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Does the sign correlation of quasiperiodic oscillations with incommensurable frequencies generate deterministic chaos?

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Contents

1. Formulation of the problem

Nonlinear dynamics is an interdisciplinary scientific direction which comprises the theory of deterministic chaos and synergetics. Chaotic oscillations have been discovered in highly diversified objects in nature, ranging from coarse mechanical systems to highly organized biological systems. At present, this research is actively being carried out and is becoming progressively more application-oriented. In particular, systems with chaotic-type oscillations underlie the development of devices for radio countermeasures and radio camouflage, confidential communications, noise radar, and medical and biological applications. The search for new ways of designing the sources of deterministic chaotic oscillations for various practical applications is therefore a topical problem.

In the July 2001 *Usp. Fiz. Nauk* [1] the paper "Dynamic chaos interference in Hamiltonian systems: experiment and potential radiophysics applications" by N V Evdokimov, V P Komolov, and P V Komolov was published. On the face of it, an interesting method of producing dynamic chaos sources was proposed in this paper. Unlike the cumbersome and complicated version of its practical implementation adduced by the authors [1], there loomed very simple versions of the implementation of this method, reliant on all-digital devices. This is precisely the reason why this paper engaged, though with delay, our special attention. However, our numerical and natural experiments have provided evidence that the initial premises of the paper referred to are incorrect and, naturally, practical operational systems relying on these premises are impossible to develop.

The aim of my paper is to report to the reader the findings of my experiments, which will supposedly avert other researchers from unjustified labor and time expenditures. Since the paper is a basis for discussion, some of its portions might have been set forth too thoroughly and in excess detail. However, we endeavored to completely eliminate the possibility of any inexactness and variant reading, with the effect that the discussion might become formal in character.

2. Summary of the basic ideas of the work to be analyzed

The paper under discussion [1] is voluminous (21 p.), and therefore we will not undertake an analysis of the hypothetical possibilities for the radiophysics applications for oscillations with incommensurable frequencies but will turn to the study of the possibility of generating dynamic chaos on the basis of the approach proposed.

The heart of this approach is as follows:

(1) Let there be two periodic rectangular wave 'meander' oscillations with circular frequencies ω_1 and ω_2, respectively.

(2) Let these frequencies be incommensurable, $\omega_2/\omega_1 = \alpha$.

(3) Let $\alpha \in \left[\sqrt{2}-1, \ldots, (\sqrt{5}-1)/2, \ldots, \sqrt{3}-1\right]$. The best value for α is the 'strong' irrational number $\alpha = (\sqrt{5}-1)/2$ — the 'golden section'.

(4) We add up these two oscillations modulo 2, i.e., perform the exclusive OR operation on them. This addition results in a discrete Poissonian stream of square pulses of random duration, which emerge at random time moments.

3. Critical review of the results of the work

First, we will make several general remarks.

In the abstract to the paper the authors state that "the sign correlation of quasiperiodic oscillations with close incommensurable frequencies forms dynamic chaos...". However, a Poissonian stream constitutes not dynamic chaos but an absolutely random stationary process [2]. What is it: our

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different understanding of the terminology? But in the first sentence of the introduction to their paper the authors make note of "dynamic chaos as a deterministic irregular motion...", i.e., they are fully aware of the difference between random processes and dynamic chaos.

Furthermore, the title of the paper implies that it is concerned with the dynamic chaos in Hamiltonian systems. And this permits the authors to extensively discuss the KAM-tori, the Arnol'd mapping, the Poincare number of rotations, etc. However, it soon turns out that the source of the oscillations with incommensurable frequencies in the experiment is a two-circuit parametric oscillator. Of course, under specific conditions the energy dissipation in such oscillators is rather low. But it is always present, i.e., this is a dissipative system rather than a conservative one. And it is common knowledge that even a vanishingly small dissipation would radically alter the properties of both linear and nonlinear oscillators! If the authors had some special considerations on this subject they should have imparted them to the reader.

In the terminological and notional aspects, the paper was written in an extremely intricate and careless manner. Reading the paper brings up a lot of questions. For instance, why of the infinite number of possible motions on the torus do the authors consider only the one generated by the Arnol'd mapping? (Many other mappings do not result in randomization!) What is "the dynamic chaos of mappings obtained from quasiperiodic oscillations"? What is "the mutual mapping of two rectangular waves"? Finally, why is a rectangular-shaped oscillation referred to as a rectangular wave? The list of such examples could be continued, but we will restrict ourselves to those above and come to the main point.

