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'Stark ladder' laser with a coherent electronic subsystem

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1. Introduction

As was shown theoretically many years ago [1, 2], the application of a static electric field E_0 to a semiconductor (or a superlattice [3, 4]) leads to a radical change in the electronic spectrum: the continuous spectrum becomes a discrete set of equidistant levels (Stark ladder, Fig. 1) with the energy separation $\hbar\omega_B$ ($\omega_B = eEa$ is the Bloch frequency, a the spacing of the crystal or the superlattice). Since then, attempts have been and are being made to employ inter-level transitions for generating an electromagnetic field. These attempts face fundamental challenges, however:

(1) Scattering and dissipation lead to the broadening of the levels.

(2) Generation, if possible, should at best occur at the surface [5] because the probabilities of transitions involving the radiation and absorption of field compensate each other.

(3) Quasi-classical theory [6] does predict amplification, however, but one with an unusual, polarization-type, line shape. The amplification is zero (changes sign) at the Bloch frequency $\omega = \omega_B$. It can therefore be expected that the fluctuations of a static field strongly suppress amplification.

(4) Electric field domains form due to the low-frequency negative differential conductivity (NDC).

As for the first problem, it has to a large extent been overcome by advances in the fabrication technology of superlattices of quantum wells, wires, and dots.

The present work proposes a way of overcoming the remaining challenges by obtaining a coherent regime in finite superlattices of N sites. The purpose of the work is to examine theoretically whether coherent generation is possible. In this regime electrons are delivered to the lattice by means of a resonance, make N coherent radiative transitions, and then are removed by means of resonance tunneling. There are three questions we are trying to answer in this work:

(1) Is such a regime in principle possible?

(2) How to obtain it?

(3) What are its properties and can problems 2, 3, and 4 be solved?

In this work it is shown that the coherent regime is possible if the time an electron spends in the structure is smaller that the decoherence time. Such a regime occurs when electrons within a sufficiently narrow energy range tunnel resonantly into a structure and when certain conditions on the structure's parameters are fulfilled.

Coherent generation turns out to be bulk generation (power is proportional to the number of wells) which has a symmetric (stable to fluctuations) gain contour and does not cause domain formation. The reason is that in a coherent situation radiative transitions do not depend on populations [7] and the NDC tends to zero.

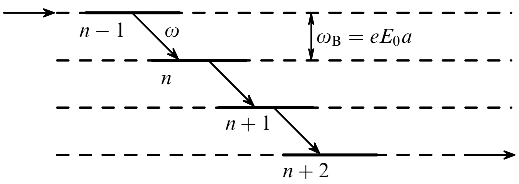

Figure 1. Single subband Stark ladder.

In this work we were able to obtain exact analytical solutions for the case of N wells. For structures with an odd number of wells the energy of the incoming electrons must simply be equal to the resonance energy of any field. That is, the energy tuning necessary for a single-well structure [7] is not needed in this case. Universal relations are found for the parameters of an N-well structure which ensure that resonance conditions are fulfilled for all the wells simultaneously.

2. The model

Specific calculations were performed for a two-miniband Stark ladder (Figs 2 and 3). In this case vertical radiative transitions occur, which lead to higher gain than does the single-miniband sublattice (diagonal transitions, see Fig. 1). The static electric field E_0 shifts the levels such that the energy $\varepsilon_{1R}^{(n-1)}$ of the lower level of the $(n-1)$ well coincides with the energy $\varepsilon_{2R}^{(n)}$ of the upper level of the n well. The difference in energy is approximately equal to the electromagnetic field frequency $E(t) = E\cos\omega t$. The amplitude of the field is determined from the equation for the field [7]

$$\frac{E}{2\tau_0} = -\frac{2\pi}{\varkappa} J_c(N), \tag{1}$$

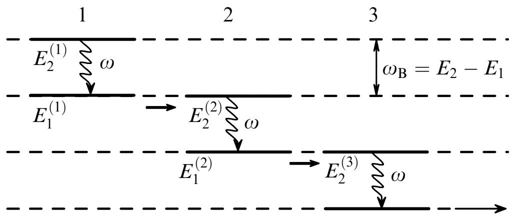

Figure 2. Two-subband Stark ladder.

