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METHODOLOGICAL NOTES

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Negative refractive index for a surface magnetostatic wave propagating through the boundary between a ferrite and ferrite-insulator-metal media

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Contents

__Abstract.__ **Refraction of a slow surface electromagnetic wave (magnetostatic wave) at the boundary between ferrite and ferrite-insulator-metal media is investigated experimentally and theoretically. The boundary is created in an yttrium iron garnet film by placing a metal plate at a certain distance from its surface. The refractive index is found to depend on the angle of incidence of the wave and can take on any positive or negative values. It is shown that in anisotropic media, in particular, in ferromagnets, due to the noncollinearity of the wave vector and the group velocity vector, negative refraction can occur not only in the earlier predicted case where the incident wave is forward and the refracted wave is backward, but also in the case where both waves are forward.**

1. Introduction

At present, considerable interest in isotropic composite media with negative electrical permittivity ε and negative magnetic permeability μ is due to several unusual effects that manifest themselves in such media. These are the inversion of the Doppler frequency shift, a change in the direction of the Cherenkov radiation, attraction of the electromagnetic radiation instead of exerting pressure from its side, and the existence of a negative refractive index at the boundary between such a medium and a vacuum [1]. Since no natural continuous media with negative ε and μ have been found so far, it has been suggested that such a medium be created artificially by alternating layers (or elements) with negative ε and positive μ and layers (elements) with positive ε and negative μ [2]. If the wavelength λ of the wave propagating in a composite medium is much larger than the layer thickness (size of the elements), then the composite medium behaves similarly to a continuous one. As a result, negative refraction angles of an electromagnetic wave have been observed at the boundary between the artificially produced composite medium consisting of layered conductive elements and the air [3].

At the same time, it has been suggested in several theoretical papers that for observing the extraordinary effects it is possible to use saturation-magnetized ferromagnets for the layer with negative magnetic permeability, which enters into the composite material. These ferromagnets possess negative effective $\mu_{\perp}$ in a certain frequency range [4 – 6].

However, it is worth noting that some of the unusual effects mentioned above can be observed without creating and using composite materials with negative ε and μ. Indeed, a negative refractive index appears not due to the negative values of ε and μ but because of the fact that an electromagnetic wave in a medium with negative ε and μ is a backward wave. For a backward wave in an isotropic medium, the momentum direction is opposite to the direction of the energy propagation. Therefore, negative refraction coefficient of the wave propagating through the boundary between two media results from the conservation of its momentum tangential component. But the momentum direction and the wave energy propagation direction (and, hence, the sign of the refraction) are fully determined by the form of the isoenergetic curves which are always circles in the isotropic media. At the same time, the conditions for negative refraction can be easily established in an anisotropic medium where isoenergetic curves can be elliptic or hyperbolic. In particular, such 'anomalous' refraction might be realized in ferromagnets where both forward and backward waves with low losses can be excited and propagate, due to the frequency

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dispersion of the magnetic permeability tensor components. For instance, anomalous refraction is observable in structures based on ferrite films or slabs under the condition that both forward and backward waves with the same frequency can propagate in neighboring layers. Moreover, calculations show that anomalous refraction is also possible in such structures for the case where both incident and refracted waves are forward ones. It is namely this case that we study in the present work.

Notice that anomalous refraction in ferrite-film structures was observed in our earlier works on the propagation of film-localized electromagnetic waves through the boundaries between various media [7, 8].

2. Definitions of forward and backward waves in an anisotropic medium

Before proceeding with our consideration, it is necessary to generalize the definitions of forward and backward waves to the case where the group velocity $\mathbf{v}$ of the wave and its wavevector $\mathbf{k}$ are not collinear, as occurs in an anisotropic medium. According to the well-known definition [9, p. 383], a forward wave constitutes a wave with the unidirectional vectors $\mathbf{v}$ and $\mathbf{k}$, while a backward wave is a wave with $\mathbf{v}$ and $\mathbf{k}$ directed oppositely. This definition is valid only for an isotropic medium. In the general case, a forward wave should be defined as one in which the scalar product of $\mathbf{v}$ and $\mathbf{k}$ is positive, $\mathbf{vk} > 0$, and a backward wave as one with $\mathbf{vk} < 0$. In the case where $\mathbf{vk} = 0$, which is possible only theoretically, the wave does not propagate and has $|\mathbf{v}| = 0$. The above definitions are most general and can be applied to determining the character of waves in any medium. Note that these criteria have been used in the literature for a long time when describing the character and name of waves in anisotropic magnetic media (examples are numerous papers about forward and backward magnetostatic waves) but still seem to be absent in textbooks and encyclopedias [1].

