

Quantum nonlocality and the absence of *a priori* values for measurable quantities in experiments with photons

A V Belinskiĭ ¶

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Abstract. This article describes real and gedanken experiments where one can observe unusual space-time behavior of elementary particles.

1. Introduction

If you are going to a shop for a watermelon, you cannot be making a world-tour or be present at an academic council at the same time. If the watermelon turned out to weigh 8 kg, neither you nor the salesman doubt that both before and after weighing its weight was exactly 8 kg and not 15 kg. However, God created this world much richer than it seems to our common sense, and such intuitive ideas about space-time are not always true for the microscopic world. Let us consider such interesting situations in more detail.

2. The absence of photon number *a priori* values before the registration of photons

Consider the following experiment. Let a light source illuminate a detector (Fig. 1). Reducing gradually the intensity of light, we approach the photon counting regime, in which the detector registers quanta, i.e., the minimum portions of energy. It is commonly believed that photocounts, or bursts of the detector's photocurrent, correspond to the arrival of photons. But is it so? Do quanta really exist in the light field? The detector measures the number of photons in the field. But does a definite value of this quantity exist before the measurement?

¶ The author is also known by the name A V Belinsky. The name used here is a transliteration under the BSI/ANSI scheme adopted by this journal.

A V Belinskiĭ Physics Department, M V Lomonosov Moscow State University, Vorob'evy Gory, 119899 Moscow, Russian Federation
 Tel. (7-095) 143 48 31
 E-mail: belinsky@inbox.ru

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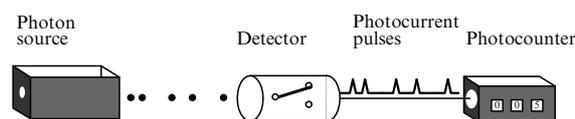


Figure 1. Schematic drawing of the apparatus for direct detection experiment.

Let us repeat the experiment many times. The radiation source can be chosen in such a way that, in some trials (realizations), one photon is registered, and in other trials, two photons are registered. What does the light field then look like? It seems that sometimes it consists of one photon, and sometimes of two photons. However, one can experimentally prove that this is not always true.

Consider an experiment (see Fig. 2) on three-order interference in nondegenerate parametric down-conversion [1]. In a transparent crystal with the quadratic nonlinearity (piezoelectric crystal), a light beam with frequency f_c generates two beams, the signal one and the idler one, with the frequencies f_a and f_b , so that $f_a + f_b = f_c$. The efficiency of the pump (c) conversion into the signal and idler beams is small, only about 0.000001%. Therefore, the main portion of

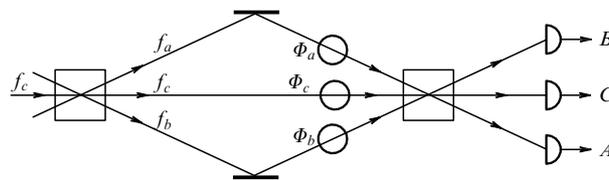


Figure 2. Schematic of an interference experiment that proves that values of measurable quantities do not exist *a priori*. Single photons at frequency f_c are fed to the input. The photocount probability for the detector A is proportional to $1 + \cos(\Phi_a + \Phi_b - \Phi_c)$, which is evidence of the simultaneous presence of the field in all three channels, i.e., of the existence of all three photons provided that they existed before the measurement (to be precise, after the first crystal). However, the energy of a single input photon provides only half of the energy of the three photons.

the pump radiation passes through the transparent crystal, so that there are three light beams at the output. In all three constituents of the radiation field, variable phase shifts Φ_a , Φ_b , and Φ_c are introduced, and then the three beams interact again in the second nonlinear crystal, which is similar to the first one. This second crystal makes an inverse conversion of the signal and idler beams into radiation at the pump frequency, and at the same time it directly converts the pump beam transmitted through the first crystal. At the output of the optical scheme, detectors register the intensities of all three beams. Figure 2 illustrates the nondegenerate case where the beams are noncollinear.