So, without citing any evidence the authors of Ref. [1] make an absolutely unjustified statement that the pulse stream obtained by their algorithm exhibits Poissonian properties. Were it true, the goal formulated might be considered as being accomplished, since both the spectrum and autocorrelation function of the Poisson stream can be analytically calculated [2]. The Poisson stream has an exponentially decaying autocorrelation function and a continuous spectrum, which makes it attractive as regards applications of a various kinds.

The subsequent presentation of our standpoint is divided into two parts. In the first part, we answer the question: Is there a Poissonian stream at the output of a modulo-2 adder (a sign correlator) for two rectangular oscillations with incommensurable frequencies? In the second part, we calculate the Fourier spectrum and autocorrelation function of the pulse stream at the output of the phase correlator and thereby establish the presence or absence of deterministic chaos.

3.1 Does the sign correlation generate a Poissonian stream?

The conditions whereby the pulse stream satisfies the Poisson distribution are very correctly set forth in Ref. [2]. There are three of them: stationarity, forgetfulness, and ordinariness. The second and the third conditions are the most strong ones. The forgetfulness implies that individual events in the stream occur independently of each other, while the ordinariness allows us to neglect the probability that two or more events occur simultaneously in a sufficiently short period of observation. The Poisson distribution does not take place unless these two conditions are fulfilled.

We consider a mathematical model of the following form:

$$\begin{aligned} x(t) &= \operatorname{sign}(\cos\omega_1 t)\,, \\ y(t) &= \operatorname{sign}\big(\cos(\omega_2 t + \varphi_0)\big)\,, \\ z(t) &= x(t) \oplus y(t)\,. \end{aligned} \tag{1}$$

Here, sign (.) is the function which returns the sign of its argument and $\oplus$ is the exclusive OR operation, in other words, a sign correlator. When the signs of $x(t)$ and $y(t)$ coincide, the function $z(t)$ is equal to zero, otherwise it is equal to unity. The ω_1-to-ω_2 frequency ratio may be arbitrary, and φ_0 is the initial phase difference, the varying of which enables defining different initial conditions. It is easily seen that model (1) operates in perfect accord with the algorithm of obtaining a Poissonian stream described in Ref. [1]. All the results given below were obtained by analyzing this model for $\omega_1 = 2\pi$ and the frequency ratio $\omega_2/\omega_1 = (\sqrt{5}-1)/2$ equal to the 'golden section'.

The pulse sequence generated by model (1) coincides completely with the sequence depicted in Fig. 1 of the paper under discussion. On the face of it, both the instants of occurrence of the pulses and their lengths seem to be perfectly independent. But is this the case?

We specify a sufficiently long observation interval $T = 5000$ and construct the dependence $z(t)$ in accordance with model (1). Calculations show that $N = 8090$ pulses are generated in the interval of time adopted, and hence the pulse stream density is $\lambda = 1.618$. We arbitrarily vary the beginning and the end of the interval of pulse number count to ascertain that λ remains invariable — hence the stream $z(t)$ under analysis is stationary. In the observation interval T we fix the instants of occurrence and termination of the pulses $\{t_k, t_{k+1}\}_{k=1}^{2N-1}$ and their duration $\{\tau_k = t_{k+1} - t_k\}_{k=1}^{N}$. The data thus obtained may be subject to statistical processing of a different kind.

We consider the randomness of pulse durations and their independence of the instant of observation. We investigated the dependence of the kth pulse duration τ_k on its number k. The dependence $\tau_k(k)$ was found to be a nonperiodic process but by no means a random one. This dependence is to be unambiguously recognized as near-periodic.

