## FROM THE HISTORY OF PHYSICS

# On the history of fluid dynamics: Russian scientific schools in the 20th century

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<u>Abstract.</u> The history of designing wind tunnels, an isolated wing, as well as both flying and non-flying machines, is reviewed. An analysis is made of those remarkable aerodynamic ideas which have been practically implemented, as well as of those, no less remarkable, ideas which have — so far remained unfulfilled. The history of theoretical fluid dynamics in Russia is represented as the history of four scientific schools: those of Zhukovsky, Friedmann, Kolmogorov, and TsAGI.

> In the beginning the gods did not at all reveal all things clearly to mortals, but by searching men in the course of time find them out better. Xenophanes

## 1. Introduction

Science is unable to provide answers to the majority of burning questions concerning the future of sciences, nations, and humanity [1-4]. But this does not mean that these questions should not be posed. Predicting the future of physics and, in particular, hydrodynamics<sup>1</sup> is always topical.

Mechanics, which had been perceived as being a perfectly completed field of knowledge, has undergone profound changes in the past century [5]. There gradually emerged in increasing numbers the remarkable properties of the evolu-

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Received 4 December 2002 Uspekhi Fizicheskikh Nauk **173** (1) 419–446 (2003) Translated by E N Ragozin; edited by M I Zel'nikov tion of dynamic systems and the decisive role of such dichotomies as stability-instability, randomness-regularity, determinism-uncertainty, linearity-nonlinearity, regularity-singularity, continuity-discreteness, symmetryasymmetry, evolution-revolution, reversibility-irreversibility, collapse-explosion, and stratification-uniformity.

Science is a flying three-horse sleigh: Rosinante (romance), Pegasus (inspiration), and Bucephalus (war).

Now it is valid to say that a new science, synergetics, which unifies different realms of natural science ranging from astronomy to biology and is largely reliant on hydrodynamics, is being formed [6]. Modern technologies are inconceivable without hydrodynamics, as is the progress of transport, power engineering, metallurgy, and of such sciences as astrophysics, biology, oceanography, and meteorology. Such terms as magnetic hydrodynamics, geological hydrodynamics, and medical hydrodynamics have long been routinely employed [7]. Hydrodynamic models are employed in quite unexpected (on the face of it) fields: classical nonrelativistic cosmology [8], physiology of humans and animals [9], and ufology [10]. In the modeling of any new phenomenon, be it atomic structure or the Big Bang, quite often a 'mechanical caftan' is tried on. We already know much about the structure of atoms and the universe, but so far cannot ab initio perform calculations of the water flow rate in a channel, the lifting force for a plane, and rocket or automobile drags.

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<sup>&</sup>lt;sup>1</sup> In lieu of the term 'hydrodynamics', in English-speaking countries use is made of a more universal word combination 'fluid dynamics'. In Russian, the root 'hydro' does not sound like 'water', does not grate upon the ears, and therefore advantage is routinely taken of a one-word term — hydrodynamics.

Gas dynamics is a division of hydrodynamics concerned with the effect of the compressibility of a medium. Aerodynamics is a division of hydrodynamics concerned with airflow past a body and therefore has aviation applications. The mechanics of continua unifies hydrodynamics and the theory of elasticity.

S K Betyaev

In accordance with the decimal number system adopted, at the beginning of every century — and especially of a millennium! — the results of cultural development are summed up; philosophers and historians revive their activities; the past is analyzed, predictions are made, and goals are set for the future [7]. "He who delves into deep mines of knowledge should, like any navvy, from time to time come up to the surface to take a breath of fresh air. It is during one of these intervals that I am writing you..." (I Newton)

History is a memory of the past. The future belongs only to the human who is in possession of the past. Everything new is firmly bound with the past, in which it originated and is routed, from which we come to know the present, and with which we feed the national pride. Not only the progress, but also the existence of civilizations is impossible without the selection of socially useful traditions. 'Only memory,' M M Bakhtin said, 'can proceed forward, and not oblivion.'

Since history is a non-Markovian process, without it there is no way of comprehending the present. In the mirror of history we seek clues to the innermost questions: what we are living for, what we belong to, and what place we occupy on the axis of historical time. F Tyutchev held that "there is not anything more humane in humans than the need to relate the past to the present."

History is not merely memory, but prognostication as well. While remembrance of the forefathers is a moral category, prognosis is a specific quantitative category, which states the relationship between the historically inevitable and the historically incidental. The mission of the history of sciences is not only to trace the great discoveries and new problems, but to provide the clues to their solution as well, enriching humanity with the experience of accumulated errors and achievements. This is precisely where its propaedeutic significance resides. Science should not be studied in isolation from its history. History is the bridge from the past to the future via the present, the adoption of the optimal route, the mechanism for transferring experience along the chain of generations, and the selection of values. This is how B Pasternak spoke about it:

> One day Georg Hegel inadvertently, And probably at random, just Called a historian a prophet Who made predictions of the past.

It would be wrong to believe that problems may exist only in those fields of physics that exhibit phenomena extremely small or extremely large in scale. They equally exist in the fields wherein phenomena are perceived by us as customary in scale and which are under the reign of the laws of classical mechanics – the part of physics which embraces theoretical mechanics, the theory of elasticity, and hydrodynamics. Hydrodynamics is largely an intuitive, incomplete science. There exists no absolute truth: each problem may be extended and improved. And this path of research, with its successes and failures, constitutes an enthralling sight, while finished and steady solutions bring cold and boredom. Paraphrasing the statement of the great mathematician D Hilbert on physics, that it is "too arduous for physicists", it is pertinent to note that hydrodynamics is too arduous for hydrodynamicists. It is interlaced with mathematics in a tight tangle.

In recent years, the ranks of research scientists (computational scientists, experimenters, theorists<sup>2</sup>) have been pressed by scientists with a world outlook (Weltanschauung) of their own (philosophers, historians, popularizers). In lieu of a onesided materialistic approach, a composite analysis is introduced, which reconciles the spiritual and materialistic principles [11, 12].

The future has many virtual scenarios, while the past only a single trajectory, which denotes the course of events that have occurred (Fig. 1). In history, however, this single trajectory is not subject to an unambiguous interpretation; far from it. It has been known that history never has a single face, a universal history never occurs, the starting factual material may be grouped and interpreted in different ways. Not being among exact sciences, history is largely based on sources whose authenticity, or at least objectiveness, is impossible to substantiate. Legends mix with facts, documents distort the truth and are falsified to please the state order and selfish rulers, memoirs of outstanding public figures are supersaturated with personal perceptions of reality. History is servile and mythologized. Its temporary, unsettled state is termed political history. It is validly assumed that histories and historians are equal in number. As Yu Trifonov said, "History blazes like a huge fire, and each of us throws into it our own brushwood." Is it possible to create an impartial history of sciences? What principles should underlie it?





Creating an unbiased history of peoples and nations has been the dream of historians of all nations and at all times, beginning with the famous seven men of wisdom. Polybius, an ancient Greek historian, stood up for the creation of a 'rational history.' Tacitus, an ancient Roman historian, believed that he wrote "*sine ira et studio*," i.e., "without anger and partiality." Despite the development of hermeneutics (the teaching of how to 'comprehend' texts and extract the truth), this dream still remains unfeasible [13, 14]. *Sine studio...* just like according to F Iskander: "What is history? Nothing. A river does not care whether a slaughter-house or a mill is constructed nearby."

A different situation arises with the history of sciences. With a more definite chronology and based on more objective material, it differs radically from the history of nations. The

<sup>&</sup>lt;sup>2</sup> According to customary scientific slang, computational scientists are referred to as 'keyboardists' and experimenters as 'trumpeters', while theorists are divided into 'classics' (those who study equations of mathematical physics), 'epsilonists' (those who take advantage of asymptotic expansion techniques), 'solitonists' (those engaged in nonlinear mechanics), and 'chaotists' (those who investigate turbulence).

introduction of cliometric techniques [15] allows it to become more objective and rest on quantitative relations, takes off the veil of ideology, the hypnosis of schools and personalities.

In line with the directives of A J Toynbee [16], an outstanding English historian, every historian should first of all specify the *field of investigations* and adopt the *method of historical analysis*. Otherwise history will become, at best, a mere enumeration of facts and memorable events or, at worst, a form of manipulating consciousness [17]. This is how the remarkable physicist H Yukawa speaks about it [18]: "I believe that approaching the past merely as a collection of bygone facts would be quite irrelevant."

As our *field of investigation* we select both theoretical and applied hydrodynamics. Applied hydrodynamics embraces aerodynamic design. The demarcation line between theoretical and experimental science is not strictly defined. The former involves the search for the laws of nature and deals with ideas, while the latter involves putting ideas into practice and the level of know-how. Applied scientists harness rather than produce scientific accomplishments; they are closer to engineering than to science.

The method of selection of outstanding scientists, schools, and discoveries relies on the use of a *citation index*. The citation index of a scientist engaged in basic and theoretical research distinguishes him from an scientist engaged in applied research. Scientists that have a citation index make up the *mainstream of science*.

In the 70s, which saw the crystallization of the idea of total 'digitization' of anything and everything in connection with general computerization, it became apparent that there is hardly any measure of usefulness of a publication other than its citation: the more frequent the citation of a paper, the weightier its contribution to the progress of science and the higher the creative activity of its author. References to the literature are indicative of the homage an author pays to his colleagues and of his scientific conscientiousness. The tradition of openly making references to the papers of predecessors emerged in the middle of the XIXth century. Since 1966, the E Garfield Institute for Scientific Information in Philadelphia (USA), which bears the name of its founder, has published thick 'Science Citation Index' volumes.

Observing the citation of scientific papers makes it possible to trace the dynamics of ideas and their penetration into neighboring fields, and to reveal the symptoms of scientific revolutions. Citations make up a chain linking all papers in a common network, where every paper is a supplement to the previous ones and in turn either closes a given branch on the tree of knowledge or gives new sprouts to it.

Citation indices are a necessity rather than the truth. They are nothing more than a crude measure of the recognition and usefulness of a scientific paper. Naturally, there exist no ideal ratings of scientists. However, under globalization of economies and cultures there exists a demand for quantitative estimates of tennis players, actors' performances, and the activities of scientists. Neither position nor status nor degree provides an unbiased criterion for learning [19]. It is known that the citation index should not be used as an incentive to working scientists, but is reasonable to use for historical analysis.<sup>3</sup> Naturally, the citation index technique is not absolute, if for no other reason than that the history of science created on this basis is subject to temporal changes. It cannot be helped: historical estimates depend on the *historical times*.

Despite the application of cliometrics, there always remains room for subjectivity in the history of science. History is written by people, and they voluntarily or involuntarily introduce 'the aroma of the epoch.' M V Nesterov, a notable Russian painter, thus spoke on the subject: "Historians Karamzin, Kostomarov, and Klyuchevsky shine brightly in historical science only because they are highly subjective."

It is appropriate to point out that the adoption of the citation index as a measure of rating is by no means justified for the assessment of the activities of a scientist engaged in applied research. In this case, we have to revert to keeping count of the scientist's papers. Not all papers, but only those published in journals with a sufficiently high rating, termed the '*impact factor*.' The distribution of such papers over the years — the *publications index* — gives an impression of the dynamics of the creative activity of an engineer or scientist engaged in applied research. Naturally, the use of the publication index is limited by secrecy. A scientist engaged in classified work is devoid of publications and hence of citation; the criterion under discussion does not apply to him.

The activities of inventors are assessed by the number of patents.

Lastly, there are scientists who enjoy the highest and most stable form of recognition. No, it is not merely Nobel Prize Laureates! It is the classics of natural science, those whose names are assigned to laws, effects, theorems, reactions, equations, etc.

Therefore, in the knowledge of science there is a measurement — a numerical assessment of the activities of scientists, scientific schools, and institutes. This approach immediately reveals the exceptional role of English scientists in the development of hydrodynamics. It should be remembered that in the past England was a great naval power and the birthplace of the first industrial revolution. In parentheses we also remark that another country — Japan — has made an inestimable contribution to painting [20]. The specific features of the development of England and Japan are largely due to their insular locations and the ensuing relative solitude, conservatism, and hence the autochthonous culture. Here are the names of outstanding English scientists written in the textbooks on hydrodynamics: G G Stokes, J Rayleigh, H Lamb, O Reynolds, W Kelvin, J S Russel, W Froude, F H Wenham, J H Michell, L Rosenhead, J I Taylor, H Goldstein, K Stewartson, F W Lanchester, K W Mangler, M J Lighthill, G K Batchelor, H K Moffatt, and F T Smith. In order that the reader not be dazzled by the luster of the names, not listed are the founders of 'whole' natural science: W Ockham, F Bacon, I Barrow, R Hooke, and I Newton.

Books have been written about each of the above English scientists. We restrict ourselves to a mention of only two of them. Sir Jeffrey Ingram Taylor (1886–1972) — grandson of G Boole, one of the founders of mathematical logic, son of the famous English painter E Taylor, nephew of E Boole (Voynich), author of the novel "The Gadfly", which is well known everywhere, especially in the USSR. Significant developments in the fields of meteorology, mathematics, statistical turbulence theory, the theory of rotating liquids, hydrodynamic stability, and experimental hydrodynamics

<sup>&</sup>lt;sup>3</sup> In the West, the citation index is used unofficially and yet extensively. In Russia, this is impeded by the scientific elite, which has no citation index and therefore fervently objects to its use. Of course, science is not sports. Learning to use the citation index correctly lies ahead.

are due to Taylor [21, 22]. He was a member both of the Royal Society (UK Academy of Sciences) and of the Academy of Sciences of the USSR.

M J Lighthill (1924–1998) was granted knighthood for the development of acoustics, wave theory, boundary-layer theory, flow theory of a non-Newtonian liquid, and biomechanics. His selected scientific papers were published in the form of a four-volume edition [23].

Scientists study the *objectively* existing laws of nature. Had Aristotle and Newton never existed, someone else would certainly have made the same discoveries sooner or later. In art, the works made are immanently *subjective*. Had Bach and Dostoevsky never existed, no one would have taken their place. In art, the personality of the contributor is more pronounced than in science. An artist does not uncover, but creates the subjective truth; a scientist does not create, but uncovers the objective truth.

## 2. Aerodynamic design

History is not an exact science. Having defined the subject, a historian selects — out of the data base which is made up of a plethora of contradictory facts — a subset consistent with his historical model (version). It is not always possible to substantiate a theory with the aid of cliometric methods, and therefore history is ambiguous: each version in its quasicyclic evolution constitutes merely a more structured new fact in the new data base, and so on infinitely. Does this process converge into the truth? It is conventional not to discuss this...