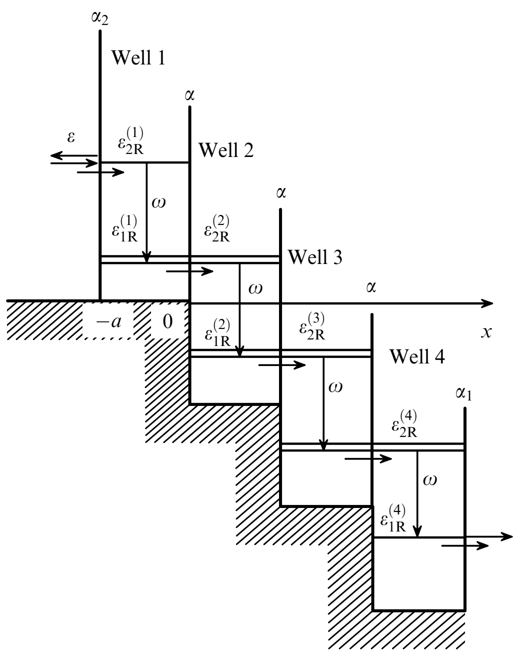

Figure 3. A structure with $N = 4$.

where J_c is the reduced polarization current, in phase with the field. The current J_c describes transitions between the levels. Here τ_0 is the photon lifetime in the cavity, and $\varkappa$ is the dielectric constant.

The current $J_c(x)$ can be found using the familiar expression

$$J(x,t) = -\mathrm{i}e\left[\Psi^* \frac{\partial \Psi}{\partial x} - \text{c.c.}\right], \tag{2}$$

where the wave function of the system $\Psi(x,t)$ obeys the Schrödinger equation

$$\mathrm{i}\,\frac{\partial \Psi}{\partial t} = -\frac{\partial^2 \Psi}{\partial x^2} + U(x)\Psi + \hat{V}(x,t)\Psi\,. \tag{3}$$

Here

$$U(x) = \alpha_2 \delta(x+a) + \sum_{n=0} \alpha\delta(x-an) + \alpha_1 \delta\big[x - a(N-1)\big] - E_0 \varphi(x)\,, \tag{4}$$

$$\varphi(x) = \sum_n \Theta\big[x-(n-1)a\big], \qquad \Theta(x) = \begin{cases} 1\,, & x>0\,,\\ 0\,, & x<0\,, \end{cases} \tag{5}$$

the α_j are the barrier powers (α_2 for the left most well, α_1 for the right most, and α in-between); $2m = \hbar = 1$. The last term in Eqn (3) describes the interaction of electrons with the electromagnetic field (A_x being the Coulomb gauge vector potential):

$$\hat{V}\Psi = 2e\mathrm{i}A_x\,\frac{\partial \Psi}{\partial x} = V\big[\exp(\mathrm{i}\omega t) - \exp(-\mathrm{i}\omega t)\big]\frac{\partial \Psi}{\partial x}\,, \qquad V = -\frac{eE}{\omega}\,. \tag{6}$$

Following Ref. [7], we seek the steady-state (stationary) solution of Eqn (3) in the form

$$\Psi(x,t) = \sum_n \sum_m \exp\big\{-\mathrm{i}t\big[\varepsilon + m\omega - E_0(n+1)\big]\big\}\,\psi_{nm}(N,x)\,, \qquad m = 0, \pm1, \pm2, \ldots, \quad 1 \leqslant n \leqslant N\,. \tag{7}$$

The function $\psi_{nm}(x,N)$ describes states with quasienergies $\varepsilon + m\omega$ in the nth well and satisfy the following system of equations:

$$\big[\varepsilon + m\omega - E_0(n+1)\big]\psi_{nm} + \frac{\mathrm{d}^2\psi_{nm}}{\mathrm{d}x^2} = V\left(\frac{\mathrm{d}\psi_{n,m-1}}{\mathrm{d}x} - \frac{\mathrm{d}\psi_{n,m+1}}{\mathrm{d}x}\right). \tag{8}$$

