relaxation dynamics of the charge qubit [17]. From these measurements we concluded that charge noise coming from two-level fluctuators plays a crucial role in qubit energy relaxation.

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PACS numbers: 74.20.De, 74.25.Dw, 74.25.Op
DOI: 10.1070/PU2004v047n09ABEH001875

Superconducting states and magnetic hysteresis in finite superconductors

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The macroscopic Ginzburg – Landau (GL) theory of superconductivity [1], evolved in 1950, is an example of the triumph of physical intuition. The theory has been successfully used to characterize the behavior of superconductors in a magnetic field and to predict many effects later verified in experiments. The key issue of the theory was the assumption that the physical state of a superconductor is described by a complex-valued function called the order parameter:

$$\Psi(\mathbf{x}) = \psi(\mathbf{x}) \exp\left[\mathrm{i}\Theta(\mathbf{x})\right],$$

where ψ is the modulus, Θ is the phase of the order parameter, and $\mathbf{x}$ is the spatial variable. The uniqueness condition imposed on $\Psi(\mathbf{x})$ implies that at any point in the superconductor the phase is determined only within the factor $2\pi m$, where $m = 0, \pm 1, \pm 2, \ldots$, with $\psi(\mathbf{x})$ at this point possibly having a singularity: $\psi(\mathbf{x}) \sim x^{|m|}$ as $x \to 0$. Later on, it was found that this singularity is associated with the presence of vortices [2] (Abrikosov, 1957) in type II superconductors, for which the value of the material parameter of the theory is $\varkappa > \varkappa_0 = 1/\sqrt{2} = 0.707$.

The division of superconductors into two groups (with $\varkappa < \varkappa_0$, and with $\varkappa > \varkappa_0$) was suggested in the original GL paper, where it was established that the free energy of the interface between the superconducting (s-) and normal (n-) states of a metal in a magnetic field (in what is known as the intermediate state) vanishes at $\varkappa = \varkappa_0$ [1], which suggests that the n-state in type I superconductors ($\varkappa < \varkappa_0$) is unstable with respect to the formation of the s-phase and leads (as the strength H of the external field diminishes) to a first-order (abrupt) phase transition from the supercooled (in the magnetic field) superconducting state to the normal state. The same researchers found that type I superconductors in a weak magnetic field exhibit what is known as the Meissner effect (the complete expulsion of a magnetic field from a superconducting material), while as the field strength grows a first-order phase transition from the superheated s-state to the normal n-state occurs. However, following Abrikosov's reasoning [2], we can also say that for $\varkappa > \varkappa_0$ the magnetic field begins to penetrate to the interior of a type II superconductor in the form of vortices (forming what is known as a mixed state), and, as the field strength grows, the normal cores of the vortices completely overlap and the superconductor passes to its normal state via a second-order phase transition (in the field $H_{c2} = \phi_0/(2\pi\xi^2)$, where $\phi_0 = hc/2e$ is the magnetic flux quantum, and ξ is the superconductor's coherence length [2]). Thus, Abrikosov [2] described the vortex mechanism by which an external magnetic field penetrates to the interior of a type II superconductor.

It must be noted at this point that the above picture of the magnetic field penetration into a superconductor was obtained in Refs [1, 2] on the basis of thermodynamic approach for uniform superconductors of infinite dimensions and without accounting for possible edge effects. Below we shall show that in finite superconductors (cylinders or plates placed in a magnetic field in a vacuum) there is another, edge, mechanism of the penetration of a magnetic field within superconductors. With this mechanism, the vortices may not even form, but the order parameter $\psi(x)$ near the superconductor's edges is strongly suppressed and a growing magnetic field begins to penetrate the superconductor almost freely near the edges. As the field strength increases still further, such an (edge) e-state is completely suppressed as the field reaches the value H_{c2}. The e-states can exist only in type II superconductors with $\varkappa > \varkappa_c = 0.93$ and of fairly small lateral dimensions (a cylinder of radius R or a plate of thickness $2D$), which are placed in a longitudinal magnetic field H. Note that as R or D grows, the edge e-layer, obviously, becomes unstable and splits into individual vortices which gradually fill the interior of a massive type II superconductor (in accordance with the mechanism described in Refs [2 – 5]), and the superconductor changes into the n-state.

We shall also show below that as the strong external magnetic field gets weaker, in finite superconductors with the values of $\varkappa$ falling in the interval $\varkappa_0 < \varkappa < \varkappa_c$, the supercooled normal $\bar{\mathrm{n}}$-state first transforms gradually into a specific p-state (precursor state), and then, it abruptly transforms into a fully superconducting Meissner (M) state in a field H_r (the subscript 'r' stands for restoration). In superconductors with $\varkappa < \varkappa_0$, no intermediate p-state forms, and, as the field weakens, the superconductor instantaneously transforms from the $\bar{\mathrm{n}}$-state to the M-state jumpwise. Thus, finite superconductors can be nominally divided into three groups: those with $\varkappa < \varkappa_0$, with $\varkappa_0 < \varkappa < \varkappa_c$, and those with $\varkappa > \varkappa_c$.