Like astronomy, which is divided into classical descriptive astronomy and astrophysics, history (according to L N Gumilev) is divided into descriptive history and prescriptive (instrumental) history. Descriptive history is a collection of events and facts, both veracious and false. Memoirs, reminiscences, archive documents, etc. (N M Karamzin, A S Pushkin, V O Klyuchevsky) may be placed into this category. In the framework of prescriptive theory, versions are suggested and studied, and the nature and causes of events and their logical links are accounted for (A J Toynbee, L N Gumilev, A T Fomenko). Hermeneutics is the methodological foundation of prescriptive history.

The history of sciences is also a science. A historian's investigation in this field involves several stages. The search for documents comes first. Next there follows their study on the basis of hermeneutics, i.e., the separation of invention from the truth employing so-called content analysis. And, lastly, comes insight, the description of events and the revelation of regularities and trends.

This year, aviation will be 100 years of age; it is conventional to consider the day of the first flight of the Wright brothers' airplane to be the birthday of aviation. Aerodynamic design as an engineering science originated much earlier. Its history is more subjective than that of aerodynamics and is helpful in understanding the laws of introduction of discoveries, in tracing the fate of ideas, in highlighting untapped possibilities, and in linking together theory and practice. Why does one idea or another remain unrealized in practice? Neither the laziness nor the incompetence of the designers of new equipment should be ruled out of the possible economic, technological, operational, etc. reasons.

Three areas should be recognized in the history of aerodynamic design: wind-tunnel design, isolated-wing design, and the design of machines as a whole.

## 2.1 Wind tunnels

The world's first wind tunnel was built in 1871 by F H Wenham (1824–1908), a construction engineer and Fellow-founder of the Royal Aeronautics Society, Great Britain. To build it he fitted a cylindrical housing to a big blower in Penn's mill in Greenwich. It was used to blow air on plane plates at different angles, as well as on airplane models.

That same year, a ballistic facility was set up by Captain V A Pashkevich, who lived in St. Petersburg. It contained all the main elements of present-day wind tunnels: a collector, a working section, a diffuser, and even a balance. However, this facility was employed to investigate the drag of shells, and not of airplanes or dirigibles.

D I Mendeleev attached great importance to aerodynamic tests of the models of flying vehicles. In his notebook of 1876, which is kept in custody in the archive of the museum of the great chemist at the St. Petersburg State University, there is a sketch of a closed-jet wind tunnel. In his program paper entitled 'On liquid drag and aeronautics' (1881), Mendeleev emphasized the groundlessness of attempts to solve the problem of medium drag in a purely theoretical way, without systematic experimental investigations. His considerations are also topical nowadays: "The insufficiency of experimental medium drag data for the complete solution of aeronautics problem is so evident that I find it impossible to pass over in silence the inevitability of new precise experiments, their objectives, and the methods and means required for their execution. This insufficiency of accurate experimental medium drag data accounts simultaneously for the poor development of general medium drag theory and the practice of aeronautics".

In 1883, a Russian engineer S S Nezhdanovsky constructed wind tunnel of original design. Not air, but the gas produced by combustion of gunpowder in a generator was blown on models.

Russia's first wind tunnel, wherein systematic investigations of body flow (weight measurements) were conducted, was constructed by K E Tsiolkovsky.

Tsiolkovsky's life is an amazing example of originality. He never graduated with a higher education — in the XIXth century, at the dawn of exact sciences, scientists without a higher education were not uncommon. Tsiolkovsky admitted: "I had no teachers at all, and therefore I had to design and create rather than perceive and learn. Directions and help were never available, much in books was obscure, and I had to explain everything by myself. In a word, there prevailed the creative element, that of self-education and originality. Throughout my life I, so to speak, have been learning to think, overcome difficulties, and solve problems and issues. For lack of textbooks and teachers, I had to found many sciences on my own." Founding sciences is, to put it mildly, an exaggeration. Tsiolkovsky had no command of the foundations of exact sciences, and it therefore is astonishing how the wealth of imagination and the aspiration for creative work in combination with logical consistency led to big accomplishments!

Tsiolkovsky constructed his first wind tunnel in 1897. A second one, more accurate and twice as large, was made in 1899 upon receiving a grant of 470 rubles from the Academy of Sciences. According to present-day terminology, it was an open-jet wind tunnel, and according to Tsiolkovsky's terminology, a blast engine (Fig. 2). A winnower-type paddle blower was employed as a fan. To produce a uniform velocity field, advantage was taken of a flow-straightening honey-



Figure 2. Tsiolkovsky's blast engine.

comb grid. In addition, Tsiolkovsky created an aerodynamic balance with quite a satisfactory accuracy of measurement.

What is the scientific value of his scientific experiments to determine the air drag for different bodies, their significance for the development of hydrodynamics? To provide an answer to this question we represent his experimental data in a dimensionless form. Of the models he tested, which were sometimes as exotic as an icosahedron, we select straight round cylinders whose axes were oriented perpendicular to the direction of unperturbed flow velocity  $u_{\infty}$ . From this point on, the subscript  $\infty$  denotes the unperturbed quantities.

Tsiolkovsky was not familiar with the similarity theory, and that is why in his tables the drag force was represented as a function of two variables:  $u_{\infty}$  and the cylinder diameter *d*. In Fig. 3 this dependence is represented as a function of one variable  $c_x = c_x$ , where the Reynolds number  $\text{Re} = u_{\infty}d/v$ and *v* is the kinematic viscosity coefficient. The reference curve is shown with a solid line. The maximum departure of Tsiolkovsky's curve (the dashed line) from the reference one amounts to 18%. In the early days of aircraft and dirigible building, this accuracy was acceptable, but the flow velocity (the Re number) was obviously low.

At the time when Tsiolkovsky was experimenting with his blast engine, wind tunnels were constructed in other countries. In England, a wind tunnel was built by H S Maxim (1840–1916), the famous inventor of a machine-gun, in



Figure 3. Dependence of  $c_x$  on Re: •  $-u_{\infty} = 1 \text{ m s}^{-1}$ ,  $\Box - u_{\infty} = 2 \text{ m s}^{-1}$ ,  $\bigtriangleup - u_{\infty} = 3 \text{ m s}^{-1}$ 

France by C Renard (1847–1945), a well-known aerodynamics specialist, and in the USA by S P Langley (1834– 1906), a notable astronomer. In Russia, the experimental base of aerodynamics was created by N E Zhukovsky: Moscow University (1902); Kuchino, the patrimonial estate of the Ryabushinskys, outstanding Russian public figures (1904); the Moscow Higher Technical School (1910); and the Central Aerohydrodynamic Institute (TsAGI) (1918). Russia was hurrying to adequately face the age of aviation [24–26].

It is likely that a supersonic velocity in a directional flow was first reached in the nozzle of a high-speed steam turbine constructed by a Swedish engineer, K G P de Laval<sup>4</sup> in 1889. By that time, shock waves had been discovered by the great German mathematician G F B Riemann (1860) 'at the tip of a pen' and photographed by the great Austrian physicist E Mach (1887). That is why the following statement of S A Chaplygin, which dates back to 1910, appears to be strange [27]: "...disregarding the compressibility of air is legitimate until the velocity anywhere in the flow reaches the speed of sound, because a *stationary motion is no longer possible* when this condition is violated" (italics introduced by me — S B).

The introduction of perforation — the permeable walls of the working section of a wind tunnel — marked a new stage in the design of near-sonic facilities. They are employed in wind tunnels to reduce the flow boundary effects on the flow past a model, which is particularly significant at near-sonic velocities. The introduction of perforation at TsAGI commenced in 1946, and the first wind tunnel with a perforated working section was put in operation in 1947. In this connection the following remark is in order. The Russian work done during the period 1945 – 1947 should be considered in view on the socalled *German trail*.

In 1945, a huge load of technical documentation was exported from Germany to Russia as reparations [28]. Furthermore, 3.5 thousand engineers also arrived, among whom 2.8 thousand (!) proved to be experts on aircraft construction. The latter were compelled to write reviews on their subjects and participate in the training of our aircraft constructors. It is an open secret that our science was behind foreign science  $^5$  and our scientists extensively used German technical solutions in aerohydrodynamics. That is precisely what the *German trail* was.

#### 2.2 Wing shape optimization

The wing is the most important aerodynamic element of a plane. The entire history of wing evolution, all the ideas aimed at its aerodynamic improvement, are merely attempts to intuitively solve the variational problem of the optimal wing. Numerical <sup>6</sup> and experimental solutions of this problem cannot be regarded for the moment as being rigorous enough and satisfying practical needs. In the general case it is not difficult to write down the equations and arrive at a

<sup>5</sup> Here is a typical anecdote of that time. An engineer approaches the TsAGI director...

- I can raise the lift of a plane by a factor of 1.5.
- Has this invention already been put into practice abroad?
- No, no!
- Then, let us wait.

<sup>6</sup> In the numerical solution, a wing is represented as a combination of a finite number of profiles, each profile being approximated by a multiparameter function.

<sup>&</sup>lt;sup>4</sup> Laval is also known as the first inventor of a cream separator.

minimum of the lift-drag ratio (aerodynamic characteristic) using a functional, but attempts to 'trace' the main approximation and sequentially improve it by directed iterations do not meet with success. Variational problems in aerodynamics are solved satisfactorily only in the framework of a nonviscous liquid model, which makes possible the linearization of the problem [29].

Thus far, flow separations have been unsatisfactorily calculated and the turbulent flows quite poorly. The theory of turbulence has not yet been constructed, and an added complication is that in computational hydrodynamics there exists the so-called *curse of the '9/4 law'*: in the solution of a direct three-dimensional problem on a turbulent liquid flow, the number of mesh nodes should be of the order of  $O(\text{Re}^{9/4})$ . This circumstance makes impossible the numerical calculation of an aircraft on present-day personal computers. One is forced to agree with the statement made by D Küchemann, a well-known aerodynamics specialist [30]: "Despite the modern advances of numerical techniques, wing design remains an art rather than an exact science, as has always been  $\langle \ldots \rangle$ , so far there are no rational and complete methods of calculation, and aircraft design is not devoid of risks quite often fraught with large expenses in all senses, because some characteristics significant in flight can in no way be calculated, measured, or predicted".

There is one more *curse* in the theory of turbulence — that of *fractal dimensionality*. A turbulent flow is inherently fractal, and we have not yet learned to deal with fractal objects.

The solution of the variational problem of an aerodynamically perfect aircraft shape  $F(x, y, z; \mathbf{M}_{\infty}, \mathbf{Re}) = 0$  so far cannot be derived theoretically. Aircraft engineers obtain it with the use of intuition and industrial experiment. It is solved in precisely the same way is the problem of shape optimization of the wing — the most important element of an aircraft. The problem is significantly simplified for a wing of high aspect ratio, since it reduces to the optimization of a wing airfoil specified in the form  $f(x, y; \mathbf{M}_{\infty}, \mathbf{Re}) = 0$ .

Streamline profile. For near-sonic values of the  $M_{\infty}$ number, the occurrence of compression shocks and the ensuing wave drag should be avoided. When a flight takes place in an incompressible liquid ( $M_{\infty} \ll 1$ ), there remain two physical effects an aircraft engineer has to reckon with: boundary layer turbulization and flow separation from the wing. The frictional stress in a laminar flow is much lower than in the turbulent one. It is therefore expedient to make the laminar part of the airfoil as long as possible, i.e., delay the laminar-to-turbulent boundary layer transition. It was found that this problem can largely be solved by selecting the profile shape f = 0. Such a profile is termed *streamline*. Its distinctive feature is the displacement of the location of highest profile thickness and concavity towards the rear edge, which ensures a rapid pressure decrease in the laminar flow part and a nearly constant value in the remaining part. This shape results in a negative pressure gradient in the greater part of the wing, which improves the flow stability.

A sharp pressure rise in the stern of the profile is inadmissible owing to a possible flow separation.

The turbulent boundary layer, on the one hand, increases the wing drag and on the other impedes the flow separation and permits attainment of a higher wing lift.

In 1939, a series of streamline profiles (the NACA series) with a favorable pressure gradient over their major part was developed by an outstanding American engineer E N Jacobs

of the Langley Research Laboratory. A special wind tunnel with a low flow turbulence had to be constructed to test them under conditions close to reality. It was possible to retain a laminar flow over 70% of the chord and reduce the wing drag by a factor of 1.5.

Similar research was also conducted in England, Japan, and Russia. The secrecy due to national security impeded international cooperation.

At present, the wings of small planes, and particularly gliders, are made up of streamline profiles. For large planes of civil aviation, streamline profiles are not taken advantage of.

The functional purpose of a wing depends on the flight mode: takeoff, cruise mode, maneuvering, and landing. Depending on the flight mode, the lifting force, the lift-drag ratio, or the maneuverability serve as functionals. Varying the wing shape in relation to the flight mode is impracticable. That is why wing optimization is accomplished by way of socalled *wing mechanization* — a complex of devices in its front and rear parts: leading-edge slats and flaps. They are diversified in design: pivoted, extension, slotted, and doubled-slotted. Mechanization is helpful in reducing adverse effects and improving aerodynamic wing characteristics.

**Sweep effect.** The effect of sweep consists of (i) the longitudinal flow being independent of the transverse flow (the autonomy principle) and (ii) the flow being twodimensional, i.e., being independent of the third (transverse) *z*-coordinate. In the simplest case of ideal liquid flow, the velocity along this coordinate is constant. Figure 4a shows an oblique wing infinite on either side. The sweep effect is theoretically valid for precisely such a wing, like for any cylindrical body.



**Figure 4.** (a) Oblique wing, (b) swept wing: *I* — region of middle effect, *2* — region of end effect, *3* — partitions.

The sweep effect was first applied to calculate a dirigible [31] by M M Munk (1924), a pupil of L Prandtl. A Busemann, also a pupil of the great Prandtl, discovered this effect when he was engaged in the classification of linear supersonic flows of an ideal gas (1928).

In 1935, in fascist Italy, whose aviation ranked as one of world's best, a congress was held on high-speed aerodynamics. From Germany there came Prandtl and from the USA T von Karman. Busemann in his report proposed the use of swept wings in a supersonic flight [30-33].

The flow past a swept wing with a high aspect ratio differs from the flow past an oblique wing in that the sweep effect breaks down in the region about the apex of the wing (1 in Fig. 4b) and in the region 2 about its side edges.

As observed by A Betz (1940), Prandtl's pupil, in order for a plane to overcome 'the sound barrier' it would not suffice to employ a swept wing; in addition there is a need to mount a radically new engine — the jet engine. The first jet-propelled aircraft was the German 'Heinkel-178' (1939). And even in 1942 the German Me-163 plane reached the speed of sound. It is likely that in this case a Mach number  $M_{\rm cr} < 1$  was reached, whereby a supersonic velocity is reached over a wing — and not in a horizontal flight, but in a hedge-hopping flight. The statements that the speed of sound was first-ever reached in the USSR (La-174 in 1948 [33] or Mig-17 in 1950 [35]) are strong exaggerations.

