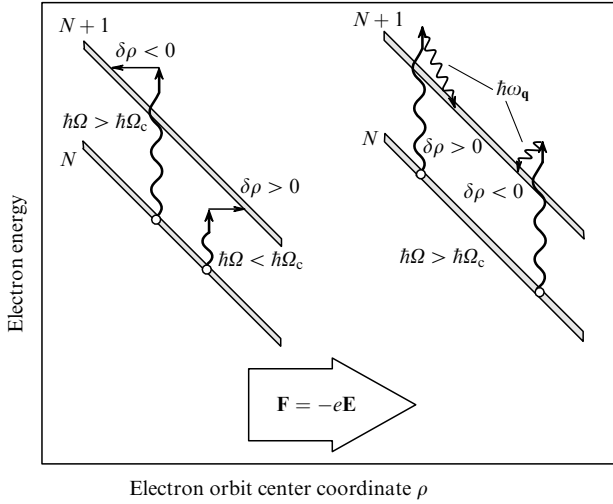


Figure 5. Inter-LL electron transitions (with the absorption of single microwave photons). Left: photon-assisted impurity scattering for both $\Omega > \Omega_c$ and $\Omega < \Omega_c$. Right: photon-assisted acoustic phonon scattering (only transitions for $\Omega > \Omega_c$ are shown).

dissipative current flowing opposite to the electric field. Indeed, if an electron absorbs A photons and makes a transition to a higher LL, a portion of the absorbed energy goes to an increase in the kinetic energy, hence, the change in the electron potential energy is $\delta\varepsilon = \hbar(\Omega - A\Omega_c)$. Thus, the energy balance for every act of photon absorption and emission yields the following equation for the displacement of the electron orbit center $\delta\rho$ [compare with Eqn (4)]:

$$F\delta\rho = A\hbar\Omega_c \pm \hbar\Omega. \quad (10)$$

As follows from Eqn (10), for processes of impurity scattering with the absorption of $A = 1, 2, 3, \dots$ photons at $\Omega > A\Omega_c$, one obtains $\delta\rho < 0$ (see Fig. 5, left-hand side). Since the incident microwave radiation is strongly non-equilibrium (its ‘temperature’ is much higher than the electron and lattice temperatures), processes of photon absorption accompanying the impurity scattering prevail over those with photon emission. As a result, the electron orbits move in the direction opposite to the electric force and, hence, ANC occurs.

To calculate the contribution of photon-assisted impurity scattering to the dissipative conductivity one can start from the following Hamiltonian:

$$\mathcal{H}_\varepsilon = -\frac{\hbar^2}{2m} \left[\frac{\partial^2}{\partial\rho^2} + \left(\frac{\partial}{\partial\xi} + i\frac{e}{\hbar c} H\rho \right)^2 \right] + eE\rho + e\mathcal{E}(t)(e_\rho\rho + e_\xi\xi). \quad (11)$$

Here, $\mathcal{E}(t) = \mathcal{E} \cos \Omega t$ is the ac electric field of microwave radiation, which is taken into account in the dipole approximation, and e_ρ and e_ξ are the components of the microwave field complex polarization vector.

Hamiltonian (11) leads to quasi-stationary states which are characterized by quasi-energies. The probability of an electron transition from state (N, ρ_k) to state (N', ρ'_k) due to the scattering by impurities accompanied with the absorption of M real photons is proportional to

