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References

1. Kroger L A, Reich C W *Nucl. Phys. A* **259** 29 (1976)
2. Reich C W, Helmer R G *Phys. Rev. Lett.* **64** 271 (1990)
3. Helmer R G, Reich C W *Phys. Rev. C* **49** 1845 (1994)
4. Burke D G et al. *Phys. Rev. C* **42** R499 (1990)
5. Strizhov V F, Tkalya E V *Zh. Eksp. Teor. Fiz.* **99** 697 (1991) [*Sov. Phys. JETP* **72** 387 (1991)]
6. Blaise J, Wyart J-F *Tables Internationales de Constantes Sélectionnées* Vol. 20 *Energy Levels and Atomic Spectra of Actinides* (Paris: Univ. Pierre et Marie Curie, 1992) p. 8
7. Tkalya E V *Pis'ma Zh. Eksp. Teor. Fiz.* **60** 619 (1994) [*JETP Lett.* **60** 627 (1994)]
8. Tkalya E V et al. *Phys. Scripta* **53** 296 (1996)
9. Dykhne A M, Tkalya E V *Pis'ma Zh. Eksp. Teor. Fiz.* **67** 233 (1998) [*JETP Lett.* **67** 251 (1998)]
10. Tkalya E V *Pis'ma Zh. Eksp. Teor. Fiz.* **71** 449 (2000) [*JETP Lett.* **71** 311 (2000)]
11. Tkalya E V, Zherikhin A N, Zhudov V I *Phys. Rev. C* **61** 064308 (2000)
12. Irwin G M, Kim K H *Phys. Rev. Lett.* **79** 990 (1997)
13. Richardson D S et al. *Phys. Rev. Lett.* **80** 3206 (1998)
14. Utter S B et al. *Phys. Rev. Lett.* **82** 505 (1999)
15. Shaw R W et al. *Phys. Rev. Lett.* **82** 1109 (1999)
16. Young J P, Shaw R W, Webb O F *Inorg. Chem.* **38** 5192 (1999)
17. Burenkov A F et al. *Prostranstvennye Raspredeleniya Energii, Vydelennoĭ v Kaskade Atomnykh Stolknoveniĭ v Tverdykh Telakh* (Spatial Distributions of Energy Released in a Cascade of Atomic Collisions in Solids) (Moscow: Energoatomizdat, 1985)
18. Pippard A B *Rep. Prog. Phys.* **23** 176 (1960)
19. Ashcroft N W, Mermin N D *Solid State Physics* (New York: Holt, Rinehart and Winston, 1976) [Translated into Russian (Moscow: Mir, 1979)]
20. Tkalya E V *Pis'ma Zh. Eksp. Teor. Fiz.* **70** 367 (1999) [*JETP Lett.* **70** 371 (1999)]
21. Ackermann R J, Thorn R J, Winslow G H *J. Opt. Soc. Am.* **49** 1107 (1959)
22. Sviridova A I, Suĭkovskaya I V *Opt. Spektrosk.* **22** 940 (1967)
23. Dykhne A M, Eremin N V, Tkalya E V *Pis'ma Zh. Eksp. Teor. Fiz.* **64** 319 (1996) [*JETP Lett.* **64** 345 (1996)]
24. Gasparro J et al. *Phys. Rev. C* **62** 064305 (2000)
25. Tkalya E V *Pis'ma Zh. Eksp. Teor. Fiz.* **55** 216 (1992) [*JETP Lett.* **55** 211 (1992)]
26. Tkalya E V et al. *Yad. Fiz.* **59** 817 (1996) [*Phys. Atom. Nucl.* **59** 779 (1996)]
27. Varlamov V O et al. *Dokl. Ross. Akad. Nauk* **346** 465 (1996) [*Phys. Dokl.* **41** 47 (1996)]
28. Dykhne A M, Tkalya E V *Pis'ma Zh. Eksp. Teor. Fiz.* **67** 521 (1998) [*JETP Lett.* **67** 549 (1998)]

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Thermalization phenomenon in hadron physics

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1. Introduction

This report describes nonequilibrium processes using the example of multiple hadron production in which the attainment of the thermodynamic equilibrium is brought about by dissipation of the kinetic energy of the colliding particles into the hadron masses. The dynamic peculiarities of these processes are limited by the need to take into account the constraints responsible for the 'nonescape' of color charge, so that, generally speaking, the formation of a thermalized state is a fairly rare event. The necessary and sufficient condition for a meaningful description of such processes has been found. By its very meaning this condition is similar to the 'correlations depression' condition which, according to N Bogolyubov, comes into play as an equilibrium state is approached by the system. Physically, such a situation occurs in highly multiple production processes. The first experimental indications of the possibility of observing thermalization phenomena are given.

