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Neutrino oscillations in three- and four-flavor schemes

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Contents

1. Introduction **117**

2. Three-flavor neutrino oscillations in matter **118**

2.1 Constant-density matter; 2.2 Arbitrary density profile; 2.3 Matter effects in $\nu_\mu \leftrightarrow \nu_\tau$ oscillations

3. Three-flavor effects in neutrino oscillations **119**

3.1 Solar neutrinos; 3.2 Atmospheric neutrinos; 3.3 Reactor antineutrinos; 3.4 Long-baseline accelerator experiments; 3.5 Supernova neutrinos

4. CP and T violation in ν oscillations in the vacuum **120**

5. CP- and T-odd effects in ν oscillations in matter **121**

5.1 CP-odd effects in matter; 5.2 T-odd effects in matter

6. The problem of U_{e3} **122**

7. Four-flavor neutrino oscillations **122**

8. Conclusions **124**

References **124**

Abstract. **We review some theoretical aspects of neutrino oscillations in the case where more than two neutrino flavors are involved. These include: approximate analytic solutions for 3-flavor (3f) oscillations in matter; matter effects in $\nu_\mu \leftrightarrow \nu_\tau$ oscillations; 3f effects in oscillations of solar, atmospheric, reactor, and supernova neutrinos and in accelerator long-baseline experiments; CP and T violation in neutrino oscillations in the vacuum and in matter; the problem of U_{e3}; and 4f oscillations.**

1. Introduction

An explanation of solar and atmospheric neutrino data in terms of neutrino oscillations [1] requires three neutrino species at least, and in fact three neutrino species are known to exist — ν_e, ν_μ, and ν_τ. If the LSND experiment is correct, then a fourth neutrino type may possibly exist, a light sterile neutrino ν_s. However, until relatively recently, most of the studies of neutrino oscillations had been performed in the 2-flavor framework. There were essentially two reasons for this: (1) simplicity — there are many fewer parameters in the 2-flavor case than in the 3-flavor one, and the expressions for the transition probabilities are much simpler and by far more tractable, and (2) the hierarchy of Δm^2 values, which allows effectively decoupling different oscillation channels. The 2-flavor approach proved to be a good first approximation, which is a consequence of the hierarchy $\Delta m^2_{\rm sol} \ll \Delta m^2_{\rm atm}$ and of the smallness of the leptonic mixing parameter $|U_{e3}|$. [2]

But the increased accuracy of the available and especially of the forthcoming neutrino data makes it very important to take even relatively small effects in neutrino oscillations into account. In addition, the experimentally favored solution of the solar neutrino problem is currently the LMA MSW one, which requires the hierarchy between $\Delta m^2_{\rm sol}$ and $\Delta m^2_{\rm atm}$ to be relatively moderate ($\Delta m^2_{\rm sol} \sim \Delta m^2_{\rm atm}/30$). Also, effects specific to $\geqslant 3$ flavor neutrino oscillations, such as CP and T violation, are now being very widely discussed. All this makes 3-flavor (or 4-flavor) analyses of neutrino oscillations mandatory.

In the present article, some theoretical issues are reviewed that pertain to neutrino oscillations in the case where more than two neutrino species are involved. We mainly concentrate on 3-flavor (3f) oscillations and only very briefly consider the 4f case. The topics that are discussed include: approximate analytic solutions for 3f oscillations in matter; matter effects in $\nu_\mu \leftrightarrow \nu_\tau$ oscillations; 3f effects in oscillations of solar, atmospheric, reactor, and supernova neutrinos and in accelerator long-baseline experiments; CP and T violation in neutrino oscillations in the vacuum and in matter; the problem of U_{e3}; and 4f oscillations.

[1] For recent reviews on neutrino oscillations, see, e.g., [1, 2].

[2] $\Delta m^2_{\rm sol} \equiv \Delta m^2_{21}$ and $\Delta m^2_{\rm atm} \equiv \Delta m^2_{31}$ are the neutrino mass squared differences responsible for the oscillations of solar and atmospheric neutrinos, respectively (see Figs 1 and 2 below). For more detail, see Section 6.1 in Ref. [1].

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2. Three-flavor neutrino oscillations in matter

Neutrino oscillations in matter are described by the Schrödinger-like evolution equation [3]

$$\mathrm{i}\frac{\mathrm{d}}{\mathrm{d}x}\begin{pmatrix}\nu_\mathrm{e}\\ \nu_\mu\\ \nu_\tau\end{pmatrix} = \left[-U\begin{pmatrix}p_1 & 0 & 0\\ 0 & p_2 & 0\\ 0 & 0 & p_3\end{pmatrix}U^\dagger + \begin{pmatrix}V(x) & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{pmatrix}\right]\begin{pmatrix}\nu_\mathrm{e}\\ \nu_\mu\\ \nu_\tau\end{pmatrix}, \quad (1)$$

where $p_i \simeq E - m_i^2/2E$ with E, p_i, and m_i being the neutrino energy, momentum, and mass, respectively,[4] and U is the leptonic mixing matrix. The effective potential $V = \sqrt{2}G_\mathrm{F}N_\mathrm{e}$ is due to the charged-current interaction of ν_e with the electrons of the medium [3]. The potentials induced by neutral current interactions are omitted from Eqn (1) because they are the same for neutrinos of all the three species and therefore do not affect neutrino oscillations. This, however, is only true in the leading (tree) order; radiative corrections induce tiny differences between the neutral current potentials of ν_e, ν_μ, and ν_τ and, in particular, result in a very small ν_μ — ν_τ potential difference $V_{\mu\tau} \sim 10^{-5}\,V$ [4]. This quantity is negligible in most situations, but may be important for supernova neutrinos.

For matter of constant density, closed-form solutions of the evolution equation can be found [5]; however, the corresponding expressions are rather complicated and not very tractable. No closed-form solutions exist for a general electron density profile $N_\mathrm{e} \neq$ const. It is therefore desirable to have approximate analytic solutions of the neutrino evolution equation. A number of such solutions have been found, most of them based on the expansions in one (or both) of the two

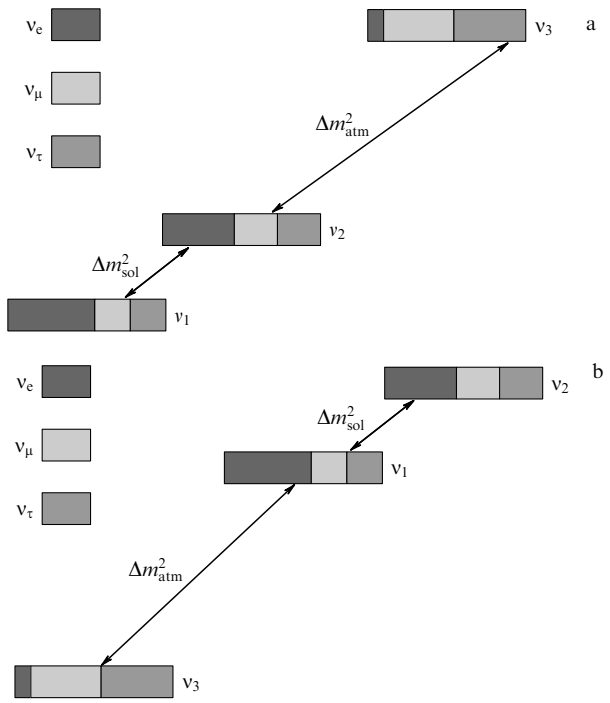

Figure 1. Normal mass hierarchy (a). Inverted mass hierarchy (b).

