Physics – Uspekhi **48** (12) 1235 – 1247 (2005)

REVIEWS OF TOPICAL PROBLEMS

PACS numbers: **04.30.–w**, **04.62.+v**, **98.80.–k**

Relic gravitational waves and cosmology [1†]

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DOI: 10.1070/PU2005v048n12ABEH005795

Contents

Abstract. The paper begins with a brief recollection of interactions of the author with Ya B Zeldovich in the context of the study of relic gravitational waves. The principles and early results on the quantum-mechanical generation of cosmological perturbations are then summarized. The expected amplitudes of relic gravitational waves differ in various frequency windows, and therefore the techniques and prospects of their detection are distinct. One section of the paper describes the present state of efforts in direct detection of relic gravitational waves. Another section is devoted to indirect detection via the anisotropy and polarization measurements of the cosmic microwave background (CMB) radiation. It is emphasized throughout the paper that the inference about the existence and expected amount of relic gravitational waves is based on a solid theoretical foundation and the best available cosmological observations. It is also explained in great detail what went wrong with the so-called 'inflationary gravitational waves', whose amount is predicted by inflationary theorists to be negligibly small, thus depriving them of any observational significance.

1. Introduction

The story of relic gravitational waves has revealed the character of Ya B Zeldovich not only as a great scientist but also as a great personality. One should remember that the beginning of the 1970s was dominated by the belief that massless particles, such as photons, neutrinos, and gravitons, cannot be created by the gravitational field of a homogeneous isotropic universe. Zeldovich shared this view and published papers supporting this picture. He was enthusiastic about cosmological particle creation [1] and contributed a lot (together with coauthors) to this subject. However, he thought that something interesting and important could only happen if the early universe was highly anisotropic.

When I showed in Refs [2, 3] that massless gravitons (gravitational waves) could, in fact, be created by the gravitational field of a homogeneous isotropic universe, considerable debate arose around this work. I argued that the coupling of gravitons to the 'external' gravitational field follows unambiguously from the equations of general relativity, and it differs from the coupling of other known massless particles to gravity. In contrast to other massless fields, this specific coupling of gravitational waves allows their superadiabatic (parametric) amplification by the 'pumping' gravitational field of a nonstationary universe. (A similar coupling to gravity can be postulated for the still hypothetical massless scalar field.) If classical gravitational waves were present before the era of amplification, they would have been amplified. But their presence is not of necessity: even if the waves are initially in their quantum-mechanical vacuum (ground) state, the state will inevitably evolve into a multiparticle state. In phenomenological language, gravitational waves are generated from their zero-point quantum oscillations.

The intense debate has finished in a surprising and very flattering way for me. It is common knowledge that it was virtually impossible to win a scientific bet against Zeldovich — he knew practically everything about physics and had tremendous physical intuition. But sometimes he would find a cute way of admitting that his previous thinking was not quite right, and that he also learned something from a debate. On this occasion it happened in the following manner.

After one of his rare trips to Eastern Europe (as far as I remember, it was Poland), Zeldovich gave me a gift. It was a poster showing a sophisticated, impressionist-style, lady. The fact that this was a poster with a sophisticated lady was not really surprising — you could expect this from Yakov

[1] Enlarged version of the report submitted to *International Conference on Cosmology and High-Energy Astrophysics dedicated to the 90th anniversary of birthday of Ya B Zeldovich, Moscow, December 20 – 24, 2004* http://hea.iki.rssi.ru/Z-90/

[†] ***Editors' Note***: The present paper by L P Grishchuk provoked a criticism that would be revealed in the letter by V N Lukash to *Physics – Uspekhi*, published in the next issue of the journal: *Phys. Usp.* **49** (1) (2006).

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Received 13 May 2005, revised 12 October 2005
Uspekhi Fizicheskikh Nauk **175** (12) 1289 – 1303 (2005)
Translated by L P Grishchuk; edited by A Radzig

Borisovich. What was surprising and flattering for me was his hand-written note at the bottom of the poster. In my translation from the Russian, it said, "*Thank you for your goal in my net*." He was hinting at my passion for football, and he knew that this comparison would be appreciated much better than any other. So, this is how a great man admits an error: he simply says "thank you for your goal in my net."

It was clear from the very beginning of the study of relic gravitational waves that the result of the amplification of a wave field should depend on the strength and time evolution of the gravitational pump field. We know little about the very early universe these days; even less was known at the beginning of the 1970s.

The best thing you can do is to consider plausible models. The simplest option is to assume [2] that the cosmological scale factor $a(\eta)$ in the expression

$$\mathrm{d}s^2 = a^2(\eta)\left[-\mathrm{d}\eta^2 + (\delta_{ij} + h_{ij})\,\mathrm{d}x^i\,\mathrm{d}x^j\right] \qquad (1)$$

consists of pieces of power-law evolution:

$$a(\eta) = l_0|\eta|^{1+\beta}\,, \qquad (2)$$

where l_0 and β are constants. Then, the perturbed Einstein equations for $h_{ij}(\eta, \mathbf{x})$ are simplified and can be solved in elementary functions. In particular, the intervals of power-law evolution (2) make tractable the effective 'potential barrier' a''/a in the gravitational wave (g.w.) equation [2]:

$$\mu'' + \mu\left[n^2 - \frac{a''}{a}\right] = 0\,, \qquad (3)$$

where the prime $'$ stands for $\mathrm{d}/\mathrm{d}\eta = (a/c)\,\mathrm{d}/\mathrm{d}t$.

Using Eqn (2) and the unperturbed Einstein equations one can also find the effective equation of state for the 'matter', whatever it is, which drives the intervals of $a(\eta)$:

$$\frac{p}{\varepsilon} = w = \frac{1-\beta}{3(1+\beta)}\,. \qquad (4)$$

The somewhat strange form of the index $1+\beta$ in Eqn (2) was motivated by a serious concern of that time: it was necessary to prove that even a small deviation from the exceptional law of evolution $a(\eta) \propto \eta$ guarantees the effect of g.w. amplification. It is only in this exceptional case that the effective potential a''/a vanishes, and therefore the superadiabatic coupling of gravitational waves to the nonstationary pump field $a(\eta)$ also vanishes. (The analogous effective potential is absent in equations for photons, massless neutrinos, and some massless scalar particles.)

The convenience of the notation utilized in Eqn (2) is that it parameterizes the exceptional case by $\beta = 0$, and deviations from this case by a small β. Indeed, it was shown in Ref. [2] that the amplitude of the generated g.w. mode is proportional to the small parameter β; but it is not zero if $\beta \neq 0$. At the same time, if β is not especially small, the amplitude of the gravitational wave $h_p(n)$, soon after the beginning of the superadibatic regime and while the wave is still in this regime, i.e., before any further processing of the amplitude, is evaluated as

$$h_p(n) \approx \frac{l_{\mathrm{Pl}}}{l_0}\left(\frac{n}{n_{\mathrm{H}}}\right)^{2+\beta}. \qquad (5)$$

Estimate (5) is approximate (we will be discussing more accurate formulas below) but it contains all the necessary physics. The underlying concepts of generation and detection of primordial gravitational waves have not changed since the first calculations [2, 3], and it is important for our further discussion to recall them.

To begin with, we note that Eqn (5) is formulated for the dimensionless amplitude h of a given g.w. mode characterized by a constant dimensionless wavenumber n. (The $h(\eta)$ and $\mu(\eta)$ mode functions are related by $h = \mu/a$.) The wavelength λ measured in units of laboratory standards (as Zeldovich used to say, measured in 'centimeters') is related to n by $\lambda(\eta) = 2\pi a(\eta)/n$. It is convenient to use (and we will always do this) such an η-parameterization of $a(\eta)$ that the present-day scale factor is $a(\eta_R) = 2l_{\mathrm{H}}$, where $l_{\mathrm{H}} = c/H(\eta_R)$ is the present-day value of the Hubble radius. Then, $n_{\mathrm{H}} = 4\pi$ is the wavenumber of the waves whose wavelength today is equal to the present-day Hubble radius. Longer waves have smaller n's, and shorter waves have larger n's.

Expression (5) is essentially a consequence of the two following assumptions. First, it is assumed that the mode under consideration has entered the superadibatic regime in the past, and is still in this regime. This means that the mode's frequency, instead of being much larger than the characteristic frequency of the pump field, became comparable with it at some instant of time in the past. Or, in cosmological context, the wavelength $\lambda(\eta)$ of the mode n, instead of being much shorter than the instantaneous Hubble radius $c/H(\eta) = a^2/a'$, became equal to it at some instant of time η_i, i.e., $\lambda_i = c/H_i$. For the scale factors entering Eqn (2), this condition leads to $(n/n_{\mathrm{H}})|\eta_i| \approx 1$.

Second, we assume that by the beginning of the super-adiabatic amplification regime at $\eta = \eta_i$, the mode is still in its vacuum state, rather than, say, in a strongly excited (multi-particle) state. That is, in the language of classical physics, the mode's amplitude near η_i was not much larger than $h_i(n) \approx l_{\mathrm{Pl}}/\lambda_i$, where l_{Pl} is the Planck length, $l_{\mathrm{Pl}} = \sqrt{G\hbar/c^3}$. This condition imposed on the amplitude follows from the requirement that initially there were only the zero-point quantum oscillations of the g.w. field, and the initial energy of the mode was equal to $(1/2)\,\hbar\omega_i$. Because of the condition $\lambda_i = c/H_i$, we can also write $h_i(n)$ as $h_i(n) \approx H_i l_{\mathrm{Pl}}/c$.

The amplitude of the mode, after the mode's entrance into the amplifying superadiabatic regime, and as long as this regime lasts, remains at the constant level $h_i(n)$, namely, $h_p(n) \approx h_i(n)$. This holds true instead of the adiabatic decrease in the amplitude $\propto 1/a(\eta)$ that would be true in the adiabatic regime. In general, the quantity H_i is different for different n's:

$$H_i \approx \frac{c}{l_0}\,\eta_i^{-(2+\beta)} \approx \frac{c}{l_0}\left(\frac{n}{n_{\mathrm{H}}}\right)^{2+\beta}.$$

Therefore, a specific dependence on n arises in the function $h_i(n)$, and this is how one arrives at Eqn (5) in a simple qualitative manner.