A thermodynamic description is appealing chiefly because it admits a complete description of a complex system with the use of a limited number of parameters. These are usually temperature, pressure, specific volume, and chemical potential. Otherwise, when a thermodynamic description is impossible (say, when the system is highly nonequilibrium), one must know $3n-4$ independent parameters in order to completely specify an n-particle distribution function.

Strictly speaking, the concept of temperature, namely the main thermodynamic parameter, is admissible only for systems (possibly, subsystems) that are in thermodynamic equilibrium, for which a homogeneous energy distribution over all the degrees of freedom is a characteristic feature. Here, the homogeneity of the energy distribution must be maintained with exponential accuracy, while the fluctuations in the neighborhood of the respective average energy (temperature) must be Gaussian. In other words, for example, the energy spectra of the particles must correspond to a Boltzmann – Gibbs distribution, while temperature fluctuations must correspond to a Gaussian distribution. Then the temperature may be considered as a 'good' parameter.

It can be stated that if a system has such a good parameter, it is in energy equilibrium in the sense that there are no macroscopic energy fluxes in it. Such a system is said to be thermalized. A detailed discussion of this problem can be found in Ref. [1].

Thus, if one uses the concept of temperature, the system is in thermal equilibrium, with the result that it is enough to know the average energy of the particles to describe it. One must bear in mind, however, that in general the system may be out of equilibrium with respect to other parameters.

Conditions of complete thermalization are not often encountered in nature. For instance, the thermodynamic description cannot generally be applied to describing biological systems, although it is known that the temperature of a biological system may be a good parameter. Thermodynamic description has limited application also in subatomic physics, while on the molecular level examples of such descriptions abound.

What then obstructs thermalization of a multiparticle system? This question may be examined fairly rigidly within the proposed S-matrix interpretation of thermodynamics (see the literature cited in Ref. [1]). For instance, in addition to ordinary kinetics, internal constraints, as a result of which not all degrees of freedom have equal status, may obstruct thermalization. In some cases, the nature of these constraints may lie hidden in the symmetry of the action or Hamiltonian. It is this situation that is realized in hadronic physics.

Distribution functions. N N Bogolyubov was the first to pose the question of how many measurable, or what is called 'partial', distribution functions are really needed to describe multiparticle systems (see the monograph [2]). Precisely, he noticed that to examine all the quantities that emerge in the description of equilibrium systems it is enough to specify the single-particle distribution function. This means that to examine the thermodynamic state of a system it is quite enough to 'keep track of' a single particle, while ignoring all the other particles. This was the beginning of the construction of the Bogolyubov – Born – Green – Kirkwood – Yvon (BBGKY) chain of equations.

Actually the same idea was used in describing multiparticle production: it was suggested that only one particle be 'tracked', while the other particles be ignored. This is what is known as the inclusive approach [3], and in its time it was a tremendous step forward. One cannot be sure, however, that knowing the single-particle distribution functions is sufficient for describing hadron production.

I would like to add here that in a certain sense the present treatment is a development of the inclusive approach, i.e., one 'keeps track of' a group of particles and ignores all the other particles. This fact greatly simplifies experimental studies, since it makes it possible to ignore unnecessary or too detailed measurements.

The phenomenon of thermalization. Actually, the question of "Whether the language of thermodynamics can be used to describe multiple production processes?" can serve as an alternative title of the present report. It requires a thorough consideration, but nevertheless the very idea of a thermodynamic description has been used by many researchers [4], since inelastic hadron scattering is a process of dissipation of the kinetic energy of the incident particles. This point was actively developed by Enrico Fermi and Lev Landau, and later by other researchers.