[3] See, e.g., [2] and references therein.

[4] We consider relativistic neutrinos.

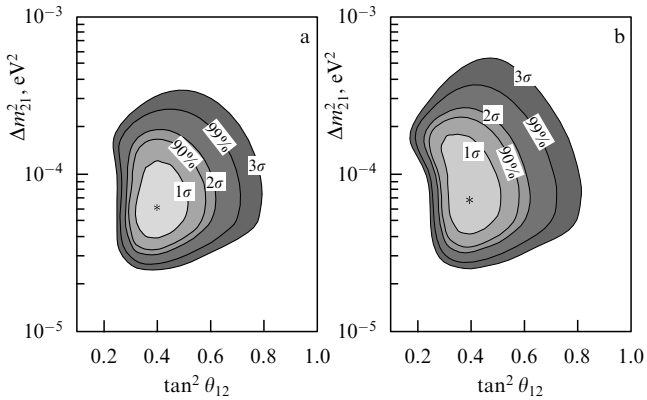

Figure 2. LMA-allowed parameter region $(\tan^2\theta_{12}, \Delta m^2_{21})$ for $\theta_{13} = 0$ (left diagram) and $\sin^2\theta_{13} = 0.04$ (right diagram) [15].

small parameters:

$$\frac{\Delta m^2_{21}}{\Delta m^2_{31}} = \frac{\Delta m^2_{\mathrm{sol}}}{\Delta m^2_{\mathrm{atm}}} \lesssim 0.1\,, \quad (2)$$

$$|U_{\mathrm{e}3}| = |\sin\theta_{13}| \lesssim 0.2\ [6]\,. \quad (3)$$

Our numbering of neutrino mass eigenstates corresponds to that in Figs 1 and 2, which also schematically show the possible neutrino mass hierarchies for three neutrino flavors and the flavor composition of neutrino mass eigenstates.

In the limits $\Delta m^2_{21} = 0$ or $U_{\mathrm{e}3} = 0$, the transition probabilities acquire an effective 2f form. When both these parameters vanish, the genuine 2f case is recovered.

2.1 Constant-density matter

In the case of constant-density matter, approximate solutions of the neutrino evolution equation were found in [7] using the expansion in $\alpha \equiv \Delta m^2_{\mathrm{sol}}/\Delta m^2_{\mathrm{atm}}$. An expansion in both α and $\sin\theta_{13}$ was used in [8]. The $\nu_\mathrm{e} \leftrightarrow \nu_\mu$ transition probability found in [8] has the general form

$$P(\nu_\mathrm{e} \leftrightarrow \nu_\mu) \sim s^2_{23}\widetilde{P}_2(\Delta m^2_{31}, \theta_{13}, N_\mathrm{e}) + c^2_{23}\widetilde{P}_2(\Delta m^2_{21}, \theta_{12}, N_\mathrm{e}) + \text{interf. term}\,, \quad (4)$$

where the quantities $\widetilde{P}_2$ are the 2f transition probabilities in matter depending on the corresponding parameters shown in the parentheses. The interference term, which is linear in both α and $\sin\theta_{13}$, describes the genuine 3f effects, both CP-conserving and CP-violating.

2.2 Arbitrary density profile

Constant-density matter is a good first approximation for long-baseline accelerator neutrino experiments (neutrinos traverse the mantle of the Earth). However, it is not very useful for describing the oscillations of solar, atmospheric, and supernova neutrinos. An alternative approach is to consider matter with an arbitrary density profile and reduce the problem to an effective 2f one plus easily calculable 3f corrections. This has been done using the expansion in α in [9] and the expansion in $\sin\theta_{13}$ in [10–12]. A different approach, based on the adiabatic approximation, was employed, e.g., in [13].

2.3 Matter effects in $\nu_\mu \leftrightarrow \nu_\tau$ oscillations

Because the matter-induced potentials for ν_μ and ν_τ are the same (neglecting the radiative corrections), the $\nu_\mu \leftrightarrow \nu_\tau$

oscillations are not affected by matter in the 2f case. This is not true in the 3f case, however; therefore, matter effects on $\nu_\mu \leftrightarrow \nu_\tau$ oscillations are a pure 3f effect. It vanishes only when both Δm^2_{21} and U_{e3} vanish.

3. Three-flavor effects in neutrino oscillations

We now discuss 3f effects in oscillations of neutrinos from various sources.

3.1 Solar neutrinos

In the 3f case, solar ν_e can in principle oscillate into either $\boldsymbol{\nu_\mu}$, $\boldsymbol{\nu_\tau}$, or some combination thereof. What do they actually oscillate to?

It is easy to answer this question. The smallness of the mixing parameter $|U_{e3}|$ implies that the mass eigenstate ν_3, separated by a large mass gap from the other two, is approximately given by

$$\nu_3 \simeq s_{23}\,\nu_\mu + c_{23}\,\nu_\tau \tag{5}$$

and, to the first approximation, does not participate in the solar neutrino oscillations. From the unitarity of the leptonic mixing matrix, it then follows that the solar neutrino oscillations are the oscillations between ν_e and a state ν' that is the linear combination of ν_μ and ν_τ orthogonal to ν_3,

$$\nu' = c_{23}\,\nu_\mu - s_{23}\,\nu_\tau\,. \tag{6}$$

Because the mixing angle θ_{23}, responsible for the atmospheric neutrino oscillations, is known to be close to 45°, Eqn (6) implies that the solar ν_e oscillate into a superposition of ν_μ and ν_τ with equal or almost equal weights.

What are the 3f effects in the oscillation probabilities? Because ν_μ and ν_τ are experimentally indistinguishable at low energies, all the observables depend on just one probability, the ν_e survival probability $P(\nu_e \to \nu_e)$. Averaging over fast oscillations due to the large mass squared difference $\Delta m^2_{\rm atm} = \Delta m^2_{31}$ yields [14]

$$P(\nu_e \to \nu_e) \simeq c^4_{13}\widetilde{P}_{2ee}(\Delta m^2_{21}, \theta_{12}, N_{\rm eff}) + s^4_{13}\,. \tag{7}$$

Here, $\widetilde{P}_{2ee}(\Delta m^2_{21}, \theta_{12}, N_{\rm eff})$ is the 2f survival probability of ν_e in matter with the effective electron density $N_{\rm eff} = c^2_{13}\,N_e$.

As follows from the CHOOZ data [6], the second term in Eqn (7), s^4_{13}, does not exceed 10^{-3}, i.e., is negligible. At the same time, the coefficient c^4_{13} of $\widetilde{P}_{2ee}$ in the first term may differ from unity by as much as $\sim 5-10\,\%$. Therefore, 3f effects may lead to an energy-independent suppression of the ν_e survival probability by up to 10%. With high-precision solar data, this must be taken into account. This is illustrated by Fig. 2 [15]: The difference between the cases $\theta_{13} = 0$ and $\sin^2\theta_{13} = 0.04$ (which is about the maximum allowed by the CHOOZ value) is quite noticeable.

3.2 Atmospheric neutrinos

(1) The dominant channel is $\nu_\mu \leftrightarrow \nu_\tau$. In the 2f limit, there are no matter effects in this channel (ignoring tiny $V_{\mu\tau}$ caused by radiative corrections). The oscillation probability is independent of the sign of Δm^2_{31}, and the normal and inverted neutrino mass hierarchies are therefore indistinguishable (cf. Figs 1a and 1b). The 3f effects result in a weak sensitivity to matter effects and to the sign of Δm^2_{31}.