Formula (5) gives the evaluation of the primordial (before further processing) g.w. spectrum $h_p(n)$. Roughly speaking, the initial vacuum spectrum $h_{\mathrm{v}}(n) \propto n$ has been transformed into the primordial spectrum $h_p(n) \sim h_{\mathrm{v}}(n)\,n^{1+\beta_i}$, where β_i characterizes the scale factor of the era when the transition from the adiabatic to superadiabatic regime took place for the given interval of wavenumbers n. However, the same mode n can sooner or later leave the amplifying regime and start oscillating again. Obviously, this reverse transition from superadiabatic to adiabatic regime is being described by the same theory.

The final amplitudes at some fixed instant of time (for example, today's amplitudes), $h_f(n)$, are related to the $h_p(n)$-amplitudes by

$$h_f(n) \sim h_p(n)\, n^{-(1+\beta_f)}\,,$$

where β_f characterizes the era when the opposite transition from the superadiabatic to adiabatic regime took place (this is why the minus sign arises in front of $1+\beta_f$ in the exponent).

The discussed amplitudes $h(n)$ are in fact the root-mean-square (r.m.s.) amplitudes of the multimode field; they determine the mean-square value of the wave field h according to the general formula

$$\langle h^2 \rangle = \int h^2_{\mathrm{rms}}(n)\, \frac{\mathrm{d}n}{n}\,.$$

It is necessary to say that in the beginning of the 1980s, the inflationary cosmological scenario governed by a scalar field [4] was gaining popularity. Its central element is the interval of de Sitter expansion, which corresponds to $\beta = -2$ in Eqn (2) (η grows from $-\infty$, $1+\beta < 0$) and $w = -1$ in Eqn (4). By the time of publication of the inflationary scenario, unusual equations of state for 'matter' driving the very early universe, including such exotic ones as $p = -\varepsilon$, $w = -1$, had already been the subject of cosmological research, most notably in the work of A D Sakharov [5].

The g.w. calculations for the special case $\beta = -2$ were performed in a number of papers (see, for example, Refs [6–9]). If $\beta = -2$, the dependence on n vanishes in the general equation (5), and the primordial (unprocessed) spectrum $h_p(n)$ becomes 'flat' — that is, n-independent. Ironically, the prospects of direct detection of the stochastic g.w. background characterized by the corresponding processed (today's) spectrum had already been explored by that time [3]; the processed spectral index for this model is $\alpha = 1$ in notations of that paper. The author of paper [3] also suggested the use of cross-correlated data from two detectors and touched upon the technique of 'drag-free satellites' that was later developed in the Laser Interferometer Space Antenna (LISA).

The generality of inflationary, quasi-de Sitter solutions was a serious concern for Zeldovich for a long time. He kept wondering about the sensitivity of inflationary solutions to the choice of initial conditions. Nobody would take the inflationary scenario seriously if it were a very contrived or unstable solution. However, it was shown [10] that inflationary type evolutions are, in fact, attractors in the space of all possible solutions of the corresponding dynamical system. This decisive property made inflationary evolutions more plausible and appealing.

2. Direct detection of relic gravitational waves

The spectrum of $h_{\mathrm{rms}}(\nu)$ expected today is depicted in Fig. 1 (for more details, see Refs [11, 12]). Almost everything in this graph is the result of the processing of the primordial spectrum during the matter-dominated and radiation-dominated stages. The postulated 'Zeldovich's epoch' governed by a very stiff effective equation of state is also present in the graph, as shown by some relatively increased power at very

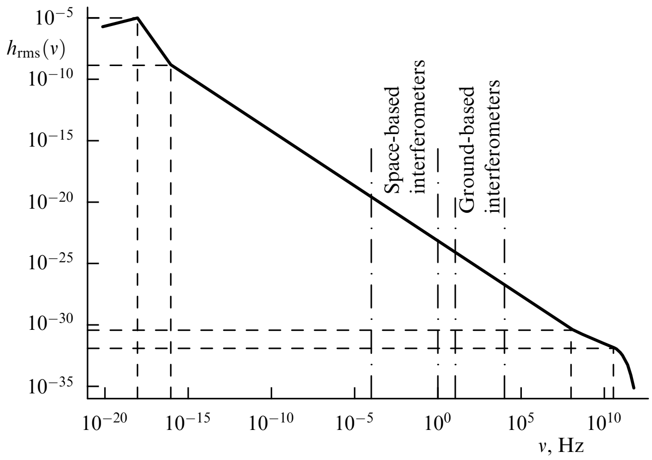

Figure 1. Envelope of the $h_{\mathrm{rms}}(\nu)$ spectrum for the case $\beta = -1.9$ (n = 1.2).

high frequencies. The primordial part of the spectrum survives only at frequencies below the present-day Hubble frequency $\nu_{\mathrm{H}} \approx 2\times10^{-18}$ Hz. The available CMB observations determine the amplitude and spectral slope of the g.w. spectrum at frequencies around ν_{H}, and this defines the spectrum at higher frequencies.

The numerical value of h_{rms} at frequencies around ν_{H} is determined by the numerical value of the observed quadrupole anisotropy of the CMB temperature. As will be shown in great detail in Section 4, it follows from the theory of cosmological perturbations that relic gravitational waves should provide a significant fraction of the observed CMB signal at very large angular scales (barring the logical possibility that the observed anisotropies have nothing at all to do with cosmological perturbations of quantum-mechanical origin).

In other words, the final theoretical results do not contain any dimensionless parameter which could be regulated in such a manner as to make the contribution of, say, density perturbations to the quadrupole anisotropy several orders of magnitude larger than the contribution from gravitational waves. These contributions must be roughly equal, but the theory cannot exclude that one of them will turn out to be a numerical factor 2–3 larger than another. Assuming that relic gravitational waves provide half of the signal, one can find from the observed $\delta T/T \approx 10^{-5}$ that $h_{\mathrm{rms}}(\nu_{\mathrm{H}}) \approx 10^{-5}$ and, hence, it follows from Eqn (5) that $l_{\mathrm{Pl}}/l_0 \approx 10^{-5}$.

The slope of the primordial g.w. spectrum is also taken from CMB observations. The commonly-used spectral index n (we denote it by a Roman letter n in order to distinguish it from the wavenumber n) is related to the parameter β appearing in Eqn (5) by the relationship $\mathrm{n} = 2\beta + 5$. The same relationship is also valid for density perturbations, to be discussed later.The current observations [13, 14] give evidence for $\mathrm{n} \approx 1$, which corresponds to $\beta \approx -2$. The particular graph in Fig. 1 is plotted for $\beta = -1.9$, n = 1.2, which tallies with the COBE data [15, 16]. (This spectral index $\mathrm{n} > 1$ implies that $w < -1$, according to Eqn (4). It is not difficult to imagine that such an effective equation of state could hold in the very early universe, if the recent supernovae observations hint at the validity of $w < -1$ even in the present-day universe !) In simple words, the position and orientation of the entire piece-wise function $h(\nu)$ is defined by the known value of the function at the point $\nu = \nu_{\mathrm{H}}$ and the known slope of the function in the vicinity of that point.

Incidentally, the initial quantum vacuum conditions for gravitational waves, at all frequencies shown in the graph, are formulated at the 'initial' instants of time, when each wavelength of interest was appreciably longer than the Planck length. Therefore, the results shown are immune to the short scale ambiguities of the so-called 'trans-Planckian' physics (see, for example, Ref. [17]). It is a different matter that the initial state at some frequencies is allowed to be a somewhat excited state, rather than the pure vacuum state, without running into a conflict with the adopted approximation of small perturbations. This exotic possibility and the corresponding modifications of the spectrum were discussed long time ago [18] (see also a related work [19]).

The graph in Fig. 1 shows the piece-wise envelope of today's g.w. spectrum. The result displayed is quite approximate. In particular, it completely ignores the inevitable oscillations in the spectrum, whose origin goes back to the gradual diminishing (squeezing) of quantum-mechanical uncertainties in the phases of the emerging waves and the macroscopic manifestation of this effect in the form of the standing-wave pattern of the generated field. (This is also related to the concept of 'particle pair creation'.) We will discuss these spectral oscillations below.

Nevertheless, the graph in Fig. 1 is convenient in that it gives simple answers to the most general questions concerning the amplitudes and spectral slopes of relic gravitational waves in various frequency intervals. For example, it points to the expected amplitude $h_{\rm rms} = 10^{-25}$ at $\nu = 10^2$ Hz. This is the level of the signal that we shall be dealing with in experimental programs. In terms of the parameter

$$\Omega_{\rm gw}(\nu) = \frac{\pi^2}{3}\, h^2(\nu)\left(\frac{\nu}{\nu_{\rm H}}\right)^2,$$

it corresponds to $\Omega_{gw} \approx 10^{-10}$ at frequency $\nu = 10^2$ Hz and in its vicinity.

Where do we stand now in the attempt to directly detect relic gravitational waves? The sensitivity of the presently operating ground-based interferometers is not good enough to reach the predicted level, but the experimenters are making a lot of progress. The data from the recently completed S3 run of LIGO [20] will probably allow us to reach the astrophysically interesting level of $\Omega_{\rm gw} \sim 10^{-4}$, as shown in Fig. 2 (courtesy of J Romano and the stochastic backgrounds group from LSC Collaboration). Fortunately, the projected sensitivity of the advanced LIGO ($\sim$ 2011) will be sufficient to reach the required level of $h_{\rm rms} \approx 10^{-25}$, $\Omega_{\rm gw} \approx 10^{-10}$, when a month-long stretch of cross-correlated data from the two independent detectors will be available.