Four record-breaking events should be recognized in the history of aviation: the attainment of  $M_{cr}$  in a hedge-hopping flight, of the  $M_{\infty} = 1$  Mach number in a hedge-hopping flight, of  $M_{cr}$  in a horizontal flight, and of  $M_{\infty} = 1$  in a horizontal flight.

The sweep effect in an ideal gas is a direct consequence of the Galilean relativity principle: the physical laws are independent of the choice of inertial coordinate system. Does it hold true in a viscous liquid? L Prandtl (1945) and V V Struminskii<sup>7</sup> (1946) showed that it is valid for a laminar flow [33]. However, the flow is turbulent for the major part of a wing. The sweep effect does not hold<sup>8</sup> for this flow, and therefore swept wings are fitted with longitudinal partitions, which are hatched in Fig. 4b.

Supercritical airfoil. For  $M_{\infty} > M_{cr}$ , on the upper wing surface there forms a supersonic inclusion (Busemann, 1941). Such mixed flows were experimentally investigated by B H Goethert (1943).

Is a shock-free supersonic-to-subsonic flow transition possible? It has been proven experimentally that such a transition is possible, and a family of wing airfoils has been designed on whose upper surfaces there is an isentropic flow compression [31].

In the 1950s, R T Whitcomb, an outstanding aircraft engineer at NASA, proposed a wing airfoil with a significantly lower drag in comparison with the conventional airfoil [36]. This wing airfoil was termed supercritical, because it enables a vehicle to attain higher flight velocities and the value  $M_{cr}$  for long-range aircraft to be raised by 0.05-0.15 units.

A supercritical airfoil (the solid line in Fig. 5) shows a less convex upper surface to reduce the supersonic flow velocity and the so-called *cut* near the trailing edge to recover the lifting force. Whitcomb stated his belief that a supersonic airfoil with an obtuse fore part can also be used for a supersonic flight, when the edge of a swept wing is subsonic.

Furthermore, Whitcomb proposed the use of a fixed slot near the trailing edge of a wing. A high-enthalpy gas jet penetrating through the slot accelerates the flow in the boundary layer at the upper airfoil surface and reduces the motion drag.

The use of supercritical airfoils in aviation also made it possible to reduce the shock stall at the ends of a helicopter rotor.





<sup>7</sup> Seek the German trail.

 $^{8}$  I do not know why.

**Boundary-layer suction.** The suction of a liquid through a permeable surface was demonstrated by Prandtl (1904) in his famous paper, wherein he came up with an essentially asymptotic ( $\text{Re} \rightarrow \infty$ ) conception of a boundary layer. This problem was subsequently investigated by Lighthill and H Glauert [30]. The purpose of suction is to delay (in the *x*-axis) either the flow separation from the wing or the laminar-to-turbulent transition. Calculations show that a substantial gain in lift-drag ratio or drag force is attained at the expense of insignificant energy losses.

Suction may be distributed evenly or discontinuously over the surface. An idea was advanced involving a two-slot gas *transfer* in the region of the compression shock closing the supersonic area on the upper airfoil profile to reduce wave drag at near-sonic velocities.

Despite obvious advantages, the boundary layer suction up to the present has not been used in aviation practice owing to the difficulties arising in the operation of suction systems.

**Busemann biplane.** At the above congress in Italy dedicated to high-speed aerodynamics, by way of an exotic example A Busemann considered the airflow past a body which theoretically possesses a zero wave drag in a linear supersonic flow. It was termed the *Busemann biplane*. Figure 6 reproduces the biplane flow schematics proposed by Busemann; compression waves are shown with solid lines and rarefaction waves with dashed lines, the body volume is hatched. The vortex wake behind this vehicle is nonexistent.



Figure 6. Busemann biplane.

Why has Busemann's elegant idea not yet been realized? Because his vehicle does not produce lifting force. Moreover, it is intended only for motion in the cruise mode, i.e., for motion at a fixed value of the  $M_{\infty}$  number and a zero angle of attack.

**V-like shape.** Why does a wing have to be V-shaped? It is believed to endow a plane with stability. But at present the dynamic stability problem is solved by other methods.

It has been found that a negatively V-shaped wing experiences a stronger lifting force due to higher-intensity vortices coming off its side edges. For a wing with a low aspect ratio there holds the law of plane sections: a three-dimensional stationary flow is equivalent to a plane transient flow. In accordance with this law, the flow past a V-shaped wing with a low aspect ratio (Fig. 7a) is equivalent to the flow past an angle (Fig. 7b). Estimates showed [37] that the lifting force  $Y(\chi)$  is maximized in this case for negative opening angles  $\chi$  (Fig. 8), and the so-called *parachute effect* occurs.

A V-shaped wing is nothing more than an approximation of the optimal wing shape with a dihedral angle. Rigorously speaking, the variational problem of the optimal wing should be posed in the selected class of functions y = f(z, x)describing the wing surface shape<sup>9</sup>. Figure 7c is a conven-

<sup>&</sup>lt;sup>9</sup> A canopy should also be the optimal shape for a parachute.



**Figure 7.** (a) V-shaped wing with a low aspect ratio, (b) cross section of a V-shaped wing with a low aspect ratio, (c) cross section of the optimal wing.



**Figure 8.** Dependence of  $Y(\chi)/Y(0)$  on the opening angle  $\chi$ ,  $\alpha/\lambda = 1$  ( $\lambda$  is the aspect ratio).

tional representation of the cross section of a wing belonging to the class of functions which are continuous along with their first derivatives. However, the gain in lifting force is not high enough to justify complicating the technology of aircraft production in order to employ it in practice.

V-shaped wings are applied in aviation: for high-wing monoplanes, i.e., for aircraft whose wings join the fuselage above its median surface, as a rule  $\chi < 0$ , and for low-wing monoplanes  $\chi > 0$ . The latter circumstance is due to the need to ensure the safety of aircraft takeoff and landing. In addition, a designer should take into account the in-flight wing deformation under the action of aeroelastic forces.

#### 2.3 Flying and non-flying machines

A well-known Leonardo da Vinci drawing depicts a prototype of a helicopter rotor. That is why Leonardo may be regarded as the author of the conception of the helicopter as a flying machine.

The conception of the plane was elaborated much later. Initially it was believed that man would become airborne like a bird with the aid of a flapping wing. However, in 1799 G P Cayley abandoned the idea of an ornithopter and came up with the classical conception of a *muscle-free* airborne vehicle [30]. It comprised all the parts of a present-day plane: the wing, the fuselage, the engine, and control surfaces. Legend has it that one of the gliders designed by Cayley successfully moved through free air long before that of O Lilienthal, the pioneer of aviation.

Cayley's conception appears to be natural, but in reality it was revolutionary in significance, and it is the origination of this conception and not the flight of the Wright brothers that the birthday of aviation should related to. However, his conception was not the only one. In 1910, a German aircraft designer and manufacturer H Junkers obtained a patent for the invention of "an aircraft consisting of one plane, which harbors all the components: engines, the crew, passengers, fuel, and the structure." Among numerous proposals, mention should be made of the airbus conception by S B Gates (1960). Gates dreamed of a widely used *fuselage-free* subsonic plane as a means of conveyance available to everybody. Despite the fact that the term 'airbus' has come to be commonplace, Gates's radical ideas need to be further developed and applied to aviation practice.

The problem of the optimal plane is an order of magnitude more complicated than the problem of the optimal wing. It is likely that the future will see the integration of the functional purposes of different elements of an aircraft and the variation of its geometry in relation to flight conditions. Owing to international terrorism, ecological problems, and the momentum-gaining globalization process, aviation is facing new problems. The order of the day is the production of an inconspicuous plane, a noiseless plane, a controllable dirigible, a miniature flying vehicle, etc.

Inconspicuous combat planes (interceptors, attack planes, and strategic bombers) are produced on the basis of high technologies involving coatings that absorb radio waves. The inconspicuousness requirement imposes limitations on the inclination of aircraft surfaces to the horizon — it should not exceed  $60^{\circ}$  in magnitude to ensure the reflection of radiation, which emanates from enemy radar stations, vertically upwards or downwards to escape detection. That is why the leading edge of the wing of an invisible plane is devoid of rounding, which significantly impairs its aerodynamics in a flight of subsonic velocity. This is a problem which invites further investigation.

**Einstein wing.** The A Einstein wing, the so-called 'cat's hunch', is an example of a curious, amusing historical incident. Truth and fiction are lavishly represented in the biography of the great scientist, as in all biographies of geniuses. His biographer C Seelig believes that in 1915 Einstein was engaged in designing planes for the German Air Transport Society [38].

His knowledge of hydrodynamics was limited to the Bernoulli equation. In the framework of this theoretical approach, the greater the curvature of the upper surface of an airfoil, the greater its lifting force. That is why Einstein designed a strongly curved wing named the 'cat's hunch'. Legend has it that the Einstein plane was constructed and even took off piloted by the German ace P H Erhard, who had a narrow escape upon landing. Einstein wrote him a letter to apologize for his contrivance in an elegant form: "This is what may happen to a man who thinks much, but reads little."

To this it may be added that Einstein, when he was working at the Bern Patent Bureau as a Counsellor of the 3rd, and later 2nd, class, together with one of his colleagues received a patent for a new type of hearing-aid, intended for a well-known lady singer, an acquaintance of his. At that time he sent her the following rhyme he had written:

> Advice to a philosopher like me: Engage yourself in techniques to digress. It's hope I am looking forward with: The labor will be crowned with success.

**Minimum wave drag bodies.** As already noted, in the context of a mathematical model of a nonviscous gas it is possible to formulate — and sometimes to solve! — the variational problem of the optimal shape of an aerodynamic body. Here are two related examples on this subject. Von



**Figure 9.** Axially symmetric minimum-drag bodies: (1) von Karman's ogive, (2) Haack–Sears body of revolution.

Karman found the optimal shape of a thin body of revolution with a given base area (Fig. 9) [39]. Its wave-drag coefficient is

$$c_x = \frac{4}{\pi} \; \frac{S(\ell)}{\ell^2} \; .$$

where  $\ell$  is the body length and  $S(\ell)$  the base area.

The optimal shape of a thin body of revolution with a given volume V was found by W R Sears and W Haack (see Fig. 9). The Sears – Haack body is closed, i.e., its base area is equal to zero. Its wave-drag coefficient is

$$c_x = 24 \, \frac{V}{\ell^3} \, .$$

The fore parts of cruise and noncruise missiles are made in the form of von Karman or Sears-Haack ogives. In this connection an important remark is in order: the fore part of the ogives is blunted, and therefore linear theory is inappropriate here and calls for refinement.

Whitcomb junction. The wing-fuselage junction area should be carefully profiled. Severe losses and large gains are possible here. Of particular significance is wing-fuselage interference in a near-sonic flow. To eliminate the production of an intense compression shock where they are joined, in 1952 Whitcomb suggested that a hollow should be made in the fuselage here, as is conventionally shown in Fig. 10. This approach is theoretically substantiated. In accordance with the near-sonic area rule (W D Hayes, 1947; K Oswatitsch, 1947), the wave drag of a wing – fuselage combination is equal to the wave drag of an axially symmetric body with the same cross sectional area distribution as the initial combination [40]. Such a body is termed equivalent. The gain is obtained when the total area of the combination is taken to be equal to the area of the optimal equivalent body. Proper choice of the junction surface has a strong effect on the flow past the entire wing.





Figure 11. Schematic diagram of cross sectional wave-rider flow (the cross sections of the wave-rider are shaded, the dashed lines show shock waves).

'Volnolet' (wave-rider) and other flying machines. A hypersonic plane should have a large volume and a strong lifting force at high Mach numbers, with liquid hydrogen being used as the proper fuel. Since the production of such a plane is highly conjectural, engineers calculate not its real configurations, but idealized ones — those obtained by matching the simplest flows: plane and conic. These configurations have been termed *wave-riders* [29, 30, 41]. The gas compression in the shocks located under the wave-rider between its edges produces the lifting force. The conception of a wave-rider was proposed by T Nonveiler (1952), and in Russia it was developed by G I Maikapar (1959) et al. Figure 11 diagrams schematically the system of compression shocks of a  $\Lambda$  wing in relation to the hollow depth; the body volume is shaded.

Unlike the wave-rider, the idea of an air-cushion ship and the idea of an 'ekranoplan' were advantageously realized [42]. A torpedo boat as a prototype of an air-cushion ship was made by an Austrian engineer, D M von Tamamul (1916). The idea of a true air-cushion ship was conceived by K E Tsiolkovsky (1927). It was built by V A Levkov (1932). The disadvantage of an air-cushion ship is that the steering gear does not come into contact with the water. That is why it has poor maneuverability and depends heavily on weather conditions. The latest designs are the 'Aviastar' crafts of the Novosibirsk Design Office.

B N Yur'ev, G P Beriev, and R L Bartini, prominent Russian men of engineering, labored over the design of an ekranoplan. The largest contribution in the design and production of the ekranoplan was made by Rostislav Alekseevich Alekseev (1919–1979), who directed the Central Hydrofoil Design Bureau in Nizhniĭ Novgorod. He made the first ekranoplan in 1961 and constructed the world's biggest aircraft, the KM ('Korabl'–Maket), in 1970, which was propelled by ten turbojet engines, had a mass of 540 t, a peak speed of 500 km h<sup>-1</sup>, and a flight altitude of 2–3 m. When the American intelligence service detected KM tests from space, they named it the 'Caspian Monster'. After the ekranoplan shipwreck in 1974, Alekseev was dismissed and his archive destroyed.

Alekseev gained high recognition — naturally, not in Russia! — and his portrait is hanging in the American Congress in the gallery of world figures of eminence who made the greatest contribution to the progress of humanity in the XXth century.

Wind-operated power plants. When it became evident that the practical harnessing of fusion energy is a matter of the distant future, interest was rekindled in the problem of harnessing wind (essentially solar) energy whose stores are practically infinite. The USA is the world's leaders as regards employment of wind-operated power plants; Germany recovers 10% of its electric power from the wind. The production rate of wind-operated power plants is rising rapidly [42], which brings to the fore the problem of aerodynamic optimization of the blades of a wind turbine.

## 2.4 Aeroelasticity

The structure of an aircraft is not rigid and undergoes deformation under the action of airflow. This effect is termed aeroelasticity [31, 43]. Aeroelasticity as a science links two disciplines of mechanics of continua: aerodynamics and strength.

