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Joint scientific session of the Physical Sciences Division of the Russian Academy of Sciences and the Joint Physical Society of the Russian Federation "Bloch Oscillations" (26 February 2003)

A joint scientific session of the Physical Sciences Division of the Russian Academy of Sciences (RAS) and the Joint Physical Society of the Russian Federation, entitled "Bloch Oscillations," was held on 26 February 2003 at the P N Lebedev Physics Institute, RAS. The following reports were presented at the session:

(1) **Suris R A, Dmitriev I A** (A F Ioffe Physical Technical Institute, RAS, St. Petersburg) "Bloch oscillations in quantum dot superlattices";

(2) **Elesin V F** (Moscow Engineering and Physics Institute, Moscow), **Kopaev Yu V** (P N Lebedev Physics Institute, RAS, Moscow) "'Stark ladder' laser with a coherent electronic subsystem";

(3) **Kop'ev P S, Zhukov A E, Ustinov V M** (A F Ioffe Physical Technical Institute, RAS, St. Petersburg), **Ignatov A A** (Institute of Physics of Microstructures, RAS, Nizhniĭ Novgorod); **D G Pavel'ev** (N I Lobachevskiĭ Nizhniĭ Novgorod State University, Nizhniĭ Novgorod) "Generation, multiplication, and detection of microwave radiation in broad miniband superlattices";

(4) **Andronov A A, Drozdov M N, Zinchenko D I** (Institute of Physics of Microstructures, RAS, Nizhniĭ Novgorod), **Marmalyuk A A** (Sigm Plus Co. Ltd., Moscow), **Nefedov I M, Nozdrin Yu N** (Institute of Physics of Microstructures, RAS, Nizhniĭ Novgorod), **Padalitsa A A** (Sigm Plus Co. Ltd., Moscow), **Sosnin A V, Ustinov A V, Shashkin V I** (Institute of Physics of Microstructures, RAS, Nizhniĭ Novgorod) "Transport in weak barrier superlattices and the problem of the terahertz Bloch oscillator".

An abridged version of the reports (except 3) is given below.

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Bloch oscillations in quantum dot superlattices

R A Suris, I A Dmitriev

1. Introduction

In the present paper it is shown that the electronic spectrum of ideal two- and three-dimensional quantum dot superlattices in a static electric field can be either discrete or continuous depending on the orientation of the field relative to the superlattice's crystallographic axes. In the case of a continuous spectrum the width of the forming transverse miniband depends exponentially on the crystallographic index of the field direction. A quantum theory of Bloch oscillation decay in such superlattices is constructed. It is shown that, unlike quantum well superlattices, the scattering of oscillating electrons in quantum dot superlattices by phonons can be strongly suppressed by properly choosing the magnitude and direction of the field. As a result, even at room temperature the decay time of Bloch oscillations can be as long as hundreds of periods.

The huge interest in quantum dots in semiconductor heterostructures is due to the fact that the discrete spectrum of carriers in a quantum dot differs considerably from bulk crystal or quantum well spectra, which are continuous. Practical advantages due to this feature are well illustrated by quantum dot injection lasers, where using an active medium with a purely discrete spectrum makes it possible to achieve a cardinal reduction in and high temperature stability for the threshold current density compared to that in quantum well lasers [1 – 6].

Another very promising approach may be to use quantum dots in unipolar devices, for example, cascade lasers, in which radiation is generated by transitions between the states of the carriers of one and the same type (for example, electrons). The concept of a unipolar semiconductor laser, first proposed as the idea of a stimulated radiation for a superlattice [7 – 9], was later embodied in the quantum well cascade laser [10, 11]. Using ordered arrays of quantum dots can possibly cause a significant increase in the efficiency of cascade lasers as well [12 – 15].

It took considerable effort to practically realize arrays of structurally perfect quantum dots of sufficiently high surface concentration and uniform size and shape [16]. Today, one can expect that fabricating perfectly periodic semiconductor structures in the form of quantum dot lattices may be around the corner. As we shall show below, in such structures terahertz Bloch oscillations with extremely high Q-factor, can be excited. The present talk presents the theoretical results of Refs [17 – 19] in which a theory of carrier localization by a static electric field in ideal 2D and 3D QDSLs and on the decay of Bloch oscillations in such superlattices was developed. We will demonstrate that, compared to quantum well superlattices, in QDSLs new attractive possibilities open up for controlling the spectrum, localization region, and the scattering of carriers by choosing the magnitude of the electric field and its orientation relative to the principal axes of the QDSL.

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2. The spectrum and localization of carriers in ideal two- and three-dimensional quantum dot superlattices in a static electric field

The system to be studied here is an array of weakly coupled, identical quantum dots that repeat themselves strictly periodically in space to form an ideal one-, two-, or three-dimensional lattice of quantum dots. In Fig. 1 a two-dimensional square QDSL is shown as an example. We assume that each quantum dot forms a potential well for electrons, similar to what occurs in laser structures. The spectrum of electrons is discrete in each well. In the absence of an electric field the electron spectrum in a superlattice is a set of minibands, due to the tunneling coupling between the quantum dots forming the superlattice. Note that a remarkable aspect of the electrical properties of layered periodic structures — the possibility of negative differential conductivity — was first pointed out by Keldysh [20]; Esaki and Tsu [21] realized this effect experimentally in layered semiconductor heterostructures — one-dimensional semiconductor superlattices. We shall assume in our work that the electric fields and resonance integrals between quantum dots are so small in magnitude that the 'isolated' miniband condition $\hbar\Omega\Delta \ll W^2$ [22] — i.e., the condition for there being no interminiband transitions due to a static electric field F — is fulfilled and that the one-miniband approximation is applicable.[1] Here $\hbar\Omega = eFa$ is the Stark frequency, i.e., the electrostatic energy gained over the superlattice period a in the direction of the electric field, Δ is the width of the miniband of allowed energies, and W is the energy gap between the minibands. Following, we specify that a miniband is formed from the quantum ground level of an isolated quantum dot.