Let the first crystal be illuminated by a single photon. It was shown in Ref. [1] that the probability of occurrence of a photocount on the detector A is proportional to $1 + \cos(\Phi_a + \Phi_b - \Phi_c)$. This expression can be understood as interference with the phase $\Phi = \Phi_a + \Phi_b - \Phi_c$. The proportionality coefficient, which we omit here, is determined by the efficiency of nonlinear transformation in the crystals. The corresponding interference experiment has been performed by Burlakov et al. [2], and the cosine dependence on the combination of phases has been confirmed.

Let us try to interpret this result in the framework of an explicit model supposing that a definite number of photons exists *a priori* (before the registration) at the output of the first crystal. For simplicity, assume the quantum efficiency of both detectors to be equal to unity.

In the first set of trials, we remove the second nonlinear crystal. Then, phase delays in the channels do not influence the result and one observes photocounts either from detectors A and B or from detector C . This picture agrees with the assumption that there is either one photon with frequency f_c or two photons with frequencies f_a and f_b at the output of the first crystal.

In the second set of trials, we put the second crystal back. Now, the photocount probabilities depend on all three phases Φ_a , Φ_b , and Φ_c .

Interference with the unit contrast given by the dependence $1 + \cos(\Phi_a + \Phi_b - \Phi_c)$ testifies that by varying the phase delay for any field component, Φ_a , Φ_b , or Φ_c , one can make $\cos(\Phi_a + \Phi_b - \Phi_c) = -1$ and, hence, completely suppress the photocounts. Let us do it, so that there are no photocounts from detector A . Then, let us shut out the light in channel C between the two crystals. Immediately, photocounts with nonzero probability appear in channel A . It follows that even if in one of the realizations of the scheme with three open channels the field in channel C were absent, there would be nonzero photocount probability in channel A . But this probability is equal to zero! Thus, the field in channel C (single pump photons) is present in *each* realization. Similarly, by shutting out the light in other channels one can prove that the radiation field is simultaneously present (there are photon pairs) in channels A and B in *each* realization. In other words, if in some realizations, with all channels open, the field were absent in at least one channel, then the probability of occurrence of a photocount on the detector A would be nonzero. It follows then that the radiation field between the crystals exists in all three channels A , B , and C in *every* realization. The cosine dependence, $1 + \cos(\Phi_a + \Phi_b - \Phi_c)$, of the photocount probability on the linear combination of all three phases also confirms this fact: it cannot be represented as a sum of probabilities $P(\Phi_a, \Phi_b)$ and $P(\Phi_c)$. Although in the experiment of Ref. [2] this harmonic dependence was observed with

a constant background, so that, strictly speaking, there were no ‘zeros’, this last reasoning is still valid.

Thus, all three photons have to be present in the field between the two crystals. But this contradicts the energy conservation law, since only one pump photon was fed to the input of the first crystal, and its energy is half that of the three photons. This experiment on the interference of field with a definite energy and indefinite photon number contradicts the model with an *a priori* number of photons. Even if one assumes the interference of ‘photon parts’ whose energies sum up to give the constant energy of the radiation field, then one has to accept that all three ‘photon parts’ are present in the free space between the crystals in Fig. 2, since the field must exist simultaneously in all three channels. Then, in the first set of trials, with the second crystal removed, detection makes these ‘photon parts’ instantly join into one photon or two photons in a random way. But this is namely the absence of photons in the light field before their registration by the detector. “A photon is the photon only if it is a detected photon” [3].

3. Quantum nonlocality

Consider next an experiment with the Mach–Zehnder interferometer (Fig. 3). Suppose there is a single-photon state at the input and let us first remove the second beam splitter which is placed before the photodetectors. The detectors will register single photocounts in one of the channels and never in two channels simultaneously, since there is only one photon at the interferometer input.