An even clearer idea of the statistical properties of the pulse stream under investigation can be gained by considering the normalized autocorrelation function $R_\tau(k)$, which characterizes the variation in the statistical relation between the pulse lengths with discrete time k:

$$R_\tau(k) = \frac{\langle\tau_n\tau_{n+k}\rangle - \langle\tau_n\rangle\langle\tau_{n+k}\rangle}{\langle\tau_n^2\rangle - \langle\tau_n\rangle^2}\,, \qquad n = 1, 2, \ldots, N\,. \tag{2}$$

To avoid misunderstandings in terminology, we note that the function (2) in statistical radio engineering is termed the correlation coefficient [2]. It also proved to be almost periodic. Not only did $R_\tau(k)$ exhibit a nondecreasing behavior (decreasing is characteristic of the Poisson process), but it could hardly be distinguished from a periodic dependence. For instance, only a careful numerical analysis allowed us to establish a slight violation of periodicity on the interval $k \in [7-27]$.

The sequence of the instants of pulse occurrence $\{t_k\}_{k=1}^{2N-1}$ was analyzed in a similar manner. We will not enlarge on the results of the investigation, for they are quite similar to the results outlined for the $\{\tau_k\}_{k=1}^{N}$ sequence. An extremely high degree of correlation was established for $\{t_k, t_m\}$ for k and m differing by arbitrarily great numbers.

Since the stationarity of the process $z(t)$ has been established, considering its properties from an arbitrary point in time is justified. We select the reference point in such a way that the instants of the onset of the first pulses in the processes $x(t)$ and $y(t)$ coincide. To do this would suffice to put $\varphi_0 = 0$. In this case, the analysis of the $z(t)$ stream is especially simple, making it possible to ascertain that all t_k and τ_k are integer-valued combinations of the pulse lengths $\tau_x = \pi/\omega_1$ of the process $x(t)$ and the pulse lengths $\tau_y = \pi/\omega_2$ of the process $y(t)$. This relation is expressed as follows:

$$\tau_k = |m\tau_x - n\tau_y|\,, \qquad t_k = m\tau_x\,, \qquad t_k = n\tau_y\,, \tag{3}$$

where m and n are arbitrary integers, and is an unambiguous indication that there is no randomness in either the instants of pulse occurrence at the sign correlator output or the duration of these pulses.

Therefore, it has been found that the pulse stream generated by the sign correlation of oscillations with incommensurable frequencies is not a Poissonian random process.

3.2 Does the sign correlation generate dynamic chaos?

The absence of Poissonian properties in the $z(t)$ stream does not necessarily imply that it is not a source of deterministic chaos, and therefore the analysis performed in Section 3.2 may not be thought of as being complete. The most reliable criterion for the random nature of any process is the positiveness of Kolmogorov–Sinai entropy, which is rather difficult to calculate in our case. However, to distinguish a chaotic process from a regular one, advantage can be taken of simpler methods. To do this it would suffice to consider the Fourier spectrum of the process or its autocorrelation function. The spectrum of a deterministic chaotic motion must necessarily have a continuous component and the autocorrelation function or its envelope must necessarily decrease, tending to zero as its argument increases.

We calculate the spectral density of the process $z(t)$. It is evident that the spectral density of a single rectangular pulse with a unity amplitude occurring at time t_k and having a duration τ_k can be described as follows:

$$S_k(\mathrm{j}\omega) = \frac{\mathrm{j}}{\omega}\left[\exp(-\mathrm{j}\omega\tau_k) - 1\right]\exp(-\mathrm{j}\omega t_k)\,, \qquad \mathrm{j} = \sqrt{-1}\,. \tag{4}$$

On the strength of the well-known theorem on the addition of spectra, the spectral density $S(\mathrm{j}\omega)$ for a train of N pulses is obtained simply by adding up all $S_k(\mathrm{j}\omega)$. Upon performing this addition and separating out the real and imaginary parts, we write down the expression which enables us to calculate the modulus of the spectral density:

$$S(\omega) = \frac{1}{\omega}\sqrt{P^2(t_k, \tau_k, \omega) + Q^2(t_k, \tau_k, \omega)}\,, \tag{5}$$

where

$$P(t_k, \tau_k, \omega) = \sum_{k=1}^{N} \cos\omega(t_k + \tau_k) - \cos\omega t_k\,,$$

$$Q(t_k, \tau_k, \omega) = \sum_{k=1}^{N} \sin\omega(t_k + \tau_k) - \sin\omega t_k\,.$$

To perform numerical calculations requires the removal of the indeterminacy arising when $\omega \to 0$. We expand the trigonometric terms in power series and restrict ourselves to first-order infinitesimal terms. Then