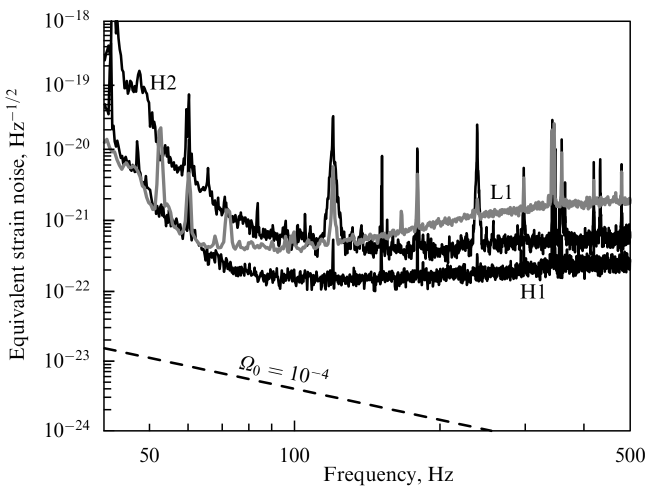

Figure 2. S3 LIGO noise curves and the expected sensitivity $\Omega_0 \sim 10^{-4}$ to stochastic gravitational waves.

The ESA – NASA space-based mission LISA ($\sim$ 2013) will have a better chance of discovering relic gravitational waves. Since the expected spectrum has larger amplitudes at lower frequencies, the detectability conditions potentially improve at lower frequencies. Figure 3 demonstrates the LISA sensitivity with a frequency resolution of $\Delta f = 3 \times 10^{-8}$ Hz, which corresponds to an observation time of 1 year. This exposure time should make it possible to resolve the g.w. lines from thousands of white dwarf binaries in our Galaxy, radiating at frequencies larger than 2×10^{-3} Hz. By removing the contribution of the binaries from the observed records, or by using sophisticated data analysis techniques without actually removing the contaminating signals from the data, one can effectively clean up the window of instrumental sensitivity at frequencies above 2×10^{-3} Hz. This window in the area of maximal sensitivity of LISA is portrayed in the graph, together with the expected level of relic gravitational waves in that window.

3. Indirect detection of relic gravitational waves via CMB anisotropies and polarization

The expected amplitudes of relic gravitational waves reach their highest level in the frequency interval from 10^{-18} to 10^{-16} Hz. This is why one has very good prospects for indirect detection of relic gravitational waves through the measurements of anisotropies in the distribution over the sky of the CMB temperature and polarization. (For an introduction to the theoretical tools of CMB physics, see, for example, Ref. [21].)

The accurately calculated power spectrum $h^2_{\rm rms}(n)$ is illustrated in Fig. 4a [22]. The spectrum is calculated at the moment of decoupling (recombination) of the CMB, with the redshift of decoupling at $z_{\rm dec} = 1100$. The derivation of the spectrum takes into account the quantum-mechanical squeezing of the wave phases, which manifests itself macroscopically in the standing-wave character of the generated gravitational waves. From the viewpoint of the underlying physics, it is this inevitable quantum-mechanical squeezing that is responsible for the oscillations in the power spectrum.

The displayed spectrum was obtained under the assumption that $\beta = -2$ (n = 1), i.e., for a flat primordial spectrum. The surviving part of the primordial flat spectrum is seen in the graph as a horizontal portion of the curve in the region of very small wavenumbers n. The normalization of the spectrum is chosen in such a way that the induced quadrupole anisotropy of the current CMB is at the level corresponding to the actually observed quadrupole [15, 13]. Specifically, the temperature function $l(l+1)C_l$ in Fig. 4b, calculated from the spectrum in Fig. 4a, gives the required value of 960 μK^2 at $l = 2$. The distribution of other induced multipoles is also given in Fig. 4b.

Figures 4a and 4b are placed one under another on purpose. This placement provides a better visual description of the fact noticed and explained previously [23]. Namely, the oscillations in the metric (gravitational field) power spectrum are entirely responsible for the oscillations in the angular power spectrum of the CMB temperature, with almost

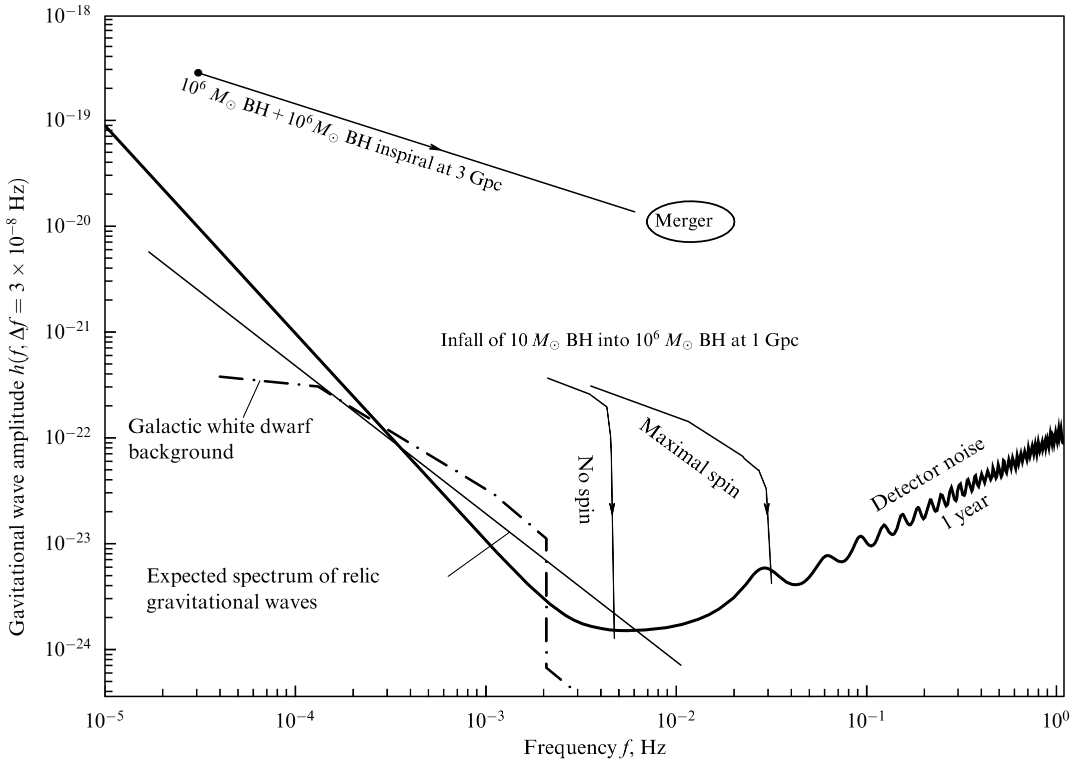

Figure 3. Various LISA sources, including black holes (BH) and relic gravitational waves.

universal correspondence between extrema in the wavenumber space n and extrema in the multipole moment space l. If there is much/little power in the gravitational field perturbations in a given interval of wavelengths, one should expect much/little power in the temperature fluctuations at the corresponding angular scale.

It is the oscillations in the metric power spectrum that are responsible for the oscillations in the l-space, and not the mysterious explanations often repeated in the literature, which claim that the peaks in the function $l(l+1)C_l$ arise because of some waves being caught (at the moment of decoupling) in their maxima or minima, while others are not. To illustrate the role of standing gravitational waves and the associated power spectrum oscillations, as compared to travelling gravitational waves with no power spectrum oscillations, it was explicitly shown [23] that the latter hypothesis does not produce oscillations in the l-space.

Incidentally, it was argued in Ref. [23] that in the case of density perturbations, the main contribution to the peaks in the temperature function $l(l+1)C_l$ can also be provided by oscillations in the metric power spectrum, rather than by the temperature variations accompanying sound waves in the photon – electron – baryon plasma at the last scattering surface. In the event of density perturbations, the metric power spectrum is mostly associated with the gravitational field of the dark matter, which dominates other matter components in terms of the gravitational field.

Oscillations in the metric power spectrum in the early universe are inevitable, and for the same reason as in the g.w. case, namely, because of the standing-wave pattern of the metric perturbations that is related to their quantum-mechanical origin. Therefore, the often-discussed 'acoustic' peaks in the l-space may well turn out to be 'gravitational' peaks. It remains to be seen how this circumstance can change inferences about cosmological parameters.

We shall now turn to the CMB polarization. (For some important papers on CMB polarization, see, for example, Refs [24 – 30].) It follows from the radiation transfer equations that the polarization of CMB is mainly determined by the first time-derivative of the metric perturbations in the interval of time when the polarization is mainly produced. Therefore, it is the power spectrum of the function $h'_{ij}(\eta, \mathbf{x})$ that is of a primary importance. Since the g.w. field itself, including its normalization, has been fully determined, the quantity of our interest is directly calculable. Figure 5a illustrates [22] the power spectrum $(h'_{\rm rms}(n)/n)^2$ calculated at the time of decoupling. The induced E and B components of polarization are shown in Fig. 5b. This graph was constructed under the usual assumptions about the recombination history, which means, in particular, that the polarization was primarily accumulated during a relatively short interval of time around $z_{\rm dec}$.

Similarly to the case of temperature anisotropies, the extrema in the graphs of Fig. 5 correspond well with each other. If there is not much power in the first time-derivative of the metric, you should not expect much power in the polarization at the corresponding angular scales. On the other hand, the region of wavenumbers $n \approx 90$, where there appears the first pronounced peak in Fig. 5a, is fully responsible for the first pronounced peak in Fig. 5b at the corresponding angular scales $l \approx 90$.

