# Current state of 'cold' antihydrogen research 

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## Contents

1. Introduction ..... 227
2. Radiative recombination. Standard theory ..... 232
3. Effects of a magnetic field on radiative recombination ..... 233
4. Stimulated radiative recombination ..... 236
5. Three-particle recombination in supermagnetized plasma ..... 239
6. Comparison of the efficiency of recombination mechanisms ..... 241
7. Deceleration and stopping of antiprotons in positron supermagnetized plasma ..... 241
8. Longitudinal diffusion of stopped antiprotons. Their confinement time in the positron cloud ..... 243
9. Lateral diffusion of antiprotons. Time of evolution of antiproton distribution in plasma. Effects of drift rotation of a positron cloud ..... 244
10. Processes accompanying the stopping of antiprotons in the positron cloud. Transverse and longitudinal heat transfer in supermagnetized plasma ..... 244
11. Perturbation of positron density distribution ..... 246
12. Cooling of supermagnetized plasma as a result of cyclotron radiation ..... 247
13. Two-temperature model of supermagnetized plasma (SMP). Longitudinal - transverse relaxation in SMP ..... 248
14. Kinetic model of antiproton stopping in the positron plasma with regard to longitudinal - transverse relaxation. Analysis of recent experiments ..... 249
15. Conclusions ..... 252
16. Appendices ..... 253I. Rate of stimulated radiative recombination. Derivation of formula (77); II. Transverse transport of energyand momentum by plasmons
References ..... 255


#### Abstract

Research on the production of cold antihydrogen atoms, aimed at directly testing the CPT invariance and the equivalence principle for antimatter, is reviewed. The properties of cold positron and electron plasmas, in particular the processes accompanying antiproton stopping, are discussed. Mechanisms for the formation of antihydrogen atoms are analyzed. The most favorable conditions for the production and confinement of cold antihydrogen atoms are appraised.


[^0]
## 1. Introduction

Antihydrogen $\overline{\mathrm{H}} \equiv \overline{\mathrm{p}} \mathrm{e}^{+}$is the simplest atom of antimatter. For the first time, 11 such atoms with an energy $\sim 1 \mathrm{GeV}$ were produced in the antiproton storage ring LEAR at CERN in the experiment [1] which lasted two months. The purpose of the experiment [1] consisted exactly of the production and the proof of the existence of antihydrogen in nature. A beam of antiprotons collided with a xenon jet having a density $3 \times 10^{13}$ atom $\mathrm{cm}^{-2}$, which served as the internal target. The collision of the antiproton with the xenon nucleus resulted in the production of an electron-positron pair, the positron being produced in one of the bound states in the field of the antiproton - mainly in the 1S state. The cross section of this process is extremely small: $\sim 6 \times 10^{-33} \mathrm{~cm}^{2}$. Nevertheless, the examination of three distinguishing features of antihydrogen (the ability to penetrate magnetic fields, which is a consequence of its electric neutrality, and the annihilation of positrons and antiprotons in the material of the detector) allowed for the reduction of the number of events that can be attributed to background down to two out of the eleven (with a confidence level of $95 \%$ ). Subsequently, 30 atoms of antihydrogen were produced in a similar experiment in the Fermilab (FNAL) [2].

Currently, experiments are aimed at the production and study of antihydrogen atoms. The leader in this field is

CERN, where the AD (antiproton decelerator) installation is used for carrying out the experiments known as ATHENA and ATRAP [3, 4]. The main object of these experiments consists in direct verification of CPT invariance by comparing the spectra of hydrogen and antihydrogen atoms [5]. According to the CPT theorem proved in Refs $[6,7]$ under the assumptions of Lorentz invariance and the locality of interactions, the properties of matter and antimatter must be the same. In particular, the spectra of $\overline{\mathrm{H}}$ and H must be similar. In the Euclidean four-dimensional world, a body can be continuously rotated in such a way that it will coincide with a body produced from the original one by inversion (reflection with respect to a point) of all its parts. In such a world, hence, the PT transformation (the so-called 'weak reflection') is an exact symmetry. In the Minkowski space, such a continuous movement of the body is not possible, because its world line cannot cross the light cone. For this reason, the exact symmetry in the assumptions of $\operatorname{Refs}[6,7]$ is CPT rather than PT. The CPT violation would imply the Lorentz invariance violation, and would suggest a change of spacetime properties on a small scale [8-11]. In particular, the detection of such a violation could help resolve the problem of the absence of antimatter in the universe.

As of today, the most precise restriction on the degree of this symmetry breaking:

$$
\begin{equation*}
\frac{\left|m\left(\mathrm{~K}^{0}\right)-m\left(\overline{\mathrm{~K}}^{0}\right)\right|}{m\left(\mathrm{~K}^{0}\right)}<10^{-18}, \tag{1}
\end{equation*}
$$

has been derived from observations of oscillations in the system of neutral kaons [12, 13], which result from CP violation [14, 15]. Observe, however, that the result (1) is only indirect [16], because it is achieved under a number of additional assumptions: the validity of the superposition principle for kaons, the phenomenological model of CP violation, etc. This circumstance greatly increases the scientific value of experiments with antihydrogen.

The authors of Refs [17, 18] proposed carrying out experiments with fast ( $E \sim 50 \mathrm{MeV}$ ) antihydrogen atoms produced by recombination in the superimposed beams of antiprotons and positrons. This technique had been earlier developed by Parkhomchuk [19] who also measured Lamb's shift for states with the principal quantum number $n=2$ (similar experiments with $E \sim 30 \mathrm{keV}$ are described in Ref. [20]). The advantage of these experiments consists in the relative simplicity of producing antihydrogen atoms, the drawback being the short time the atom resides in the analyzing spectroscopic device. In this respect, the ATHENA and ATRAP experiments employ the opposite approach: the measurement of energy of certain radiative transitions in practically nonmoving antihydrogen atoms. This task can be tentatively divided into several stages. The first stage is associated with the development of techniques and the construction of equipment for storage of slow antiprotons and positrons. The second stage, which is currently under way, deals with the study of the mechanisms of recombination of positrons and antiprotons with the purpose of finding the optimal conditions for producing slow antihydrogen atoms. The third stage consists of the development of methods for confinement of the cold antihydrogen atoms. Finally, the last stage will be devoted to the spectroscopic studies of atoms.