(2) The subdominant channels are the $\nu_e \leftrightarrow \nu_{\mu,\tau}$ transitions. Contributions of these oscillation channels to the number of μ-like events are not fundamental and are difficult to observe. For e-like events, one could *a priori* expect significant oscillation effects,[5] but these effects are in fact strongly suppressed because of the specific composition of the atmospheric neutrino flux and proximity of the mixing angle θ_{23} to 45°. Indeed, in the 2f limits, we find

$$\frac{F_e - F^0_e}{F^0_e} = (rs^2_{23} - 1)\,\widetilde{P}_2(\Delta m^2_{31}, \theta_{13}, V)\,, \tag{8}$$

in the limit as $\Delta m^2_{21} \to 0$ [9], and

$$\frac{F_e - F^0_e}{F^0_e} = (rc^2_{23} - 1)\widetilde{P}_2(\Delta m^2_{21}, \theta_{12}, V)\,, \tag{9}$$

in the limit as $s_{13} \to 0$ [10]. Here, F^0_e and F_e are the respective ν_e fluxes in the absence and in the presence of the oscillations and $r \equiv F^0_\mu/F^0_e$. At low energies, $r \simeq 2$; also, we know that $s^2_{23} \simeq c^2_{23} \simeq 1/2$. Therefore, the factors $(rs^2_{23} - 1)$ and $(rc^2_{23} - 1)$ in Eqns (8) and (9) are very small and strongly suppress the oscillation effects even if the transition probabilities $\widetilde{P}_2$ are close to unity. This happens because of the strong cancelations of the transitions from and to the ν_e state.

All this looks like a conspiracy to hide the oscillation effects on the e-like events! This conspiracy is broken by the 3f effects, however. Keeping both Δm^2_{21} and s_{13} in the leading order yields [12]

$$\begin{aligned}\frac{F_e - F^0_e}{F^0_e} &\simeq (r\,s^2_{23} - 1)\,\widetilde{P}_2(\Delta m^2_{31}, \theta_{13})\\ &\quad + (r\,c^2_{23} - 1)\,\widetilde{P}_2(\Delta m^2_{21}, \theta_{12}) - 2s_{13}\,s_{23}\,c_{23}\,r\ \mathrm{Re}\,(\widetilde{A}^*_{ee}\,\widetilde{A}_{\mu e})\,.\end{aligned} \tag{10}$$

Here, $\widetilde{A}_{ee}$ and $\widetilde{A}_{\mu e}$ are the ν_e survival and transition amplitudes in the rotated basis $\tilde{\nu} \approx U_{13}(\theta_{13})^\dagger U_{23}(\theta_{23})^\dagger \nu_{\rm fl}$, where $U_{ij}(\theta_{ij})$ is the rotation matrix in the ij plane and $\nu_{\rm fl}$ is the neutrino state in the flavor basis. The interference term, which represents the genuinely 3f effects, is not suppressed by the flavor composition of the atmospheric neutrino flux; it can reach a few per cent and may be responsible (at least, partially) for some excess of the upward-going sub-GeV e-like events observed at Super-Kamiokande [12].

3.3 Reactor antineutrinos

Because the average energy of reactor $\bar{\nu}_e$ is $E \simeq 3$ MeV, for intermediate-baseline experiments such as CHOOZ and Palo Verde ($L \simeq 1$ km), we have

$$\frac{\Delta m^2_{31}}{4E}\,L \simeq 1\,, \qquad \frac{\Delta m^2_{21}}{4E}\,L \ll 1\,. \tag{11}$$

This justifies the use of the one mass scale dominance approximation, which gives a pure 2f result:

$$P(\bar{\nu}_e \to \bar{\nu}_e) = 1 - \sin^2 2\theta_{13}\sin^2\left(\frac{\Delta m^2_{31}}{4E}\,L\right). \tag{12}$$

But in the case of the LMA solution of the solar neutrino problem, at a high enough confidence level, Δm^2_{21} can be comparable with Δm^2_{31}, and the second condition in (11) may not be satisfied. In such a situation, the 3f effects coming through the subdominant mass-squared difference Δm^2_{21} should be taken into account. The analyses [16] show that

[5] μ-like and e-like events are those in which the observed Cherenkov radiation can be identified with high confidence as being due to the production of muons and electrons, respectively. They are associated with the charged-current interactions of muon and electron neutrinos in the detector.

the constraints on $|U_{e3}|$ derived from the CHOOZ experiment become slightly more stringent in that case. However, the new SNO data [17] disfavor large values of Δm^2_{21} and therefore make this possibility less likely.

For KamLAND, which is a very long baseline reactor experiment ($L \simeq 170$ km), the relations

$$\frac{\Delta m^2_{31}}{4E}\, L \gg 1\,, \quad \frac{\Delta m^2_{21}}{4E}\, L \gtrsim 1\,. \tag{13}$$

hold for the LMA solution. Averaging over the fast oscillations driven by $\Delta m^2_{31} = \Delta m^2_{\rm atm}$ yields

$$P(\bar{\nu}_e \to \bar{\nu}_e) = c^4_{13}\, P_{2\bar{e}\bar{e}}(\Delta m^2_{21}, \theta_{12}) + s^4_{13}\,. \tag{14}$$

This has the same form as Eqn (7), except that the 2f survival probability $P_{2\bar{e}\bar{e}}$ has to be calculated in the vacuum rather than in matter; it is in fact given by Eqn (12). Probability (14) can differ from 2f probability (12) by up to $\sim 10\%$.

3.4 Long-baseline accelerator experiments

(1) Disappearance of ν_μ. 3f effects can result in up to $\sim 10\%$ correction to the disappearance probability, mainly due to the factor c^4_{13} in the effective amplitude of the $\nu_\mu \leftrightarrow \nu_\tau$ oscillations,

$$(\sin^2 2\theta_{\mu\tau})_{\rm eff} \equiv c^4_{13}\, \sin^2 2\theta_{23}\,. \tag{15}$$

Another manifestation of 3-flavorness is given by small matter effects in $\nu_\mu \leftrightarrow \nu_\tau$ oscillations. The same applies to the appearance of ν_τ in experiments with conventional neutrino beams. Disappearance of ν_μ also receives contributions from the subdominant $\nu_\mu \leftrightarrow \nu_e$ oscillations.

(2) Appearance of ν_μ at neutrino factories; appearance of ν_e at neutrino factories and in experiments with the conventional neutrino beams. These are driven by $\nu_e \leftrightarrow \nu_{\mu,\tau}$ oscillations. There are two channels through which these subdominant oscillations can proceed – those governed by the parameters $(\theta_{13}, \Delta m^2_{31})$ and $(\theta_{12}, \Delta m^2_{21})$. For typical energies of long-baseline accelerator experiments, a few GeV to tens of GeV, and assuming the LMA solution of the solar neutrino problem, one finds that the two channels compete for θ_{13} in the range $3\times10^{-3} \lesssim \theta_{13} \lesssim 3\times10^{-2}$; otherwise, one of them dominates.

Unlike in the case of atmospheric neutrinos, there is no suppression of the oscillation effects on the ν_e flux due to the flavor composition of the original flux.

The dependence of the oscillation probabilities on the CP-violating phase $\delta_{\rm CP}$ (terms proportional to $\sin\delta_{\rm CP}$ and $\cos\delta_{\rm CP}$) comes from the interference terms and is a pure 3f effect. 3f effects will be especially important for future experiments at neutrino factories that are designed for precision measurements of neutrino parameters.