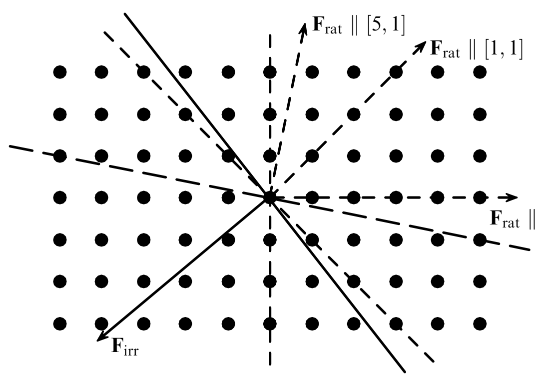

Figure 1. Examples of rational ($F_{\rm rat}$) and irrational ($F_{\rm irr}$) field directions in a square two-dimensional quantum dot superlattice.

For a chosen QDSL miniband described by the Hamiltonian $\hat{H}_0$ in the absence of an electric field, the Schrödinger equation for an electron in a static electric field has the form

$$\hat{H}_F \Psi = (\hat{H}_0 + e\mathbf{F}\cdot\mathbf{r})\Psi = E\Psi\,. \tag{1}$$

This equation is conveniently solved by taking as the basis the miniband Wannier functions, whose strong-coupling form is close to wave functions in isolated quantum dots and is given by the relation [23]

$$|\,\boldsymbol{\rho}\rangle = \frac{1}{\sqrt{N}}\sum_{\mathbf{K}} \exp\left[\mathrm{i}\mathbf{K}(\mathbf{r}-\boldsymbol{\rho})\right] u_{\mathbf{K}}(\mathbf{r})\,, \tag{2}$$

where N is the total number of sites in the QDSL, $\boldsymbol{\rho} = \sum_i n_i \mathbf{a}_i$ are site vectors, $\mathbf{a}_i$ are the QDSL basis vectors, the summation is over all allowed values of the wave vector $\mathbf{K}$ within the QDSL's first Brillouin zone, and $\exp(\mathrm{i}\mathbf{K}\mathbf{r})\,u_{\mathbf{K}}(\mathbf{r})$ are the Bloch eigenfunctions of the Hamiltonian $\hat{H}_0$. From Eqn (1), expanding the wave function in terms of the Wannier functions $\Psi = \sum C_{\boldsymbol{\rho}}\,|\,\boldsymbol{\rho}\rangle$ we obtain for the amplitudes $C_{\boldsymbol{\rho}}$ the equation

$$(E - e\mathbf{F}\cdot\boldsymbol{\rho})\,C_{\boldsymbol{\rho}} - \sum_{\boldsymbol{\rho}_1} \frac{1}{4}\,\Delta_{\boldsymbol{\rho}-\boldsymbol{\rho}_1} C_{\boldsymbol{\rho}_1} = 0\,, \tag{3}$$

where the quantities $\Delta_{\boldsymbol{\rho}}/4$ are the field modified resonance integrals linking quantum dots one QDSL vector $\boldsymbol{\rho}$ apart.

The solutions of Eqn (3) are qualitatively different for two classes of electric field directions [17]. If all the quantities $\mathbf{F}\cdot\mathbf{a}_i/\mathbf{F}\cdot\mathbf{a}_k$, $i \neq k$ — the ratios of the field projections onto the QDSL basis vectors — are irrational for a given field orientation ($F_{\rm irr}$ in Fig. 1), then the electric potentials of the QDSL sites turn out to be different. The spectrum in this case is discrete and forms a two-dimensional (2D QDSL) or three-dimensional (3D QDSL) Wannier – Stark ladder as follows:

$$\varepsilon_{\mathbf{R}} = -e\mathbf{F}\cdot\mathbf{R} \;= -\sum n_i(e\mathbf{F}\cdot\mathbf{a}_i)\,. \tag{4}$$

When the ratio of the projections of the electric field onto the QDSL basis vectors is a rational number (a rational field direction), then chains of quantum dots with the same electric potential form in the direction perpendicular to the field (Fig. 1). Neglecting interdot tunneling in these chains leads to a degenerate discrete spectrum of carriers in the form of Eqn (4). The inclusion of resonance integrals between the dots in the chains leads to qualitative changes in the spectrum and the wave functions. The motion in transverse chains is not quantized and transverse minibands form. The resonance integrals between quantum dots in transverse chains decrease exponentially with increasing quantum dot separation, so that *the width of the transverse bands depends exponentially on the field direction*: the longer the period of the transverse quantum dot chains, $d_\perp$, the smaller the width of the transverse miniband, $\Delta_\perp$, which is four times the resonance integral between the nearest quantum dots in transverse chains (Figs 1, 2). Taking into account the quantization of the motion of the carriers along the field direction, the spectrum forms a one-dimensional Stark ladder of states (Fig. 3b)

$$\varepsilon_N(k) = \frac{1}{2}\,\Delta_\perp \cos(kd_\perp) - NeFd_\parallel\,, \qquad \Delta_\perp \propto \exp(-\alpha d_\perp)\,, \tag{5}$$

where the number N labels the steps of the Stark ladder, k is the wave vector of the transverse motion along the chains, $d_\parallel$ is the separation between transverse QD chains, and α is a positive coefficient dependent on material parameters and the geometry of a particular structure.