In Fig. 6, we combine some of the expected signals from relic gravitational waves. They are encoded in the CMB anisotropies and polarization. This figure also includes a possible polarization bump, discussed previously by other authors, at very small l's. This feature arises because of the extended reionization period in the relatively late universe, around $z_{\rm reion} \approx 17$. In agreement with the explanations given above, the amplitude and position of this bump in the l-space are determined by the amplitude and position of the first

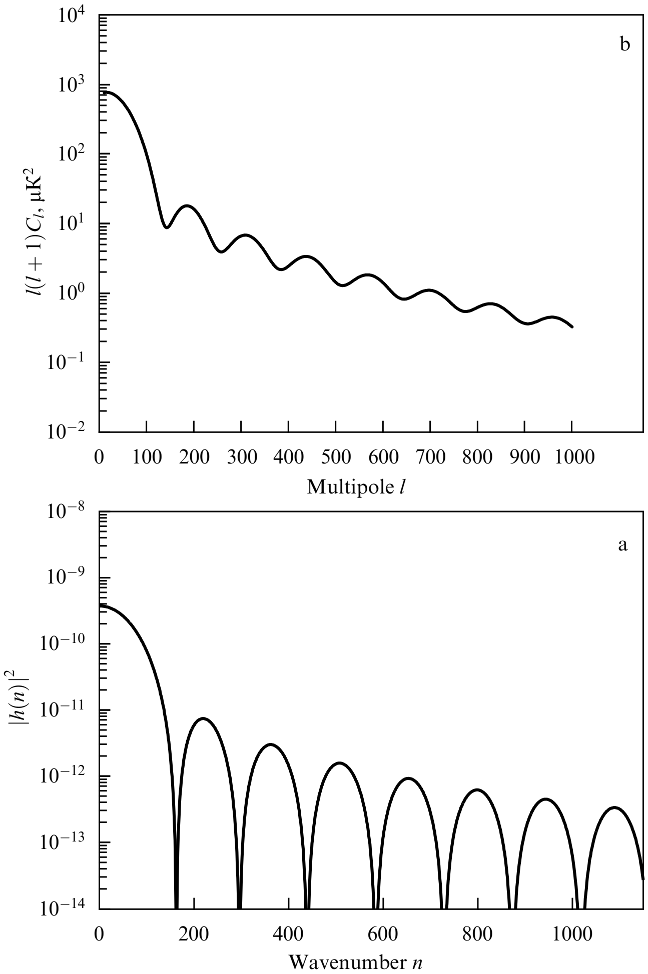

Figure 4. (a) Power spectrum of gravitational waves at decoupling. (b) CMB temperature angular power spectrum.

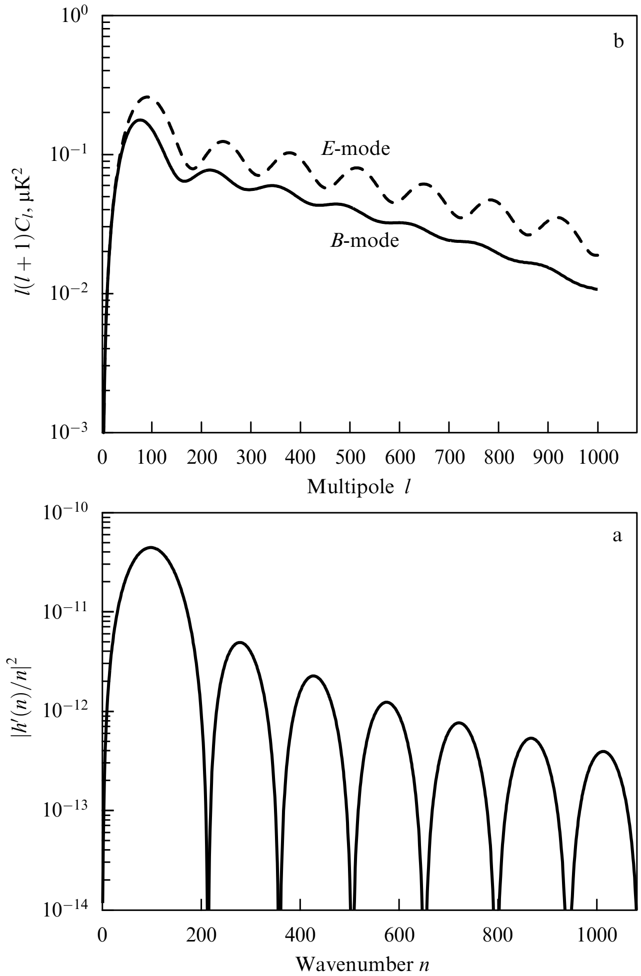

Figure 5. (a) Power spectrum of the first time-derivative of g.w. metric. (b) Angular power spectrum for CMB polarization.

maximum in the power spectrum $(h'/n)^2$ of the function $h'_{ij}(\eta, \mathbf{x})$ calculated at z_{reion}.

The resulting graphs in Fig. 5b and Fig. 6 are qualitatively similar to the graphs plotted by other authors before us. However, we take the responsibility of claiming that the numerical level of, say, the B component of polarization, shown in our graphs, is what the observers should expect to see in the sky. Of course, this statement assumes that the observed large-scale CMB anisotropies are caused by cosmological perturbations of quantum-mechanical origin, and not by something else.

The true level of the B signal can be somewhat higher or somewhat lower than the theoretical level shown in our figures. But the signal cannot be, say, several orders of magnitude lower than the one shown in our graphs. In contrast, the inflationary literature claims that the amount of 'inflationary gravitational waves' vanishes in the limit of the flat primordial spectrum $\beta = -2$ (n = 1). Therefore, the most likely level of the B-mode signal produced by 'inflationary gravitational waves' is close to zero. This would make their detection impossible in any foreseeable future. It is a pity that many of our experimenter colleagues, being guided by the wrong theory, are accepting their defeat even before having started to build instruments aimed at detecting relic gravitational waves via the B component of polarization.

Their logic seems to be the following, "We would like to discover the fundamentally important relic gravitational waves, but we were told by inflationists many times that this is very unlikely to happen, so we agreed to feel satisfied even if we succeed only in putting some limits on, say, polarization

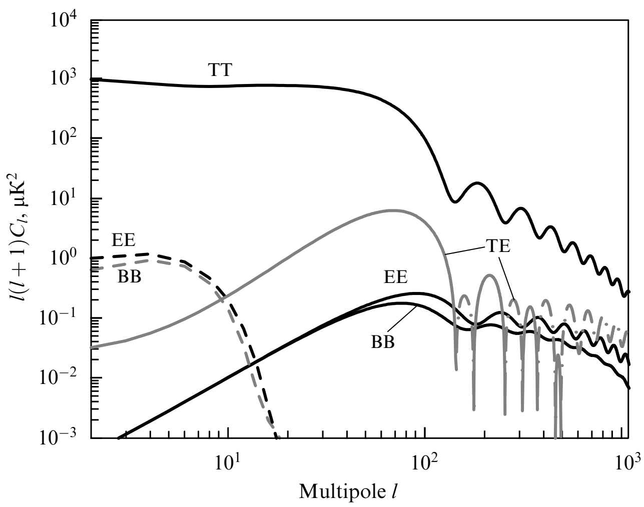

Figure 6. Expected numerical level of anisotropy and polarization induced by relic gravitational waves.

properties of dust in the surrounding cosmos''. The author of this contribution fears that in a complex experiment like the *B*-mode detection, this kind of logic can only lead to overlooking the important signal that the experiment originally targeted.

Concluding this section I would like to say as a witness that Zeldovich suggested using the CMB polarization as a g.w. discriminator, as early as in the very beginning of the 1980s. This was clearly stated in private conversations, but I am not aware of any written records.

4. The false 'standard inflationary result'. How to correctly quantize a cosmological harmonic oscillator

Why bother about relic gravitational waves if inflationists claim that the amount of relic gravitational waves (inflationists and followers call them 'inflationary gravitational waves') should be zero or almost zero? This claim is a direct consequence of the so-called 'standard inflationary result', which is the main contribution of inflationary theorists to the subject of practical, rather than imaginary, cosmology.

In the inflationary scenario, the 'initial' era of the universe expansion is driven by a scalar field φ with the scalar field potential $V(\varphi)$. It is in this era that the initial quantum vacuum conditions for cosmological perturbations are being formulated. The inflationary solutions for the scale factor $a(\eta)$ are close to the de Sitter evolution characterized by $\beta = -2$ in Eqn (2). The effective equation of state for the scalar field always assumes the form $\varepsilon + p \geqslant 0$, so that for the power-law intervals of expansion driven by the scalar field, the parameter β can only be $\beta \leqslant -2$ [see Eqn (4)]. Therefore, one expects the primordial spectrum of the generated metric perturbations to be almost flat, i.e., the primordial spectral index n should be close to $\mathrm{n} = 1$, with $\mathrm{n} \leqslant 1$.

The beginning of the amplifying superadiabatic regime for the given mode of perturbations is often called the 'first Hubble radius crossing', while the end of this regime for the given mode is often called the 'second Hubble radius crossing'. The 'standard inflationary result' is formulated for cosmological perturbations termed density perturbations (scalar, *S*, perturbations) as opposed to the gravitational waves (tensor, *T*, perturbations) considered in Section 1.

The 'standard inflationary result' states that the final (second crossing, $\bar{f}$) amplitudes of quantities describing density perturbations are related to the initial (first crossing, *i*) values of φ and other quantities, according to the estimates

$$\left(\frac{\delta\rho}{\rho}\right)_{\bar{f}} \sim (h_S)_{\bar{f}} \sim (\zeta)_{\bar{f}} \approx (\zeta)_i \sim \left(\frac{H^2}{\dot{\varphi}}\right)_i \sim \left(\frac{V^{3/2}}{V_{,\varphi}}\right)_i \sim \frac{H_i}{(1-\mathrm{n})^{1/2}} \,. \tag{6}$$

The numerator of the last term on the r.h.s. of Eqn (6) is the value of the Hubble parameter taken at the instant of time when the given mode enters the superadiabatic regime. This is the same quantity H_i which defines the g.w. ('tensor') metric amplitude, as described in Section 1. Since we suppose to start with the initial vacuum quantum state for all cosmological perturbations, one would expect that the results for density perturbations should be similar to the results for gravitational waves. One would expect that the amplitude h_S of the generated 'scalar' metric perturbations should be finite and small, and of the same order of magnitude as the amplitude h_T of 'tensor' metric perturbations.

However, according to the 'standard inflationary result', this is very far from being the case. The denominator of the last term in Eqn (6) contains a new factor $(1-\mathrm{n})^{1/2}$. This factor goes to zero in the limit of the most interesting and observationally preferred possibility of the flat (Harrison–Zeldovich–Peebles) primordial spectrum $\mathrm{n} = 1$. Correspondingly, the amplitudes of the generated density perturbations go to infinity, according to the predictions of inflationary theorists, in the limit of the flat spectrum. (By now, the 'standard inflationary result' (6) has been cited, used, praised, reformulated, popularized, etc. in hundreds of inflationary publications, so it has become 'accepted by way of repetition'.)

