system do not constitute a complete set of operators needed for universal quantum computing. Nor should they: as shown in Ref. [13], the complete set of exact quantum operators can only be realized in a system with non-Abelian topological order — in a system, in other words, where excitations are non-Abelian anyons. Our lattice, however, produces the simplest Abelian anyons, as already explained above. Thus, the proposed model can be considered as a model of an ideal quantum memory, but as yet not a model of an ideal quantum computer. A theory of Josephson lattice-based topologically-ordered lattice systems with a non-Abelian gauge group has been given recently in Ref. [29].

6. Conclusions

In the present talk, which is the development of the ideas of Ref. [14], we offer a new type of a Josephson lattice, capable of acting as an 'ideal' quantum memory. Compared to the original version outlined in Ref. [14], there are a number of important advantages to the new system, namely: (i) it operates in the parameter range $E_J \gg E_C$, thereby reducing the level of poorly controlled electric noise due to offset charges in the insulating substrate; (ii) it employs only one type of Josephson junctions and is therefore much simpler to fabricate; (iii) in the ideal case it is exactly 2^K-fold degenerate, whereas in the version in Ref. [14] degeneracy was achieved asymptotically with the size of the system, with accuracy $\exp(-cL)$, where $c \sim 1$; for the new system a similar constant is calculated, including small perturbations, to be of the order of $\ln(r/\epsilon) \gg 1$, i.e., an array of smaller size will suffice to achieve the desired accuracy.

We have greatly benefited from numerous discussions with G Blatter, D A Ivanov, A S Ioselevich, S E Korshunov, A I Larkin, A Millis, B Pannetier, and E Serret when working on the idea outlined here.

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Electrodynamics of materials with negative index of refraction

V G Veselago

Over the past few years there has been an avalanche of progress in a new branch of electrodynamics — electrodynamics of materials with negative refraction. Experiments in this area were pioneered by a group of physicists at the University of California at San Diego, USA [1, 2]. They demonstrated the unusual electrodynamic properties of some composite materials, which can be explained purely formally by ascribing a negative index of refraction n to these materials. These composites are the assemblies of small metallic elements arranged into strictly regular crystal-like geometric structures. The structures can be considered continuous for wavelengths considerably longer than the size of and the separation between its constituent elements. The UCSD experiments were performed in the centimeter wavelength range on composites with element size and separation typically of the order of 7 to 10 mm.

The key experimental finding of the San Diego study was a rather unusual manifestation of Snell's law of refraction for such materials. In Fig. 1 is shown the passage of a light ray through the interface between two media with indices of refraction n_1 and n_2. If we take $n_1 = 1$ (without loss of generality) then, customarily, a refracted ray takes the path *1–4*. In the San Diego experiments the ray took the path *1–3*.

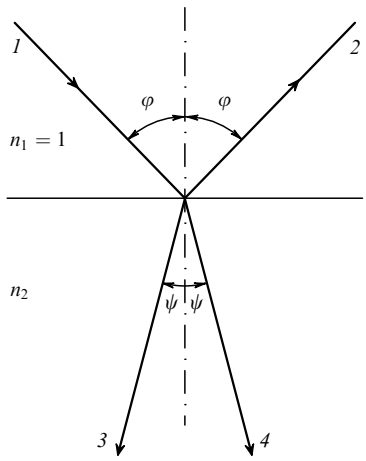

Figure 1. Refraction of light at the interface between two media. The paths *1–4* and *1–3* are taken by the incident and refracted rays in the cases $n_2 > 0$ and $n_2 < 0$, respectively.

This path satisfies Snell's law if we assume that $n_2 < 0$. Snell's law itself,

$$\frac{\sin\varphi}{\sin\psi} = \frac{n_2}{n_1} = n_{21}\,, \tag{1}$$

remains unchanged by this step.

Along with Snell's law, some other aspects of electrodynamics and optics, in particular the Doppler effect, the Cherenkov effect, Fresnel formulas, and Fermat's principle, manifest themselves in an unusual way in materials with $n < 0$. A fairly comprehensive introduction to electrodynamics of materials with $n < 0$ is given, in particular, in Refs [3–6]. The important point these papers make is that in materials with a negative index of refraction the dielectric permittivity ε and the magnetic permeability μ are also negative. Importantly, all the results obtained in these papers are valid for isotropic materials, for which the quantities n, ε, and μ are scalars.