3.5 Supernova neutrinos

In supernovae, matter density varies in a very wide range, and the conditions for the three MSW [3] resonances are satisfied (taking into account that $V_{\mu\tau} \neq 0$ due to radiative corrections); see Fig. 3. In this figure, various neutrino energy levels in matter are shown as functions of the electron number density. The dashed lines represent the energy levels of flavor eigenstates, the dotted lines correspond to the energy levels of ν'_μ and ν'_τ, which are the linear combinations of ν_μ and ν_τ that diagonalize the Hamiltonian of the neutrino system at very high densities, and the solid lines represent the exact neutrino matter eigenstates ν_{1m}, ν_{2m}, and ν_{3m} (see [18, 19] for more details). The level crossing points correspond to the MSW

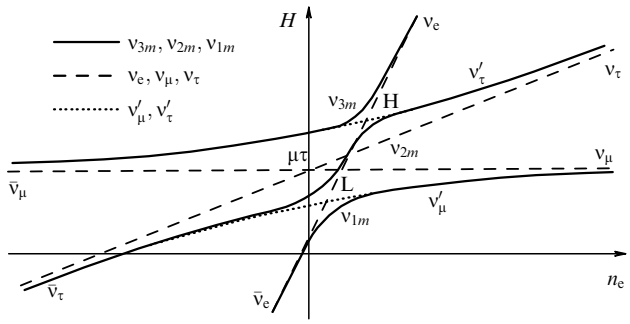

Figure 3. Energy level crossing scheme for supernova neutrinos.

resonances. The hierarchy $\Delta m^2_{21} \ll \Delta m^2_{31}$ leads to the approximate factorization of transition dynamics at the resonances, and therefore the transitions are effectively 2f ones in the first approximation. But the observable effects of the supernova neutrino oscillations depend on the transitions between all the three neutrino species.

Supernova neutrinos can propagate significant distances inside the Earth before reaching the detector. The matter effects on the oscillations of supernova neutrinos inside the Earth depend crucially on the sequence of the neutrino flavor conversions in the supernova, which, in turn, depends on the sign of Δm^2_{31} and is very sensitive to the value of the leptonic mixing parameter U_{e3}. Thus, the Earth matter effects on supernova neutrinos can be used to determine the sign of Δm^2_{31} and to measure $|U_{e3}|$ to a very high accuracy ($\sim 10^{-3}$) [20].

The transitions due to the $\nu_\mu - \nu_\tau$ potential difference $V_{\mu\tau}$ caused by radiative corrections may have observable consequences if the originally produced ν_μ and ν_τ fluxes are not exactly the same [19].

4. CP and T violation in ν oscillations in the vacuum

In this section, we consider the evolution of the neutrino system in time rather than in space. For relativistic neutrinos, both descriptions are equivalent, at least in the plane-wave approximation. The probability of $\nu_a \to \nu_b$ oscillations in the vacuum is given by

$$P(\nu_a, t_0 \to \nu_b; t) = \Big|\sum_i U_{bi} \exp\big[-{\rm i}E_i(t-t_0)\big]\, U^*_{ai}\Big|^2. \tag{16}$$

In the general case of n flavors, the leptonic mixing matrix U_{ai} depends on $(n-1)(n-2)/2$ Dirac-type CP-violating phases $\{\delta_{\rm CP}\}$ (see, e.g., [21, 22]). If neutrinos are Majorana particles, there are $n-1$ additional, so-called Majorana-type CP-violating phases. However, they do not affect neutrino oscillations and therefore are not discussed here.

Under CP transformation, neutrinos are replaced by their antiparticles ($\nu_{a,b} \leftrightarrow \bar{\nu}_{a,b}$), which is equivalent to the complex conjugation of U_{ai}:

$$\begin{aligned} {\rm CP}: \quad & \nu_{a,b} \leftrightarrow \bar{\nu}_{a,b} \\ & \Leftrightarrow U_{ai} \to U^*_{ai} \quad \big(\{\delta_{\rm CP}\} \to -\{\delta_{\rm CP}\}\big)\,. \end{aligned} \tag{17}$$

Time reversal transformation interchanges the initial and final evolution times t_0 and t in Eqn (16), i.e., corresponds to evolution "backwards in time". As follows from Eqn (16), the interchange $t_0 \rightleftarrows t$ is equivalent to complex conjugation of the exponential factors in the oscillation amplitude. Because the

transition probability only depends on the modulus of the amplitude, this is equivalent to complex conjugation of the factors U_{bi} and U^*_{ai}, which in turn amounts to interchanging a $\rightleftarrows$ b. Thus, instead of evolution "backwards in time", we can consider evolution forward in time, but between the interchanged initial and final flavors:

$$\begin{aligned} \mathrm{T}: \quad & t_0 \rightleftarrows t \Leftrightarrow \nu_a \leftrightarrow \nu_b \\ & \Rightarrow U_{ai} \to U^*_{ai} \quad \left(\{\delta_{CP}\} \to -\{\delta_{CP}\}\right). \end{aligned} \tag{18}$$

Under the combined action of CP and T, we have

$$\begin{aligned} \mathrm{CPT}: \quad & \nu_{a,b} \leftrightarrow \bar{\nu}_{a,b}\,, \quad t_0 \rightleftarrows t\,(\nu_a \leftrightarrow \nu_b) \\ & \Rightarrow P\,(\nu_a \to \nu_b) \to P\,(\bar{\nu}_b \to \bar{\nu}_a)\,. \end{aligned} \tag{19}$$

From CPT invariance, it follows that CP violation implies T violation and vice versa.

CP and T violations can be characterized by the probability differences

$$\Delta P^{CP}_{ab} \equiv P\,(\nu_a \to \nu_b) - P\,(\bar{\nu}_a \to \bar{\nu}_b)\,, \tag{20}$$

$$\Delta P^{T}_{ab} \equiv P\,(\nu_a \to \nu_b) - P\,(\nu_b \to \nu_a)\,. \tag{21}$$

From CPT invariance, it follows that the CP- and T-violating probability differences coincide, and that the survival probabilities have no CP asymmetry:

$$\Delta P^{CP}_{ab} = \Delta P^{T}_{ab}\,, \qquad \Delta P^{CP}_{aa} = 0\,. \tag{22}$$

CP and T violations are absent in the 2f case, and therefore any observable violation of these symmetries in neutrino oscillations in the vacuum would be a pure $\geqslant$ 3f effect.

In the 3f case, there is only one CP-violating Dirac-type phase δ_{CP} and hence only one CP-odd (and T-odd) probability difference,

$$\Delta P^{CP}_{e\mu} = \Delta P^{CP}_{\mu\tau} = \Delta P^{CP}_{\tau e} \equiv \Delta P\,, \tag{23}$$

where

$$\begin{aligned} \Delta P = & -4 s_{12}\, c_{12}\, s_{13}\, c^2_{13}\, s_{23}\, c_{23}\, \sin\delta_{CP} \\ & \times \left[\sin\left(\frac{\Delta m^2_{12}}{2E} L\right) + \sin\left(\frac{\Delta m^2_{23}}{2E} L\right) + \sin\left(\frac{\Delta m^2_{31}}{2E} L\right)\right]. \end{aligned} \tag{24}$$

It vanishes when at least one $\Delta m^2_{ij} = 0$; when at least one $\theta_{ij} = 0$ or 90°; when $\delta_{CP} = 0$ or 180°; in the averaging regime; or in the limit $L \to 0$ (as L^3). Clearly, this quantity is very difficult to observe.