Detailed reviews of ideas and achievements in this field can be found in Refs [21, 22]. A number of experiments
dealing with the proton stopping in a cloud of cold electrons [23] and the stopping of antiprotons in a cloud of positrons [24-26] have been carried out by now. It was found that protons and antiprotons do not cool down to low temperatures. Apart from that, it was not possible to observe an easily detected production of antihydrogen atoms. These mysterious circumstances stimulated the writing of this paper, which is mainly concerned with the detailed review of studies pertaining to the second stage. A critical analysis of the pertinent physical processes is necessary both for establishing the reason for the absence of recombinations in the abovementioned experiments (at the time of publication of this review the antihydrogen atoms have already been produced, see Appendix at the end of this paper), and for planning the studies at the third stage.

The AD installation (see the details in Refs [21, 22]) is the storage ring of antiprotons which are generated in a target bombarded by a beam of protons with a momentum of $26 \mathrm{GeV} / c$, coming from the proton synchrotron. The target is irradiated approximately once per minute with bunches containing $\sim 5 \times 10^{13}$ protons. Then a special pulse magnetic lens is used to form from the antiprotons born in the target a secondary bunch of $5 \times 10^{7}$ particles with a momentum of $3.57 \mathrm{GeV} / c$, which enters the AD . The antiprotons in AD are decelerated in three steps:

$$
3.57 \rightarrow 2 \rightarrow 0.3 \rightarrow 0.1 \mathrm{GeV} / c
$$

using the technique of electron cooling. The exit beam of antiprotons from the AD supplies about $\sim 5 \times 10^{7}$ antiprotons per minute to the experimental zone, and has a momentum spread of $\Delta p / p \sim 0.001$, energy 5.9 MeV (momentum $0.1 \mathrm{GeV} / c$ ), and pulses of length 200 to 500 ns at a repetition rate of several pulses per minute. After several stages of electron cooling and passage through stopping foils and gas layers, the antiproton energy is brought down to $\sim 3 \mathrm{keV}$. Then, by opening and closing the input electrostatic barrier (electrode A in Fig. 1), the antiprotons are captured in the Penning trap [27, 28]. Thus, the motion of antiprotons along the axis of the trap is blocked by applying a negative electric potential to electrodes A and B, while electrode C is grounded or carries a positive potential. In the transverse direction, the particles are confined by the homogeneous magnetic field

$$
\begin{equation*}
H \sim 3 \mathrm{~T} \tag{2}
\end{equation*}
$$

directed along the axis of the trap. The antiprotons captured in the trap are further cooled by the electrons. The electrons are cooled as a result of cyclotron radiation. By now such a


Figure 1. Penning trap (section in the plane passing through the axis of rotation).
technique of capturing antiprotons is well developed [ $3,4,21$, 22, 29]. Then, the potential on one of the stopping electrodes is reversed for a short time. As a result, the light (and therefore fast) electrons escape from the trap, while the antiprotons remain.

The Penning-Malmberg trap is also used for capturing and confining the charged particles [30]. It is a modification of the Penning trap and differs from the latter in the shape of the electrodes. The electrodes in this case are coaxial hollow cylinders placed in the uniform magnetic field directed along the axis (Fig. 2). There can be as many electrodes as necessary, which allows the creation of the desired profile of the electric potential along the axis. In particular, Fig. 3 shows the electric potential in the nested Penning trap [4]. The positrons are stored and confined in zone B, the antiprotons in zones A and C. To accomplish antiproton-positron recombination, the potentials in zones A and C are changed as shown by arrows in the diagram.


Figure 2. Penning-Malmberg trap.


Figure 3. Electric (negative) potential along the axis of the nested Penning trap.

The possible use of the combined traps or the so-called Paul-Penning traps is also considered [28,31]. In this case, an alternating potential at a frequency of several megahertz is applied, together with a constant potential, to the electrodes of the Penning trap designed for the confinement of antiprotons (Fig. 1). This idea consists in the use of the Kapitza effect (see, for example, paragraph 30 in monograph [32]) implying that in a rapidly oscillating field the particle possesses additional potential energy which is proportional to the square of the amplitude of the field. Since this potential energy is inversely proportional to the square of the mass of the particle, it is easy to choose such conditions under which it will confine the positrons and has practically no effect on the antiprotons. In this way, the Paul-Penning trap can be used for confining antiprotons and positrons at the same time. The obvious disadvantage of these traps lies in the fact that in the course of oscillations the positrons develop high velocities with respect to antiprotons, which hampers their mutual recombination.

Various methods of electron and positron accumulation in Penning traps have been developed by now [33-40]. As a
rule, their common feature is the utilization of radioisotopes as the source of positrons and the use of solid-state (in particular, single crystal) positron moderators serving as either reflecting or transmitting media. In this way, it is possible to produce positrons with an energy of $\sim 1 \mathrm{eV}$, whose number amounts to approximately $10^{-4}$ (the moderation efficiency) from the totality of positrons emitted by the radioactive source. According to Mills and Gullikson [41], the moderation efficiency increases tenfold when crystals of noble gases are used as moderators.

The decelerated positrons are stored in the Penning trap. In work [38] they were captured in the trap by the technique proposed in Refs [34, 35], which somewhat resembles the method of stochastic deceleration: the external oscillatory LCR circuit coupled to the trap is tuned in resonance with one of the periodical degrees of freedom of the magnetron drift motion of positrons in the fields of the trap. The positron energy attenuation achieved in this way is not strong, so it is only possible to capture a small portion of the positrons entering the trap.