The negative value of n also corresponds to the fact that the wave vector $\mathbf{k}$ and the Poynting vector $\mathbf{S}$ in such materials are antiparallel or, equivalently, that the phase velocity is antiparallel to the group velocity.

This is verified by simply writing down the Maxwell equations and the Poynting vector expression for the case of uniform plane waves in an isotropic medium,

$$\begin{aligned} [\mathbf{k}\times\mathbf{E}] &= \frac{\omega}{c}\,\mu\mathbf{H}\,,\\ [\mathbf{k}\times\mathbf{H}] &= -\frac{\omega}{c}\,\varepsilon\mathbf{E}\,,\\ \mathbf{S} &= [\mathbf{E}\times\mathbf{H}]\,. \end{aligned} \tag{2}$$

It is easily seen that simultaneously changing the sign of ε and μ translates the right handed vector triplet $\mathbf{k}$, $\mathbf{E}$, and $\mathbf{H}$ into the left handed one. It is for this reason that such composites are called left-handed materials (LHMs) in the English language literature.

Thus it can be argued that isotropic media with both ε and μ negative exhibit negative refraction (or have a negative value of n, which is the same) and that the phase and group velocities in them are aligned antiparallel. Also the reverse is true: if an isotropic material has a negative value of the index of refraction n, then it has both ε and μ negative, and the phase and group velocities are in opposite directions.

It should be noted that, by itself, the notion of oppositely directed phase and group velocities is not anything new. It was discussed, in particular, in the long-standing work of L I Mandelstam [7]. In addition, there are electronic devices that have long been known (for example, the backward wave tube, BWT), in which the phase velocity is opposite to the direction of the energy flow. In recent years there has been a good deal of discussion about the properties of so-called photonic crystals [8], another system where the vectors $\mathbf{k}$ and $\mathbf{S}$ can be made to be oppositely directed. However, photonic crystals are generally essentially anisotropic and cannot be characterized by a scalar index n. The same is true for BWT-type devices.

The advent of materials with a negative value of n raises a very important question about all those laws and formulas in electrodynamics, optics, and in related engineering sciences that involve the index of refraction n: to what measure is their validity preserved upon the transition to $n < 0$? When straightforwardly replacing $n \to -n$, can we always expect to arrive at a correct result as we do in the case of Snell's law? The answer is generally negative because most laws and formulas of electrodynamics and optics correspond to the case of an apriori nonmagnetic material with magnetic permeability $\mu = 1$. The result of such 'nonmagnetic approximation' is that many formulas originally containing μ cardinally change upon the substitution $\mu = 1$ and it is only in this nonmagnetic approximation that they retain their validity. The following table illustrates the situation.

From this table it can be seen that there are three groups of physical laws and effects whose formulation changes differently upon transition from nonmagnetic approximation formulas to exact expressions. The first group includes Snell's law and the Doppler and Cherenkov effects, in whose formulas the usual nonmagnetic approximation expression $n = \sqrt{\varepsilon}$ should simply be replaced by $n = \sqrt{\varepsilon\mu}$, and if ε and μ are both negative, then n should also be taken with the minus sign.

The second group includes light refraction and reflection laws — in particular, the Fresnel formulas. In these, the transition from the nonmagnetic approximation to exact theory is achieved by replacing $n = \sqrt{\varepsilon}$ not by $n = \sqrt{\varepsilon\mu}$, but by $\sqrt{\varepsilon/\mu} = 1/z$, where $z = \sqrt{\mu/\varepsilon}$ is the wave impedance of the medium. The wave impedance has the dimension Ohm and, together with the speed of light, is a unique characteristic of the medium. It can be seen from the table that, among other things, the condition for there being no light reflection at the flat interface between two media changes significantly on departure from the nonmagnetic approximation. What this condition now requires is that the wave impedances of the two