The phenomenon of multiple production. Several remarks concerning the history of the physics of multiparticle production are in order. The time of birth of this area of research should be placed in the period from 1927 to 1930. It was then that D V Skobel'tsyn discovered showers of particles in cosmic rays for the first time. The work of G V Vatagin, who theoretically predicted the possibility of producing several secondary particles at high production energies, also belongs to this period. The first direct observations of inelastic processes were made by a group of scientists from the P N Lebedev Physics Institute of the Soviet Academy of Sciences in the Pamir Mountains expedition of 1945 – 1946 (V I Veksler and others).

After Cecil F Powell's discovery of the π meson in 1947, it became clear that showers in cosmic rays stem from the interaction of high-energy particles (starting at several gigaelectron-volts), as a result of which many pions are produced. The study of the multiparticle production in cosmic rays was the first step and is associated with the works of our outstanding scientists (S N Vernov, V L Ginzburg, G T Zatsepin, A E Chudakov, S N Nikol'skiĭ, G B Zhdanov, the Alikhan'yan brothers, and others) and foreign scientists (L Janossy, D Morrison, B Andersson, and others).

Later on, extensive studies of such processes were carried out in the accelerators of CERN, Fermilab (Tevatron), and BNL (RHIC) and in Russia at the Serpukhov and Dubna facilities.

2. On multiple production theory

The main prediction of the common statistical Fermi – Landau model is that a system of interacting particles will reach equilibrium with the surrounding 'medium'. In the problems considered here, vacuum is the medium, which means that the average particle multiplicity must be proportional to the total energy of the colliding particles, i.e., close to the threshold value of the multiplicity. The experiment shows that this is far from the actual situation, however. The data on average particle multiplicity show that such behavior is far from reality and, most probably, complete thermalization is not reached in hadron – hadron collisions. The point is that, as noted in Ref. [5], the non-Abelian gauge symmetry, which controls the dynamics of hadrons, impedes complete thermalization, at least in the relatively early stages.

It must also be noted that, following the assumptions of Fermi and Landau, thermalization occurs when the particle multiplicity measured in units of average multiplicity is very high. Hence, special attention is paid to the case of very high multiplicity (VHM) of the particles produced.

It is important to study the phenomenological indications of thermalization, and this is the *main goal* of our investigations. Here the predictions of statistical models are expected to be compared with the experimental data. The crucial point is the study of necessary and sufficient conditions for thermalization in hadronic processes.

3. Phenomenology of the statistical description

I will begin by noting that the statistical approach is gaining wide acceptance in describing inelastic collisions of precisely heavy ions. *The theoretical basis* was developed in the works of J Schwinger, L V Keldysh, and others. This made it possible for J Manjavidze and me to formulate what is known as the *S*-matrix interpretation of thermodynamics, which in turn enabled us to find the necessary and sufficient conditions for the thermalization of a state produced in accelerator experiments [1, 13]. Under certain restrictions, our formulation coincides with the Schwinger – Keldysh field theory in real time and at a finite temperature [6]. A more detailed relationship to the thermodynamics of the states observed can be established by using the formalism of Wigner functions as interpreted by Carruthers and Zachariasen [6]. It should be emphasized that the latter approach makes it possible to expand our field formalism so that it encompasses the quantum statistics of condensed media.

Unfortunately, it is impossible in a brief report to tell about all the methods and literature. Most of the papers published in 1999 to 2002 are devoted to central collisions of heavy ions:

(i) It was noticed that the theoretical analysis of the production of secondary particles in central Au – Au collisions at RHIC energies, based on the thermal model, is in good agreement with the experimental data [7].

(ii) It was shown that the advanced statistical model suggests that chemical equilibrium can be reached in Pb – Pb collisions at SPS energies [8].

(iii) The application of the statistical model suggests that a single parameter controls the process of production of different particles and their momentum spectra [9].

However, despite the progress achieved in the statistical description of heavy-ion collisions, it is still advisable to critically analyze the problem of thermalization and to quantitatively show that the statistical description is applicable.

Structure of the phase volume of produced particles. First, there must be a clear picture of the kinematics of multiparticle production. This picture will make it possible to find the kinematic region where thermalized states are most likely to exist. This is also useful in determining the conditions of the dynamics necessary for thermalization to occur.