It is well known that the main contribution to laser generation comes from two resonance levels that differ in energy by the frequency ω. In the present case, for the nth well these are the upper level with energy $\varepsilon_{2\mathrm{R}}^{(n)}$ and the lower level with $\varepsilon_{1\mathrm{R}}^{(n)}$. The wave functions corresponding to these levels are $\psi_{n2}(x)$ and $\psi_{n1}(x)$, so that the wave function (7) reduces to two terms for each well,

$$\Psi(x,t) = \psi_{n2}\exp\big\{-\mathrm{i}t\big[\varepsilon - E_0(n-1)\big]\big\} + \psi_{n1}\exp\big\{-\mathrm{i}t\big[\varepsilon - \omega - E_0(n-1)\big]\big\}\,, \qquad a(n-2) \leqslant x \leqslant (n-1)a\,, \quad 1 \leqslant n \leqslant N\,. \tag{9}$$

The functions ψ_{n2} and ψ_{n1} satisfy the system of equations

$$\big[\varepsilon - E_0(n-1)\big]\psi_{n2} + \frac{\mathrm{d}^2\psi_{n2}}{\mathrm{d}x^2} = V\,\frac{\mathrm{d}\psi_{n1}}{\mathrm{d}x}\,, \tag{10}$$

$$\big[\varepsilon - \omega - E_0(n-1)\big]\psi_{n1} + \frac{\mathrm{d}^2\psi_{n1}}{\mathrm{d}x^2} = -V\,\frac{\mathrm{d}\psi_{n2}}{\mathrm{d}x} \tag{11}$$

with appropriate boundary conditions [7]. The boundary conditions describe the flow of electrons from $x = -\infty$, their reflection, the exit to $x = +\infty$, as well as the continuity of the wave functions and the jump in their derivatives at the boundaries of the quantum wells.

Formally, the problem reduces to the solution of a set of $4N$ nonhomogeneous algebraic equations. In this work, the exact solutions, the shape of the wave function, and the currents for any N are obtained.

3. Single-well structure

The problem for $N = 1$ was solved earlier in Ref. [7]. The current $J_c(1)$ and equations for the field have the form

$$J_c(1) = \int_{-a}^{0} \mathrm{d}x\, J_c(1,x) = E\,\frac{\Gamma^2 Q\eta}{\big|\tilde{\Delta}(1)\big|^2}\,, \tag{12}$$

$$f(1,\xi) \equiv \big|\tilde{\Delta}(1)\big|^2 = \tilde{Q}\Gamma^2\,, \qquad f(1,\xi) = (\lambda^2 + \Gamma^2 - \xi^2)^2 + 4\Gamma^2\xi^2\,, \tag{13}$$

where $\tilde{Q}$ is the pump current, Γ is the width of the resonance levels,

$$\tilde{\Delta}(1) \approx \big[\tilde{\Delta}_{12}\Delta_{34} + \lambda^2 b(1)\big]\,, \qquad \lambda^2 = \frac{16p_1p_2e^2E^2}{\omega^2}\,, \tag{14}$$

$\tilde{\Delta}_{12}$ and Δ_{34} are determinants, $\xi = \varepsilon + \varepsilon_{2\mathrm{R}}$, and $b(1)$ is a constant.

The dependence of the current $J_c(1)$ and the laser field λ on ξ is determined by the square of the resonant determinant $f(1,\xi)$. The minimum of $f(1,\xi)$ corresponds to the maximum of the current and laser field. The equation for the extreme values of ξ has two solutions. The first, $\xi_1 = 0$, corresponds to the minimum of f for $\Lambda < \Gamma$ and to the maximum for $\Lambda > \Gamma$. The second,

$$\xi_2^2 = \lambda^2 - \Gamma^2\,, \qquad \lambda > \Gamma\,, \tag{15}$$

yields the minimum of $f(1,\xi)$ for the strong field case $\lambda > \Gamma$.