Consider the deformation of an airplane wing. The lifting force not only bends the wing, but twists it as well. This brings about an increase in the angle of attack and hence in the lifting force and the torsion of the wing. The wing elasticity resists, within certain limits, this deformation. Since the lift of an airfoil increases with flight speed and the elastic forces are independent of speed, there exists a critical flight speed whereby these forces are equal. It is termed the *divergence speed*. At higher speeds there sets in an elastic instability and the destruction of the wing. This effect is observed not only for a wing, but also for aerodynamic control devices (ailerons, elevators, rudders, etc.).

Among the numerous manifestations of aeroelasticity, of greatest significance is *flutter* — self-excited vibrations of elastic structures. Multi-faceted flutter is the most challenging problem of aeroelasticity, for it involves various types of resonances between aerodynamic waves, flexural-and-torsional wing vibrations, and vibrations of control surfaces.

From the viewpoint of vibration theory, an aircraft is a self-oscillatory system wherein the incident flow serves as the source of energy and the feedback is effected by elasticity.

Aircraft designers encountered the manifestations of aeroelasticity even before the flight of the Wright brothers. The first aeroelasticity-related plane crash occurred to a motor monoplane of S P Langley, a professor of the Smithsonian Institution (USA), in 1903. His plane crashed and fell into the Potomac river owing to wing disruption caused by aeroelastic torsional divergence.

The successful flight of the biplane of the Wright brothers and the failure of the Langley monoplane underlay the adherence to biplanes early in the development of aviation. Biplane wings exhibited a higher torsional rigidity than monoplane wings. However, it so developed that the most acute problem for biplanes was the empennage flutter problem. This is precisely the reason why the 'Handley Page' bomber crashed at the beginning of the First World War. F W Lanchester (1916) was among the first involved in the determination of the causes of its crash. Many lives and planes were lost before a way of combating flutter was found. Engineers straight away proposed an increase in structure rigidity and variations in wing thickness, wing mass distribution, and the location of the engines. In the 1920s, theorists W Birnbaum, H Wagner, H J Küssner) constructed a mathematical flutter model, which reduces to a system of ordinary differential equation. The first exact solution of the problem was derived by T G Theodorsen in 1934<sup>10</sup>.

Mathematical aeroelasticity models were empirical and simplified. Designers reposed no trust in theorists. This is the reason why statistical and dynamic aeroelasticity tests of aircraft were introduced in aviation practice. Advances in aeroelasticity research made it possible to change from biplanes to monoplanes in the 1930s. However, even now, when the flight velocities of airplanes and helicopters have risen and rockets have made their appearance, aeroelasticity problems have not been removed from the agenda. They are also topical in fields unrelated to aviation and rocket design. In 1940, flutter was responsible for the destruction of the steel suspension bridge in Tacoma (USA). Since then, civil engineers place special emphasis on the aeroelasticity of bridges, smokestacks, turbine blades, high cranes, and other structures subject to wind loading.

The flap of flags and sails in the wind and the 'singing' of power transmission lines are all manifestations of aeroelasticity.

## 3. Hydrodynamics in Russia

Despite the fact that Russia embarked on the path of industrial, and hence scientific and engineering, development later than other developed countries (the Chaadaev paradox), late in the XIXth century and early in the XXth century the names of outstanding Russian researchers engaged in the natural sciences received world-wide recognition: D I Mendeleev, I P Pavlov, I I Mechnikov, I M Sechenov, A G Stoletov, S P Timoshenko, I G Bubnov, and many others. At that time hydrodynamics in Russia was still in its infancy. N E Zhukovsky (1847–1921) is rightfully regarded as its founder. Before him, this science in Russia was pursued only by talented lone enthusiasts.

#### 3.1 Past history

The great L Euler (1707-1783), a member of the St. Petersburg Academy of Sciences, was the first to construct a mathematical model of liquid flow. He came up with a system of equations which has retained its relevance up to the present time and bears his name: the Eulerian system.

A F Popov (1815–1879), a staff professor at Kazan University and N I Lobachevsky's pupil, investigated waves in an incompressible liquid.

I S Gromeka (1851–1889), a staff professor at Kazan University, studied a class of incompressible liquid flows whereby the vortex lines are either perpendicular to the trajectories or parallel to them. The results of his investigation, which were published in *Uchenye Zapiski Kazanskogo Universiteta* (Scholarly Notes of Kazan University), remained unknown to the scientific community at large. Eight years later, an Italian scientist, E Beltrami, published a paper on this subject. Today, a flow with vortex lines corresponding to the trajectories are referred to as the *Beltrami flow* rather than the *Gromeka flow*. But the memory of Gromeka has been preserved — one of the forms of equations of motion is termed the Gromeka–Lamb equation.

In the middle of the XIXth century, at St. Petersburg University an excellent school of mathematical science was formed: P L Chebyshev, M V Ostrogradsky, V Ya Bunyakovsky, A M Lyapunov, A A Markov, V A Steklov, Yu V Sokhotsky, and D K Bobylev. Many of them were involved in the solution of hydrodynamic problems.

D K Bobylev (1842–1917), A A Friedmann's teacher, derived a closed-form solution of the problem of a symmetric jet flow of an ideal liquid past a wedge. Today this flow is commonly referred to as the *Bobylev flow*.

<sup>&</sup>lt;sup>10</sup> In Russia, the work on flutter began after some delay — in 1930. E P Grossman was a pioneer in this field.

A M Lyapunov (1857–1918) studied figures of equilibrium of a uniformly rotating liquid, investigated their stability, and made a significant contribution to the potential theory.

M V Ostrogradsky (1801–1861) continued A L Cauchy's and S D Poisson's investigation of wave propagation in a cylindrical basin.

The name Yu V Sokhotsky (1842–1927), a professor at St. Petersburg University, is related to the theory of boundary-value problems, which underlies the theory of piecewise potential flows. It would be difficult to name the author who first took advantage of the Cauchy-type integral. However, it is valid to say that Sokhotsky was the first to investigate it. In his Doctoral Thesis (1873) he proved the theorem on the limiting values of a Cauchy-type integral [44]. Subsequently, his findings were undeservedly forgotten, only to be derived anew by a Yugoslavian mathematician I Plemelj in 1908. The Sokhotsky theorem is frequently and unfairly referred to as the Sokhotsky–Plemelj theorem or even as the Plemelj theorem.

Boris Aleksandrovich Bakhmet'ev (1880–1951) busied himself with hydrodynamics in the XXth century. He was born in Tiflis. Upon receiving his education in Petersburg and Zurich he became a civil engineer. In 1912 Bakhmet'ev published a monograph concerned with liquid flow in an open channel. His brilliant education and breeding allowed him to become the ambassador to the USA under the Russian Provisional Government. After the October Revolution he emigrated to America and became proprietor of a large match factory. Having become a capitalist, he would return to hydrodynamics studies and published original monographs on hydraulics and turbulent flow dynamics. He helped 'firstwave' Russian immigrants to settle down in New York.

In passing we note that Ivan (Vano) Il'ich Nikuradze (1894–1979), an outstanding hydrorodynamicist and a pupil of Prandtl, was also born in Georgia.

Four scientific schools have left an indelible mark on Russian hydrodynamics: those of Zhukovsky, Friedmann, Kolmogorov, and TsAGI [45, 46].

#### 3.2 Zhukovsky school

Early in the XXth century the theory of airfoils was enriched with two outstanding discoveries: the condition of velocity finiteness at the trailing edge of an airfoil and the formula which establishes a direct proportionality between velocity circulation and the lifting force acting on the wing. Participants in both of these discoveries were N E Zhukovsky and well-known German mathematician M W Kutta (1867– 1944), a privat-docent of the Higher Technical School in the German town of Stuttgart.

The name of S A Chaplygin, Zhukovsky's pupil, came to be related to these discoveries of the 1940s, when a drive was launched in our country to combat so-called cosmopolitanism — an attempt to prove that 'Russia is the origin of elephants.' It was proposed, for instance, to refer to the Mach number<sup>11</sup> as the Maĭevskiĭ number and to refer to the finite velocity condition at a wedge-shaped edge as the Chaplygin condition or even as the Chaplygin–Zhukovsky–Kutta condition. In fact Chaplygin merely generalized the results



Nikolaĭ Egorovich Zhukovsky

obtained by Zhukovsky and Kutta. Kutta was the first to point to the velocity finiteness condition, but his result referred to the specific case of a flow past an airfoil. Kutta was the first to obtain the formula for the lifting force, but again in the special case of an airfoil and not in the elegant form which was subsequently determined by Zhukovsky: the lifting force is equal to the product of the velocity of airfoil motion, the circulation, and the air density.

The nomenclature is hard to change; it is only in the hands of authors of good textbooks. But statistics shows that the velocity finiteness condition is nowadays termed the Kutta condition and the theorem of the lifting force the Zhukovsky theorem. A compromise solution is beginning to emerge.

The Zhukovsky formula proved to be universal: not only did it define the lifting force experienced by an airfoil, but it also explained the motion of a rotating cylinder in a flow the so-called Magnus effect. This formula alone would have been sufficient for entering Zhukovsky's name forever in the annals of hydrodynamics.

Zhukovsky regarded H Helmholtz as his teacher. "Modern aerodynamics," he said, "owes its origin to Helmholtz."

Among Zhukovsky's pupils, an indelible mark in hydrodynamics was left by D P Ryabushinsky, S A Chaplygin, and V P Vetchinkin. In 1902 Zhukovsky constructed a suctiontype closed-jet wind tunnel. Its length was 7 m. By that time it had become clear that improving experimental accuracy necessitated large tunnels, expensive equipment, and hence significant material expenditures. The University was short of money. A helping hand was lent by a rich Russian businessman, Dmitriĭ Pavlovich Ryabushinsky (1882–1962), who in

<sup>&</sup>lt;sup>11</sup> The Mach number was used even by Euler. It was brought into use not by Mach himself, but by J Ackeret. By the way, the term 'Reynolds number' was introduced not by Reynolds himself, but by A Sommerfeld many years after Reynolds' investigations.



Dmitriĭ Pavlovich Ryabushinsky

1904 established an aerodynamic institute in Kuchino, the family estate situated near Moscow. Ryabushinsky was one of the younger children of the proprietor of the biggest Russian commercial and industrial firm, 'P M Ryabushinsky and Sons Co.' He graduated successfully from the Practical Academy of Commercial Sciences in Moscow, where the lecturer in mechanics was the 'father of Russian hydrodynamics' Zhukovsky. Ryabushinsky chose science as his life work, graduating with distinction at the age of thirty from his second higher educational institution — the Physicomathematical Department of Moscow University — staying to work there at Zhukovsky's Chair. His first scientific paper was concerned with the theory of helicopter flight.

Russia's first aerodynamic institute in Kuchino was the precursor of TsAGI. Experiments were carried out at the highest level of engineering possible at that time. For instance, advantage was taken of photograph — a novelty in world practice in those days. An excellent building was constructed in Kuchino and a large wind tunnel was assembled. As the scientific supervisor, Ryabushinsky invited Zhukovsky — his teacher. Measurement of the forces acting on a body embedded in an airflow was the focus of attention. I I Sikorsky, the 'father of Russian aviation,' spoke of the Kuchino laboratory as 'the renowned nest of aviation science.'

Before long, relations between teacher and pupil deteriorated. This is how M Arlazorov, author of the book *Zhukovsky* [Zhizn' Zamechatel'nykh Lyudei (Lives of Outstanding People series), Moscow, 1959] describes the quarrel between Zhukovsky and Ryabushinsky, the 'inveterate



Sergeĭ Alekseevich Chaplygin

financial wolf,' an 'operator of a wide-ranging enterprise': "A year had not yet passed when Ryabushinsky transformed from a suppliant and an admirer into a sovereign. He was jealous of Zhukovsky's fame and tried to lay down his own conditions." Although it would be illegitimate to judge the past from the present-day standpoint, it is pertinent to note that Zhukovsky showed excessive adherence to principles. He was a difficult-natured man. And only the weak are never jealous... After the 1917 revolution, Ryabushinsky was arrested, and his institute was confiscated. It was only due to his friends' aid that he was able to escape from prison and flee abroad <sup>12</sup>. A committee consisting of Zhukovsky, Chaplygin, et al., pronounced that the institute was fit for conducting aerophysics research. A prototype for TsAGI, the Kuchino Institute existed up to 1921, after which it was transferred to geophysicists.

Ryabushinsky emigrated to France to become the head of the aerodynamic laboratory at the Sorbonne.

An outstanding Russian scientist and engineer and a Corresponding Member of the French Academy, Ryabushinsky combined the outstanding talents of a scientist, an engineer, and even a writer. He is known not only for his imperishable work in the field of hydrodynamics, but also for the invention of a thermal wire anemometer, which is indispensable in aerohydrodynamic experiments up to the present time. In 1916 he proposed an original weapon — a light portable weapon for aiming and launching armor-

<sup>&</sup>lt;sup>12</sup> Attempts to free his sisters did not meet with success — they perished in the concentration camp located in the Solovetskie isles.



Vladimir Petrovich Vetchinkin

piercing rockets, which was subsequently termed a bazooka and would try to promote it without success. Only a quarter of a century later was the bazooka added to the armory of the USA and other countries.

Ryabushinsky made a fundamental contribution to dimensional theory: the proof, which he derived in 1911 and which was subsequently generalized by an English scientist, E Buckingham, and given the name ' $\pi$  theorem', entered all textbooks of continuum mechanics. The jet flow scheme proposed by Ryabushinsky (1919) bears his name and is familiar to every hydrodynamicist. He made a significant contribution to wave theory and cavity flow research. Although the two hundred papers written by Ryabushinsky are hard to study, even nowadays they attract the attention of researchers.

In recognition of his scientific merits, Ryabushinsky was elected a Corresponding Member of the French Academy of Sciences. The memory of a great Russian scientist has been perpetuated — his name has come back to us after a lingering oblivion to be entered in the Golden Book of Russian emigration [48].

Stephan Drzewieski (1844–1938), an engineer and scientist of Polish descent, was another famous pupil of Zhukovsky. He also emigrated to France, where he worked with Eiffel. His contribution to aerodynamic propeller theory is quite significant.

Among the scientific papers of yet another pupil of Zhukovsky, there is one which occupies an exceptional place. This is the doctoral thesis of Chaplygin, published in 1902 [49]. Shock waves had been discovered by that time, but gas dynamics was not yet recognized as a science. Chaplygin thesis marked its inception. Such notions as 'the Chaplygin gas' and 'the Chaplygin equation' were brought into common scientific practice.

The works on propeller theory and aircraft strength design performed by V P Vetchinkin (1888–1950) — one of the originators of the applied science concerned with aircraft flight dynamics are widely known.