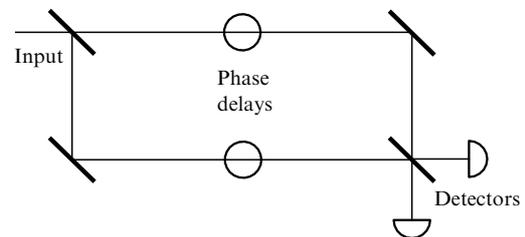


Figure 3. Schematic of the Mach–Zehnder interferometer.

Let us put the beam splitter back. The probability of occurrence of photocounts on the detectors is given by the harmonic function $1 \pm \cos(\Phi_1 - \Phi_2)$, where Φ_1 and Φ_2 are the phase delays in the interferometer arms. The sign depends on the detector that gives the photocounts. This harmonic dependence cannot be represented as a sum of two probabilities, $P(\Phi_1) + P(\Phi_2)$. Consequently, after the first beam splitter, the photon seems to be present in both interferometer arms simultaneously, although at the first stage of the experiment it existed only in one arm and never in both. It is this unusual spatial behavior that is called *quantum nonlocality*. It cannot be explained from the viewpoint of intuitive common sense, which is usually valid in the macroscopic world. Most probably, the reason is that quantum state vectors belong to the Hilbert vector space which does not necessarily imply space locality.

Let us consider another example [4] of quantum nonlocality (see Fig. 4). In some crystals, one can realize the so-called type-II parametric interaction where linearly polarized photons of laser radiation decompose into pairs of scattered photons with mutually orthogonal polarizations. Creation of

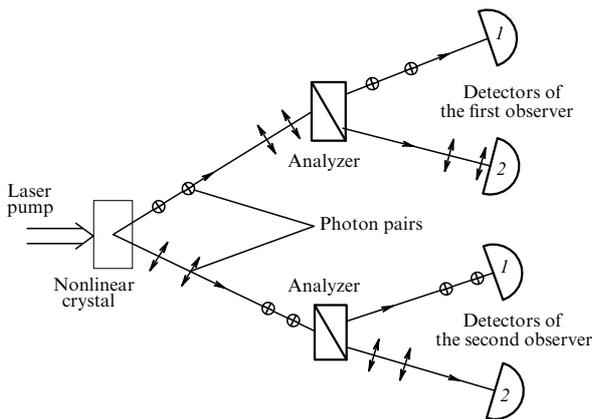


Figure 4. An experiment with simultaneously emitted photon pairs: although photons in the figure are shown with definite polarizations, in reality, neither photon of a pair has definite polarization before any of them is registered. Still, their polarizations always turn out to be mutually orthogonal. At the point in time when one photon of the pair is registered, the state of the other one is instantly changed and it acquires a definite polarization.

a photon pair takes place at one time but the polarization for each photon is not fixed beforehand. For instance, after the analyzer, a photon of a pair can go to detector 2 of the first observer, which means its polarization is in the plane of the figure, while the second photon of the same pair will then *necessarily* go to detector 1 of the second observer, which corresponds to its orthogonal polarization. With the probability 1/2, the opposite case can happen, when detector 1 of the first observer and detector 2 of the second observer come into action in parallel. If we correctly understand the results of the experiment considered in Section 2, then no photon of a pair has a definite polarization *a priori*, i.e. before at least one photon of the pair is detected. At the instant of registration, when one of the detectors fires, the so-called quantum state reduction takes place: if the second photon of the pair has not reached the detector, then with the probability unity it acquires polarization orthogonal to the one detected for the first photon in the pair. According to the common viewpoint, which is confirmed by experiment, reduction occurs instantly (certainly, within the limits determined by the experimental technique). The photons of a pair can bounce apart by a distance of several kilometers but the ‘information’ about the result of the first photon detection instantly changes the quantum state of the second one — it becomes the state with a definite polarization.