$$|\mathcal{M}_i^{(M)}|^2 = J_M^2(\xi_\Omega) |\mathcal{M}_i|^2. \quad (12)$$

Here, $J_M(x)$ is the Bessel function, $\xi_\Omega \propto L_\varepsilon$ where $L_\varepsilon = (e\mathcal{E}/m\Omega^2) \sqrt{\Phi(\Omega/\Omega_c)}$ is the amplitude of classical oscillations of the electron orbit center in the crossed dc electric and magnetic fields under the ac microwave electric field \mathcal{E} , and, neglecting the effects of microwave radiation polarization, $\Phi(\omega) = \omega(1 + \omega^2)/(1 - \omega^2)$. Actually, the appearance of the Bessel functions in Eqn (12) is the result of the scattering matrix element calculation using the exact wave functions of electrons in both dc and ac fields. This can be attributed to the processes of absorption and emission of an arbitrary number of virtual photons in each process involving M real photons. Equation (12) corresponds to the following two features of the photon-assisted impurity scattering processes [53]. First, $|\mathcal{M}_i^{(M)}|^2$ and, consequently, the probability of the processes involving M real photons is not proportional to $|\mathcal{E}|^{2M}$ — it is a more complex function of \mathcal{E} due to the Bessel function dependence. Naturally, at low microwave powers (low ac electric fields), the matrix element for single photon processes becomes $|\mathcal{M}_i^{(1)}|^2 \propto J_1^2(\xi_\Omega) \propto \xi_\Omega^2 \propto |\mathcal{E}|^2$. Second, the probability of the impurity scattering without the absorption or emission of real photons ($M = 0$), i.e., elastic impurity scattering depends, nevertheless, on the microwave field.

Dependence (12) was actually used for calculations of dissipative conductivity associated with photon-assisted impurity scattering in various papers [27–30, 35]. Function $\Phi(\Omega/\Omega_c)$ exhibits a singularity at $\Omega = \Omega_c$. This singularity is removed due to either the properties of the Bessel functions [$J_M(x)$ approaches zero as $x \rightarrow 0$] or the effect of collisions on the electron cyclotron motion.

In the electric field $E > E_b$, $E_b^{(F)}$, the LL broadening can be neglected. In this case, for the dissipative current associated with photon-assisted impurity scattering in a 2DES, one can arrive at a formula similar to that obtained in Ref. [9]:

$$J_D^{\text{inter}} \propto \Gamma_i \sum_{A, M > 0} \left| I_A^M \left[\varepsilon, T, \frac{\hbar(N\Omega_c - M\Omega)}{e|E|L} \right] \right|^2 \times \frac{\hbar(A\Omega_c - M\Omega)}{e|E|EL} \exp \left[-\frac{\hbar^2(A\Omega_c - M\Omega)^2}{2e^2 E^2 L^2} \right]. \quad (13)$$

Here, factor $|I_A^M|^2$ determines the contribution of the electron transitions between LLs with a change in the LL index for the value equal to A and with the absorption or emission of M real photons. At low intensities of microwave radiation, $|I_A^M|^2 \propto E^{2M} \propto P^M$, where $P \propto E^2$ is the microwave power. In particular, for a 2DES with a small filling number, at $E \ll E_c$ when the inter-LL processes without absorption or emission of photons provide an exponentially small contribution, assuming that the microwave intensity is moderate (so that only single-photon processes with $M = 1$ should be taken into account), one obtains the following simple formula [compare with Eqn (6)]:

$$J_D^{\text{inter}} \propto P \frac{\Gamma_i(\Omega_c - \Omega)}{e^2 |E| EL^2} \exp \left[-\frac{\hbar^2(\Omega_c - \Omega)^2}{2e^2 E^2 L^2} \right]. \quad (14)$$

Equation (13) leads to oscillations of the dissipative current (i.e., the microwave photoconductivity) as a function of the cyclotron frequency Ω_c and microwave radiation frequency Ω . The oscillations are associated with the dominance of those terms in Eqn (13) which correspond to the vicinity of the cyclotron resonance and its harmonics:

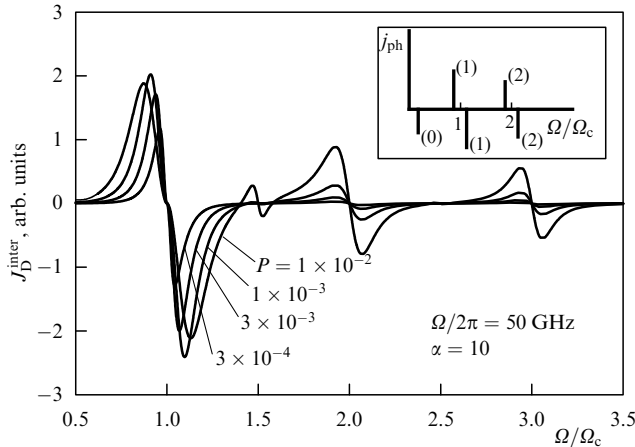


Figure 6. Microwave photocurrent as a function of inverse cyclotron frequency Ω/Ω_c calculated for different normalized microwave powers P , radiation frequency $\Omega/2\pi = 50$ GHz, and broadening parameter $a = 10$ [28]. The inset [9] shows positions of maxima and minima of the photocurrent associated with single photon absorption ($M = 1$). The numbers in parentheses are the resonance indices A .

$A\Omega_c \approx M\Omega$. One can see that each resonant term vanishes at the exact resonance. However, in the vicinity of each resonance, the microwave photoconductivity exhibits paired maximum (at $A\Omega_c$ somewhat exceeding $M\Omega$) and minimum (at $A\Omega_c$ somewhat less than $M\Omega$). As follows from Eqn (13), the microwave photocurrent in the minima is negative. Hence, if its absolute value exceeds the contributions of nonresonant transitions including the intra-LL transitions, the net current is directed opposite to the electric field that corresponds to ANC. The positions of the maxima and minima in question are schematically shown in the inset in Fig. 6 [9]. When $E \ll E_c$, $E_c^{(F)}$, the maxima and minima are well pronounced. In the case of small filling numbers, the maxima/minima of microwave photoconductivity correspond to the detuning $A\Omega_c - M\Omega \approx \pm eEL_F/\hbar$. Considering the specific dependence of the matrix elements at large LL indices and, hence, the dependence of $|I_A^M(\varepsilon, T, \eta)|^2$ on its arguments, one can find that in a degenerate 2DES with $N_F \gg 1$, the hops of the electron Larmor orbit centers with $\delta\rho \sim L_F \gg L$ are most important. As a result, for the positions of the maxima/minima one can obtain $A\Omega_c - M\Omega \approx \pm eEL_F/\hbar$.

In the case of a weak electric field when $E \ll E_b$, $E_b^{(F)}$, the shape of oscillations of the microwave photocurrent and, in particular, the positions of the maxima and minima are determined by the LL broadening. In such a case, the dissipative current in the presence of microwave radiation can be presented as [28]

$$J_D^{\text{inter}} \propto E\Gamma_i \sum_{A,M} \frac{|\bar{I}_A^M(\varepsilon, T)|^2 (A\Omega_c - M\Omega)}{[(A\Omega_c - M\Omega)^2 + \Gamma^2]^2}, \quad (15)$$

where the factor $|\bar{I}_A^M|^2$ plays the same role as factor $|I_A^M|^2$ in Eqn (13).

Figure 6 demonstrates the microwave photocurrent calculated for the frequency (magnetic field) range near the cyclotron resonance [20] corresponding to the electron transitions shown in Fig. 5 (left-hand side). One can see relatively weak maximum and minimum near $\Omega/\Omega_c \approx 1.5$ associated with two-photon absorption which become distinguishable at elevated microwave powers. Similar oscillating

dependences were obtained by Durst et al. [26] and Lei et al. [29, 30]. As seen from Fig. 6, the magnitudes of the microwave photoconductivity maxima and minima increase with increasing microwave power P in a rather wide range of P . As a result, at a certain microwave power the negative photocurrent associated with photon-assisted impurity scattering accompanied by electron inter-LL transitions can surpass the current due to the intra-LL transitions. In this case, the net dissipative current in the vicinity of the photocurrent minima can become negative, which corresponds to ANC with all its consequences. At elevated microwave powers, the increase in the magnitudes of the maxima and minima slows down. A further increase in microwave power can even lead to a decrease in these magnitudes. The calculated dependences of the microwave photocurrent on Ω_c , Ω , and P like those demonstrated in Fig. 6 are consistent with the experimental data.