The various edge effects have been described in detail in Refs [6 – 17]; below we only touch on the essential points of the research. For the sake of simplicity, we examine the case of cylindrical geometry, where there are no vortices in the superconductor, i.e., $m = 0$. The self-consistent solutions of the system of nonlinear GL equations for the order parameter $\psi(\mathbf{x})$ and the dimensionless magnetic-field potential $a(\mathbf{x})$ can be found by applying the iteration technique [6]. (Note that the results do not depend on the method of calculation.)

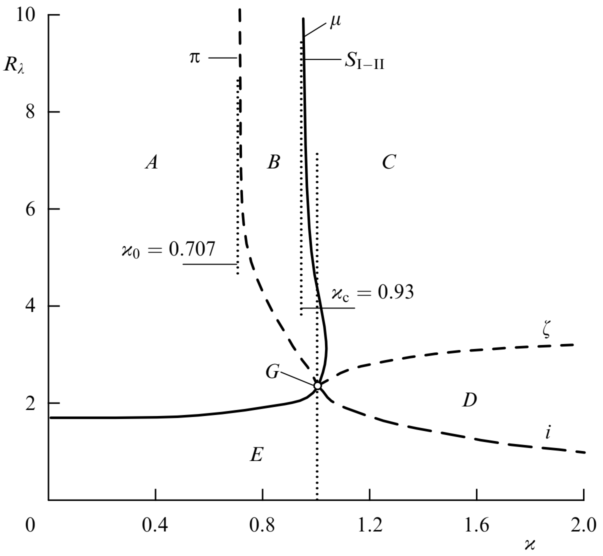

Figure 1. The state diagram of a cylinder at $m = 0$. The critical curves (π, μ, ζ, and i) in the ($\varkappa$, R_λ)-plane of parameters separate the regions of existence (A, B, C, D, and E) of the different superconducting states. They all intersect at a single point G.

These equations and the boundary conditions associated with them can be found in Refs [6–17]. For a unit of length x we can either take ξ (the coherence length) or λ (the field penetration depth), which enter the GL equations with equal reason. For a unit of field strength we can either take $H_\xi = \phi_0/(2\pi\xi^2) \equiv H_{c2}$ or $H_\lambda = \phi_0/(2\pi\lambda^2) = H_\xi/\varkappa^2$. Only one-dimensional solutions (or states) are considered, and these depend not only on the radial coordinate x but also on the parameters of the problem, namely, $\varkappa$, R, and H. It is convenient to present the results of the analysis in the form of a diagram.

Figure 1 depicts the state diagram of a cylinder in the plane of the parameters $\varkappa$ and $R_\lambda = R/\lambda$. Each point in this plane corresponds to a specific superconductor whose states depend on the strength H of the external field. The various relationships can be visualized if we 'poke' a hole at each point ($\varkappa, R_\lambda$) and look through them underneath the plane. After studying these relationships, we can find the four critical curves (π, μ, ζ, and i) separating the ($\varkappa, R_\lambda$)-plane into five regions (A, B, C, D, and E), in each of which states with a characteristic behavior of the order parameter $\psi(x)$ and the magnetization $4\pi M(H) = \bar{B} - H$ (here, $\bar{B}$ is the average magnetic field inside the sample) are realized. Below we explain the meaning of these states, curves, and regions.

Furthermore, we distinguish between two modes in which the external field H acts on a superconductor: the field-enhancement (FE) mode, when the superconductor was initially in the superconducting state in a zero field and the field subsequently increased, and the field-reduction (FR) mode, when the superconductor was initially in the normal state in a strong field and the field subsequently decreased. The sequences of states that emerge in these two modes are different, which suggests there is hysteresis in the system.

Figure 2 schematically depicts the behavior of the order parameter $\psi_0(h)$ at the center of the cylinder in the different regions specified in Fig. 1 as a function of the normalized field strength h (arbitrary normalization). In region A, the Meissner (M) state with $\psi_0 \sim 1$ is realized in a weak field ($h \sim 0$ and $R_\lambda \gg 1$); the external field is screened and does not penetrate into the sample. The superconducting M-state remains present as the field strength increases up to a certain value h_1, at which the M-state is destroyed by a first-order phase transition M $\to$ n. As the field is reduced, the n-state passes into the supercooled $\bar{\text{n}}$-state ($\psi \equiv 0$) which is stable (under small fluctuations) down to point $\dot{h}_\text{r}$, where it loses its stability and the M-state is restored (the jump $\bar{\text{n}} \to$ M). For $h < \dot{h}_\text{r}$, the normal state ($\dot{\text{n}}$, with $\psi \equiv 0$) is dynamically unstable, since it possesses a positive time increment.