The discussion so far has been concerned with the ideal QDSL. However, in a system of such technological complexity, with many heteroboundaries in it, a certain spread in

[1] The isolated miniband condition as formulated here is obtained from the results of Ref. [22] by noting that the effective mass of an electron in a miniband is related to the miniband width by the relation $m \approx \hbar^2/\Delta a^2$.

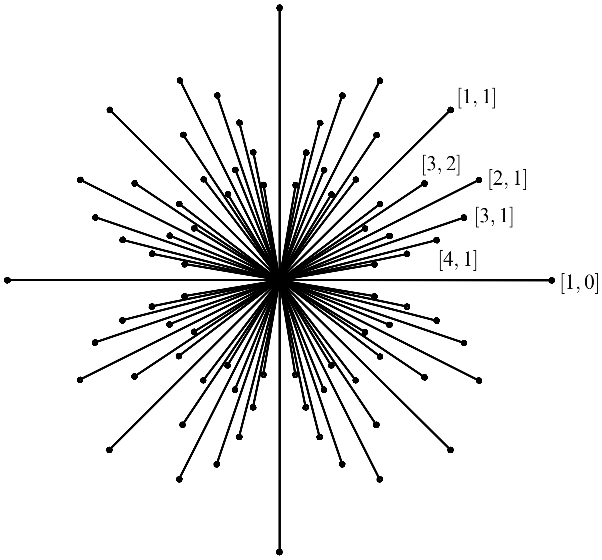

Figure 2. Variation of the transverse miniband width with the crystallographic index of the electric field direction for a two-dimensional quantum dot superlattice (on the logarithmic scale).

parameters is inevitable, creating dispersion in the dimensional quantization energies and in the resonance integrals involved. In addition, real structures are spatially limited and the number of rational field directions is finite. It has been shown [24] that for weak diagonal disorder in a strong coupling-model chain, an electron is localized on the length $l_{\rm loc} = d(\Delta_\perp/2)^2/\langle\delta\varepsilon^2\rangle$, where d is the chain period, and $\langle\delta\varepsilon^2\rangle$ is the mean square dispersion of the quantization levels in a QD. Let us denote by L the linear dimension of the QDSL in the direction transverse to the field. Clearly one can speak of a continuous transverse spectrum only if the condition $l_{\rm loc} \gg L \gg d$ is fulfilled. If, however, the relationship between the dispersion and the widths of the transverse minibands $\Delta_\perp$ is such that $l_{\rm loc} < d$, complete localization occurs in the direction transverse to the electric field, making the corresponding resonance integral negligible. In the intermediate region $d < l_{\rm loc} < L$ the localization length in the transverse direction is determined by the amount of spread in quantization levels. We conclude that in real QDSLs delocalization in the direction normal to the electric field does not occur for all rational field directions but only for several principal directions. Except for these principal rational directions, in a strong enough field the nearest-neighbor approximation is sufficiently accurate to describe the spectrum and wave functions.

3. Bloch oscillations

The problem of an electron traveling in an ideally periodic crystal lattice in a static uniform electric field has attracted physicists' attention since the very beginning of quantum solid state theory. In the fundamental work of 1928 [25], where Bloch laid the foundations of this theory, he showed, using quasi-classical arguments, that in the absence of scattering and transitions between allowed energy bands an electron in such a system undergoes oscillations (both in momentum and coordinate spaces) at a frequency

$$f = \frac{eFa}{h}\,, \tag{6}$$

where F is the electric field strength, a is the lattice spacing in the electric field direction, and h is the Planck constant.

Let us apply this simplest picture to the case of QDSLs. The spectrum of a given QDSL miniband in the strong coupling approximation has the form

$$\mathcal{E}(\mathbf{K}) = \sum_{\mathbf{R}} \frac{\Delta_{\mathbf{R}}}{4} \exp\left(\mathrm{i}\mathbf{K}\cdot\mathbf{R}\right), \tag{7}$$

where $\mathbf{K}$ is the wave vector lying within the first Brillouin zone of the QDSL. Now if an electric field is switched on instantaneously, electrons are distributed over miniband Bloch states in equilibrium initially. Under the action of the field the electrons will move in $\mathbf{K}$ space at a constant velocity $e\mathbf{F}t/\hbar$, undergoing Bragg reflections on the Brillouin zone boundary — which corresponds to oscillations in coordinate space. Applying the laws of quasi-classical dynamics