3. Statement of the problem. The experimental scheme

To produce structures with various dispersion properties, we used epitaxial films of yttrium iron garnet (YIG) on gadolinium gallium garnet (GGG) substrates. In these films, both forward and backward slow electromagnetic waves can be easily excited and propagate with low losses. Such waves are called magnetostatic waves (MSW) in the literature, because due to their small phase velocity ($\sim 10^7 - 10^8$ cm s^{-1}), they can be described without the retarding terms in the Maxwell equations, i.e., using the magnetostatic approximation. Since both the wavelength of magnetostatic waves in the microwave range ($\lambda \sim 50$ μm – 2 mm) and the thickness of YIG films ($\sim 10 - 20$ μm) are rather small, forming the interface between the edges of different films is technically challenging. In practice, it is rather difficult to obtain films with very flat straight edges and to join two films without a space between them. Therefore, for our experiment we created two media with different dispersion properties using only one film.

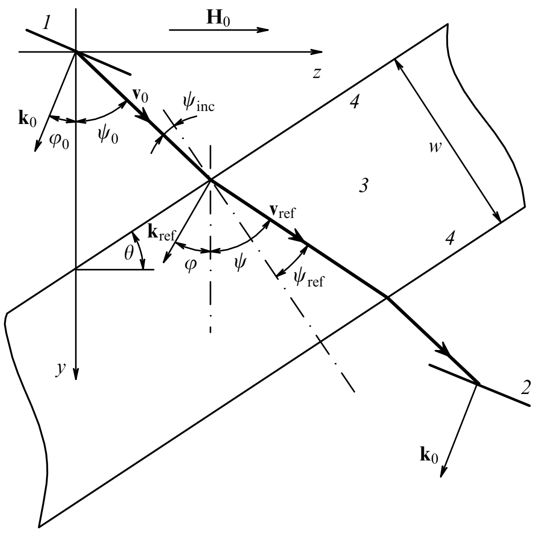

Figure 1. Geometry of the experiment for the case of 'normal' refraction of the wave at the F/FIM interface: *1* and *2* — the excitation and receiving transducers, respectively, *3* — the FIM medium, and *4* — the F/FIM interfaces.

Schematic of the experiment in the plane of the ferrite film is shown in Fig. 1.

The film used in the experiment was a gallium-doped YIG film in the form of a circle 60 mm in diameter and having $4\pi M_0 = 840$ G, $2\Delta H = 0.6$ Oe, and thickness $s = 16$ μm. The film was magnetized to saturation by a tangent uniform magnetic field with $H_0 \approx 570$ Oe.

The surface MSW was excited and detected by mobile antennae whose transducers (*1* and *2* in Fig. 1) were made of gilded tungsten wire of thickness 12 μm and length 3.5 mm. It is convenient to represent the results in a Cartesian frame of reference with the origin at the center of the excitation transducer and the z-axis parallel to the $\mathbf{H}_0$ vector.

At a distance d from the YIG-film surface, a polished rectangular copper plate with width $w = 6$ mm and length 80 mm was placed parallel to the film. Thus, the free part of the film surface with the neighboring space constituted the 'ferrite' (F) medium, while the part of the film surface and the air gap under the metal plate the 'ferrite-insulator-metal' (FIM) medium [2] (*3* in Fig. 1). Projections of the metal plate onto the film surface (*4* in Fig. 1) could be considered as the F/FIM interfaces. It should be noted, however, that in the vicinity of such boundaries the wavelength λ varies smoothly within some distance on the order of the wavelength, and not stepwise, as in the case of wave propagation through an ideal plane interface. In this sense, the F/FIM interfaces can be called 'distributed'. Still, under the conditions $d \ll w$ and $\lambda \ll w$, the F/FIM interfaces can be considered as infinitely thin.

By means of special mechanical apparatus, both antennae and the metal plate could be moved in the plane of the YIG film and rotated around the axis orthogonal to the film surface. Rotation of the antennae enabled us to vary the wavevector $\mathbf{k}_0$ direction (the φ_0 angle) and the group velocity

[1] For instance, it is pointed out on page 384 of Ref. [9] that "in an anisotropic medium, the notions of forward and backward waves are fully applicable only to specific directions related to the principal axes of the susceptibility or deformation tensors".