5. CP- and T-odd effects in ν oscillations in matter

For neutrino oscillations in matter, CP transformation (substitution $\nu_a \leftrightarrow \bar{\nu}_a$) implies not only complex conjugating the leptonic mixing matrix, but also flipping the sign of the matter-induced neutrino potentials:

$$\begin{aligned} \mathrm{CP}: \quad & U_{ai} \to U^*_{ai} \quad \left(\{\delta_{CP}\} \to -\{\delta_{CP}\}\right), \\ & V(r) \to -V(r)\,. \end{aligned} \tag{25}$$

It can be shown [11] that in matter with an arbitrary density profile, as well as in the vacuum, the action of time reversal on neutrino oscillations is equivalent to interchanging the initial and final neutrino flavors. It is also equivalent to complex conjugating U_{ai} and replacing the matter density profile with the reverse one:

$$\begin{aligned} \mathrm{T}: \quad & U_{ai} \to U^*_{ai} \quad \left(\{\delta_{CP}\} \to -\{\delta_{CP}\}\right), \\ & V(r) \to \tilde{V}(r)\,. \end{aligned} \tag{26}$$

Here

$$\widetilde{V}(r) = \sqrt{2}\, G_F \widetilde{N}(r)\,, \tag{27}$$

with $\widetilde{N}(r)$ being the reverse profile, i.e., the profile that corresponds to the interchanged positions of the neutrino source and detector. In the case of symmetric matter density profiles (e.g., constant-density matter), $\widetilde{N}(r) = N(r)$.

An important point is that the very presence of matter (with unequal numbers of particles and antiparticles) violates C, CP, and CPT, leading to CP-odd effects in neutrino oscillations even in the absence of the fundamental CP-violating phases $\{\delta_{CP}\}$. This fake (extrinsic) CP violation may complicate the study of the fundamental (intrinsic) one.

5.1 CP-odd effects in matter

Unlike in the vacuum, CP-odd effects in neutrino oscillations in matter exist even in the 2f case (in the case of three or more flavors, even when all $\{\delta_{CP}\} = 0$):

$$P\,(\nu_a \to \nu_b) \neq P\,(\bar{\nu}_a \to \bar{\nu}_b)\,. \tag{28}$$

This is actually a well-known fact – for example, the MSW effect can enhance the $\nu_e \leftrightarrow \nu_\mu$ oscillations and suppress the $\bar{\nu}_e \leftrightarrow \bar{\nu}_\mu$ ones or vice versa. Moreover, the survival probabilities in matter are not CP-invariant:

$$P\,(\nu_a \to \nu_a) \neq P\,(\bar{\nu}_a \to \bar{\nu}_a)\,. \tag{29}$$

Disentangling the fundamental CP violation from the matter-induced one in long-baseline experiments requires measuring the energy dependence of the oscillated signal or the signals at two baselines, which is a difficult task. The (difficult) alternatives are:

- long-baseline experiments at relatively low energies and moderate baselines ($E \simeq 0.1 - 1$ GeV, $L \simeq 100 - 1000$ km) [23], with matter effects negligible.
- Indirect measurements through CP-even terms $\sim \cos\delta_{CP}$ [24] or the area of the leptonic unitarity triangle [25].

CP-odd effects cannot be studied in the supernova neutrino experiments because of the experimental indistinguishability of low-energy ν_μ and ν_τ.

5.2 T-odd effects in matter

Because CPT is not conserved in matter, CP and T violations are no longer directly connected (although some relations between them still exist [11, 26]). Therefore, T-odd effects in neutrino oscillations in matter deserve an independent study. Its characteristic features are:

- Matter does not necessarily induce T-odd effects (only asymmetric matter with $\widetilde{N}(r) \neq N(r)$ does).
- There is no T violation (either fundamental or matter-induced) in the 2f case. This is a simple consequence of unitarity. For example, for the (ν_e, ν_μ) system, we have

$$P_{ee} + P_{e\mu} = 1\,, \quad P_{ee} + P_{\mu e} = 1\,, \tag{30}$$

which implies that $P_{e\mu} = P_{\mu e}$.

- In the 3f case, there is only one T-odd probability difference for ν's (and one for $\bar{\nu}$'s), irrespective of the matter density profile:

$$\Delta P^{T}_{e\mu} = \Delta P^{T}_{\mu\tau} = \Delta P^{T}_{\tau e}\,; \tag{31}$$

this is a consequence of 3f unitarity [27].

The matter-induced T-odd effects are very interesting, pure $\geqslant$ 3f matter effects, absent in symmetric matter (in

particular, in constant-density matter). They do not vanish in the regime of complete averaging of neutrino oscillations [11]. They may fake the fundamental T violation and complicate its study, i.e., the extraction of $\delta_{\rm CP}$ from the experiment. The matter-induced T-odd effects vanish when either $U_{\rm e3} = 0$ or $\Delta m^2_{21} = 0$ (i.e., in the 2f limits) and are therefore doubly suppressed by both these small parameters. This implies that perturbation theory can be used to obtain analytic expressions for the T-odd probability differences. The general structure of these differences is

$$\Delta P^{\rm T}_{{\rm e}\mu} = Y \sin\delta_{\rm CP} + X \cos\delta_{\rm CP}\,, \tag{32}$$

were the first term ($\propto \sin\delta_{\rm CP}$) is due to the fundamental T violation and the second term is due to the matter-induced one.

In the adiabatic approximation, one finds [11]

$$X = J_{\rm eff} \times (\text{oscillating terms})\,,$$

where

$$J_{\rm eff} = s_{12}\, c_{12}\, s_{13}\, c^2_{13}\, s_{23}\, c_{23}\, \frac{\sin(2\theta_1 - 2\theta_2)}{\sin 2\theta_{12}}\,. \tag{33}$$

Here, θ_1 and θ_2 are the mixing angles in matter in the (1-2) sector (i.e., the in-matter analogues of the mixing angle θ_{12}) at the initial and final points of neutrino evolution, respectively; $\theta_1 - \theta_2$ is therefore a measure of the asymmetry of the density profile.

$J_{\rm eff}$ is to be compared with the vacuum Jarlskog invariant

$$J = s_{12}\, c_{12}\, s_{13}\, c^2_{13}\, s_{23}\, c_{23} \sin\delta_{\rm CP}\,. \tag{34}$$

We see that the factor $\sin(2\theta_1 - 2\theta_2)/\sin 2\theta_{12}$ in $J_{\rm eff}$ plays the same role as the factor $\sin\delta_{\rm CP}$ in J.

In asymmetric matter, both fundamental and matter-induced T violations contribute to the T-odd probability differences $\Delta P^{\rm T}_{ab}$. This may hinder the experimental determination of the fundamental CP- and T-violating phase $\delta_{\rm CP}$. In particular, in the long-baseline accelerator experiments, one has to take into account that the Earth's density profile is not perfectly spherically symmetric. Strictly speaking, extracting the fundamental T violation requires measuring

$$P_{\rm dir}(\nu_{\rm a} \to \nu_{\rm b}) - P_{\rm rev}(\nu_{\rm b} \to \nu_{\rm a})\,, \tag{35}$$

where $P_{\rm dir}$ and $P_{\rm rev}$ correspond to the direct and reverse matter density profiles. (An interesting point is that even the survival probabilities $P_{\mu\mu}$ and $P_{\tau\tau}$ can be used for that [28]. The ν_e survival probability $P_{\rm ee}$ is an exception because it is independent of $\delta_{\rm CP}$ in the 3f case [29 30]. This, however, is not true if ν_s is present [11].)