In Refs [35, 36], the positrons loose their excess energy by exciting the nitrogen molecules admitted into the trap. Such a scheme, adopted in the ATHENA experiment [1], is characterized by a high ( $\sim 30 \%$ ) capture probability of positrons. In Ref. [37], the positron capture is performed with ions rather than with neutral molecules, and the authors of Ref. [40] propose using electrons for this purpose.

In Refs [42, 43] it was found that, along with the slow positrons, the atoms of positronium in the ground state and in the excited states escape from the moderators. This phenomenon was used by Estrada et al. [39] for realizing a new efficient method of positron capture in the nested Penning trap. According to the model adopted in Ref. [39], the atoms of positronium reach zone B (Fig. 3) and are ionized there by the strong electric field. With a large probability, the positrons remain in the potential well of zone $B$, whereas the electrons escape. As noted by Estrada et al. [39], this model calls for comprehensive theoretical analysis.

As indicated above, the main experimental task today is the detection of positron-antiproton recombination. At the next stage it will be necessary to construct a system capable of capturing and confining the produced atoms of antihydrogen. Best suited for this purposed is presumably the IoffePritchard trap [44]. A magnetic field $H$, whose magnitude attains its minimum $H_{0}$ in the middle (a maximum of the magnetic field in the space outside of the conductors is not possible, which follows from the Maxwell equations) is created in such a trap. For atoms of antihydrogen, in which the positron spin is directed against the magnetic field, this point corresponds to the minimum of the potential energy $u=M_{\mathrm{B}}\left(H-H_{0}\right)$. Such atoms may be confined in the IoffePritchard trap as long as their kinetic energy is low enough:

$$
\begin{equation*}
\varepsilon<u_{\max } \sim 1 \mathrm{~K}, \tag{3}
\end{equation*}
$$

where for the magnetic field we substituted the typical value (2) achieved in steady-state conditions. As far as the atoms with the opposite orientation of spin are concerned, they are not confined in the trap.

According to the estimates made in Refs [4, 45], for spectroscopic measurements it suffices to have

$$
\begin{equation*}
N_{\overline{\mathrm{H}}} \sim 1000 \tag{4}
\end{equation*}
$$

antihydrogen atoms in the Ioffe-Pritchard trap. Especially interesting is the measurement of frequency $\omega_{0}$ of the
transition $2 \mathrm{~s} \rightarrow 1 \mathrm{~s}$. This forbidden transition possesses a small natural width

$$
\frac{\Delta \omega_{0}}{\omega_{0}}=10^{-15}
$$

which is determined by the time $\tau=1 / \Delta \omega_{0}=0.125 \mathrm{~s}$ of the two-photon transition $2 \mathrm{~s} \rightarrow 1 \mathrm{~s}$. In these experiments, the signal-to-noise ratio can be minimized, allowing one to determine the position of the line center with an accuracy of $f \sim 10^{-3}$ of the line width. This implies that the study of the transition $2 \mathrm{~s} \rightarrow$ 1s allows us to achieve an accuracy of

$$
\begin{equation*}
\frac{\delta \omega_{0}}{\omega_{0}} \sim 10^{-18} \tag{5}
\end{equation*}
$$

which is comparable to the value (1) obtained for kaons. The measurement of $\omega_{0}$ will be tentatively based on the method of the two-photon Doppler-free spectroscopy [46, 47]. A laser beam of frequency $\omega \approx \omega_{0} / 2$ passes through the gas along the $x$-axis in the forward direction (beam 1), and, bounced from the mirror, goes back through the gas (beam 2). In the rest frame of a certain antihydrogen atom moving along the $x$-axis at velocity $v_{x}$, the frequencies of beams 1 and 2 are equal, respectively, to

$$
\omega_{1}=\omega-k v_{x}, \quad \omega_{2}=\omega+k v_{x}
$$

where $k=2 \pi c / \omega$ is the wave vector. These two photons are absorbed by the atom and transfer it from the 1s state into the 2 s state. Then, under the external electric fields, an impurity of a 2 p state is induced in such an atom, which results in a fast transition $2 \mathrm{p} \rightarrow 1 \mathrm{~s}$, and the emitted quantum is detected. We see that the velocity of the atom drops out from the condition of resonance $\omega_{1}+\omega_{2}=\omega_{0}$ in the linear approximation with respect to $v / c$. This implies that only the quadratic Doppler effect remains, which gives the transition width

$$
\begin{equation*}
\frac{\Delta \omega_{\mathrm{D}}}{\omega_{0}}=\left(\frac{v_{\mathrm{T}}}{c}\right)^{2}=10^{-12} \bar{T} \tag{6}
\end{equation*}
$$

where $v_{\mathrm{T}}=\sqrt{T / M}$ is the thermal velocity of the atom, $M$ is its mass, $c$ is the velocity of light, $\bar{T}=T / T_{0}$, with $T$ being the temperature of the gas, and $T_{0}=10 \mathrm{~K}$ [hereinafter we use the Hartree atomic units (a.u.) $\hbar=m_{\mathrm{e}}=e=1$, unless indicated otherwise]. By comparing Eqns (5) and (6) we see that in order to achieve the ultimate accuracy (5), the antihydrogen atoms must have a temperature of $T \sim 10^{-5} \mathrm{~K}$, and taking into account the above-mentioned factor $f$ we get a less arduous condition

$$
\begin{equation*}
T \sim 10^{-2} \mathrm{~K} \tag{7}
\end{equation*}
$$

The methods for cooling atoms to the temperature (7) are discussed by Walraven [44]. The ultimate temperature $T_{1}$ achievable by laser cooling is determined by the atomic recoil upon spontaneous emission of photons. For the transition $2 \mathrm{p} \rightarrow 1 \mathrm{~s}$ we get $T_{1}=0.003 \mathrm{~K}$, which is even lower than the required temperature (7).