[2] We deliberately do not mention the GGG substrate and the neighboring half-space, since they have almost no effect on the phenomena under study. Indeed, the MSW energy is localized near the ferrite film and the wave decays exponentially as the distance from the film increases.

vector $\mathbf{v}_0$ direction (the ψ_0 angle) in the F medium. The angles φ_0 and ψ_0 in the F medium and similar angles φ and ψ in the FIM medium are reckoned from the optic axis (the y-axis in Fig. 1). This approach is typical for studies of MSW, since the vectors $\mathbf{k}_0$ and $\mathbf{v}_0$ have the same direction for a wave propagating along the optic axis. The angles of incidence (ψ_{inc}) and refraction (ψ_{ref}), as is usual in optics and electrodynamics, were reckoned from the normal to the interface surface. Orientation of the latter (the angle θ) was defined with respect to the optic axis y. As one will see from further consideration, this way of introducing the angles is most convenient for both describing the wave refraction and correlation of the parameters sought in the yz-plane and in the plane of $k_y k_z$ wavenumbers (isofrequency plane). All angles were assumed to be positive in the counterclockwise direction. Evidently, in this case the angles of incidence and refraction of the wave, ψ_{inc} and ψ_{ref}, are related to the interface orientation θ and the group velocity orientation ψ via the following expressions

$$\psi_{\text{inc}} = \psi_0 - \theta \,, \tag{1}$$

$$\psi_{\text{ref}} = \psi - \theta \,. \tag{2}$$

The angles and distances were measured by means of a microscope placed above the YIG film.

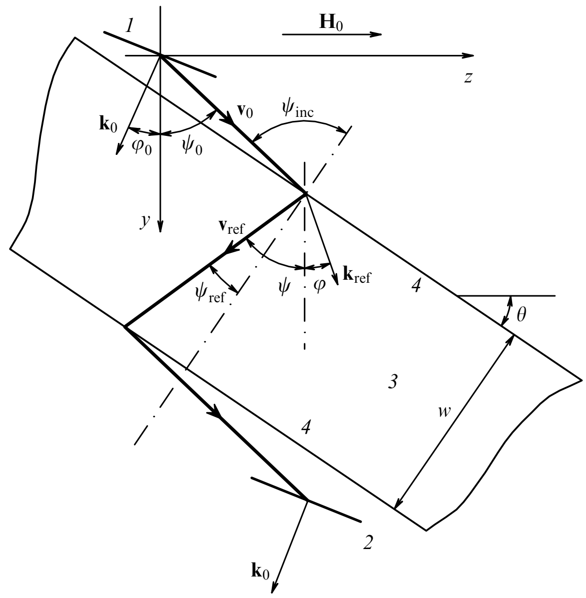

Figure 3. Geometry of the experiment for the case of anomalous wave refraction at the F/FIM interface: *1* and *2* — the excitation and receiving transducers, *3* — the FIM medium, and *4* — the F/FIM interfaces.

4. Calculated and measured results

For the analysis of the wave refraction, consider the wavevector surface which in the two-dimensional case under study is given by isofrequency curves. Using the theoretical approach of Refs [10, 11], we have calculated the isofrequency curves in an FIM medium (Fig. 2). The results showed that wave refraction varying from normal to anomalous could be observed at the F/FIM interface, for instance, for an incident wave with frequency $f_0 = 2610$ MHz, wavelength $\lambda_0 = 0.33$ mm, the angles $\varphi_0 = -21°$, $\psi_0 = 45°$, and the insulator spacing $d_0 = 100$ μm. Then, for the F/FIM interface oriented at $\theta = 35°$, one will observe normal wave refraction (see Fig. 1) with the angle of incidence $\psi_{\text{inc}} = \psi_0 - \theta = 10°$ and the angle of refraction $\psi_{\text{ref}} = 22°$. For the F/FIM interface oriented at $\theta = -35°$, there will be anomalous wave refraction (Fig. 3) with the angle of incidence $\psi_{\text{inc}} = \psi_0 - \theta = 80°$ and the angle of refraction $\psi_{\text{ref}} = -20°$. (As an example, Fig. 2 shows the orientation of the interface, the wavevector, and the group velocity vector of the refracted wave for the case of anomalous refraction at $\theta = -35°$.)