In practical terms, it would certainly be difficult to measure the quantity in (35): it would not be easy, for example, to move CERN to Gran Sasso and the Gran Sasso Laboratory to CERN. Fortunately, this is not actually necessary — matter-induced T-odd effects due to the imperfect sphericity of the Earth's density distribution are very small. They cannot spoil the determination of $\delta_{\rm CP}$ if the error in $\delta_{\rm CP}$ is $> 1\%$ at 99% C.L. [11].

Can we study T violation in neutrino oscillation experimentally? Because of problems with the detection of $e^{\pm}$, this seems to be difficult, but probably not impossible. Studying matter-induced T-odd effects would be a harder task. These effects are expected to be negligible in terrestrial experiments. They cannot be observed in the supernova neutrino oscillations because of the experimental indistinguishability of low-energy ν_μ and ν_τ. They could, however, affect the signal from GeV neutrinos produced in the annihilations of WIMPs inside the Sun [31].

6. The problem of $U_{\rm e3}$

The leptonic mixing parameter $U_{\rm e3}$ plays a very special role in neutrino physics. It is of particular interest for a number of reasons.

First, it is the least known of leptonic mixing parameters: while we have (relatively small) allowed ranges for the other two mixing parameters, we only know an upper bound on $|U_{\rm e3}|$. Its smallness, which looks strange in light of the fact that the other two mixing parameters, θ_{12} and θ_{23}, are apparently large,[6] remains essentially unexplained. (There are, however, some ideas that relate the smallness of $|U_{\rm e3}|$ to that of $\Delta m^2_{\rm sol}/\Delta m^2_{\rm atm}$ [33–35].)

The smallness of $U_{\rm e3}$ is likely to be the bottleneck for studying the fundamental CP and T violation effects and matter-induced T-odd effects in neutrino oscillations. The same applies to the determination of the sign of Δm^2_{31} in future long-baseline experiments, which would allow discriminating between the normal and inverted neutrino mass hierarchies. Therefore, it would be vitally important to know how small $U_{\rm e3}$ actually is.

The parameter $U_{\rm e3}$ can be efficiently used to discriminate between various neutrino mass models [36, 37]. It is one of the main parameters that drives the subdominant oscillations of atmospheric neutrinos and is important for their study. It also governs the Earth matter effects on supernova neutrino oscillations.

Finally, $U_{\rm e3}$ apparently provides us with the only opportunity to see the "canonical" MSW effect. While matter effects can be important even in the case of large vacuum mixing angles, the most spectacular phenomenon, strong enhancement of mixing by matter, can only occur if the vacuum mixing angle is small. From what we know now, it seems that the only small leptonic mixing parameter is $U_{\rm e3}$.

All this makes measuring $U_{\rm e3}$ one of the most important problems in neutrino physics.

7. Four-flavor neutrino oscillations

If the LSND experiment is correct, the oscillation interpretation of solar, atmospheric, and accelerator neutrino data would require three distinct values of Δm^2: $\Delta m^2_{\rm sol} \ll \Delta m^2_{\rm atm} \ll \Delta m^2_{\rm LSND}$. This would imply the existence of at least four light neutrino species, $\nu_{\rm e}$, ν_μ, ν_τ, and $\nu_{\rm s}$. A possible alternative is a strong CPT violation in the neutrino sector, leading to inequalities of Δm^2 in the neutrino and antineutrino sectors [38]; this possibility is not discussed here.

In general, the 4f neutrino oscillations are described by six mixing angles θ_{ij}, three Dirac-type CP-violating phases, and three values of Δm^2_{ij}, i.e., they are quite complicated. Fortunately, there is a simplification: The data allow only two classes of 4f schemes, the so-called $(3+1)$ and $(2+2)$ schemes.

In the $(3+1)$ schemes (Fig. 4a), three neutrino mass eigenstates are close to each other, while the fourth one is

[6] Analyses of solar, atmospheric, and reactor neutrino data yield the following allowed ranges at 90% CL: $\tan^2\theta_{12} = 0.42^{+0.2}_{-0.1}$, $\tan\theta_{23} = 1.0^{+0.35}_{-0.25}$, $\sin\theta_{13} < 0.2$ [6, 17, 32].

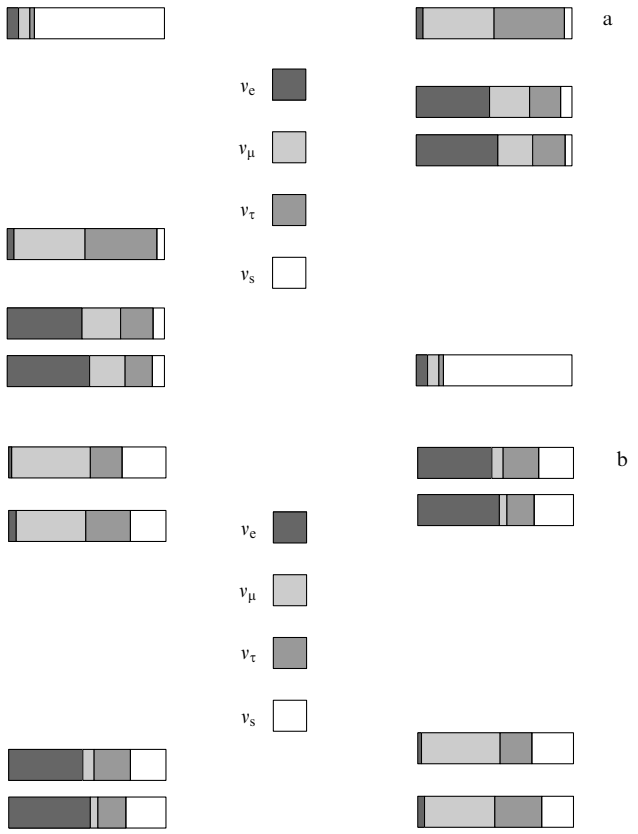

Figure 4. 3 + 1 schemes (a) and 2 + 2 schemes (b).

separated from them by a large mass gap. This mass eigenstate is predominantly ν_s with small admixtures of the active neutrinos,

$$\nu_4 \simeq \nu_s + \mathcal{O}(\epsilon) \cdot (\nu_e, \nu_\mu, \nu_\tau)\,, \quad \epsilon \ll 1\,, \tag{36}$$

whereas ν_1, ν_2, and ν_3 are the usual linear combinations of ν_e, ν_μ, and ν_τ, plus small ($\sim \epsilon$) admixtures of ν_s. In this scheme, the amplitude of the $\nu_\mu \to \nu_e$ oscillations at LSND is

$$\sin^2 2\theta_{\rm LSND} = 4\,|U_{e4}\,U_{\mu 4}|^2 \sim \epsilon^4\,. \tag{37}$$

Strong upper bounds on $|U_{e4}|$ and $|U_{\mu 4}|$ from the $\bar{\nu}_e$ and ν_μ disappearance experiments make it rather difficult to fit the LSND data in the (3 + 1) schemes [39].

This is illustrated by Fig. 5. One can see that the LSND result contradicts the data of the short-baseline disappearance experiments at 95% C.L.; there are allowed regions at 99% C.L., but they are very small.