$$\hbar\mathbf{v}(\mathbf{K}) = \nabla_{\mathbf{K}}\mathcal{E}(\mathbf{K})\,, \qquad \frac{\partial}{\partial t}(\hbar\mathbf{K}) = e\mathbf{F}\,, \tag{8}$$

to the electrons and using Eqn (7), the current of Bloch oscillations is found to be

$$\begin{aligned}\mathbf{j}(t) &= en_{\rm e}\int \mathrm{d}\mathbf{K}\,\mathbf{v}\left(\mathbf{K}+\frac{e\mathbf{F}t}{\hbar}\right) f(\mathbf{K}) \\ &= \sum_{\mathbf{R}} \frac{en_{\rm e}\Delta_{\mathbf{R}}\mathbf{R}}{2\hbar}\left\langle\cos\left(\mathbf{K}\cdot\mathbf{R}\right)\right\rangle \sin\left(\Omega_{\mathbf{R}}t\right),\end{aligned} \tag{9}$$

where $\Omega_{\mathbf{R}} = e\mathbf{F}\cdot\mathbf{R} = \sum n_i(e\mathbf{F}\cdot\mathbf{a}_i) = \sum n_i\Omega_i$ are the QDSL Stark frequencies; $n_{\rm e}$ is the electron density; $f(\mathbf{K})$ is the initial electron distribution function normalized to one electron; and the angular brackets indicate averaging with the distribution function.

From Eqn (9) it can be seen that, compared to layered quantum well superlattices, QDSLs offer significant advantages for the practical use of Bloch oscillations. In layered superlattices oscillations occur at a single frequency of $\Omega = eFa/\hbar$, with a the superlattice spacing. In QDSLs, the spectrum of the oscillations consists of two (2D QDSL) or three (3D QDSL) basic frequencies Ω_i (the resonance integrals $\Delta_{\mathbf{R}}/4$ decrease exponentially with quantum dot separation $|\mathbf{R}|$ and, consequently, the amplitude of other harmonics is exponentially smaller than that of the basic components). By controlling the magnitude of the electric field and its orientation relative to the basic vectors of the QDSL, the basic frequencies can be changed independently [12].

Another possible way to describe Bloch oscillations is based on the Stark representation of the eigenfunctions of the Hamiltonian $\hat{H}_F$ for a QDSL in a static electric field. In this formalism, Bloch oscillations are quantum beats between the states of the Stark ladder. Suppose that at the initial time the wave function of an electron is a coherent mix of Stark states (5),

$$\Psi(t=0) = \sum \chi^0_{N,k}|N,k\rangle\,. \tag{10}$$

The temporal evolution of Stark states is known, allowing us to write down the solution of the non-stationary Schrödinger equation straightaway in the from

$$\Psi(t) = \sum \chi^0_{N,k}\exp\left(-\frac{\mathrm{i}}{\hbar}\,\varepsilon^k_N t\right)|N,k\rangle\,. \tag{11}$$

Because the levels of the Stark ladder are equidistant, this implies that the quantum beats result in the electron density oscillating at the Stark frequency in each quantum dot. When the electric field is switched on instantaneously, the electrons are distributed over the Bloch states of a QDSL miniband in equilibrium initially. In this case one can obtain an expression for the current identical to that obtained earlier in the quasi-classical approximation (9), but with a different meaning. In this picture Bloch oscillations are quantum beats arising due to the coherent excitation of carriers to Stark ladder states.

It is known that the phenomenon of Bloch oscillations is very hard to observe experimentally because of the extremely severe requirement that scattering over the oscillation period be weak,

$$\tau_{\mathrm{eff}}^{-1} < \frac{\Omega}{2\pi} = \frac{eFa}{h} , \tag{12}$$

where τ_{eff}^{-1} is the effective collision rate. Therefore, predicted theoretically as long ago as 1928 by Bloch [25], these oscillations were not observed until the early 1990s — specifically, in ideally pure, periodic layered heterostructures in the form of $A^{\mathrm{III}}B^{\mathrm{V}}$ quantum dot superlattices [26, 27]. The periodicity scale of superlattices is tens of times larger than the interatomic separation, allowing the condition (12) to be fulfilled already at quite realistic electric fields of tens of kV cm^{-1}; the oscillation frequency then is of the order of terahertz. It is this fact — and particularly the promise of creating sources and detectors of terahertz radiation — which explains the great applied interest in the phenomenon of Bloch oscillations.

However, in layered superlattices, as well as in bulk semiconductors, whatever the magnitude of the electric field applied, carriers are strongly scattered by lattice vibrations — an unavoidable factor which leads to the fast decay of the oscillations. Even in the very low temperature range $T \simeq 10$ K the lifetime of Bloch oscillations is only ten oscillation periods [28]. We will show later on that, unlike layered superlattices, in QDSLs all scattering channels can be strongly suppressed by effectively controlling and manipulating the electronic spectrum by varying the magnitude and direction of a static electric field. Our analysis shows that in technologically perfect QDSLs, hundreds of Bloch oscillation periods can be observed at room temperature. By comparison, in layered superlattices it takes one period for Bloch oscillations to practically completely decay at room temperature [29].

