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Superluminal localized structures of electromagnetic radiation

N N Rozanov‡

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Contents

Abstract. **This paper analyzes localized invariable-form structures of electromagnetic radiation in vacuum and soliton-like light structures traveling superluminally in a nonlinear medium. Both types of structures are concluded to be unstable to small perturbations.**

1. Introduction

In connection with the approach of the centenary of the publication of Einstein's first paper on the theory of relativity [1], it is well to bear in mind that there are several subjects in this theory which are presumably not quite readily perceived and from time to time give rise to bursts of scientific publications. Among these subjects is the question of motion with superluminal velocities (more precisely, with velocities exceeding the speed of light in vacuum c).

As repeatedly noted in the literature, the possibility of suchlike superluminal motion does not contradict the theory of relativity and these motions exist in nature and have been observed experimentally. 'Prohibited' are those motions which involve signal (information) transfer with a superluminal velocity, the prohibition being related to the violation of the causality principle [2]. Leaving aside hypothetical particles — 'tachyons', whose velocity always exceeds the speed of light c [2] and the acceptance of the existence of which would indeed generate problems with causality — the examples, which have been known for decades, reduce to the following (they are analyzed in review [3]). First, there is a light pulse made up of plane waves with the same wave vector direction which is obliquely incident on a plane screen from a vacuum [4]. If the angle of incidence (the angle between the wave vector and the normal to the screen) is denoted by ψ, the boundary of the illuminated screen area will travel along the screen with a velocity $v = c/\sin\psi > c$. Kindred examples are different versions of a spotlight — the spot produced by a searchlight (a pulsar in astrophysics) rotating with a constant angular velocity Ω on a screen located far enough from the searchlight [3, 5]. Second, there is the case of media with frequency dispersion and appreciable absorption or amplification, where both the phase velocity (at which a surface of constant phase travels in a monochromatic wave) and the group velocity (at which the part of a pulse which carries the bulk of energy travels) may exceed c. In this case, it is pertinent to note that the phase velocity does not correspond to the motion of any physical carrier and the group velocity is an approximate notion, which pertains to pulses whose spectral width is small in comparison with the carrier frequency and whose shape is conserved fairly well [6].

The closer the carrier frequency to the absorption or amplification resonances of the medium, the greater the violation of this approximation, i.e., the approximation is violated precisely in those cases where the group velocity exceeds c. Velocities of $6-9\,c$ were experimentally observed for a laser pulse in an amplifying medium [7]. Here, it is actually a question of pulse shape variation: the leading edge of the pulse travels through the medium with an atomic level population inversion and decreases it to experience the strongest amplification, so that the leading edge becomes progressively steeper [8, 9]. Close to this group is the following gedanken experiment. Along a straight line we arrange participants with their own lamps and synchronized watches and agree upon the time of sequential actuation of each lamp such that the interval is shorter than the time the light takes to propagate the distance between the lamps. Then, a superluminal propagation of radiation and energy will be observed. But in none of these examples does the superluminal motion signify a signal transfer with a superluminal velocity. Indeed, in the former group of examples, the spotlight does not transfer information between different points of the screen. In the latter group, the superluminal motion of the peak of the

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‡ The author is also known by the name N N Rosanov. The name used here is a transliteration under the BSI/ANSI scheme adopted by this journal.

N N Rozanov Research Institute for Laser Physics,
Birzhevaya liniya 12, 199034 St.-Petersburg, Russian Federation
Tel. (7-812) 328 10 93
E-mail: nrosanov@yahoo.com

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pulse is possible only for a medium 'prepared by the prepulse', i.e., by the smooth leading edge of the pulse preceding the peak. It should be remembered that a sharp pulse edge, in front of which the field vanishes or possesses a discontinuity, travels with the speed of light in vacuum c [10]. These are precisely the pulses associated with the limiting velocity of information transfer (in the presence of noise, the description of signal extraction against the background noise would necessitate detailing the detector properties, but these questions are not considered here).