Laser cooling of hydrogen atoms to a temperature of $T=0.008 \mathrm{~K}$ in the Ioffe-Pritchard trap was first accomplished in Refs [48, 49] using a hydrogen laser operating on the $2 \mathrm{p} \rightarrow 1 \mathrm{~s}$ transition. Doppler-free measurements of $2 \mathrm{~s} \rightarrow 1 \mathrm{~s}$ splitting in a hydrogen atom have also been done in
atomic beams [50, 51] and in the trap [52]. The resolution achieved was $\delta \omega_{0} / \omega_{0} \sim 3 \times 10^{-13}$ at an atomic temperature of approximately 1 K , which agrees with the boundary estimate (6).

After the production of antihydrogen atoms, the most serious task will be their capture and confinement in the Ioffe-Pritchard trap. The real scheme of this procedure was proposed by Walraven [44]: the overlap zone of the positron and the antiproton clouds, where the recombination takes place, must occur within a large Ioffe - Pritchard trap, near the minimum of the magnetic field. As will be shown in Sections $2-5$, the recoil of antihydrogen atoms in the course of recombination is not strong, and therefore $\varepsilon \sim T$, where $T$ is the temperature of the positron-antiproton plasma. Consequently, according to Eqn (3), the condition of capture and confinement of antihydrogen can be written in the form

$$
\begin{equation*}
T<u_{\max } \sim 1 \mathrm{~K} \tag{8}
\end{equation*}
$$

This condition becomes much less severe, viz.

$$
\begin{equation*}
T<100 \mathrm{~K} \tag{9}
\end{equation*}
$$

if we recall that the antihydrogen atoms $\overline{\mathrm{H}}_{n}$ are mainly born in the excited states, $n \sim 30$ (see Sections $2-5$ ), in which the orbital magnetic moment is quite large: $\mu_{z} \sim M_{\mathrm{B}} n^{2} \sim 1000 M_{\mathrm{B}}$. For such atoms at $\mu_{z}<0$, we have $u_{\max } \sim 1000 \mathrm{~K}$, and therefore, according to condition (9), they are not able to escape from the trap. At subsequent transitions (which mainly belong to the cascade radiative transitions), the condition $\mu_{z}<0$ is preserved with good accuracy, which ensures confinement of the atoms suffering deexcitation.

The main problem in this method of capture and confinement is the cooling of such atoms. Laser cooling will hardly be suitable (as a matter of fact, we believe that the same applies to the cooling of ground-state atoms under conditions of a real experiment with antihydrogen). According to simple preliminary estimates, antihydrogen can be cooled by the atoms of noble gases $\mathrm{X}=\mathrm{He}, \mathrm{Ne}, \mathrm{Ar}$, etc. Indeed, the annihilation cross section of a slow positron and the electrons of atom X is of the order of $\sigma_{1} \sim 10^{-22} \mathrm{~cm}^{2}$ [53]. Here we have taken into account the motion of electrons in the atom, and therefore for the relative velocity of electron and positron we take $v \sim 1$ a.u. The $\overline{\mathrm{p}}+\mathrm{p}$ annihilation cross section at $\beta=v / c \ll 1$ is $\sigma_{2} \sim 0.5 \times 10^{-27} \beta^{-2} \mathrm{~cm}^{2}[54-56]$, where $v$ is the relative velocity of colliding particles. With the impact parameter $\rho>1$, the trajectory in the system $\overline{\mathrm{p}}+\mathrm{X}$ is practically rectilinear because of shielding of the nuclear field by electrons. We are interested in the annihilation cross section in the system $\overline{\mathrm{p}}+\mathrm{X}$ at low energies $M v^{2} \ll 1$, where $M$ is the proton mass. For $\rho<1$, the antiproton approaching the atom X to distance $\sim 1$ a.u. is accelerated to the kinetic energy $\sim 1$ a.u. Hence we conclude that the annihilation cross section in the system $\bar{p}+X$ is of order

$$
\sigma_{3} \sim \sigma_{2}\left(M v^{2}=1\right) \sim 10^{-19} \mathrm{~cm}^{2}
$$

The annihilation cross section of particles in the system $\overline{\mathrm{H}}_{n}+\mathrm{X}$ is of the order of $\sigma_{\mathrm{a}}=\sigma_{1}+\sigma_{3} \approx \sigma_{3}$, and is therefore small compared with the elastic scattering cross section $\sigma_{\mathrm{e}} \sim 10^{-16} \mathrm{~cm}^{2}$ of these particles. If the polarization attraction is taken into account, both the cross sections increase, but their ratio remains more or less the same: $\sigma_{\mathrm{a}} / \sigma_{\mathrm{e}} \sim 10^{-3}$. Hence we conclude that annihilation in the course of cooling
can be neglected. The capture of protons in the reactions of the type

$$
\begin{equation*}
\overline{\mathrm{p}}+\mathrm{X} \rightarrow\left(\overline{\mathrm{p}} \mathrm{X}^{+}\right)+\mathrm{e}, \tag{10}
\end{equation*}
$$

characterized by the cross section $\sigma_{\mathrm{c}}$, can also be neglected [57-59]:

$$
\begin{equation*}
\frac{\sigma_{\mathrm{c}}}{\sigma_{\mathrm{e}}}<10^{-3} . \tag{11}
\end{equation*}
$$