As pointed out above, microwave radiation can affect intra-LL elastic impurity scattering processes (involving no real photons) in 2DESs. This effect was discussed recently [28]. The mathematically similar case of a 3DES was considered long ago [53]. Such an effect can be explained by the following: an increase in the ac electric field leads to an increase in the amplitude of the electron Larmor orbit and, consequently, to smearing of the electron wave function. Due to this, the integral of the wave functions before and after electron scattering and the impurity potential decrease, which causes a decrease in the scattering probability. Indeed, according to Eqn (12), the matrix element of impurity scattering without absorption or emission of real photons ($M = 0$) is $|\mathcal{M}_i^{(0)}|^2 = J_0^2(\xi_\Omega) |\mathcal{M}_i|^2$. Consequently, this matrix element decreases (exhibiting damping oscillations) with increasing ξ_Ω and even vanishes at $\xi_\Omega = 2.4$. This implies that microwave radiation can effectively suppress inter-LL impurity scattering, resulting in a decrease in the dissipative conductivity outside the cyclotron resonance and its harmonics (compare with the effect of radiation-induced suppression of tunneling between quantum wells [54]). This effect can be particularly important at relatively low microwave frequencies $\Omega < \Omega_c$, when the dissipative conductivity is mainly due to intra-LL impurity scattering processes. The effect of suppression of the intra-LL dissipative conductivity and its Shubnikov–de Haas oscillations in 2DESs with rather high electron mobility was recently observed by Dorozhkin et al. [55] and Mani [56]. The modulation of the Shubnikov–de Haas oscillations by microwaves was also observed by other authors (see, in particular, [57]). One can show that the dissipative conductivity σ (which in the case under consideration is primarily associated with intra-LL electron scattering processes, therefore, $\sigma = \sigma^{\text{intra}}$) can be presented as

$$\sigma_D \simeq \sigma_D^0 \left[1 - \left(\frac{P}{\bar{P}} \right) \mathcal{F} \left(\frac{\Omega_c}{\Omega} \right) \right]. \quad (16)$$

Here, σ_D^0 is the dissipative conductivity without microwave radiation [given, e.g., by (8) and exhibiting pronounced Shubnikov–de Haas oscillations] and $\bar{P} = m\Omega^4\hbar/4\pi\alpha\varepsilon_F$ is the characteristic microwave power, where $\alpha = e^2/\hbar c = 1/137$. According to Eqn (16), the averaged part of the dissipative conductivity and the amplitude of the Shubnikov–de Haas oscillations markedly decrease when the microwave power P becomes comparable to \bar{P} . As follows from Eqn (16) and as shown in Fig. 7, the effect of suppression of the dissipative conductivity and its oscillations in a fixed magnetic field (i.e., fixed cyclotron frequency) reinforces with

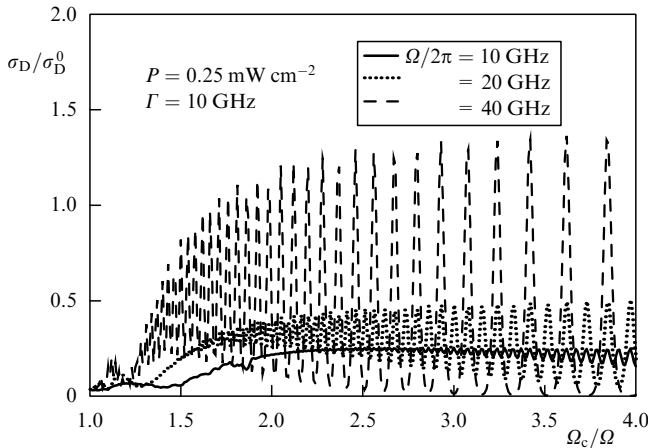


Figure 7. Magnetic-field dependences of conductivity associated with intra-LL processes at different microwave frequencies.