Fairly recently, a series of papers about localized (retaining limited dimensions and invariable in form) electromagnetic radiation structures in homogeneous linear media made its appearance in the literature; in several publications these polychromatic structures are referred to as X waves (see, for instance, Refs [11, 12] and references therein). In a sense, these works are a continuation of the subject of 'nondivergent' (Bessel) beams (see Ref. [13] and its detailed criticism expressed in Ref. [14]); Ref. [11] also bears references to the related acoustic X waves not considered below. While the localization of light in an inhomogeneous linear medium is universally known (for instance, in an optical fiber), the necessary condition for the spatial (non-one-dimensional) light localization in a homogeneous medium is commonly believed to be the presence of optical nonlinearity (soliton-like structures). That is why the question of whether it is possible to suppress diffraction and localize light in a linear medium deserves, in our view, additional discussion.

A still more extraordinary property of the aforementioned localized structures in vacuum is their superluminal velocity, i.e., the fact that they travel as a whole with a constant velocity V exceeding the speed of light in vacuum c. We emphasize that, owing to the invariance of the structure form, we need not introduce the concept of group velocity: the superluminal transfer of the fixed distributions of electric and magnetic field intensities rigorously corresponds to the superluminal transfer of all field characteristics as well, including energy characteristics. However, as explained in Section 3, this does not necessarily imply the superluminal velocity of information transfer. For the same reason, such behavior is qualitatively different from that discussed above for amplifying media, where the superluminal group velocity or the superluminal velocity of motion of the peak of pulse intensity is associated with a significant change of the pulse shape during propagation. Also related to this subject are the conclusions of Ref. [15] about the possibility of the existence of stable superluminal solitons in nonequilibrium nonlinear media. That is why the question of whether the existence of such structures corresponds to the principle of limiting velocity of information transfer should be given more careful consideration.

The above questions are the subject of our paper. In the new wave of publications on superluminal velocities, so basic a matter as the stability of a localized structure receives, in our opinion, inadequate consideration. For 'conventional' solitons (conservative, i.e., in media with negligible dissipation [16, 17]), a small external perturbation (with a sufficiently low energy) leads to a perturbation of the soliton that is also small even for arbitrarily long propagation distances. Even more stable are dissipative solitons (autosolitons) realized in media with the gain and loss of energy: in this case, a small perturbation of soliton shape decays in the course of propagation [17–19]. The same requirement of at least a 'weak' stability (as in the case of conventional solitons) is naturally also imposed upon superluminal localized optical structures in vacuum and in a continuous medium. However, we will arrive at the conclusion that all these structures are unstable and therefore possess a limited lifetime.

2. Superluminal localized radiation structures in vacuum

We begin with the discussion of the very possibility of localized electromagnetic radiation structures in vacuum in the framework of the classical Maxwell equations. To take into account the vector nature of electromagnetic radiation, it is conveniently characterized by the vector potential $\mathbf{A}$. The electric $\mathbf{E}$ and magnetic $\mathbf{H}$ field intensities are expressed in terms of the vector potential by the relations (the scalar potential is taken to be equal to zero)

$$\mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{A}}{\partial t}, \qquad \mathbf{H} = \operatorname{rot}\mathbf{A}. \tag{1}$$

In the one-dimensional geometry, when the fields are plane waves and the potential depends only on one spatial coordinate z and on the time t, the Maxwell equations reduce to the one-dimensional wave equation. Its general solution given by D'Alembert corresponds to the superposition of two pulses of arbitrary shape invariable during propagation, which travel in opposite directions with a velocity c:

$$\mathbf{A} = \mathbf{A}_{+}(z - ct) + \mathbf{A}_{-}(z + ct). \tag{2}$$

In the one-dimensional vacuum, localized light structures can therefore exist, provided that they move with a velocity c. This corresponds to the absence of dispersion (the frequency dependence of optical properties) of a vacuum. Note that we are treating the vacuum in a purely classical way, in the framework of the Maxwell equations. Although the physical — electron-positron — vacuum possesses nonlinearity as well as spatial and frequency dispersion owing to the quantum-electrodynamic effects of vacuum polarization, these vacuum properties can manifest themselves in modern experiments on unique laser facilities affording ultrahigh power densities [20–22].