Another pupil of Zhukovsky, A N Tupolev, came to be an outstanding Russian aircraft designer.

#### 3.3 Friedmann School

The Zhukovsky School was concerned with wave theory, the impact of a body against a free surface, transient ideal liquid flows, the underwater-wing theory, and gliding. L G Loit-syansky recalled [50] that Zhukovsky and his pupils would stay aloof from new lines of research — from investigating viscous liquid dynamics.

Quite different principles underlay the activities of the Friedmann school. Unlike the Zhukovsky School, wherein national aviation sciences were constructed, Friedmann's pupils approached hydrodynamics, not from the side of aviation applications, but from the geophysics side. A A Friedmann astonishes by the breadth of his interests. His expanding universe model was initially turned down then later supported by Einstein. Friedmann's second passion was dynamic meteorology, of which he was the founder. The theorems and equations he derived acquired fundamental significance for weather forecasting. In 1920 Friedmann



Aleksandr Aleksandrovich Friedmann

started working at the Main Physical Observatory of St. Petersburg. Hydrodynamics was Friedmann's third passion. His interest lay in the most basic hydrodynamic problem — turbulence. Together with L V Keller he derived in a rigorous mathematical way the infinite system of equations for momenta [51].

After Friedmann' death, his pupil N E Kochin became the head of the St. Petersburg Hydrodynamic School. He pursued the modeling of cyclones (1923), investigations of discontinuity surfaces (1924), the wave motion of a heavy liquid, and vortex stability. Kochin derived an exact solution to the linear problem of the flow past a circular airfoil parallel to its plane (1940) independently of the well-known German hydrodynamicist K Krienes, who found the solution to this problem in a more general case — for an elliptic airfoil. Several generations of hydrodynamicists in Russia were raised with Kochin's textbooks. His removal to Moscow and work at TsAGI marked the continuity of the hydrodynamic schools of Friedmann and TsAGI.

L G Loitsyansky, another of Friedmann's pupils, was engaged in the development of laminar flow theory. In turbulence theory he discovered an invariant (1939) conserved in the course of degeneration of uniform and isotropic turbulence. True, it was discovered later that this result applies only to a hypothetical situation [52].

#### 3.4 Kolmogorov school

Basic discoveries in many fields of mathematics are due to A N Kolmogorov (1903–1987); his role in the history of hydrodynamics is unique, for he is the central figure among turbulence theory development scientists. Experts are well aware of Kolmogorov's stature, Kolmogorov's hypotheses, the Kolmogorov–Landau debate, and the Kolmogorov– Arnol'd–Moser theory in nonlinear dynamics.

Kolmogorov spoke humorously in 1983 that one of his pupils controlled the atmosphere and another the oceans. He meant Academician A M Obukhov, the Director of the Institute of Atmospheric Physics of the USSR Academy of Sciences, and Corresponding Member A S Monin, the Director of the Institute of Oceanology of the USSR Academy of Sciences [53]. The Institute of Atmospheric Physics was established on the basis of the turbulence research laboratory previously supervised by Kolmogorov.

The decomposition of the velocity pulsation spectrum into low- and high-frequency parts is referred to as the Obukhov expansion. Obukhov derived a law, which is referred to as the Kolmogorov–Obukhov law, independently of Kolmogorov and starting from different considerations. He also came up with a logarithmically normal structure of fluctuations.

Monin generalized the von Karman-Howarth equation to the case of anisotropic turbulence. This equation now is referred to as the von Karman-Howarth-Monin equation.

When developing the Kolmogorov theory, A M Yaglom considered the general class of random cascade models.

M D Millionshchikov advanced the hypothesis that the fourth-order semi-invariant is nullified. Seven years later W Heisenberg came up with the same idea. 'The minus fivefourths law', or the 'Millionshchikov law' defines the damping of velocity pulsations with time.

V I Arnol'd investigated the stability of stationary ideal liquid flows; with the aid of the Lie group theory he found that such a flow is an infinite-dimensional analog of the rotation of a solid.



Andreĭ Nikolaevich Kolmogorov

Inherent in the Kolmogorov school was a deep comprehension of the problem and the mathematical rigor of its solution. Science historians believe that the Russian School of Mathematical Science headed by Kolmogorov occupied the leading position in the world in the 1930s – 1940s. There came into effect the 'blackboard' rule: Russian scientists meets with success where no instruments more complicated than chalk and blackboard are required <sup>13</sup>.

#### 3.5 ... and other schools

After the October Revolution of 1917, the Bolsheviks dispersed the Parliament (Constituent Assembly), suppressed the activity of democratic parties, and proclaimed the 'red terror' on September 5, 1918. The Bolsheviks made a start on the construction of concentration camps for differently minded people and introduced the system of

<sup>&</sup>lt;sup>13</sup> The reason lies in education. Even F M Dostoevsky was aware of it: "Where education commenced with engineering (in Russia the reforms of Peter the Great), there never emerged Aristotles. Quite the reverse, an extraordinary narrowing and scarcity of ideas were observed. Where a start was made with Aristotle (the Renaissance, 15th century), there immediately occurred great technical discoveries (book printing, gunpowder)  $\langle ... \rangle$  and the broadening of the human mind (the discovery of America, the Reformation, astronomical discoveries, etc.)."

taking hostages, to which tens of thousands of people fell prey — primarily men of science and cultural workers. The implementation of one of the biggest repressive actions of the totalitarian regime began.

With an atmosphere of fear and terror setting in, Russian intellectuals, including scientists, engineers, and professors, began to leave Russia. They rushed to the frontier, forming the so-called *first emigration wave*, which peaked in 1920–1925 and amounted to a total of 1,160,000 people, according to official data. Some refugees made their way via Odessa, others via China.

On the one hand, emigration from Russia aided in preserving and developing its art, science, and technology. On the other hand, the synthesis of Euro-american and Russian cultures favored the progress of western civilization. Some of the immigrants remained in Europe hoping for the forthcoming end of Bolshevism; others, possessing selfreliance and confident in their ability to survive in the unusual conditions of cut-throat competition, rushed to America. It was precisely there that a pleiad of brilliant figures of Russian aviation science and technology settled down [48].

Next to the Sikorsky firm, which came to be a center for Russian emigrant engineers, the most prominent in the West was the aviation firm of A N Prokof'ev–Seversky, a former captain of the Baltic Fleet, holder of the Order of St. George, and the inventor of an automatic bomb-sight and a device for in-flight aircraft refueling. His successor, A M Kartveli, a former Russian artillery officer, constructed 'Thunderbolt' one of the best interceptors of the Second World War, which was delivered to many countries, including Russia, in the context of military aid.

Heavy aircraft were produced by the firm of M Strukov, a former captain of the tsarist army. The founder of flight dynamics G A Botezat, a former professor at St. Petersburg University, produced a helicopter which was first added to the armory of the USA. A Russian engineer, I Makhonin, produced planes with variable-geometry wings in France. Fully unveiled abroad were the talents of Zhukovsky's pupils: the helicopter designer N Florin, the engineer M Vatter, and TsAGI founders I A Rubinskiĭ and G I Luk'yanov. A Kolchak officer Aleksandr Aleksandrovich Nikol'skiĭ became an unrivaled expert on the theory of rotary-wing aircraft, the founder of helicopter education in the USA.

Zhukovsky did not go abroad: he was in his seventies. Moreover, he had his life-work — TsAGI, the dream of reviving Russian aviation.

As the Stalin dictatorship consolidated, the cases of deportation abroad became scarce — this butcher preferred that the intellectuals be shot or exhausted in the Gulag, and even that barges with prisoners be sunk at sea <sup>14</sup>.

In 1937–1938 many scientists found themselves in the Gulag and later in Beriya's 'sharashkas'. At TsAGI, for instance, they shot N M Kharlamov, the head of TsAGI, V I Chekalov, the head of the 8th Department, and E M Furmanov, Deputy Head of the Personnel Training Department. There was a dearth of experts. It came to the point where the Scientific Council of TsAGI was headed by a Baltic Fleet sailor, Ivan Petrov, who was said to have stormed

the Winter Palace in 1917. Subordinate to him were. S A Chaplygin and other outstanding scientists of the Institute. V O Klyuchevsky said: "History does not teach anybody anything. History punishes us for unlearned lessons."

Against the background of these events, our successes in mathematics, physics, aircraft building, and astronautics are particularly astonishing!

The practice of deporting differently minded people was restored during Brezhnev's reign, when over 170 people were deprived of citizenship between 1966 and 1988.

The second emigration wave of scientists rose after the revolution of 1991. In the intervening period between the two waves, only individual scientists found their way through the iron curtain to the West. G A Gamov, a theoretical physicist and Friedmann's pupil, remained there after an official journey in 1933. V G Levich, one of the founders of physicochemical hydrodynamics and Landau's pupil, found himself there as a prisoner of conscience in 1978. E A Novikov, Obukhov's pupil, remained in Japan after a conference on hydrodynamics held there in 1982, later moving to the USA. Already by the end of 1991 Israel had accepted more than six thousand scientists from Russia [54].

The second emigration wave involved almost all wellknown Russian hydrodynamicists: M A Gol'dshtik, M G Goman, A L Gonor, G M Zaslavskiĭ, V R Kuznetsov, A A Praskovskiĭ, A I Ruban, O S Ryzhov, V A Sabel'nikov, S N Timoshin, V N Trigub, S I Chernyshenko, and V N Shtern. This emigration took place under conditions of the free transfer of minds. In connection with the fall of the 'iron curtain', the notion of emigration completely lost its former significance. The means of information are such that having residence in any part of the world ensures access to open knowledge, which defines basic science.

Russian science has grown old: the average age of a scientist engaged in the military industrial complex has approached the pensionable age<sup>15</sup>. This notwithstanding, aviation is bound to develop in Russia, since we are endowed with one seventh of the entire sky; as is shipbuilding, since we face two oceans; as is automobile construction, since we are situated on two continents... and not by borrowing, but on the basis of our own applied science and industry!

An inestimable contribution in hydrodynamics was made by mathematicians (GIMarchuk, VPMaslov, NNYanenko) and physicists (of Gaponov-Grekhov's school in Nizhniĭ Novgorod, V O Zakharov, Ya B Zel'dovich, P L Kapitsa, L D Landau).

<sup>&</sup>lt;sup>14</sup> Such a case was described by A I Solzhenitsyn. However, it was not an isolated incident. This was a system which deserves further investigation. My grandfather Matvei Fedorovich Betyaev was sunk near the Solovetskie isles; the steamship which was carrying from imprisonment the uncle of my spouse, who had served his sentence, was blown up in full of numerous witnesses at the exit of Nagaevo Bay.

<sup>&</sup>lt;sup>15</sup> V I Arnol'd [55] believes that the creativity of a scientist peaks at the age of 27. N Bourbaki fires members of his team who have reached the age of 50. Naturally, every scientist has a critical age of his own, when his creative activity attains a maximum. Hence, there also exists an age of peak activity averaged over the entire scientific community. In reality the situation is more complex: there are several 'singular points' in scientific activity. The phase of creative activity comes to a close approximately after the age of thirty, the phase of comprehension of *radically new ideas* after forty, the phase of creative lecturing after fifty, and the phase of ideological leadership of a laboratory after sixty. And, however regrettable it may sound, there comes a phase when a human begins to impede the introduction of new ideas.

It is assumed [56] that the average lifetime of scientific monographs amounts to 25 years and that of scientific papers to five years. However, outstanding monographs and papers live forever. Ancient wisdom states: *"Habet sua fata libelis in capitia lectores"* ("books have their fate in readers' heads").

Moscow University was devoid of a united school of hydrodynamics. However, outstanding scientists have worked there.

L I Sedov (1907–1999), Chaplygin's pupil, initially worked at TsAGI and subsequently headed the Hydromechanics Chair at Moscow University. He developed the theory of self-similar flows, derived the closed-form solution of the problem of a strong point-like explosion in an ideal gas, and made a contribution the axiomatics of continuum mechanics and to relativistic mechanics.

A A II'yushin (1911–1998) headed the Chair of Elasticity of the Department of Mechanics and Mathematics and took part in projects aimed at developing Soviet atomic and hydrogen bombs. Independently of Tsien (1946) he established the law of plane sections in the hypersonic flow theory. In the annotation to his paper published in 1956 it is pointed out that the paper was classified and published in a limited number of copies in 1948; the delay in publication is due to secrecy. II'yushin's textbook of continuum mechanics ran into three editions and has retained relevance up to the present time.

## 4. TsAGI school

The TsAGI School came to be a natural continuation of Zhukovsky's school. TsAGI was assigned the chief role in the creation of Soviet aviation (up-to-date at that time), and this mission was adequately accomplished. There existed no single leader, and so independent subschools evolved. No new USSR aircraft could take off without TsAGI 'approval'.

By the end of the XXth century the attitude toward engineering experts somewhat changed. In the scientific institutes of the Russian military industrial complex, the position of a scientist came to be reputed as being more prestigious than that of an engineer. Even early in the XXth century, both A N Krylov and S P Timoshenko took pride in bearing the high title of engineer! For the words 'engineer' and 'genius' have a common root.

Of course, it is impossible to establish a strict line of demarcation between the notions of an engineer-researcher and a scientist. A scientist deals with scientific problems and an engineer with technical ones. A scientist writes papers and an engineer writes reports. A scientist has a citation index <sup>16</sup> and an engineer does noes not. At the same time, an engineer's activities are different from those of a designer or an inventor. A designer brings into existence the appearance of a product, its technical documentation. An inventor suggests new solutions to technical problems and has inventor's certificates or patents.

Additional confusion is brought about by the fact that engineers may be of two types. In aviation, a research engineer is engaged in aerodynamic experiments and a production engineer in aircraft construction.

Here, we are dealing with research engineers. The dividing line between the spheres of activity of a research engineer and clearly demarcated an applied scientist is not, it is blurred. N E Zhukovsky and his pupil D P Ryabushinsky, while being outstanding scientists and engineers, mastered both theoretical and experimental techniques. V N Chelomei, a Designerin-Chief, was a notable scientist [57, 58]. While these examples are exceptions, combining the work of an engineer and an inventor is a rule.

## 4.1 B N Yur'ev

Boris Nikolaevich Yur'ev (1889–1957) was born into the family of an artillery officer in Smolensk. From his father, in whose study there stood a work bench, he inherited his love for working with his hands and for various handicrafts. He graduated from the Second Moscow Military School with a commendation for good conduct and progress. Like Zhu-kovsky, his teacher, he lectured in the Moscow Higher Technical School (MVTU), in which he established an aerodynamic laboratory [59].