As will be demonstrated below, the divergence in Eqn (6) is not a violation, suddenly descending upon us from the 'blue sky', of the adopted approximation of small linear perturbations. This is a manifestation of the incorrect theory. Even if the spectral index n is not very close to 1, and you combine n with a reasonable H_i in order to obtain, for example, a small number 10^{-5} for the r.h.s. of Eqn (6), this will not make your theory correct. This will be just an acceptable number accidentally following from the wrong formula. You will have to pay a heavy price in some other places.

An attempt to derive physically meaningful consequences from this formula can only lead to mistakes. The current literature is full of incorrect far-reaching physical conclusions derived from this wrong theory. This is a kind of situation which L D Landau used to describe sarcastically in the following words, ''*If you assume that the derivative of the function* $\sin x$ *is* $\ln x$*, rather than* $\cos x$*, you can make many wonderful discoveries....*''

In inflationary literature, the 'zero in the denominator' factor $\sqrt{1-\mathrm{n}}$ appears in many different guises. It is often written in equivalent forms, such as $(\dot{\varphi}/H)_i$, $(V_{,\varphi}/V)_i$, $(H_{,\varphi}/H)_i$, $(1+w_i)^{1/2}$, etc. Inflationists are routinely hiding their absurd prediction of infinitely large amplitudes of density perturbations that should take place in the limit of the flat spectrum, $\mathrm{n} \to 1$. They divide the g.w. amplitude h_T by the predicted divergent amplitude h_S. This division produces the so-called 'tensor-to-scalar ratio', or 'consistency relation': $h_T/h_S \approx (1-\mathrm{n})^{1/2}$. The quantity H_i, common for the *T* and *S* perturbations, cancels out in the composed ratio, and the 'zero in the denominator' factor is transferred to the numerator of the final expression. It is then declared that the metric amplitude h_S of density perturbations is determined by the observed CMB anisotropies, and, therefore, the inflationary 'consistency relation' demands that the g.w. amplitude h_T must vanish in the limit $\mathrm{n} \to 1$.

In other words, instead of being horrified by the fact that their theory predicts arbitrarily large amplitudes of density perturbations (and, hence, the theory is in complete disagreement with observations, because the data analysis shows no catastrophic increase in the amplitude when the tested spectral index approaches $\mathrm{n} = 1$), supporters of the inflationary approach to science systematically claim that their theory is in 'spectacular agreement' with observations, and it is gravitational waves that should vanish.

If this were true, there would not be much sense in attempting to detect primordial gravitational waves, as the observations persistently point toward $\mathrm{n} \approx 1$, including $\mathrm{n} = 1$. It is quite common to hear these days enthusiastic promises of inflationary believers to detect 'inflationary gravitational waves' in the 'not-so-distant future' via the measurement of B-mode polarization of CMB. But from other papers by the same authors it follows that there is no reason even to try. If you trust and cite inflationary formulas, the expected amount of 'inflationary gravitational waves' should be very small or zero. You can only hope to be extremely lucky if you suggest detecting them, even in the quite distant future, for example, with the proposed mission called Big Bang Observer. And nobody should be surprised if you find nothing, because n = 1 is at the heart of all claims, both theoretical and observational. Moreover, most loyal inflationists would say that this was exactly what they had always been predicting.

To demonstrate the incorrectness of inflationary conclusions, we shall now concentrate on the 'zero in the denominator' factor. We will have to recall the quantization procedure for gravitational waves and density perturbations. It is necessary to remind the reader that some inflationists and their supporters insisted for many years on the claim that the dramatic difference in the final numerical values of h_T and h_S arises not because of the initial conditions, but because of the subsequent evolution of perturbations.

Specifically, they claimed that classical long-wavelength 'scalar' metric perturbations are capable of experiencing, in contrast to gravitational waves, a 'big amplification during reheating' (for a critical discussion, see Ref. [31]). But it now looks as if the fallacy of this proposition has become clear even to its most ardent proponents. Therefore, we shall now focus on the issue of quantum mechanics and initial conditions.

The perturbed gravitational field for all three sorts of cosmological perturbations (scalar, vector, and tensor) is described by Eqn (1). For simplicity, we will consider spatially flat cosmologies, whose radius of spatial curvature is infinite. However, if the radius of spatial curvature is finite but, say, only a factor of 10 longer than $l_{\rm H}$, very little will change in our analysis.

The metric perturbations $h_{ij}(\eta, \mathbf{x})$ can be expanded over spatial Fourier harmonics labelled by the wave vector $\mathbf{n}$:

$$h_{ij}(\eta, \mathbf{x}) = \frac{\mathcal{C}}{(2\pi)^{3/2}} \int_{-\infty}^{\infty} d^3\mathbf{n} \sum_{s=1,2} p_{ij}^s(\mathbf{n}) \times \frac{1}{\sqrt{2n}} \left[h_n^s(\eta) \exp(\mathrm{i}\mathbf{n}\cdot\mathbf{x})\, c_{\mathbf{n}}^s + h_n^{s*}(\eta) \exp(-\mathrm{i}\mathbf{n}\cdot\mathbf{x})\, c_{\mathbf{n}}^{s\dagger} \right]. \quad (7)$$

The three sorts of cosmological perturbations are different in that they have three different sorts of polarization tensors $p_{ij}^s(\mathbf{n})$, and each of them is characterized by two different polarization states $s = 1, 2$. The 'scalar' and 'vector' metric perturbations are always accompanied by perturbations in density and/or velocity of matter. The normalization constant $\mathcal{C}$ is determined by quantum mechanics, and the derivation of its numerical value is one of the aims of our discussion.

Let us recall the procedure of quantization of gravitational waves. Let us consider an individual g.w. mode $\mathbf{n}$. The time-dependent mode functions $h_n^s(\eta)$ can be written down as

$$h_n^s(\eta) = \frac{1}{a(\eta)}\, \mu_n^s(\eta)\,. \quad (8)$$

For each s and $\mathbf{n}$, the g.w. mode functions $\mu(\eta)$ satisfy the familiar equation (3).

The action for each mode has the form

$$S = \int L\, d\eta\,, \quad (9)$$

where the g.w. Lagrangian L is given by the expression [32]

$$L_{\rm gw} = \frac{1}{2c\varkappa}\, n^{-3} a^2 \left[\left(\frac{\mu}{a}\right)'^2 - n^2 \left(\frac{\mu}{a}\right)^2 \right], \quad (10)$$

$$\varkappa = \frac{8\pi G}{c^4}\,.$$

The Euler – Lagrange equation

$$\frac{\partial L}{\partial h} - \frac{d}{d\eta} \frac{\partial L}{\partial h'} = 0$$

for the dimensionless g.w. variable $h = \mu/a$ brings us to the equation of motion

$$h'' + 2\frac{a'}{a} h' + n^2 h = 0\,, \quad (11)$$

which is equivalent to Eqn (3).

In order to move from 3-dimensional Fourier components to the usual description in terms of an individual oscillator with frequency n, we will be working with the quantity $\bar{h}$ introduced according to the definition

$$\bar{h} = \frac{a_0}{n(c\varkappa)^{1/2}}\, h = \left(\frac{\hbar}{8\pi}\right)^{1/2} \frac{a_0}{l_{\rm Pl}\, n}\, h = \left(\frac{\hbar}{32\pi^3}\right)^{1/2} \frac{\lambda_0}{l_{\rm Pl}}\, h\,. \quad (12)$$

where a_0 is a constant. This constant a_0 reflects the value of the scale factor $a(\eta)$ at some instant of time $\eta = \eta_0$ where the initial conditions are being formulated, and $\lambda_0 = 2\pi a_0/n$.

In terms of $\bar{h}$, the Lagrangian (10) takes the form

$$L_{\rm gw} = \frac{1}{2n} \left(\frac{a}{a_0}\right)^2 \left[(\bar{h}')^2 - n^2 \bar{h}^2 \right]. \quad (13)$$

The quantity $\bar{h} = q$ plays the role of the 'position' variable, while the canonically conjugate 'momentum' variable p is given by

$$p = \frac{\partial L}{\partial \bar{h}'} = \frac{1}{n} \left(\frac{a}{a_0}\right)^2 \bar{h}'\,. \quad (14)$$

In the distant past, at times near η_0, and before η_i, when a given mode entered the superadiabatic regime, the g.w. amplitude behaved according to the law

$$h(\eta) \propto \frac{1}{a(\eta)} \exp\big(-\mathrm{i}n(\eta - \eta_0)\big)\,.$$

The time-derivative of $a(\eta)$ can be neglected, i.e., $a'/a \ll n$. Then, we promote q and p to the status of quantum-mechanical operators, denote them by bold-face letters, and write down their asymptotic expressions

$$\mathbf{q} = \left(\frac{\hbar}{2}\right)^{1/2} \frac{a_0}{a} \times \left[\mathbf{c} \exp\big(-\mathrm{i}n(\eta - \eta_0)\big) + \mathbf{c}^\dagger \exp\big(\mathrm{i}n(\eta - \eta_0)\big) \right], \quad (15)$$

$$\mathbf{p} = \mathrm{i} \left(\frac{\hbar}{2}\right)^{1/2} \frac{a}{a_0} \times \left[-\mathbf{c} \exp\big(-\mathrm{i}n(\eta - \eta_0)\big) + \mathbf{c}^\dagger \exp\big(\mathrm{i}n(\eta - \eta_0)\big) \right]. \quad (16)$$

The commutation relationships for the $\mathbf{q}$, $\mathbf{p}$ operators, and for the annihilation and creation operators $\mathbf{c}$, $\mathbf{c}^\dagger$, are expressed as follows

$$[\,\mathbf{q}, \mathbf{p}\,] = \mathrm{i}\hbar\,, \quad [\,\mathbf{c}, \mathbf{c}^\dagger\,] = 1\,.$$

The initial vacuum state $|0\rangle$ is defined by the condition

$$\mathbf{c}|0\rangle = 0\,.$$

This is indeed a genuine vacuum state of a simple harmonic oscillator, which gives at $\eta = \eta_0$ the following relationships

$$\langle 0|\,\mathbf{q}^2|\,0\rangle = \langle 0\,|\,\mathbf{p}^2|\,0\rangle = \frac{\hbar}{2}\,, \quad \Delta\mathbf{q}\Delta\mathbf{p} = \frac{\hbar}{2}\,.$$

The root-mean-square value of $\mathbf{q}$ in the vacuum state is $q_{\mathrm{rms}} = \sqrt{\hbar/2}$. Combining this number with the definition (12) we arrive at

$$h_{\mathrm{rms}} = \big(\langle 0|\,\mathbf{h}^2|0\rangle\big)^{1/2} = \sqrt{2}(2\pi)^{3/2}\,\frac{l_{\mathrm{Pl}}}{\lambda_0}\,. \qquad (17)$$

Extrapolating the initial time η_0 up to the boundary between the adiabatic and superadiabatic regimes at $\eta = \eta_i$, we derive the estimate $h_{\mathrm{rms}} \sim l_{\mathrm{Pl}}/\lambda_i$. It is this evaluation that was used in Ref. [2] and in Section 1. More accurate calculations along these lines produce $\mathcal{C} = (16\pi)^{1/2}\,l_{\mathrm{Pl}}$ in expression (7) for gravitational waves.

