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Analysis of a new electron-runaway mechanism and record-high runaway-electron currents achieved in dense-gas discharges †

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Contents

Abstract. **It is shown that the 'new' runaway criterion for electrons in dense gases suggested by Tarasenko and Yakovlenko (*Usp. Fiz. Nauk* 174 953 (2004) [*Phys. Usp.* 47 887 (2004)]) is actually not a criterion, and the 'upper' curve of the $U(Pd)$ dependence does not exist. Only the Z-shaped segment of $U(Pd)$ in the region of small Pd, known for helium since the early 1930s, agrees with the reality. The decrease in the ionization coefficient with E/P increasing and the existence of $Pd_{\min}$ for helium have been known since the same time. Doubt is cast upon the 'record' runaway-electron currents at $P = 1$ atm. The acceleration mechanism suggested in the above article has been known for a long time, and the interpretation of the 'record' runaway-electron currents on this basis is the result of fitting the data to the formula that implies the lack of electron multiplication but is 'understood' by Tarasenko and Yakovlenko as a runaway criterion. Nothing new has been added to the mechanism of volumetric discharge formation, but mistakes have been made.**

† A comment on "The electron runaway mechanism in dense gases and the production of high-power subnanosecond electron beams" by V F Tarasenko and S I Yakovlenko, *Usp. Fiz. Nauk* **174** 953 (2004) [*Phys. Usp.* **47** 887 (2004)].

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"There are more things between cathode and anode than are dreamt of in your philosophy."
H Raether

1. Introduction

A number of papers published in 2003 – 2004 and describing experimental and theoretical studies of runaway electrons (REs) in dense gases reported results that could have far-reaching implications [1 – 12]. In the opinion of their authors, these studies have formed a new comprehension of the formation mechanism of RE beams in gases.

(1) "In contrast to the commonly used local electron-runaway criteria," a 'nonlocal' criterion is suggested [1, 2, 10].

(2) The 'critical voltage' is represented as a function of the pressure P times the interelectrode gap d; this function "has an additional upper branch that indicates that no self-sustained discharge occurs if the high voltage is applied to the electrodes sufficiently fast" [1, 2].

(3) Subnanosecond RE beams were obtained in gas discharges at atmospheric pressure with current amplitudes two orders of magnitude higher than in experiments at the Russian Federal Nuclear Center — All-Russia Research Institute of Experimental Physics (RFNC — VNIIEF) [14 – 19]: the RE fraction reported in Refs [3 – 9, 12] comprised several percent, in contrast to tenths of a percent reported in Refs [14 – 19].

(4) "...A volumetric discharge was formed in the absence of an (*external* — L B) preionization source... It is shown that the mechanism... responsible for the volumetric discharge involves the generation of fast electrons...," which "...are efficient in preionizing the gas between the cathode and anode..." [8].

The authors of Refs [1 – 13] claim the novelty of their findings that subverts the results obtained over a few decades.

The studies pioneered at the VNIIEF have dealt over several decades with dense-gas discharges that develop in the

RE regime, mainly at $P = 1$ atm. Although the high-voltage pulse generators employed in the VNIIEF experiments have parameters similar to those used in Refs [3–9], the number of REs was always two orders of magnitude less than in Refs [3–9], under identical conditions in the gas diodes. Extensive experimental studies based on electrical and optical techniques have been carried out. Most of their results have not been published, because, to ensure their proper reliability, the configuration of the experiments has been varied and the measurements repeated many times. The published studies (see Refs [18, 19] and the references therein) addressed the space–time evolution of the optical emission of the discharge, emission spectra measured with time resolving, measuring the space–time parameters and energy distributions of the REs, determining the moment of RE generation, investigating the X-ray emission of the REs, etc. Based on a coherent interpretation of various aspects of the studied discharges, a self-consistent model was suggested for the development of the discharge and for electron acceleration.

The interpretation of their experiments by the authors of Refs [1–12] is purely speculative. Only integrated-emission images and oscillograms of voltage and RE current are interpreted; other data that could provide more objective information on the discharge dynamics — emission spectra, the space–time evolution of the emission, RE spectra, etc. — are not available to the authors. The authors claim that the RE beam "...forms at the stage when the plasma produced at the cathode approaches *closely* the anode" [4, 8,12], but this is merely proclaimed, being demonstrated neither experimentally nor by numerical simulations of the discharge dynamics in a self-consistent field. The conventional Townsend coefficient reduced to unit pressure, $\alpha_{\mathrm{T}}(E, P)/P$, is denoted by $\xi(E/P)$ by the authors of Refs [1, 2, 12]; however, they call $\xi(E/P)$ a 'universal function,' thus imparting some new sense to it, such that the generation of REs by dense-gas discharges does not seem to be describable without using this sense.

The discoverers of the new criterion [1, 2, 10, 12] proceed from the statement that "...the electron-runaway mechanism in gas-discharge plasmas is, in contrast to the common view, completely different from that in fully ionized plasmas," and they endue the term runaway with a meaning different from that of the term acceleration (see, e.g., Ref. [1], p. 268). It would therefore be reasonable to introduce a new term, because acceleration has been implied by runaway from the very beginning [20, 21]; more precisely, if relativistic energies are included, the term runaway has been assigned to a continuous gain of energy by the electron that '*runs away from collisions*' [22, 31] with atomic particles but *experiences nevertheless some number of collisions*, including ionizing ones: "...an electron is runaway if it does not circulate through all energy states available to it at a given E/N, but on average moves towards high-energy states" [22]. Here, N is the atomic-particle number density. It is noteworthy that the RE phenomenon was predicted not by Giovanelly in 1949 [23], as the authors of Refs [1, 12] believe, but by Wilson in 1924 [20], and the term 'runaway electron' was introduced by Eddington in 1926 [21]. The fundamental difference between the RE phenomena in fully ionized plasmas and weakly ionized gases is not in the fact that, in plasmas, "the Coulomb cross sections exhibit a quadratic decrease with the increase of the energy," as Tkachev and Yakovlenko [1] believe, but in the fact that, in a highly ionized plasma, collective degrees of freedom are excited and limit the acceleration even if the directed velocity of electrons only slightly exceeds their thermal velocity. As for the interactions of electrons with atomic particles, they are of a Coulomb nature irrespective of whether these particles are ionized, and "a quadratic decrease with the increase of the energy" is exhibited by the cross sections of the elastic collisions in which scattering by the nucleus dominates, while the cross sections of inelastic interactions with atomic electrons decrease in inverse proportion to the energy [24–26].

Because any plasma is produced from a nonionized substance, I cannot agree with the statement that "...the fundamental difference between the runaway phenomena in a Coulomb plasma and a gas... stems from the fact that, in a fully ionized plasma, new electrons are not born... but rather the available electrons are accelerated" [1, 2]. It is another matter that, to my knowledge, the known models of the RE phenomenon in strongly ionized plasmas ignore the transient process and, from the very beginning, describe a runaway process with the drag due to elastic collisions of electrons with ions, usually protons, and Ref. [27] may be an exception.

The present article analyzes the new theory of REs and the results of numerical simulations (which are not easy to scrutinize, because many details remain hidden to the reader in computer calculations). An attempt is made to evaluate the reliability of the experimental results reported in Refs [3–9, 12]; this is a nearly hopeless affair: the purity of the experiment must be assessed, which is usually a matter of the experimenter's qualification and thoroughness. In the case of Refs [3–9] analyzed here, this task is entangled by the incompleteness of the description of the experiments and by the carelessness of the presentation. For example, Ref. [3] contains the following passage: "...the design of the diode was similar (*to what*? — L B), and it is described in Ref. [6]." However, two substantially different cathodes are described in Ref. [6]: the electric field was nearly uniform over the gap in the case of one of these cathodes but strongly nonuniform in the case of the other. The papers analyzed here do not present oscillograms of the total current, RE-absorption curves, etc. Reference [7] reports that a 140-A current of a beam of 150 keV electrons was recorded in a 'helium-filled' diode, but the pressure is not specified. Figure 3 in Ref. [6] shows oscillograms of the currents of the REs generated by air discharges at $P = 1$ atm. The currents were recorded behind the anode either in the air or in a vacuum, but what can be inferred from these oscillograms if the air and vacuum records refer to experiments with different generators and radically different cathode configurations?

The title of Ref. [12] announces "the formation of powerful subnanosecond electron beams," although there is no formation; instead, a spontaneous generation of RE pulses or fluxes, rather than beams, occurs due to the simple application of a sufficiently high-voltage pulse to the gas-filled gap.

2. The new criterion for electrons runaway in dense gases

2.1 General remarks

(1) In criticizing the traditional 'local' criterion of electron runaway in dense gases, the authors of Refs [1, 2, 10, 12] confuse two processes — *the gain of energy* by the electrons and the electron *multiplication*.

(2) The high-energy part of the energy distribution function (EDF) of REs cannot be described in terms of the quantities averaged over the EDF of all free electrons.

(3) The runaway criterion and the runaway energy threshold were introduced many years ago for clarity; they are meaningful in a deterministic description of the kinetics of REs, which is not quite adequate to their nature. Nowadays, when numerical simulations are accessible, stochastic techniques of description should be developed as an approach adequate to the nature of the phenomenon.

2.2 On criticism aimed at the traditional, 'local' electron-runaway criterion

The authors of Refs [10, 12], as they "...address themselves on the basic steps in deriving the local criterion...," write as follows citing Raizer [28]: "It is believed that the distribution is nearly monoenergetic in a steady-state electron flow from the cathode to the anode." This is not true; on p. 74, as an approach to a description of REs, Raizer describes "...the approximation of 'monoenergetic' electrons. It is assumed that... electrons of the same, definite energy are present at any point..." [28]. This approximation is ensured by the behavior of the cross sections "...at high energies..." [28]. Raizer means an approximation appropriate precisely for the description of REs, but in no way for the 'steady-state flow' as a whole.

The authors of Refs [10, 12], criticizing the traditional runaway criterion, use the following equation of balance of energy ε in the direction of the x coordinate, which is measured from the cathode:

$$\frac{\mathrm{d}\varepsilon}{\mathrm{d}x} = eE - F(\varepsilon)\,, \tag{1}$$

where the Bethe formula for the specific electron energy losses is used to calculate the drag force $F(\varepsilon)$. In the opinion of the authors, "the usual approach yields a local criterion for the electric field strength, which, as is commonly believed, determines a condition for the generation of numerous REs. This criterion specifies that the field strength must exceed the value at which the energy acquired by the electron over its free path becomes equal to the maximum energy loss for the ionization of the gas" [12]. By the traditional local criterion, the authors of Refs [1, 10, 12] imply the inequality

$$E > E_{\mathrm{cr1}} = \frac{F_{\max}}{e}\,, \tag{2}$$

where E is the local field strength and $F_{\max}$ is the maximum value of $F(\varepsilon)$. They write that if condition (2) is satisfied, "...all electrons, according to the usual point of view, are continuously accelerated." Obviously, if condition (2) is satisfied, even electrons with zero initial energy are accelerated, because $\mathrm{d}\varepsilon/\mathrm{d}x > 0$; however, the process is stochastic, and therefore the satisfaction of condition (2) is restricted in space and time by the growing conductivity of the gas, and hence only "...some fraction of electrons... can be continuously accelerated up to the anode" [29].

The runaway condition for electrons in a gas in the form of the inequality

$$\frac{eE}{P} > \frac{L_1(\varepsilon)}{\mu} \tag{3}$$

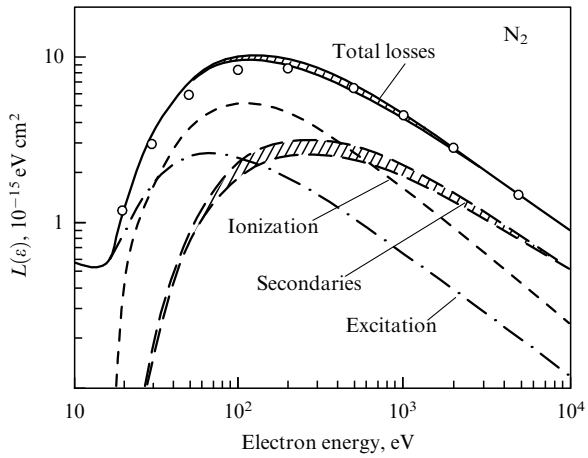

Figure 1. Specific energy lost by electrons in nitrogen [30], $L(\varepsilon) = L_1(\varepsilon)P/N$.

was originally written in terms of the function of the specific electron energy losses, $L_1(\varepsilon)$, at an energy ε (Fig. 1); for this function, the semiempirical formulas in Ref. [30] were used with the addition of the mean cosine of the scattering angle μ [18]. At low energies, $L_1(\varepsilon)$ is much more accurate than the prediction of the Bethe formula. But the presence of the ε dependence in condition (3) is more important because it sets an energy threshold of runaway, $\varepsilon_{\mathrm{th}}(E/P)$, while condition (2) is a particular case in which $\varepsilon_{\mathrm{th}} = 0$ (see below for details). For example, if condition (2) is satisfied for all electrons during some time interval near the maximum of the voltage pulse, the threshold $\varepsilon_{\mathrm{th}}$ grows during the pulse damping, and electrons with progressively higher energies can be accelerated.

In the opinion of Tarasenko and Yakovlenko [12], the formula

$$eE_{\mathrm{cr1}} = \frac{4\pi e^3 ZN}{2.72I}$$

for a gas is 'qualitatively similar' to the formula

$$eE_{\mathrm{cr1}} = \frac{4\pi e^3 \Lambda N_{\mathrm{i}}}{T}$$

for a fully ionized hydrogen plasma with an ion concentration N_{i} and a temperature T. These formulas are similar if the quantity $2.72\Lambda I$ is regarded as the temperature; for nitrogen, for example, it is equal to 2200 eV, which is absolutely unrealistic. Actually, in a fully ionized plasma, $eE_{\mathrm{cr1}} = 0.214\,(4\pi e^3 \Lambda N_{\mathrm{i}}/T)$ [31, 73], and the Bethe formula averaged over a Maxwellian EDF yields a formula that differs by a factor of Z only [18, 19]. It should be noted that, no matter how small the critical field strength may be, the high conductivity of the fully ionized plasma prevents the development of this field and, in contrast to the opinion of Tarasenko and Yakovlenko [12], the 'hurtling' regime in which all electrons are accelerated is impossible; however, 'partial runaway' can occur in weaker fields, with electrons of energies $\varepsilon > \varepsilon_{\mathrm{th}}(E)$ involved in the acceleration process [31]. This restriction is also valid for gas discharge.

2.2.1 A limitation on the mean RE energy set by electron multiplication. Based on the equation for the mean electron

Table. Nitrogen, $P = 100$ Torr [12].

E/P, V cm^{-1} Torr^{-1}	α_T/P, cm^{-1} Torr^{-1} [12]	$(eE/P)/(\alpha_T/P)$, keV	α_T, cm^{-1}	α_T^{-1}, cm	E, MV cm^{-1}
10^3	10	0.1	10^3	10^{-3}	0.1
10^4	2	5	200	5×10^{-3}	1
2×10^4	0.5	40	50	2×10^{-2}	2
0.8×10^5	10^{-3}	0.8×10^5	0.1	10	8

energy,

$$\frac{\mathrm{d}\langle\varepsilon\rangle}{\mathrm{d}x} = eE - F\big(\langle\varepsilon\rangle\big) - \alpha_T\langle\varepsilon\rangle\,, \tag{4}$$

the authors of Refs [10, 12] argue that "...even provided the condition $E > E_{\mathrm{crl}}$ is satisfied, the mean electron energy will in no way grow indefinitely with x, even if the drag force is completely neglected," because Eqn (4) "...contains a term that describes the 'spreading' of the energy gained by the electrons from the field over all electrons, including secondaries. Therefore, even if the drag in the gas medium is neglected [at $F(\varepsilon) = 0$], the mean electron energy will be restricted, $\langle\varepsilon\rangle < \langle\varepsilon\rangle_{\max} = eE/\alpha_T$. This means that Eqn (1) cannot be treated as an equation for the mean electron energy, and the distribution of electrons cannot be considered monoenergetic." However, if the drag force is neglected, condition (2) acquires the form $E > 0$.

Equation (1), as the approximation of *one* 'mean' electron, is in fact a 'monoenergetic' approximation for the entire electron flow; if so interpreted, it can in no way describe high-energy EDF 'tails.' To describe *high-energy REs* in the 'monoenergetic' approximation, one has to average only the 'tails'; of course, Eqn (1) can be used for this purpose.

The low efficiency of energy 'spreading' in strong fields can be judged by the dependences of α_T/P on E/P obtained by the authors of the new runaway criterion [12]. The table presents the energy $(eE/P)/(\alpha_T/P)$ calculated for nitrogen, based on the dependence of α_T/P on E/P at $P = 100$ Torr as given in Fig. 3a in Ref. [12]. As E/P is increased 80 times, α_T/P decreases by a factor of 10^4, precisely because of the decrease in $\sigma(\varepsilon)$ with increasing ε, while $(eE/P)/(\alpha_T/P)$ increases almost 10^6 times. Because α_T/P decreases with the increase in E/P, the rapidly growing energy $(eE/P)/(\alpha_T/P)$ acquired by the electron over its path of length α_T^{-1} is spread over a progressively more slowly growing number of electrons, which become runaway with increasing probability. For $10^3 - 10^4$ V cm^{-1} Torr^{-1}, this energy does not exceed several keV; however, in fields with such E/P values, numerous REs with energies close to the applied voltage are indeed generated [18, 19]. Producing a field with the strength $E = 1$ MV cm^{-1} in gas is complete fantasy, but the results given in the last row of the table exceed all reasonable limits: even at the pressure $P = 100$ Torr, for which Ref. [12] gives computation results, the strength is close to the intra-atomic values.

We now check the correctness of Eqn (4) obtained by the authors of Refs [10, 12] by eliminating $\mathrm{d}n/\mathrm{d}x$ in the 'energy conservation law' [12],

$$\frac{\mathrm{d}n\langle\varepsilon\rangle}{\mathrm{d}x} = neE - nF\big(\langle\varepsilon\rangle\big)\,, \tag{5}$$

using the balance equation for the electron concentration n,

$$\frac{\mathrm{d}n}{\mathrm{d}x} = \alpha_T n\,, \tag{6}$$

where $\alpha_T n$ is responsible for ionizing collisions. These are truncated equations for the EDF moments [31], in which $\langle\varepsilon\rangle$ *is the mean energy not of high-energy REs only but of all electrons*.