decreasing microwave radiation frequency. Setting the electron effective mass $m = 6 \times 10^{-29}$ g (GaAs) and the electron sheet density $\Sigma = 3 \times 10^{11}$ cm $^{-2}$ (as in Ref. [56]), at the microwave frequency $f = \Omega/2\pi = 20$ GHz (so that $\varepsilon_F/\hbar\Omega = 125$), one can find $\bar{P} = 0.7$ mW cm $^{-2}$. Formula (16) is valid at microwave intensities that are not too large. At elevated microwave powers, σ_D becomes a nonlinear function of P/\bar{P} .

The electron–phonon interaction can affect the transport phenomena in 2DESs with very high electron mobility. Due to this, photon-assisted scattering of electrons by acoustic phonons can contribute to microwave photoconductivity in 2DESs. This mechanism was considered recently [32–34] as a possible alternative to photon-assisted impurity scattering in an attempt to explain the high sensitivity of photoconductivity oscillations and ZR- and ZC-states to temperature. Some electron transitions associated with photon-assisted acoustic scattering are schematically shown in Fig. 5 (right-hand side). As shown in Refs [33, 34], photon-assisted acoustic scattering of electrons also results in an oscillating dependence of the microwave photoconductivity as a function of the cyclotron and radiation frequencies with maxima and minima around the cyclotron resonance and its harmonics. Moreover, this mechanism can provide a negative contribution to the dissipative current (in the minima). However, the positions of maxima and minima correspond to $\Lambda\Omega_c \leq M\Omega$ and $\Lambda\Omega_c \geq M\Omega$, respectively. This is in contrast to the photon-assisted impurity scattering mechanism of microwave photoconductivity and results of experimental observations. Therefore, one can conclude that photon-assisted acoustic scattering of electrons is not responsible for ANC and its consequences under the conditions of the experiments on ZR and ZC states. However, this mechanism can be one cause of the suppression of ANC with increasing temperature when the maxima of microwave photoconductivity due to photon-assisted acoustic interaction interfere with the minima associated with photon-assisted impurity scattering.

4. Discussion

The suppression of microwave photoconductivity oscillations leading to the vanishing of ANC (and, hence, ZR- and ZC-states) with increasing temperature as well as nonlinear dependences of the photoconductivity maxima and minima on the microwave power can be also attributed to an increase in the LL-broadening with the (electron) temperature and an

intensification of scattering processes involving the stimulated emission of microwave photons [39, 40].

As mentioned above, the span of the variations of the dc electric field strength in the domain structures formed due to ANC and the average electric field in the 2DES are essentially determined by the value $E = E_0$ at which the net dissipative current becomes zero (see Fig. 1). One can say that there are two possible mechanisms responsible for a steep increase in the local dissipative current at elevated local electric fields. These mechanisms are the ‘switch-on’ of the inter-LL impurity scattering (in this case, $E_0 \approx E_c$ in 2DESs with few occupied LLs and $E_0 \approx E_c^{(F)}$ in 2DESs with large filling numbers) and the ‘switch-on’ of the intra-LL acoustic phonon emission when the electron Hall drift velocity exceeds the speed of sound (in this case, $E_0 \approx E_a$). Disregarding the origin of the change in the dissipative current sign, the average value of the dc electric field throughout the 2DES should be $\langle E \rangle < E$ or even $\langle E \rangle \ll E$.

