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Quantum phase transitions

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Contents

Abstract. Continuous phase transitions that occur at zero temperature as a result of quantum fluctuations required by Heisenberg's uncertainty principle are called quantum phase transitions. In the present paper an elementary introduction to quantum phase transitions is given. A few experimental examples from the physics of heavy-fermion systems and itinerant ferromagnets are described.

1. Introduction

Recent studies in the field of strongly correlated electron systems have largely been concentrated on the so-called quantum phase transitions or quantum critical phenomena. And although a number of monographs and numerous reviews [1 – 9] have already been devoted to this comparatively new problem, it is still very far from being completely resolved.

As distinct from classical phase transitions, quantum phase transitions result from nonthermal quantum fluctuations, which arise due to the uncertainty principle [1]. The concept of quantum phase transitions was first introduced by J Hertz in 1976 [10] [2], who showed that in view of the inextricable relation between the static and dynamic properties of a quantum system its time characteristics significantly affect the behavior of the substance in the critical region at $T = 0$ and the effective dimension of a quantum system always exceeds its spatial dimension [3]. The latter fact largely influences the behavior of a substance in the critical region.

Quantum phase transitions in a pure form only occur at $T = 0$, although their effect on the properties of the substance can spread to the region of finite temperatures. We will emphasize that here and in what follows we only deal with continuous or second-order phase transitions. First-order quantum phase transitions (e.g., helium melting) do not have a fluctuation region and are of no interest in this context.

Further, we will need a brief review of classical phase transitions, which will be presented in the next section.

2. Brief review of phase transitions

The theory of second-order phase transitions developed by Landau is based on the concept of the order parameter η (for more details concerning second-order phase transitions see [14 – 16]). In the framework of the Landau theory the order parameter η near the transition point behaves as

$$\eta \propto |t|^{1/2}, \tag{1}$$

where $t = (T - T_c)/T_c$.

Heat capacity, the coefficient of thermal expansion, and compressibility experience finite jumps at the transition point.

In the general case, t in (1) should be replaced by the quantity δ which is a certain dimensionless distance to the phase transition point and is expressed, depending on the particular case, in terms of pressure, magnetic field strength, concentration, etc. The behavior of the thermodynamic (physical) quantities $y_1, \ldots, y_N$ in the vicinity of a phase transition point is typically expressed by power functions of the form

$$y \propto |\delta|^{-x}, \tag{2}$$

[1] Although generally accepted, this formulation should obviously be explained. It means that quantum fluctuations, which destroy the long-range order in a system, are controlled by non-thermal parameters like pressure, concentration, magnetic field etc.

[2] Part of the results of that paper were revised in Ref. [11].

[3] This circumstance was probably first noted in Ref. [12] (see also [13]) in the analysis of the quantum two-dimensional Ising model with a spin 1/2 in a transverse magnetic field.

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where x is the so-called critical exponent determined from the relation

$$x \equiv \lim_{\delta \to 0} \frac{\ln y(\delta)}{\ln \delta} . \tag{3}$$

With such a definition, the critical indices of 'nonsingular' quantities, quantities with logarithmic divergence, or quantities with a 'peak-like' singularity turn out to be equal to zero.

In a simple version, the Landau theory totally ignores spatial fluctuations of the order parameter and is therefore invalid in a region close to a phase transition. However, in some cases the fluctuation region is so narrow that it cannot be resolved in a real experiment. The simplest way to allow for fluctuations is to expand the thermodynamic potential, not only in the power series of the order parameter, but also in its gradients. The first significant term of the gradient expansion is of the form

$$\delta \Phi \propto (\Delta \eta)^2 \simeq \xi^2 \left(\frac{\partial \eta}{\partial x} \right)^2 . \tag{4}$$

The quantity ξ with dimensionality of length is called the correlation length and characterizes the spatial inhomogeneity of the system. In the framework of the Landau theory the correlation length has the form

$$\xi = \xi_0 |t|^{-1/2} . \tag{5}$$

In the general case the correlation length is written as

$$\xi = \xi_0 |t|^{-\nu} . \tag{6}$$

Using relation (4) one can estimate the width of the fluctuation region and, correspondingly, the range of applicability of the Landau theory from the expression for the relative mean-square order parameter fluctuation (see [17, 18]),

$$\frac{(\Delta \eta)^2}{\eta^2} = \frac{T_c^2}{\Delta C_p \xi_0^d (T - T_c)^{2-d/2}} , \tag{7}$$

where d is space dimensionality.

Expression (7) also establishes an important relation between the space dimensionality and the intensity of fluctuations. From (7) it follows that for $d \geqslant 4$ the order parameter fluctuations are finite for any t and, accordingly, diverge for $d < 4$. The dimension $d = 4$ is in this case called the upper critical dimension — d_c^+. A lower critical dimension d_c^- also exists, for which the long-range order is absent in the system at any finite temperature.