4. Scattering of carriers in layered and quantum dot superlattices

Let us now perform a layered vs QD superlattice analysis of possible ways to suppress scattering. In Fig. 3a is shown the spectrum of a layered superlattice in an electric field. The paraboloids on each of the Stark ladder's steps describe the momentum dependence of the electron energy in the plane of the quantum well. It is owing to the broad energy spectrum for electron motion along the quantum dots that the scattering remains strong for any magnitude of the electric field. Due to the energy overlap of the states in different ladder steps, both elastic scattering and that with the participation of optical and acoustic phonons are possible.

The situation in a QDSL is cardinally different. Here, as we have seen, the width of the transverse bands can be changed by varying field orientation relative to the QDSL's principal axes thus effectively allowing control of the QDSL

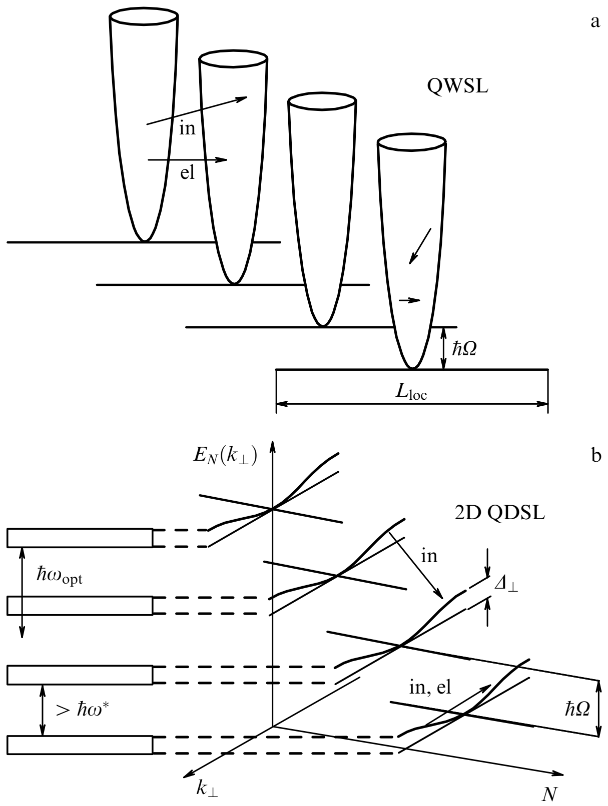

Figure 3. Inelastic (in) and elastic (el) scattering channels (a) in a layered quantum well superlattice (LQWSL) in a static electric field and (b) in a two-dimensional quantum dot superlattice for rational field directions ($\hbar\Omega = eFa$ is the Stark ladder level separation, $\Delta_\perp$ is the width of the transverse minibands, $\hbar\omega_{\mathrm{opt}}$ is the optical phonon energy, $\hbar\omega^* = \hbar s\pi/R_{\mathrm{D}}$ is the upper limit on the energy of actual acoustic phonons).

spectrum — and hence of the scattering.

Indeed, from Fig. 3b it is seen that scattering with the participation of optical phonons is not possible within a transverse miniband if its width becomes less than the energy of the optical phonons:

$$\hbar\omega_0 > \Delta_\perp . \tag{13}$$

Suppose now that the transverse bands have no overlap in energy and that for all natural n the condition

$$n\Omega + \frac{\Delta_\perp}{2\hbar} < \omega_0 < (n+1)\Omega - \frac{\Delta_\perp}{2\hbar} \tag{14}$$

is fulfilled, where $\hbar\Omega = eFa_\parallel$ is the separation between the Stark ladder steps and $\Delta_\perp$ is the width of the transverse bands. Then the interband processes of both elastic and optical phonon scattering also become completely suppressed (Fig. 3b).

Moreover, it turns out that miniband scattering on acoustic phonons can be arbitrarily strongly suppressed. This can be seen by noting certain aspects of the interaction with acoustic phonons in QDSLs.

The wave function of an electron in the Stark ladder state (5) contains two characteristic scales: the characteristic length for carrier localization by the electric field, $L_{\mathrm{loc}} = \Delta_\parallel/eF$, and the characteristic scale of the Wannier function, i.e., in the strong coupling approximation, the size of the quantum dot

R_D. It is this latter — the smallest — scale which determines the upper limit for the wave vectors of the phonons effectively interacting with electrons. It has been shown [18] that the probability of a photon's being emitted or radiated by an electron in a QDSL is proportional to the form factor

$$Q(q) = \left|\langle \boldsymbol{\rho}| \exp \mathrm{i}\mathbf{q}\cdot\mathbf{r}|\boldsymbol{\rho}\rangle\right|^2 \approx \begin{cases} 1\,, & q \ll q^*\,, \\ \left(\dfrac{q}{q^*}\right)^{-\beta}\,, & q \gg q^*\,, \end{cases} \qquad (15)$$

where $q^* = \pi/R_D$, and that in Eqn (15) the exponent $\beta = 8$ if the Wannier function falls off exponentially outside the quantum dot and has no discontinuities in its first derivative. But if electron effective masses inside and outside of the dot are different, then the discontinuity wave yields $\beta = 6$.

Such strong dependence of the form factor on the phonon wave vector makes it possible to virtually completely suppress scattering on acoustic phonons between the transverse minibands of the Stark ladder (Fig. 3b). Indeed, from Eqn (15) it follows that if the energy gap between the transverse bands exceeds the energy of the actual acoustic phonons $\hbar\omega^* \equiv \hbar s\pi/R_D$ determined by the form factor, i.e., if

$$\hbar\Omega - \Delta_\perp > \hbar\omega^*\,, \qquad (16)$$

then the probability of acoustic phonon scattering between the minibands falls off as $(\hbar\Omega - \Delta_\perp)^{-\beta}$, $\beta \geqslant 6$ as the energy spread increases.