Let us now make certain that the Maxwell equations do allow the existence of non-spreading ('non-diffracting') radiation structures in three-dimensional space, which travel in vacuum with a constant superluminal velocity $V > c$ (see Refs [11, 12]). At each point in time, the field can be represented as the superposition (the Fourier integral) of uniform plane waves with (real) wave vectors $\mathbf{k}$. Then, for any point in time the components of the vector potential corresponding to an individual plane (monochromatic) wave are of the form $\tilde{\mathbf{A}}(\mathbf{k})\exp(\mathrm{i}\mathbf{kr} - \mathrm{i}\omega t)$ in the complex representation. In this case, from the Maxwell equations there follow the transverse-type condition $\tilde{\mathbf{A}}(\mathbf{k})\mathbf{k} = 0$ and the dispersion relation $k^2 = \omega^2/c^2$ between the frequency ω and the wave vector $\mathbf{k}$. The general solution of the wave equation for the vector potential is defined by the spectral decomposition of the form

$$\mathbf{A}(\mathbf{r}, t) = \int \tilde{\mathbf{A}}(\mathbf{k})\exp(\mathrm{i}\mathbf{kr} - \mathrm{i}\omega t)\,\mathrm{d}\mathbf{k}.$$

Our interest is with localized field structures (with a finite energy) traveling along the z-axis with an invariable velocity V and possessing invariable form (the broader class of 'weakly varying' waves was considered in Refs [23, 24] and references therein). For them, the vector potential has the form

$$\mathbf{A}(x, y, z, t) = \mathbf{A}(x, y, z - Vt) \,. \tag{3}$$

We introduce the longitudinal z and transverse $\mathbf{r}_\perp = (x, y)$ coordinates and the corresponding wave vector components $k_\parallel$ and $\mathbf{k}_\perp = (k_x, k_y)$. Applying conditions (3) to the partial plane waves with amplitudes of the form

$$\tilde{\mathbf{A}}(\mathbf{k}) \exp\left(\mathrm{i}\mathbf{k}_\perp \mathbf{r}_\perp\right) \exp\left[\mathrm{i}k_\parallel \left(z - \frac{\omega}{k_\parallel}\, t\right)\right],$$

we obtain the relation

$$\omega = V k_\parallel \,. \tag{4}$$

Then, the longitudinal and transverse components of the wave vectors obey the following dispersion relation:

$$k_\perp^2 = \left(\frac{V^2}{c^2} - 1\right) k_\parallel^2 \,. \tag{5}$$

From relation (5) it follows that stationary structures may only be superluminal: $V^2 > c^2$. In cylindrical coordinates (r, φ, z) the wave superpositions now take on the form

$$\mathbf{A}(r, \varphi, z - Vt) = \int \tilde{\mathbf{A}}(k_\parallel, \alpha) \exp\left\{\mathrm{i}k_\parallel \left[\left(\left(\frac{V}{c}\right)^2 - 1\right)^{1/2} \times r\cos(\alpha - \varphi) + (z - Vt)\right]\right\} \mathrm{d}k_\parallel \,\mathrm{d}\alpha \,. \tag{6}$$

Integration with respect to $k_\parallel$ is performed between infinite limits and with respect to the angle α over the interval of length 2π. The field distributions defined by expression (6) are exact solutions to the Maxwell equations for an arbitrary weight function $\tilde{\mathbf{A}}(k_\parallel, \alpha)$ (subject to the above transverse-type condition). Dependent on the form of this function is the distribution localization property related to the finiteness of the energy, for which purpose it would suffice to impose the normalization condition

$$\int \left|\tilde{\mathbf{A}}(k_\parallel, \alpha)\right|^2 \mathrm{d}k_\parallel \,\mathrm{d}\alpha < \infty \,. \tag{7}$$