A consistent formal derivation of the total Hamiltonian, including the terms describing the interaction of the oscillator with the external field, is presented in Ref. [33] by equations (19)–(24) there. Technically, the derivation is based on the canonical pair $q = \mu$, $p = \partial L/\partial\mu'$. The Hamiltonian associated with the Lagrangian (13) has the form

$$\mathbf{H}(\eta) = n\mathbf{c}^\dagger\,\mathbf{c} + \sigma\mathbf{c}^{\dagger 2} + \sigma^*\mathbf{c}^2\,, \qquad (18)$$

where the coupling to the external field is given by the function $\sigma(\eta) = (\mathrm{i}/2)(a'/a)$. In the same Ref. [33], one can also find the Heisenberg equations of motion for the Heisenberg operators $\mathbf{c}(\eta)$, $\mathbf{c}^\dagger(\eta)$, and their connection to classical equation (3). The asymptotic expressions for the Heisenberg operators

$$\mathbf{c}(\eta) = \mathbf{c}\,\exp\big(-\mathrm{i}n(\eta - \eta_0)\big)\,, \quad \mathbf{c}^\dagger(\eta) = \mathbf{c}^\dagger\exp\big(\mathrm{i}n(\eta - \eta_0)\big)$$

enter into formulas (15), (16). Clearly, the vacuum state $|0\rangle$, defined as $\mathbf{c}(\eta)|0\rangle = 0$, minimizes the oscillator's energy (18).

A rigorous quantum-mechanical Schrödinger evolution of the initial vacuum state of cosmological perturbations transforms this state into a strongly squeezed (multiparticle) vacuum state [32], but we focus here only on the initial quantum state which defines the quantum-mechanical normalization of our classical mode functions.

We shall now switch to density perturbations.

For each mode $\mathbf{n}$ of density perturbations (S-perturbations), the mode's metric components h_{ij} entering Eqn (1) can be written out as

$$h_{ij} = h(\eta)\,Q\delta_{ij} + h_l(\eta)\,n^{-2}Q_{,ij}\,,$$

where the spatial eigenfunctions are $Q = \exp(\pm\,\mathrm{i}\,\mathbf{n}\mathbf{x})$. Therefore, the metric components associated with density perturbations are characterized by two polarization amplitudes $h(\eta)$ and $h_l(\eta)$. If the initial era is driven by an arbitrary scalar field φ, there appears a third unknown function — the amplitude $\varphi_1(\eta)$ of the scalar field perturbation:

$$\varphi = \varphi_0(\eta) + \varphi_1(\eta)\,Q\,.$$

One often considers the so-called minimally coupled to gravity scalar field φ, with the energy–momentum tensor

$$T_{\mu\nu} = \varphi_{,\mu}\,\varphi_{,\nu} - g_{\mu\nu}\left[\frac{1}{2}\,g^{\alpha\beta}\varphi_{,\alpha}\,\varphi_{,\beta} + V(\varphi)\right].$$

The coupling of scalar fields to gravity is still a matter of ambiguity, and the very possibility of quantum-mechanical generation of density perturbations relies on an extra hypothesis, but we suppose that we were lucky and the coupling was such as we need. The three unknown functions $h(\eta)$, $h_l(\eta)$, and $\varphi_1(\eta)$ should be found from the perturbed Einstein equations augmented by the appropriate initial conditions dictated by quantum mechanics.

It is important to note that inflationary theorists are still struggling with the basic equations for density perturbations. In inflationary papers, you will often see equations containing complicated combinations of metric perturbations mixed up with the unperturbed and/or perturbed functions of the scalar field φ and $V(\varphi)$.

Inflationists are still engaged in endless discussions about the shape of the scalar field potential $V(\varphi)$, and what it could mean for countless inflationary models. However, this state of affairs is simply a reflection of the fact that the equations have not been properly transformed and simplified. Since the underlying physics is the interaction of a cosmological harmonic oscillator with the gravitational pump field, mathematically the equations should reveal this themselves. And indeed they do.

It was shown in the paper [34] that, for any potential $V(\varphi)$, there exists only one second-order differential equation to be solved:

$$\mu'' + \mu\left[n^2 - \frac{(a\sqrt{\gamma})''}{a\sqrt{\gamma}}\right] = 0\,, \qquad (19)$$

where the function $\mu(\eta)$ represents the single dynamical degree of freedom, describing S-perturbations. The effective potential barrier $(a\sqrt{\gamma})''/a\sqrt{\gamma}$ depends only on $a(\eta)$ and its derivatives, in full analogy with the g.w. oscillator [see Eqn (3)]. The time-dependent function γ ($\gamma(\eta)$ or $\gamma(t)$) is defined in the following way

$$\gamma = 1 + \left(\frac{a}{a'}\right)' = -\frac{c}{a}\,\frac{H'}{H^2} = -\frac{\dot{H}}{H^2}\,.$$

As soon as the appropriate solution for $\mu(\eta)$ is found, all three functions describing density perturbations are easily calculable:

$$h(\eta) = \frac{1}{c}\,H(\eta)\left[\int_{\eta_0}^{\eta}\mu\sqrt{\gamma}\,\mathrm{d}\eta + C_i\right],$$

$$h_l'(\eta) = \frac{a}{a'}\left[h'' - \frac{H''}{H'}\,h' + n^2 h\right],$$

$$\varphi_1(\eta) = \frac{\sqrt{\gamma}}{(2\varkappa)^{1/2}}\left[\frac{\mu}{a\sqrt{\gamma}} - h\right].$$

The constant C_i reflects the remaining coordinate freedom within the class of synchronous coordinate systems. (Another constant comes out from the integration of the above-given equation for $h_l{}'$.) The function μ does not depend on this remaining coordinate freedom, and the constant C_i cancels out in the expression defining $\mu(\eta)$ in terms of $h(\eta)$:

$$\frac{\mu}{a\sqrt{\gamma}} = h - \frac{H}{H'}\, h' .$$

The function $\mu/a\sqrt{\gamma}$ is that part of the scalar metric amplitude $h(\eta)$ which does not depend on the remaining coordinate freedom ('gauge-invariant' metric perturbation).

In the short-wavelength regime, the function μ describing density perturbations behaves as $\mu \propto \exp(-\mathrm{i}\, n\eta)$. This is the same behavior as in the case of the function μ describing gravitational waves. This similarity between the respective functions μ (μ_T and μ_S) is valid only in the sense of their asymptotic η-time dependence, but not in the sense of their overall numerical normalization (see below).

In the long-wavelength regime, the dominant solution to Eqn (19) is $\mu \propto a\sqrt{\gamma}$. The quantity which remains constant in this regime is $\mu/a\sqrt{\gamma}$. It is this physically relevant variable that takes over from the analogous variable $h = \mu/a$ in the g.w. problem. We introduce the notation

$$\frac{\mu}{a\sqrt{\gamma}} = \zeta\,, \tag{20}$$

where μ satisfies Eqn (19).

To make contact with earlier works, it should be mentioned that the previously introduced quantity

$$\zeta_{\mathrm{BST}} = \frac{2}{3}\,\frac{(a/a')\,\Phi' + \Phi}{1+w} + \Phi\,,$$

where Φ is Bardeen's potential, and BST stands for Bardeen, Steinhardt, Turner [35], can be reduced to our variable ζ (20) up to the numerical coefficient $-(1/2)$. Our quantity μ for density perturbations can also be related to the variable u_{CLMS}, where CLMS stands for Chibisov, Lukash, Mukhanov, Sasaki [36 – 38].

In preparation for quantization, we should first identify the inflationary 'zero in the denominator' factor. The unperturbed Einstein equations for the coupled system of gravitational and scalar fields require that [34]

$$\varkappa(\varphi_0{}')^2 = 2\left(\frac{a'}{a}\right)^2 \gamma\,.$$

Therefore, the 'zero in the denominator' factor

$$\left(\frac{\dot{\varphi}_0}{H}\right)_i = \left(\frac{2}{\varkappa}\right)^{1/2} (\sqrt{\gamma})_i$$

is expressed in the form of very small values of the dimensionless function $\sqrt{\gamma}$.

Within the approximation of power-law scale factors (2), the function γ reduces to a set of constants

$$\gamma = \frac{2+\beta}{1+\beta}\,, \quad 1 + w = \frac{2}{3}\,\gamma\,.$$

The constant γ degenerates to zero in the limit of the evolution law with $\beta = -2$ — that is, in the limit of the gravitational pump field which is responsible for the generation of primordial cosmological perturbations with flat spectrum $\mathrm{n} = 1$. So, we are especially interested in the very small values of $\sqrt{\gamma}$.