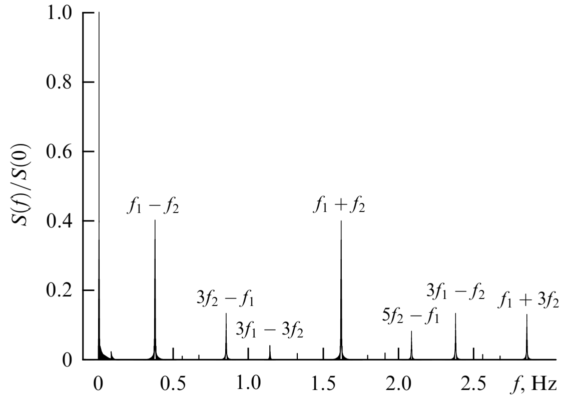

Figure 1. Portion of the normalized spectral density $S(f)$ of the pulse stream at the output of the sign correlator.

$$S(0) = \sum_{k=1}^{N} \tau_k\,. \tag{6}$$

Since the spectral density increases with N, it is appropriate to introduce into consideration its normalized value $S(f)/S(0)$, $f = \omega/2\pi$. This is precisely the quantity given in Fig. 1, whence it follows that the spectral density constitutes a set of narrow nonoverlapping spectral lines. The continuous component is missing from the spectrum, and hence the process at the output of the sign correlator is immanently quite regular and not chaotic. The frequencies of the spectral components unambiguously correspond to the nonequidistant combination frequencies of the form

$$|mf_1 \pm nf_2|\,, \qquad f_{1,2} = \frac{\omega_{1,2}}{2\pi}\,, \tag{7}$$

where m and n are integers.

On the strength of the well-known Wiener–Khintchine theorem, the autocorrelation function $R_z(\tau)$ of the process $z(t)$ is related in a unique fashion to its spectral density (5). However, for the completeness of our statement, it was numerically calculated with the use of relation (2) written for continuous argument values,

$$R_z(\tau) = \frac{\langle z(t)z(t+\tau)\rangle - \langle z(t)\rangle\langle z(t+\tau)\rangle}{\langle z^2(t)\rangle - \langle z(t)\rangle^2}\,, \qquad t \in [0-T]\,, \tag{8}$$

and is plotted in Fig. 2.

The autocorrelation function is almost periodic in character and does not decay with τ, and therefore the Kolmogorov–Sinai entropy is equal to zero. And so $z(t)$ is also a near periodic and totally regular process, but by no means a deterministic chaotic one.

From the standpoint of the basics of nonlinear dynamics our results seem to be quite natural. Any dynamic system with chaotic behavior should continuously reproduce information. But the system under discussion merely effects the functional transformation of two periodic processes. It is evident that no new information is reproduced by the system, and therefore its Kolmogorov–Sinai entropy is equal to zero for any sufficiently long observation interval.

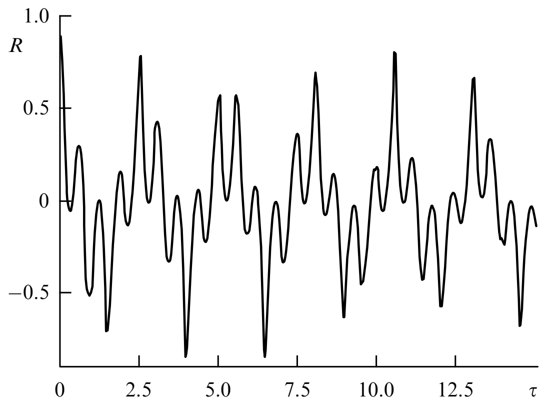

Figure 2. Normalized autocorrelation function of the pulse stream at the output of the sign correlator.