Is it possible to speak in this case about the superluminal transmission of information via spontaneous parametric down-conversion? Apparently not, because a connection between remote observers of photon pairs requires, in addition to photodetectors, a ‘telephone’ line. Indeed, without the data about the result of the first photon detection, the second photon observer sees in fact a random signal with equally probable (1/2) polarization.

4. On the Bell paradox

In a simplified form, one of the experiments on verifying Bell’s theorem for two observers is depicted in Fig. 5. A light source simultaneously emits pairs of photons, so that one photon is sent to observer *A*, and the other one to observer *B*. Each

observer has a measuring instrument that registers photons. It can operate in two regimes, which are shown symbolically as two positions of each switch, similarly to radio range switches. When the instrument registers a photon, we obtain binary information like ‘yes’ or ‘no’. It is convenient to denote the results of measurement as +1 or –1. The observers write down the measurement results indicating the arrival times of the photons and the readings, +1 or –1. If the photon registration occurred with the switch ‘up’, then the result +1 of the first observer is written as $A = +1$, and if the switch was ‘down’, then one writes the result $A' = +1$. For the second observer, everything is similar. The observers do not communicate (a brick wall is symbolically put up between them). The measurement protocols are sent to the coordinator (a circle on the right). The coordinator takes the results of simultaneous measurements and combines them into products of the form AB or AB' (four possible combinations), depending on the regime of photon measurements. In discussions with the observers before the experiment, they are informed when they should switch the regime. The products are averaged, and the so-called Bell inequality is constructed:

$$|S| \leq 1, \quad \text{where } S = \frac{\langle AB \rangle + \langle A'B \rangle + \langle AB' \rangle - \langle A'B' \rangle}{2}.$$

Here angle brackets denote averaging.

Deriving Bell’s inequalities is very simple [3]. Suppose that the result of measurements for any emitted photon pair is completely predetermined by the light source at the instant of their emission, and the source is not influenced by the measuring instruments and the observers. Then all possible results of measurements (values of A , A' , B , and B') are predetermined. Since their possible values are equal to +1 or –1, the value of $(AB + A'B + AB' - A'B')/2$ is also equal to +1 or –1, respectively, and the averaging leaves S in the interval $[-1, +1]$.

Under certain conditions, Bell’s inequality can be violated (see, for instance, Refs [5–11]). This means that the photons of a pair behave not as independent objects but as a correlated system, i.e., the result of photon registration by the first observer (+1 or –1) seems to immediately become ‘known’ to the second photon, although the photons may have been separated by a very large distance. For instance, in the experiments by Tittel et al. [8, 9], the distance between the observers A and B was more than 10 km. The so-called Franson’s interference scheme was used, in which each observer traps his photon and feeds it to the input of a Mach–Zehnder interferometer. The path difference in the

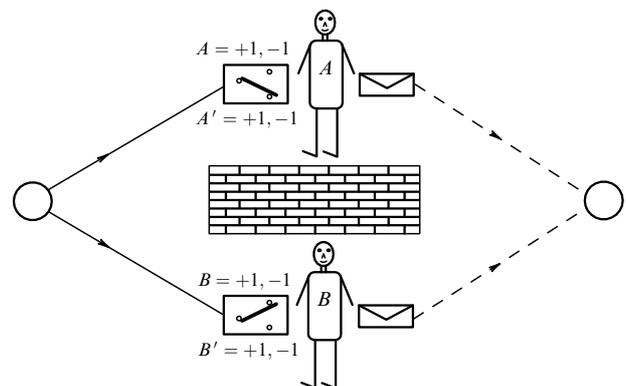


Figure 5. Schematic of the Bell inequality testing.

interferometer arms was made larger than the coherence length, so that usual single-photon interference, which we discussed at the beginning of Section 3 (see Fig. 3), was suppressed. Nevertheless, quantum correlation between the photons flown apart by 10 km was still present, and the Bell inequality was violated.