It was shown in Ref. [34] that the dynamical problem for the scalar-field-driven S-perturbations can be obtained from the dynamical problem for gravitational waves by simple substitutions: $a(\eta) \to a(\eta)\sqrt{\gamma(\eta)}$, and $\mu_T(\eta) \to \mu_S(\eta)$. (This is not a conjecture, but this is a rule whose validity was established after a thorough analysis of these two problems separately.) Each of these substitutions is valid up to an arbitrary constant factor. Using these substitutions, one obtains the S-equation (19) from the T-equation (3), and the physically relevant variable $\zeta = \mu_S/a\sqrt{\gamma}$ for S-perturbations from the g.w. variable $h = \mu_T/a$.

Moving from the 3-dimensional Fourier components of the field ζ to an individual oscillator with frequency n, we introduce the quantity $\bar{\zeta}$ according to the same rule (12) that was used earlier when we introduced $\bar{h}$. Namely, we introduce

$$\bar{\zeta} = \frac{a_0}{n(c\varkappa)^{1/2}}\,\zeta = \left(\frac{\hbar}{8\pi}\right)^{1/2} \frac{a_0}{l_{\mathrm{Pl}}\, n}\,\zeta = \left(\frac{\hbar}{32\pi^3}\right)^{1/2} \frac{\lambda_0}{l_{\mathrm{Pl}}}\,\zeta\,. \tag{21}$$

The application of the substitutions $a \to \tilde{a} = a\sqrt{\gamma}$, and $\bar{h} \to \bar{\zeta}$ to the g.w. Lagrangian (13) gives rise to the Lagrangian L_{dp} for the single dynamical degree of freedom, describing S-perturbations:

$$L_{\mathrm{dp}} = \frac{1}{2n}\left(\frac{a\sqrt{\gamma}}{a_0\sqrt{\gamma_0}}\right)^2 \left[(\bar{\zeta}')^2 - n^2\bar{\zeta}^2\right]. \tag{22}$$

Obviously, the Euler – Lagrange equation

$$\zeta'' + 2\,\frac{(a\sqrt{\gamma})'}{a\sqrt{\gamma}}\,\zeta' + n^2\zeta = 0\,, \tag{23}$$

derivable from the Lagrangian (22) in terms of the independent variable ζ, is equivalent to Eqn (19) which is the Euler – Lagrange equation derivable from the Lagrangian (22) in terms of the independent variable μ_S. The Lagrangian (22) should be used for quantization. The Lagrangian itself, as well as the action and the Hamiltonian, does not degenerate in the limit $\gamma \to 0$, i.e., in the limit of the most interesting background gravitational field in the form of the de Sitter metric, $\gamma = 0$.

We shall start with the analysis of the paper [39] which, together with Ref. [40], is sometimes referred to as the most recent work that contains a rigorous mathematical derivation of the 'standard inflationary result'. The author of these papers uses slightly different notations, such as $a^2 = \exp(2\rho)$ and $\varphi = \phi$. In his notation, the quantity $\dot{\varphi}_0/H$ is $\dot{\phi}_*/\dot{\rho}_*$, so that the 'zero in the denominator' factor appears as $\dot{\phi}_*/\dot{\rho}_*$, where the asterisk means 'the time of horizon crossing'.

As a 'useful example to keep in mind' for quantization of density perturbations, the author suggests the artificial model of a test massless scalar field f in the de Sitter space. But the Lagrangian, classical solutions, and quantization procedure for the field f are identical to the g.w. case that we recalled above, so that his variable f is our h for gravitational waves. His Lagrangian (2.12) for density perturbations coincides in structure with our Lagrangian (22), and we discuss one and the same observable quantity ζ.

It is worthwhile to quote explicitly the attempted rigorous proof [39]: "Since the action (2.12) also contains a factor $\dot{\phi}/\dot{\rho}$

we also have to set its value equal to the value at horizon crossing, this factor only appears in normalizing the classical solution. In other words, near horizon crossing we set

$$f = \frac{\dot{\phi}}{\dot{\rho}}\,\zeta\,,$$

where f is a canonically normalized field in de Sitter space. This produces the well known result...." And the author immediately writes down the square of the 'standard inflationary result', with the square of the factor $\dot{\phi}_*/\dot{\rho}_*$ in the denominator of the final expression.

Let us try to traverse in practice the path to the 'well known result'. (To be fair to the author, the derivation of the 'standard inflationary result' does not appear to be the main purpose of his paper [39], so my criticism does not imply anything about other statements in that paper.) The factor $\dot{\phi}/\dot{\rho}$ in expression (2.12) of the cited paper is our factor $\sqrt{\gamma}$ in Eqn (22). It is recommended [39] to combine the results for the g.w. variable h with the prescription $\zeta = (1/\sqrt{\gamma})\,h$. So, instead of Eqn (15), we would have to write down

$$\mathbf{q} = \bar{\zeta} = \left(\frac{\hbar}{2}\right)^{1/2} \frac{\tilde{a}_0}{\tilde{a}}\,\frac{1}{\sqrt{\gamma}} \times \left[\mathbf{b}\,\exp\big(-\,\mathrm{i}n\,(\eta-\eta_0)\big) + \mathbf{b}^{\dagger}\exp\big(\mathrm{i}n\,(\eta-\eta_0)\big)\right]. \qquad (24)$$

The canonically conjugate momentum seems to be

$$p = \frac{\partial L}{\partial \bar{\zeta}'} = \frac{1}{n}\left(\frac{\tilde{a}}{\tilde{a}_0}\right)^2 \gamma\,\bar{\zeta}'\,. \qquad (25)$$

The time derivative of γ should be neglected, as γ is either a constant or a slowly varying function at times near η_0. Therefore, we would have to write, instead of Eqn (16), the following relationship

$$\mathbf{p} = \mathrm{i}\left(\frac{\hbar}{2}\right)^{1/2} \frac{\tilde{a}}{\tilde{a}_0}\,\sqrt{\gamma} \times \left[-\,\mathbf{b}\,\exp\big(-\,\mathrm{i}n\,(\eta-\eta_0)\big) + \mathbf{b}^{\dagger}\exp\big(\mathrm{i}n(\eta-\eta_0)\big)\right]. \qquad (26)$$

The commutation relations are given by

$$[\mathbf{q},\mathbf{p}] = \mathrm{i}\hbar\,, \quad [\mathbf{b},\mathbf{b}^{\dagger}] = 1\,.$$

One is encouraged and tempted to think that the quantum state $|0_s\rangle$, annihilated by $\mathbf{b}$, namely

$$\mathbf{b}|\,0_{\mathrm{s}}\rangle = 0\,,$$

is the vacuum state of the field ζ, i.e., the ground state of the Hamiltonian associated with the Lagrangian (22). The calculation of the mean-square value of $\bar{\zeta}$ at $\eta = \eta_0$ produces the result

$$\langle 0_{\mathrm{s}}|\,\mathbf{q}^2|\,0_{\mathrm{s}}\rangle = \frac{\hbar}{2}\,\frac{1}{\gamma_0}\,,$$

in which the 'zero in the denominator' factor $\sqrt{\gamma}$ is manifestly present and squared, as the 'well-known result' prescribes.

In the limit of very small $\sqrt{\gamma}$, one obtains the divergence of initial amplitudes, which is in the heart of all inflationary predictions. (In the published version [40] of the e-paper [39], the road to the 'well-known result' recommends, possibly due to a misprint, the diametrically opposite prescription

$$\zeta = \frac{\dot{\phi}}{\dot{\rho}}\,f,$$

which would send the factor γ to the numerator of the above expression. It looks as though the 'rigorous' inflationary predictions fluctuate between zero and infinity.)

In inflationary literature, the power spectrum $P_{\mathcal{R}}(k)$ of curvature perturbations is usually written in the form

$$P_{\mathcal{R}}(k) = \frac{k^3}{2\pi^2}\,\frac{|u_k|^2}{z^2}\,,$$

where

$$z = a\,\frac{\dot{\varphi}}{H} = a\sqrt{\gamma}\,\left(\frac{2}{\varkappa}\right)^{1/2},$$

and u_k are the mode functions ($u_k = \mu_n$ in our notations) satisfying Eqn (19) with the initial conditions

$$u_k = \frac{1}{\sqrt{2k}}\exp(-\mathrm{i}k\eta) \quad \text{for} \quad \eta \to -\infty\,. \qquad (27)$$

As one can see from the expression for $P_{\mathcal{R}}(k)$, in inflationary theory, which is based on the initial conditions (27), the divergence of $P_{\mathcal{R}}(k)$ in the limit of very small $\sqrt{\gamma}$ is present from the very beginning of the evolution of the perturbations. To put it differently, the divergence takes place from the very early high-frequency regime, where by the physical statement of the problem we were supposed to choose a minimal amplitude of the 'gauge-invariant' metric perturbations ζ (or, in other words, a minimal amplitude of the curvature perturbations ζ).

The crucial point of our discussion is that the temptation to interpret $|0_{\mathrm{s}}\rangle$ as the vacuum state for the field ζ is, in fact, a grave error. The calculation of the mean-square value of the canonically conjugate momentum $\mathbf{p}$ gives

$$\langle 0_{\mathrm{s}}|\,\mathbf{p}^2|\,0_{\mathrm{s}}\rangle = \frac{\hbar}{2}\,\gamma_0\,,$$

so that the factor $\sqrt{\gamma}$ drops out of the uncertainty relation

$$\Delta\mathbf{q}\,\Delta\mathbf{p} = \frac{\hbar}{2}\,.$$

The derived numbers clearly indicate that the quantum state $|0_{\mathrm{s}}\rangle$ is not a genuine (ordinary) vacuum state $|0\rangle$ for the dynamical variable ζ, but, on the contrary, is a multiparticle (squeezed vacuum) state. That is why we have used the subscript s.