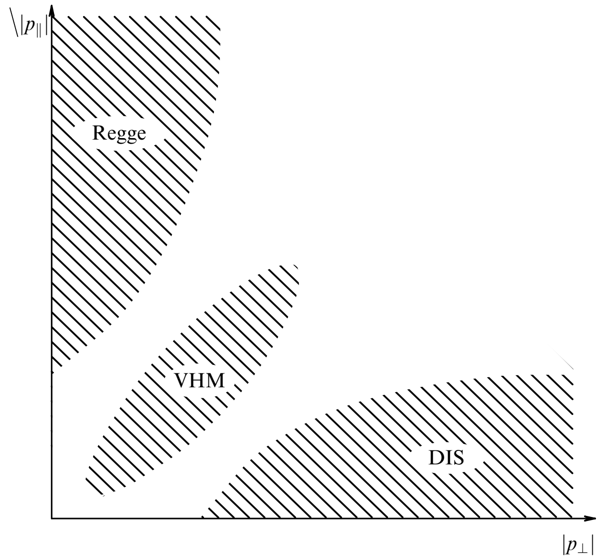

Figure 1.

Let us examine the projection of the phase space onto the plane of the longitudinal ($p_{\|}$) and transverse ($p_{\perp}$) momenta (Fig. 1). The following regions can be isolated in this plane:

(a) Models of the multiperipheral type belong to the region known as the Regge region [10]. Such kinematics are characterized by a small average value of the transverse momentum that is independent of the initial energy and particle multiplicity; the Balitskiĭ – Fadin – Kuraev – Lipatov (BFKL) approach works well in this region.

(b) Deep inelastic scattering (DIS) belongs to the region where perturbative QCD (or pQCD based on the logarithmic approximation) operates. In this case, the transverse momentum of the produced hadrons is much higher than the longitudinal momentum [11] (the Dokshitzer – Gribov – Lipatov – Altarelli – Parisi, or DGLAP, approach).

(c) It is natural to assume that thermodynamics should be placed somewhere between the two regions, Regge and DIS, since in this case the transverse and longitudinal momenta of the particles produced are comparable and one can expect that equilibrium is established with respect to these degrees of freedom. This gives rise to the VHM region in Fig. 1.

Apparently, the final state relevant to processes of inelastic collisions of heavy ions belongs to this intermediate region.

A new perturbation theory has been developed [13] to describe the VHM region, namely the topological QCD (tQCD).

4. Necessary and sufficient condition for thermalization

Our conclusion implies that the presence of well-defined thermodynamic parameters is the necessary and sufficient condition for a thermodynamic description to be valid. This condition was used in work [14] to prove that if the inequality

$$\left|K_l(E,n)\right|^{2/l} \ll K_2(E,n)\,, \quad l = 3, 4, \ldots, \tag{1}$$

which guarantees the smallness of the higher-order central moments

$$K_l(E,n) = \left\langle \prod_{k=1}^{l} (\varepsilon_k - \langle \varepsilon \rangle) \right\rangle, \tag{2}$$

with ε_k being the energy of the kth particle, is valid on the scale of the variance of the energy distribution, $K_2(E,n)$, then the temperature is a good integral parameter and the statistical description is possible. Hence we must first check whether this inequality is satisfied.

Averaging in formula (2) is done over the observed energy spectra, with the number of produced particles being fixed. There is a certain analogy between the above condition (1) and the correlations depression condition which was proposed by N N Bogolyubov for statistical physics. I would like to add that the derivation of condition (1) is general and depends only slightly on the dynamic details.

The following *scenario* can be proposed of how a thermalized state is achieved in hadron – hadron interactions with increasing particle multiplicity. First, it must be noted that for a very large number of particles to be produced the interaction between colliding hadrons must be central. This makes it possible to predict the increase in the average transverse momentum of produced particles with multiplicity, which is corroborated by the data obtained in the E735 experiments in the Tevatron accelerator (Fermilab). What I have just said can be illustrated in terms of the particle multiplicity distribution. Thus, let us see what happens when multiplicity increases:

(i) Multiperipheral models are valid up to $n_s \sim \bar{n}(E)^2$.

(ii) 'Hard' processes contribute at large values of multiplicity, $n > n_s$.

(iii) However, the leading logarithm approach (LLA) can be used for values of n such that $n_s < n < n_h$.

(iv) Thermalization sets in at very high multiplicities, and all momenta of the secondary particles are comparable.

(v) The asymptotic region is the region of the 'ideal-gas approximation', where the momenta of the particles are much smaller than their masses.