Below, we describe an accurate procedure of combining the balance equations for the energy density and concentration, after a book by Golant et al. [31]. We immediately note, however, that the term $-\alpha_T\langle\varepsilon\rangle$ appears wrongly in Eqn (4), because the drag force F represented by the Bethe formula in Ref. [12] already contains the energy ε_s transmitted to the secondary electron, rather than only the mean threshold energies for ionization, $\varepsilon_{\mathrm{ion}}$, and excitation, $\varepsilon_{\mathrm{ex}}$, of atomic particles [25, 26, 32] [see Eqn (26) below]. The physical inadequacy of Eqn (4) is evident: a 'mean' electron completely loses its energy over the mean path between two consecutive ionization events, $1/\alpha_T$, and this loss is complemented by a loss due to the drag force $F\big(\langle\varepsilon\rangle\big)$, which is obviously unrealistic. This incorrectness results from the fact that neither primary nor secondary electrons are considered in terms of the hydrodynamic approximation, which is expressed by the original Eqn (5); instead, only an electron gas with the energy density $n\langle\varepsilon\rangle$ is treated. Therefore, $F\big(\langle\varepsilon\rangle\big)$ in Eqn (5) is not the full drag force but only its fraction responsible for the losses $\varepsilon_{\mathrm{ion}}$ and $\varepsilon_{\mathrm{ex}}$; see below, Eqn (14) with Eqn (18) taken into account.

2.2.2 Equations for the EDF moments. The rigorous equations for the zeroth and the second moments of the EDF written as $\Phi(\mathbf{r}, \mathbf{v}) = n(\mathbf{r}, t) f(\mathbf{p}, t)$ are [31]

$$\frac{\partial n}{\partial t} + \operatorname{div} n\mathbf{u} = \frac{\delta n}{\delta t}\,, \tag{7}$$

$$\frac{\partial n\langle\varepsilon\rangle}{\partial t} + \operatorname{div}\langle\mathbf{v}\varepsilon\rangle + n\mathbf{u}e\mathbf{E} = \frac{\delta\big(n\langle\varepsilon\rangle\big)}{\delta t}\,, \tag{8}$$

where $\mathbf{v}$ is the velocity, $\mathbf{u}$ is the directional (hydrodynamic) velocity ($\mathbf{u}\uparrow\downarrow\mathbf{E}$), $\mathbf{p}$ is the momentum of the electron, $e > 0$ is the elementary charge, $\int f(\mathbf{p}, t)\,\mathrm{d}\mathbf{p} = 1$, $\langle\varepsilon\rangle = \int \varepsilon f(\mathbf{p}, t)\,\mathrm{d}\mathbf{p}$, and $\langle\mathbf{v}\varepsilon\rangle = \int \mathbf{v}\varepsilon f(\mathbf{p})\,\mathrm{d}\mathbf{p}$.

We perform our further analysis with an emphasis on the ionizing collisions of the electrons. We omit the angular dependence and by the EDF we mean $f(p, t)$. Then, if we neglect the small losses due to elastic collisions, we obtain collision integrals that are otherwise accurate:

$$\frac{\delta n}{\delta t} = n\,4\pi\int_0^\infty \big(\mathrm{St}_{\mathrm{ion}}\{f(p,t)\} + \mathrm{St}_{\mathrm{ex}}\{f(p,t)\}\big)\,p^2\,\mathrm{d}p\,, \tag{9}$$

$$\frac{\delta n\langle\varepsilon\rangle}{\delta t} = n\,4\pi\int_0^\infty \varepsilon\big(\mathrm{St}_{\mathrm{ex}}\{f(p,t)\} + \mathrm{St}_{\mathrm{ion}}\{f(p,t)\}\big)\,p^2\,\mathrm{d}p\,, \tag{10}$$

where

$$\mathrm{St}_{\mathrm{ex}} = Nv\Big[f(p', t)\,\sigma_{\mathrm{ex}}(p')\left(\frac{p'}{p}\right)^2 - f(p, t)\,\sigma_{\mathrm{ex}}(p)\Big]\,, \tag{11}$$

$$\mathrm{St}_{\mathrm{ion}} = Nv \int_{\varepsilon+\varepsilon_{\mathrm{ion}}}^{\infty} \mathrm{d}\varepsilon' \left(\sigma_{\varepsilon'}(\varepsilon',\varepsilon)\right)_{\mathrm{ion}} \left(\frac{p'}{p}\right)^2 f(p',t) - Nv\,\sigma_{\mathrm{ion}}(\varepsilon) f(p,t) \tag{12}$$

are operators that describe the excitation and ionization of atomic particles by electron impact, respectively [32, 33]; σ_{ex} is the total excitation cross section; and $\left(\sigma_{\varepsilon'}(\varepsilon',\varepsilon)\right)_{\mathrm{ion}}$ is the differential ionization cross section. The primes mark the pre-interaction quantities. We substitute Eqns (11) and (12) in Eqns (9) and (10) to express the collision integrals as

$$\frac{\delta n}{\delta t} = n\langle \nu_{\mathrm{ion}}\rangle\,, \tag{13}$$

$$\frac{\delta n\langle\varepsilon\rangle}{\delta t} = -n\varepsilon_{\mathrm{ex}}\langle\nu_{\mathrm{ex}}\rangle + nN\int_{\varepsilon_{\mathrm{ion}}}^{\infty} \mathrm{d}\varepsilon'\, v' f(\varepsilon') \int_0^{\varepsilon'-\varepsilon_{\mathrm{ion}}} \mathrm{d}\varepsilon\, \varepsilon \left(\sigma_{\varepsilon'}(\varepsilon',\varepsilon)\right)_{\mathrm{ion}} - nN\int_{\varepsilon_{\mathrm{ion}}}^{\infty} \mathrm{d}\varepsilon\, \varepsilon v\, \sigma_{\mathrm{ion}}(\varepsilon) f(\varepsilon)\,. \tag{14}$$

We reduce these expressions to forms that can be compared with the collision terms in Eqns (5) and (6). The order of integration is changed here in the double integral and the relations $\varepsilon' - \varepsilon = \varepsilon_{\mathrm{ex}}$, $\mathrm{d}\varepsilon = v\,\mathrm{d}p$, and $4\pi f(p',t)\,p'^2 = v f(\varepsilon',t)$ are used. Here, the excitation and ionization frequencies, ν_{ex} and ν_{ion}, and the total ionization cross section σ_{ion} are determined by the integrals

$$\langle\nu_{\mathrm{ex}}\rangle = N\int_{\varepsilon_{\mathrm{ex}}}^{\infty} \sigma_{\mathrm{ex}}(\varepsilon)\, v f(\varepsilon)\,\mathrm{d}\varepsilon\,, \tag{15}$$

$$\langle\nu_{\mathrm{ion}}\rangle = N\int_{\varepsilon_{\mathrm{ion}}}^{\infty} \sigma_{\mathrm{ion}}(\varepsilon)\, v f(\varepsilon)\,\mathrm{d}\varepsilon\,, \tag{16}$$

$$\sigma_{\mathrm{ion}}(\varepsilon) = \int_0^{(\varepsilon-\varepsilon_{\mathrm{ion}})/2} \mathrm{d}\varepsilon' \left(\sigma_{\varepsilon}(\varepsilon,\varepsilon')\right)_{\mathrm{ion}} = \frac{1}{2}\int_0^{\varepsilon-\varepsilon_{\mathrm{ion}}} \mathrm{d}\varepsilon' \left(\sigma_{\varepsilon}(\varepsilon,\varepsilon')\right)_{\mathrm{ion}}\,. \tag{17}$$

We recall that the Townsend ionization coefficient, which appears in Eqn (4), is $\alpha_{\mathrm{T}} = \langle\nu_{\mathrm{ion}}\rangle/u$.

2.2.3 Energy lost by the free-electron gas in ionizing collisions. We apply the mean-value theorem to the inner integral in the double integral in Eqn (14) to find that, for the ionization component of the collision integral,

$$nN\int_{\varepsilon_{\mathrm{ion}}}^{\infty} \mathrm{d}\varepsilon'\, v' f(\varepsilon')\,\frac{\varepsilon'-\varepsilon_{\mathrm{ion}}}{2}\,2\sigma_{\mathrm{ion}}(\varepsilon') - nN\int_{\varepsilon_{\mathrm{ion}}}^{\infty} \mathrm{d}\varepsilon\, \varepsilon v\, \sigma_{\mathrm{ion}}(\varepsilon) f(\varepsilon) = -nN\varepsilon_{\mathrm{ion}}\int_{\varepsilon_{\mathrm{ion}}}^{\infty} \mathrm{d}\varepsilon'\, v' f(\varepsilon')\,\sigma_{\mathrm{ion}}(\varepsilon') = -n\langle\nu_{\mathrm{ion}}\rangle\varepsilon_{\mathrm{ion}}\,, \tag{18}$$

where $\varepsilon_{\mathrm{ion}}\langle\nu_{\mathrm{ion}}\rangle$ is the mean power of the drag force due to the energy loss equal to the ionization threshold. Therefore, the energy density $n\langle\varepsilon\rangle$ of the free-electron gas is obviously reduced due to the ionizing collisions over the time $\Delta t = 1/\langle\nu_{\mathrm{ion}}\rangle$ by the quantity $n\varepsilon_{\mathrm{ion}}$. The remaining energy acquired by the electrons from the field (minus the excitation losses) is indeed "...spread over all electrons, including secondaries..." [10, 12]. This cannot, however, be claimed in a mean-value description, as the authors of Refs [10, 12] do based on Eqn (4), because, as is worth repeating, a continuous-medium description deals with neither primary nor secondary electrons but only with the electron gas as an entity.

Thus, we have shown that Eqn (5) *does not include the total drag force. Its component responsible for the transmission of the energy* ε_{s} *to a secondary electron is missing in Eqn* (5).

2.2.4 Energy lost by a 'mean' electron. We combine Eqns (7) and (8), following the authors of Refs [10, 12], but according to the book by Golant et al. [31]:

$$\frac{\partial\langle\varepsilon\rangle}{\partial t} + \frac{\mathrm{div}\, n\langle\mathbf{v}\varepsilon\rangle}{n} - \langle\varepsilon\rangle\frac{\mathrm{div}\, n\mathbf{u}}{n} = -\mathbf{u}e\mathbf{E} + \frac{\delta\left(\langle\varepsilon\rangle\right)}{\delta t}\,, \tag{19}$$

where

$$\frac{\delta\left(\langle\varepsilon\rangle\right)}{\delta t} = \frac{1}{n}\left(\frac{\delta\left(n\langle\varepsilon\rangle\right)}{\delta t} - \frac{\delta n}{\delta t}\,\langle\varepsilon\rangle\right)\,. \tag{20}$$

(1) The explicit form of collision integral (20) with Eqns (13) and (14) taken into account is

$$\frac{\delta\langle\varepsilon\rangle}{\delta t} = -\varepsilon_{\mathrm{ex}}\langle\nu_{\mathrm{ex}}\rangle + N\int_{\varepsilon_{\mathrm{ion}}}^{\infty} \mathrm{d}\varepsilon'\, v' f(\varepsilon') \int_0^{\varepsilon'-\varepsilon_{\mathrm{ion}}} \mathrm{d}\varepsilon\, \varepsilon \left(\sigma_{\varepsilon'}(\varepsilon',\varepsilon)\right)_{\mathrm{ion}} - N\int_{\varepsilon_{\mathrm{ion}}}^{\infty} \mathrm{d}\varepsilon\, \varepsilon v\, \sigma_{\mathrm{ion}}(\varepsilon) f(\varepsilon) - \langle\varepsilon\rangle\langle\nu_{\mathrm{ion}}\rangle\,. \tag{21}$$

In contrast to Eqn (4), three terms, one of which is positive, are here responsible for ionization; therefore, the conclusion of the authors of Refs [10, 12] that "...the mean electron energy... is limited..." to the quantity eE/α_{T} is not obvious.

(2) Equation (21) reduces, in view of Eqn (18), to the form

$$\frac{\delta\langle\varepsilon\rangle}{\delta t} = -\varepsilon_{\mathrm{ex}}\,\langle\nu_{\mathrm{ex}}\rangle - \left(\varepsilon_{\mathrm{ion}} + \langle\varepsilon\rangle\right)\langle\nu_{\mathrm{ion}}\rangle\,. \tag{22}$$

As in Eqn (4), the energy lost by a 'mean' electron in a mean ionizing event here exceeds its energy by a quantity equal to the ionization threshold. This contradiction results from the fact that the mean power of the ionizing drag force, $\left\langle F_{\mathrm{ion}}(\varepsilon)v\right\rangle$, is replaced with the product of mean quantities $\left(\varepsilon_{\mathrm{ion}} + \langle\varepsilon\rangle\right)\langle\nu_{\mathrm{ion}}\rangle$.

(3) To separate $\left\langle F(\varepsilon)v\right\rangle$ explicitly, we use the reduced form of the ionization operator with the 'weak' interactions segregated [32, 34, 35],

$$\mathrm{St}_{\mathrm{ion}} = \frac{1}{p^2}\frac{\partial}{\partial p}\,p^2 F_{\mathrm{ion}}(\varepsilon) f(p) + Nv\int_{2\varepsilon+\varepsilon_{\mathrm{ion}}}^{\infty} \mathrm{d}\varepsilon' \left(\sigma_{\varepsilon'}(\varepsilon',\varepsilon)\right)_{\mathrm{ion}} \left(\frac{p'}{p}\right)^2 f(p') \tag{23}$$

and the differential representation for the operator $\mathrm{St}_{\mathrm{ex}}$ [32],

$$\mathrm{St}_{\mathrm{ex}} = \frac{1}{p^2}\frac{\partial}{\partial p}\,p^2 f(p,t)\,F_{\mathrm{ex}}(p)\,. \tag{24}$$

Then the integral in (20) becomes

$$\frac{\delta\langle\varepsilon\rangle}{\delta t} = -\big\langle F(\varepsilon)v\big\rangle + N\int_{\varepsilon_{\rm ion}}^{\infty} \mathrm{d}\varepsilon'\, v' f(\varepsilon') \times \int_0^{(\varepsilon'-\varepsilon_{\rm ion})/2} \big(\sigma_{\varepsilon'}(\varepsilon',\varepsilon)\big)_{\rm ion}\varepsilon\,\mathrm{d}\varepsilon - \langle\varepsilon\rangle\langle\nu_{\rm ion}\rangle\,, \qquad (25)$$

where the total drag force includes the ionization losses by the primary electron, $\varepsilon_{\rm ion}+\varepsilon_{\rm s}$, and the losses due to excitation of atomic particles [25, 26, 32],

$$F(\varepsilon) = F_{\rm ion} + F_{\rm ex} = N\int_0^{(\varepsilon-\varepsilon_{\rm ion})/2} (\varepsilon_{\rm ion}+\varepsilon_{\rm s})\big(\sigma_\varepsilon(\varepsilon,\varepsilon_{\rm s})\big)_{\rm ion}\,\mathrm{d}\varepsilon_{\rm s} + N\sum_i \varepsilon_{\rm ex}^{(i)}\,\sigma_{\rm ex}^{(i)}(\varepsilon)\,. \qquad (26)$$

Equation (4) is a crude approximation to the stationary version of the accurate Eqn (8) with collision integral (21). The following is more important for our analysis: if we separate the energy $\langle\varepsilon\rangle$ in the integral in Eqn (25) according to the mean-value theorem, we obtain an expression *exactly compensating* the term $-\langle\varepsilon\rangle\langle\nu_{\rm ion}\rangle$, with only the mean power of the total drag force $-\big\langle F(\varepsilon)v\big\rangle$ remaining in the losses.

The conclusion of the authors of Refs [10, 12] *that* "*...the mean electron energy is ...limited...*" *to the quantity* $eE/\alpha_{\rm T}$ *is incorrect and results from carelessly handling the mean quantities and ignoring the fact that* $F(\varepsilon)$ *already contains* $\varepsilon_{\rm s}$. It is senseless to make far-reaching conclusions in the physics of REs on the basis of the equations for the EDF moments — especially truncated ones, because they are obtained by integrating over all energies $\varepsilon\in(0,\infty)$. An accurate description of REs is only possible in the framework of stochastic approaches, which correspond to the nature of the phenomenon, and, what is extremely important, with the angular scattering necessarily taken into account.

Of course, an approximate description can be given in terms of mean quantities, based on at least two groups of equations, one for REs with averaging over the interval $[\varepsilon_{\rm th},\infty)$ and the other for electrons in the subthreshold region with averaging over the interval $(0,\varepsilon_{\rm th}]$, if $F_{\rm min} < eE < F_{\rm max}$. For example, Babich and Kutsyk [36] use a three-group approximation, with the group $[0,\varepsilon_{\rm th}]$ divided into two subgroups. According to stochastic simulations of the avalanche of relativistic REs for $F_{\rm min} < eE \ll F_{\rm max}$, the larger the eE in this region, the smaller the mean RE energy $\langle\varepsilon\rangle$ because of the lowering of the threshold $\varepsilon_{\rm th}$, but the larger the number of REs [37, 38], with $\langle\varepsilon\rangle$ depending extremely weakly on E in the interval $eE/F_{\rm min} = 2-8$. The last statement is clear from the expression $\langle\varepsilon\rangle_{\rm approx} \approx (eE - F_{\rm min})\lambda_{\rm e}(E)$, according to which $\langle\varepsilon\rangle$ can be estimated as the energy acquired by an electron with a small initial energy ε_0 ($\varepsilon_{\rm th} \leqslant \varepsilon_0 \ll \langle\varepsilon\rangle$) over the distance equal to the RE-multiplication length $\lambda_{\rm e}$ ($\varepsilon \geqslant \varepsilon_{\rm th}$), which decreases with increasing E. The energy gain in the field competes with the production of secondary electrons but, in contrast to the case of Eqn (5), only of secondary REs, the overwhelming majority of which have initial energies much smaller than $\langle\varepsilon\rangle$ but larger than $\varepsilon_{\rm th}$. As this takes place, an enormous number of electrons are born in the subthreshold energy region; however, averaging over them taken together with the REs is not the correct procedure. If this is done, following the authors of Refs [10, 12], no avalanches of relativistic REs can be obtained. The EDF moments can be used to describe REs if $eE > F_{\rm max}$ at $\varepsilon_{\rm th} = 0$, such that any secondary electron with an arbitrarily small initial energy continues gaining energy, but high-energy REs can then be masked by the large number of low-energy REs. This is not the case if $eE \gg F_{\rm max}$.

2.3 The Townsend ionization regime

The abstract to Ref. [12] claims: "It is shown that the Townsend electron-multiplication mechanism can operate even if the fields are strong and the ionization drag of electrons can be neglected." How can the multiplication of electrons be discussed if the ionization is neglected? Tkachev and Yakovlenko [1, 2] believe that in gas discharges, "the electron runaway is consistent with the Townsend ionization mechanism..., so that the energy acquired in the field is balanced by losses for the ionization and excitation of the gas." Such a situation occurs, however, in any gas discharge where no REs are generated. It is not clear how electrons can run away, i.e., accelerate, if the acquired energy is immediately 'balanced.' The impression is that something else is meant by runaway! The authors are consistent in applying the concept of the 'Townsend ionization mechanism'; on this basis, they interpret the results of their experiments [3–9] carried out in the region of large *Pd* and large overvoltages relative to the static breakdown voltage, where the Townsend mechanism does not operate. Even at small overvoltages in dense gases, the streamer breakdown mechanism in a self-consistent electric field is realized; at large overvoltages achievable using voltage pulses with subnanosecond fronts, the breakdown is always controlled by the gas preionization due to the flow of REs accelerated at the front of the primary avalanches and streamers that are initiated by the field emission (FE) and develop near the cathode.