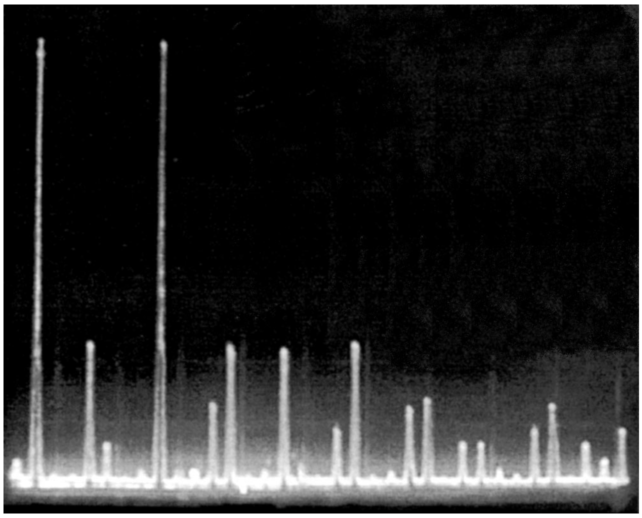

Figure 3. Spectrum of the pulse stream at the output of the sign correlator for $f_2/f_1 \cong (\sqrt{5}-1)/2$. When comparing with Fig. 1, account should be taken of the fact that the constant component is absent in Fig. 3. The two highest-amplitude harmonics correspond to the frequencies $f_1 - f_2$ and $f_1 + f_2$.

3.3 Natural experiment

What can the inadequacy of model (1) with respect to a practical functioning dynamic system consist of? Only that in the circumstances of a natural experiment it is impossible, in principle, to get rid of various fluctuation processes, which constantly introduce uncertainty into experimental conditions. Under certain conditions, this uncertainty may result in the loss of predictability of the state of a dynamic system.

Here, it is pertinent to make two remarks. First, in order for a deterministic chaos mode to exist in a dynamic system, no external or internal fluctuations are required whatsoever, because the chaos is generated by the properties inherent in the system itself. Second, the uncertainty always exists in numerical experiments as well: in our case, it is necessarily caused by the finiteness of digital representation in a computer and the finite step value of the sampling of the solutions of model (1). Nevertheless, errors may have the effect that the results of natural and numerical experiments become incomparable. Therefore, we supplement our numerical analysis with the results of a natural experiment.

The experimental facility which we developed consisted only of digital elements and comprised two RC self-excited oscillators, two frequency dividers, and a modulo-2 adder. The first self-excited oscillator operated at a fixed frequency $f_1 = 100$ kHz and the second one was tunable in the $f_2 \in 40-80$ kHz range. This was the way to attain the frequency ratios recommended in Ref. [1]. Next, the oscillation frequencies of the self-excited oscillators were divided by 2 using two D-triggers to acquire the shape of a 'meander'. Upon frequency division, the square pulses were delivered to a modulo-2 adder, which produced at its output the desired pulse stream.

The experimental results can be formulated very briefly. Under no ratio between the frequencies f_1 and f_2 was it possible to observe irregular pulse sequences. The spectrum consisted of lines that made up a set of nonequidistant harmonic components with frequencies defined by relation (7).

By way of example, Fig. 3 shows a spectrum analyzer screen photograph obtained for a frequency ratio equal to the 'golden section'. Its complete identity to the spectrum calculated by relation (5) is compelling.

The authors of Ref. [1] also adduce their experimental results. In particular, they record an autocorrelation function close to the δ function (Fig. 6c from Ref. [1]), though a screen photograph of a physical instrument — the spectrum analyzer — would be more desirable. We are not saying that the results presented in the paper were not obtained in reality, it is more likely that we are dealing with a misinterpretation of the results. It is rather safe to assume that the chaotic process resulted from an external periodic action on the nonlinear oscillatory system with a five-dimensional phase space (two RLC oscillatory circuits coupled by a nonlinear capacitance). With consideration for the external action, the overall dimensionality of the phase space is equal to six. It is noteworthy that dynamic chaos also showed up in simpler systems containing a nonlinear capacitance. In this case, the ratio between the resonance frequencies of the RLC circuits are of no fundamental importance. But this notion interferes with the statement of the authors of Ref. [1] about the periodic nature of the oscillations at the output of the first and second circuits of the parametric oscillator.

4. Conclusion

In our work, with the use of numerical and natural experiments we have shown that the sign correlation of two oscillations with incommensurable frequencies cannot lead to the generation of either a Poissonian stream or a stream exhibiting the properties of dynamic chaos. The radical disagreement of our results with the results of Ref. [1] has no reasonable explanation and can be regarded as a fundamental fallacy of one of the parts and an invitation to the reader to participate in further discussion.

The author expresses his gratitude to the reviewer, whose exact comment and remarks aided in augmenting the level of the present paper.

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