In region B in the FR mode, the restoration of the superconducting M-state begins in the field h_p with a second-order phase transition from the supercooled $\bar{\text{n}}$-state into the intermediate p-state which exists down to point h_r, at which the p-state loses its stability and the M-state is abruptly restored. Notice that the p-state is metastable, since along with this state there exists an M-state with a larger order parameter ψ and a lower free energy. In region B, in addition to these solutions, there appears yet another branch of solutions (u), which, however, is absolutely unstable. (Small deviations from the u-state have a positive increment, i.e., increase with the passage of time. The instability of u-solutions is also evident from the fact that the derivative $\mathrm{d}\psi_0/\mathrm{d}h > 0$ is positive on the mathematical u-branch, i.e., the external field enhances rather than suppresses the superconductivity, as it does to the physical M-branch where $\mathrm{d}\psi_0/\mathrm{d}h < 0$. The arrows in Fig. 2 indicate the bifurcation points of the solutions, where within a small neighborhood of the fields h_r, h_p, and h_1 there are two states that differ very little from each other.)

At the boundary $A = B$ of regions A and B, the bifurcation points h_r and h_p lie along a straight vertical line ($h_\text{r} = h_\text{p}$), which corresponds to values of $\varkappa$ that belong to the line π in Fig. 1. This line indicates the boundary at which the superconducting p-state disappears in the FR mode and superconductivity is restored via a jumpwise transition from the $\bar{\text{n}}$-state to the M-state. The asymptote of π line for $R \gg \lambda$ coincides with $\varkappa_0 = 1/\sqrt{2}$.

In region C in the FE mode, there is a jump (in field h_1) from the M-state to the e-state with a finite amplitude $\psi_0 > 0$, and this amplitude vanishes in the maximum field h_2 (this field coincides in strength with $H_{c2} \equiv H_\xi$). In the FR mode, the e-state gradually transforms into a depressed (d-) state, from which it is restored jumpwise to the M-state in field h_r. There is no supercooled $\bar{\text{n}}$-state in region C.

At the boundary $B = C$ of regions B and C, the bifurcation points h_1, h_p, and h_2 lie along a single straight line ($h_1 = h_\text{p} = h_2$), which corresponds to values of $\varkappa$ that belong to the line μ in Fig. 1. This line indicates the boundary of existence of the supercooled $\bar{\text{n}}$-state. The asymptote of μ line for $R \gg \lambda$ coincides with $\varkappa_\text{c} \approx 0.93$.

At the boundary $C = D$ of regions C and D, the jump M $\to$ e (which occurs in region C) disappears, as does the hysteretic d-state. The bifurcation points h_1 and h_r then lie along a straight line $h_1 = h_\text{r}$, and the function $\psi_0(h)$ acquires a reversible (nonhysteretic) shape, with the derivative $\mathrm{d}\psi_0/\mathrm{d}h = \infty$ at point ζ. The corresponding values of $\varkappa$ indicate the hysteresis boundary (curve ζ in Fig. 1).

In region D, there appears an inflection point i in curves $\psi_0(h)$ with a finite value of the derivative, $\mathrm{d}\psi_0/\mathrm{d}h < \infty$. At the boundary $D = E$ of the regions D and E, the inflection point i moves down onto the horizontal axis ($\psi_0 = 0$ and $h_i = h_2$; curve i in Fig. 1 corresponds to these points). In region E, the curves $\psi_0(h)$ monotonically decrease and have no inflection points.