The following statement is fundamental to Refs [1–12] and includes, in the authors' opinion, the 'runaway criterion': "...REs are generated as the characteristic multiplication length (the inverse Townsend coefficient) becomes comparable to or larger than the distance between the electrodes (*?* — L B)" [1, 2]. Where do electrons come from if no multiplication occurs over the distance *d*? The authors contradict themselves: it turns out that in a gas discharge, as "...in a fully ionized plasma, new electrons are not born... but the available ones are accelerated" [1, 2], without any difference. How can the electron runaway be realized "...in the framework of the Townsend ionization mechanism...," if ionization by electrons is completely absent? To a certain extent, the quoted statement is applicable, for example, to the 'open discharge' under reduced pressures, which, in essence, develops consistently with the external source of electrons [39–41]; in general, however, this statement is wrong, especially for discharges in dense gases at high overvoltages, which were present in the experiments described in Refs [3–9].

Tkachev and Yakovlenko [1], analyzing the 'Townsend multiplication mechanism,' use the kinetic equation (KE), without any electric field included, to obtain an exponential growth in the number of electrons with a coordinate; in doing so, they demonstrate their incomprehension of the meaning of the KE components: in their opinion, "...the situation in which the *inflow* of electrons to the given *velocity* interval due to inelastic collisions is balanced by their *outflow* from the given point of *space* is described by the equation..." (*my*

emphasis — L B)

$$v_x \frac{\partial}{\partial x}\big(n_e(x)\big) f(x,v) = n_e(x) N \int \sigma_{n.st}(v,v') v' f(x,v')\,dv' , \tag{27}$$

where $n_e(x)$ is the electron number density, v is the velocity, $f(x,v)$ is the local velocity distribution function of electrons at a point $x \in [0,d]$, and $\sigma_{n.st}(v,v')$ is the cross section of inelastic collisions. How can the inflow of a fluid ('electrons' [1]) into one reservoir ('velocity interval' [1]) be balanced by an outflow from another ('the given point of space [1]) if these 'reservoirs,' figuratively speaking, are not connected and differ in their physical dimension? Actually, Eqn (27) means that the time variation of $n_e(x) f(x,v)$ in the given element of the phase volume $dx\,dv$ due to the *inflow* and *outflow* of electrons in configuration space is exactly equal to the *source* power [the sink is ignored in Eqn (27)] controlled by all inelastic collisions.

The treatment of the 'Townsend mechanism' based on Eqn (27) is completely wrong, because the field strength E is absent in this equation. The Townsend mechanism, as the streamer or any other local mechanism, implies that for a given E, the distribution function $\Phi(x,v) = n_e(x) f(x,v)$ is time-independent, i.e., all electrons that found themselves in a given physically infinitesimal volume relax to the EDF determined by the local E within a time interval much shorter than the characteristic times of the macroscopic processes — first of all, the time of variations in the local E due to the accumulation of space charges. In the corresponding stationary KE,

$$v \frac{\partial \Phi(x,v)}{\partial x} + eE \frac{\partial \Phi(x,v)}{\partial v} = N\big[\mathrm{St}_{el}\{\Phi\} + \mathrm{St}_{ex}\{\Phi\} + \mathrm{St}_{ion}\{\Phi\}\big] , \tag{28}$$

each of the respective operators $\mathrm{St}_{el}\{\Phi\}$, $\mathrm{St}_{ex}\{\Phi\}$, and $\mathrm{St}_{ion}\{\Phi\}$ of elastic, exciting, and ionizing collisions consists of two components responsible for the inflow and outflow of electrons in $dx\,dv$ (see, e.g., Refs [31–36]); this is not the case for Eqn (27), where only the source is present. The subsequent procedure is not to 'demonstrate' an exponential multiplication in the absence of a field, as the authors of Ref. [1] do, but, on the contrary, to set (see, e.g., Ref. [42])

$$\Phi(x,v) = n_e(x) f(x,v) = \exp(\alpha_T x)\, f(v) , \tag{29}$$

leading to the equation

$$\frac{\alpha_T}{N} v f(v) + \frac{eE}{mN} \frac{\partial f(v)}{\partial v} = \mathrm{St}_{el}\{f\} + \mathrm{St}_{ex}\{f\} + \mathrm{St}_{ion}\{f\} , \tag{30}$$

which is to be solved for α_T/N as a function of E/N. Other approaches have also been developed, but the local electric field is taken into account in any case.

Finally, in their derivation of the trivial equation $dn_e/dx = \alpha_T n_e$ for the exponential growth in the number of electrons along the x axis from Eqn (27), the authors use in fact a truncated procedure of deriving the equation for the zeroth EDF moment,

$$\frac{\partial n_e}{\partial t} + \mathrm{div}\, n_e \mathbf{v} = \nu_{ion} n_e ,$$

from the time-dependent KE [31]. If the EDF is t-independent, $\partial n_e/\partial t = 0$, then the equation remaining in the one-dimensional case is $dn_e/dx = \alpha_T n_e$. The authors prove precisely this relationship, which is, however, generally known [24, 28].

The ionization frequency ν_{ion} depends on E/N as the integral (16) of the ionization cross section $\sigma_{ion}(\varepsilon)$ but not of "...the total cross section of electron-neutral inelastic collisions..." [1], $\sigma_{n.col.}(v)$, as in Eqns (2) of Ref. [1]; there, apropos, ν_{ion} is even independent of E/N, because the force term eE does not appear in Eqn (27). The Townsend coefficient is the root of the equation

$$\nu_{ion} - \alpha_T v_d + \alpha_T^2 D = 0 ,$$

where D is the electron diffusion coefficient [42] rather than $\alpha_T = \nu_{ion}/v_d$.

Tkachev and Yakovlenko [1] claim that "...the following two basic assumptions underline Townsend's model: (a) a compensation for the electrons produced at a given x by their drift, which is precisely implied by Eqn (27), and (b) the x-independence of the form of the electron velocity distribution." This is not the case: in the Townsend mechanism of breakdown in a uniform external field U/d [24, 28],

(a) no distortion of the external field due to space charges is present, and therefore an exponential multiplication of electrons occurs over the entire gap of width d at a constant $\alpha_T(U/Pd)$ — in contrast, e.g., to streamer models, in which α_T is x-dependent because of the effect of space charges;

(b) the ionization in the gas volume develops consistently with the γ processes at the cathode; and

(c) the breakdown develops as a series of sequentially generated avalanches maintained by the γ processes.

True, in Refs [10, 12], their authors write: "At arbitrarily high field strengths, the Townsend regime... takes place for the overwhelming majority of electrons. Two features are characteristic of it. First, the number of ionization events grows exponentially with the distance from the cathode. Second, the mean velocity and energy of electrons do not depend on this distance." Indeed, the first and the second statements are true, but the whole is nonsense, because the Townsend regime is restricted to relatively small Pd and E/P. At sufficiently large values of these parameters, the streamer breakdown mechanism occurs, with cathode processes unimportant and bulk charges playing a vital role. The breakdown in overvoltaged gaps was discussed above.

We note that the EDF and therefore $\alpha_T\big(E(x)/P\big)$ are x-dependent in a nonuniform field; however, the Townsend breakdown mechanism can operate until the RE contribution becomes large, such that the discharge cannot be adequately described in terms of the local $\alpha_T\big(E(x)/P\big)$ (see, e.g., Refs [28, 43]). Moreover, the nonlocal character of $\alpha_T\big(E(x)/P\big)$ becomes very important in nonuniform fields with very large E/P, as shown in Ref. [44] for the region of the cathode potential drop in a glow discharge (helium, $U_{cath} = 150$ V, $E/P \in [0; 231]$ V cm^{-1} Torr^{-1}), although the fraction of electrons with energies in the region of decreasing cross sections is extremely small because of the small U_{cath}. The Townsend mechanism is realized, crudely, in the region $Pd \leqslant 200$ [24] or $Pd \leqslant 1000$ Torr cm [28]; but the authors of Refs [1, 12] have 'extended' this region beyond 1000 Torr cm (Fig. 2) and to unbelievable overvoltages.

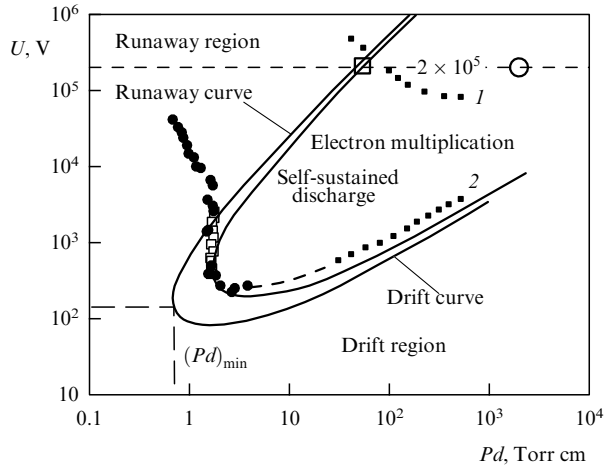

Figure 2. Dependences $U(Pd)$ for helium in a uniform field. The figure is taken from Ref. [12]. "Dashed curve, experimental data from Ref. [28]; circles, Penning's experiments [52]. The large circle corresponds to the maximum voltage in the experiments of Tarasenko et al. [4] at the atmospheric pressure and a distance between the electrodes of $d = 28$ mm. The large square corresponds to the case where the 'plasma cathode' has approached the cathode to a distance of $d = 0.7$ mm" [12]. The additional heavy squares represent the measurements of $U(Pd, \tau_U)$ published in Refs [18, 19]: *1*, a large overvoltage (the voltage-pulse rise time is $\tau_U < 0.5$ ns); *2*, a static regime ($\tau_U \to \infty$).

2.4 The new, 'nonlocal' electron-runaway criterion

The formula

$$\alpha_{\mathrm{T}}(E_{\mathrm{cr}}, P)d = Pd\,\frac{\alpha_{\mathrm{T}}}{P} \equiv Pd\,\xi\left(\frac{E_{\mathrm{cr}}}{P}\right) = 1 \tag{31}$$

is suggested by the authors of Refs [1, 2, 10, 12] as a new, 'nonlocal' runaway criterion. The quantity E_{cr} is not defined in Ref. [1], but this is seemingly a voltage that ensures an 'appreciable number of REs' [1]. It is already clear from Refs [10, 12] that the corresponding "$U_{\mathrm{cr}}(Pd)$ curve... is universal for the given gas"; the authors call it 'the outflow curve of electrons.' Likely, $U_{\mathrm{cr}} = E_{\mathrm{cr}}d$. Tarasenko and Yakovlenko [12] emphasize that "...the value of E_{cr}/P depends on Pd, in contrast to E_{cr1}/P (*see formula* (2) — L B), which is determined by a local criterion and depends on the parameters of neutrals only." However, the runaway criterion was never specified by formula (2). This is done by the authors of Ref. [12]. Our criterion (3) depends on the energy of a particular electron and the direction of its motion with respect to the direction of the electric force, which makes it radically different from Eqn (31), where α_{T} is the result of averaging all free electrons over the EDF.

The value of Eqn (4) lies in the fact that its publication elucidated the meaning attributed by Tkachev and Yakovlenko [1, 2] to criterion (31). Before, the treatment of Eqn (31) as an electron-runaway condition appeared completely absurd, because this formula means that the number of electrons in the interelectrode gap of length d increases by a factor of e; it is not a runaway criterion because it does not contain dynamical quantities. The 'runaway criterion' (31) is trivial:

$$d = \frac{1}{\alpha_{\mathrm{T}}} \approx \frac{v_{\mathrm{d}}}{\nu_{\mathrm{ion}}} \approx \frac{v_{\mathrm{d}}}{v_{\mathrm{Te}}}\,\lambda_{\mathrm{ion}} \leqslant \lambda_{\mathrm{ion}}\,. \tag{32}$$

Moreover, this criterion is fairly flexible: "Generally, the choice of the right-hand sides of the expressions (31) is somewhat arbitrary. The right-hand sides can be set equal, e.g., to π or $1/\pi$ instead of unity. However, the choice of the constant in the right-hand side is clearly unimportant" [12, p. 960].

The authors of Refs [1, 2, 10, 12] regard relation (31) as nonlocal, but this is not the case, because α_{T} depends on the local strength $E = E_{\mathrm{cr}}$. They write: "It is usually believed that REs emerge if E/P exceeds some critical value independent of d." They associate the nonlocality precisely with the presence of the interelectrode distance d in 'criterion' (31). This is, however, completely absurd. If we apply a high voltage, e.g., hundreds of kV, to a pointed cathode, we obtain a large flux of accelerated electrons, whatever the distance to the anode may be, because the potential drop occurs near the cathode, and therefore $eE \sim eU/r \gg F_{\max}$ (where r is the curvature radius of the point).

It cannot be understood how to combine the "...criterion of the emergence of an appreciable number of REs..." in Eqn (31), which in essence implies the absence of multiplication, with the claim that in the runaway-electron regime, "...the Townsend electron-multiplication mechanism remains valid for given E and P if d is sufficiently large" [1]; compared to what is d large if it is bounded according to condition (31)? By the Townsend multiplication, the authors mean the exponential multiplication with $\alpha_{\mathrm{T}} = \mathrm{const}$ at a given E/P. The avalanche of relativistic electrons [34, 35, 37, 38, 45] also grows exponentially, but how is this fact related to the Townsend breakdown mechanism? The runaway phenomenon is characterized precisely by the fact that the multiplication is controlled not by the local E/N, to which the EDF has been relaxed, but by the prehistory of the kinetics of the electrons, with their energy and angular distributions at a given point $\mathbf{r}_2$ determined not by the local $E(\mathbf{r}_2)$ but by the potential drop

$$\Delta\varphi = \int_{\mathbf{r}_1}^{\mathbf{r}_2} \mathbf{E}(\mathbf{r})\,\mathrm{d}\mathbf{r}$$

along the displacement of the electron, $\mathbf{r}_2 - \mathbf{r}_1$, and by all interactions along the trajectory corresponding to this displacement. One should use either a consistent stochastic description (KE, Monte Carlo techniques, etc.) or a deterministic description, for example, in the continuous-medium approximation. In the latter case, it is necessary to use equations for at least the first two EDF moments [31]. The treatment of Eqn (31) as the runaway criterion confuses two processes — multiplication, which can be described in the continuous-medium approximation by the equation

$$\frac{\partial n_{\mathrm{e}}}{\partial t} + \operatorname{div} n_{\mathrm{e}}\mathbf{v} = \nu_{\mathrm{ion}} n_{\mathrm{e}}\,,$$

and accelerated motion [motion is completely missing from the 'runaway criterion' (31)], which can be deterministically described in terms of the equation for the first EDF moment; the simplest representation of this equation is

$$\frac{\mathrm{d}\mathbf{p}}{\mathrm{d}t} = -e\mathbf{E} - \frac{\mathbf{p}}{p}\,F(\varepsilon)\,, \tag{33}$$

where semiempirical energy-loss functions per unit path are reasonable to use for the effective drag force $F(\varepsilon)$ in the nonrelativistic region of energies below the validity limit for

the Born approximation (see Fig. 1) [18, 19, 29, 46]. Most certainly, the equation of motion is used in the modification of the particle technique employed for numerical simulations in Refs [1, 10, 12], but why does it not appear in the runaway criterion?

The authors of Refs [1, 10, 12] regard the runaway criterion (2) as local. Actually, electron-acceleration criteria similar to those given by formulas (2) and (3) should be satisfied in some region of space; this precisely implies nonlocality and the need to solve Eqn (33). For this reason, as Ul'yanov and Chulkov emphasize, α_T "...is determined not only by the local value of the parameter E/P, which is constant throughout the gap (*the case of a uniform external field is meant* — L B) but also by the distance (*i.e.,* $\Delta\varphi$ *and all interactions* — L B) traversed by the electron... as well as by the processes at the anode" [47]. If $F(\varepsilon)$ is approximated by a smooth function in the low-energy region, then, for $eE < F_{\max}$ and $eE > F_{\min}$, the equation $eE = F(\varepsilon)$ has three roots, $\varepsilon_1 < \varepsilon_2 < \varepsilon_3$ [48]. The root ε_1, which is located in the low-energy region, corresponds to a stable state. The idea of locality is applicable here: electrons move directionally at the drift velocity v_d, which is less than the chaotic ('thermal') velocity:

$$v_d = v_d(\varepsilon_1) \ll v_{Te}(\varepsilon_1)\,.$$

The root ε_2 is in the region of decreasing cross sections. It corresponds to an absolutely unstable state, being the runaway threshold ε_{th}; the latter was introduced long ago by Gurevich [49] and, later, in our paper [29] in terms of semiempirical loss functions per unit path [30]: the electron is continuously accelerated in the region $\varepsilon > \varepsilon_{th}$. The third stable state ε_3 is realized in the relativistic region, which we do not discuss here. The notion of ε_{th} is used in solving the KE to separate the runaway region from the 'reservoir' [50] of slow electrons. Criterion (2) implies that $\varepsilon_{th} = 0$.

By means of a numerical simulation of the 'Townsend ionization regime' in helium using a particle technique, Tkachev and Yakovlenko [1] have shown that 'virtually no' REs are present for E/P up to 5000 V cm^{-1} Torr^{-1} if $d \gg \alpha_T^{-1}$. This is a very strange result, because the maximum energy loss in helium is 67 eV cm^{-1} Torr^{-1} [30]. The definition of RE is important in a stochastic description, since a strict runaway threshold ε_{th} cannot be introduced. For example, in the Monte Carlo numerical simulations described in Ref. [50], electrons that had reached 4 keV were considered to be runaway. In helium at $P = 1$ atm and $E = 200$ kV cm^{-1} ($E/P \approx 270$ V cm^{-1} Torr^{-1}), all electrons reached 4 keV within 0.03 ns. In the numerical simulations [1] carried out along the coordinate $x \in [0, d]$, it is reasonable to assign the term runaway to electrons with ε close to $e\varphi(x)$, i.e., to eU at the anode. According to Fig. 2f in Ref. [1], the fraction of electrons with $\varepsilon \geqslant 10$ keV is very large, $\sim \exp(-10/4.2)$, although $\alpha_T d \approx 9$.