The fulfillment of the conditions (13), (14), and (16) leaves scattering on acoustic phonons within the transverse bands the only effective scattering channel for the ideal QDSL. Generally speaking, it is not at all obvious that such scattering should lead to the decay of the Bloch oscillations because these relate to motion along the electric field direction, and in this channel only the transverse motion of the carriers is scattered. So in order to estimate the oscillation decay rate for scattering within transverse minibands, it is not at all sufficient to calculate transition probabilities, and a more rigorous and consistent theory is needed. Because Bloch oscillations are beats between Stark ladder states in the Stark representation, the decay of oscillations in this representation is a consequence of the fact that these states lose coherence when a phonon is emitted or radiated in a transition of an electron from one of them to another. A natural way to describe the decay of Bloch oscillations then is through density matrix formalism: it is the non-diagonal elements of the density matrix which describe the degree of coherence of the states.

5. Decay theory of superlattice Bloch oscillations. The quantum kinetic equation describing phase relaxation

In constructing the quantum relaxation equation to describe the decay of oscillations we [18] mainly follow Kohn and Luttinger's procedure [30] which was applied, in particular, to the kinetics of carriers in layered superlattices [7–9, 31]. We assume that scattering is weak over the oscillation period (12). This assumption makes it possible to obtain a closed system of linear equations for the quantities $\rho^{k,k}_{N,N'}$, the elements of the density matrix in the Stark representation (5), which are nondiagonal in the Stark indices N, N' and diagonal in the wave vectors k of the transverse band motion. In the absence of scattering these elements oscillate at the Stark frequency, $\rho^{k,k}_{N,N'} = \rho^{k,k}_{N,N'}\big|_{t=0} \exp\left[-\mathrm{i}(N-N')\Omega t\right]$, which, for the case of an electric field switched on instantaneously, yields the same expression (9) earlier obtained in the quasi-classical limit for the oscillation current.

Because the general equations of Ref. [18] are quite difficult to analyze, we will only consider here the case in which the electric field is along one of the basic vectors of a two-dimensional rectangular QDSL and in which the nearest neighbor approximation is applicable. For an electric field switched on instantaneously the density matrix is spatially uniform,

$$\rho^{k,k}_{N+n,\,N'+n} = \rho^{k,k}_{N,N'}\,, \qquad (17)$$

and the current density in the nearest neighbor approximation is expressed in terms of the only density matrix element present, $\rho^{k,k}_{N,N+1}$, which describes quantum beats between the nearest Stark ladder steps,

$$j = -e\,\mathrm{tr}\,(\hat{\rho}\hat{v}) \propto \int \mathrm{d}k\ \mathrm{Im}\,\rho^{k,k}_{N,N+1}\,. \qquad (18)$$

The equation for $\rho^{k,k}_{N,N+1}(t)$ has the following form [18]:

$$\left(-\frac{\partial}{\partial t} - \mathrm{i}\Omega\right)\rho^{k,k}_{N,N+1} = \sum_{q,k'}\left[W_q^{k,k'}\rho^{k,k}_{N,N+1} - \cos\left(q_\parallel a_\parallel\right) W_q^{k',k}\rho^{k',k'}_{N,N+1}\right]. \qquad (19)$$

Here $\Omega = eFa_\parallel/\hbar$ is the Stark frequency, $a_\parallel$ is the QDSL spacing in the direction of the electric field, and q and $q_\parallel$ are the phonon wave vector and its component along the electric field, respectively. The right-hand side of Eqn (19) describes the loss of coherence between the Stark states due to the interaction with phonons (with wave vector q). The first term on the right corresponds to the movement of electrons from states $|N,k\rangle$, $|N+1,k\rangle$ to any other states with probability $W_q^{k,k'}$. In this process $\rho^{k,k}_{N,N+1}$ decreases simply due to the decrease in the number of electrons with transverse wave vector k which participate in the coherent mixing, and the phase factor is absent. On the contrary, for electron transitions to states $|N,k\rangle$, $|N+1,k\rangle$ it is important what phase they acquire when interacting with a phonon described by the factor $\cos q_\parallel a_\parallel$.

Suppose that the non-uniformity scale of the scattering potential (in our particular case, the phonon wavelength) in the direction of the electric field greatly exceeds the QDSL spacing or, in the general case, $q_\parallel a_\parallel$ is a multiple of 2π. Then the longitudinal wave vector of an electron is virtually unchanged by scattering, and the phase factor $\cos q_\parallel a_\parallel$ is close to unity. Noting now that the oscillation current (18) is proportional to the integral of the function $\rho^{k,k}_{N,N+1}$ over transverse quasi-momenta and integrating both sides of Eqn (19) over k under the condition $q_\parallel = 0$ we discover that the right-hand sides of these equations integrate up to absolute zero: the Bloch oscillation current does not decay under these conditions. This reflects the specific nature of phase scattering: there is no loss of coherence between Stark states $|N\rangle$ and $|N+1\rangle$ if the perturbation affects both of them equally. In particular, the oscillations do not decay if a scattering process changes only the electron wave vector in the direction perpendicular to the electric field and if $q_\parallel = 0$. In the quasi-classical picture $q_\parallel$ is the longitudinal momentum an electron loses (acquires) when emitting (absorbing) a phonon. Thus, the oscillation decay rate has much the same meaning as the scattering rate of the longitudinal electron quasi-momentum.