This fact can be explained in several ways. The first explanation consists of the formal statement that the parameters of photons simply have no measurable values. This concept suggests the absence of *a priori* values for the parameters of a photon. However, in this case we still face the question: can there be any relation between nonexistent parameters?

The second explanation implies a mysterious connection, of unknown origin, between flying apart photons. This connection is ‘switched on’ instantly between remote objects, i.e., the above-mentioned quantum nonlocality manifests itself.

The third way to explain the experiment is to assume that one of the two particles ‘lives’ in ‘negative time’, from the future to the past. In other words, it is born in the detector and travels to the source. At the instant of time it meets the source, the other particle is born. Since the first particle looks like that on a movie going backward, it seems to us that both particles are simultaneously born in the radiation source.

However, one can look at these experiments from a different viewpoint. The reasoning presented above is based on the traditional concepts of space and time in which a light field exists in reality. But these assumptions are not evident to all physicists. For instance, Professor Yu S Vladimirov from Moscow State University develops in his monographs [12, 13] the theory according to which there is no general space and time in the microscopic world. Such an approach seems to resolve quantum paradoxes, since in the absence of time in this world, the very concept of *a priori* existence disappears. Time (and space) appears only as a result of averaging the ‘individual times’ of many elementary particles composing a macroscopic object. Still, a detailed development of Vladimirov’s theory is still ahead.

Most important is the fact that the possibilities of existence of objects beyond the space–time is seriously discussed in the scientific community. In this connection, let us stress once again that the vectors of quantum states, which belong to the Hilbert vector space, are not subject to standard space–time restrictions.

5. Bell’s theorem with an account for losses

The experimental violation of Bell’s inequality, observed by Aspect et al. [5] and disproving the theory of hidden parameters, has been criticized lately. The existence of losses in a real situation allows the results to be formally explained by the local theory of hidden parameters (see, for instance, Refs [6,7] and the literature cited therein). In this section, it is shown that the experiments carried out in Ref. [5] can be rehabilitated.

If the local theory of hidden parameters is valid, then the results of measurements in the scheme of Fig. 5 are predetermined at the instant of time when two elementary particles are emitted by the source; they can be described by four-dimensional joint probabilities

$$P_{AA'BB'}(a, a', b, b') = \int_{A(a, a', b, b')} P(\lambda) d\lambda, \quad (1)$$

which are the probabilities of a sort of simultaneous measurement of all four quantities. Here, capital letters denote the measured quantities, and small letters denote their values (+1 or –1). Let $A(a, a', b, b')$ be the subset of the complete set $\{\lambda\}$ of the hidden parameters of the source, for which the measured quantities take the values a, a', b, b' , and $P(\lambda)$ be the probability density distribution for the hidden parameters. For brevity, let us denote the four-dimensional probabilities by their corresponding values, for instance,

$$P_{AA'BB'}(a = +1, a' = -1, b = -1, b' = +1) = (+ - - +).$$

There are 2^4 joint probabilities of this kind. Their sum is equal to unity, and each of them lies within the interval $[0, +1]$. Then, the moment

$$\langle AB \rangle = \sum_1^{16} ab P_{AA'BB'}(a, a', b, b'),$$

and analogously for the other three moments. If one substitutes these moments in the expression for Bell’s observable

$$S = \frac{\langle AB \rangle + \langle A'B \rangle + \langle AB' \rangle - \langle A'B' \rangle}{2},$$

which was introduced in the previous section, then one can easily verify the Bell inequality in the Clauser–Horne–Shimony–Holt form:

$$|\langle AB \rangle + \langle A'B \rangle + \langle AB' \rangle - \langle A'B' \rangle| \leq 2. \quad (2)$$

This is the second way of deriving it. Quantum theory predicts violation of Bell’s inequality, and the experiments [5] confirm this prediction. However, quantum efficiencies of the detectors used in those experiments were less than unity, and in fact, trichotomic variables $a, a', b, b' = 0, \pm 1$ were measured. In this case, inequality (2) can be violated even in the framework of the local theory of hidden parameters, since additional combinations of joint probabilities are possible. Their number becomes equal to 3^4 . Specific examples are given in Refs [6, 7].