In the framework of the model under consideration, one can provide a plausible explanation as to why the effect of ANC and related ZR- and ZC-states are observed only under the conditions $\mu \sim 10^7$ cm 2 V $^{-1}$ s $^{-1}$, $T \sim 1$ K, and $H \sim 0.1$ T. The reasons for this, in our view, are as follows:

(a) *Weakness of electron scattering on remote donors and acoustic phonons and small ratio T/ε_F .* This results in small LL broadening, small dark conductivity associated with intra-LL electron transitions, and pronounced separation of LLs in fairly weak magnetic fields.

(b) *The cyclotron resonance at rather low microwave frequencies (due to small but pronounced separation of LLs).* This provides a strong effect of the microwave ac electric field on electron motion.

(c) *Large electron cyclotron radius (due to weak magnetic fields and, consequently, large filling numbers).* This leads to long-distance hops of the electron Larmor orbit centers caused by impurity scattering, particularly on short-range residual impurities, and a strong influence of the microwave ac electric field on such hops.

(d) *Strong photon-assisted electron scattering by impurities due to (b) and (c).* As a result, ANC occurs at relatively low microwave powers (the required power $\sim \Omega^3$, i.e., it steeply decreases with decreasing microwave frequency).

Although the plausibility of the scenario of the occurrence of ZR- and ZC-states in 2DESs is based on the concept of ANC associated with photon-assisted impurity scattering, further experimental and theoretical studies are required. First of all, this necessitates accurate measurements of the residual resistance and conductance in the ZR- and ZC-states, respectively; investigation of the photoconductivity, ANC, and ZR- and ZC-states in 2DESs in higher magnetic fields and, therefore, at higher radiation frequencies (and its higher powers); and direct observation of the electric-field domain structures under the conditions of ANC. Perhaps such domain structures can be visualized using the shift of interband magnetoabsorption in the electric field [41].

5. Conclusion

Thus, the main experimental facts concerning the oscillatory behavior of the microwave photoconductivity in 2DESs with high electron mobility and the appearance of ZR- and ZC-states can be explained invoking the concept of ANC associated with photon-assisted impurity scattering and affected by acoustic phonon scattering and electron–electron scattering.

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Millimeter-wave response in the magnetoconductivity of highly perfect two-dimensional electron systems

S I Dorozhkin

1. State of the art

This talk reviews the state of the art research on photo-response in the magnetoconductivity of 2D electron systems, with emphasis on the experimental aspects of the phenomenon and including the latest results from the cond-mat preprint archive; and presents a photoresponse model involving a radiation-induced nonequilibrium in the electron distribution function. It also puts forward the hypothesis that

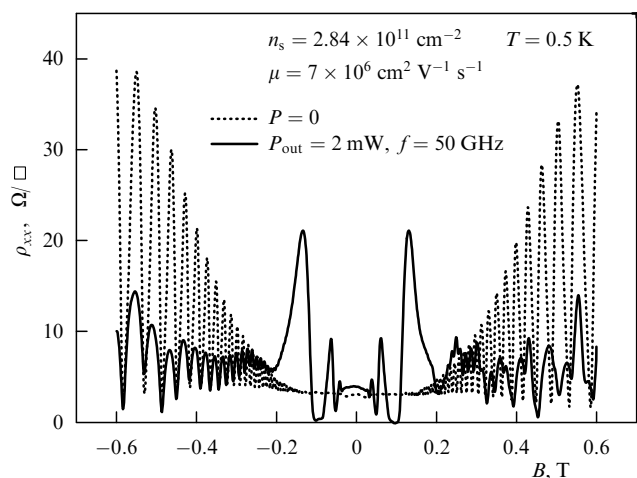


Figure 1. Magnetoconductivity ρ_{xx} of a 2D electronic system near a single GaAs/AlGaAs heterojunction in the absence of radiation (dotted curve) and in the presence of 50 GHz radiation (solid curve). Experimental parameters are shown in the figure. Here and hereafter the generator output power is indicated as radiation power (2 mW).