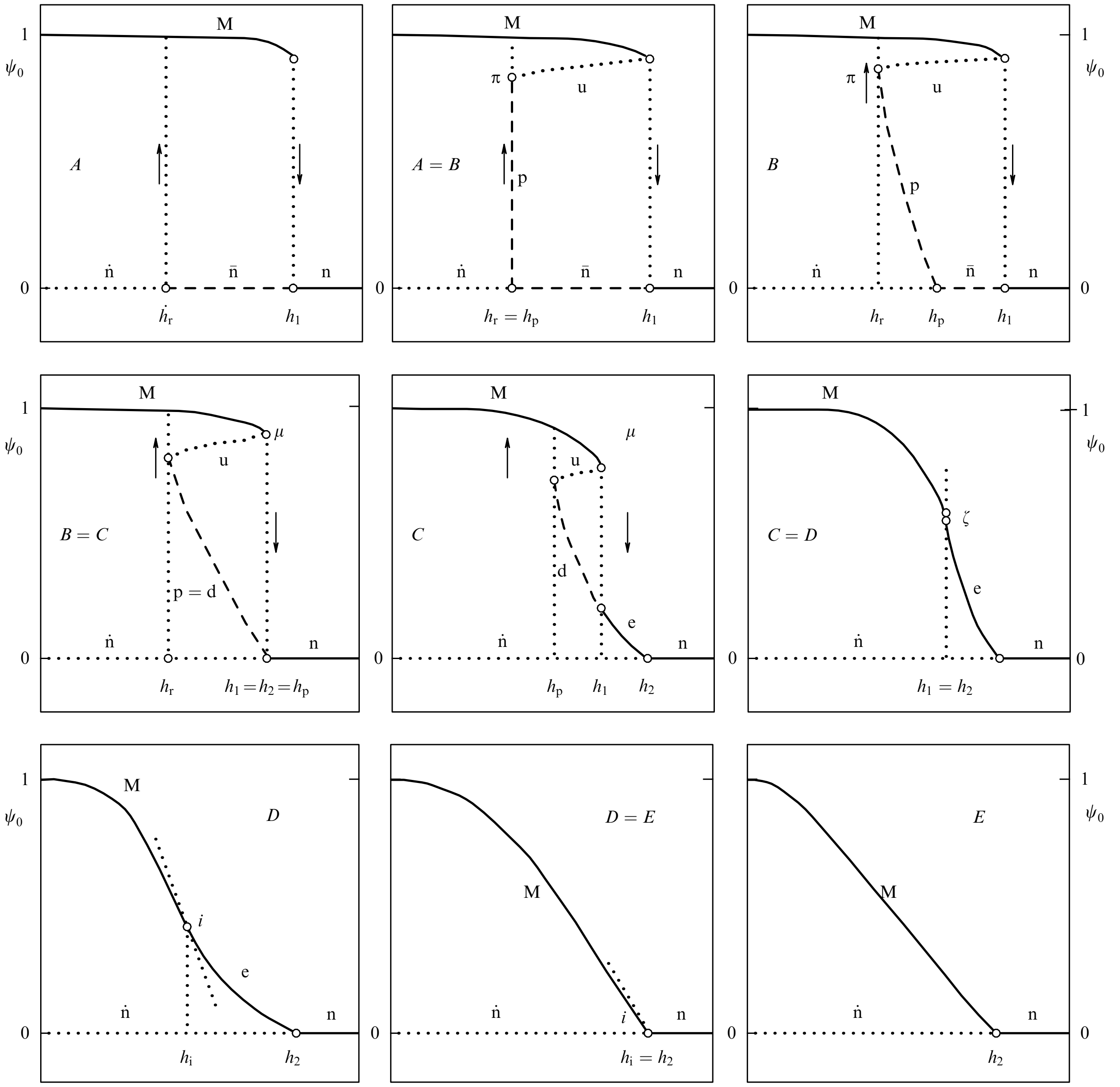

Figure 2. Characteristic behavior (shown schematically) of the order parameter $\psi_0(h)$ in different regions (A, B, C, D, and E) of Fig. 1. Solid curves depict the stable states (M, e, and n), dashed curves depict the metastable (hysteretic) states (n̄, p, and d), and the dotted curves correspond to the absolutely unstable states (u and ṅ).

What we have just said can be illustrated by Fig. 3 which depicts the coordinate dependences of the order parameter $\psi(x)$ and the field $b(x)$ (normalized to H_ξ) for a cylinder with $R_\lambda = 7$ and $\varkappa = 1.2$ (Figs 3a and b) and $\varkappa = 0.8$ (Figs 3c and d). Clearly, at $\varkappa = 1.2$ (region C) in the FE mode, the order parameter, which initially had the form $\psi(x) \equiv 1$ (at $h = 0$), gradually acquires the form of a Meissner curve M (solid curve), which at $h_1 = 0.7692$ is abruptly transformed into an edge e-state. The meaning of an e-state is revealed most vividly from the $b(x)$ curves where, clearly, at $h = h_1$ the M-state (in which the external field h_1 is screened and is kept out of the cylinder almost perfectly) is replaced by an e-state (in which the external field h_1 almost freely penetrates the cylinder at its edge but superconductivity is retained at the center). Thus, the production of an e-state constitutes an additional edge mechanism by which the magnetic field penetrates a superconductor, and this mechanism differs from the ordinary vortex one [2]. As the strength of the field h increases still further, the amplitude of the e-state gradually decreases (see curves e′) and at $h_2 = 1.0013$ the e-state disappears by transforming into an n-state. In the FR mode, the n-state again produces an e-state. This state gradually transforms into a d-state, which in a field $h_r = 0.6418$ abruptly transforms into an M-state (the dashed curves in Figs 3a and b). Thus, the d $\to$ M transitions constitute an additional mechanism of magnetic field expulsion from the superconductor, and this mechanism is unrelated to the motion of vortices across the sample's boundary.