5. Predictions of the generators of events

PYTHIA generator. According to our scenario, three particle multiplicity regions can be specified (Fig. 2):

(A) It can be concluded that the processes built into PYTHIA cannot predict even a tendency toward equilibrium. PYTHIA can be used only in this region.

(B) The transition region to a thermalized state. VHM belongs to this region.

(C) The limiting thermalization region: $(K_3^{2/3}/K_2) \sim 1/n$.

The analysis made in Ref. [15], which corresponds to the multiplicity region A, suggests that the dynamical models forming the base of PYTHIA cannot even predict the tendency toward thermalization. This conclusion corrobo-

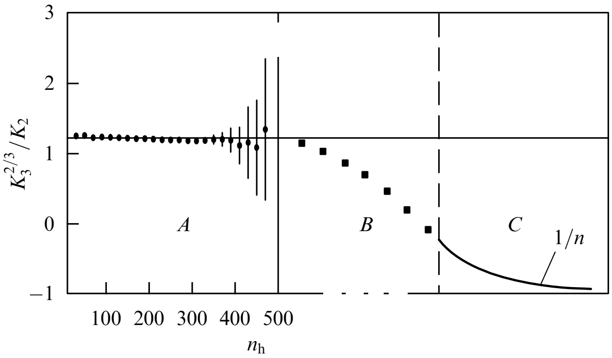

Figure 2.

rates our prediction that thermalization cannot occur in the Regge or DIS regions (see Fig. 1).

HIJING generator. This generator predicts a certain tendency toward thermalization, which can be explained by its ability to allow for multiple rescattering. In the scattering of heavy ions, thermalization may set in at smaller particle multiplicities on the scale of the average multiplicity values.

6. What must be measured?

(i) The problem of detecting the phenomenon of thermalization in inelastic events is probably the most important one. For this we must measure the $K_3^{2/3}/K_2$ ratio.

(ii) This ratio will also make it possible to quantitatively study the range within which the LLA is valid for pQCD.

(iii) It is also important to study the ratio of the average values of the momenta of produced particles, $\langle p_{\|}\rangle/\langle p_{\perp}\rangle \to \pi/4$. Thus, if our assumption that in the transition to equilibrium the interaction becomes central is true, this ratio should tend to $\pi/4$ from above.

(iv) In conclusion, if the $K_3^{2/3}/K_2$ ratio is smaller than unity, we can also introduce the so-called 'chemical potential'

$$\mu(E,n) = -\langle\varepsilon\rangle \ln \frac{\sigma_n(E)}{\sigma_{\text{tot}}} . \tag{3}$$

Such an interpretation will make it possible to directly analyze the contributions from different mechanisms of multiparticle production and to observe phase transitions.

The following experiments are in the planning stage. In agreement with our suggestions, the program of these experiments includes the study of VHM and thermalization.

The 'Thermalization' experiment (the U-70 accelerator, Protvino, Russia). The goals of the experiment are

(1) to determine the effect of multiparticle Bose – Einstein correlations on thermalization;

(2) to investigate the role of resonance excitations as equilibrium (with respect to temperature) sets in;

(3) to study the tail of the multiplicity distribution in order to (a) establish the applicability of the S-matrix approach, and (b) find the chemical potential of the system.

The study of VHM processes at low energies has certain advantages, since in view of the Koba – Nielsen – Olesen (KNO) scaling, even when slightly violated, we can get rather close to the kinematic threshold which is equal to 69 pions at 70 GeV. Thus, there is a high probability of producing a 'cold' and quite dense state of pions.

This is an entirely new area of research that is being actively studied at Protvino in the accelerator of the Institute of High-Energy Physics (the 'Thermalization' experiment; P F Ermolov, V A Nikitin, and others).

The CDF (Fermilab), STAR (RHIC), and ATLAS (LHC) experiments. [1] The goals of these experiments are

(1) to establish and study the properties of a thermalized state;

(2) to investigate collective phenomena (of the phase-transition type) in the equilibrium system;

(3) to study the 'low-x' region in order to render more precise the range of applicability of QCD.

These experiments, in contrast to the low-energy experiment at Protvino, should answer the question of whether there can actually be a relativistic thermalized state, i.e., a thermalized state in which the particle momenta are much larger than the particle masses. The search for such a state under the specified kinematic restrictions will make it possible to get rid of such background effects as, say, Bose – Einstein and resonance correlations.