In the (2 + 2) schemes (Fig. 4b), there are two pairs of mass eigenstates with relatively small mass squared differences $\Delta m^2_{\rm sol}$ and $\Delta m^2_{\rm atm}$ between the states within the pairs and a large separation ($\Delta m^2_{\rm LSND}$) between the two pairs. The ν_μ state is predominantly in the pair responsible for the atmospheric neutrino oscillations, whereas ν_e is mainly in the pair responsible for $\nu_{\rm sol}$ oscillations,

$$\text{osc. } \nu_{\rm atm}: \quad \nu_\mu \leftrightarrow \nu'\,,$$
$$\text{osc. } \nu_{\rm sol}: \quad \nu_e \leftrightarrow \nu''\,,$$

where

$$\nu' \simeq c_\xi\,\nu_\tau + s_\xi\,\nu_s + \mathcal{O}(\epsilon)\cdot\nu_e\,,$$
$$\nu'' \simeq -s_\xi\,\nu_\tau + c_\xi\,\nu_s + \mathcal{O}(\epsilon)\cdot\nu_\mu\,. \tag{38}$$

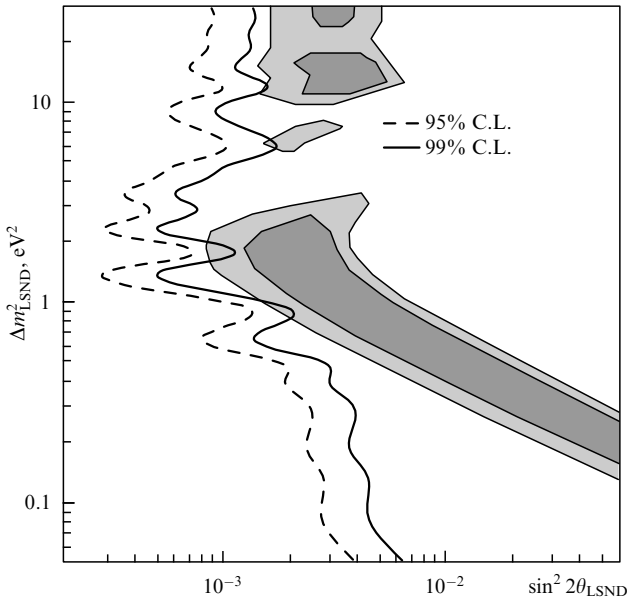

Figure 5. LSND-allowed regions (shaded areas) and short-baseline excluded regions (to the right of the solid and dashed curves) on the ($\sin^2 2\theta_{\rm LSND}$, $\Delta m^2_{\rm LSND}$) plane [39].

The amplitude of the $\nu_\mu \to \nu_e$ oscillations at LSND is

$$\sin^2 2\theta_{\rm LSND} \sim \epsilon^2\,, \tag{39}$$

and is therefore only of the second order in ϵ and, unlike in the (3 + 1) case, the LSND data can be easily fitted.

However, the (2 + 2) schemes suffer from a different problem. In these schemes, the fractions of ν_s involved in the oscillations of atmospheric and solar neutrinos must sum to unity with good accuracy [40]:

$$\left|\langle \nu_s|\nu''\rangle\right|^2 + \left|\langle \nu_s|\nu'\rangle\right|^2 \simeq c_\xi^2 + s_\xi^2 = 1\,. \tag{40}$$

This sum rule is in conflict with the atmospheric and solar neutrino data. Indeed, the Super-Kamiokande atmospheric neutrino data strongly prefer $\nu_\mu \to \nu_\tau$ over $\nu_\mu \to \nu_s$ oscillations, leading to the upper limits $\sin^2\xi < 0.20$ at 90% C.L. and $\sin^2\xi < 0.26$ at 99% C.L. [41].

At the same time, the solar neutrino experiments also prefer the $\nu_e \to \nu_{\rm active}$ oscillation interpretation over the $\nu_e \to \nu_s$ one. The (pre-SNO neutral current) solar neutrino data imply that $\sin^2\xi > 0.7$ (90% C.L.); $\sin^2\xi > 0.48$ (99% C.L.) for the LMA solution of the solar neutrino problem [42].

Therefore, sum rule (40) is violated, i.e., the (2 + 2) scenarios are also strongly disfavored by the data. The recently published SNO neutral current data [17] strengthen this conclusion; it has been pointed out in [43] that 2 + 2 schemes are actually ruled out now. See, however, the discussion in [44], where it was argued that the corrections to sum rule (40) may not be negligible and may weaken the case against the 2 + 2 schemes.[7]

[7] After this review was submitted for publication, the SNO Collaboration published its salt-phase neutral current data [45]. These data put more stringent limits on the possible fraction of sterile neutrinos involved in the solar neutrino oscillations: $\cos^2\xi < 0.31$ (at 99% CL) [46]. As a result, the (2 + 2) schemes are now ruled out at the 5.8σ level. The (3 + 1) schemes remain disfavored by the short-baseline data at the 3.2σ level [46].

In the 4f case, there may be interesting matter effects on neutrino oscillations [47]. CP violation is potentially much richer than in the 3f case: there are several CP-violating observables, and large CP-odd effects are possible (in general, there is no suppression due to small $\Delta m^2_{\rm sol}$). Large T violation (both fundamental and matter-induced) can also occur.

8. Conclusions

3f effects in solar, atmospheric, reactor, and supernova neutrino oscillations and in long-baseline accelerator neutrino experiments may be quite important. They can lead to up to $\sim 10\%$ correction to the oscillation probabilities and to specific effects absent in the 2f case. The manifestations of $\geqslant 3$ flavors in neutrino oscillations include fundamental CP violation and T violation, matter-induced T-odd effects, matter effects in $\nu_\mu \leftrightarrow \nu_\tau$ oscillations, and specific CP- and T-conserving interference terms (proportional to the sines of three different mixing angles) in oscillation probabilities. The leptonic mixing parameter U_{e3} plays a very special role and its study is of great interest.

In the 4f case, large CP violation and (both fundamental and matter-induced) T-odd effects are possible. However, 4f scenarios are strongly disfavored by the data.