In 1914 he participated in military operations at the front, was taken prisoner, and returned to Russia in 1918. Yur'ev took part in the establishment of TsAGI and constructed an original wind tunnel there in 1923. During the Second World War Yur'ev established an aerodynamic laboratory in Ekaterinburg, which comprised four wind tunnels.

Yur'ev was granted many patents. Up to the present time use is made of an *automatic warp device* — a mechanism for controlling a helicopter — which he invented. He created the first Soviet helicopter which set the world record in altitude.

Yur'ev was the Editor of the first volume of *Rukovodstvo* dlya Konstruktorov (Designer's Handbook) (1940), which



Boris Nikolaevich Yur'ev

<sup>&</sup>lt;sup>16</sup> I have made approximate calculations of the *fractional* citation indices of several Russian scientists: N E Zhukovsky -98.5, S A Chaplygin -27, A A Friedmann -32, N E Kochin -15, M V Keldysh -21, S A Khristianovich -11.2, F I Frankl' -36.3, V V Struminskiĭ -15.4, A A Nikol'skiĭ -26.8. The fractional index accounts for the presence of coauthors: each paper is assessed as 1/N points, where N is the number of coauthors. Included are the papers published in *reviewed* journals only.

became an indispensable manual on the desks of Russian aircraft designers. However, the prime object of Yur'ev's research was the helicopter. At present, the so-called Sabinin – Yur'ev pulsed rotor theory is obsolete, since it has been supplanted by numerical experiments. However, Yur'ev's works on the history of aerodynamics are of interest even today, despite the fact that for certain reasons they do not reflect the activities of first-wave emigrants.

For his work in the fields of science and engineering and his activity in personnel training, Yur'ev was awarded two State Prizes and decorated with two Orders of Lenin, the First-Degree Order of the Patriotic War, the Order of the Red Star, and other medals. He was given the rank of lieutenantgeneral of engineering-aviation service.

B N Yur'ev was simultaneously a scientist and an engineer, but the latter activity prevailed in his biography. Among Yur'ev's works [60], his investigations in the field of experimental aerodynamics [61] are still in demand.

## 4.2 A K Martynov

Appolinarii Konstantinovich Martynov (1901–1991), Yur'ev's pupil, was born into a gentry family. His father worked as a physician. A K Martynov could fluently speak French and German. He began his work life as a railroad metalworker in 1918. In 1920 he served as a Red Army man in the armed forces in the Third Moscow Regimental District. That same year he entered the Moscow Higher Technical School, being taken onto the staff of TsAGI as an engineer upon graduation in 1925. From 1939 to 1941 Martynov headed the Second (the most important one!) Division of TsAGI, which was concerned with aircraft aerodynamics, and in 1950–1972 headed the Helicopter Division [62].

Martynov was the Editor of Spravochnik dlya Konstruktorov (Designer's Reference Book) and of Rukovodstvo dlya Konstruktorov (Designer's Handbook). Moreover, he was a member of the editorial board of the Tekhnika Vozdushnogo Flota (Air Fleet Engineering) journal. Martynov was the supervisor of post-graduate study at TsAGI. His primary invention was a six-component aerodynamic balance for propeller tests, which is employed in wind tunnel experiments to the present day.

Being a professor, Martynov lectured at the Moscow Higher Technical School and the Moscow Institute of Aviation. However, since he was not a member of the Communist Party, his fate was particularly vulnerable to the action of sneaky informers and libellous anonymous letters. He retired in 1989 holding the rank of leading (and not principal!) researcher.

Martynov left a significant legacy: among the four books he wrote, a textbook for higher technical schools, which was translated into Chinese, English, and Romanian, still remains a relevant and unparalleled example of educational literature [63].

#### 4.3 S A Khristianovich

Sergeĭ Alekseevich Khristianovich (1908–2001) worked at TsAGI from 1937 to 1953. He was my supervisor during my post-graduate study. This is how he recalled his youth in those days.



Appolinarii Konstantinovich Martynov



Sergeĭ Alekseevich Khristianovich

Our family had to flee to the south. In Rostov-on-Don, all my nearest and dearest — mother, father, elder sister — died of typhus almost simultaneously. I was left quite alone and as a homeless child was running through the snow barefooted. I happened to have luck. Professor David Ivanovich Ilovaiskii lent me a helping hand. He made me give up trade and enter a nautical school. However, I did not complete my study there, for I exchanged letters with my aunt, who lived in Leningrad, and went to her place during my vacation. In Leningrad I fell seriously ill with spotted fever (typhus) and stayed in that city for many years. I graduated from Leningrad State University to join a very good collective body of the Institute of Hydrology. Then I came by invitation to the Academy of Sciences in Moscow, only to learn later that all my colleagues had been shot. I went on a geological expedition to Central Asia. I would not tell precisely what good it did, but I benefited greatly. I was able to observe how research, measurements, and design are conducted, and how hydrogeologists work. It was extremely interesting to visit those localities.

"Why did I leave TsAGI? Because something was wrong there. I did not feel needed. I was accustomed to being respected at TsAGI. At that time it was headed by A I Makarevskii—a gentle man who would not interfere with anything. They told me directly I was no longer needed— the wind tunnel was in service and all the necessary tests had been passed.

"Yet another annoying event fostered my resignation. I was accused of stealing a secret document — a memorandum on a projectile-plane capable of reaching as far as America. This was reported to Stalin. Having asked who wrote the memorandum, he concluded: 'Why should he steal it once he has written it'?"

'SAKh' — this is what his colleagues and subordinates called him — is among those researchers whose scientific and engineering activities are balanced, as in the case of Zhukovsky and many of his pupils. This is clearly attested to by the extraordinary fact that he simultaneously defended two theses in 1938: one on physicomathematics and the ot technical science. He is author and coauthor of five monographs [64].

The range of SAKh's engineering interests was extraordinarily wide. He was engaged in the gas flow in the Laval nozzle and in an ejector, and in the application of ejectors in gas-collecting circuits. He studied the hydraulic disruption of an oil-bearing stratum, the collapse of roofing in a store, the motion of subsoil waters, as well as the sudden ejection of coal and gas. Khristianovich produced high-speed wind tunnels: the subsonic T-106 wind tunnel constructed in 1943 is the seat of experiments pursued to the present day. In the T-112 wind tunnel, constructed in 1946, advantage was taken of perforation for the first time and a transonic transition was accomplished.

## 4.4 F I Frankl'

The name of the outstanding hydrodynamicist Feliks Isidorovich Frankl' has wrongly been partially forgotten, even though experts are aware how high his *citation index* is. He luckily escaped repression both from the fascists and the communists.

4.4.1 Life. Frankl' (1905-1961) was born to a rich Jewish family in Austria, and in 1927 graduated from the Department of Mathematics of Vienna University. Initially he was a confirmed communist and participated actively in the international working-class movement, and joined the Austrian Communist Party in 1928. It is not known whether Frankl' was escaping from the rising fascism, but in 1929 (Hitler came to power in 1933) he emigrated to the USSR to start work as a research worker in the Communist Academy attached to the Central Executive Committee of the USSR. Before long, Frankl' realized he had found his way to the wrong place and moved to work at TsAGI. There he became a member of the Communist Party of the Soviet Union. The main period of his scientific activity is related to TsAGI. Among his coauthors were such outstanding scientists as S A Khristianovich, I A Kibel', and M V Keldysh.

In 1944 Frankl' transferred to work at the Dzerzhinskiĭ Artillery Academy, where he was engaged in gas dynamics as before: the supersonic flow past elongated bodies of revolution, flow in nozzles, and supersonic jet efflux.

It is likely that Frankl' moved to the city of Frunze (now Bishkek) to escape the persecution of Jews in 1951, where he headed a chair at Kirghiz State University.



Feliks Isidorovich Frankl'

In 1959 Frankl' was already working at Kabardian-Balkar State University (the town of Nal'chik). There, like in Kirghizia, he engaged in vigorous pedagogical activity: dozens of Ph.D theses were upheld under his supervision.

Possessing a phenomenally retentive memory, Frankl' spoke many European languages. He was quick to learn Russian and could recite by heart "Slovo o Polku Igoreve" (The Song of Igor's Campaign). He could easily read Euler's original papers, which were published in Latin in those days, and was a connoisseur of his works.

Frankl' died in the prime of his life; his death was an irreplaceable loss to science throughout the world.

**4.4.2 Time.** Upon accession to power, the fascists pursued a genocidal policy towards Jews and other 'non-Aryan' nations. Science itself came to be Aryan<sup>17</sup>. Only Germans could officially be treated as coryphaei: Hilbert in mathematics and Planck in physics. The division of scientists into pure (Aryan) and impure (non-Aryan) was inculcated into the consciousness of Germans not only by fuhrers, but by Nazi scientists as well. The mathematician Biberbach, for instance, wrote that the witty findings of Lagrange, a *non-Aryan*, are a disgrace to mathematics and are due to the structure of his aquiline nose. As for Weierstrass's works, they are lofty science, because he is an Aryan with a straight nose.

Understanding that scientists throughout the world hung on every word of the great Hilbert, the fascists suggested that he should make a report entitled "National socialism and mathematics" at the public session of the Berlin Academy of Sciences. Hilbert's report was ultimately brief: "They say that national socialism and mathematics are inimical to each other. This is nonsense: they simply have nothing in common."

While abroad, it was hard to realize that socialism in the USSR was a savage dictatorship. Many eminent foreign figures were unable to perceive the murderous nature of socialism behind its facade. Frankl' was among them at the time he selected the USSR as his second homeland.

The persecution of Jews in the USSR was launched in the 1930s, when Stalin decided to obliterate Bolsheviks — the socalled 'Lenin guard', which consisted primarily of Jews. The second anti-Semitic wave was generated in the late 1940s, under the guise of the struggle against *bourgeois cosmopolitanism*. Frankl' was its involuntary eyewitness.

**4.4.3 Creative work.** How do scientific papers go out of date? Small details sink into oblivion. There persist ideas, paradigms, and the *spirit of time*.

Frankl', like Landau and others of their outstanding contemporaries, though, could not be familiar with those perturbation theory techniques which were developed during the second half of the past century and which now largely determine the face of hydrodynamics. Nor did they wield computational techniques, for computers were unavailable at that time. In this connection the papers of the old masters should be read selectively, omitting what has not stood the test of time. Unclaimed are Frankl's papers on general topology and quantum dynamics.

His investigations are distinguished by mathematical rigor, and he sometimes formulated a seemingly applied problem in the 'theorematic language' [65]. However, in his works on boundary layer, advantage was taken of techniques which are now recognized as obsolete: the so-called Pohlhausen method in the theory of laminar boundary layers and the semi-empirical methods in the theory of turbulent boundary layers. Nevertheless, in the TsAGI school, where neither Zhukovsky nor Chaplygin pursued the Prandtl theory, his investigations were innovative.

**Gas dynamics.** The gas dynamics boom which commenced in the 1930s was not simply a tribute to fashion. The production of high-speed aircraft and high-velocity wind tunnels was already on the agenda. Gas dynamics became the most important line of Frankl's work at TsAGI. A monograph on this subject published in collaboration with S A Khristianovich and R N Alekseeva played a significant part in the dissemination of the knowledge of gas dynamic [66].

Frankl' investigated the existence and uniqueness of problems for the solution of nonlinear gas dynamic equations as applied to the flow past an airfoil or an axially symmetric body. He extended the Zhukovsky theorem of the lifting force of an airfoil to the case of compressible gas. Such methods of mathematical physics as the method of characteristics, the method of distributed sources, potential theory, and expansion into series were used validly in contemporary gas dynamics. Taking advantage of these methods, Frankl' derived the solution of the problem of supersonic flow past a revolving sharpened figure and the problem of an incompressible liquid flow past bodies (with or without a channel) close in shape to axially symmetric ones [67].

Frankl' managed to linearize the complex problem of the motion of propeller blades at high translational and rotational velocities. He showed the *Froude – Finsterwalder theorem* to be invalid in this case.

**Transonic flows** <sup>18</sup>. K Guderley, von Karman, and Frankl' are recognized as the founders of transonic flow theory — the pearl of gas dynamics [68]. The works of Frankl' in this field are dedicated to direct and inverse problems of transonic profile flow, direct and inverse problems of the flow in the Laval nozzle throat, and the flow structure in the vicinity of the end of a *standing* compression shock located inside the flow region.

The transonic flow equation

$$\left[k - (\gamma + 1)\varphi_x\right]\varphi_{xx} + \varphi_{yy} = 0\,,$$

where  $\varphi$  is the velocity potential,  $\gamma$  is the specific heat ratio, and k is the transonic similarity parameter, is referred to as the Karman–Guderley equation, though Frankl' was the first to come up with the independent derivation of it.

Taking advantage of the hodograph technique, he constructed (1945) Laval nozzles which exhibit a shock-free flow.

Frankl' was the first (1947) to call attention to the possible mathematical incorrectness of the problem of stationary flow around a body with a shock-free local supersonic region on the basis of the uniqueness theorem for the solution of the *generalized Tricomi problem* for Chaplygin-type equations. These assumptions, the so-called *Frankl' arguments*, underlie the proof of the theorem of the correctness of this solution

<sup>&</sup>lt;sup>17</sup> The theoretical sciences were regarded as 'Jewish': Marxism, theoretical physics, etc. Experiment was considered as the main physical toolkit.

<sup>&</sup>lt;sup>18</sup> The term 'transonic' was invented by Professors von Karman and H L Dryden. The English spelling 'transonic', with one 's', is grammatically wrong. However, Karman believed that grammatical rules do not apply to hydrodynamics.

performed subsequently by Cathleen Morawetz (1953) in the USA.

Frankl' and Guderley proved independently the existence and uniqueness of the solution of the far-field problem for a transonic flow (1947). Along the way they had to wittily 'bypass' a nontrivial singularity of the Tricomi equation in the canonical elliptic form.

In the post-war works of Frankl' and his followers, the *Tricomi problem* of a maximum-discharge jet flow from a nozzle received an exhaustive solution with the aid of the Fourier technique. The *generalized Tricomi problem* was subsequently solved numerically.

Taking advantage of the hodograph technique, Frankl' reduced the inverse problem of the flow around some theretofore unknown profiles in the presence of a local supersonic region closed by a normal (1956) or oblique (1957) compression shock to the *generalized Tricomi problem*. These problems were termed the *Frankl' shock problems*. At present, their numerical solution has been derived, i.e., the streamlined profile shape has been found.

**Mathematical models of natural processes.** The mathematical model of a phenomenon is constructed on the basis of a physical model and intuition. Frankl' was an unrivalled 'modeler' — he constructed models of the following:

(1) a side water intake from fast rivers,

- (2) an oblique hydraulic jump,
- (3) supercritical flow in a chute,
- (4) liquid flow with sediments in suspension,
- (5) sand waves,
- (6) bora,
- (7) the motion of a cold air layer over rugged terrain,
- (8) planning of artificially irrigated fields.