Moreover, of certain interest are QCD studies in the region where partons have high virtuality but low momenta. Such kinematics are inherent in VHM processes.

[1] In principle, the study of the VHM region and the thermalization phenomenon is also possible in other experiments in the LHC, RHIC, and Tevatron accelerators.

7. Analysis of experimental data

At present we have started analyzing the first data on the $K_3^{2/3}/K_2$ ratio gathered in the STAR and CDF experiments. The results are preliminary and are not corroborated by sufficient statistics, although they do contain VHM events (on the scale of the average multiplicity values). There is, however, a certain tendency toward reduction of the $K^{2/3}/K_2$ ratio as the particle multiplicity grows.

8. Conclusions

In conclusion I would like to point out the following.

(i) There are certain indications that thermalization manifests itself in heavy-ion collisions.

(ii) Making quantitative estimates of the VHM kinematics region is impossible if we remain within LLA in pQCD. Moreover, the existing dynamical models cannot predict even the tendency toward equilibrium.

(iii) It is important that an S-matrix interpretation of the necessary and sufficient condition for thermalization has been found. This makes it possible for us to show that thermalization must occur at least in deep asymptotics with respect to multiplicity.

(iv) That is why we proposed a strong-coupling perturbation theory, namely tQCD. The theory describes perturbations that retain the topology of Yang – Mills fields. What is important is that it incorporates pQCD as the short-wave approximation.

Experimental approaches to VHM physics are being actively discussed from various standpoints by such collaborations as ATLAS, CDF, and STAR, to name just a few.

From the practical viewpoint, building a 'fast' generator of events with VHM based on tQCD is today the most important task. The problems of an effective trigger must also be thoroughly studied. There is still insufficient experience in analyzing VHM events.

At the end of my report, I would also like to say that only the statistical approach can provide a complete description of the inelastic hadron – hadron collisions. A more general statement is in order, however: the fact that we have a multiparticle system is not enough to justify the use of the thermodynamic approach. Attempts to do just the opposite have often been made in different areas of research. Attention should be focused on the methods that make it possible to find the necessary and sufficient conditions for a thermodynamic description to be valid.

References

doi 1. Manjavidze J, Sissakian A *Phys. Rep.* **346** 1 (2001)
2. Bogolyubov N N *Problemy Dinamicheskoĭ Teorii v Statisticheskoĭ Fizike* (Problems of Dynamical Theory in Statistical Physics) (Moscow – Leningrad: Gostekhizdat, 1946) [Translated into English as *The Dynamical Theory in Statistical Physics* (Delhi: Hidustan Publ. Corp., 1965)]