Note that at the tricritical point

$$\frac{(\Delta \eta)^2}{\eta^2} \propto \frac{T_c}{\xi_0^d (T - T_c)^{(3-d)/2}} , \tag{8}$$

the upper critical dimension is $d_c^+ = 3$.

Let us now focus our attention on the relation between static and dynamic phenomena in the critical region:

$$\tau \propto \xi^z , \tag{9}$$

where τ is the relaxation time of the order parameter and z is the dynamic critical exponent.

It is appropriate to denote τ as ξ_τ and rewrite (9) in the form

$$\xi_\tau \propto \xi^z . \tag{10}$$

Relations (6) and (10) imply

$$\xi_\tau \propto |t|^{-z\nu} \tag{11}$$

or

$$\omega^* \propto |t|^{z\nu} , \tag{12}$$

where ω^* is the characteristic frequency of fluctuations.

3. What are quantum phase transitions?

Phase transitions occurring at $T = 0$ upon variation of variables determining the intensity of quantum fluctuations are called quantum phase transitions.

We will consider Fig. 1, where $T_c(P)$ is the line of a continuous phase transition corresponding, for example, to a certain magnetic transformation. At a normal pressure, the phase transition temperature has an entirely finite value. An increase in pressure leads to a progressive fall in the transition temperature, down to $T = 0$ K at a certain critical pressure P_c (another control parameter, for instance, the magnetic field, concentration, etc., may be used here instead of pressure). The 'disordered' phase due to quantum fluctuations at $T = 0$ is a realization of 'quantum' disorder, which is essentially different from 'classical' disorder. In particular, for a magnetic quantum phase transition, the paramagnetic phase cannot be treated as a system of individual spins fluctuating, for example, between 'up' and 'down' states in real time. The ground state of a quantum paramagnet is described by the wave function, which is a quantum superposition of these states and therefore possesses zero entropy [1].

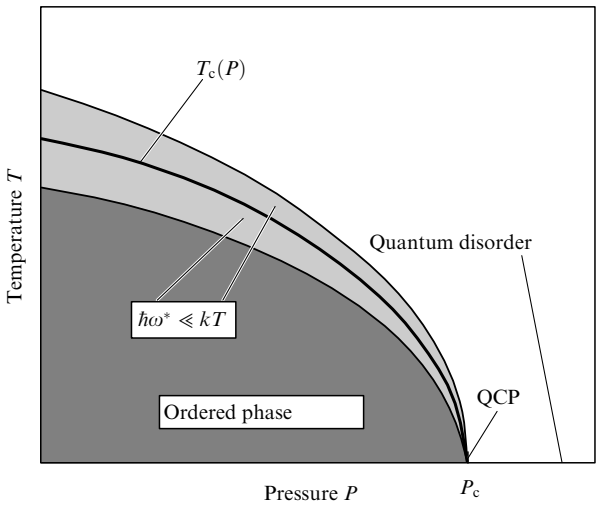

Figure 1. Schematic phase diagram of a substance undergoing a second-order phase transition where there is a negative slope of the transition curve $dT/dP < 0$. ω^* is the characteristic frequency of fluctuations, which vanishes at the phase transition point. The region around the phase transition line $T_c(P)$ is the region of classical fluctuations. $dT/dP = \infty$ at the quantum critical point (QCP) for $P = P_c$.

In this case, the quantum phase transition at $T = 0$ is a limiting case of the classical phase transition that occurs at $T \neq 0$. However, as has been mentioned above, the situation is possible (a system in a state of the lower critical dimension) where a phase transition may occur at $T = 0$ [4] only.

Phase transitions occurring at $T > 0$ can always be described in the framework of classical statistical mechanics (henceforth, the description of a phase transition is understood as an analysis of the behavior of the order parameter, correlation functions, and thermodynamic quantities in the vicinity of T_c). This also applies to such essentially quantum phenomena as superfluidity and superconductivity. The cause can readily be understood with the help of relation (12), implying that as $t \to 0$, that is, in the nearest neighborhood of T_c, the following inequality always holds:

$$\hbar\omega^* \ll kT, \tag{13}$$

which corresponds to the classical behavior of critical fluctuations. According to (13) the selected region around the phase transition line is the region of classical fluctuations [19, 20]. This naturally does not mean that quantum mechanics plays no role in this case. Quantum mechanics determines the very existence of the order parameter, whereas its behavior in the critical region [5] at $T > 0$ is controlled precisely by the classical thermal fluctuations.

