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METHODOLOGICAL NOTES

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Photons traveling at the speed of light in a two-level atomic medium as a source of Cherenkov radiation cones

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Contents

Abstract. **This paper discusses the mechanisms responsible for the emergence of a cone radiation structure observed in the nearly resonance interaction of a laser radiation beam (of frequency ν) with a two-level atomic medium (the case $\nu < \nu_{01}$, where ν_{01} is the energy-level transition frequency). Use is made of a model which takes into account the axial symmetry of laser radiation beams and the fact that the transverse intensity distribution of the laser beams is Gaussian, and therefore the probability of nonlinear optical processes affecting the propagation of light is maximal on the axis of the laser beam.**

1. Introduction

The 1930's – 1940's saw the experimental discovery and theoretical investigation of Vavilov – Cherenkov radiation (see the reviews [1, 2].) This effect was first observed in the study of luminescence properties of the solutions of uranyl salts excited by gamma-ray beams of radium. However, it was decided that the new kind of radiation was caused not by the gamma-ray photons but by the Compton electrons knocked out by the photons. The Compton electrons travel with a superluminal velocity, which exceeds the phase velocity of light in the medium $V = c/n(\nu)$.

I M Frank [3] showed that dipoles propagating along the cone axis may also cause this kind of radiation to emerge. To do this it would suffice to direct a broad light beam with a plane front onto the surface of a medium with a refractive index $n > 1$ at an angle to the surface. In doing this the point of intersection of the wave front with the surface of the medium moves with a superluminal velocity (which corresponds to the superluminal motion of the dipoles), and in this case the Cherenkov cone is in fact caused by the photons of the radiation which is incident on the surface and excites the dipoles of the medium.

We revert to the moving particles of the early theoretical and experimental investigations to set off Vavilov – Cherenkov radiation where it is induced by a particle traveling along the axis of an empty bore (the refractive index $n = 1$) of small radius inside a cylinder filled with a medium whose refractive index $n \neq 1$ [4, 5]. Our report is concerned with precisely this slot geometry, with the difference that not the particles but a beam of 'superluminal' photons — the source of Cherenkov radiation — will propagate along the cone axis.

Prior to the advent of lasers, however, no experiments were reported on the observation of Vavilov – Cherenkov radiation induced by the photons of a light beam. According to early suppositions, the Cherenkov-type radiation could be observed by way of laser-induced nonlinear interaction of light waves in the medium, as reported in Refs [6 – 9]. In this scheme, the part of the source responsible for the Cherenkov radiation was assigned to the nonlinear polarization propagating with a superluminal velocity through the medium. In this case, the divergence angle of the radiation cone at the output of the nonlinear medium can be calculated from the condition

$$\cos\theta_1 = \frac{K_{\mathrm{non}}}{K}\,, \tag{1}$$

where K, K_{non} are the wavenumbers of the divergent light wave and the nonlinear polarization wave induced in the medium on the axis of the laser beam.

The experimental results reported in Ref. [10] lent support to the validity of this model. The issue remained open as to whether photons collected into a beam can be the source of the Cherenkov cone. Finally, this possibility was stated in Ref. [11]. The possibility of photons propagating through a medium with a 'superluminal' velocity has been possible to realize under conditions of multiphoton bleaching of atomic potassium vapor in the spectral region where the refractive index $n(\nu) > 1$. Of course, the term 'superluminal' velocity

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applies to the case where the velocity of light propagation in the ambient medium is $V = c/n(\nu)$ and the velocity of the 'superluminal' photons does not exceed the speed of light in free space, c.

2. On the feasibility of satisfying the conditions for the excitation of Vavilov – Cherenkov radiation with the aid of a laser beam

A. As a rule, the increase in the pump frequency ν in the region of normal dispersion is associated with a rise in the refractive index $n(\nu)$ of a gaseous medium (for instance, of atomic potassium vapor near the lines of the principal doublet D_1, D_2). This dependence breaks down in the range of an absorption line ν_{01} — these are the so-called portions of the spectrum with anomalous dispersion, where the refractive index $n(\nu) > 1$ for $\nu < \nu_{01}$ and $n(\nu) < 1$ for $\nu > \nu_{01}$. Upon further increase in frequency, the refractive index increases to soon become a quantity greater than unity.