From Eqn (13) we obtain the laser power λ^2, whose dependence on the pump current $\tilde{Q}$ is linear

$$P(1) = \lambda^2 = \frac{\tilde{Q}}{4} \tag{16}$$

for $\xi = \xi_2$, and the square root,

$$P(1) = \Gamma\left(\sqrt{\tilde{Q}} - \Gamma\right)$$

for $\xi = \xi_1 = 0$.

The physical meaning of the solution ξ_2 is clarified by calculating the reflection coefficient

$$R(\xi) = \frac{(\xi^2 + \Gamma^2 - \lambda^2)^2}{\big|\tilde{\Delta}(1)\big|^2}\,, \qquad R(\xi_2) = 0\,. \tag{17}$$

From this it follows that the energy $\varepsilon_2 = \varepsilon_{2R} + \xi_2$ coincides with the resonant energy of the structure in an alternating field.

4. Two-well structure

Many fundamental features of the multi-well structure emerge already for $N = 2$. Let us find the value of the reduced current in the first well. We have

$$J_{1c}(2) = \frac{1}{a}\int_{-a}^{0} J_{1c}(2, x)\,dx\,, \tag{18}$$

$$J_{1c}(2) \sim \left[\frac{i|\tilde{\varDelta}_{12}|\Phi_0 + \lambda^2 i\tilde{\varDelta}_{12}}{|\varDelta(2)|^2}\right] + \text{c.c.}\,, \tag{19}$$

where

$$\varDelta(2) \sim \left[\tilde{\varDelta}_{12}\varDelta_{34}\Phi_0 + \lambda^2 b(2)\right], \tag{20}$$

$$\Phi_0 = 4\big[\alpha\sin(p_1 a)\cos(p_2 a) + p_1\cos(p_1 a)\sin(p_2 a) + p_2\cos(p_2 a)\sin(p_1 a)\big]\,, \tag{21}$$

and $b(2)$ is the function of ξ. It is not difficult to see that the first term in Eqn (19) vanishes, and the only contribution comes from the second term, which is proportional to the square of the field λ^2 and to the imaginary part of $\tilde{\varDelta}_{12}$,

$$J_{1c}(2) \sim \frac{\lambda^2\,\mathrm{Im}\,\tilde{\varDelta}_{12}^{*}}{|\varDelta(2)|^2}\,. \tag{22}$$

This fundamentally important result implies that the decay causing radiative interlevel transitions in the first well is induced by the current in the second (right) well and is proportional to λ^2. This result will persist for any number of wells $N \geqslant 2$, but the decay will be proportional to $\lambda^{2(N-1)}$. Because there are no dissipative processes in a coherent system, it is the 'current-transmitted' decay of the rightmost well which is the cause of generation.

Let us proceed to the analysis of the current $J_{1c}(2)$ of the first well in the two-well system. The key quantity here is the resonant determinant $\varDelta(2)$, Eqn (20). The fundamental difference between $\varDelta(2)$ and $\varDelta(1)$ [see Eqn (14)] is the appearance of a new elemental resonance determinant Φ_0, Eqn (21). The vanishing of Φ_0,

$$\Phi_0 = 0\,, \tag{23}$$

gives the equation for the spectrum of two tunneling-coupled quantum wells. Equation (23) has two solutions, antisymmetric (with no energy shift) and symmetric. The energy of the latter solution shifts downward, and the levels split. The upper level in the first well and the lower level in the second one also shift downward in energy because of the finite values of α_2 and α_1, the powers of the end barriers. Their shift is determined by the equations

$$\mathrm{Re}\,\tilde{\varDelta}_{12} = 0\,, \qquad \mathrm{Re}\,\varDelta_{34} = 0\,. \tag{24}$$

We can achieve the simultaneous fulfillment of the resonance conditions and Eqns (23) and (24) by satisfying the following relations:

$$\alpha_2 = \frac{4\alpha}{5}\,, \qquad \alpha_1 = \frac{\alpha}{5}\,, \qquad \alpha_1 + \alpha_2 = \alpha \tag{25}$$

for any α. These relations retain their validity for multi-well structures, in which case α_2 and α_1 are the powers of the first and the last wells, respectively.