It is a known fact [1, 4, 10, 12, 13, 19, 20] that the quantum statistical problem of a phase transition in a d-dimensional space at $T = 0$ can be reduced to the classical problem [6] with effective dimension $(d + 1)$. Imaginary time in the interval $[0, -i\hbar\beta]$, where $\beta = 1/kT$, stands as an additional coordinate here. Generally, the coordinate space of such a system is finite in the direction of time, but as $T \to 0$ the time interval becomes infinite and the system acquires all features of a classical system in a $(d + 1)$- dimensional space. However, when it concerns the critical properties of the system, its effective dimension appears to be equal to $d + z$, where z is the dynamic exponent. So, the effective dimension of a quantum system in the critical region at $T = 0$ may appear to equal or even exceed the upper critical dimension d_c^+ with all ensuing consequences (see above). This effect is illustrated in Fig. 2, which shows the results of the analysis of the evolution of the exponent β corresponding to the critical behavior of the order parameter (magnetization in this case) upon the phase transition in the antiferromagnet $MnCl_2 \cdot 4H_2O$ [21]. One can see in Fig. 2 that, as the temperature drops, the exponent β increases and tends to the mean-field value $\beta = 0.5$.

[4] This is the case, for example, with the one-dimensional Ising model.

[5] Quantum fluctuations are undoubtedly very important at distances on the order of interatomic ones, but fluctuations of much larger scale, with a correlation length of dozens and hundreds of interatomic spacings, which control the behavior of the system in the critical region at $T > 0$, are described adequately within classical statistical mechanics [4, 5, 19, 20].

[6] Recall in this connection that in the framework of fluctuation theory the values of critical exponents characterizing a phase transition are independent of the microscopic nature of a substance, but are defined by the symmetry of the Hamiltonian and the dimensionality of space, which determines the particular class of universality [15, 16, 19]. It has turned out that the critical exponents of the Ising quantum model in a transverse magnetic field at $T = 0$ correspond to a higher space dimension than the initial dimension of the problem [12, 13]. It is precisely this situation that was explained in the pioneering paper by Hertz [10].

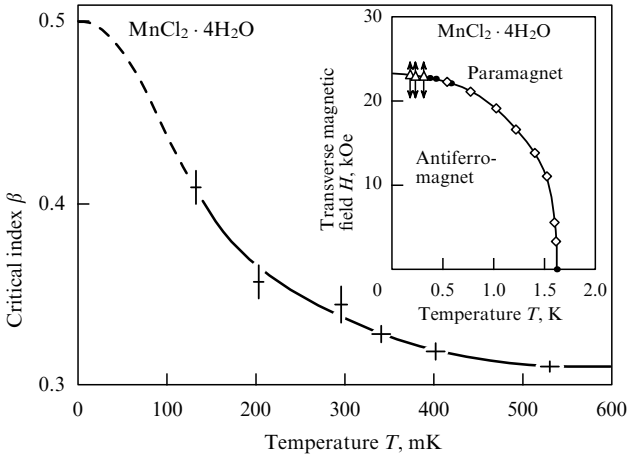

Figure 2. Dependence of the critical exponent β determining the behavior of the order parameter on the temperature of measurement for the antiferromagnet $MnCl_2 \cdot 4H_2O$ [21]. The inset gives the corresponding phase diagram in magnetic field H — temperature T coordinates.

One should also bear in mind that for $\xi_\tau < L_\tau$, where $L_\tau = \hbar/kT$ is the time extent of the space-time continuum, the system is unaware of being at a finite temperature and behaves as if it were in a $(d + 1)$-dimensional space. The lines $\xi_\tau = L_\tau$ conditionally divide the phase diagram of the substance with a quantum critical point (QCP) into regions with different effective dimensions. The same lines mark the crossover between phenomena occurring at small and large times characterized by the correlation time ξ_τ (Fig. 3). We should stress that for $\xi_\tau < L_\tau$ the description of the phenom-

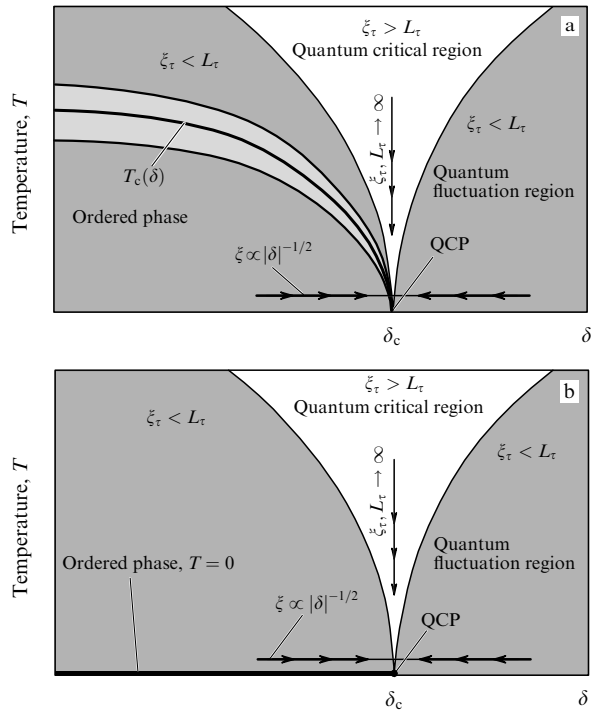

Figure 3. Schematic phase diagram of a substance in the vicinity of a quantum critical point: (a) ordered phase exists at finite temperatures; (b) ordered phase exists at $T = 0$ only. ξ is the correlation length, $\xi_\tau \propto \xi^z$ is the correlation time, $L_\tau = \hbar/kT$ is the extent of time coordinate. The lines $\xi_\tau = L_\tau$ correspond to the quantum-classical crossover.

ena requires a quantum-mechanical approach, whereas for $\xi_\tau > L_\tau$ the corresponding phenomena become classical because of the loss of phase coherence.