Tarasenko and Yakovlenko [12] are bewitched by the 'fairly good,' in their mind, agreement they achieved between E_{cr1}/P and $(E/P)_{\max}$ at which α_T reaches its maximum; for example, for nitrogen, $E_{cr1}/P \approx 590$ V cm^{-1} Torr^{-1} and $(E/P)_{\max} = 1500$ V cm^{-1} Torr^{-1} [12]. They write: "Clearly, the quantity E_{cr1} in reality determines not the condition of continuous acceleration with increasing x for the bulk of electrons but rather the condition of the decrease of the Townsend multiplication coefficient at $E > E_{cr1}$. In this sense, the above $E_{\max}$ values are simply improved E_{cr1} values." Actually, no agreement can be noted: there is a difference by a factor of 2.5 or, more precisely, of 4.2, because, in reality, $E_{cr1}/P = L_{1,\max} = 356$ V cm^{-1} Torr^{-1} [18, 19, 30] (see Fig. 1). The fact that E_{cr1}/P and $(E/P)_{\max}$ nevertheless differ severalfold rather than in orders of magnitude results from the domination of the ionization process in the neighborhood of the energy-loss maximum $L(\varepsilon)$ or, equivalently, the maximum of the drag force $F(\varepsilon)$. As can be seen from Fig. 1, the ionization losses in nitrogen beyond 30 eV exceed the excitation losses.

Tkachev and Yakovlenko [1], discussing the RE phenomenon based on the numerical simulations, write that at $d < \alpha_T^{-1}$, "...an appreciable portion of electrons are continuously accelerated...," i.e., they nevertheless mean acceleration by 'runaway'! However, where do these REs come from, if $\alpha_T d < 1$? If we compare the captions to Fig. 2 ($d = 15$ mm) and Fig. 4 ($d = 1$ mm) in Ref. [1], we can conclude that Fig. 4 [1] simply represents the initial segment of the graphs given in Fig. 2 [1], where the exponential ('Townsend' [1]) multiplication regime is established; this is evidenced by the unsteady EDF in Fig. 4 [1], which is given by noise, and hence no runaway occurs.

The fact that $u_x \ll u_\perp$ in weak fields and $u_x \gg u_\perp$ in strong fields can be explained in a trivial and generally known manner. The result that the components of the electron velocity in the directions along and across the electric-force vector prove to be equal at $E/P = 350$ V cm^{-1} Torr^{-1} is confusing, because the energy-loss maximum for helium is 67 eV cm^{-1} Torr^{-1} [30]. As noted above, according to Monte Carlo simulations without renormalizations, the fraction of electrons with the energy 4 keV became equal to unity at $E/P \approx 270$ V cm^{-1} Torr^{-1} within 0.03 ns [50]. Angular scattering cannot compensate for the focusing effect of such a strong field.

2.5 The inadequacy of the Townsend coefficient at large E/P

In the context of the question under discussion, the concept of the local ionization coefficient is completely inadequate at very large E/P: in a strong field, even if it is uniform, the energy distribution of electrons varies in space so strongly that the quantity $\alpha_T(E/P)$ does not remain constant throughout the entire gap. In strong fields, "...the ionization coefficient at a given point x coincides with the inverse ionization free pass of the electron... at the energy $\varepsilon(x)$ corresponding to this point" [28].

Tarasenko and Yakovlenko [12], based on 'numerical simulations,' argue that the "...notion of the Townsend coefficient does not lose its sense even at $E > E_{cr1}$... ." Of course, α_T can be formally calculated for any E/P, but this quantity is physically meaningless as a function of E/P at large E/P. In particular, even in a uniform field of a very high strength, the EDF proves to be nonlocal, and the ionization frequency

$$\langle \nu_{ion}(\mathbf{r}) \rangle = N \int_{\varepsilon_{ion}}^{\infty} \sigma_{ion}(\varepsilon) v f(\varepsilon, \mathbf{r})\, d\varepsilon$$

and the directional velocity vary in space. As a result, $\alpha_T = \langle \nu_{ion} \rangle / v_d$ turns out to depend on $\mathbf{r}$, but the coefficient α_T calculated as $N_e^{-1}(dN_e/dx)$ [12] (where N_e is the local number of electrons) remains constant throughout the gas-discharge gap, because $E/P = \text{const}$. For this reason, the calculations of $\alpha_T(E/P)$ carried out in many published studies have always been restricted to reasonable E/P values. For the

same reason, in the region of strong fields, the local ionization frequency $\langle \nu_{\rm ion} \rangle$ or the local ionization free path must be used [28].

The values $E/P = 10^4 - 10^5$ V cm^{-1} Torr^{-1} for which the dependence $\alpha_{\rm T}(E/P)$ at $P = 100$ Torr is calculated [12] (see the table) are fantastic. It suffices to look at the last column of the table. In the experiments [3–9], the breakdown occurred in the centimeter-sized gap at the front of the open-circuit voltage pulse within a fraction of a nanosecond, such that the voltage over the gap did not even reach 100 kV. Even with the use of pulses of high voltage in the megavolt range with rise times $\ll 1$ ns, for which no generators are available, the breakdown in the gas would occur well before the field achieves the strength $E \sim 1$ MV cm^{-1}.

Tkachev and Yakovlenko [1] argue that the formula

$$\alpha_{\rm T}\left(\frac{E}{P}\right) = 4.4P \exp\left(-\frac{14P}{E}\right) \tag{34}$$

for helium, given in their paper with a reference to the monograph by Raizer [28], "...is only valid for relatively low reduced field strengths $E/P < 200$ V cm^{-1} Torr^{-1}" [1]. However, nobody has ever claimed that this formula is valid for any E/P. It is clearly written in the same monograph on p. 74 that "...this formula is not applicable to very strong fields." For approximation formulas, the interval of E/P where they are valid is always indicated (see, e.g., Refs [24, 28]). In particular, Ref. [44] gives an approximation for the known measurements in helium [51], and Raizer's monograph [28] presents three other approximations:

$$\alpha_{\rm T}\left(\frac{E}{P}\right) = 6.5P \exp\left(-16.4\sqrt{\frac{P}{E}}\right) \text{ [44]},$$

$$\alpha_{\rm T}\left(\frac{E}{P}\right) = 3P \exp\left(-\frac{34P}{E}\right)$$

$$\text{for } \frac{E}{P} = 20-150 \text{ V cm}^{-1} \text{ Torr}^{-1} \text{ [28]}, \tag{35}$$

$$\alpha_{\rm T}\left(\frac{E}{P}\right) = 4.4P \exp\left(-14\sqrt{\frac{P}{E}}\right)$$

$$\text{for } \frac{E}{P} < 100 \text{ V cm}^{-1} \text{ Torr}^{-1} \text{ [28]}.$$

Tkachev and Yakovlenko [1], quoting Ref. [28], obviously confuse these formulas and omit their validity regions. In the next paper [2], they give the last formula in the correct form but ignore the validity region again. Tarasenko and Yakovlenko act similarly in their review [12].

The authors of Refs [1, 2, 10, 12] have 'discovered' that the function $\alpha_{\rm T}(E/P)$, after passing its maximum, decreases with the further growth of E/P, and this is due to the decrease in $\sigma_{\rm ion}(\varepsilon)$ in the region of sufficiently large ε; however, as long ago as 1931, Penning accounted for the Z-shaped segment of the breakdown voltage curve $U_{\rm br}(Pd)$ in helium for small Pd (Fig. 3) based on these dependences $\alpha_{\rm T}(E/P)$ and $\sigma_{\rm ion}(\varepsilon)$ [52]. In particular, this is described in the widely known monograph by Meek and Craggs [53]. The above behavior of $\alpha_{\rm T}(E/P)$ related to the decrease in $\sigma_{\rm ion}(\varepsilon)$ is described by Raizer in his monograph [28, p. 74]. The empirical dependences $\alpha_{\rm T}(E/P)$ with a 'plateau' have long been known [24, 28, 54], and it is clear from them that $\alpha_{\rm T}$ must decrease as E/P increases further. We mention only a few of the multitude of studies where the dependence

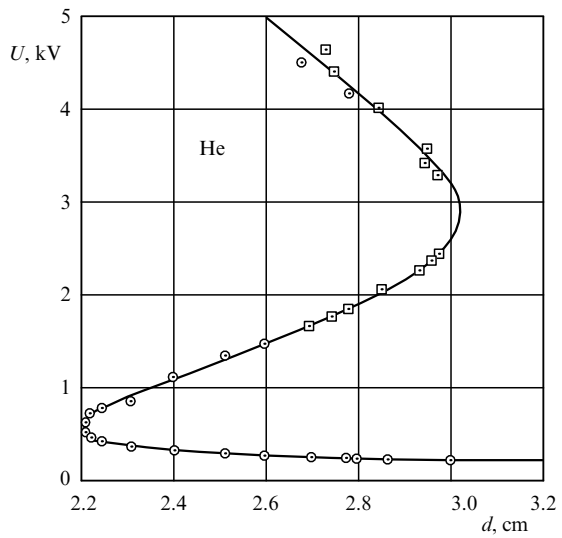

Figure 3. Dependence of the breakdown voltage $U_{\rm br}$ in helium on d for Pd below the values corresponding to the minimum breakdown voltage, $P = 0.84$ Torr [52, 53].

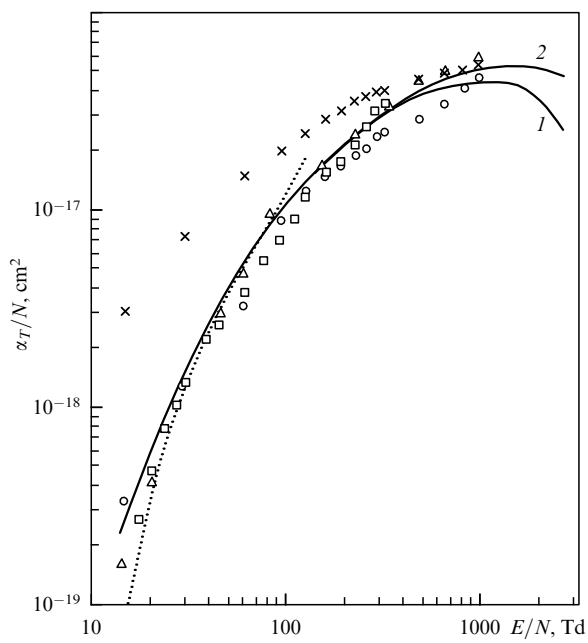

Figure 4. The Townsend ionization coefficient $\alpha_{\rm T}$ for helium. Solid curves represent the calculations in Ref. [44] (*1*, the elastic-scattering cross section is used; *2*, isotropic scattering). Measurements of $\alpha_{\rm T}$ are presented — in particular, triangles for Ref. [51] and ×'s for measurements in commercially pure helium. 1 Td = 0.338 V cm^{-1} Torr^{-1}.

$\alpha_{\rm T}(E/P)$ is calculated or, which is the same thing, the dependence of $\alpha_{\rm T}/N$ on E/N, with a maximum, was calculated. In particular, this was done by numerical Monte Carlo simulations for nitrogen [22], helium (Fig. 4), and neon [55]. The simulations in Ref. [55] were done with the angular-scattering anisotropy taken into account and, possibly for this reason, the maximum of $\alpha_{\rm T}$ in helium was found to be reached at $E/P \approx 350$ V cm^{-1} Torr^{-1} rather than at 200 V cm^{-1} Torr^{-1}, as in Refs [1, 2], or in the region < 100 V cm^{-1} Torr^{-1}, as in Ref. [12]. The maximum of $\alpha_{\rm T}(E/P)$ was obtained using the KE technique, e.g., in Ref. [72], where the accuracy is limited by the weak-anisotropy approximation.

To the authors of Refs [10, 12], "...it is unclear in advance to what extent the notion of the Townsend coefficient is applicable to an electronegative gas... ." For electronegative gases, as is known, the effective Townsend coefficient [28] is used as the difference between the Townsend ionization coefficient and the attachment coefficient η, i.e., $\alpha_{\text{eff}} = \alpha_{\text{T}} - \eta$, which is negative at small E/P. The authors of Refs [10, 12] reduce their clarifications to the notion of the negative Townsend coefficient and to calculating the dependence of the modulus $|\alpha_{\text{T}} - \eta|$ on E/P for sulphur hexafluoride, which is shown in Figs 3 [10] and 4 [12]. The authors ignore the extensive literature that presents measurements and calculations of α_{eff}, α_{T}, η, and the transport coefficients for sulphur hexafluoride, borrowing only the cross sections of the elementary interactions of electrons.

Tarasenko and Yakovlenko [1] write as follows: "If the relativistic effects are taken into account (see Fig. 6), the dependence... α_{T} as a function of E/P drops sharply after passing its maximum at $E/P \approx 263$ kV Torr^{-1} cm^{-1}, but reaches thereafter an almost constant value and then starts growing slowly. This takes place at $E/P \approx 6.6$ MV Torr^{-1} cm^{-1}... ." However, α_{T} in Fig. 6 in Ref. [12] reaches its maximum at $E/P < 100$ V cm^{-1} Torr^{-1}, while the calculations were done for the values up to $E/P = 30$ kV cm^{-1} Torr^{-1}. The quotation is taken from Section 2.2.4 of the paper "Electron runaway at relativistic velocities" [12]. Nothing is said about the runaway phenomenon in this section, while the applicability of the Townsend coefficient is analyzed again but, this time, "...at relativistic velocities of the electrons" [12]. In essence, no analysis is presented: merely, the number of ionization events per unit path is calculated. There is no reason to carry out such an investigation. Indeed, according to Fig. 6a in Ref. [12], for $E/P = 30$ kV cm^{-1} Torr^{-1}, $\alpha_{\text{T}}/P \approx 5 \times 10^{-3}$ cm^{-1} Torr^{-1} and $(eE/P)/(\alpha_{\text{T}}/P) \approx 6$ MeV. Because of the emission of electrons from the cathode, it is impossible to maintain a voltage of 6 MV over a nanosecond at pressures of several dozen or hundred Torr. We emphasize that this voltage drops over one ionization length, $\alpha_{\text{T}}^{-1} \approx 2-20$ cm, which, according to criterion (31), is equal to the length d of the interelectrode gap. If we follow the recommendation of the authors [12, p. 960] and set the right-hand side of Eqn (31) equal to $1/\pi$, we find that the voltage drops over the length $\pi d > d$!

2.6 Upper branch of the *Pd*-dependence of the breakdown voltage. The minimum *Pd* at which the breakdown is possible

Upon a 'comparison' of formula (31) where α_{T} is written as $P\xi(E_{\text{cr}}/P)$ and ξ is specified by formulas (7) and (8) in Refs [1] and [12], respectively, with the condition for discharge ignition,

$$Pd\,\xi\left(\frac{E_{\text{br}}}{P}\right) = L\,, \tag{36}$$

which is actually valid for only $Pd \leqslant 200$ Torr cm, the authors of Refs [1, 2, 12] obtain "...a relationship between the outflow and ignition curves..." [1, p. 267; 2, p. 59; 12, p. 960],

$$U_{\text{br}}(Pd) = L\,U_{\text{cr}}\left(\frac{Pd}{L}\right), \tag{37}$$

where U_{br} is the breakdown voltage and L is a function of the coefficient γ of secondary electron emission from the cathode,

$$L = \ln\left(1 + \frac{1}{\gamma}\right) = 2.89\,. \tag{38}$$

The criteria of runaway (31) and breakdown (36) essentially express the same condition because of the weak dependence of L on γ. As quoted above, the authors of Ref. [12] themselves consider the right-hand side of Eqn (31) to be fairly arbitrary.

We note that γ is a function of E/P. In particular, Ul'yanov and Chulkov [47] use $\gamma \in [0.27; 1.38]$ for

$$\frac{E}{P} \in \sim [1000; 8000] \text{ V cm}^{-1} \text{ Torr}^{-1}\,,$$

and therefore $L < 1.45$, which is substantially less than the value given by Eqn (38). Moreover, as we can judge by Fig. 2 in Ref. [2] and Fig. 5 in Ref. [1] (see also Fig. 2 in this article), the authors believe that condition (36) is valid for any Pd, which is absolutely wrong because, even in the case of a breakdown at a dc voltage, beginning with $Pd \approx 200$ Torr cm, it is necessary at least to take the fields of the space charges into account [24, 28, 53, 56].

The claim by the authors of Ref. [1] that they have discovered "...an upper branch [$U_{\text{br}}(Pd)$ — L B] determined by the drop of α_{T} with the increase of E/P..." is groundless. The nonmonotonic behavior of the curve of $U_{\text{br}}(Pd)$ for helium was discovered by Penning in 1931 [47, 52, 53]. At small Pd, the curve is indeed bent upward and rightward, but quickly turns again upward and leftward, forming a Z-shaped segment; this is due to the dependence of γ on E/P and switching-on the ionization of the gas by ions and fast atoms, which was taken into account by Penning and ignored by the authors of Refs [1, 2, 10, 12].

The numerical simulation of $U_{\text{br}}(Pd)$ in helium in the region of small Pd was done by Ul'yanov and Chulkov using a Monte Carlo technique with the allowance for gas ionization by electrons, ions, and fast atoms and with the inclusion of γ processes at the cathode due to ions (γ_{i}), fast atoms (γ_{a}), and atoms in metastable states (γ_{m}); the dependence of γ on E/P was also taken into account [47]. In the case where $\gamma_{\text{a}} = 0$, Ul'yanov and Chulkov obtained an 'upper branch' of $U_{\text{br}}(Pd)$, which was explained by them as in Refs [1, 2] but much earlier and with a reference to Penning's study. The inclusion of $\gamma_{\text{a}}(E/P)$ enabled Ul'yanov and Chulkov to obtain a complete Z-shaped segment of the $U_{\text{br}}(Pd)$ dependence. In helium, γ_{a} begins varying in the upper part of the $U_{\text{br}}(Pd)$ curve, which is directed rightward, ultimately forming the Z-shaped segment; but for most gases, γ_{a} starts changing before the decrease in the ionization cross section comes into effect, and hence $U_{\text{br}}(Pd)$ does not turn rightward. In particular, analogous simulations for hydrogen in the region of small Pd yielded the usual, smooth Paschen curve [57]. Tkachev and Yakovlenko [2] write: "Likely, the segment of the Paschen curve that is located left of the Pd_{min} point reflects a different discharge-ignition mechanism, which weakly depends on the electron multiplication in gas." Indeed, all processes in a gas and at the electrodes that involve ions, fast atoms, and photons must be taken into account, as was done in Refs [47, 57], and the analysis should not be restricted to the simple exponential multiplication of electrons in the gas. The Z-shaped $U_{\text{br}}(Pd)$ dependence in helium was observed by Guseva [58] and Dikidzhi and Klyarfel'd [59]. The authors of Refs [2, 4] mention the

experimental studies by Penning and Dikidzhi and Klyarfel'd but still ignore Ul'yanov and Chulkov's calculations.