At $\varkappa = 0.8$ (region B) in the FE mode, the M-state (solid curves in Figs 3c and d) abruptly produces in the field $h_1 = 1.2358$ an n-state, while in the FR mode the n-state first transforms into a supercooled (hysteretic) n̄-state, which

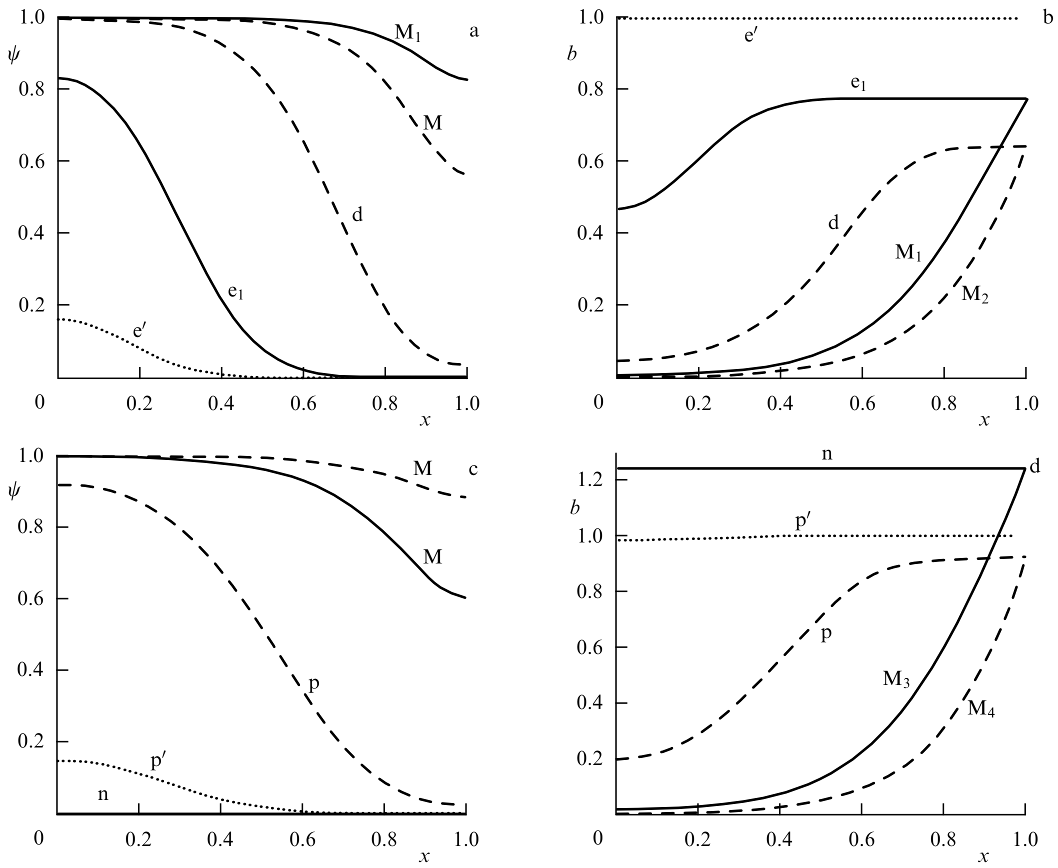

Figure 3. The coordinate functions $\psi(x)$ and $b(x)$ in a cylinder with $R_\lambda = 7$: (a, b) $\varkappa = 1.2$, region C, the FE mode, and (c, d) $\varkappa = 0.8$, region B, the FR mode. The notation is explained in the main text.

first produces a p$'$-state with a small amplitude in the field $h_{\rm p} = 1.0009$, and this state at $h_{\rm r} = 0.9219$ abruptly transforms into an M-state (the dashed curves). Thus, in the p $\to$ M transitions an additional (vortex-free) mechanism of magnetic field penetration into a superconductor is also realized, a mechanism caused by the dynamic transformation of the p-state at an instability point of the solution.

Incidentally, the p-state $\psi(x)$ in Fig. 3c resembles an (s, n)-wall by appearance [1]. It can be shown (e.g., see Refs [15–17]) that as $R_\lambda \to \infty$, $\varkappa \to \varkappa_0 = 1/\sqrt{2}$, and $h \to 1$ (i.e., $H \to H_\xi$) the last of the p-states [denoted by π in Fig. 2 ($A = B$)] coincides with the (s, n)-wall examined by Ginzburg and Landau [1], with the free energy of this state vanishing ($\sigma_{\rm s,n} = 0$), and can be described analytically by Bogomol'nyĭ's degenerate equations [18–20]. Thus, the (s, n)-wall constitutes a particular case of metastable p-states existing in region B in the FR mode.