Figure 3 presents schematically the phase diagrams of substances possessing QCP for which two cases are possible: the ordered phase exists at $T = 0$ only (Fig. 3a) or the ordered phase exists also at $T > 0$ (Fig. 3b). The lines corresponding to the condition $\xi_\tau = L_\tau$ separate regions with predominantly classical and predominantly quantum fluctuations.

Thus, in the phase diagrams one can distinguish between regions corresponding to ordered and quantum-disordered (quantum-fluctuation) states of a substance and a region of mixed nature, referred to as the quantum critical region. Since practically always $d + z \geqslant 4$, the behavior of the correlation function in the former two regions corresponds to the Gaussian case $\xi \propto |\delta|^{-1/2}$. In the mixed region, along the trajectory $\delta = 0$ the behavior of the correlation function is controlled exclusively by the temperature, which indicates the absence of any other energy scales. For metallic systems it means that the Fermi energy E_F (or the Fermi temperature T_F) no longer plays the role of a universal scaling factor in the description of the electronic properties of materials in a quantum critical region, and accordingly the absolute temperature assumes this role as the energy scale. As will be seen below, this situation is due to the divergence of the effective mass of carriers at the QCP. It is this fact that determines the so-called non-Fermi-liquid behavior [22 – 24]. In systems whose effective dimension is less than the upper critical dimension, $d < d_c^+$, the absolute temperature determines the scale of all phenomena in the quantum critical region. This situation is described in the English literature as E/T or ω/T scaling (see, e.g., [6, 7]).

Section 4 presents a description of experimental examples illustrating some of the above statements. However, we will first give necessary explanations concerning the thermodynamics of phase diagrams of substances with quantum critical behavior. Figure 1 schematically shows a particular case where the phase transition line $T_c(P)$ at $T > 0$ has a negative slope, $dT_c/dP < 0$. This type of behavior is rather widespread but not the only possible. Cases are possible when dT_c/dP reverses sign at a certain positive pressure or when the derivative dT_c/dP is always positive. The latter case is of no interest in the present context. Naturally, the aforesaid is also valid when a control parameter other than pressure is considered.

We shall note further that the slope of the phase transition curve tends to infinity with approaching the QCP[7] ($dT_c/dP \to \infty$ as $T_c \to 0$). Similarly, for the phase diagram in coordinates T-H (H is the magnetic field) one can write $dT_c/dH \to \infty$ as $T_c \to 0$. If the concentration e of the impurity element is taken as the 'control' parameter, the situation becomes somewhat more complicated, but when the impurity, creating the so-called 'chemical pressure', plays to an extent a passive role, we can again write $dT_c/dc \to \infty$ as $T_c \to 0$. It should also be recalled that, as follows from the Nernst heat theorem, all temperature derivatives of thermodynamic quantities are equal to zero at the absolute zero of temperature[8]. Accordingly, the amplitudes of anomalies (jumps) of specific heat, thermal expansion coefficients, etc. observed during a high-temperature phase transition tend to zero when approaching the QCP. Hence, the results of measurements of specific heat and other thermal quantities along a trajectory corresponding to the critical coordinate $\delta = \delta_c$ are not perturbed by the nearness of the phase transition line.

Concluding this section, we will emphasize that in the classical case the phase transition temperature does not become zero for a finite value of the parameter δ, and therefore the very shape of the phase transition curves shown in Figs 1 and 3 is a manifestation of quantum effects.

4. Examples of systems with quantum critical behavior

4.1 Heavy-fermion compounds[9] $CeCu_{6-x}Au_x$, $YbRh_2Si_2$, $YbRh_2(Si,Ge)_2$, $CePd_2Si_2$, and $CeIn_3$

(i) The compound $CeCu_{6-x}Au_x$ has an antiferromagnetic ground state for $x < 0.1$ with a Neel temperature T_N that rises linearly with increasing x up to $x = 1$ (Fig. 4) [26, 27]. In the neighborhood of the quantum critical point for $x = x_c \approx 0.1$ the behavior clearly differs from that of a Fermi liquid[10] [27 – 29]. In particular, in the quantum critical region the heat capacity varies as $C/T = a\ln(T_0/T)$ and the static magnetic susceptibility depends on the temperature