The above properties are embodied in the classical model of a harmonic oscillator developed by Lorentz [12]. According to this model, for an atomic system residing in the ground (unexcited) state, in the frequency range $\nu < \nu_{01}$ the dispersion is positive and the refractive index $n(\nu) > 1$. For the second solution obtained in the same frequency range ($\nu < \nu_{01}$) for an inverted medium [13, 14], the dispersion is negative and the refractive index $n(\nu) < 1$.

In considering an uninverted medium it can be seen that the propagation velocity for the radiation of frequency ν in the specified frequency range ($\nu < \nu_{01}$) in the linear case (for low intensities E of the light field) becomes lower owing to the increase in $n(\nu)$ as the radiation frequency approaches the absorption line from the low-frequency side. The velocity decreases according to the relation

$$V = \frac{c}{n(\nu)} \, . \tag{2}$$

If we assume that switching on the radiation interacting with the atoms drives the medium into a state with an approximately 50% inversion (the state of dynamic dispersion suppression), irrespective of the frequency its refractive index $n(\nu) \to 1$ throughout the frequency range. In this (the third) case, different-frequency photons would propagate through such a medium with the same velocity — the speed of light c.

In the case of weak light fields, which do not saturate the atomic transition, the propagation velocity of 'bluer' photons near the absorption line in the frequency range $\nu < \nu_{01}$ turns out to be lower [because of the growth of the refractive index $n(\nu) > 1$ in the region of normal dispersion] than that of lower-frequency photons shifted to the red (Stokes) region of the spectrum. If we could accelerate the 'bluer' (anti-Stokes) photons in such a medium by letting them pass through specially prepared 'slots' [4, 5, 15] in which the refractive index $n(\nu) \to 1$ owing to dispersion suppression (saturation), we would then gain a superluminal source capable of generating the Cherenkov radiation cone.

This opportunity appears when we recall that in the near-resonance interaction between laser beams and a two-level atomic medium with the levels of opposite parity, an extremely important role is played by nonlinear-optical processes — three-photon electronic Raman scattering (RS) [16 – 18] and six-photon parametric scattering (SPS) [10, 19]. Nonlinear-optical near-resonance processes (with the participation of three photons of the pump radiation in an elementary act), which are allowed in the medium of two-level atoms [20] and obey the energy and momentum conservation laws, can be represented in the following way.

The $\nu < \nu_{01}$ case, where ν_{01} is the resonance transition frequency:

$$\nu + \nu + \nu = \nu_{\mathrm{s}} + \nu_{\mathrm{as}} + \nu_{\mathrm{as}} \, , \tag{3}$$

where

$$\nu_{\mathrm{s}} = 2\nu - \nu_{01} \, , \tag{4}$$

$$\nu_{\mathrm{as}} = \frac{\nu + \nu_{01}}{2} \, . \tag{5}$$

The appearance of the new spectral components ν_{s} and ν_{as} in the radiation interacting with the resonance medium points to the possibility of a situation where the velocities of propagation of the photons at frequencies ν, ν_{s}, and ν_{as} and their directions of propagation are different, especially when the transverse intensity distribution of the laser beam is nonuniform (normally Gaussian). Some distance away from the beam axis, the velocity of radiation propagation at these frequencies is defined by the relations

$$V_{\mathrm{s}} = \frac{c}{n(\nu_{\mathrm{s}})} \, , \quad V_{\mathrm{as}} = \frac{c}{n(\nu_{\mathrm{as}})} \, .$$

For instance, in Ref. [10] it was experimentally shown that the output radiation component of frequency ν_{as} propagates along the generatrix of a cone. The opening angle θ_1 of this cone (Fig. 1a) obeys the Cherenkov condition [see expression (1)], which coincides with the momentum conservation law for the six-photon parametric process (3) $3\mathbf{K} = \mathbf{K}_{\mathrm{s}} + 2\mathbf{K}_{\mathrm{as}}$:

$$\cos\theta_1 = \frac{K_{\mathrm{non}}}{K_{\mathrm{as}}} \, . \tag{6}$$

Here, K_{non} is the wavenumber of the nonlinear polarization at the ν_{as} frequency propagating through the medium; its value can be calculated from the formula

$$K_{\mathrm{non}} = \frac{3K - K_{\mathrm{s}}}{2} \, , \tag{7}$$

where $K_{\mathrm{as}} = 2\pi n_{\mathrm{as}}/\lambda_{\mathrm{as}}$, $K_{\mathrm{s}} = 2\pi n_{\mathrm{s}}/\lambda_{\mathrm{s}}$, and $K = 2\pi n/\lambda$ are the wavenumbers; λ_i and n_i are the corresponding wavelengths and refractive indices; and λ_{s} is the wavelength of the three-photon electronic RS corresponding to the process (4). The emergence of radiation with the wavelength λ_{as} is caused by stimulated three-photon atomic transition from an excited level to the ground one according to expression (5) [10, 19].