Figure 4 shows the critical fields of a cylinder with $\varkappa = 0.8$ and $\varkappa = 1.2$ as functions of the radius R_λ. Following is an explanation of the notation used in this figure.

At $\varkappa = 0.8$ (see Fig. 4a) and large R_λ, the maximum field strength, up to which in the FE mode there is still a superconducting M-state, coincides with the field h_1 of a first-order phase transition. When $h > h_1$, there is only the n-state. In the FR mode there appears the supercooled (hysteretic) $\bar{\rm n}$-state which exists down to the field $h_{\rm p}$, where a metastable p-state is created as a result of a first-order phase transition. Such p-states exist down to the field $h_{\rm r}$, where they lose stability and an M-state is abruptly restored. When $h < h_{\rm r}$, the normal ($\dot{\rm n}$) solutions are absolutely unstable and only stable M-states exist. As R_λ decreases, the region where p-states can exist narrows, and at point π such a region ceases to exist. (The critical curve π in Fig. 1 consists of critical π-points found in a similar manner for other values of $\varkappa$.) The field h_1 and the region of the supercooled $\bar{\rm n}$-state exist down to point ζ, at which the hysteretic $\bar{\rm n}$-state disappears. (The curve ζ in Fig. 1 indicates the hysteresis boundary and consists of ζ-points found for other values of $\varkappa$.) Field $\dot{h}_{\rm r}$ corresponds to the dynamic-instability points of the supercooled $\bar{\rm n}$-state, where the M-state is immediately restored as a result of a first-order phase transition, without the formation of an intermediate p-state. For small values of R_λ (below point ζ), the transition from the M-state to the n-state (and reversely) occurs by way of a reversible second-order phase transition in the field h_2.

At $\varkappa = 1.2$ (see Fig. 4b) and large R_λ, as the field strength increases (the FE mode) there occurs a jump from the M-state to an e-state (in the field h_1) followed by a second-order phase transition e $\to$ n (in the field h_2). In the FR mode, an e-state is again created (in the field h_2), which then gradually transforms into a d-state. This state exists down to the field $h_{\rm r}$, where the M-state is restored jumpwise. As R_λ decreases, the region where hysteretic d-states can exist narrows and disappears at point ζ. (Curve ζ in Fig. 1, which is the hysteresis boundary, consists of such critical points ζ.) For $R_\lambda < \zeta$, the jumps (j) related to d-states are replaced by inflection points (i). Curve $h_i(h)$ corresponds to the fields at which the inflection point of the function $\psi_0(h)$ (i.e., ${\rm d}^2\psi_0/{\rm d}h^2 = 0$) occurs for $\psi_0 > 0$. At point i, the region of the inflections of the function $\psi_0(h)$ [and the magnetization

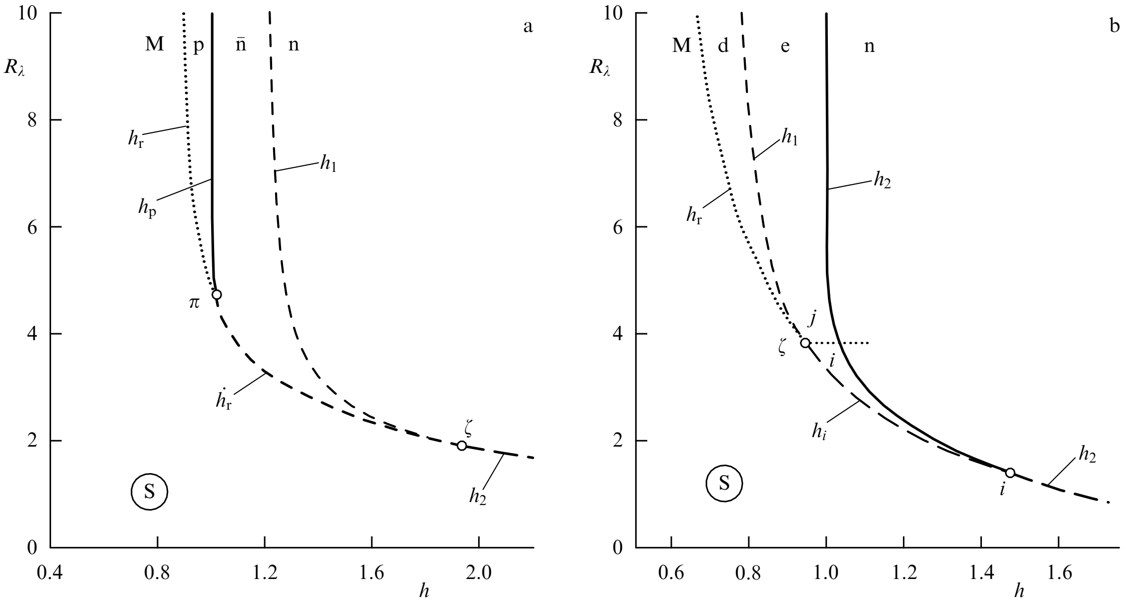

Figure 4. Critical fields of a cylinder: (a) $\varkappa = 0.8$ (region *B*), and (b) $\varkappa = 1.2$ (region *C*). All explanations are given in the main text.