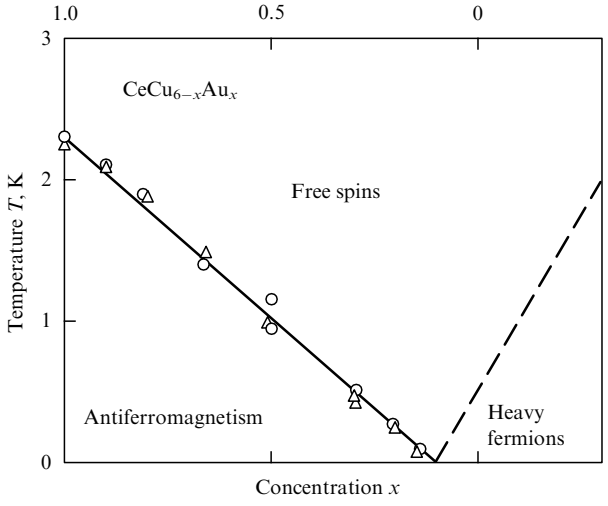

Figure 4. Concentration phase diagram of $CeCu_{6-x}Au_x$ [28]. Separate regions are named according to the nomenclature used in paper [28]. In terms of the diagrams presented in Fig. 3a, b, the free-spin region corresponds to the quantum-critical area.

[7] This conclusion is a corollary of the Nernst heat theorem stating that the entropy S is identically equal to zero at $T = 0$. From this, for first-order phase transitions we obtain directly from the Clausius – Clapeyron equation $dT_c/dP \to \Delta V/\Delta S$ (ΔV and ΔS are volume and entropy jumps upon the phase transition), $dT_c/dP \to \infty$ as $T_c \to 0$. For second-order phase transitions one should use one of the Ehrenfest equations: $dT_c/dP = \Delta(dV/dP)/\Delta(dS/dP)$. From the identity $S \equiv 0$ at $T = 0$ we again obtain $dT_c/dP \to \infty$ as $T_c \to 0$. However, the temperature at which the corresponding effects are observed can be fairly low.

[8] In this connection, the statement made in [25] about the divergence of the coefficient of thermal expansion at a quantum critical point looks rather strange.

[9] Heavy-fermion compounds constitute a class of metallic materials with a strong electron correlation that possess a chemically ordered lattice of magnetic ions (a Kondo lattice). In these compounds, conduction electrons interact with local magnetic moments of the ions. As a result, the effective mass of conduction electrons becomes very large [26].

[10] In the case of a normal Fermi liquid we have $C \sim \gamma T$ or $C/T = \text{const}$; $\chi = \text{const}$ and $\Delta\rho \sim T^2$.

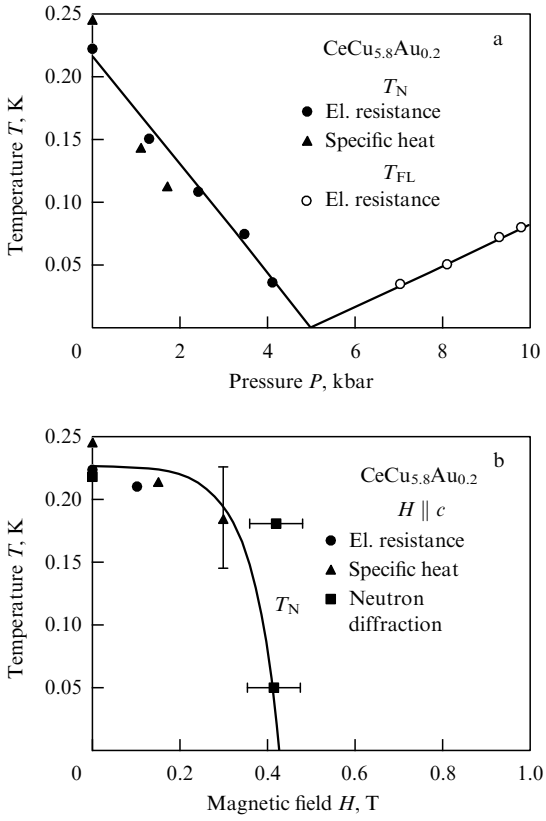

Figure 5. Phase diagram of the low-temperature antiferromagnet $CeCu_{5.5}Au_{0.2}$: (a) at high pressures; (b) in a magnetic field. T_{FL} marks the boundary of the Fermi-liquid regime [29].

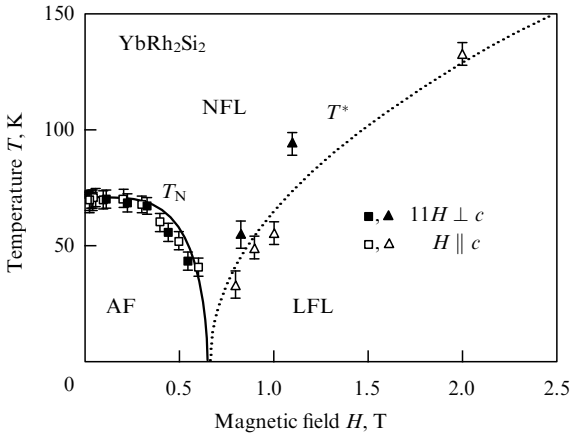

Figure 6. Phase diagram of $YbRh_2Si_2$ in a magnetic field [30]. T_N is the Neel temperature and T^* is the boundary of Fermi-liquid behavior. The values of the corresponding quantities in the perpendicular field are multiplied by 11. AF is an antiferromagnet, NFL is a non-Fermi liquid, and LFL is a Landau Fermi liquid.