The dependence of the current on the electron energy ξ is determined only by the absolute value squared of the reduced determinant $|\tilde{\varDelta}(2, \xi)|^2$,

$$f(2,\xi) = |\tilde{\varDelta}(2,\xi)|^2 = \xi^2\left(\xi^2 - \lambda^2 - \frac{\Gamma_1^2}{2}\right)^2 + \frac{9\Gamma_1^2}{4}\left(\xi^2 - \frac{2}{5}\lambda^2\right)^2. \tag{26}$$

The optimum energies of the delivered electrons ξ are found from the minimum condition for $f(2, \xi)$ to be

$$\xi_1 = 0\,, \qquad \xi_2^2 \simeq \frac{\lambda^2}{3}\,, \qquad \xi_3^2 \simeq \lambda^2\,, \qquad \lambda \gg \Gamma_1\,. \tag{27}$$

The solutions ξ_1 and ξ_3 correspond to the minimum of $f(2, \xi)$, and ξ_2 to the maximum of f. Unlike the single-well structure, the minimum of $f(2)$ for $\xi_1 = 0$ occurs for any λ. Besides, it is more convenient in that no electron energy tuning is required. It is the solution $\xi_1 = 0$ which is of most interest.

Comparing the current of the first well $J_{1c}(2, 0)$ with that of a single-well structure we see that they are practically identical,

$$J_{1c}(2, 0) = \frac{8}{9}\,J_{1c}(1, \lambda)\,. \tag{28}$$

Note that the solutions ξ_1 and ξ_3 lead to the linear dependence of the generation power on the pump current, whereas ξ_2 leads to the square root dependence. Thus, the reduced current of the two-well structure is equal to that of the first well, $J_c(2) = J_{1c}(2)$. Because the currents $J_{1c}(2, 0)$ and $J_c(1, \lambda)$ are identical to within a factor of 8/9 according to Eqn (28), it follows that the field generated by the two-well structure at $\xi = 0$ is given by the formula (16) with $\tilde{Q} \to (8/9)\tilde{Q}$. The total power $P(2)$ is naturally twice that,

$$P(2) = 2\,\frac{2\tilde{Q}}{9}\,. \tag{29}$$

5. Even-membered structures with $N \geqslant 4$

We start with the four-well structure. In the approximation adopted we have

$$\varDelta(4) \sim \tilde{\varDelta}_{12}\varDelta_{34}\Phi_0^3 + \lambda^2 b(4) + \lambda^4 c(4)\,, \tag{30}$$

where $b(4)$ and $c(4)$ are the functions of ξ. The structure of the determinant has a clear physical meaning. The first term is the product of the determinants of the end wells and of the three determinants of the collectivized levels of the inner wells. Of fundamental importance is the fact that the second (λ^2) term also contains the determinant Φ_0. In addition, as is the case with $\varDelta(3)$, the terms with λ^4 must be retained.

It can be shown that the solution $\xi_1 = 0$ corresponds to the minimum of $|\varDelta(4)|^2$ and is optimal. This property is common to structures with even N.

It turns out that for the optimal solution $\xi_1 = 0$, $\Phi_0 = 0$ the wave function of the first well of the four-well structure is identical to that of the two-well structure. Hence, the reduced current of the $N = 4$ structure is

$$J_c(4, 0) = J_{1c}(2, 0)\,. \tag{31}$$

This property is valid for any even N, so that the reduced current of such a structure has the form

$$J_c(N, 0) = J_{1c}(2, 0)\,. \tag{32}$$

By extending Eqn (29) we obtain the total power of the N-well structure,

$$P(N) = N\,\frac{2\tilde{Q}}{9}\,, \qquad (33)$$

i.e., is proportional to the number of wells N.

6. Conclusions

In this work we have proven that coherent generation is possible in a structure containing any number of wells in the absence of dissipation processes. As is known, the emission of a photon requires decay, which in large systems is due to interaction with phonons. In the structure studied, decay is due to the exit of electrons from the lowest level of the rightmost well. The stationary interband current 'transmits' this decay to all the wells and causes the emission of photons in each well. Electrons delivered by resonant tunnelling make N transitions from upper to lower levels, retain their phase in doing so, and this process is independent of level population. Hence, amplification and generation in such a structure are bulk phenomena, unlike the non-coherent situation [5]. The total generation power is proportional to the number of wells N.