3. Logunov A A, Mestvirishvili M A, Nguyen Van Hieu *Phys. Lett. B* **25** 611 (1967); Feynman R P *Phys. Rev. Lett.* **23** 1415 (1969); Logunov A A, Mestvirishvili M A, Preprint TH-1707 (Genève: CERN, 1973)
4. Fermi E *Prog. Theor. Phys.* **5** 570 (1950); Landau L D *Izv. Akad. Nauk SSSR Ser. Fiz.* **17** 51 (1953) [Translated into English in *Collected Papers of L D Landau* (Ed. D ter Haar) (Oxford: Pergamon Press, 1965) p. 569]; Feinberg E L *Phys. Rep.* **5** 237 (1972); Hagedorn R *Nuovo Cimento* **35** 216 (1965); Andreev I V, Dremin I M *Usp. Fiz. Nauk* **122** 37 (1977) [*Sov. Phys. Usp.* **20** 381 (1977)]; Matveev V A, Muradyan R M, Tavkhelidze A N *Lett. Nuovo Cimento* **5** 907 (1972); Bialas A et al. *Nucl. Phys. B* **11** 479 (1969)
5. Manjavidze J, Sissakian A *J. Math. Phys.* **42** 641 (2001)
6. Schwinger J *J. Math. Phys.* **5** 1606 (1964); Keldysh L V *Zh. Eksp. Teor. Fiz.* **47** 1515 (1964) [*Sov. Phys. JETP* **20** 1018 (1965)]; Niemi A J, Semenoff G W *Ann. Phys.* (New York) **152** 105 (1984); Carruthers P, Zachariasen F *Phys. Rev. D* **13** 950 (1976)
7. Becattini F et al. *Phys. Rev. C* **64** 024901 (2001); Becattini F, Bellucci L, Passaleva G *Nucl. Phys. B Proc. Suppl.* **92** 137 (2001); Becattini F *Nucl. Phys. A* **702** 336 (2002); Braun-Munzinger P, Heppe I, Stachel J *Phys. Lett. B* **465** 15 (1999); Heinz U W, Kolb P F, in *Proc. of the 18th Winter Workshop on Nuclear Dynamics, Nassau, Bahamas, Jan. 20–27, 2002* (Eds R Bellwied, J Harris, W Bauer) (Debrecen: EP Systema, 2002) p. 205; hep-ph/0204061
8. Heinz U *Nucl. Phys. A* **661** (Suppl. 999) 140c (1999); Braun-Munzinger P et al. *Phys. Lett. B* **518** 41 (2001); Oeschler H *J. Phys. G: Nucl. Part. Phys.* **27** 257 (2001); Zhong-Dao Lu, hep-ph/0207029; Baier R et al. *Phys. Lett. B* **539** 46 (2002)
9. Elliott J B et al. *Phys. Rev. Lett.* **85** 1194 (2000); Tsallis C, in *Nonextensive Statistical Mechanics and Its Applications* (Lecture Notes in Physics, Vol. 560, Eds S Abe, Y Okamoto) (Berlin: Springer-Verlag, 2001) p. 3; Kozlov G A *New J. Phys.* **4** 23 (2002); Kharzeev D, in *QCD Perspectives on Hot and Dense Matter: Proc. of the NATO Adv. Study Institute, Cargèse, France, Aug. 6–18, 2001* (NATO Sci. Series, Ser. II, Vol. 87, Eds J-P Blaizot, E Iancu) (Boston: Kluwer Acad. Publ., 2002); hep-ph/0204014; Shuryak E, hep-ph/0205031; Dremin I M, Nechitailo V A, hep-ph/0207068; Alexopoulos T et al. *Phys. Lett. B* **528** 43 (2002); hep-ex/0201030
10. Gribov V N, in *Fizika Atomnogo Yadra i Elementarnykh Chastits: Materialy VIII Zimneĭ Shkoly LIYaF* (Physics of Atomic Nuclei and Elementary Particles: Proc. of the 8th Leningrad Institute of Nuclear Physics Winter Workshop) Pt. 2 (Leningrad: Izd. LIYaF, 1973) p. 5; Baker M, Ter-Martirosyan K A *Phys. Rep.* **28** 1 (1976); Kuraev E A, Lipatov L N, Fadin V S *Zh. Eksp. Teor. Fiz.* **71** 840 (1976) [*Sov. Phys. JETP* **44** 443 (1976)]; Lipatov L N *Yad. Fiz.* **20** 181 (1974) [*Sov. J. Nucl. Phys.* **20** 94 (1975)]; Gribov V N, Lipatov L N *Yad. Fiz.* **15** 781, 1218 (1972) [*Sov. J. Nucl. Phys.* **15** 438, 675 (1972)]
11. Altarelli G, Parisi G *Nucl. Phys. B* **126** 298 (1977); Andreev I V *Khromodinamika i Zhestkie Protsessy pri Vysokikh Energiyakh* (Chromodynamics and Hard Processes at High Energies) (Moscow: Nauka, 1981)
12. Gribov L V, Levin E M, Ryskin M G *Phys. Rep.* **100** 1 (1983)
13. Mandzhavidze I D, Sissakian A N *Teor. Mat. Fiz.* **130** 179 (2002) [*Theor. Math. Phys.* **130** 153 (2002)]
14. Mandjavidze J, Sissakian A, in *Bogolyubov Conf.* "*Problems of Theoretical and Mathematical Physics", Moscow–Dubna–Kyiv, Sept. 27–Oct. 6, 1999*; *Fiz. Elem. Chastits At. Yadra* **31** (7A) 104 (2000)
15. Manjavidze J, Sissakian A, in *31st Intern. Conf. on High Energy Physics: ICHEP02, Amsterdam, July 24–31, 2002*; http://www.ichep02.nl/Transparencies/QCDS/QCDS-1/qcds-1-4.sissakian.ppt