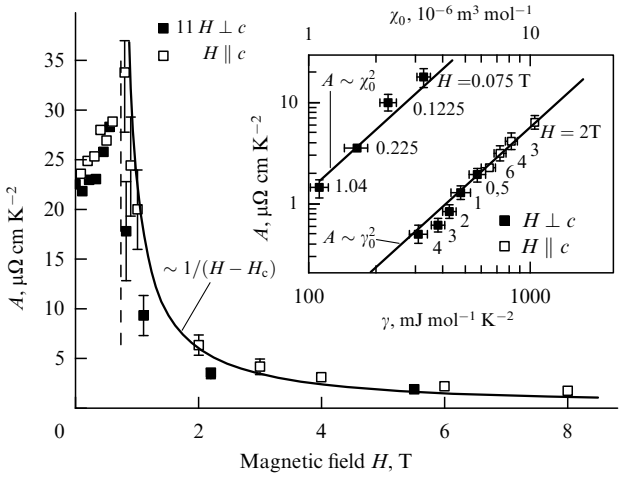

Figure 7. Dependence of the coefficient $A = \Delta\rho/T^2$ on the magnetic field for $YbRh_2Si_2$ [30]. The data for $H \perp c$ are multiplied by 11. The dashed line marks the H_c values, the solid curve corresponds to the dependence $\sim 1/(H - H_c)$. The inset gives the dependencies $A(\gamma_0)$ and $A(\chi_0)$ on a double logarithmic scale.

according to the expression $\chi^{-1} = \chi_0^{-1} + cT^a$, $a < 1$; the electrical resistance behaves quasilinearly, namely, $\Delta\rho = \rho - \rho_0 \sim T^m$, $m \approx 1$. For the compound $CeCu_{6-x}Au_x$ the quantum critical point can be reached under hydrostatic pressure (e.g., for $x = 0.2$ the critical pressure is $P_c \approx 5$ kbar, Fig. 5a), and the corresponding temperature dependences have essentially the same form as dependences obtained in a 'chemical' realization of the QCP. The situation is different when the quantum critical point in $CeCu_{6-x}Au_x$ is attained through a variation of the magnetic field (Fig. 5b). In this case, specific heat in the quantum critical region varies according to the expression $C/T = \gamma_0 + a''T^{0.5}$ and the electrical resistance is described by the formula $\rho = \rho_0 + A''T^{1.5}$. The authors of paper [29] believe that the magnetic field has a substantial effect on the character of fluctuations in the compound $CeCu_{5.8}Au_{0.2}$ at a QCP. We note, however, that the shape of the phase transition line $T_N(P)$ contradicts the Nernst heat theorem, and it therefore can not be excluded that a more accurate determination of the QCP coordinate may affect the conclusions drawn in [29].

(ii) As was shown in paper [30], the heavy-fermion antiferromagnet $YbRh_2Si_2$ can be transformed into a paramagnetic state by a comparatively small magnetic field $H_c \approx 0.6$ T (Fig. 6). Electrical resistance in the antiferromagnetic state is best described by the expression $\Delta\rho = AT^2$ with a very large coefficient $A = 22$ μΩ cm K^{-2} in the region $20 \leqslant T \leqslant 60$ mK and $H = 0$. In a magnetic field $H = H_c$ the temperature dependence of the resistance follows the linear law up to the lowest attainable temperatures (20 mK). For $H > H_c$ and $T < T^*$ (see Fig. 6), electrical resistance is again described by the Fermi-liquid expression $\Delta\rho = AT^2$. Being proportional to the electron-electron scattering cross section, the coefficient A diverges as $A(H) \propto 1/(H - H_c)$ when $H \to H_c$ (Fig. 7). An analysis of the behavior of electrical resistance and specific heat in longitudinal and transverse magnetic fields suggests the quasiparticle effective mass divergence $1/(H - H_c)^{1/2}$ when $H \to H_c$. This conclusion was confirmed in a study of the doped compound $YbRh_2(Si_{0.95}Ge_{0.05})_2$ in weak magnetic fields [31]. In this case, a small germanium impurity expands the lattice to shift the magnetic coordinate of the QCP nearer to the value $H = 0$, which makes it possible to check whether any new property appears in a QCP obtained in a strong magnetic field. In the particular case of $YbRh_2(Si,Ge)_2$ the properties of QCPs with coordinates $H_c \approx 0$ and $H_c \neq 0$ turn out to be identical. In both compounds in the quantum critical region for $H = H_c$ the magnetic susceptibility behaves like $\chi^{-1} \propto T^{\alpha}$, where $\alpha \approx 0.75$ (0.3–1.5 K), and the coefficient of electronic specific heat C_{el}/T diverges logarithmically at