Different examples of suchlike distributions are given in Ref. [11]. Upon expanding the potential into the Fourier series

$$\tilde{\mathbf{A}}(k_\parallel, \alpha) = \sum_{m=-\infty}^{\infty} \tilde{\mathbf{A}}_m(k_\parallel) \exp(\mathrm{i}m\alpha) \,,$$

we can write expression (6) in the form

$$\mathbf{A}(r, \varphi, z - Vt) = -2\pi \sum_{m=-\infty}^{\infty} \mathrm{i}^m \exp(\mathrm{i}m\varphi) \times \int \tilde{\mathbf{A}}_m(k_\parallel)\, J_m(k_\perp r_\perp) \exp\left[\mathrm{i}k_\parallel (z - Vt)\right] \mathrm{d}k_\parallel \,, \tag{8}$$

where J_m is the Bessel function.

Therefore, exact solutions of the Maxwell equations describe stationary blobs of electromagnetic fields possessing finite or normalizable energy and propagating with a constant superluminal velocity. One can easily see from expression (6) that the magnitude of velocity affects only the scale of coordinates and time. If the new variables

$$\rho = r\sqrt{\left(\frac{V}{c}\right)^2 - 1} \,, \qquad \varsigma = z - Vt \tag{9}$$

are introduced, the velocity V will not appear in the expressions for the field distributions whatsoever:

$$\mathbf{A}(\rho, \varphi, \varsigma) = \int \tilde{\mathbf{A}}(k_\parallel, \alpha) \exp\left\{\mathrm{i}k_\parallel \left[\rho\cos(\alpha - \varphi) + \varsigma\right]\right\} \mathrm{d}k_\parallel \,\mathrm{d}\alpha \,. \tag{10}$$

The absence of diffraction spreading can be explained as follows. The diffraction of an ordinary (for instance, Gaussian) beam of monochromatic radiation takes place owing to the difference (for a fixed frequency of the light) of the longitudinal components of the wave vector or the phase velocities of individual plane waves into which the beam in the initial section is decomposed. As a consequence, misphasing of these plane waves with path length z occurs, which is accompanied by diffraction beam spreading. When the field is made up of plane waves with different frequencies ω whose traveling direction ensures the fulfillment of relation (4), the longitudinal components of the phase velocities for all such waves coincide, so that they remain phased-in during the subsequent propagation as well. It is valid to say that producing a localized structure requires illuminating a sufficiently large spatial volume (the characteristic dimension is L) with broadband radiation with a specially selected relation between the partial frequencies and the wave vectors in such a way that the interference of these waves is constructive in a local domain and destructive in the remaining one. The domain of constructive interference will travel with the velocity V through the illuminated volume.

One can see from this reasoning that such superluminal structures are idealized objects which require for their realization the 'illumination' of a vacuum throughout infinite space. When only a finite domain with dimensions L is 'illuminated', as is inevitably the case in a real experiment, the boundary between the illuminated and non-illuminated domains will travel with the speed of light in vacuum c, so that the lifetime of the localized structure is limited by a value $\sim L/(V - c)$ (recall that $V > c$). The structure persists as long as its center is far from the boundaries of the illuminated region (which travel with a velocity c lower than the velocity of travel of the main part of the structure V).

3. On the stability of superluminal structures

A more general realizability criterion for the structures of the type under consideration consists of the requirement that they should be immune to small perturbations. It is easy to see that suchlike structures are unstable. Indeed, one version of a small perturbation of an ideal (extending throughout the entire space) structure is truncation of the edges of this structure at an arbitrarily low intensity level, which corresponds to experimental realization. In this case, the perturbation energy will also be arbitrarily low. However, the truncation front will, in accordance with the aforesaid, propagate with the speed of light in vacuum c, with the result that the structure lifetime will once again be limited to the quantity $\sim L/(V-c)$, where L is the characteristic dimension of the 'truncated' structure. The lifetime increases when $V \to c$, but in this case the structure dimensions increase as well.