As found out in the present study, there are a number of requirements for the generation to be effective: the resonance condition for each of the wells, choosing the optimal energy of the supplied electrons, a narrow enough electron energy distribution ($\Delta\varepsilon$), and, finally, coherence conditions for the electronic subsystem.

It is shown that these requirements can be met in superlattices of quantum wells and wires and, especially, in quantum dot superlattices.

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Transport in weak barrier superlattices and the problem of the terahertz Bloch oscillator

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1. Introduction

The subject matter of this work is the transport properties of weak barrier semiconductor superlattices and whether these superlattices can be used as a basis for creating the terahertz Bloch oscillator. Because of interminiband tunneling occurring in strong fields, the current is a growing function of the field. At the same time, in such superlattices tunneling and Bloch oscillations can lead to dynamic negative differential conductivity in the terahertz frequency range. It is pointed out that such a system is a no-inversion laser because the laser transition occurs between equally populated (Wannier–Stark) levels. Monte Carlo calculations for weak barrier n-GaAs–GaAlAs superlattices are presented which show that dynamic negative conductivity can exist in the frequency range of 1–7 THz for superlattices with moderate mobility at 77 K. The first experimental results on the transport properties of such superlattices are presented.

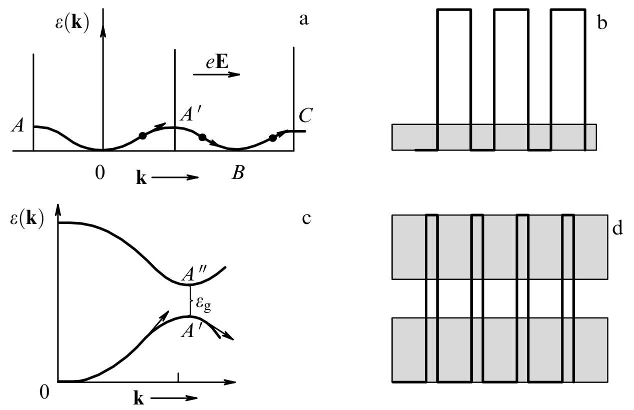

Figure 1. Superlattices with wide (b) and narrow (d) forbidden bands; (a) shows Bloch oscillations, and (c) shows the Zener breakdown.

The idea of a Bloch oscillator (or generator) dates back to the work of Bloch (1928), Kroemer (1954), Keldysh (1962), Esaki and Tsu (1970), a.o., and is based on the following argument (see also Ref. [1]). If an electric field E applied along an axis of a semiconductor superlattice (SL) with period d is strong enough that an electron moves nearly unscattered between Brillouin zone boundaries ($A-A'$ in Fig. 1a) within one energy band, then an electron performs Bloch oscillations (BOs) at the (Bloch) frequency

$$\omega_{\mathrm{B}} = \frac{eEd}{\hbar}\,.$$

Taking $E = 3$ kV cm^{-1}, $d = 150$ Å we obtain the frequency $f_{\mathrm{B}} = \omega_{\mathrm{B}}/2\pi = 1$ THz. This frequency is continuously tuned by the applied electric field, creating prerequisites for developing a universal tunable radiation source (generator) in the terahertz range. Clearly, the fact that oscillations exist does not guarantee that they can be used for radiation generation purposes. It is negative conductivity near the Bloch frequency which makes the generator.

Negative conductivity is a traditional topic in the study of transport in semiconductor superlattices in the presence of strong electric fields (see, for example, a review in Ref. [2]). Most studies in this area consider transport *in superlattices with strong barriers and wide forbidden* (*mini*) *bands*. In this case Bloch oscillations in strong electric fields actually involve transport within one miniband only as shown in Fig. 1a. Negative differential conductivity (NDC) in such systems occurs in the frequency range from $\omega = 0$ to ω_{B} (Ktitorov, Simin, Sindalovskiĭ [3], Fig. 2b). A Bloch oscillator (generator) might look as follows (Fig. 3a). On a conducting