It is worth noting the 'limiting-voltage effect' that restricts the 'upper-curve' breakdown according to Kolbychev [60]. In contrast to Refs [1, 2], where the consideration is given in terms of $\alpha_T(E/P)$, Kolbychev directly uses the $\sigma_{ion}(\varepsilon)$ dependence with the energy ε depending on x. We emphasize that Tkachev and Yakovlenko, like Kolbychev, did not consider the ionization of gas by atomic particles, which play an extremely important role in the left-hand branch of the Paschen curves, and the possibility of FE, which can remove the restriction on the 'upper-curve' breakdown. The following should also be noted. According to Penning and also Ul'yanov and Chulkov, the allowance for the dependence of γ on E/P is of fundamental importance for the $U_{br}(Pd)$ curve to be bent upward and rightward. In contrast, the prediction of the 'upper curve' by Tkachev and Yakovlenko [1, 2] and of the 'limiting-voltage effect' by Kolbychev [60] were made for $\gamma = \text{const}$.

Tkachev and Yakovlenko believe that "...the discovery of the minimum Pd_{min} value at which the ignition of a self-sustained discharge is still possible ($Pd_{min} = 1.8$ Torr cm for helium)" was made by them [1]. But the value $Pd_{min} \approx 1.85$ Torr cm for helium was in fact obtained experimentally by Penning [52], Guseva [58], and Dikidzhi and Klyarfel'd [59]. This Pd_{min} should be distinguished from the Pd value at which $U_{br}(Pd)$ passes through its minimum. According to calculations by Ul'yanov and Chulkov, $Pd_{min} \approx 0.8$ Torr cm. The discrepancy with experiment can occur because even minor admixtures of other gases to helium strongly affect the $U_{br}(Pd)$ dependence, since the associative-ionization reactions $A + A^* \to A_2^+ + e^-$ are involved [24] [see Fig. 4 for the results of measuring $\alpha_T(E/N)$ in commercially pure helium]. It is surprising that Tkachev and Yakovlenko [1] managed to obtain $Pd_{min} \approx 1.8$ Torr cm from Eqn (36) with $\gamma = \text{const}$. The point is that commercially pure helium was most likely used in the experiments in Refs [52, 58, 59]. True, a value that is an order of magnitude smaller, $Pd_{min} \approx 0.17$ Torr cm, is reported in Ref. [2]. Apparently, this is Pd at which the $U_{cr}(Pd)$ curve, rather than $U_{br}(Pd)$, turns to the right. As can be judged by the above quotation from Ref. [1], the authors believe that Pd_{min} is inherent in all gases. Actually, as noted above, it is very unusual and is related to varying γ_a.

Tarasenko and Yakovlenko [12] already acknowledge that "as shown by Penning as long ago as 1932,... the Paschen curve for helium forms a loop with a turning point at $Pd \approx 1.5$ Torr cm" and agree that "...Penning's suggestion was correct," although "...his point of view was not widely supported." Penning's interpretation of the Z-shaped segment of the $U_{br}(Pd)$ curve for helium is generally accepted, but Penning did not exclusively associate its origin with the existence of a maximum in the ionization cross section, as Tarasenko and Yakovlenko [12] believe. As for the 'upper curve' of $U_{br}(Pd)$ in the interpretation of the authors of Refs [1, 2, 4, 10, 12], it does not exist. Breakdown does not develop in the small region of Pd between the middle and upper portions of the Z-shaped segment of the dc $U_{br}(Pd)$ curve, this segment being reliably detected for helium only. However, if the applied voltage is above the dc $U_{br}(Pd)$ curve, and above the upper part of the Z-shaped segment for helium in particular, breakdown can develop at any Pd, at sufficiently large overvoltages with respect to $U_{br}(Pd)$ — in a regime with REs involved.

Tarasenko and Yakovlenko [12], differentiating Eqn (31) under the condition $d(Pd)/dU_{cr} = 0$, obtain that $\xi' = 0$, which "...corresponds precisely... to the E/P value at which α_T/P passes through its maximum." However, the differentiation of Eqn (36) under the condition $d(Pd)/dU_{br} = 0$ leads to the same conclusion.

In the opinion of Tkachev and Yakovlenko [1], to observe the upper curve, "...one has to elevate the voltage between the electrodes rapidly enough, before the ionization wave has reached the anode and the plasma has short-circuited the electrodes." However, in VNIIEF experiments with voltage pulses up to 300 kV and rise time $\tau_U < 0.5$ ns [16–19], the volumetric discharge developed at relatively small Pd values in a helium-filled gap with a hemispherical working surface of the cathode ($r_{cath} = 2$ cm) at $d = 1$ cm (i.e., in a virtually uniform field), well above the 'upper curve.' It can be seen from Fig. 2 that the $U(Pd)$ curve measured by us [18, 19] intersects the 'upper' $U_{cr}(Pd)$ and $U_{br}(Pd)$ curves, penetrating the region where, according to Refs [1, 2, 4, 10, 12], a self-sustained discharge develops due to the Townsend mechanism, and REs are not generated. In the VNIIEF experiments, intense RE fluxes were recorded in this region, which were necessarily involved in the development of the breakdown. On the other hand, in the region over the $U_{cr}(Pd)$ and $U_{br}(Pd)$ segments where, according to the idea of the authors of Refs [1, 2, 4, 10, 12], the discharge cannot ignite, breakdown in the RE regime developed in the VNIIEF experiments. This also refers to other gases [18, 19].

In Fig. 5 in Ref. [1], the authors of Ref. [12] show the Paschen curve $U(Pd)$ for static breakdown in helium (see Fig. 2), with a reference to Raizer's monograph [28], in such a way that a decrease in Pd, crudely, from 1 to 0.8 Torr cm increases the breakdown voltage U from 1 to 100 kV; actually, no such interval in $U(Pd)$ is present in Ref. [28]. However, at a constant voltage of 50–450 kV, a breakdown in gaps with $d = 0.2$–20 mm occurs in a deep vacuum. Under the action of 150–2500 kV microsecond pulses, breakdown occurs in vacuum gaps with $d = 1.7$–100 mm [61]. The results of

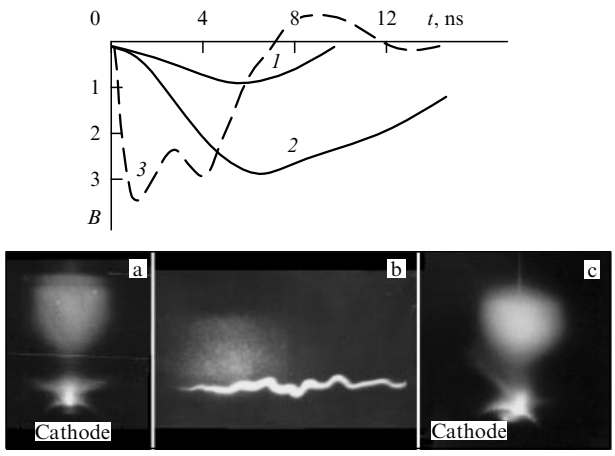

Figure 5. Space–time evolution of diffuse volumetric discharge in air [18, 19] at $P = 1$ atm; the interelectrode distance is $d = 15$ mm, a pointed conical cathode with the apex curvature radius $r_{cath} = 3$ mm is used, the open-circuit voltage of the generator is $U_{idle} \approx 270$ kV, and the voltage-pulse rise time is $\tau_{idle} < 0.5$ ns. Here, B is the film-blackening density conditioned by the diffuse glow (*1*) and the emission of the dense near-cathode plasma (*2*); *3*, the oscillogram of the gas-discharge current. Bottom: photograph of the discharge (a, c) and the corresponding streak photograph (b).

measurements and numerical simulations of $U(Pd)$, given in numerous publications, cover a range from static conditions (e.g., in Refs [18, 19, 28, 47, 52, 53, 57–59]) to high overvoltages [18, 19], and extrapolating them to the left, into the vacuum region, allows reaching the mentioned voltage ranges. Essentially, the upper branch of $U(Pd)$ depicted by the authors of Refs [1, 2, 10, 12] goes to the right and upward, to high pressures and voltages; therefore, it cannot reach the vacuum-breakdown region. Thus, it does not reflect any objective reality unless it turns again leftward and downward, to low pressures and voltages.

3. The role of runaway electrons in the development of volumetric discharges in dense gases

In the review article of Ref. [12] and in Ref. [8], the known formation mechanism of volumetric discharges in dense gases with RE-pulse preionization [16–19] is, on the one hand, claimed as a new one and, on the other hand, is discussed speculatively and self-contradictorily based on integrated photographs of the discharge appearance. In contrast, in our studies, the mechanism is constructed based on experimental investigations of the space–time evolution of the discharge optical emission and its spectra, as well as on the measured parameters of the REs and X-ray emission [16–19]. The statement that "...the reasons and conditions for the formation of a volumetric discharge in a nonuniform nanosecond... discharge field have not yet been investigated, and the specific energy inputs did not exceed 100 MW cm^{-3}" [8, 12] appears puzzling at the very least. The authors themselves cite review [18], where, in particular, "...the formation mechanism of volumetric discharge in a nonuniform nanosecond... field" [8] is described — in the same terms as in Refs [8, 12] but in more detail and with more comprehensive calculations. The energy inputs in our studies and in Refs [8, 12] must be the same because the parameters in the high-voltage generators and conditions in the gas-discharge diodes were virtually identical.

In their investigation of the role of accelerating processes in the formation of volumetric discharges, Kostyrya et al. [8] have completely repeated the VNIIEF studies without adding any novelty to the experiment; only the electrode configuration was different. They found that "over a wide range... of conditions, a volumetric discharge develops between the tubular, sharp-edged cathode and the anode... in the form of diffuse cones or jets..., and bright spots emerge only near the cathode, at the voltage-pulse front" [8, 12]; however, the last statement has not yet been verified experimentally. Exactly the same pattern was observed in our experiments [16–19] in both weakly and strongly nonuniform fields: "...one or several bright plasmoids with apparent sizes $\ll d$ form at the cathode, and the remaining volume, up to the anode... is filled with a diffuse glow" [18]. Figure 5 presents two photographs of the integrated discharge glow and the time sweep of the glow [18, 19].

The authors of Refs [8, 12] model the near-cathode plasmoid by means of an ideally conducting bulge, ignoring the analysis of the effect of the field distribution in the neighborhood of the ellipsoidal plasma channel on the plasmoid evolution carried out by Lozanskii and Firsov [24]. The accurate formula for the potential of the ellipsoidal bulge of the cathode is written, but without using it, the energy ε acquired by the electron in the enhanced field is estimated to be eUa/d. Why is ε not directly estimated by multiplying the field strength near the apex of the bulge, e.g., an ellipsoidal one,

$$E_{\mathrm{b}} \approx \frac{U}{d}\,\frac{(a/r)^2}{\Lambda(a/b)} = \frac{U}{d}\,\frac{a}{\Lambda(a/b)b}\,, \tag{39}$$

by $\sim (2-3)b$? Here, a and r are the major and minor semiaxes of the ellipsoid and b is the curvature radius of the apex. Ultimately, the authors of Refs [8, 12] act precisely in this way but without taking the formfactor $\Lambda(a/b) \sim 10$ into account. But is there any need to take the field enhancement by the space charge of plasmoids into account in the case of a sharp-edged foil cathode? The energy acquired by an RE near the apex of motionless plasmoids was calculated in Refs [29, 46, 62] but with the inclusion of the dependence of voltage on the distance from the apex and consistently with the energy losses by the electrons (see also Refs [18, 19]).

The following remark is much more important. In local discharge models, starting from Raether's classic model [24, 28, 56], the ionization develops *consistently* with the electron acceleration at the avalanche (streamer) front: the field soliton, i.e., the region with a permanently growing maximum strength $E_{\mathrm{f\,max}}$, and the electrons drift at a growing speed $v_{\mathrm{d}}\big(E_{\mathrm{f}}(t)\big)$ due to gas ionization by the electrons trapped by the soliton. This is the *drift self-acceleration* of electrons, which can be realized if the EDF has time to completely relax to the local field $f(\varepsilon, t) = f\big(\varepsilon\big(E_{\mathrm{f}}(t)\big)\big)$. As was already done in Refs [18, 19, 29, 46, 62], the authors of Refs [8, 12] also consider the polarizational *acceleration of REs* in the filed of a *stationary* soliton. In a nonlocal model, however, the self-consistency results in the polarizational *self-acceleration of REs*, which involves *accelerated* motion of the soliton [18, 19]. This is a direct extrapolation of Raether's model [24, 56] to the region of strong fields $E_{\mathrm{f}}(t)$ for which the EDF has no time to relax to the local field at the channel front [18, 19]. The aim of the study in Ref. [3] was "...to find the formation mechanism of an electron beam at small... E/P in a diode filled with air to the atmospheric pressure." In essence, however, the authors describe the very simple acceleration mechanism whose principle was suggested by us [29] and which was further developed taking the self-consistent motion of the REs and the field soliton into account [18, 19].

The authors of Refs [8, 12] claim that "...it has been proved that... an electron beam is generated at the stage when the plasma formed near the cathode approaches the anode to a small distance" [8, p. 32; 12, p. 954]. But does this not contradict the statement that "the volumetric discharges are formed due to the preionization by fast electrons" [8, p. 37]? The point is that the authors of Ref. [8], as they estimate the RE energy near the apex of the bulge for $U = 100$ kV and $d = 28$ mm to be $\varepsilon \approx eUa/d \approx 2-5$ keV (1–4 keV in Ref. [12]), assume that the length of the bulge is $a \approx 1$ mm $\ll d$. Thus, what is primary — the plasma in the volume or the RE beam? This question is clarified in Ref. [12]: in the authors' opinion, the electron accelerated at the front of the near-cathode plasmoids (plasma 'bulges' at the cathode) are primary. They do not constitute "...the powerful electron beam" [12] recorded in Refs [3–9]. They merely preionize the gas over the entire gap, thus forming an ionization wave; after it approaches the anode, "...the criterion (31) comes to be satisfied in a thin layer between the plasma formed in the volume and the anode, so that a powerful electron beam is generated" [12]. However, the frequency of

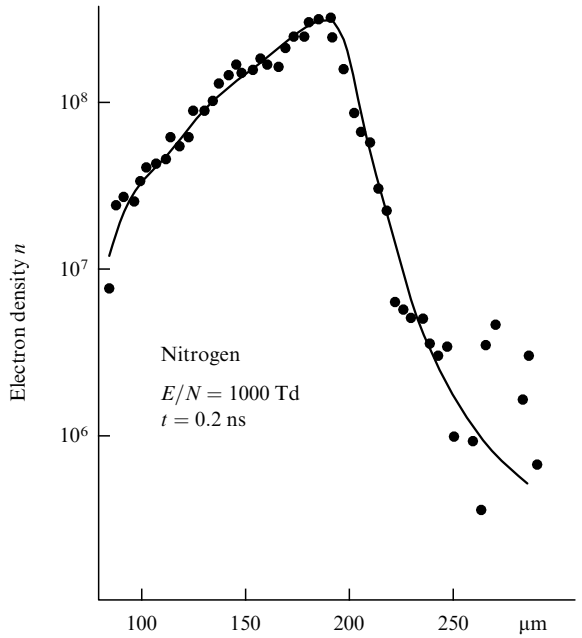

Figure 6. Electron density in the avalanche, integrated in the direction normal to the external-field vector; nitrogen, $P \approx 280$ Torr, $E/P \approx 334$ V cm^{-1} Torr^{-1} [63].

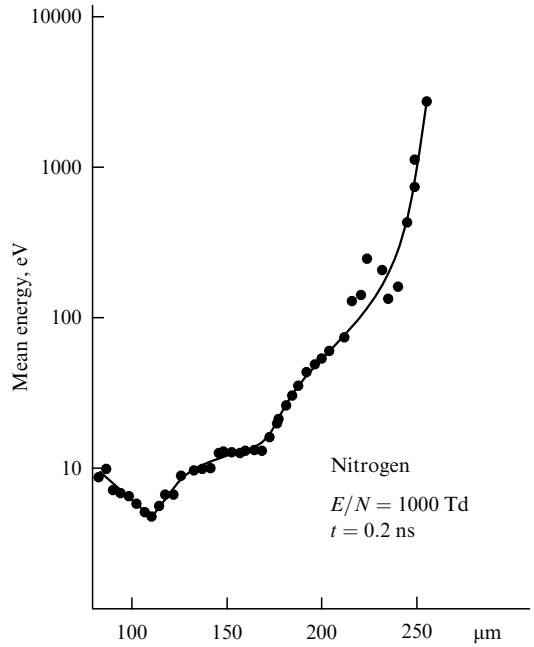

Figure 7. Electron energy in the avalanche, integrated in the direction normal to the external-field vector; nitrogen, $P \approx 280$ Torr, $E/P \approx 334$ V cm^{-1} Torr^{-1} [63].

ionization by 1 – 4 keV electrons is less than the maximum frequency by a factor of only 0.9 – 0.7; therefore, such electrons, efficiently ionizing the gas at the front of the channel, simply cannot leave this front to preionize the gap. The channel propagates self-consistently with the electron acceleration at its front, until the REs reach high energies such that the ionization frequency becomes insufficient to attach the REs to the channel front [18, 19].

Tarasenko and Yakovlenko apparently strive to fit the formation mechanism of the volumetric discharge suggested by them to criterion (31). Would it not be more logical to assume (keeping in mind Raether's criterion and its extrapolation to the region of very high overvoltages [18, 19]) that the plasma 'bulge' at the cathode (the avalanche, near-cathode streamer, and channel) develops self-consistently with the acceleration of electrons at its front [18, 19]? (The electron acceleration *can* initially be described in the drift approximation as the growth of the drift velocity v_d consistent with the field strengthening at the front of the channel, a rigorous description in terms of 'pure' acceleration to be used as a increases.)

According to the results of a Monte Carlo simulation of the initial stage of the ionization development in nitrogen in a self-consistent electric field at $N = 10^{19}$ cm^{-3} ($P \approx 280$ Torr) and at a reduced external-field strength $E/N = 1000$ Td ($E/P \approx 334$ V cm^{-1} Torr^{-1}), REs appear by a time of 0.2 ns as a pronounced leader at the avalanche front, at the distance 200 – 300 μm from the cathode (Fig. 6) [63]. Their mean energy reaches several keV, exceeding the mean electron energy behind the ionization front in the plasma by some orders of magnitude (Fig. 7) [63]. Incidentally, we note in the context of Ref. [13] that numerical simulations of the avalanche have been done in an enormous number of studies, in particular, taking the self-consistent field into account (see, e.g., Refs [71, 54]). Tkachev and Yakovlenko [13] have thus obtained nothing new concerning the avalanche structure.