Similar superluminal localized structures of electromagnetic radiation are also possible in continuous media. Here, an additional limitation is the requirement that the absorption in the medium can be neglected (the frequency interval involved should belong to the medium transparency range). Furthermore, the difference between linear media and the vacuum will lie with the form of dispersion relation. By repeating the previous reasoning about truncation of the distant edges of the structure, it is easy to verify that all superluminal structures — both in linear and nonlinear media — are unstable and possess a finite lifetime, its upper limit coinciding with the estimate made in the vacuum case. It is also pertinent to note that one of the most general physical equations — the equation of wave front motion [10], in which the speed of light in vacuum naturally appears as the highest possible velocity of propagation of any signals — serves as an important verification criterion for material equations of nonlinear electrodynamics and optics, which is especially topical in the case of extremely short pulses. Indeed, the conventional material relation for the refractive index n of a medium with the Kerr nonlinearity is of the form $n = n_0 + n_2 I$, where n_0 is the linear refractive index, I is the radiation intensity, and n_2 is the nonlinearity coefficient, which may be either positive or negative for various media. Then, as is easily seen, the velocity of the wave front for $n_2 < 0$ could exceed the speed of light in vacuum. This means that the material relation in use has limited applicability and the dispersion (finite response time) of the nonlinear refractive index should be included into consideration [25].

A significant difference of the above superluminal localized structures from the ordinary 'subluminal' soliton-like radiation packets consists of precisely the fact that the latter are stable to small perturbations. Indeed, from the stability of, for instance, dissipative (in a medium with nonlinear amplification and absorption) subluminal solitons to truncation of their distant edges there follows the completion (of formation) of these edges during propagation, which is impossible for superluminal structures. It is easy to verify that superluminal soliton stability would imply the possibility of transferring information with a superluminal velocity, which is ruled out by the general principles of the theory of relativity. It is noteworthy that Sazonov in his review paper [15] expressed the opposite viewpoint that superluminal solitons may be stable in nonequilibrium media. In our view, this is due to an incomplete stability analysis and the neglect of the most dangerous perturbations in that review. Soliton stability is also incompletely treated in a recent paper [26] (in the analysis of transverse effects by the averaged-Lagrangian technique, small-scale perturbations, i.e., the effects like small-scale self-focusing [27], are not considered).

4. Conclusion

Thus, although localized (invariable in form) superluminal structures (X waves) exist as exact solutions of the Maxwell equations in vacuum, they are idealized objects, whose realization necessitates the 'illumination' of the entire infinite space and which are not stable to small perturbations. From a broader viewpoint, any superluminal radiation structures in linear and nonlinear media alike are unstable. The real (stable) localization of light in a homogeneous medium is possible only in the nonlinear regime for subluminal-velocity solitons. However, this does not signify that localized linear distributions cannot be realized, for instance in a vacuum or a diluted gas, for a limited time interval (lifetime). Such structures are not merely a 'freak of light and shade'. Quite the opposite. These experiments appear to be interesting from the general physical standpoint as well as due to the possibility of ultrashort action of a high-intensity burst of electromagnetic radiation on different objects. In principle, the superluminal travel of the burst of electromagnetic radiation in vacuum may give rise to Vavilov – Cherenkov emission of gravitational waves. This follows from the (linearized) Einstein gravitational equations, in which the superluminal source is represented by the energy-momentum electromagnetic field tensor [28]. This effect leads to energy losses in the burst and is thereby responsible for the additional limitation of its lifetime. However, because of the extreme smallness of the effect its manifestation may be tangible only for structures of cosmic (astrophysical) scale. In laboratory conditions, the superluminal propagation velocity of a field burst opens up fresh real possibilities, supposedly including applications (see the discussion of Vavilov – Cherenkov radiation in the superluminal motion of a spotlight along a screen [3, 5]). The use of laser supercontinuum generation sources [29, 30] shows promise for the production of laboratory superluminal localized structures of the X-wave type, which necessitate broadband coherent radiation.

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