The reason is that the electrons remain in the bound state at any distance $R$ between the antiproton and the nucleus of the noble gas atom. In particular, at $R=0$ in the case where $\mathrm{X}=\mathrm{He}, \mathrm{Ne}, \mathrm{Ar}, \mathrm{Kr}, \mathrm{Xe}$ we get respectively the negative ions $\mathrm{H}^{-}(0.75), \mathrm{F}^{-}$(3.4), $\mathrm{Cl}^{-}$(3.6), $\mathrm{Br}^{-}$(3.4), $\mathrm{I}^{-}$(3), where in parentheses we indicated the ionization potential $I$ of the outer electron (the electron affinity) in electron-volts, quoted from handbook [60]. In Refs [57-59], the capture cross section $\sigma_{c}$ was calculated using the adiabatic perturbation theory in the classical trajectory approximation for heavy particles. It was also demonstrated that all these approximations are valid, while the approximations used by Briggs et al. [61] lead to a considerable overestimation of the cross sections $\sigma_{\mathrm{c}}$. Thus, the estimate (11) is good for $\mathrm{X}=\mathrm{Ne}, \mathrm{Ar}, \mathrm{Kr}, \mathrm{Xe}$ owing to the large value of $I$. For helium, the situation is more complicated. Owing to the small values of $I$ and the electron excitation energies of the molecular ion ( $\overline{\mathrm{p}} \mathrm{H}$ ), the process (10) follows a different mechanism characterized by the reaction

$$
\begin{equation*}
\overline{\mathrm{p}}+\mathrm{He} \rightarrow(\overline{\mathrm{p}} \mathrm{He})^{*} \rightarrow(\overline{\mathrm{p}} \mathrm{He})^{+}+\mathrm{e} . \tag{12}
\end{equation*}
$$

As the antiproton approaches a helium atom, the outer electron goes to the excited state (for $R \rightarrow \infty$, this state transforms to $\overline{\mathrm{p}}+\mathrm{He}^{*}$ ). For a short time, the antiproton turns out to be in the bound state. Then this state is stabilized by the emission of the electron. By nature, the reaction (12) is close to the dielectron recombination known in plasma physics. In scientific literature it would have been referred to as 'capture by Feshbach resonances'. We are not aware of the existence of calculations pertinent to this reaction. A rough estimate for the cross section of reaction (12) gives

$$
\begin{equation*}
\sigma_{\mathrm{c}} \sim 10^{-17} \mathrm{~cm}^{2} \tag{13}
\end{equation*}
$$

which is an order of magnitude greater than the results reported in Refs [57-59]. The estimate (13) agrees with the results of experiments performed by Kottmann [62] on the 'muon bottle' at low pressures $p=6$ Torr in the mixtures $\mathrm{He}+\mathrm{H}_{2}$.

On the strength of arguments developed above we may conclude that neon is preferable to helium for cooling antihydrogen.

According to the theorem proved by O'Neil [63], the cloud of charged particles in the trap will be stable in the presence of ideal axial symmetry. In the Ioffe - Pritchard trap [64], such a symmetry is lacking, and therefore the problem of stability of the antiproton-positron plasma in the inhomogeneous magnetic field of such a trap calls for special investigation. One might expect that any deviation from axial symmetry will cause gradual drift spreading of plasma in the direction transverse to the magnetic field. The first results of such an analysis of plasma stability in the Ioffe-Pritchard trap are encouraging: they indicate that plasma is stable owing to the existence of stable trajectories of charged particles in the field of this trap.

It would be natural to expect that the production of antihydrogen will open new fields of research. In addition to the validation of CPT invariance, we should mention verification of the equivalence principle for antimatter [3, 4, $16,21,22$ - which means answering the question of whether antihydrogen falls with the same gravitational acceleration as hydrogen. It should be emphasized that such experiments cannot be staged with antiprotons because the forces caused by accidental electric fields are much stronger than gravity force. Gravitational experiments with antihydrogen have not yet been worked out in detail; certain suggestions can be found in Refs [65-68]. In particular, in Ref. [65] it was proposed to measure the equilibrium density distribution of atoms throughout the height of the trap, and use the Boltzmann formula for calculating the acceleration of gravity for antihydrogen.

It could be argued that since CPT invariance implies symmetry between matter and antimatter, then it follows that matter and antimatter will fall in the terrestrial field with the same acceleration, namely there is no need to check the equivalence principle. Such a statement, however, is not correct. CPT invariance assumes that this transformation is applied not to a particular object, but to the entire universe. Feasibility of CPT invariance says that the acceleration of gravity will be the same when antimatter falls on the antiEarth. Here, a certain reservation is due - the validity of this last statement requires that the gravitational field described by a tensor of second rank (spin two) was CPT invariant and renormalizable. It is known, however, that this field defies renormalization.

The fact that the CPT theorem is applied to the whole universe can be illustrated with a simple example. Consider a speck of dust floating in an isolated flask filled with gas. For the sake of simplicity, we just want to check whether this system features T invariance (also known as time reversibility). The floating particle experiences the Stokes viscous force which tends to stop the particle. The equation of motion of the particle, which includes this force, is expressly irreversible, which means that T invariance is violated in this equation. In practice, this means that if at some time $(t=0)$ we reverse the particle velocity, the particle will not move with acceleration, as might be deduced from the principle of time reversibility. Rather, it will slow down again. The fault in this reasoning is as follows: in order to check T invariance at $t=0$, we should also reverse the velocities of all molecules in the flask. Then the particle will accelerate, i.e. T invariance demonstrates its practicability. In real life, however, such an experiment cannot be staged, because it is not possible to achieve complete thermal and mechanical isolation of the flask. Even a very weak external influence on the flask, taking into consideration the Lyapunov instability of the trajectories of the colliding particles, will lead to exponential growth of the deflections from these trajectories. As a result, even if we manage to reverse the velocities of all molecules in the flask, a speck of dust will still be decelerated. Curiously enough, these nontrivial circumstances are from the outset taken into account by the Navier-Stokes equations which must be used for solving the problem of the motion of the probe particle. The contemporary views on the problem of irreversibility in nature are presented in detail in book [69].

This brings us to the end of the introductory section, and we embark on discussing the physics of production of cold antihydrogen atoms.

## 2. Radiative recombination. Standard theory

This section is in a certain sense auxiliary, and actually serves as a prelude to Section 3. Here we try to highlight on a qualitative level some features of the process of radiative recombination in the absence of a magnetic field $(H=0)$. This will help us to comprehend the rather complicated physics of radiative recombination in a strong magnetic field.