$4\pi M(h)$ as well] disappears. (Curve *i* in Fig. 1 consists of the respective critical points.)

Similar calculations can be done for the case of a flat plate placed in a parallel magnetic field in the absence of vortices [14–17]. The state diagram, the solutions, and the critical fields for a plate are identical to those depicted in Figs 1–4 for a cylinder. Thick superconducting plates (with $D_\lambda = D/\lambda \gg 1$) can also be divided into three groups: with $\varkappa < \varkappa_0$, with $\varkappa_0 < \varkappa < \varkappa_c$, and with $\varkappa > \varkappa_c$, where $\varkappa_0 = 1/\sqrt{2} = 0.707$ and $\varkappa_c = 0.93$. For plates with smaller values of D_λ, we can again isolate five regions (*A*, *B*, *C*, *D*, and *E*) similar to those shown in Fig. 1.

One-dimensional GL equations can be used for describing the states of a cylinder with an arbitrary number *m* of vortices on the axis (here $m > 0$ is the vortex strength). In this case, as noted earlier, the size of the normal core of the vortex increases with *m* as x^m, so that the residual superconducting state [with $\psi(x) \neq 0$] is pushed out to the surface of the sample, where it is confined by the external magnetic field (e.g., see Ref. [11]). As a result, as *m* increases in massive samples (with a fixed but large radius *R*), there can exist what is known as surface superconductivity, which is retained up to the maximum field H_{c3} of surface superconductivity [21, 22]:

$$H_{c3} = 1.69 H_{c2}\,, \qquad H_{c2} \equiv H_\xi = \frac{\phi_0}{2\pi\xi^2}\,. \tag{1}$$

The existence of surface superconductivity, predicted in Refs [21, 22], has been corroborated by numerous experiments.

The following remark of a methodical nature is in order. Equation (1) can be written down in an equivalent form by using a different normalization:

$$H_{c3} = 1.69\sqrt{2}\,\varkappa H_{cb} = 2.4\varkappa H_{cb}\,, \tag{2}$$

where $H_{cb} = \phi_0/(2\pi\sqrt{2}\,\lambda\xi) = H_\xi/(\sqrt{2}\,\varkappa)$ is the thermodynamic critical field of a massive superconductor [1]. Interpreting expression (2) as an equation in $\varkappa$, a number of researchers (see Refs [23–27] and Refs [2–5]) have concluded that there exists a 'specific' value $\varkappa_* = (2.4)^{-1} = 0.417$ that separates type I superconductors with $H_{c3} < H_{cb}$ (for $\varkappa < \varkappa_*$) from the superconductors with $H_{c3} > H_{cb}$ (for $\varkappa > \varkappa_*$). However, such a conclusion is based on a misunderstanding. The fact is that the field H_{cb} in expression (2) depends on $\varkappa$, in view of which Eqn (2) is actually independent of $\varkappa$ and, therefore, no new information can be extracted from it, except the information that expression (1) already contains. According to formula (1), there are two critical fields of strengths H_{c2} and H_{c3} for a bulk superconductor and the ratio of these field strengths depends neither on the scale of the field measurement nor on $\varkappa$. Thus, the 'specific' value $\varkappa_* = 0.417$ has no special meaning in itself, neither physically nor mathematically. This is evident from Fig. 1, in which the value $\varkappa_* = 0.417$ has no special meaning, i.e., it is a regular point of the GL equations.

In conclusion, it must be noted that the above behavior of the order parameter $\psi_0(H)$ and the similar special features in the behavior of the magnetization $4\pi M(H)$ of small enough samples can, in principle, be examined in experiments. The extracted information (the jumps between states, hysteresis phenomena, the hysteresis boundaries, the inflection points in the magnetization curves, the exact values of the critical fields for the transitions between states, etc.) can probably be used to refine the values of the parameters *R* and $\varkappa$ in real samples. Such information can obviously be useful, especially in connection with the prospects of fabricating small-sized superconducting devices based on mesoscopic superconductors. Hence, there is a need for further theoretical and experimental investigations into the above problems.

The author is grateful to V L Ginzburg for the interest in this work and for valuable remarks, and to V G Zharkov, A Yu Tsvetkov, A L Dyshko, and N B Konyukhova for useful discussions. The work was made possible by the financial support of the Russian Foundation for Basic Research (grant No. 02-02-16285).