Kostyrya et al. [8] estimate the overlapping time of the avalanches to be $\lambda_{\rm ion}(\varepsilon)/v_{\rm d}$, but this is merely the time required for an electron drifting at the speed $v_{\rm d}$ to traverse the distance equal to the RE free path between two ionization events, $\lambda_{\rm ion}(\varepsilon)$; this time is not related in any way to the avalanches. To see whether the avalanches overlap, $\lambda_{\rm ion}(\varepsilon)$ should be compared with the path $z_{\rm cr}$ traversed by the RE-initiated avalanche until the Raether avalanche – streamer transition occurs [24, 53, 56]: $z_{\rm cr}(E)/\lambda_{\rm ion}(\varepsilon) = \sigma_{\rm ion}(\varepsilon) N z_{\rm cr}(E) \gg 1$. More precisely, the volume $\sim \pi r_{\rm av}^2 z_{\rm cr}$ (where $r_{\rm av}$ is the avalanche radius) must contain a large number, $N_{\rm sec}$, of secondary electrons due to ionization by an RE flux of $N_{\rm e}/S$ [18, 19]:

$$N_{\rm sec} \approx \frac{N_{\rm e}}{S}\, \sigma_{\rm ion}(\varepsilon) N \pi r_{\rm av}^2 z_{\rm cr} \gg 1\,. \qquad (40)$$

In this case, the avalanches do not overlap, the contracted channel does not develop, and the discharge acquires a diffuse appearance. Tarasenko et al. [3] estimate $z_{\rm cr}$, but it is not known for what purpose. Is this done to show that $z_{\rm cr} \ll d$ at high overvoltages? But this is generally known [18, 19, 54]. Moreover, the authors of Refs [3,12] ignore the fact that the mechanism of electron acceleration at the front of the developing near-cathode plasmoids has already been understood as a *self-consistent process* [18, 19] and present some elements of a very simple mechanism lacking self-consistency, as was done in early studies [29, 46]. The anomalous-energy electrons (AEEs, $\varepsilon > eU_{\rm max}$, where $U_{\rm max}$ is the maximum voltage [18, 19]) are contrasted to the electrons "...in a mean-energy beam..." [3]. In an external field of strength $eE < F_{\rm max}$, however, no mechanism of RE generation *at the streamer-channel front* other than that responsible for AEE generation can operate: since Raether's time, it has been known as the *self-consistent process of polarizational self-acceleration* [18, 19]. It is another thing that the RE energy must not exceed

$eU_{\max}$, as was observed in discharges in air at $P = 1$ atm under the action of microsecond voltage pulses [18, 19].

In the opinion of Kostyrya et al. [8], "...the direction... that the field has near the apex (*of the plasma 'bulge' at the cathode* — L B) results in a nearly isotropic emission of electrons... . However, the field turns the electrons toward the anode." It is unclear why a strongly nonuniform field near the apex of the 'bulge,' which decreases with the increase in the angle measured from the symmetry axis, gives rise to isotropic emission, but the same field "...turns the electrons toward the anode... ." It is also unclear how the electrons can be created "...with *chaotically directed* velocities" [10]. The effect of nonuniformities in the field on the runaway of electrons near the front of the avalanches (streamers) was studied by Kunhardt and Byszewski [46] (see also Refs [16, 18]). Its essence is as follows [19]. The strength of the space-charge field $\mathbf{E}_{\mathrm{p}}(\mathbf{r}, t)$ at the avalanche (streamer) front is a decreasing function of the angle ψ between the vectors of the external-field strength $\mathbf{E}_0$ and the space-charge field $\mathbf{E}_{\mathrm{p}}(\mathbf{r}, t)$. For illustration, let

$$\mathbf{E}_{\mathrm{p}}(\mathbf{r}, t) = \mathbf{E}_{\mathrm{p}}(x_a, t) \cos\psi \,, \tag{41}$$

where $\cos\psi = (\mathbf{E}_0, \mathbf{E}_{\mathrm{p}})/E_0 E_{\mathrm{p}}$ and $x_a(t)$ is the intersection point of the avalanche front and its symmetry axis. Because $\mathbf{E}_0 \uparrow\uparrow \mathbf{E}_{\mathrm{p}}(x_a, t)$, the self-consistent field is maximum, $E_{\mathrm{f\,max}} = E_0 + E_{\mathrm{p}}(x_a, t)$, at the point x_a. The streamer-like development pattern of ionization persists until the electrons at small ψ reach energy values that exceed 1 keV with certainty, in which case the ionization frequency decreases slowly. At later stages, the breakdown develops in the form of a channel with a 'corona' at large ψ angles [18, 19]. The runaway threshold $\varepsilon_{\mathrm{th}}$ is minimal at the point x_a. The runaway criterion gradually 'extends' along the entire frontal surface. As the point is displaced from the avalanche axis to large ψ angles, the local number of REs decreases. Actually, the electrons accelerate as they leave some volume ('injection zone' [46]) rather than receding from the frontal surface. As a result, an 'injection cone' of ψ angles develops such that the maximum number of REs is near its symmetry axis [46]. The accelerated electrons run away from the moving frontal region where the field strength is maximum and then relax to the local energy $\varepsilon(E_0)$ determined by the external field E_0: they become 'trapped' [46] at various distances from the front, and these distances are larger for smaller ψ angles.

In the opinion of Kostyrya et al. [8], "...the sources of numerous overlapping trajectories are separate plasmoids at the cathode"; however, our studies demonstrated the development of a volumetric discharge from one plasmoid at the cathode, as can be seen from Fig. 5.

The meaning of the estimate obtained in Ref. [8] for the electron density n_{e} in the plasma at which the screening length for the external field l_{sh} is equal to d remains unclear. A timespan of $d/v_{\mathrm{d}} \approx 18$ ns is needed to achieve the equality $l_{\mathrm{sh}} = d$; it is much longer than the duration of the discharge, and hence no screening occurs. In contrast, the estimate

$$\frac{E}{l_{\mathrm{sh}}} \approx \frac{e n_{\mathrm{p}}}{\varepsilon_0} \approx \frac{e N_{\mathrm{e}} \sigma_{\mathrm{ion}}(\varepsilon) N}{S \varepsilon_0} \frac{\varepsilon_{\mathrm{sec}}}{\varepsilon_{\mathrm{in}}} \approx \frac{I_{\mathrm{RE}} \Delta t_{\mathrm{RE}} \sigma_{\mathrm{ion}}(\varepsilon) N}{S \varepsilon_0} \frac{\varepsilon_{\mathrm{sec}}}{\varepsilon_{\mathrm{in}}} \tag{42}$$

of the screening of a field $E \sim 100$ kV cm^{-1} due to the plasma produced in air [3, 8] by an RE pulse with a current $I_{\mathrm{RE}} \sim 100$ A and duration $\Delta t_{\mathrm{RE}} \sim 1$ ns [8] or with a current $I_{\mathrm{RE}} \sim 30$ A and duration $\Delta t_{\mathrm{RE}} \sim 0.4$ ns [3] yields $l_{\mathrm{sh}} \approx 0.6 \times 10^{-3}$ cm or $l_{\mathrm{sh}} \approx 5 \times 10^{-3}$ cm, respectively, if the RE energy is $\varepsilon \sim 10$ keV, $\sigma_{\mathrm{ion}}(\varepsilon) \sim 10^{-17}$ cm^2, the ratio of the mean energy of the secondary electrons to the 'ionization cost' is $\varepsilon_{\mathrm{sec}}/\varepsilon_{\mathrm{in}} \approx 5$ [18, 19], and the discharge cross section is $S \sim 10$ cm^2 [5]. Because $l_{\mathrm{sh}} \ll d$ and the characteristic screening time is $t_{\mathrm{sh}} = l_{\mathrm{sh}}/v_{\mathrm{d}} \sim 0.02-0.2$ ns for the drift speed $v_{\mathrm{d}} \approx 3 \times 10^7$ cm s^{-1} [54, 65], the plasma produced *solely* due to RE-pulse preionization shields the field, and hence avalanches do not develop. The bulk charge forms easily and simply! Because $v_{\mathrm{d}} = \mu\,(E/P)$, where μ is the mobility of electrons, the shielding time t_{sh}, according to Eqn (42), is virtually independent of E/P but is determined by the fundamental parameters of the gas and by the parameters of the REs.

The above estimate of the screening raises serious doubts about the reality of the RE parameters obtained for the so-called avalanche-electron-beam-initiated volumetric discharge (AEBIVD) [3, 8], because the RE generation is restricted to the time of field screening by the plasma produced by the REs themselves, which is much shorter than Δt_{RE} in Ref. [8].

In the VNIIEF experiments [18, 19] with air at $P = 1$ atm, only REs of anomalous energies were recorded; their number N_{e} was a few orders of magnitude smaller, and therefore preionization did not shield the field, avalanches could develop and, overlapping one another before transforming into streamers, could form a volumetric discharge. This was directly noted in Refs [18, 19], although estimates were not given: indeed, according to Eqns (40) and (42), $N_{\mathrm{sec}} \geqslant 20-200$ and $l_{\mathrm{sh}} \simeq 1-10$ cm $\gtrsim d$ for the AEE number $N_{\mathrm{e}} = 10^9$, air pressure $P = 1$ atm, $d = 1$ cm, $S = 1-10$ cm^2, $E = 100$ kV cm^{-1}, $\varepsilon(\mathrm{AEE}) = 300$ keV, $\sigma_{\mathrm{ion}}(\varepsilon) \geqslant 5 \times 10^{-19}$ cm^2, and $r_{\mathrm{av}} \approx 1/2\alpha_{\mathrm{T}}$ [23]. The degree of plasma ionization by the AEE pulse is

$$i = \frac{n_{\mathrm{p}}}{N} \approx \frac{N_{\mathrm{e}}\,\sigma_{\mathrm{ion}}(\varepsilon)}{S} \frac{\varepsilon_{\mathrm{sec}}}{\varepsilon_{\mathrm{in}}} \sim 10^{-8} - 10^{-9} \,.$$

In the region of the diffuse glow of volumetric discharge, only emission in nitrogen molecular bands was recorded, while no ion-line emission was detected. In view of the instrumental sensitivity, this last fact implies that $i < 10^{-5}$ [16–19], which is much larger than the above estimate for i. The ionization degree 10^{-5} corresponds to $l_{\mathrm{sh}} \approx 10^{-4}$ cm $\ll d$.

The above estimates were based on experimental data without using the doubtful hypothesis that the gas was preionized by electrons that had been accelerated at the front of the near-cathode channel to energies of 1–4 keV and then detached from this front.

Possibly, it is the termination of the avalanche enhancement due to the field shielding that the authors of Refs [3, 9] implied, claiming that "...the stabilization of the discharge-current amplitude at high fields in the gap... could be due to an increase in the energy losses of secondary electrons as they pass through the plasma produced by the developing discharge and also due to the recombination process." If so, which secondary electrons are meant? Does this statement imply that the current did not vary over several nanoseconds when the voltage remained, on average, constant [3, 4, 9, 12] (Fig. 8), because E is small in the plasma and electrons do not multiply? Recombination is efficient if $n_{\mathrm{p}}\beta\Delta t \sim 1$, which is satisfied in nitrogen (with the recombination coefficient $\beta \approx 10^{-7}$ cm^3 s^{-1} [64]) for $\Delta t \sim 1$ ns and for a plasma of an unlikely density $n_{\mathrm{p}} \sim 10^{16}$ cm^{-3}: according to Eqn (42), the

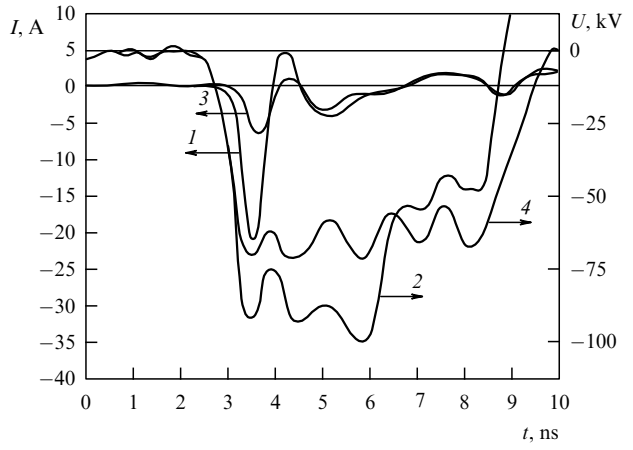

Figure 8. Oscillograms of RE-current pulses behind Al–Be foil of thickness 40 μm (*1*, *3*) and oscillograms of voltage (*2*, *4*): air at $P = 1$ atm, $d = 16$ mm. The open-circuit voltage is $U_{\rm idle} = 260$ kV (oscillograms *1*, *2*) or 155 kV (oscillograms *3*, *4*) [4, 9, 12].

concentration $n_{\rm p} \sim 5 \times (10^{11} - 10^{12})$ cm^{-3} produced by an RE pulse is sufficient to shield the field. In Refs [3, 4, 9, 12], the voltage remained high over 5–6 ns. Therefore, the field strength E in the near-electrode regions was more than sufficient to accelerate electrons even in a uniform field, although the field was shielded away from the electrodes. Thus, the RE current was nevertheless much smaller, as in the VNIIEF experiments, and, within some time interval after the termination of the RE pulse, avalanches developed and formed a volumetric charge, the current increased dramatically, the voltage decreased consistently, etc. — as usual [65].

Kostyrya et al. [8], interpreting the results of experiments with *argon, xenon, and air* in a *strongly nonuniform* field, also make estimates for *helium* in a *uniform* field. However, the field enhanced near the cathode due to the geometry of the latter substantially facilitates the acceleration of electrons compared to the case of a uniform field. There is no need to take the enhancement of the field by the space charge of plasmoids into account. Moreover, plasmoids at the edge of the cathode (foil, $h = 50$ μm [8]), most likely, reduced the field because their size was $\gg 50$ μm: we recall the effect of polarity in corona discharges due to the shielding of the pointed cathode by a positive space charge. At $U = 100$ kV and the formfactor $\Lambda \sim 10$, the field strength at the edge $\sim U/\Lambda h \sim 2 \times 10^6$ V cm^{-1} was sufficient for an intense FE and explosive electron emission (EEE) in the nanosecond range, in view of the field enhancement by microscopic irregularities [65–67]. Because the current of accelerated electrons at pressures below 0.01 Torr constituted the entire current in the diode, it cannot be ruled out that REs at $P = 1$ atm are simply a fraction of the emission electrons that statistically reach the anode. It is claimed in Refs [3, 7] that an RE beam was obtained "...at small values of the parameter $E/P \sim 0.1$ kV cm^{-1} Torr^{-1}, which were well below the critical ones for the runaway-electron effect [65]." But E/P in the near-cathode region is at least one order of magnitude larger. For the conditions of Fig. 1 in Ref. [3], $E/P \approx 80$ kV cm^{-1} Torr^{-1} at the voltage-pulse maximum. Then, $v_{\rm d} \approx 3 \times 10^7$ cm s^{-1} [24, 54], $\alpha_{\rm T} \approx 160$ cm^{-1} [24, 54], and $\alpha_{\rm T} v_{\rm d} \Delta t \approx 5$ for $\Delta t = 1$ ns. This means that the avalanche in the mean field cannot reach the critical size within ~ 1 ns.

The authors of Ref. [3] write that '...the field increases... near the cathode," but they do not take this fact (which is important under the conditions of their experiments) into account. Actually, the RE beam forms at the front of the near-cathode plasmoid in a strong field. According to our calculations, even in a weakly nonuniform external field (with the working surface of the smooth cathode having the curvature radius $r_{\rm cath} = 6$ mm), away from the cathode, avalanches did not develop almost at all if $U_{\rm max} = 240$ kV and $d = 2$ cm within ~ 0.5 ns, which is equal to the rise time of the discharge current pulse [18, p. 69; 19, p. 229]; they did not develop at all at $U_{\rm max} \approx 100$ kV and $d = 1.5$ cm within ~ 1 ns [16], but developed intensely near the cathode. To the authors of Ref. [3], however, "...it is clear that the number of electrons in the beam should be substantially smaller than the number of electrons in the avalanches, and the AEE number should be substantially less than the number of electrons in a mean-energy beam." In this context, we present the summary from Refs [16, 19]. In any cross section of the gap outside the near-cathode plasma region, the conduction current of the bulk charge consists of three components,

$$I(x) = I_{\rm RE}(x) + I_{\rm sec}(x) + I_{\rm av}(x)\,, \tag{43}$$

where the electron-avalanche current $I_{\rm av}(x)$ has a maximum, whose position in a particular pulse displaces toward the anode as the voltage $U(t)$ grows. In a cross section in the vicinity of the near-cathode plasma region, the current consists of the RE current $I_{\rm RE}$ in dark space and the coronal-plasma current. All three components are present near the anode, but the avalanche current $I_{\rm av}(x)$ is zero in strongly nonuniform configurations. Therefore, the conductivity of nanosecond volumetric discharges at high overvoltages is determined by the 'mean-energy' REs [3] [$I_{\rm RE}(x)$ current] and secondary electrons [$I_{\rm sec}(x)$ current] along with the gas preionization by the AEE pulse, with subsequent avalanche-like multiplication in both cases. The contribution of one mechanism or another depends on the particular conditions — first of all, on the field geometry and the voltage achieved at the gap.

4. Record runaway-electron currents

4.1 The efficiency of runaway-electron generation

Because the studies analyzed here report unusually large amplitudes of the RE currents, Alekseev et al. [7] estimated the fractions of REs. In their opinion, for $\sim 1\%$ of electrons to run away, an electron must acquire, over its free path, an energy equal to the double ionization energy $\varepsilon_{\rm ion}$ ($2\varepsilon_{\rm ion}$ is the potential energy of an atomic electron in the field of an ion [24]), and hence the RE beam is formed provided the condition

$$U > U_{\rm cr1} \equiv N d \sigma_0 2\varepsilon_{\rm ion} \tag{44}$$

is satisfied; here, σ_0 is the ionization cross section at $\varepsilon = 2\varepsilon_{\rm ion}$. This inequality can be equivalently written as

$$\frac{E}{P} > N_1 \sigma_0 2\varepsilon_{\rm ion}\,, \tag{45}$$

where $N_1 = 3.54 \times 10^{16}$ cm^{-3} Torr^{-1}. For example, the right-hand side of formula (45) for nitrogen is 60 eV cm^{-1} Torr^{-1}, which only slightly exceeds the quantity $E/P \approx 45$ V cm^{-1} Torr^{-1} needed for a static breakdown and is

much less than the maximum electron inelastic losses, 356 V cm^{-1} Torr^{-1} [18, 19 29, 30]. Not only can the RE fraction not be $\sim 1\%$ at 60 V cm^{-1} Torr^{-1}, but the runaway phenomenon is highly improbable. For example, in air discharges under the action of microsecond voltage pulses with $U_{\max} = 70$ kV, in configurations with $r_{\text{cath}} = 3$ mm and $d = 2$ cm, RE fractions as small as 5×10^{-7} were recorded [18, 19]. This is despite the fact that the reduced field strengths E/P calculated for a hyperbolic approximation of the cathode without allowances for space charges were 200 and 70 V cm^{-1} Torr^{-1} at the cathode apex and at the distance r_{cath} from it, respectively, over ~ 100 ns preceding the beginning of the voltage collapse.