Consider first the process of radiative recombination (RR)

$$
\begin{equation*}
\mathrm{e}^{+}+\overline{\mathrm{p}} \rightarrow \overline{\mathrm{H}}_{n l}+\gamma \tag{14}
\end{equation*}
$$

with the formation of highly excited Rydberg states

$$
\begin{equation*}
n \gg 1 . \tag{15}
\end{equation*}
$$

Of special interest for the production of antihydrogen is the case of low positron energy

$$
\begin{equation*}
\varepsilon=\frac{p^{2}}{2}=\frac{v^{2}}{2} \ll 1, \tag{16}
\end{equation*}
$$

to which the Kramers approximation can be applied [70-72]. The slow positron is captured by an antiproton at the impact parameters

$$
\begin{equation*}
\rho \sim \frac{n^{2 / 3}}{v} \tag{17}
\end{equation*}
$$

and angular momenta

$$
\begin{equation*}
l \sim n^{2 / 3} \ll n, \tag{18}
\end{equation*}
$$

which are characteristic of the long-range Coulomb attraction between these particles. Then the positron follows the near-parabolic trajectory, approaching the antiproton to the distance

$$
\begin{equation*}
r_{\mathrm{m}} \sim n^{4 / 3} \tag{19}
\end{equation*}
$$

and accelerating to the kinetic energy

$$
\begin{equation*}
\varepsilon_{\mathrm{k}} \sim n^{-4 / 3} . \tag{20}
\end{equation*}
$$

Remaining within this region for the period of time

$$
t_{0} \sim \frac{r_{\mathrm{m}}}{\sqrt{\varepsilon_{\mathrm{k}}}} \sim n^{2},
$$

the positron experiences there the highest acceleration, emits a photon with a frequency

$$
\begin{equation*}
\omega=\frac{p^{2}}{2}+\frac{1}{2 n^{2}} \approx \frac{1}{2 n^{2}} \sim \frac{1}{t_{0}} \tag{21}
\end{equation*}
$$

and is captured in the highly elongated Keplerian elliptical orbit with a bond energy

$$
\begin{equation*}
I_{n}=\frac{1}{2 n^{2}}, \tag{22}
\end{equation*}
$$

a major semiaxis

$$
\begin{equation*}
R_{n}=n^{2}, \tag{23}
\end{equation*}
$$

and an eccentricity $e$ :

$$
\begin{equation*}
1-e \approx \frac{l^{2}}{2 n^{2}} \sim n^{-2 / 3} \ll 1 . \tag{24}
\end{equation*}
$$

According to the quantum-mechanical Fermi golden rule [53, 73], the cross section of radiative recombination (14) into the state $(n l)$ is given by

$$
\begin{equation*}
\sigma_{\mathrm{R}}(n l)=\frac{4 \omega^{3}}{3 c^{2} v}(2 l+1) d^{2} \tag{25}
\end{equation*}
$$

where the notation is introduced:

$$
\begin{align*}
& d^{2}=\frac{1}{2 l+1} \sum_{m}\left|\mathbf{r}_{\mathrm{f}}\right|^{2},  \tag{26}\\
& \mathbf{r}_{\mathrm{fi}}=\langle n l m| \mathbf{r}|\mathbf{p}\rangle .
\end{align*}
$$

The total cross section of radiative recombination is defined by the expressions

$$
\begin{align*}
& \sigma_{\mathrm{R}}=\sum_{n l} \sigma_{\mathrm{R}}(n l)=\sum_{n} \frac{4 \omega^{3}}{3 c^{3} v} B \approx \int \frac{4 \omega^{3} n^{3}}{3 c^{3} v} B \mathrm{~d} \omega, \\
& B=\sum_{l}(2 l+1) d^{2} . \tag{28}
\end{align*}
$$

In devising formula (27), we used the summation rule

$$
\begin{equation*}
\sum_{n}=\int \mathrm{d} n=\int n^{3} \mathrm{~d} \omega \tag{29}
\end{equation*}
$$

which holds true in domain (15).
From Eqns (20), (21), and (15) we deduce
$\omega \ll \varepsilon_{\mathrm{k}}$.

Together with constraint (15), this ensures the validity of classical electrodynamics [74], according to which

$$
\begin{equation*}
\sigma_{\mathrm{R}}=\int \frac{16 \pi}{3 \sqrt{3} c^{3} v^{2}} \frac{\mathrm{~d} \omega}{\omega} \tag{31}
\end{equation*}
$$

[as a matter of fact, however, Eqn (30) follows from Eqn (15), and so formula (15) represents actually the only assumption]. Comparing Eqns (27) and (31), we find the expression for a square of the transition matrix element

$$
\begin{equation*}
B=\frac{4 \pi}{\sqrt{3} \omega^{4} n^{3} v}, \tag{32}
\end{equation*}
$$

which we shall need shortly.
Cross sections of radiative recombination into states ( $n l$ ) and $n$ are

$$
\begin{align*}
& \sigma_{\mathrm{R}}(n l)=\frac{2 \pi(2 l+1) Q}{3 c^{3} v^{2} n^{7 / 3}},  \tag{33}\\
& \sigma_{\mathrm{R}}(n)=\sum_{l} \sigma_{\mathrm{R}}(n l)=\frac{4 \omega^{3} B}{3 c^{3} v}=\frac{16 \pi}{3 \sqrt{3} c^{3} \omega n^{3} v^{2}} . \tag{34}
\end{align*}
$$