References

1. Bilenky S M, Giunti C, Grimus W *Prog. Part. Nucl. Phys.* **43** 1 (1999); hep-ph/9812360
2. Gonzalez-Garcia M C, Nir Y *Rev. Mod. Phys.* **75** 345 (2003); hep-ph/0202058
3. Mikheev S P, Smirnov A Yu *Yad. Fiz.* **42** 1441 (1985) [*Sov. J. Nucl. Phys.* **42** 913 (1985)]; Wolfenstein L *Phys. Rev. D* **17** 2369 (1978)
4. Botella F J, Lim C-S, Marciano W J *Phys. Rev. D* **35** 896 (1987)
5. Barger V D et al. *Phys. Rev. D* **22** 2718 (1980); Zaglauer H W, Schwarzer K H *Z. Phys. C* **40** 273 (1988); Ohlsson T, Snellman H *J. Math. Phys.* **41** 2768 (2000); Erratum, ibid. **42** 2345 (2001); hep-ph/9910546; *Phys. Lett. B* **474** 153 (2000); hep-ph/9912295; Kimura K, Takamura A, Yokomakura H *Phys. Lett. B* **537** 86 (2002); hep-ph/0203099
6. Apollonio M et al. (CHOOZ Collab.) *Phys. Lett. B* **420** 397 (1998); hep-ex/9711002; *Phys. Lett. B* **466** 415 (1999); hep-ex/9907037
7. Yasuda O *Acta Phys. Pol. B* **30** 3089 (1999); hep-ph/9910428; Freund M et al. *Nucl. Phys. B* **578** 27 (2000); hep-ph/9912457; Freund M, Huber P, Lindner M *Nucl. Phys. B* **615** 331 (2001); hep-ph/0105071; Mocioiu I, Shrock R *JHEP* **0111** 050 (2001); hep-ph/0106139
8. Cervera A et al. *Nucl. Phys. B* **579** 17 (2000); Erratum, ibid. **593** 731 (2001); hep-ph/0002108
9. Akhmedov E Kh et al. *Nucl. Phys. B* **542** 3 (1999); hep-ph/9808270
10. Peres O L G, Smirnov A Yu *Phys. Lett. B* **456** 204 (1999); hep-ph/9902312
11. Akhmedov E Kh et al. *Nucl. Phys. B* **608** 394 (2001); hep-ph/0105029
12. Peres O L G, Smirnov A Yu *Nucl. Phys. B*: *Proc. Suppl.* **110** 355 (2002), hep-ph/0201069
13. Kuo T K, Pantaleone J *Phys. Rev. D* **35** 3432 (1987); Ohlsson T, Snellman H *Eur. Phys. J. C* **20** 507 (2001); hep-ph/0103252
14. Lim C-S, in *Proc. of the BNL Neutrino Workshop on Opportunities for Neutrino Physics, Upton, NY, Feb. 5 – 7, 1987* (Ed. M J Murtagh), Report No. BNL-52079, C87/02/05 (Long Island: BNL, 1987) p. 111; Preprint BNL-39675 (scanned version at http://www-lib.kek.jp/cgi-bin/img_index?8710120)
15. de Holanda P C, Smirnov A Yu *Phys. Rev. D* **66** 113005 (2002); hep-ph/0205241
16. Gonzalez-Garcia M C et al. *Phys. Rev. D* **63** 033005 (2001); hep-ph/0009350; Bilenky S M, Nicolo D, Petcov S T *Phys. Lett. B* **538** 77 (2002); hep-ph/0112216; Gonzalez-Garcia M C, Maltoni M *Eur. Phys. J. C* **26** 417 (2003); hep-ph/0202218
17. Ahmad Q R et al. (SNO Collab.) *Phys. Rev. Lett.* **89** 011301 (2002); nucl-ex/0204008
18. Dighe A S, Smirnov A Yu *Phys. Rev. D* **62** 033007 (2000); hep-ph/9907423
19. Akhmedov E Kh, Lunardini C, Smirnov A Yu *Nucl. Phys. B* **643** 339 (2002); hep-ph/0204091
20. Lunardini C, Smirnov A Yu *Phys. Rev. D* **63** 073009 (2001); hep-ph/0009356; *Nucl. Phys. B* **616** 307 (2001); hep-ph/0106149; Minakata H, Nunokawa H *Phys. Lett. B* **504** 301 (2001); hep-ph/0010240; Barger V, Marfatia D, Wood B P *Phys. Lett. B* **532** 19 (2002); hep-ph/0202158
21. Kobzarev I Yu et al. *Yad. Fiz.* **32** 1590 (1980) [*Sov. J. Nucl. Phys.* **32** 823 (1980)]
22. Pirogov Yu F *Eur. Phys. J. C* **17** 407 (2000); hep-ph/0002299
23. Koike M, Sato J *Phys. Rev. D* **61** 073012 (2000); Erratum, ibid. **62** 079903 (2000); hep-ph/9909469; Minakata H, Nunokawa H *Phys. Lett. B* **495** 369 (2000); hep-ph/0004114; *JHEP* **0110** 001 (2001); hep-ph/0108085
24. Lipari P *Phys. Rev. D* **64** 033002 (2001); hep-ph/0102046
25. Farzan Y, Smirnov A Yu *Phys. Rev. D* **65** 113001 (2002); hep-ph/0201105; Aguilar-Saavedra J A, Branco G C *Phys. Rev. D* **62** 096009 (2000); hep-ph/0007025; Sato J *Nucl. Instrum. Meth. A* **472** 434 (2000); hep-ph/0008056
26. Minakata H, Nunokawa H, Parke S *Phys. Lett. B* **537** 249 (2002); hep-ph/0204171
27. Krastev P I, Petcov S T *Phys. Lett. B* **205** 84 (1988)
28. Fishbane P M, Kaus P *Phys. Lett. B* **506** 275 (2001); hep-ph/0012088
29. Kuo T K, Pantaleone J *Phys. Lett. B* **198** 406 (1987)
30. Minakata H, Watanabe S *Phys. Lett. B* **468** 256 (1999); hep-ph/9906530
31. de Gouvêa A *Phys. Rev. D* **63** 093003 (2001); hep-ph/0006157
32. Nakaya T (Super-Kamiokande Collab.) *eConf* **C020620** SAAT01 (2002); hep-ex/0209036; Fukuda S et al. (Super-Kamiokande Collab.) *Phys. Lett. B* **539** 179 (2002); hep-ex/0205075; Eguchi K et al. (KamLAND Collab.) *Phys. Rev. Lett.* **90** 021802 (2003); hep-ex/0212021
33. Akhmedov E Kh, Branco G C, Rebelo M N *Phys. Rev. Lett.* **84** 3535 (2000); hep-ph/9912205
34. King S F *JHEP* **0209** 011 (2002); hep-ph/0204360
35. Kuchimanchi R, Mohapatra R N *Phys. Lett. B* **552** 198 (2003); hep-ph/0207373
36. Barr S M, Dorsner I *Nucl. Phys. B* **585** 79 (2000); hep-ph/0003058
37. Tanimoto M *Nucl. Instrum. Meth. A* **503** 157 (2001); hep-ph/0106064
38. Murayama H, Yanagida T *Phys. Lett. B* **520** 263 (2001); hep-ph/0010178; Barenboim G et al. *JHEP* **0210** 001 (2002); hep-ph/0108199; *Phys. Lett. B* **537** 227 (2002); hep-ph/0203261
39. Maltoni M, Schwetz T, Valle J W F *Phys. Lett. B* **518** 252 (2001); hep-ph/0107150; *Phys. Rev. D* **65** 093004 (2002); hep-ph/0112103
40. Peres O L G, Smirnov A Yu *Nucl. Phys. B* **599** 3 (2001); hep-ph/0011054
41. Shiozawa M, talk at *XXth Intern. Conf. on Neutrino Physics and Astrophysics* (*Neutrino 2002*), *Munich, Germany, 25 – 30 May 2002*; transparencies at http://neutrino2002.ph.tum.de/pages/transparencies/shiozawa/index.html
42. Gonzalez-Garcia M C, Maltoni M, Peña-Garay C *Phys. Rev. D* **64** 093001 (2001); hep-ph/0105269
43. Maltoni M et al. *Nucl. Phys. B* **643** 321 (2002); hep-ph/0207157
44. Päs H, Song L, Weiler T J *Phys. Rev. D* **67** 073019 (2003); hep-ph/0209373
45. Ahmad Q R et al. (SNO Collab.), nucl-ex/0309004
46. Schwetz T, talk at the *Intern. Workshop on Astroparticle and High Energy Physics, AHEP-2003, October 14 – 18, 2003, Valencia, Spain*; TUM-HEP-534/03; hep-ph/0311217
47. Dooling D et al. *Phys. Rev. D* **61** 073011 (2000); hep-ph/9908513