We note that the concept of a 'free path' for REs is meaningful only as a formal quantity, the reciprocal of the cross section times the density of atomic particles; its physical meaning is not determinate, because the electron moves between successive collisions in a strong electric field and substantially changes its energy.

Tkachev and Yakovlenko [2] make the following very strong claim: "If an external electric field is present in the plasma, some fraction of electrons will **always** be accelerated." This only seems so! We invite the authors of Ref. [2] to estimate the time needed to accelerate at least one electron to an energy comparable with the applied voltage in air plasma at atmospheric pressure, e.g., with the ionization degree 10^{-7} and in a field of strength 1 V cm^{-1}.

"The runaway of a considerable fraction ($\sim 50\%$) occurs..." provided that [7]

$$\frac{mN}{eE}\int_0^{\sqrt{2U/m}} \sigma_{\text{ion}}(v)v\,\mathrm{d}v > 1\,; \tag{46}$$

this conditions appears fairly strange if rewritten in the form

$$eE < \int_0^{\sqrt{2meE}} \nu_{\text{ion}}(v)\,\mathrm{d}p \approx \langle \nu_{\text{ion}}\rangle\, p_{\max} = \langle \nu_{\text{ion}}\rangle \sqrt{2meU}\,. \tag{47}$$

Relation (47) does not involve the EDF and, therefore, cannot predict RE fractions.

We rewrite formula (47) in a different way, as

$$eE\langle\lambda_{\text{ion}}\rangle = \frac{eE}{N\langle\sigma_{\text{ion}}\rangle} < \int_0^{\sqrt{2eU/m}} mv\,\mathrm{d}v \approx eU\,. \tag{48}$$

Indeed, an electron in a dense gas, as it moves between successive ionization events, acquires an energy below eU, but how is this fact related to the runaway phenomenon, especially to the runaway of 50% of electrons? Inequality (48) can be represented in form (32), i.e., $\langle\lambda_{\text{ion}}\rangle < d$, which bears no relation to the gas discharge with electron-produced ionization.

Alekseev et al. [7], approximating the ionization cross sections by the expression $\sigma_{\text{ion}} = \sigma_1\varepsilon_1/\varepsilon$, derived another runaway condition from formula (46):

$$U > U_{\text{cr2}} \equiv \frac{Nd\sigma_1\varepsilon_1 \ln(eU/\varepsilon_1)}{e}\,, \tag{49}$$

where the inequality sign does not correspond to the sign in condition (46). For helium, this inequality yields

$$\frac{E_{\text{cr2}}}{P} \approx 3.4 \text{ kV cm}^{-1}\text{ Torr}^{-1}\,,$$

which is overstated by more than an order of magnitude. Numerical simulations based on a particle technique [7] yielded an RE fraction of 50% relative to the total number of electrons at the anode for $E/P = 5$ kV cm^{-1} Torr^{-1}. This E/P value appears to be strongly overstated, because the maximum energy loss in helium is 67 eV cm^{-1} Torr^{-1} [30]. In particular, according to Monte Carlo calculations without renormalizations, the fraction of REs ($\varepsilon \geqslant 4$ keV) in helium at $E = 100$ and 200 kV cm^{-1} and $P = 1$ atm reaches unity within ~ 0.15 and 0.25 ns, respectively [50].

Condition (49) can be rewritten as

$$eE > \frac{\varepsilon \ln(eU/\varepsilon_1)}{\lambda_{\text{ion}}(\varepsilon)}\,, \tag{50}$$

wherefrom the runaway threshold ε_{th}, rather than the RE fraction, could be crudely estimated; however, what is wrong with directly using the functions of energy losses per unit path [18, 19, 30] (see Fig. 1), as was already done in Ref. [29], or at least the Bethe formula, which includes not only ionization but all inelastic interactions?

It turns out that "...for a cathode... of foil, the beam current grew as the pressure increased from 150 to 160 Torr, the parameter E/P simultaneously decreasing from 0.5 to 0.1 V cm^{-1} Torr^{-1}" [7]. This is a very strange result. The authors suggest it be compared with the results of numerical simulations (Fig. 1 in Ref. [7]; Fig. 15 in Ref. [12]) according to which the RE current increases with the decrease in E/P. In fact, however, Fig. 1 in Ref. [7] and Fig. 15 in Ref. [12] imply that the RE fraction relative to the total number of electrons at the anode, $\eta = N_{\text{RE}}/N_{\text{tot, anod}}$, decreases with increasing pressure. According to the above-mentioned figures, as the pressure grows, increases are observed in the "...currents of REs initiated by one electron, $\eta(j/j_0)$, where j_0 is the current from the cathode and j is the current to the anode." However, can a dimensionless quantity be a current? Does it grow with the increase in P although E/P decreases? If it is written as

$$\eta\,\frac{j}{j_0} \approx \frac{N_{\text{RE}}}{N_{\text{tot, anod}}}\,\frac{N_{\text{tot, anod}}/\Delta t}{N_{\text{tot, cath}}/\Delta t} = \frac{N_{\text{RE}}}{N_{\text{tot, cath}}}\,,$$

where $N_{\text{tot, cath}}$ is the total number of electrons emitted by the cathode within a time Δt, it becomes clear that this quantity decreases with increasing P, because the RE fraction relative to the number of emitted electrons naturally decreases with the increase in P.

In the section 'Efficiency curves for the electron-beam formation,' Tarasenko and Yakovlenko [12] develop the idea that the right-hand side of Eqn (31) is arbitrary and, "...assuming, for example, that $\alpha_{\text{T}}d = A = \text{const}$...," obtain "...a simple relationship..." between U_{cr} and the quantity U'_{cr} (prime) that characterizes the efficiency: $U_{\text{cr}}(Pd) = U'_{\text{cr}}(Pd/A)/A$. Based on this relationship, they plot 'constant-efficiency' curves for REs with $\varepsilon \geqslant 2U/3$. In helium, for example, at $Pd = 20$ Torr cm and $U = 100$ kV, an efficiency of $\eta = 80\%$ can be achieved (Figs 7 and 15 in [12]). In the VNIIEF experiments, however, the maximum efficiency of RE generation did not exceed 10% ($P = 22$ Torr, $d = 1$ cm, $U_{\text{idle}} = 300$ kV, $\varepsilon \geqslant 50-60$ keV), and it was achieved only after making incisions in the working surface of the cathode to enhance the emission.

4.2 Formation of a runaway-electron beam

To avoid a breakdown, the voltage should be increased so rapidly that near-cathode plasmoids have no time to develop;

this, however, is impossible at voltages ~ 100 kV because of the FE, which is virtually an inertialess process. As a result, an RE pulse preionizing the gap is generated self-consistently with the development of the discharge. Tarasenko et al. [4] claim that "...to observe the upper branch in a dense gas, the voltage must be raised to a hundred kilovolts within a fraction of a nanosecond." Actually, only the Z-shaped segment of the $U_{\rm br}(Pd)$ curve is realistic for a dc breakdown at small Pd values, and the extension of the 'upper curve' to the range of dozens, hundreds, or even thousands of Torr cm [12] results from the neglect of many elementary processes that involve atomic particles, FEE, EEE, and the fields of space charges. The calculation of $U_{\rm br}(Pd)$ at large Pd based on the Townsend mechanism, with insignificant space-charge effects, is totally incorrect, all the more so in the case of dense gases and high overvoltages. The experiments described in Refs [4, 12] were carried out in strongly nonuniform fields, while the 'upper curve' of $U_{\rm br}(Pd)$ was calculated for a uniform field, which the authors themselves regard as "...a very crude approximation..."; if so, what is the information carried by the point in Fig. 5 in Ref. [4] and Fig. 7 in Ref. [12] (see our Fig. 2) that corresponds to "...the maximum voltage in the presented experiments ([4] — L B) for $U_{\max} = 200$ kV, $P = 1$ atm, and $d = 28$ mm..."?

Based on the ionization-wave speed, the authors estimate the duration of the RE pulse to be

$$\frac{0.7\ \text{mm}}{10^{10}\ \text{cm s}^{-1}} \sim 0.01\ \text{ns}\,,$$

where 0.7 mm is the distance between the plasma front and the anode at the time when the RE beam forms, in the authors' opinion. However, for RE currents $I_{\rm RE} = 40$ A [4], 35 A, and 75 A [12], such a duration corresponds to the electron number $N_{\rm e} \simeq (2.5-5) \times 10^9$, which is close to the AEE number recorded during the VNIIEF experiments in air at $P = 1$ atm [18, 19]. Is this compatible with the constantly emphasized claim that an electron runaway process some orders of magnitude more efficient was realized in the experiments described in Refs [3–9]? In the authors' opinion, the total 40 A RE current of duration $\sim 0.3-1$ ns resulted from the formation of several, more precisely, $\sim (0.3-1)/0.01 = 30-100$, channels that approached the anode. If, however, only one channel reaches the anode, this suffices for the gap to be short-circuited, after which the voltage collapses and the other channels do not reach the anode. Alternatively, if the channels develop nearly simultaneously, the estimate of 0.01 ns remains valid, although it was obtained without considering the fact that the channels develop consistently with the acceleration of electrons at their fronts. It is likely that the short-circuiting by one channel in the VNIIEF experiments at pressures close to atmospheric pressure and in the configuration with emission-enhancing dents on the anode surface prevented a substantial enhancement of the efficiency of RE generation [1, 19].

Alekseev et al. [7] "...observed four characteristic regimes..." of discharge in helium.

(1) $P = 0.1-1$ Torr, $E/P > 70$ kV cm^{-1} Torr^{-1}. This is 'the known regime of acceleration' with an RE current exceeding 1 A [7].

(2) $P = 1-10$ Torr, $E/P > 7$ kV cm^{-1} Torr^{-1}. In this regime, an RE beam was recorded in only some pulses and only for configurations with strongly nonuniform field (the foil cathode) [7].

(3) $P = 10-40$ Torr, $E/P > 1.5$ kV cm^{-1} Torr^{-1}. An RE current of up to 30% of the RE current in the first regime was recorded in the configuration with the strongly nonuniform field [7].

(4) $P > 100$ Torr, $E/P < 0.7$ kV cm^{-1} Torr^{-1}.

The experiments described in Refs [3–9, 12] were carried out with two cathode configurations — a tube cathode of thin (50 μm) metallic foil (or three coaxial tubes as a variant) and a convex pellet-shaped graphite cathode. In the context of understanding the RE effect in dense gases, the second configuration, in which the field is nearly uniform, is of interest. In the case of the foil cathode, increases in the pressure implied in essence a deterioration of the efficiency of the EEE operation regime of the accelerating vacuum tube [66, 67]. RE generation in dense gases with such a configuration is interesting in the context of engineering applications, but the generation mechanism is trivial. In the configuration with the pellet-shaped cathode, REs were not present in the first three regimes, but a 140 A RE current with the maximum electron energy 150 keV was recorded in the fourth regime [7]. In the former regimes (with small Pd values), the breakdown from the graphite cathode could develop over a 'long path,' i.e., to the chamber case; most likely, REs were generated but not recorded. The instability of the recording of REs in the second regime, in the configuration with the strongly nonuniform field, is evidence for 'long-path' breakdowns. According to the notion of the authors of Ref. [3], the development of an avalanche-electron-beam-initiated volumetric discharge with the ultrashort-avalanche-electron-beam effect requires 10^6 initiating electrons. In the nanosecond range, the breakdown is initiated by FE [18, 19, 54, 66]; while such intense (10^6) FE from the tube cathode within the time interval ~ 1 ns is possible due to microasperities on the cathode, a smooth cathode can actually ensure the emission of only individual electrons — most likely when the voltage over the gap reaches its maximum value. A fortiori, FE is impossible in a 'prepulse' [3].

References [6, 9, 12] present the results of measuring the energy distribution of REs using the technique of absorption in metallic foils. The computational procedure of this technique allows obtaining only a general idea of the electron energy. In the high-energy range, because of straggling, electrons can be found where they are not actually present. This calls into question the report of the authors of Refs [6, 7] that they observed AEEs. These are not detectable in the energy distributions of REs presented in Refs [6, 9] either. In the VNIIEF experiments, the energy distribution of REs was measured using the magnetic spectrometry technique [18, 19] (Fig. 9). Applying this technique to measuring the spectra of RE fluxes that are two orders of magnitude more intense than in the VNIIEF experiments would pose no difficulties.

"In the beam-formation mechanism...," Tarasenko and Yakovlenko [12] identify "...two phases, ... the phase of formation of the volumetric discharge and the subsequent... phase of beam generation.... At the front of the voltage pulse, plasma is generated that short-circuits the gap within a time of the order of a nanosecond.... Such a plasma cathode approaching the anode effectively reduces... d. As a result, conditions similar to those for the runaway curve are realized..." [12, p. 964]. "After reaching the maximum beam current, the conditions for the formation of an electron beam in the gas diode are violated very quickly, although no substantial voltage variations occur" [12, p. 963]. If the

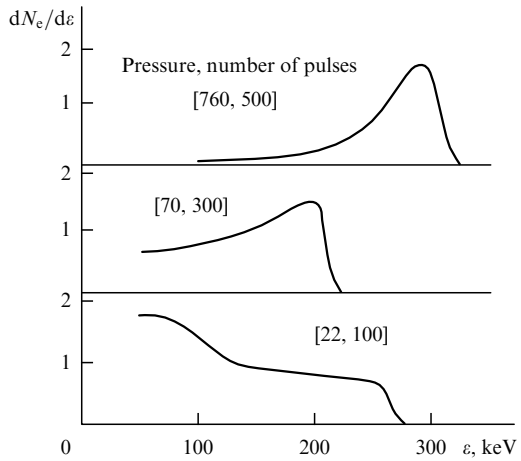

Figure 9. Energy distribution of REs in air discharges [18, 19]. The open-circuit voltage is $U_{\rm idle} = 270$ kV, $\tau_{\rm idle} < 0.5$ ns, $d = 20$ mm, $r_{\rm cath} = 200$ μm; a plane anode of aluminum foil with the thickness 8 μm is used.

voltage does not collapse after the plasma short-circuits the gap (which is the case, as can be judged by the oscillograms given in Refs [3, 4, 9, 12] — see Fig. 8), then the field strength in the near-electrode layers is so high that the termination of RE generation is completely impossible.

The REs are generated at the beginning of the voltage pulse; it is reasonable to ask whether the voltage can be quasi-constant, as the oscillograms given in Refs [3, 4, 9, 12] (illustrated by our Fig. 8) indicate. Corresponding oscillograms of the total discharge current are not presented in any of Refs [3–9, 12]. For this reason, we estimate the conduction current $I_{\rm disch}$ that results from preionization by the RE pulse with the current $I_{\rm RE}$:

$$I_{\rm disch} \approx e n_{\rm p} v_{\rm d} S \approx e\,\frac{I_{\rm RE}\Delta t_{\rm RE}\,\sigma_{\rm ion}(\varepsilon) N}{eS}\,\frac{\varepsilon_{\rm sec}}{\varepsilon_{\rm in}}\,v_{\rm d} S$$

$$\approx I_{\rm RE}\Delta t_{\rm RE}\,\sigma_{\rm ion}(\varepsilon)\,N\,\frac{\varepsilon_{\rm sec}}{\varepsilon_{\rm in}}\,v_{\rm d}\,. \qquad (51)$$

According to Ref. [7], $I_{\rm RE} = 140$ A in helium if the configuration with the graphite electrode is used. Let $P = 1$ atm. We set $\Delta t_{\rm RE} \sim 0.5$ ns, $\sigma_{\rm ion}(\varepsilon) \sim 10^{-17}$ cm^2, $v_{\rm d} = 5 \times 10^7$ cm s^{-1} [1], and $\varepsilon_{\rm sec}/\varepsilon_{\rm in} \approx 3$ to obtain $I_{\rm disch} \approx 3$ kA, which is close to the current $I_{\rm disch,\,exp} \approx 5-6$ kA measured at P from 20 Torr to 1 atm for both electrode configurations (Fig. 1 in Ref. [7], Fig. 13 in Ref. [12]). The estimate of $I_{\rm disch}$ is consistent with to the estimate of the screening of field (42): if it is overstated, no screening occurs, and an avalanche-like preionization growth $\exp(\alpha_{\rm T} v_{\rm d}\Delta t)$ occurs. Within $\Delta t \sim 0.1$ ns, for $E/P = 100$ V cm^{-1} Torr^{-1}, we have $\alpha_{\rm T} v_{\rm d}\Delta t \approx 3.5$, which is sufficient for $I_{\rm disch,\,exp} \approx 5-6$ kA. A current of 5–6 kA was achieved at the front of the open-circuit voltage pulse near the voltage maximum in the working regime. Therefore, the voltage drop at the internal resistance of the generator equal to $R = 30\ \Omega$ [7, 12] was $RI_{\rm disch,\,exp} \approx 150-180$ kV. How could the voltage over the gas-discharge gap be quasi-constant ($\sim 60-100$ kV) or even grow over several nanoseconds? Either the voltage was improperly measured or, more likely, the RE current $I_{\rm RE}$ was strongly over-estimated.

Because "...the beam is generated at the voltage-pulse front..." [3, 4, 12], does the measured RE current not include the eddy current (which could account for the strong disagreement with our results)? In particular, the charging current of the interelectrode capacitance is

$$C\,\frac{{\rm d}U}{{\rm d}t} \approx C\,\frac{U_{\rm max}}{\tau_U} \approx 10-300\ {\rm A}$$

for $C = 0.1-1$ pF, $U_{\rm max} = 100$ kV, and the pulse-rise time $\tau_U = 0.3-1$ ns. True, the authors immediately note that "...the beam is recorded ~ 0.5 ns after the voltage pulse is applied..." [3], but how could the beam be generated at the front in view of $\tau_U = 0.3$ ns?

It is not clear why $I_{\rm RE}$ as a function of the open-circuit voltage $U_{\rm idle}$ of the generator has a maximum, although the maximum values of the current $I_{\rm disch}$ and voltage U over the gap grow monotonically and slowly with $U_{\rm idle}$ [4, 9, 12]. The point is that all processes in a gas, including RE generation, are directly and self-consistently related to $I_{\rm disch}$ and $U_{\rm max}$, but only indirectly to $U_{\rm idle}$. In the VNIIEF experiments, the number of anomalous-energy REs was virtually independent of $I_{\rm disch}$ and $U_{\rm max}$ [18, 19]. In Refs [4, 9, 12], as $U_{\rm idle}$ varied from 160 to 260 kV, the current $I_{\rm RE}$ grew from 20 to 28 A, i.e., it varied rather weakly; thus, within the conceivable measurement accuracy and in view of the varying duration $\Delta t_{\rm RE}$, the RE number could be considered constant. For $U_{\rm idle} = 340$ kV, however, $I_{\rm RE} \approx 5$ A $\ll 28$ A, which cannot be accounted for by measurement errors.