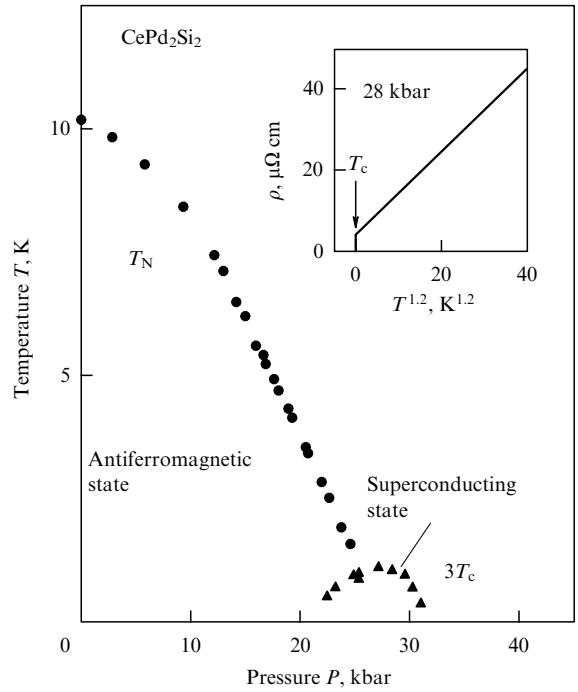

Figure 8. Phase diagram of $CePd_2Si_2$ [32]. Superconductivity occurs in a narrow pressure range in the region where the Neel temperature T_N tends to zero. For clarity, the superconducting transition temperatures T_c are multiplied by 3. The inset illustrates the non-Fermi-liquid behavior of $CePd_2Si_2$ electrical resistance for $P = 28$ kbar.

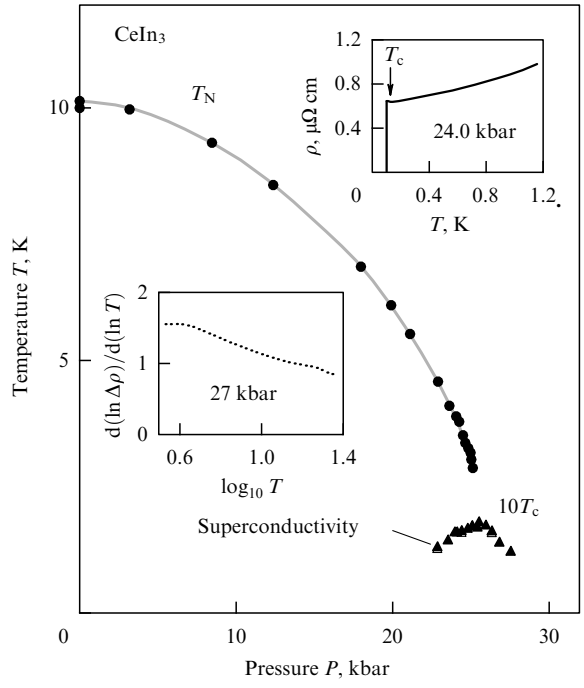

Figure 9. Phase diagram of $CeIn_3$ [32]. The upper inset shows that the superconducting transition is completed even at a pressure below critical. The lower inset demonstrates that the electrical resistance in the normal state varies as $T^{1.6\pm0.2}$ at a pressure above critical. The superconducting transition temperatures are multiplied by 10.

temperatures between 0.3 and 10 K. The electrical resistance exhibits a linear temperature dependence in the range of 0.02 to 0.5 K.

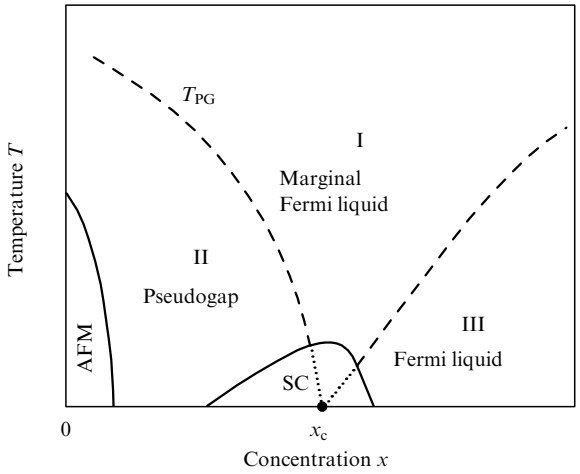

Figure 10. Generalized phase diagram of superconducting cuprates with hole doping [9]. T_{PG} is the boundary of the pseudogap state. AFM is the antiferromagnetic phase, SC is the superconducting phase, x_0 is a virtual quantum critical point. The properties of cuprates in region I resemble the properties of heavy-fermion metals in the quantum-critical region.