**Relativistic gas dynamics.** Being a magnificent mathematician, Frankl' gained an excellent understanding of subtle issues of the general relativity theory. Several of his papers are concerned with the initial value problem. His discussion with V A Fock exhibited the complexity of this problem, which has no solution in the general form. Only in the special case, with the introduction of so-called *harmonic coordinates*, can this problem be solved correctly.

In Bishkek, Frankl' founded a scientific school of relativistic gas dynamics. In the framework of the general relativity theory he derived the equations of motion and energy conservation in Riemannian space, in Riemannian space, and that vortices and that vortices neither emerge nor vanish in the absence of friction, as in classical gas dynamics. He generalized the Kelvin theorem to the case of relativistic flow.

Frankl' investigated potential relativistic flows. He revealed the analogy between the flow of an ultrarelativistic or photon gas and a conventional ideal gas in the case when the heat capacity ratio is equal to two. He calculated the thrust of a photon rocket engine with the inclusion of the absorption and emission of light.

## 4.5 V V Struminskii

Vladimir Vasil'evich Struminskiĭ (1914–1998) was born in Orenburg into the family of a teacher. Prior to entering the Physics Department of Moscow State University, he worked as a metalworker and later as a turner. In 1938 he graduated from the University with distinction and entered postgraduate courses [69]. In May of 1941, upon successful defense of his thesis, Struminskiĭ was sent to work in TsAGI (the town of Zhukovsky). At that time, the majority of engineers had an uncompromising and proud temper. Oh, this character! At first it favored him in his career — even in 1953 Struminskiĭ became the head of the Second Division and Deputy Chief of TsAGI. Subsequently his relations with the authorities deteriorated, and in 1962 he was transferred to a post as head of a department. At that time he engaged in vigorous activities in the USSR Academy of Sciences, and in 1966 accepted the post of Director of the Institute of Theoretical and Applied Mechanics established in the Siberian Division of the USSR Academy of Sciences, for that post ensured the conferment of the title of a Full Member of the Academy of Sciences. After five years of work in Siberia he was discharged from office and returned to Moscow to engage in the problems of chemical technology.

Struminskiĭ was a versatile scientist. Aeromechanics and power engineering, physics and chemistry, ecology and quantum mechanics, aviation and cosmonautics, technology and mechanical engineering, economics and philosophy these are only some of the fields in which he worked actively. But his main accomplishment was the development of the USSR's first swept-wing aircraft and overcoming the *sound barrier*. His objective was the design of an optimally shaped airfoil with the aid of a multiparameter wind-tunnel experiment.

Struminskiĭ was among ardent advocates of the introduction of swept wings. New ideas always have to struggle against old ones. The opponents of swept-wing application were trying to introduce diamond-shaped or triangular wings.



However, even in 1947, pilot swept-wing interceptors were constructed in the Lavochkin, Yakovlev, and Mikoyan Design Offices. The application of optimal wings in passenger and military aviation significantly improved the aerodynamic performance of Russian aircraft.

In the fifties there commenced a debate, which has not faded away, on the fate of TsAGI. What should the Institute be: an arsenal of wind tunnels or a 'silicon valley'? Struminskiĭ adhered to the former viewpoint and Khristianovich to the latter. At that time <sup>19</sup> there was no way of deciding between the two alternatives on the basis of accurate economic calculations. Volleys were exchanged for nothing...

Complexes of unique experimental facilities were produced under Struminskii's supervision: low-turbulence subsonic and supersonic wind tunnels (in the towns of Zhukovsky and Novosibirsk), supersonic and hypersonic intermittent wind tunnels, hypersonic pulsed facilities for high Reynolds numbers, vacuum and cryogenic wind tunnels.

In addition to the engineering activities, Struminskiĭ was engaged in science and has a rather high citation index.

#### 4.6 G L Grodzovskiĭ

Georgiĭ L'vovich Grodzovskiĭ (1923–1985) was notable for his exceptional breadth of interests. Here is a far from complete list of the objects of his activity: perforated walls of wind tunnels, magnetic suspension of models, the laser Doppler velocimeter, and the 'Yantar' plasma ion engine. Inspiring his like-minded colleagues with his ideas, he was able to unite large teams of scientists and engineers, remain a tireless romanticist, and get down to grandiose projects. The TsAGI authorities would humiliate him in every possible way, even making use of people of questionable repute to badger him. Dismissing him from a post as head of a division undermined his health once and for all.

In 1946, a start was made in the USSR on the investigation of supersonic gas flow within perforated boundaries, when a group of engineers under Khristianovich's supervision for the first time took advantage of perforated walls in a wind tunnel to accomplish a transonic transition. In 1949–1951, G L Grodzovskiĭ, A A Nikol'skiĭ, and G I Taganov performed a series of comprehensive investigations to study the effect of perforated boundaries on the structure and nature of a supersonic flow [70]. Grodzovskiĭ performed an experimental investigation of flow flattening in nozzles with perforated walls, the interaction of a plane-parallel flow with perforation, and the so-called *autosuction of a wind tunnel*.

Investigations of a new way of fixing models in wind tunnels — magnetic suspension — commenced independently in the French ONERA aerodynamics center and in the USSR in the 1940s. This method involves using a magnetic field in lieu of model-supporting devices. Grodzovskiĭ pioneered the employment of magnetic suspension in our country [71]. Work in this direction is being continued nowadays.

In the 1970s, the problem of rapid and accurate measurements of physical gas flow parameters became acute. The optimal and most progressive technique was the use of laser anemometry based on the Doppler effect, which enabled the measurement of airflow velocity with a laser beam reflected from a foreign particle seeded in the flow [72]. The advantage of a laser Doppler anemometer (LDA) is that it is an optical



Georgiĭ L'vovich Grodzovskiĭ

contactless technique, which does not perturb the flow. Implementing this idea, i.e., producing a high-frequency measuring complex for wind tunnels, called for the solution of several complex engineering problems in the field of aerodynamics, optics, electronics, and informatics [73]. For this purpose Grodzovskiĭ united and headed a team of engineers and scientists from different Russian scientific research institutes, educational institutions, and factories of those times. In 1981, a laser Doppler velocimeter was applied in the subsonic wind tunnel at TsAGI.

Grodzovskiĭ brought into existence a prototype of a space flight sustainer (cruise engine), thereby realizing the idea of K E Tsiolkovsky and F A Tsander of using the air in the upper layers of the atmosphere in an economical propulsor [74]. During the 1966–1971 period, a series of launches was made of the Grodzovskiĭ-designed ionospheric 'Yantar' laboratory, which was separable from the geophysical rocket head. The rocket thrust was equal to 1 g and the outlet jet velocity to 140 km s<sup>-1</sup>.

## 4.7 A A Nikol'skiĭ

Aleksandr Aleksandrovich Nikol'skiĭ, an outstanding Russian hydrodynamicist, is a full namesake of the abovementioned founder of helicopter education in the USA who emigrated there [75].

**4.7.1 Life.** Nikol'skiĭ was born on February 13, 1919 in the Nizhniĭ Baskunchak station in Astrakhan region into the

<sup>&</sup>lt;sup>19</sup> The question now is different. Will the military industrial complex, including the TsAGI, accommodate itself to the market conditions or remain a council of the elite and aged scientists?!

family of an orthodox priest. His surname is traditional for the clergy. When militant atheists came to power in 1917, his father started teaching mathematics and physics at school. Subsequently he was awarded the title of Honored Teacher of the RSFSR (Russian Soviet Federal Socialist Republic). His mother taught Russian and literature in the same school. His family gave Aleksandr a good upbringing and humanitarian education. His favorite poet was N Gumilev, forbidden and therefore little known in the Soviet time. Nikol'skii's favorite work was Gumilev's poem "Magic Violin" dedicated to Valeriĭ Bryusov:

He who takes it once in his imperative hands, the tranquil light of his eyes will be gone forever. Ghosts of Hades enjoy listening to these regal sounds, Violent wolves roam the path of violinists.

In 1936 Nikol'skiĭ entered the Department of Mechanics and Mathematics of Moscow State University. Initially he was listless in his studies and was keen on chess, Moscow, and literature. Then he began to study science seriously, became a Stalin scholarship holder, and graduated with distinction in two specialties simultaneously: mathematics and physics. In 1941 he was assigned to the Aircraft (Second) Division of TsAGI, where he was given an exemption from military service. During the war, the Institute was evacuated to Kazan' and Novosibirsk. Nikol'skiĭ found himself in Novosibirsk. It was precisely there that Chaplygin died in 1942, as if passing on the baton to a new generation of scientists.

In 1943 Nikol'skiĭ returned, together with TsAGI, to the Stakhanovo settlement (the town of Zhukovsky since 1947). In 1946, upon graduation from the TsAGI post-graduate courses and defense of his thesis, he obtained the scientific degree of Candidate of Physical and Mathematical Sciences. During the war and postwar times, scientific careers were quickly made. Furthermore, Nikol'skiĭ stood out from his colleagues because of his abilities and educational level. His chiefs and supervisors were Martynov and Khristianovich.

When reading Nikol'skii's works [76] one is convinced that he may be regarded as the successor of Chaplygin, Frankl', and, of foreign scientists, Busemann. This is precisely what he was called behind his back — Pusemann — with a hint at his paunch, which caught one's eye from a distance.

In 1949 Nikol'skiĭ became a Doctor of Physical and Mathematical Sciences and the Head of Department in the Second Division. That same year he started working simultaneously at the Institute of Mechanics of the USSR Academy of Sciences.

If the style of Nikol'skii's scientific leadership is to be characterized with a single word, 'freedom' immediately crosses one's mind. Freedom in everything, be it presence in the office or the choice of the subject of investigation. Freedom implies independence in making decisions, which distinguishes a real scientist from an imitation scientist. Among his learners, M D Ladyzhenskii [77] was considered to hold the greatest promise, but he died early. Both of them, the teacher and his pupil, exhibited a wonderful capacity for seeing and 'sensing' the properties of gas dynamic equations — the 'longest' and most complicated equations of theoretical physics.

There is one common feature in Nikol'skiĭ and I Newton. It is well known that Newton was not fond of delivering lectures. Students did not attend his lessons. Should two or



Aleksandr Aleksandrovich Nikol'skiĭ

three students come, Newton would unwillingly start lecturing, repeatedly hesitating and making mistakes. Nikol'skiĭ showed a similar lack of skill in lecturing. One might get the impression that he never spent a minute preparing for his lectures.

Naturally, tokens of encouragement of those times fell to his lot in full: certificates of good work, prizes, medals, decorations, and even ... penalties. Here is a curious extract from the personal file on Nikol'skiĭ.

1. A reprimand for absence from duty (1943).

2. Severe reprimand and warning for leaving classified material in his locked desk (1947).

3. A 1950 Stalin Prize.

Here, we see a combination of reprimands and awards. That is how the impetuous life of the young scientist commenced.

At that time scientists were held in respect: the prize money enabled Nikol'skiĭ to buy a 'Pobeda' car, and a cottage in the center of the town was allocated to his family.

During the war- and postwar years, the routine of secrecy was tough. Nikol'skiĭ published his first paper in the open press in 1944. *Sbornik Teoreticheskikh Statei po Aerodinamike* (A Collection of Theoretical Papers on Aerodynamics) [78], which saw the light of day in 1957, contained 26 papers, seven (!) of which belonged to Nikol'skiĭ. This monograph, which was often referred to as *A Collection of Nikol'skiĭ's Papers* in jest, has played a significant part in the development of gas dynamics. In 1960 Nikol'skiĭ filled the post of Director of the Institute of Mechanics of the USSR Academy of Sciences. At first he established a new publication — *Inzhenernyi Zhurnal* (Engineering Journal), became a member of the editorial boards of other scientific journals, and was engaged in intense scientific work: nine published papers in 1961, five in 1962, and five in 1963. However, the directorship did not turn into a sinecure for the forty five year old scientist. Being by nature and upbringing a person of high moral standards incapable of servility, flattery, and intrigue, Nikol'skiĭ was not a person suitable for leadership duties.

Initially Nikol'skii was denied conferment of the academic status of Corresponding Member of the USSR Academy of Sciences, and subsequently he was compelled to leave the post of Director 'of his own accord'. Such was the drama of the scientist. From 1965 to 1967, Nikol'skiĭ was the head of a laboratory at the Computer Center of the USSR Academy of Sciences, and later reverted to TsAGI before long to fill the post of Deputy Head of the same Second Division. By that time the scientists of that Division, which was the face of TsAGI, had moved to other Divisions owing to short-sighted politico-administrative decisions, a start was made on the reorientation of the Institute to rocketrelated problems, aviation problems being relegated to the background. Nikol'skii was facing the problem of augmenting the level of theoretical investigations in the Division, and he undertook to develop the most relevant field in aircraft aerodynamics - the theory of detached flows. The point is that the theories which neglected the flow separation from an aircraft wing sharply contradicted the data of wind-tunnel and flight experiments. There arose a demand to elucidate the mechanism of separation — one of the most complicated hydrodynamic phenomena. Nikol'skiĭ considered his papers on the theory of detached flows to be his most significant.

The second period of his activity at TsAGI was not as fruitful as the first one. Both his age and setbacks to his career led to comprehension of what was going on...

Nikol'skiĭ suffered from ciliary arrhythmia — that form of arrhythmia whereby the blood flow in the vessels becomes unpredictable and turbulent. He died of heart failure on June 12, 1976.

**4.7.2 Time.** Like all exact sciences, theoretical hydrodynamics owes its origin to Newton. As already mentioned, Euler came up with the first paradigm by constructing the mathematical model of nonviscous laminar flow. The second paradigm is due to C L Navier and G G Stokes — the mathematical model of the laminar flow of a viscous liquid. The third paradigm — the model of turbulence — has not yet been constructed.

The theory of theoretical hydrodynamics is simple: the first paradigm was studied in the XIXth century, the second one in the XXth century, and the third is still under study. The development of the first paradigm, as applied to gas dynamics, was completed at the TsAGI school.

In the 1950s-1970s, L I Sedov held sway over the Division of Mechanics of the USSR Academy of Sciences. It was hardly possible to become a Corresponding Member of the USSR Academy of Sciences without his recommendation. Papers not bearing a reference to his book on the theory of dimension were hard to publish in academic journals on mechanics. This was hinted at to Nikol'skiĭ almost straight away. The book itself was written at a high scientific standard, but the role of developing the dimensional theory and the theory of self-similar solutions was assigned entirely to its

author. References to either E Buckingham [80], one of the founders of similarity theory, or K G Guderley [81], the founder of the theory of one-dimensional self-similar flows, were lacking <sup>20</sup>.