B. Three-photon processes involving radiation at frequencies (4) and (5) provide equilibrium between the operating laser radiation and the two-level medium and accordingly the dynamic dispersion suppression [$n(\nu) \to 1$]. In the case of a near-resonance interaction between the intense transition-saturating monochromatic radiation and the two-level atoms (0, 1), the well-known Einstein scheme [21] needs supplementation.

Let 0 correspond to the ground state of the atom and 1 to the excited one. When the frequency of laser radiation is tuned off the resonance transition frequency ($\nu \neq \nu_{01}$) to the low-frequency side of the spectrum, along with the spontaneous emission (the 1 → 0 transition), single-photon absorp-

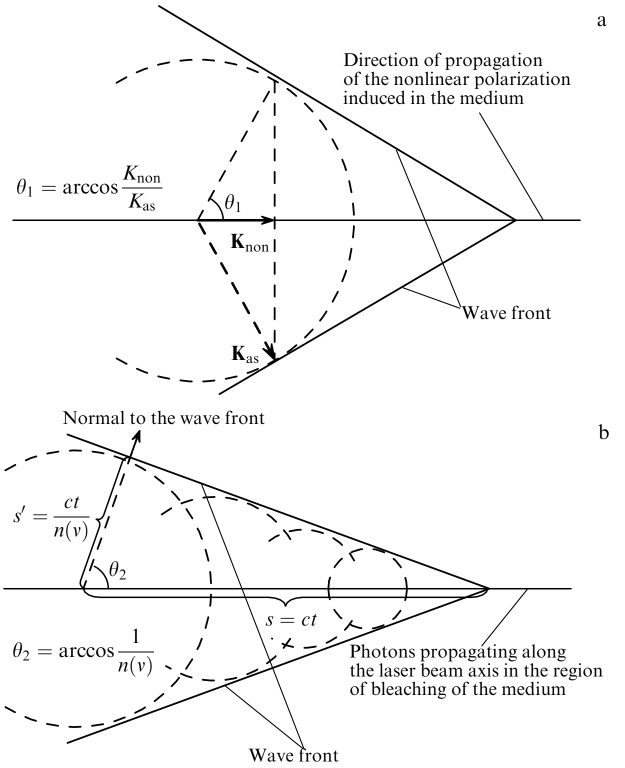

Figure 1. Formation of the wave front of Cherenkov radiation arising from (a) the nonlinear polarization with the wave vector $\mathbf{K}_{\rm non}$ propagating through the medium and (b) the photons traveling along the laser beam axis in the region of bleaching of the medium. The refractive index of the medium is $n(\nu) > 1$.

tion (0 → 1), and stimulated emission (1 → 0), this scheme, first, needs to be supplemented with the process of three-photon RS at the $\nu_{\rm s}$ frequency according to relation (4) for the atomic ground-to-excited state transition (0 → 1). Second, to maintain equilibrium, the stimulated excited-to-ground state atomic transition (1 → 0) in a single elementary act may occur when the excited atom absorbs one photon (of the pump ν) with the simultaneous emission of two new photons at the $\nu_{\rm as}$ frequency defined by relation (5).

We assume that the single-photon absorption and stimulated emission can be neglected (in our case, $\nu \neq \nu_{01}$). Then, the new condition for the equilibrium interaction of near-resonance radiation with the atoms implies that the number of three-photon transitions of the atomic system from the ground state 0 to the excited state 1 according to relation (4) should be equal to the number of three-photon transitions from state 1 to state 0 according to relation (5), if the spontaneous transitions from level 1 to level 0 are neglected.

In our case, the condition of equilibrium, or dynamic dispersion suppression, takes on the form

$$n_0 C_{0\to 1}\, u(\nu)\, \mathrm{d}t = n_1 \left[C_{1\to 0}\, u(\nu) + A_{1\to 0} \right] \mathrm{d}t\,, \tag{8}$$

where n_0 is the number of atoms in the ground energy state 0; n_1 is the number of atoms in the excited energy state 1; $C_{0\to 1}$ is the probability of a three-photon transition from the ground state 0 to the excited state 1 according to relation (4); $C_{1\to 0}$ is the probability of a three-photon transition from the excited state 1 to the ground state 0 according to relation (5); $A_{1\to 0}$ is the spontaneous 1 → 0 transition probability; and $u(\nu)$ is the radiation density at the pump frequency.