(iii) $CePd_2Si_2$, and $CeIn_3$ [32] are heavy-fermion compounds with an intriguing phase diagram (Figs 8, 9). One can see from the figures that the phase diagrams of these compounds are similar. The antiferromagnetic phase transition temperature T_N drops in both cases as pressure increases. In both cases, the phase transition curve should have crossed the pressure axis with the formation of a QCP crowning the antiferromagnetic-paramagnetic transition. However, at low temperatures both curves actually end at the top of the 'superconducting dome', which evidently points to the genetic relation between quantum critical phenomena and superconductivity in these magnetic systems. In this connection it is of interest to compare the phase diagrams of Figs 8 and 9 with the generalized phase diagram of superconductive cuprates (Fig. 10) [9]. Isomorphism of the phase diagrams becomes obvious if we assume the pseudogap line T_{PG} to be in a sense equivalent to the magnetic phase transition curve.

It is noteworthy that the temperature dependence of the electrical resistance of both substances in the quantum critical region does not correspond to Fermi-liquid behavior.

4.2 Itinerant ferromagnets UGe_2 and $ZrZn_2$

The properties of the ferromagnetic compound UGe_2 [33–35] represent another interesting example of the relation among quantum critical phenomena, magnetism, and superconductivity. As Fig. 11 shows, a small superconducting domain in this case lies entirely in the region of ferromagnetic phase stability near the phase boundary. It is of importance that, as established with the help of elastic scattering of neutrons, ferromagnetic ordering is preserved in the transition to a superconducting state. At low temperatures, a ferromagnetic phase transition becomes a first-order phase transition, and therefore the role of quantum fluctuations in the possible formation of a triplet superconducting state in UGe_2 seems to be vague. Useful information in this respect can be gained from the phase diagram of another ferromagnetic superconductor, $ZrZn_2$ [36] (Fig. 12). The relation between ferromagnetism and

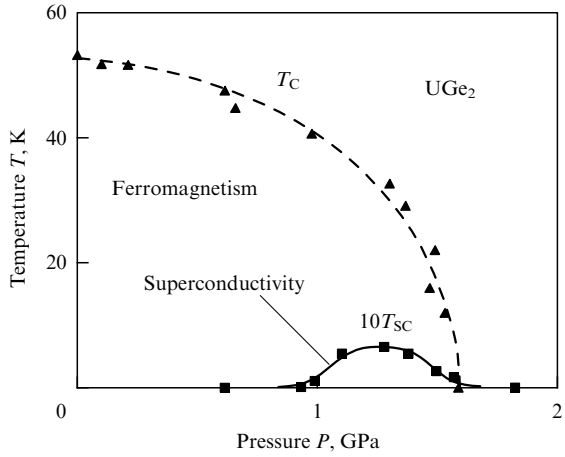

Figure 11. Phase diagram of UGe_2 [33]. T_C is the Curie temperature, T_{SC} is the superconducting transition temperature (T_{SC} values are multiplied by 10).

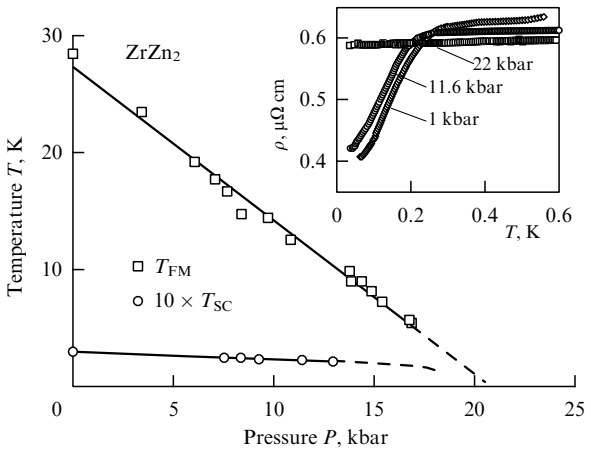

Figure 12. Pressure dependence of ferromagnetic (T_{FM}) and superconducting (T_{SC}) transition temperatures in $ZrZn_2$ [36] (T_{SC} values are multiplied by 10).

superconductivity seems obvious in this case[11], but the quantum critical fluctuations have apparently nothing to do with this.

With this it seems expedient to conclude the present review, which may to some extent be called an introduction to the subject. Its goal is to draw the attention of Russian researchers to an important and rapidly developing trend in the physics of strongly correlated systems. Although not exhaustive, the list of references includes the most important original studies and reviews. The present survey is based on a talk given at the seminar 'Strongly correlated electron systems and quantum critical phenomena' held on April 11, 2003 in Troitsk.

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[11] The recent study of the phase diagram of $ZrZn_2$ [37] is apparently indicative of the absence of a direct relation between superconductivity and magnetism in this compound because a superconducting phase transition is also observed in the paramagnetic phase stability region.