To demonstrate how the states $|0\rangle$ and $|0_{\mathrm{s}}\rangle$ are related, we shall first transform the operators. Let us introduce the annihilation and creation operators $\mathbf{c}, \mathbf{c}^{\dagger}$ according to the Bogolyubov transformation

$$\mathbf{b} = u\mathbf{c} + v\mathbf{c}^{\dagger}\,, \quad \mathbf{b}^{\dagger} = u^*\mathbf{c}^{\dagger} + v^*\mathbf{c}\,, \qquad (28)$$

where

$$u = \cosh r\,, \quad v = \exp\,(\mathrm{i}2\phi)\,\sinh r\,. \qquad (29)$$

The parameters r and ϕ are called squeeze parameters.

Let us assign the following values to r and ϕ:

$$\exp(2r) = \gamma\,, \quad \phi = n(\eta - \eta_0) \tag{30a}$$

or

$$\exp(-2r) = \gamma\,, \quad \phi = n(\eta - \eta_0) + \frac{\pi}{2}\,. \tag{30b}$$

We shall now apply the substitution (28), together with Eqns (29) and (30), to Eqns (24) and (26). The factor $1/\sqrt{\gamma}$ cancels out in Eqn (24), and the factor $\sqrt{\gamma}$ cancels out in Eqn (26). In terms of $\mathbf{c}, \mathbf{c}^\dagger$, the operators $\mathbf{q}, \mathbf{p}$ will take the final forms

$$\mathbf{q} = \left(\frac{\hbar}{2}\right)^{1/2} \frac{\tilde{a}_0}{\tilde{a}} \left[\mathbf{c} \exp\left(-\mathrm{i}n(\eta - \eta_0)\right) + \mathbf{c}^\dagger \exp\left(\mathrm{i}n(\eta - \eta_0)\right)\right], \tag{31}$$

$$\mathbf{p} = \mathrm{i}\left(\frac{\hbar}{2}\right)^{1/2} \frac{\tilde{a}}{\tilde{a}_0} \left[-\mathbf{c} \exp\left(-\mathrm{i}n(\eta - \eta_0)\right) + \mathbf{c}^\dagger \exp\left(\mathrm{i}n(\eta - \eta_0)\right)\right]. \tag{32}$$

The genuine vacuum state for the variable ζ (i.e., the ground state of the corresponding Hamiltonian) is defined by the condition

$$\mathbf{c}|0\rangle = 0\,.$$

Calculating the mean-square values of $\mathbf{q}$ and its canonically conjugate momentum $\mathbf{p}$, we find

$$\langle 0|\,\mathbf{q}^2|0\rangle = \langle 0|\mathbf{p}^2|0\rangle = \frac{\hbar}{2}\,, \quad \Delta\mathbf{q}\,\Delta\mathbf{p} = \frac{\hbar}{2}\,,$$

as it should be.

Taking into account the definition (21), we finally derive the initial r.m.s. value of the variable $\zeta = \mu/a\sqrt{\gamma}$:

$$\zeta_{\mathrm{rms}} = \left(\langle 0|\zeta^2|0\rangle\right)^{1/2} = \sqrt{2}(2\pi)^{3/2}\,\frac{l_{\mathrm{Pl}}}{\lambda_0}\,. \tag{33}$$

Extrapolating the initial time η_0 up to the boundary between the adiabatic and superadiabatic regimes at $\eta = \eta_i$, we arrive at the estimate

$$\left(\frac{\mu}{a\sqrt{\gamma}}\right)_{\mathrm{rms}} \sim \frac{l_{\mathrm{Pl}}}{\lambda_i}\,.$$

This evaluation, plus the constancy of the quantity $\mu/a\sqrt{\gamma}$ throughout the long-wavelength regime, is the foundation of the result according to which the final (at the end of the long-wavelength regime) amplitudes of gravitational waves and density perturbations should be roughly equal to each other [34].

There is no dimensional parameter which could be regulated in such a way as to make one of the amplitudes several orders of magnitude larger than another. In terms of the 'scalar' and 'tensor' metric amplitudes, this means that $h_T/h_S \approx 1$ for all γ's. More accurate calculation along the same lines produces $\mathcal{C} = \sqrt{24\pi}\,l_{\mathrm{Pl}}$ in expression (7) for density perturbations.

Certainly, the correct quantization procedure (31), (32), as opposed to the incorrect (inflationary) procedure (24), (26), could be formulated from the outset of quantization. Mathematically, the Lagrangians (13) and (22) are alike, if in expression (13) one means $\tilde{a}$ by a, and replaces h with ζ.

The derivation of the Hamiltonian for S-perturbations repeats its derivation for gravitational waves. Using the canonical pair $q = \mu$, $p = \partial L/\partial\mu'$ for μ_S, we arrive at the Hamiltonian (compare with Eqn (98) in Ref. [34])

$$\mathbf{H}(\eta) = n\mathbf{c}^\dagger\mathbf{c} + \sigma\mathbf{c}^{\dagger\,2} + \sigma^*\mathbf{c}^2\,, \tag{34}$$

where the coupling to the external field is now given by the function $\sigma(\eta) = (\mathrm{i}/2)(\tilde{a}'/\tilde{a})$.

The Heisenberg equations of motion for the Heisenberg operators $\mathbf{c}(\eta)$, $\mathbf{c}^\dagger(\eta)$ lead to classical equations (19). The asymptotic expressions for the Heisenberg operators

$$\mathbf{c}(\eta) = \mathbf{c}\exp\left(-\mathrm{i}n(\eta - \eta_0)\right), \quad \mathbf{c}^\dagger(\eta) = \mathbf{c}^\dagger \exp\left(\mathrm{i}n(\eta - \eta_0)\right)$$

are participating in Eqns (31), (32). Clearly, the vacuum state $|0\rangle$, defined as $\mathbf{c}(\eta)|0\rangle = 0$, minimizes the oscillator's energy (34).

Since at times near η_0 the coefficients a/a_0 and $\tilde{a}/\tilde{a}_0$ are close to 1, the equality of the initial values for h_{rms} and ζ_{rms} follows already from the simple comparison of the Lagrangians (13) and (22).

The relationship between the above-mentioned genuine vacuum state $|0\rangle$ and the squeezed vacuum state $|0_{\mathrm{s}}\rangle$ is determined by the action of the squeeze operator $S(r,\phi)$ on $|0\rangle$:

$$|0_{\mathrm{s}}\rangle = S(r,\phi)|0\rangle\,,$$

where

$$S(r,\phi) = \exp\left[\frac{1}{2}\,r\Big(\exp(-\mathrm{i}2\phi)\,\mathbf{c}^2 - \exp(\mathrm{i}2\phi)\,\mathbf{c}^{\dagger\,2}\Big)\right].$$

The mean number of quanta in the squeezed vacuum state is given by

$$\langle 0_s|\mathbf{c}^\dagger\mathbf{c}|0_{\mathrm{s}}\rangle = \sinh^2 r = \frac{1-\gamma}{2\sqrt{\gamma}}\,.$$

This is a huge and divergent number, when the 'zero in the denominator' factor $\sqrt{\gamma}$ goes to zero. Therefore, the 'standard inflationary result' for S-perturbations is based on the wrong initial conditions, according to which the initial amplitude of the ζ-perturbations can be arbitrarily large from the very beginning of their evolution.

Moreover, the initial amplitude is assumed to go to infinity in the most interesting limit $\sqrt{\gamma} \to 0$ and $\mathrm{n} \to 1$. If $\sqrt{\gamma}$ does not deviate from 1 too much, then the mean number of quanta in the squeezed vacuum state is acceptably small, and the wrong initial conditions give results sufficiently close to the correct ones. However, as in the Landau example mentioned above, if the wrong formula gives acceptable answers for some range of x, this does not make the wrong theory the correct one. (Finally, if $\sqrt{\gamma} = 1$, then $a(t) \propto t$, $a(\eta) \propto \exp\eta$, and $w = -1/3$. From this model of cosmological evolution, the study of relic gravitational waves has begun in the first paper of Ref. [2].)

In terms of the classical mode functions, it is the function $\mu/a\sqrt{\gamma}$ that should satisfy the classical version of the initial conditions (31), and not the function μ/a, which is postulated by the inflationary requirement (27). They both are the so-called 'gauge-invariant' variables, but their physical meaning is drastically different. The original derivations of the 'well-known result' were guided simply by the visual analogy between the function $u = \mu$ in the theory of density perturbations and the function μ in the theory of gravitational waves already developed by that time.

The assumption of arbitrarily large initial amplitudes of curvature perturbations or, technically speaking, the choice of the initial multiparticle squeezed vacuum state $|0_s\rangle$ for ζ, instead of the ordinary vacuum sate $|0\rangle$, is the origin of the absurd 'standard inflationary result'. Certainly, this wrong assumption cannot be the basis of observational predictions for cosmology.

5. Conclusions

The grossly incorrect predictions of inflationary theorists should not be the reason for doubts about the existence and expected amount of relic gravitational waves. The generation of relic gravitational waves is based on the validity of general relativity and quantum mechanics in a safe cosmological regime where quantization of the background gravitational field is not necessary.

In our numerical evaluations, we also assumed that the observed large-angular-scale anisotropies of CMB are caused by cosmological perturbations of quantum-mechanical origin. This is not necessarily true, but it would be quite disastrous if it proved to be untrue.

It is quite a challenge to imagine that the natural and unavoidable quantum-mechanical generation of cosmological perturbations is less effective than anything else. In any case, if relic gravitational waves are not discovered at the (relatively high) level described in this contribution, the implications will be much more serious than the rejection of one inflationary model or another. The reality of our time is such that if the proposal is not properly 'sexed-up', it is not very likely to be funded. But the ultimate truth lies in the fact that real physics of the very early universe is much more exciting than the artificial hullaballoo over popular words such as 'inflation' or 'inflationary gravitational waves'.

Hopefully, relic gravitational waves will be discovered in experiments which are already in the well-developed stage. I personally would think that this is likely to happen first in dedicated ground-based observations, such as the recently approved Cardiff–Cambridge–Oxford collaboration CLOVER [41]. Let us hope this will indeed be the case.

Acknowledgments. I am grateful to D Baskaran, J Romano, and especially M Mensky for fruitful discussions and help, and to P Steinhardt for calling my attention to the paper [40] and for the accompanying intense and useful correspondence.

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