Eventually, the following should be borne in mind. In a medium residing in such a near-resonance light field there occurs an equilibrium superposition of atomic states and/or the atoms themselves in the ground and excited states as well as the simultaneous participation in the process of at least three waves interacting with each other even though the radiation at the input to the medium may be monochromatic.

C. We enlarge on yet another radiation component emanating from the medium and propagating by the generatrixes of the second cone. To this end we take advantage of the model of a two-component, two-mode ('slotted') medium.

The intensity profile of the laser beam incident on the cell is close to Gaussian, the intensity peaking on the beam axis.

The refractive index of a medium whose optical properties are modeled by a harmonic oscillator is normally $n(\nu) > 1$ near the resonance in the frequency region $\nu < \nu_{01}$ in a weak field. The combination process (4) is able, above all on the beam axis, to transfer a part of the electrons from level 0 to level 1. As mentioned above, the photons emerging at the $\nu_{\rm s}$ frequency overtake the pump photons ν. However, the medium changes its properties owing to the equalization of the populations at the beam axis. On the beam axis there emerges a spatial 'slot' [4, 5, 15] containing a medium with properties changed owing to the dynamic dispersion suppression — the second mode of the medium $n(\nu) = 1$. That is why along the beam axis the pump photons of frequency ν begin to pursue with a velocity c (the speed of light) the photons of frequency $\nu_{\rm s}$, while off the beam axis, where (in accordance with the transverse Gaussian beam intensity distribution) the photon density decreases, their velocity in the weak field corresponds to the value $c/n(\nu)$. According to the Cherenkov condition [15], for the opening angle of the second Cherenkov cone we can write

$$\cos\theta_2 = \frac{c}{n(\nu)c} = \frac{1}{n(\nu)}\,. \tag{9}$$

The 'superluminal' photons at the pump frequency ν propagating along the beam axis (along the spatial 'slot' induced in the medium) are capable of making a contribution to the radiation propagating by the generatrixes of the Cherenkov cone (the second cone) now at the pump frequency ν (Fig. 1b).

We emphasize that there are no 'superluminal' photons at the $\nu_{\rm as}$ frequency. The function of a superluminal source is fulfilled by the nonlinear polarization induced in the atomic vapor.

3. On the experimental observation of cone components

When a 10-ns-long pulse of laser radiation with an energy of 3–5 mJ is directed into a cell with atomic potassium vapor, observed in the specified spectral region ($\nu < \nu_{01}$) is, as a rule, a supercontinuum — the broadening of the frequency spectrum of the pump radiation transmitted through the cell up to the frequencies $\nu_{\rm as}$ (5) and $\nu_{\rm s}$ (4), which normally limit the further broadening of the spectrum. In these experiments, the low-frequency shift of laser radiation frequency relative to the frequency of the 4S(1/2)–4P(3/2) two-level transition investigated in the potassium vapor is 10–30 cm^{-1} and the vapor temperature 200–250 °C. As a rule, the spectrum broadening is asymmetric: when the pump frequency ν is lower than the resonance transition frequency ν_{01}, the spectrum at the cell output is broadened primarily to the Stokes region $2(\nu_{\mathrm{as}} - \nu) = (\nu - \nu_{\mathrm{s}})$. The broadening itself is generally attributed to the effect of amplitude-phase modulation [18, 22].

Counting in favor of the model proposed in our work is the dependence of the opening angle of the cone on the magnitude of the shift of laser radiation frequency relative to the resonance transition frequency and on the density of the atomic vapor, which was discovered in Ref. [10]. When the recording system at the output of the cell allows the recording of the cone components of the output radiation, it is possible to realize the above-discussed possibilities for obtaining two cones of the Cherenkov type. The experimental setup and the description of the conditions for obtaining such frequency-angular spectra are presented by the author in detail elsewhere [10, 11, 19].

4. Conclusion

Therefore, laser beams with Gaussian transverse intensity distribution propagating through a cell with a two-level medium and having frequencies below the resonance transition frequency may be responsible for the emergence of two radiation cones at the output of the cell (the first cone is at the ν_{as} frequency and the second one at the ν frequency). For the numerical evaluation of the opening angles of the cones, the conditions (6) and (9) were obtained.

The scattering processes on the high-frequency side of the transition progress differently, and in a special case they can be reduced to the diffraction of tunneling pump photons on a small aperture [11].

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