Table

Physics	Nonmagnetic approximation	Exact formula
Snell's law, Doppler effect, Cherenkov effect $n = \sqrt{\varepsilon} \to n = \sqrt{\varepsilon\mu}$, if ε and $\mu < 0$, then $n < 0$	$\dfrac{\sin\varphi}{\sin\psi} = n_{21} = \sqrt{\dfrac{\varepsilon_2}{\varepsilon_1}}$	$\dfrac{\sin\varphi}{\sin\psi} = n_{21} = \sqrt{\dfrac{\varepsilon_2\mu_2}{\varepsilon_1\mu_1}}$
Fresnel formulas $n = \sqrt{\varepsilon} \to \dfrac{1}{z} = \sqrt{\dfrac{\varepsilon}{\mu}}$	$r_\perp = \dfrac{n_1\cos\varphi - n_2\cos\psi}{n_1\cos\varphi + n_2\cos\psi}$	$r_\perp = \dfrac{z_2\cos\varphi - z_1\cos\psi}{z_2\cos\varphi + z_1\cos\psi}$
Reflection coefficient for normal incidence on an interface	$r = \dfrac{n_1 - n_2}{n_1 + n_2}$	$r = \dfrac{z_2 - z_1}{z_2 + z_1}$
No-reflection condition	$n_1 = n_2$	$z_1 = z_2$
Brewster law	$\tan\varphi = n$	$\tan\varphi = \sqrt{\dfrac{\varepsilon_2}{\varepsilon_1}\,\dfrac{\varepsilon_2\mu_1 - \varepsilon_1\mu_2}{\varepsilon_2\mu_2 - \varepsilon_1\mu_1}}$

media, not their indices of refraction, be equal. It is important to emphasize that, unlike n, the wave impedance z remains positive for negative values of ε and μ.

And, finally, the third group of relations that depend on n and change significantly upon transition from nonmagnetic to exact formulas includes, in particular, the Brewster angle formula $\tan\varphi = n$. The exact expression for the Brewster angle is given in the last row of the table. The important point to note is that the expression under the square root in this exact formula remains unaltered by changing the signs of ε and μ simultaneously in one of the media. It must be remembered that the Brewster angle formula given in the table corresponds to one particular polarization of the light. The formula for the polarization perpendicular to this one is obtained from the formula in the table by the replacements $\varepsilon \to \mu$ and $\mu \to \varepsilon$ in the expression under the square root. Thus, reflection at the Brewster angle always takes place, for any permittivity and permeability values — but only for one of the two possible polarizations the incident light can have.

The introduction of the concept of negative refraction of index also has made it possible to improve the formulation of such a fundamental law as Fermat's principle. A detailed discussion of this question has been given recently in Ref. [9], where it is shown that for an electromagnetic wave traveling through materials with n of either sign, the correct formulation of Fermat's principle is the requirement that the total length of the optical path,

$$\delta L = \delta \int n \, \mathrm{d}l = 0 \,, \tag{3}$$

be an extremum. In this expression (which is in fact the eikonal) one integrates along the actual path along which the light ray travels. Such an approach assumes that the length of the optical path traveled by an electromagnetic wave in a medium with negative n is also negative. This implies, in particular, that in some cases the total length of the optical path can be negative or even zero, although clearly neither the geometrical length of the light propagation path nor the light propagation time itself are by any means zero.

This is exactly the situation which occurs when light travels through a plane-parallel plate made of material with $\varepsilon = \mu = n = -1$. Such a plate, as can be seen from Fig. 2, can focus radiation from a point source located on the other side of the plate to a point. From Fig. 2 it is seen that the path Am traveled by the light from the source to the plate and the path nB from the plate to the image add up to give the path mn the light travels within the plate,

$$Am + nB = mn \,. \tag{4}$$

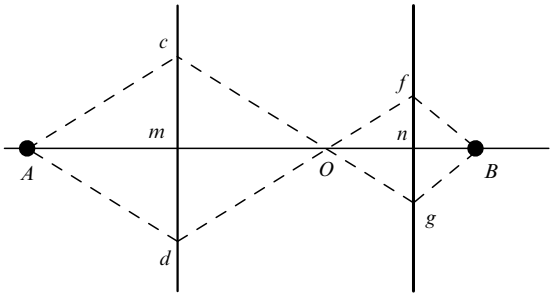

Figure 2. Propagation of light from object A to image B through a plane-parallel plate of material with $\varepsilon = \mu = n = -1$, in a vacuum.