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PACS numbers: 74.25.Ha, **74.81.–g**
DOI: 10.1070/PU2004v047n09ABEH001876

Coexistence of ferromagnetism and nonuniform superconductivity

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(1) Superconductivity and ferromagnetism appear to be antagonists in relation to a magnetic field: a superconductor expels a magnetic field (the Meissner–Ochsenfeld effect), while a ferromagnet concentrates such a field. Hence, it is more appropriate to use the name 'antiferromagnet' in relation to superconductors than to substances commonly known as antiferromagnets. The first attempt to tackle the problem of the coexistence of these states was made by V L Ginzburg [1] in 1956, even before the Bardeen–Cooper–Schrieffer (BCS) microscopic theory appeared [2].

According to the work [1], coexistence is possible if the critical magnetic field H_c is higher than the magnetic induction I. From the microscopic point of view [2], the magnitude of H_c is determined in most cases by the effect that this magnetic field (and the induction) has on the orbital motion of pairs. Moreover, due to pairing with oppositely directed spins, Zeeman splitting also suppresses superconductivity (the paramagnetic effect), and it is this splitting that is predominant [3].

When the superconducting transition temperature T_c is much higher than the ferromagnetic transition temperature T_m, the magnetic state is nonuniform in the coexistence region [4]. (A discussion of the existing theoretical and experimental results can be found in the review [5].)

When $T_c \leqslant T_m$, there exists a narrow interval of values of the magnetization I, where the superconducting state proves to be nonuniform in the state coexistence conditions [6, 7].

Currently, a large number of works have appeared (e.g., see Refs [8, 9]) in which the coexistence of superconductivity and ferromagnetism has been observed in layered cuprate $RuSr_2GdCu_2O_8$ compounds, in which T_m is much higher than T_c ($T_m = 132$ K, and $T_c = 46$ K). Such a T_m-to-T_c ratio is unacceptable for the simple spherical Fermi surface which lies at the base of the model discussed by Larkin and Ovchinnikov [6] as well as by Fulde and Ferrell [7]. As is well known, the uniform superconducting state is insensitive to the shape of the Fermi surface [2, 3], and the existence of such a state requires that $\varepsilon_\sigma(\mathbf{p}) = \varepsilon_{-\sigma}(-\mathbf{p})$. The nesting condition $\varepsilon_\sigma(\mathbf{p}) = -\varepsilon_{-\sigma'}(\mathbf{p}+\mathbf{q})$ is preferable for electron–hole pairing (the insulating state), while the mirror nesting condition $\varepsilon_\sigma(\mathbf{K}/2+\mathbf{p}) = \varepsilon_{-\sigma}(\mathbf{K}/2-\mathbf{p})$ is preferable for the realization of superconducting pairing with a large total momentum $\mathbf{K}$ in the case of electron–electron repulsion. On the other hand, coexistence of ferromagnetism and a nonuniform superconducting state, with the magnetic-nesting condition

$$\varepsilon_\sigma(\mathbf{p}) = \varepsilon_{-\sigma}\left(-\mathbf{p} + \frac{\mathbf{n}I}{v_F}\right) \qquad (1)$$

being met for an electron dispersion law with spin σ in a selected direction $\mathbf{n}$, is possible for an arbitrarily large magnetization I (here v_F is the Fermi velocity). Thus, in the given situation the main mechanism by which magnetization suppresses superconductivity is the orbital mechanism examined by Ginzburg [1].

In the present work, we show that the processes of hopping to third-sphere centers, which were ignored in Refs [10, 11] but which exceed the processes of hopping to second-sphere centers, drastically change the situation concerning the state coexistence. Furthermore, we show that a superconducting state with a large total momentum of the pairs [12] can coexist with a ferromagnetic state at high enough magnetizations.

(2) We select a simple model that meets the magnetic-nesting condition (1), namely, a two-dimensional model of an electronic spectrum corresponding to the constant energy lines in the form of squares within a certain energy interval (on the order of the cutoff energy ω of the attractive interaction V) (Fig. 1). Assuming that ω ($\omega = \omega_{ph}$ for electron–phonon coupling) is small compared to the Fermi energy ε_F, we can write down the equation for the order parameter Δ [$\Delta(\mathbf{r}) = |\Delta| \exp(\mathrm{i}\mathbf{q}\mathbf{r})$, where $\mathbf{q}$ is the pair momentum] within the BCS theory at $T = 0$ in the form

$$\frac{1}{\lambda} = \int_0^\omega \frac{\mathrm{d}\xi}{\sqrt{\xi^2 + |\Delta|^2}} \left\{1 - \frac{1}{2}\left[n(\varepsilon + I + Q) + n(\varepsilon + I - Q) + n(\varepsilon - I - Q) + n(\varepsilon - I + Q)\right]\right\}, \qquad (2)$$