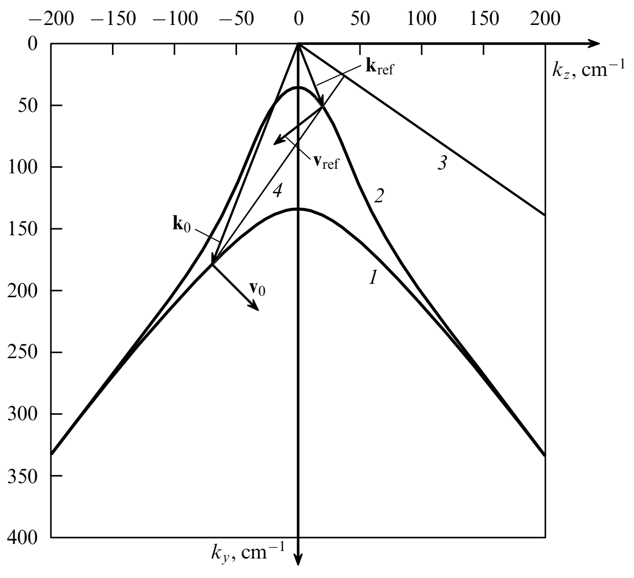

Figure 2. Isofrequency curves for the surface MSW with the frequency 2610 MHz: *1* — for a free ferrite film (the FIM medium at $d \to \infty$), *2* — for the FIM medium at $d = 100$ μm, *3* — orientation of the interface at $\theta = -35°$, *4* — perpendicular dropped from the end of the $\mathbf{k}_0$ vector to the interface for $\theta = -35°$ (illustrating the conservation of the tangential momentum component).

These calculations have been confirmed in experiments. Below, we briefly describe the method employed for measuring the angles of incidence and refraction. The receiving and excitation transducers were fixed, so that the perpendiculars to their apertures formed an angle φ_0 with the y-axis. The frequency was set to be f_0. First, the angle ψ_0 of the wave propagation in the F medium was measured in the absence of the metal plate. To do this, the receiving antenna was transposed along the z-axis until its position corresponded to the maximum of the registered signal; then the angle between the y-axis and the straight line connecting the centers of both transducers was measured. Next, the metal plate was fixed at a distance d_0 from the YIG film at a certain angle θ, and the new position of the receiving antenna, corresponding to the maximal signal, was determined. For this new position, we measured the projection l_y of the distance between the centers of transducers onto the y-axis and the angle β between the y-axis and the straight line connecting the transducer centers. After taking some auxiliary geometric constructions (not shown in Figs 1 and 3), one can obtain the following formula for the refraction angle ψ_{ref}:

$$\psi_{\text{ref}} = \arctan \frac{\tan\psi_0 + l_y \cos\theta(\tan\beta - \tan\psi_0)/w}{1 - l_y \sin\theta(\tan\beta - \tan\psi_0)/w} - \theta \,, \tag{3}$$

where w is the extent of the FIM medium, which is equal to the width of the metal plate.