At that time bureaucratization of the scientific and design enterprises of the military-industrial-complex was occurring. Under the conditions of the single-party system, where party members could be removed, there emerged a special class of leaders — the party elite ('nomenclature'). Its serried ranks were made up of unsuccessful scientists and teachers, unceremonious persons without talent, and charming swindlers [54, 82]. Proper cadre rotation, divergence of opinion, and free circulation of ideas were lacking.

The problem of the emergence of nomenclature in a totalitarian society is not new: it has been investigated by such outstanding XXth century thinkers as English economist F Hayek, American journalist H W Liepmann, German philosopher K Jaspers, and many other scholars.

By the 1980s it turned out that TsAGI was not among the top hundred Russian scientific centers with the highest citation indices. At that distant time, money did not count and profitability was not a factor to be seriously considered. It came to the point where the Chief of TsAGI, G P Svishchev, and the Deputy Chiefs started implementing a project equivalent in scale to the notorious diversion of Siberian rivers — the construction of a 'second TsAGI' in the environs of the town of Ul'yanovsk...

A good example of the amateurism of the nomenclature is Academician A A Dorodnitsyn's proposal that aerodynamic experiments be replaced with computing experiments, but thank God! — his proposal was rejected. He also established 'computation centers' at TsAGI and the Academy of Sciences, despite the fact that even then it was evident that the development of computer engineering would take the path of personalization. Nikol'skiĭ always took a stand against such — enclose the following word in quotation marks! — ideas.

A talented scientist and a charming person, Nikol'skiĭ properly represented Russian science abroad, being a member of the Presidium of the International Union of Theoretical and Applied Mechanics (IUTAM) and therefore able to go abroad despite the 'iron curtain'. He was personally familiar with the most prominent hydrodynamicists of that time, Theodore von Karman and Klaus Oswatitsch, and the well-known publisher Maxwell. The families of Nikol'skiĭ and Antonio Ferri were on friendly terms. It was precisely Nikol'skiĭ whom an outstanding American hydromechanics specialist, Milton Van Dyke, trusted to edit the Russian translation of his book on asymptotic methods, which has come to be a classic [83]. The foreword to the book ended with the words: "I have failed to properly reflect the significant contribution of Soviet researchers in the development of the subject. Fortunately, my old friend (italics mine -S B), Professor Nikol'skiĭ, will write the foreword to the translation, and I hope he will also provide additional information on the works of Soviet scientists."

4.7.3 Creative work. The outstanding Russian mechanics scientist Nikol'skiĭ always adhered to the position of strict

 $^{20}$  As already noted, Lord Buckingham proved the so-called ' $\pi$ -theorem'. Guderley was the first to solve a self-similar problem. That was the onedimensional problem on a cylindrical or spherical shock wave converging to the center — the so-called self-similar problem of the second kind. mathematics and constructed mathematical models based on fundamental mechanics principles, thereby preferring an axiomatic approach to an empirical one. He was neither a 'keyboardist' — a computational scientist, for computers were unavailable at that time, — nor an 'epsilonist'. He wielded asymptotology at the intuitive level. He was not a 'chaotist' — one engaged in the problem of chaos, i.e., turbulence. He had no time for these problems. He was just a 'classic'. He was destined to complete the development of gas dynamics — a branch of hydrodynamics. Furthermore, he constructed the mathematical models of complex dynamic phenomena.

Let us consider some fragments of his creative work.

Nikol'skii — Taganov theorem. This theorem (sometimes referred to as the *monotonicity law*) was established in 1946 and appears nowadays in all textbooks on the theory of transonic flows.

When moving along a transition line so that the subsonic velocity domain is on the left, the velocity vector will monotonically turn clockwise.

Based on this theorem, the authors determined the limiting value of the Mach number in the incident flow, after which a potential flow with a local supersonic region becomes impossible.

Does a continuous transonic flow exist on an airfoil? In 1956–1958 this problem was solved by the American woman scientist C S Morawetz, mentioned above [84].

Supersonic conic flow. Supersonic conic flows were discovered by an outstanding German scientist, A Busemann, a pupil of Prandtl [85]. He investigated two types of conic flows: the cone flow (Fig. 12a) and the flow in a converging nozzle (Fig. 12b). In both cases he derived exact solutions to arrive at the following conclusion: "there exist only two types of axially symmetric conic flows: the flow in a contracting nozzle and the flow past conic vertices embedded in an axially symmetric airflow." Despite Busemann's statement, in 1946 Nikol'skiĭ discovered a third type of axially symmetric supersonic conic flow — an external flow past the narrowing part of a body of revolution between the rays OA and OB (Fig. 12c). He also derived the exact solution of the problem.

At present, the investigation of conic flows is being continued; it has been extended to the subsonic velocity range with the inclusion of vorticity, aerodynamic twist, and even viscosity [86]. The so-called conic turbulence, i.e., the chaotic mode of a conic flow, remains unexplored.



Figure 12. Supersonic conic flows: (a) cone flow (Busemann, 1929), (b) flow in a converging nozzle (Busemann, 1942), (c) flow past the trailing segment of an axially symmetric body (Nikol'skiĭ, 1946).

**Optimal aerodynamic body with a channel.** In the context of linear theory, in 1950 Nikol'skiĭ determined the shape of a body with a channel exhibiting the lowest external wave drag in supersonic flow. In this work, advantage was taken for the first time of an original technique to reduce the number of independent variables owing to the crossing over to the characteristic control contour. This work served as a source of ideas for the investigation of wings and bodies of revolution with a minimal drag, which were performed by his pupils and followers: V N Zhigulev, Yu L Zhilin, M N Kogan, and Yu D Shmyglevskiĭ [29].

At the present time, the theory of aerodynamically perfect forms has reached an impasse. The point is that a nonempirical solution of the problem (the Navier–Stokes equation) of flow past a body of a given shape has not been obtained. If we intend to be consistent it should be admitted that the theory of aerodynamically perfect forms should be treated only after this problem is solved employing the rapidly developing DNS or LES techniques<sup>21</sup>. In this sense, Nikol'skii's work outstripped his time.

Nikol'skiĭ conditions at permeable boundaries. To reduce the effect of wind-tunnel walls on the flow past a model, especially significant at near-sonic velocities, advantage is taken of perforation, as noted above. Calculating the gas flow near a permeable wall requires specifying boundary conditions at the wall. Nikol'skiĭ (1951) derived such a condition for a longitudinally slotted wall: u' = 0, where u' is the perturbation of the longitudinal velocity component. The Nikol'skiĭ condition implies that the pressure perturbation at the wall is constant. It was thereby shown that the effect of longitudinal slots on the flow in the linear theory is equivalent to the effect of a free boundary. Grodzovskiĭ et al. [70] devoted the fourth chapter of their book to Nikol'skiī's work on the conditions at permeable boundaries.

The general form of a linear boundary condition at permeable boundaries (u' = kv', where v' is the perturbation of the transverse velocity component) is known a priori, prior to the solution of the problem. The solution of the linear problem of gas flow near a permeable surface is needed only to determine the dependence of parameter k on the Mach number  $M_{\infty}$  and wall penetration factor. Symbolically, the complete linear problem on the flow over a wing in permeable boundaries was solved by Natal'ya Marevtseva, the daughter of professor Nikol'skii [87].

According to modern views, the complete problem of the flow past a body placed inside a contour with a small-scale permeability is split into external and internal problems [88]. The external one is the problem of flow over a body with boundary conditions at permeable and impermeable segments of the wall. The internal one is the problem of gas flow through the permeable wall; the body flown over is absent here — it is as if moved off to infinity. The boundary conditions for the external problem, which are the outer limit of inner expansion, are determined from the solution of the internal problem. Unfortunately, to this day the solution of the internal problem has not been obtained.

**Nikol'skiĭ equation or Birkhoff – Rott equation?** We specify the shape of a vortex sheet in the *xy*-plane in the parametric form:

$$z(\Gamma, t) = x(\Gamma, t) + iy(\Gamma, t),$$

<sup>21</sup> Direct Numerical Simulation (DNS); Large Eddy Simulation (LES).

selecting as a parameter the circulation  $\Gamma$  of a piece of the vortex sheet measured from some fixed point where  $\Gamma = 0$ . Then, the equation for the evolution of the plane vortex sheet <sup>22</sup> with time *t* is of the form

$$\frac{\partial \bar{z}(\Gamma,t)}{\partial t} = \frac{1}{2\pi i} \int_0^{\Gamma} \frac{d\Gamma'}{z(\Gamma,t) - z(\Gamma',t)} + u_0(z,t), \qquad (1)$$

where  $u_0$  is the velocity induced by outside factors, the bar implies a complex conjugate quantity, and the integral is the Cauchy-value integral.

This integro-differential equation is referred to as the Birkhoff–Rott equation [90]. It was derived under rather general assumptions by G Birkhoff in 1962, but it was used by N Rott even in 1956. Nikol'skiĭ employed Eqn (1) in 1957. He was one year late, but it was employed even earlier by L Anton (1939).

At present, the equation for the time evolution of contact discontinuity has been derived with the inclusion of gravity, different properties of the liquids in contact, and surface tension and has been generalized to the case of threedimensional motion [91]. It is fundamentally significant that the Cauchy problem for Eqn (1) has been proven to be illdefined, and a particular manifestation of this incorrectness is Moore's paradox, which is associated with the manifestation of singularity for a finite time [34, 90].

Nikol'skiĭ flow. If the circulation is represented as a  $\delta$  function, i.e., the vorticity is concentrated on a point, the integral in Eqn (1) disappears and the equation becomes algebraic. In this case, the vortex sheets are replaced with point vortices. Nikol'skiĭ's achievement consists not in the fact that he proposed considering flows with point vortices — this had been done many times before! — but in the fact that he showed (1957) it to be appropriate for the exact modeling of a detached self-similar flow for a self-similarity index value n = 1/2. The plane Nikol'skiĭ flow describes the detached flow over a body for which translational motion and affine expansion with velocities proportional to  $t^{-1/2}$  set in at t = 0 (Fig. 13).

The spatial stationary Nikol'skiĭ flow describes the flow past sharpened bodies shaped according to a power law [92]. Figure 14 shows a power-law-shaped wing bent according to the same power law — the so-called Nikol'skiĭ ski.



**Figure 13.** Plane Nikol'skiĭ flow:  $at^{1/2}$  — the affine expansion law,  $bt^{1/2}$  — the law of translational motion,  $\Gamma_1$  — the circulation of a point vortex separated from the sharp edge A,  $\Gamma_2$  — the circulation of a point vortex separated from the smooth surface.

 $^{22}$  S Hawking believes that each formula in a popular article halves the number of readers. This conclusion does not apply to readers with a mathematical cast of mind — they understand symbols more easily than words.

As S S Averintsev said, "We live in the epoch where all words have already been said." However, not all formulas have been written yet.



Figure 14. Three-dimensional stationary Nikol'skiĭ flow.

At present, numerical techniques make it possible to calculate the evolution of a self-similar vortex sheet separating from a body at any values of self-similarity index n. The degenerate solution found by Nikol'skiĭ remains the only solution of this problem derived in a closed form.

**Nikol'skiĭ paradox.** Nikol'skiĭ showed (1961) the following: when a body moves in a limited volume of revolving liquid, the drag force is proportional not to the acceleration, as in a non-revolving liquid, and not to the velocity, as in an infinite liquid, but to the path (!) covered by the body.

The transient slow motion of a body in a revolving liquid is associated with the formation of Taylor–Proudman columns [93], which is responsible for the paradoxical behavior of the drag force. The details of this phenomenon are still unknown.

**Exact solutions of the Boltzmann equation.** The Boltzmann equation for the distribution function f(x, y, z; u, v, w; t), where x, y, and z are coordinates and u, v, w are velocities, is of the form

$$\frac{\partial f}{\partial t} + u \frac{\partial f}{\partial x} + v \frac{\partial f}{\partial y} + w \frac{\partial f}{\partial z} = I, \qquad (2)$$

where *I* is the collision integral.

The exact solutions of this equation known at that time corresponded to either the case of a constant macroscopic gas velocity  $\mathbf{u} = \text{const}$  or the case of gas rotation as a solid. Nikol'skiĭ (1963) discovered a continuous and spatially uniform transformation group

$$f = f_0 \left( 0, 0, 0; u - \frac{x}{t}, v - \frac{y}{t}, w - \frac{z}{t}; t \right),$$
(3)

which reduces Eqn (2) to a simple form

$$\frac{\partial f_0}{\partial \tau} = I, \tag{4}$$

where the new variable  $\tau$  is a function of time only.

Cases have been analyzed in which the velocity distribution tends to the Maxwellian one for  $t \to \infty$ .

The Nikol'skiĭ transformation (3) is valid when  $\mathbf{u} = \mathbf{r}/t$ , where  $\mathbf{r}$  is the radius vector. The flow is referred to as collapse when the time is taken to be in the interval  $-\infty < t \le 0$  and explosion when the time is taken to be in the interval  $0 \le t < \infty$ . The Nikol'skiĭ transformation [94] endowed the (exact) solutions of Eqn (4) with new significance. However, its practical application is limited, since the density and temperature are time-dependent in a special way.

The attitude toward the so-called exact solutions is now of two kinds, especially so if they were obtained intuitively, when they suddenly dawned upon the author, rather than with the aid of a resourceful theoretical group analysis of differential equations [95]. On the one hand, not every exact solution describes exact flow properties. On the other hand, every exact solution describes the turbulent flow mode, like any integrable system [96].

The outstanding Russian hydrodynamicist Nikol'skii passed to immortality in the prime of his talent, never to see time catch up with him.

## 5. Conclusions

In the second half of the last century, a new generation of highly qualified specialists came into aviation. Graduates of Moscow State University and the Moscow Institute of Physics and Technology superseded S A Chaplygin's companions. The techniques of aerodynamic experimentation changed, making it complex, combining wind-tunnel, natural, and computer approaches. The subject matter of research broadened. In particular, V I Ponomarev, who was pursuing research in the context of the theory of turbulence, showed that the asymptotic structure of a turbulent boundary layer is three-zonal [97]. However, Russian hydrodynamics, along with Russia itself, has entered a period of crisis. Even though aircraft engineers, unlike aerodynamics scientists, have not gone to the West to seek their fortune, so-called internal emigration — young specialists going into business – has occurred. The bond of the times has disintegrated, and the average age of a Russian aircraft engineer has exceeded the critical point.

It remains only to guess whether Russia will see the revival of aviation, which is so necessary for the county's revival...

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