This view can easily lead to misunderstanding, however. In a strong electric field, motion along the field is quantized, and the concept of longitudinal quasi-momentum loses its physical meaning. It may seem from what has been said that in the absence of transitions between the transverse Stark ladder bands, Eqn (5), scattering within these bands should not lead to the decay of the oscillations because this scattering is the one affecting the carriers' transverse motion. But this is not so. In transitions within the transverse bands the longitudinal momentum of the phonon emitted or absorbed by an electron may have any value, $q_\| \neq 0$. The conservation law for the longitudinal momentum component of the QDSL as a whole is obeyed as before, but the recoil momentum is now acquired not by the electric-field-localized electron but by the superlattice as a whole. Thus, for scattering within the transverse bands $\cos q_\| a_\| \neq 1$, and such scattering does lead to the decay of the oscillations.

It should be noted that in the derivation of the quantum kinetic equation it was assumed that at zero time a coherent mixing of Stark states forms. Hence, because of the weak scattering, the amplitude of the Bloch oscillation current is much larger than the direct current, and this latter can be neglected. On the contrary, at large times, when coherence between the Stark states is already completely destroyed, the right-hand side of Eqn (19) is dominated by the diagonal elements of the density matrix, which are neglected in the present work, and non-diagonal elements are different from zero only to the extent of the scattering. The procedure for deriving equations for the density matrix at large times and for calculating direct current in layered superlattices while accounting for the electric-field-induced electron heating is described in depth in Ref. [31] and is suitable equally well for calculating the $I-V$ curve of a QDSL.

6. Decay of Bloch oscillations in a one-dimensional quantum dot chain

Equation (19), describing the decay of Bloch oscillations that arise due to the sudden switch-on of an electric field, takes the following simple form for a one-dimensional chain of quantum dots with the period a:

$$\left(\frac{\partial}{\partial t} + \mathrm{i}\Omega + \gamma\right) \rho_{N,N+1}(t) = 0 \,. \tag{20}$$

Bloch oscillations decay exponentially in time, with the decrement

$$\gamma = \frac{\pi}{\hbar^2} \sum_{n,q} (2\nu_q + 1) \left| V^q_{N+1,N+1+n} - V^q_{N,N+n} \right|^2 \delta(n\Omega - \omega_q) \,. \tag{21}$$

Here, as before, q denotes the phonon wave vector, V_n^q is the matrix element for the transition to n chain spacings by spontaneously emitting a phonon, and the ν_q are phonon occupation numbers. The expression for the decay decrement contains the difference of the interaction matrix elements in neighboring Stark states losing coherence in accordance with Eqn (21). Unlike the usual probability of an electron making a transition by absorbing or emitting a phonon, W_q^n, the expression for γ contains the phase factor $1 - \cos q_\| a$: from Eqns (19) and (21) it follows that $\gamma = \sum_{n,q} (1 - \cos q_\| a) W_q^n$. This reflects the specific nature of phase scattering: coherence between the states $|N\rangle$ and $|N+1\rangle$ survives if the scattering potential affects both states equally.

Because of the absence of transverse minibands it is possible by increasing the field to achieve (in an ideal one-dimensional QDSL) an arbitrarily long lifetime for Bloch oscillations: if the conditions (13), (14), and (16) are fulfilled, the rate of scattering is proportional to $F^{-\beta}$, $\beta > 6$. This actually means that in experimental conditions the decay of oscillations for $\Omega \gg s\pi/R_{\mathrm{D}}$ will be determined by the imperfection of the real structure rather than by scattering on phonons.

7. Decay of Bloch oscillations in two- and three-dimensional quantum dot superlattices

In two- and three-dimensional QDSLs, because carriers can move perpendicularly to the electric field, the decay of Bloch oscillations is no longer exponential, and the decay decrement itself turns out to be a function of time. The decay rate, $\Gamma(t) = -\partial/\partial t[j(t)/j_0(t)]$, where $j_0(t)$ is the non-scattering current oscillation, is, as before, determined by $\gamma(k)$, a quantity which is similar in structure and physical meaning to the oscillation decay decrement γ, Eqn (20), in one-dimensional QDSLs, but which now depends on the wave vector k characterizing the motion of an electron in the transverse miniband. However, besides $\gamma(k)$, the quantum relaxation equation in 2D and 3D QDSLs contains terms that account for mixing processes in transverse minibands. This mixing in itself does not lead to oscillation decay but its effect on the decay time dependence turns out to be significant. Reference [19] examines the time dependence of the oscillation decay rate and analyzes in detail how the decay rate depends on the width of the transverse minibands and on the electric field for the case of the scattering of carriers on acoustic phonons in the transverse minibands of the Stark ladder. Here, we will only present results on the maximum (in time) oscillation current decay rate for the case in which (a) the electric field is switched on instantaneously along one of the basis vectors of a rectangular 2D QDSL, (b) the conditions (13), (14), and (16) are fulfilled, and (c) only scattering on acoustic phonons in the transverse minibands is of significance.