Similar relations are valid for any other possible light propagation path, for example $AcgB$ or $AdfB$. But because indices of refraction inside and outside the plate are, respectively, $n = -1$ and $n = +1$, the total optical path for the light traveling from point A to point B is zero for any possible propagation path according to expression (3). At the same time, the light propagation time from A to B is essentially nonzero as already mentioned above.

Now focusing light from a point source on the one side of the plate to a point on the other does not mean that the plate is a lens. Such a plate is an ideal optical device which transfers the image of an object from the space of objects to that of images without any distortion. But such a transfer is only possible for objects whose distance from the plate does not exceed the plate's thickness. The plate clearly cannot focus a parallel beam of light, which comes from infinity, to a point. However, the properties of such a plate are undoubtedly very interesting and might have practical implications.

An important point to have in mind when evaluating negative index of refraction materials is that they are bound to be frequency dispersive. To see this, note that if ε and μ are both zero, then in the absence of dispersion the total energy of the material,

$$W = \varepsilon \mathbf{E}^2 + \mu \mathbf{H}^2 \,, \tag{5}$$

is negative. In the presence of frequency dispersion, however, the expression (5) is written somewhat differently,

$$W = \frac{\partial(\varepsilon\omega)}{\partial\omega}\,\mathbf{E}^2 + \frac{\partial(\mu\omega)}{\partial\omega}\,\mathbf{H}^2 \,. \tag{6}$$

It is readily seen that the derivatives $\partial(\mu\omega)/\partial\omega$ and $\partial(\varepsilon\omega)/\partial\omega$ are positive if the frequency dispersion relations for ε and μ are taken in the following sufficiently general form:

$$\mu = 1 - \frac{A_\mathrm{m}^2}{\omega^2} \,, \tag{7}$$

$$\varepsilon = 1 - \frac{A_\mathrm{e}^2}{\omega^2} \,. \tag{8}$$

If we set

$$A_\mathrm{e}^2 = A_\mathrm{m}^2 = A^2 > \omega^2 \,, \tag{9}$$

then the index of refraction will be negative, and the phase velocity

$$v_\mathrm{ph} = \frac{c}{1 - A^2/\omega^2}$$

and the group velocity

$$v_\mathrm{gr} = \frac{c}{1 + A^2/\omega^2}$$

will be related by the relation

$$\frac{c}{v_\mathrm{ph}} + \frac{c}{v_\mathrm{gr}} = 2 \,. \tag{10}$$

For waves propagating in a negatively dispersive medium we must put a minus sign in front of the wave vector $\mathbf{k}$. In absorbing media, however, the vector $\mathbf{k}$ has not only a real

part but also an imaginary part. The reason is that expressions for ε and μ have imaginary parts. So, the question arises: should the sign in front of the imaginary part of the wave vector be changed if the sign in front of its real part is changed?

Let us write the expressions for ε and μ in the form

$$\varepsilon = \varepsilon' + \mathrm{i}\varepsilon'', \qquad \mu = \mu' + \mathrm{i}\mu''. \tag{11}$$

It is readily seen that for low damping the expression for **k** becomes

$$k = k' + \mathrm{i}k'' = \sqrt{(\varepsilon' + \mathrm{i}\varepsilon'')(\mu' + \mathrm{i}\mu'')}$$
$$\approx \sqrt{\varepsilon'\mu'}\left[1 + \frac{\mathrm{i}}{2}\left(\frac{\varepsilon''}{\varepsilon'} + \frac{\mu''}{\mu'}\right)\right]. \tag{12}$$

From equation (12) it is readily seen that, by itself, changing the signs of the real parts of ε and μ does not automatically change the sign of the imaginary part of the wave vector. To achieve this, it is necessary to change the signs of the imaginary parts of ε and μ, which corresponds to the transition from a positively to a negatively absorbing material — similar to what takes place in quantum amplifiers, for example. Such a transition generally has no relation to the possible transition from ordinary materials with positive refraction to negative refraction materials.