In Refs [4, 9, 12], the results of experiments in strongly nonuniform fields, where the strength of the external field near the cathode already guarantees the generation of REs, are interpreted in terms of the same idea that the electron runaway or, more precisely, the satisfaction of criterion (31) requires that the plasma approach the anode to a distance $\ll d$. The estimates based on field amplification at the front of the intergrowing channel are themselves inconsistent: in any gas at $E/P \sim 1000$ V cm^{-1} Torr^{-1}, electrons run away at the front of a channel of length $a = 8$ mm, but the authors use the concept of drift, estimating "...the propagation speed of ionization..." to be $\sim 3 \times 10^9$ cm s^{-1}; this figure corresponds to $\varepsilon \sim 2.5$ keV, which belongs to the runaway region. The idea of Tarasenko et al. [4] implies the electron-multiplication magnitude $\exp(\alpha_{\rm T} a) \sim \exp(80)$, which is unlikely even at $\alpha_{\rm T} = 100$ cm^{-1} — a value strongly under-estimated for the experimental conditions in Refs [3–9]. For example, in air, if $E/P \sim 100-1000$ V cm^{-1} Torr^{-1} and $P = 750$ Torr, then $\alpha_{\rm T} \sim 1000-8000$ cm^{-1} [24]. The contradiction can be removed by recalling that the description of dense-gas breakdown in terms of only the local Townsend coefficient $\alpha_{\rm T}$ as an EDF-averaged quantity is not adequate for very strong fields, where REs play a crucial role in the discharge dynamics. The contradiction seems to be resolved in Ref. [12] by including the preionization of the gas-discharge gap, by electrons accelerated at the front of the near-cathode channel to the energies 1–4 keV, in the formation mechanism of the volumetric discharge; however, as noted above, a new contradiction emerges.

In their idea of electron acceleration at the front of plasma 'jets' (*why not channels*? — L B) approaching the anode, the authors of Refs [4, 12] proceed from the integrated photographs of the discharge [8, 9, 12]. Why do they not assume that the 'jets' are a secondary entity with respect to RE generation? This would be more natural, because the electrons accelerate self-consistently with the development of the channels. Moreover, the channels could even form after the breakdown-formation stage, when the voltage decreases. In their attempt at explaining the generation of REs in their

experiments using the 'new runaway criterion' and 'the upper branch' of the $U_{\rm br}(Pd)$ dependence (which were obtained with the space-charge fields ignored), the above authors, making estimates, nevertheless fictitiously consider these fields using the distance of the channel front from the anode as d. However, the meaning of the $U_{\rm br}(Pd)$ curves lies in the fact that they represent a relation between the macroscopic parameters U, P, d, and, in general, also the voltage-pulse rise time τ_U [18, 19] (where, for emphasis, d is the interelectrode distance), while the microscopic processes and space-charge effects (including the distance to which the channel is propagated) are hidden in the relations $U_{\rm br}(Pd, \tau_U)$ themselves.

The RE-generation mechanism suggested in Refs [1, 2, 12] is based on two assumptions:

(1) the concept of the Townsend coefficient is valid for any E/P;

(2) at the end of the bulk-discharge formation stage, after the plasma front has approached the anode to a distance $\sim \alpha_{\rm T}^{-1}$, the entire applied voltage U drops in a near-anode layer of thickness $\alpha_{\rm T}^{-1}$, where the electrons acquire the energy eU.

We show that these assumptions, taken together with the results of measurements and of Tarasenko and Yakovlenko's calculations [12], are inherently inconsistent. We make estimates for the conditions under which the oscillograms of the RE current and voltage [4, 9, 12] shown in Fig. 8 were obtained: air at $P = 1$ atm and $U \sim 100$ kV. The equality between the plasma conduction current $I_{\rm cond} = en_{\rm e}v_{\rm d}S$ and the eddy current

$$I_{\rm ed} = \varepsilon_0 \frac{\partial E_{\rm anod}}{\partial t} S = \varepsilon_0 U \frac{1}{x^2} \frac{{\rm d}x}{{\rm d}t} S \approx \varepsilon_0 U \alpha_{\rm T}^2 u S$$

in a layer of thickness $\sim \alpha_{\rm T}^{-1}$ ahead of the plasma front traveling toward the anode at the speed u implies that

$$n_{\rm e} \approx \frac{\varepsilon_0 U}{e} \frac{u}{v_{\rm d}} \left(\frac{\alpha_{\rm T}}{P}\right)^2 P^2 . \qquad (52)$$

According to assumption (2) above, $E_{\rm anod} = U/\alpha_{\rm T}^{-1}$ in the near-anode layer, wherefrom the relation $\alpha_{\rm T}/P = E_{\rm anod}/UP$ follows; it is consistent with the dependence of $\alpha_{\rm T}/P$ on E/P shown in Fig. 3a in Ref. [12] provided that $E/P \approx 30$ kV cm^{-1} Torr^{-1} and $\alpha_{\rm T}/P \approx 0.3$ cm^{-1} Torr^{-1} (Fig. 10). We now make two estimates.

(1) The electron drift velocity $v_{\rm d}$ in a plasma with a shielded field is much less than the velocity in an unperturbed external field, $v_{\rm d}(U/Pd) \sim 10^7$ cm s^{-1} [54], and decreases in the process of plasma polarization. According to Fig. 3b in Ref. [12], $u \sim 10^{10}$ cm s^{-1} for $E/P \approx 30$ kV cm^{-1} Torr^{-1}. Thus, we obtain the estimate $n_{\rm e} \gg 10^{18}$ cm^{-3} for the electron density in the plasma, which is totally unrealistic because the molecular number density is $N = 2.7 \times 10^{19}$ cm^{-3}. The estimated $n_{\rm e}$ is in sharp disagreement with the plasma ionization degree $i < 10^{-5}$ for a volumetric discharge in air at $P = 1$ atm [17–19]. The measured total discharge current $I_{\rm disch,\,exp} \approx 6$ kA (Fig. 1 in Ref. [7], Fig. 13 in Ref. [12]) and $n_{\rm e} = 10^{18}$ cm^{-3} at $S \sim 10$ cm^2 [5] correspond to $v_{\rm d} \approx 4 \times 10^3$ cm s^{-1}.

(2) On the other hand, the current in the outer circuit must be equal to the eddy current in a layer of thickness $\sim \alpha_{\rm T}^{-1}$. The estimate $I_{\rm ed} \approx 50$ MA for $P = 1$ atm and the same S exceeds the measured discharge current by many orders of magnitude.

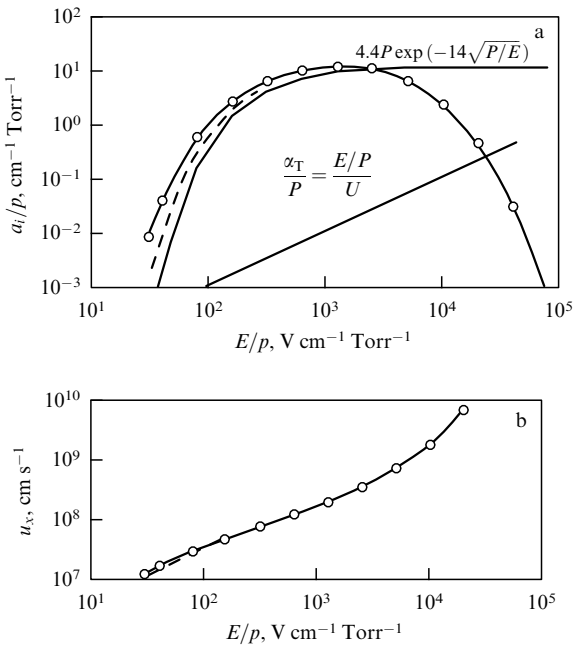

Figure 10. Dependence of (a) the ionization coefficient and (b) the directional velocity of electrons on E/P for nitrogen [12]. Circles: results of numerical computations for $P = 100$ Torr [12]. The line $\alpha_{\rm T}/P = E/UP$ is drawn for $U = 100$ kV.

Alekseev et al. [5] write that in the experiments described in Ref. [18], "...the (RE — L B) beam currents are small and, as a rule, are measured by the X-ray intensity." This is not the case: neither in the VNIIEF experiments nor in studies by other research teams (see Refs [19, 62, 65] and the references therein) were weak currents and RE-energy distributions measured using such an unreliable method as that based on the X-ray-emission parameters. In the VNIIEF experiments, the pulse duration, effective energy, and RE-energy distributions were measured directly [18, 19]. These parameters were used to calculate the X-ray spectra and radiation doses, which proved to be close to the measured ones.

5. Conclusion

In view of the aforesaid, the claims by the authors of Refs [1–12] that they have 'shown' something, 'revealed' something, etc., are groundless. The authors have not "...shown... that the critical voltage... at which the REs constitute a substantial fraction is a universal... function..." Pd; this is merely declared, and the 'universal' function $\xi(E/P)$ is itself the known function $\alpha_{\rm T}(E/P)/P$, which, according to the groundless opinion of the authors of Refs [1, 2, 12], has constantly been described by preceding generations of researchers in terms of Eqn (4) in Refs [1, 2]; as is known, this formula is valid in a restricted range of E/P values.

On p. 960 in Ref. [12], the authors write: "The departure curve $U_{\rm cr}(Pd)$ is a universal characteristic of the gas while the ignition curve $U_{\rm br}(Pd)$ depends on the model that describes the ignition of the discharge, in particular, on the properties of the electrodes." This is not correct; the $U_{\rm br}(Pd)$ curves are fundamental precisely because they take into account not only the electron-impact ionization but also numerous other processes, including processes at the electrodes. In contrast, $U_{\rm cr}(Pd)$, being a consequence of Eqn (31) with the constant on the right that is regarded by the authors of Ref. [12] as

fairly arbitrary, includes only the electron-impact ionization of the gas, without accounting for many elementary processes that involve ions, photons, and fast atoms; it also ignores the fact that the discharge develops in a self-consistent electric field and consistently with the processes at the electrodes. Thus, FE can equally well be considered fundamental, but EEE [66, 67] cannot; however, nature is such that FEE at very high current densities is replaced with EEE, which turns out to be as fundamental as FEE at low current densities. Furthermore, it is completely absurd to claim that $U_{\rm br}(Pd)$ depends on the model while $U_{\rm cr}(Pd)$ does not, being universal. Similarly, linear equation (6), from which criterion (31) and, therefore, the $U_{\rm cr}(Pd)$ curve follow, cannot be considered universal: the linear law of development is highly restricted for any process. The $U_{\rm br}(Pd)$ curves were measured in many gases with electrodes made of various materials, and the corresponding model may or may not be adequate.

The views of the authors of Refs [1, 2, 12] could be related to an 'open discharge,' i.e., to a discharge at $P \ll 1$ atm with $\alpha_{\rm T} d < 1$, but their claim that criterion (31) is a fundamental novelty is not justified in any way. The substantiation of the 'Townsend ionization mechanism' is not actually a substantiation, and the interpretation of the discharge dynamics in dense gases (up to $P = 1$ atm) at very high overvoltages in terms of this mechanism is not adequate, disagreeing with the results of a large number of experiments [18, 19, 24, 28, 53, 54, 56, 62, 65, 66]. The interpretation of experiments on REs in dense-gas discharges at high overvoltages based on the inequality $\alpha_{\rm T} d < 1$, as well as the interpretation of the results of numerical simulations, is not convincing. If Tkachev and Yakovlenko [1, 2] planned to develop a model combining the multiplication and accelerated motion of electrons, they needed to use the equation of motion and the balance equation for the number of electrons instead of relation (31), which essentially represents the absence of electron-impact ionization. As regards the statement that "...the runaway phenomenon... is not conditioned by the predominance of acceleration over collisional drag..." [1], indeed, "it has generally been assumed" [1] since the publication of papers by Wilson [20] and Eddington [21] that the runaway is equivalent to the accelerated or uniform motion of the electron with its energy conserved in the region of decreasing cross sections of elementary interactions or, more precisely, according to Ref. [48], in the energy range $\varepsilon \in [\varepsilon_{\rm th}, \varepsilon_3]$. The essence of the phenomenon is trivial, the predominance of the accelerating force over the dissipative force ('runaway from collisions' [22]), but the description of the discharge dynamics with REs involved is a complex problem even in a one-dimensional formulation; the corresponding computer simulations are extremely labor-consuming, because the kinetics of a dramatically growing number of electrons in a self-consistent field should be modeled for a wide energy range. Fairly complete models and corresponding numerical calculations are available (see, e.g., Refs [18, 19, 22, 28, 57, 62, 63, 65, 66, 68–70] and the references therein).

The electron-acceleration mechanism in dense gaseous media at the fronts of polarizing plasma channels suggested by the authors of Refs [3–8, 12] was published long ago, the interpretation of the experimental results on the basis of this mechanism in the framework of the 'nonlocal' runaway criterion (31) does not correspond to the physics of gas-discharge processes, and invoking the simplest version of polarizational acceleration is by itself a piece of retrogression. The above authors do not add any novelty to the formation mechanism of volumetric discharges in dense gases compared to the previously published studies [16–19], and they make mistakes.

Alekseev et al. [7] claim that they "have recorded electrons of anomalously high energies (in excess of eU), the observation of which was reported previously [14, 18]." Indeed, in 1974, Tarasova et al. [14] reported the generation of AEEs, based also on the absorption curves; however, Babich et al., in their review article of 1990 [18], presented the results of thorough experimental investigations: AEE spectra were measured (see Fig. 9), the instant of generation was determined, the dependences of the AEE number $N_{\rm e}$ and energy on the electric parameters of the discharge and of the high-voltage pulse generators were researched, a check experiment measuring the electron energy was done based on the retarding-potential technique, and the bremsstrahlung of the AEEs was investigated. All the results obtained were interpreted in the framework of a consistent mechanism of polarizational RE self-acceleration, which is an extrapolation of Raether's streamer mechanism to the high-overvoltage range. We note that the plasma front shape is not important for the polarizational RE self-acceleration: the front may not necessarily be a plane, as the authors of Ref. [12] desire. The formation mechanism of volumetric discharges in air at $P = 1$ atm with preionization by an AEE pulse is based on experimentally revealed facts [18, 19]:

(1) AEEs are generated in the near-cathode region during the growth of the conduction current, which is the main argument in favor of preionization by the AEE pulse;

(2) the AEE-pulse duration is $\Delta t_{\rm e} < 0.5$ ns — most likely, $\Delta t_{\rm e} \approx 0.05$ ns;

(3) AEEs are distributed over a narrow energy range, 180–320 kV for $d = 0.5$–3.5 cm;

(4) the AEE number is $N_{\rm e} \approx 10^9$.

The authors of Refs [3, 8] propose that the revealed "...effect of electron-beam formation due to avalanche multiplication be termed the UAEB effect (UAEB means 'ultrashort avalanche electron beam') and the volumetric discharge initiated by the beam, the AEBIVD discharge (AEBIVD means 'avalanche-electron-beam-initiated volumetric discharge')" [3]. However, the discharge regimes described in Refs [3–9, 11, 12] do not present any fundamental novelty compared to the discharge regimes in dense gases at high overvoltages, which have been the subject of long-standing research [18, 19]. The authors of Refs [1–9] admit arbitrariness in their claims concerning the RE fractions. They believe that "...regimes differing from those described in Refs [18, 28, 66] by some parameter values have been revealed." However, no new discharge regime is presented in Refs [3–9]! This is the same regime: an RE beam is generated at the front of the voltage pulse or, more precisely, during the current-pulse rise.

The novelty to which the authors could claim is the unusually high efficiency of RE generation: seemingly, they managed to obtain, other than AEEs, 'normal' REs with energies $\varepsilon < eU_{\rm max}$ from the discharge as well; the feasibility of the generation of such REs was directly noted in Refs [18, 19], but nobody managed to directly detect them: the energy spectra of REs in air discharges at $P = 1$ atm measured using the magnetic spectroscopy technique simply did not contain REs in the energy range $\Delta\varepsilon \approx 50$–$150$ keV [18, 19] (see Fig. 9). However, did the authors of Refs [3–9] actually manage to do this? At voltages much higher than in Refs [3–9], only AEEs were recorded in all experiments at $P = 1$ atm described in Refs [18, 19], although the layers of substance the electron

passed to reach the detector were no thicker than in Refs [3–9]. Efficient generation of RE flows starts upon reducing the pressure below 100 Torr: 'normal' REs with energies ranging from 50 to 200 keV appear in the energy spectra (see Fig. 9), and the RE-pulse duration increases with the decrease in pressure [18, 19], approaching the duration of the high-voltage pulse.

The RE beam was recorded at the beginning of the voltage pulse, while the voltage itself does not virtually decrease or even increases over several nanoseconds [3, 4, 9]. If the field in the volume is shielded by the plasma, its strength in the near-electrode regions should remain enormously high as long as the high voltage is applied. Why is the RE-pulse duration much shorter than the duration of the voltage pulse rather than being equal to it? In view of the powerful electromagnetic radiation emanating from the nanosecond generators and the discharge, it would be interesting to see oscillograms of the RE current, voltage, and the total discharge current — in such a version of the experiment that REs be completely absorbed by the material that transmits the electromagnetic disturbance.

In essence, all of Refs [3–9, 12] report only one experimental finding: in the authors' opinion, high-voltage nanosecond discharges under atmospheric pressure are shown to be able to generate RE currents two orders of magnitude larger than in the experiments described in Refs [18, 19]. Unfortunately, the scarcity of experimental information does not permit carefully analyzing this result and assessing its reliability. The authors present the amplitude value of the total discharge current and its density at the anode but no oscillograms of this important parameter are available. The RE energy was determined based on the absorption in metallic foils, but an absorption curve for electrons and a description of the technique of recovery of the electron-energy distribution have not found a place in any of the seven published papers dedicated to this experiment.

The authors of Refs [1, 2, 10, 12] ignore many processes that are fundamental to the breakdown mechanism, reducing it to the highly limited exponential multiplication of electrons and the no less limited Paschen's similarity law: given Pd, the breakdown develops at small d and large P in a different way compared to the opposite case. For example, if $P = 100$ Torr and $d = 0.001$ cm, the breakdown should involve FE from microasperities, especially at high overvoltages, while this is impossible at $P = 0.001$ Torr and $d = 100$ cm, although $Pd = 0.1$ Torr cm in both cases. We would like to hope that the authors of Ref. [12] will take into account the complexity and diversity of microscopic processes responsible for the developments of the gas discharge. It would be especially desirable that the 'new' runaway criterion be revised and accurate measurements of RE currents in discharges at atmospheric pressure be carried out. If the RE currents are as large as claimed in Refs [3–7, 12] and the voltage does not collapse after the RE pulse, then the time is not far off when it will be possible, simply by applying high-voltage pulses between two electrodes in the open air, to obtain electron and X-ray fluxes comparable to those presently achievable only in high-tech evacuated accelerating tubes.

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