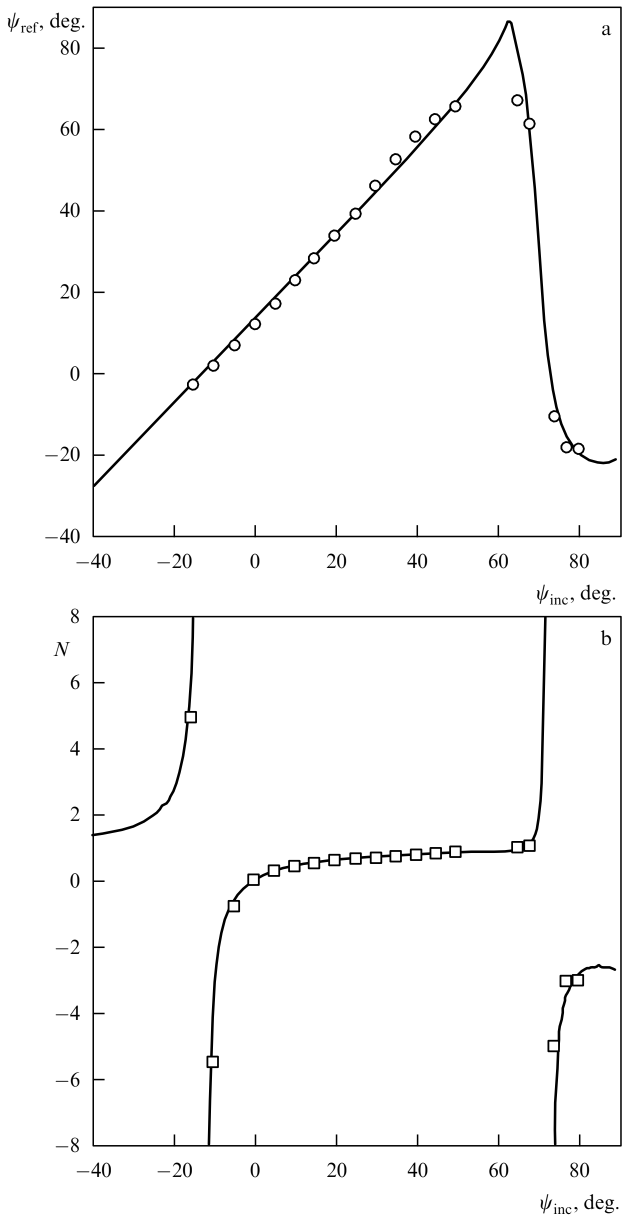

Figure 4. Refraction of a surface MSW at the F/FIM interface for $d = 100$ μm, the parameters of the incident wave being $f_0 = 2610$ MHz, $\lambda_0 = 0.33$ mm, $\varphi_0 = -21°$, and $\psi_0 = 45°$. Solid lines show theoretical results, points denote the results of measurement: (a) the angle of refraction as a function of the angle of incidence, $\psi_{ref}(\psi_{inc})$, and (b) the refractive index as a function of the angle of incidence, $N(\psi_{inc})$.

For an isotropic medium, it is not important whether the dependence $\psi_{ref}(\psi_{inc})$ was measured by rotating the interface or by rotating the transmission antenna. In contrast to this case, for an anisotropic medium these two methods would lead to completely different results. Clearly, it is more convenient to apply the first method, since in this case the *parameters of the incident wave* (the wavelength λ_0, the vectors $\mathbf{k}_0$, $\mathbf{v}_0$, and the attendant angles φ_0, ψ_0) *remain constant*, while in the second method setting each new angle of incidence leads to a change in the incident wave parameters (because of the antenna orientations varying with respect to the homogeneous magnetic field $\mathbf{H}_0$). For this reason, our measurements were performed using the first method.

Thus, the angles ψ_{ref} were determined from formula (3), and the angles ψ_{inc} from formula (1) for all possible orientations of the metal plate.

The dependences $\psi_{ref}(\psi_{inc})$ predicted theoretically and established experimentally are shown in Fig. 4a. Evidently, in the case under study the refractive index N calculated as

$$N = \frac{\sin\psi_{inc}}{\sin\psi_{ref}} \qquad (4)$$

is not constant, as in the case of isotropic media, but depends on ψ_{inc} and ψ_{ref}. The dependences $N(\psi_{inc})$ calculated from Eqn (4) and measured in experiment are shown in Fig. 4b.

One can see from Fig. 4 that the experimental evidence is in good agreement with the theoretical prediction. Let us briefly comment on some essential features typical for the case under study and not observed for wave refraction in isotropic media.

(1) As Fig. 4a suggests, the dependence $\psi_{ref}(\psi_{inc})$ does not pass through the point with the coordinates $(0; 0)$. In other words, a wave incident normally to the interface deflects from the normal to the boundary after refraction. This is caused by the fact that the vectors $\mathbf{k}_0$ and $\mathbf{v}_0$ in the incident wave are not collinear. If the incident wave had collinear vectors $\mathbf{k}_0$ and $\mathbf{v}_0$ (for instance, if the wavenumber $\mathbf{k}_0$ of the incident wave were directed along k_y, see Fig. 2), then the dependence $\psi_{ref}(\psi_{inc})$ would pass through the $(0; 0)$ point. In addition, the dependence $\psi_{ref}(\psi_{inc})$ possesses extremum points, which are not observed for wave refraction in isotropic media.

(2) Dependence $N(\psi_{inc})$ (Fig. 4b) display regions with a negative refractive index. The refractive index changes its sign from positive to negative at $\psi_{inc} = 0$, and from negative to positive at $\psi_{ref} = 0$ (Fig. 4a, b); therefore, the values of $\psi_{inc} = 0$ and $\psi_{ref} = 0$ bound the intervals where N takes on only positive values or only negative values. At the same time, for the refraction at the boundary between isotropic media, the refractive index N is either always positive or always negative.