Here, the notation was introduced:

$$
\begin{aligned}
& Q=\left[\phi^{\prime}(z)\right]^{2}+z \phi^{2}(z)=\frac{\mathrm{d}\left(\phi \phi^{\prime}\right)}{\mathrm{d} z} \\
& z=l^{2}(2 n)^{-4 / 3}
\end{aligned}
$$

where $\phi(z)$ is the Airy function defined as [74, 75]

$$
\begin{equation*}
\phi(z)=\frac{1}{\sqrt{\pi}} \int_{0}^{\infty} \mathrm{d} t \cos \left(z t+\frac{1}{3} t^{3}\right) . \tag{35}
\end{equation*}
$$

From formulas (25) and (33) for the cross sections we conclude that in the Kramers approximation we have

$$
\begin{equation*}
d^{2}=\frac{2^{17 / 3} \pi n^{11 / 3}}{v} Q . \tag{36}
\end{equation*}
$$

Within the positron plasma there is an electric field that can ionize the emerging atoms. Because of shielding, the component of the electric field $E_{\|}$parallel to the magnetic field is small. The positrons are magnetized; they move along the magnetic field lines, and their transverse motion is hampered. Because of this, shielding in the crosswise direction is absent, and the transverse electric field is nonzero. Its typical strength in the experiments is

$$
\begin{equation*}
E_{\perp} \sim 30 \frac{\mathrm{~V}}{\mathrm{~cm}} \tag{37}
\end{equation*}
$$

This field destroys the emerging atom if the inequality

$$
E_{\perp} R_{n}>\frac{1}{2 n^{2}}
$$

is satisfied, i.e., for

$$
\begin{equation*}
n>n_{c} \approx 100 \tag{38}
\end{equation*}
$$

In the coordinate system comoving the atom, there is an electric field

$$
E_{\mathrm{L}} \sim \frac{v_{\mathrm{T}}}{c} H \sim 3 \frac{\mathrm{~V}}{\mathrm{~cm}} .
$$

Since $E_{\mathrm{L}}<E_{\perp}$, the ionization caused by this field (Lorentz ionization) is insignificant in the states with $n<n_{\mathrm{c}}$ that are stable with respect to $E_{\perp}$.

From Eqns (27), (32), and (38) we conclude that the total cross section of radiative recombination is equal to

$$
\begin{equation*}
\sigma_{\mathrm{R}}=\frac{32 \pi \Lambda}{3 \sqrt{3} c^{2} v^{2}} \tag{39}
\end{equation*}
$$

where

$$
\begin{aligned}
& \Lambda=\sum_{n<n_{\mathrm{c}}} \frac{1}{n\left(1+n^{2} v^{2}\right)} \approx \ln n_{1}, \\
& n_{1}=\min \left(n_{\mathrm{c}} ; \frac{1}{\sqrt{T}}\right) .
\end{aligned}
$$

For $n \gg 1$, formula (34) coincides with the quantummechanical result [76]. At $n=1$, from Eqn (34) we get

$$
\sigma_{\mathrm{R}}(1)=A c^{-3} v^{-2}, \quad A=19.3
$$

whereas the exact formula yields $A=15.4[77,53]$. We see that even at $n=1$ the expression (34) gives an adequate result.

As concerns the error in expression (39), it is logarithmic and approximates $1 / \Lambda$.

The radiative recombination coefficient (in units of the CGS electrostatic system) is written as

$$
\begin{equation*}
\alpha_{\mathrm{r}}=\left\langle v \sigma_{\mathrm{R}}\right\rangle=\frac{32 \sqrt{2 \pi} \alpha r_{\mathrm{e}}^{2} c^{2} \Lambda}{3 \sqrt{3} v_{\mathrm{Te}}}, \tag{40}
\end{equation*}
$$

and for the Maxwellian positron distribution function at plasma temperatures

$$
T=10^{-4}, \quad 10^{-2}, \quad 1 \mathrm{eV}
$$

it gives respectively

$$
\alpha_{r}=1 \times 10^{-10}, \quad 1 \times 10^{-11}, \quad 1 \times 10^{-12} \mathrm{~cm}^{3} \mathrm{~s}^{-1}
$$

In formula (40), $\alpha=e^{2} /(\hbar c) \approx 1 / 137, r_{\mathrm{e}}=e^{2} /\left(m c^{2}\right), m$ is the electron mass, $v_{\mathrm{Te}}=\sqrt{T / m}$, and $\Lambda=\ln \left(E_{0} / T\right)^{1 / 2}$.

## 3. Effects of a magnetic field on radiative recombination

In the absence of a magnetic field $(H=0)$, the process of radiative recombination is governed by two characteristic scales: the Thomson radius $R_{\mathrm{T}}=e^{2} / \varepsilon_{\mathrm{k}}$, and the Bohr radius $a_{0}=\hbar^{2} /\left(m_{\mathrm{e}} e^{2}\right)$. The latter arises naturally in the consideration of radiative recombination to the lower levels $n \sim 1$ : such transitions provide the main contribution to $\sigma_{\mathrm{R}}$. A positron following the trajectory with the impact parameter

$$
\rho=\left(r_{\mathrm{m}} R_{\mathrm{T}}\right)^{1 / 2}
$$

nears the antiproton to the distance of closest approach, $r_{\mathrm{m}}$, after which it recedes to infinity with the probability close to 1. With some small probability $W_{\mathrm{R}}$, radiative recombination to levels $n \sim 1$ takes place on this trajectory, therefore

$$
r_{\mathrm{m}} \sim R_{n} \sim a_{0}, \quad \sigma_{\mathrm{R}} \sim \pi \rho^{2} W_{\mathrm{R}} .
$$

In the region $r \sim r_{\mathrm{m}} \sim 1$, the positron moves with a velocity $\sim 1$, with an acceleration $\sim 1$, and for a period of time $t_{0} \sim 1$, and it produces radiation with an intensity [74]

$$
I \sim \frac{(\ddot{\mathbf{d}})^{2}}{c^{3}} \sim \frac{1}{c^{3}} .
$$

It emits a photon of frequency $\omega \sim 1 / t_{0}$ with the probability

$$
W_{\mathrm{R}} \sim I \frac{t_{0}}{\hbar \omega} \sim \frac{1}{c^{3}} \sim \alpha^{3},
$$

where $\alpha=e^{2} /(\hbar c) \approx 1 / 137$. Therefore, in accordance with formula (34), one obtains

$$
\sigma_{\mathrm{R}} \sim \alpha^{3} a_{0} R_{\mathrm{T}} \sim c^{-3} v^{-2}
$$

Transitions to the levels with $n \gg 1$ add to this formula an extra factor $\Lambda \sim 2-3$.