The way we assess the implications of this new concept, a negative refraction material, crucially depends on whether such materials are really available. This question already arose during the publication of Refs [3, 4]. We had at one time devoted much effort to obtaining a negative refraction material using the magnetic semiconductor $CdCr_2Se_4$ as a basis, but met with no success because of considerable technological difficulties in synthesizing this material. Nor did our analysis [5] of the properties of the exotic mixture of electric and magnetic charges produce anything worth mentioning here.

A breakthrough came, as already mentioned in the beginning of this talk, with the announcement in Refs [1, 2] of a composite material which could be characterized by negative values of ε and μ and hence by a negative value of n. The material consisted of many copper posts and rings arranged in a strict geometric pattern. The posts and rings were in facts antennas which responded to an electric and a magnetic field, respectively. The size of and separation between these elements was smaller than the wavelength, and the system as a whole had negative effective values of ε and μ.

Reference [2] reported the direct measurement of the angle of refraction for a prism made of this composite, which showed the relation (1) with negative n to be entirely valid for this material. These experiments were repeated by at least two independent teams with the same positive result later on [10, 11].

The somewhat unusual electrodynamics of the newly-emerging class of materials has led to a number of controversial statements in the literature. For example, it is argued [12] that negative refraction takes place only for the phase velocity and that the group velocity under all circumstances obeys the usual positive-n law of refraction. The authors are not embarrassed by the fact that differently-directed phase and group velocities is a typical feature of optically anisotropic media which apriori cannot have a scalar index of refraction. The mistake of the authors of Ref. [12] is that they confused the direction of the group velocity with that of the perpendicular to the surface of constant amplitude when considering amplitude-modulated wave propagation in a medium. This mistake is analyzed in sufficient detail and explained in Ref. [13].

There is one more question which arises in close connection with the emergence of negative refraction materials. This is the problem of overcoming the diffraction limit or, equally, of amplifying the so-called evanescent modes, to use a somewhat different terminology. The first to address this problem was Pendry [14], who argued that negative refraction materials can support waves in which k_z, the wave vector component along the propagation direction, is purely imaginary,

$$k_z^2 = \frac{\omega^2}{c^2} - k_x^2 < 0. \tag{13}$$

This inequality holds for very large k_x, i.e., for very short wavelengths.

In materials with a positive value of n the amplitude of such waves (evanescent modes) decays exponentially along the z axis in accordance with Eqn (13), and this is precisely why optical systems cannot image objects much less than a wavelength in size. However, it is argued in Ref. [14] and in many other works that followed that in negative refraction materials waves with large k_x are amplified rather than attenuated. This statement is equivalent to choosing in the relation

$$k_z = \pm\mathrm{i}\sqrt{k_x^2 - \frac{\omega^2}{c^2}} \tag{14}$$

the sign '$-$' in front of the imaginary root — rather than '$+$' as usual. The author of Ref. [14] introduced the concept of a 'superlens' for a device similar to that depicted in Fig. 2 and argued that there is no classical diffraction limit constraint for this device.

Statements like this are perhaps most convincingly disproved by the electron modeling study [15], which shows that in the case of negative n evanescent modes can only propagate for distances much smaller than the wavelength — the same as usual. This, however, does not rule out the need for fully understanding the propagation of such modes in the case of negative n.

We are now just at the beginning of the path leading us to a new area of electrodynamics, a fascinating and promising one. The number of researchers and research groups and organizations involved in this theme is avalanching, and so is the number of publications in the area. The interested reader is referred to the Internet web site *http://physics.ucsd.edu/~drs/left_home.htm* for a very comprehensive collection of materials on the subject. The author's papers [3–6] are available at *http://zhurnal.ape.relarn.ru/~vgv* in both Russian and English.

Note in proof

At a recent seminar on negative refraction materials held in Arlington, USA, the creation of composite materials capable of operating at frequencies of up to 300 GHz was reported, and a talk on the first experiments aimed at the fabrication of a composite magnetic material with individual elements about 35 μm in size was presented.

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