(3) Within the whole range of angles ψ_{inc} and ψ_{ref}, including the regions where $N < 0$, both the incident wave and the refracted wave are forward ones. From the analysis of our results we infer that anomalous refraction with both incident and refracted waves being forward is only possible under the condition that the vectors **k** and **v** are noncollinear. This condition should be satisfied for both waves or at least for one of them. Indeed, if we used an incident wave with $\mathbf{k}_0$ parallel to the k_y-axis (see Fig. 2), we could still observe anomalous refraction. This fact can be explained as follows. The wavenumber **k** and the group velocity vector **v** bear the responsibility for quite different functions: while the vector **k** 'accounts' for the momentum conservation, the energy propagation direction is 'determined' by the vector **v** which is connected with **k** via the dispersion relation. This principal distinction between the vectors **k** and **v** manifests itself most obviously in anisotropic media where **k** and **v** are noncollinear.

In connection with the dependences in Fig. 4, we should also explain why no experimental points have been obtained for the angles $\psi_{inc} < -20°$ and in the vicinities of $\psi_{inc} \approx 60°$ and $\psi_{inc} \approx 70°$.

For $\psi_{inc} < -20°$, there are no experimental points because of the low intensity of the refracted wave. Indeed, for $\psi_{inc} < -45°$, the coefficient R of reflection from the F/FIM interface is equal to 1 and the refracted wave is absent; when $-45° < \psi_{inc} < -20°$, the coefficient R is still large, and only for $\psi_{inc} > -20°$ the reflection is sufficiently reduced, so that the refracted wave can be registered by means of the available transducers and other equipment. A description of the behavior of the reflection coefficient R and the results of its measurement can be found in Ref. [12].

The absence of experimental points in the vicinity of $\psi_{inc} \approx 60°$ (Fig. 4a) is caused by the fact that the calculated values of ψ_{ref} are in this case close to $\approx 80°$ or even higher. Therefore, the path length of the wave beam in the FIM medium is increased more than ten times compared to the extent w of the FIM medium (equal to 6 mm). Thus, the path length becomes on the order of 60 mm, which is comparable to the YIG-film diameter, and the measurement becomes impossible. Besides, this increase in the path length leads to a considerable increase in wave losses; for this reason, it is impossible to observe refraction with the angles ψ_{ref} larger than $\approx 70°$.

In addition to conventional measurement errors, there are other reasons for the slight difference between the measured and calculated values of ψ_{ref}.

(1) Since the F/FIM interfaces are 'distributed', the change in the wavelength at the interface is not stepwise; most probably, the change occurs over a distance comparable to the wavelength ($\lambda_0 = 0.33$ mm). Therefore, the actual extent of the FIM medium can be somewhat smaller than the width of the metal plate. This leads to additional errors in the calculations based on formula (3).

(2) As evident from Fig. 2, the wavelength λ_{ref} in the refracted wave beam can be much larger than in the incident beam. This leads to a rapid expansion of the refracted beam in the course of its propagation; as a result, the accuracy of adjusting the receiver to the maximum of the signal and, accordingly, the accuracy of measuring ψ_{ref} is reduced.

In conclusion, it should be emphasized that refraction of an MSW at an F/FIM interface essentially depends on the parameters of the media, the orientation of the interface with respect to the $\mathbf{H}_0$ vector, and the parameters of the wave. In the context of this paper it is impossible to consider the effect of all these parameters on the wave refraction. Therefore, our aim was to give a very general description of the anomalous refraction in ferrite films and to consider the basic distinctions between this effect and wave refraction in isotropic media.

5. Conclusion

We have carried out experimental and theoretical investigations of the refraction of slow electromagnetic waves (magnetostatic waves) at the boundary between a ferrite and ferrite-insulator-metal media. This boundary was created in a monocrystalline yttrium iron garnet film magnetized to saturation by a tangent homogeneous magnetic field. Investigation showed that, in contrast to refraction at the interface between isotropic media, in our case the refractive index N is not constant but depends on the angle of incidence of the wave and can take on any negative or positive values. It assumes the values of $N = 0$, $N \to -\infty$, and $N \to +\infty$ in the cases where the incident wave is normal to the interface, and the refracted wave is not (or vice versa). It is found that anisotropic media and, in particular, plane-parallel structures based on ferrite films can provide negative values of the refractive index not only in the case realized in isotropic media where the incident wave is forward and the refracted wave is backward, but also in the case where both waves are forward. Such a possibility arises in anisotropic media due to the fact that, in the general case, the wavenumber of a wave is noncollinear to its group velocity vector.

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