Under these conditions the oscillation decay rate $\Gamma(t)$ is maximum immediately after the electric field is suddenly switched on, and tends to a constant value at large times. The dependence of the maximum oscillation decay rate $\Gamma(0)$ on the degree of field-induced electron localization and the transverse miniband width at room temperature is shown in Fig. 4. It is seen that in the range of parameters of interest the decay rate varies by two orders of magnitude. Knowing this dependence it is possible, by choosing the magnitude and direction of the electric field in 2D and 3D QDSLs, to strongly suppress scattering and to increase the lifetime of Bloch oscillations. The fulfillment of the condition $\hbar\Omega - \Delta_\perp > \hbar\omega^*$ for the suppression of scattering on acoustic phonons between transverse minibands implies, for the parameters chosen, that the frequency of Bloch oscillations $f_{\mathrm{BO}} = \Omega/2\pi > 10^{12}$ Hz. Then from Fig. 4b it follows that the lifetime of Bloch oscillations in a QDSL can reach hundreds of oscillation periods at room temperature. Such strong scattering suppression is fundamentally unachievable in layered superlattices, where it takes on the order of one period for oscillations to decay at room temperature [29].

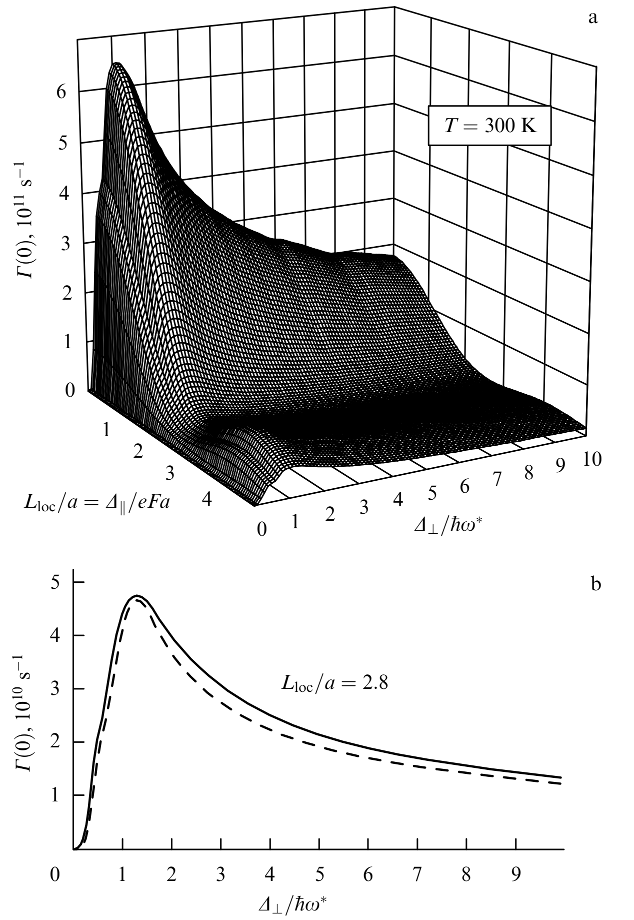

Figure 4. (a) Maximum decay rate of Bloch oscillations $\Gamma(0)$ at room temperature in a two-dimensional square GaAs QDSL with a period $a = 100$ Å against the electron localization length in an electric field (expressed in units of the superlattice spacing a) and the transverse miniband width $\Delta_\perp$ (in units of $\hbar\omega^* = \hbar s\pi/R_D = 4.3$ meV); $R_D = 25$ Å is the QD size. (b) The same for a fixed localization length of 3 superlattice spacings (upper curve); the lower curve corresponds to the asymptotic large-time value $\Gamma(\infty)$ of the oscillation decay rate.

8. Conclusions

We have seen that quantum dot superlattices allow control of scattering processes. It has been shown that the lifetime of Bloch oscillations in such superlattices can be increased by several orders of magnitude compared to layered superlattices of quantum wells, and that it can reach hundreds of oscillation periods at room temperature. Such strong scattering suppression becomes possible due to a new means, absent in layered superlattices, of controlling the electronic spectrum — by changing not only the magnitude of the applied electric field but also its orientation relative to the principal axes of a two- or three-dimensional superlattice of quantum dots.

Specifically, changing the magnitude and direction of an electric field in a quantum dot superlattice makes it possible:

— to widely vary the width of the transverse minibands corresponding to motion in the direction perpendicular to the electric field (using the fact that the width depends exponentially on the field orientation);

— to completely suppress single-optical-phonon scattering processes determining the scattering rate of carriers in layered III – V superlattices;

— to virtually completely suppress scattering on acoustic phonons between transverse minibands on various steps of Stark's ladder of states; and

— by properly choosing the magnitude and direction of the electric field, to strongly suppress scattering on acoustic phonons in the transverse minibands.

Moreover, unlike layered superlattices, in quantum dot superlattices in two and three dimensions the spectrum of Bloch oscillations consists of two or three basic frequencies that can be retuned independently by changing the magnitude and direction of the electric field.

All these features make two- and three-dimensional quantum dot superlattices very attractive for electronic and optoelectronic device applications.

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