However, certain restrictions can be imposed on joint four-dimensional probabilities. Indeed, the probabilities that detectors of the observers A and B come into action are equal to their quantum efficiencies η_a and η_b , which are assumed to take into account all possible losses. Suppose that the quantum efficiencies do not depend on the set of hidden parameters $\{\lambda\}$ (validity of this assumption will be discussed below). Then, one finds

$$(\pm \pm \pm \pm)_{\eta < 1} = \eta_a^2 \eta_b^2 (\pm \pm \pm \pm)_{\eta = 1},$$

since the four-dimensional probability is a probability of four photocounts. Sixteen joint probabilities with one zero can also be expressed in terms of the probabilities for registering particles by ideal detectors, for instance,

$$(0 \pm \pm \pm)_{\eta < 1} = (1 - \eta_a) \eta_a \eta_b^2 [(\pm \pm \pm \pm)_{\eta = 1} + (- \pm \pm \pm)_{\eta = 1}].$$

Thirty-two joint probabilities with two zeros are calculated similarly to the following example

$$(0 \pm 0 \pm)_{\eta < 1} = (1 - \eta_a) \eta_a (1 - \eta_b) \eta_b [(\pm \pm \pm \pm)_{\eta = 1} + (- \pm \pm \pm)_{\eta = 1} + (\pm \pm - \pm)_{\eta = 1} + (- \pm - \pm)_{\eta = 1}],$$

and each one from 16 joint probabilities with three zeros is equal to the sum of 8 probabilities calculated for ideal detectors and multiplied by $(1 - \eta_a)^2(1 - \eta_b)\eta_b$ or by $(1 - \eta_a)\eta_a(1 - \eta_b)^2$, depending on the positions of the zeros. The remaining probability, which corresponds to the instance when no detector fires, is given by

$$(0000) = (1 - \eta_a)^2(1 - \eta_b)^2 \sum_{\pm\pm\pm\pm} (\pm \pm \pm \pm).$$

Now, we can find the moments

$$\langle AB \rangle_{\eta < 1} = \eta_a \eta_b \langle AB \rangle_{\eta = 1}, \quad (3)$$

and analogously for the other three moments. Let us also define the combination

$$\langle |AB| \rangle_{\eta < 1} = \eta_a \eta_b. \quad (4)$$

Therefore, the experiment with ideal detectors can be described using the moments for a scheme with real detectors, for instance,

$$\langle AB \rangle_{\eta = 1} = \frac{\langle AB \rangle_{\eta < 1}}{\langle |AB| \rangle_{\eta < 1}} = \frac{\sum_M ab}{M}, \quad (5)$$

where M is the number of simultaneous detections of particles by both observers, i.e., when single photocounts are discarded. But this is exactly the way the results of experiments [5] were processed.

Let us return to the validity of the assumption that the detectors' quantum efficiencies are independent of the hidden parameters $\{\lambda\}$. If this assumption is true, then the final relations (3) and (4) are also true, and they can be easily verified in experiment by introducing controllable losses into the channels of observers A and B . If the test confirms these relations, then one can use relation (5). Although, strictly speaking, it is not possible to test relations (3) and (4) in the whole range of quantum efficiencies, from zero to unity. Still, relations (3) and (4) are rather evident, and are hardly doubted by specialists. Note that all possible losses in the channels are included into quantum efficiencies. Thus, using the moments measured in real experiment, one can reconstruct the moments that would be obtained by an ideal measuring instrument with no losses.

During the last 10 years, intensive efforts have been made to experimentally disprove the theory of hidden parameters. The experiments involve high-efficiency detectors and, therefore, are very expensive (see, for instance, Ref. [11]). Considerations given here allow one to bring down the requirements to the detectors' quantum efficiency for the experimental disproof of the local theory of hidden parameters.

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