In the presence of a magnetic field there is a third characteristic scale, the Larmor radius, which describes the cyclotron motion of the positron:

$$
\begin{equation*}
r_{H}=\frac{v_{\perp}}{\omega_{H}} \sim 500 \tag{41}
\end{equation*}
$$

where $\omega_{H}=e H /\left(m_{\mathrm{e}} c\right) \sim 10^{-5}$ is the Larmor frequency. Hereinafter we make numerical estimates for typical experimental values defined by expression (2), and also for

$$
\begin{equation*}
T \sim 10 \mathrm{~K}, \quad n_{\mathrm{e}^{+}} \equiv n_{\mathrm{e}} \sim 10^{8} \mathrm{~cm}^{-3} . \tag{42}
\end{equation*}
$$

Under these conditions, the following relationships are satisfied:

$$
r_{H} \gtrdot a_{0}, \quad R_{\mathrm{T}} \gtrdot a_{0},
$$

and therefore only two limiting cases are possible:

$$
\begin{align*}
& a_{0} \ll R_{\mathrm{T}} \ll r_{H},  \tag{43}\\
& a_{0} \ll r_{H} \ll R_{\mathrm{T}} . \tag{44}
\end{align*}
$$

The results obtained in Section 2 relate to the case (43) characteristic of conventional hot plasma. The magnetic field in these conditions has very little effect on the process of radiative recombination [78]. Condition (44), however, holds true in the experiments with antihydrogen. Such a plasma displays certain unusual features that distinguish it sharply from hot plasma. We shall call such plasma the 'supermagnetized plasma' (SMP), since the term 'magnetized plasma' is already in use and refers to the case $[79,80]$

$$
\begin{equation*}
\frac{\omega_{H}}{v_{\mathrm{e}}} \gg 1, \quad r_{H} \gg R_{\mathrm{T}}, \tag{45}
\end{equation*}
$$

where $v_{\mathrm{e}}=v_{\mathrm{ee}}+v_{\mathrm{ei}}$ is the total collision frequency of positrons in the processes $\mathrm{e}^{+}+\mathrm{e}^{+}, \mathrm{e}^{+}+\overline{\mathrm{p}}$. Observe that in the experiments in question the plasma is considered as ideal:

$$
\begin{equation*}
f=\frac{e^{2} n_{\mathrm{e}}^{1 / 3}}{T} \ll 1, \tag{46}
\end{equation*}
$$

and therefore

$$
\frac{v_{\mathrm{e}}}{\omega_{H}}=\frac{r_{H}}{R_{\mathrm{T}}} f^{3} \ll 1,
$$

which means that condition (45) is fulfilled with certainty if condition (44) is met.

At this point it is worth noting that, unlike our case, the method of electron cooling of beams [81-83] deals with nonideal plasma, which is due to the small typical values of the temperature for the longitudinal motion of electrons.

The smallness of the Larmor radius (44) means that the magnetic field considerably changes the pattern of binary collisions in SMP. The particles move along the magnetic field lines ( $z$-axis) like strung beads. The characteristic length of free motion of the particle (mean free path) approximates

$$
\lambda \sim \frac{1}{n_{\mathrm{e}} R_{\mathrm{T}}^{2}} .
$$

By virtue of Eqn (46), one finds

$$
\frac{\lambda}{R_{\mathrm{T}}} \gg 1,
$$

which means that collisions are infrequent, and for the most part the particles move freely.

Let us consider the collision of a positron with an antiproton. Owing to its large mass, the latter may be regarded as immobile and fixed at the origin. In the
cylindrical coordinates $(z, r, \varphi)$, the Lagrangian of a positron moving in a homogeneous magnetic field $\mathbf{H}$ and an electric field $\mathbf{E}$ of the antiproton with a charge $z_{0}=1$ assumes the form

$$
\begin{equation*}
L_{0}=\frac{1}{2} \dot{r}^{2}+\frac{1}{2} \dot{z}^{2}+\frac{1}{2} r^{2} \dot{\varphi}^{2}+\frac{1}{2} \omega_{H} r^{2} \dot{\varphi}+\frac{z_{0}}{\sqrt{r^{2}+z^{2}}} \tag{47}
\end{equation*}
$$

where $r=\sqrt{x^{2}+y^{2}}$, and $\varphi$ is the azimuth angle $(\tan \varphi=y / x)$.

According to Eqn (47), the canonical momentum is conserved, viz.

$$
\begin{equation*}
L=\frac{\partial L_{0}}{\partial \dot{\varphi}}=\frac{1}{2} \omega_{H} r^{2}+r^{2} \dot{\varphi}=\text { const }, \tag{48}
\end{equation*}
$$

along with the energy

$$
\begin{equation*}
E=\frac{1}{2} p_{r}^{2}+\frac{1}{2} p_{z}^{2}+u(r, z)=\text { const } \tag{49}
\end{equation*}
$$

where

$$
\begin{equation*}
u(r, z)=\frac{1}{2}\left(\frac{L}{r}-\frac{1}{2} \omega_{H} r\right)^{2}-\frac{z_{0}}{\sqrt{r^{2}+z^{2}}} \tag{50}
\end{equation*}
$$

is the effective potential energy, $p_{r}=\dot{r}$, and $p_{z}=\dot{z}$.
When $t \rightarrow-\infty$, the trajectory of the positron colliding with an antiproton is described by the equations

$$
\begin{align*}
& x=r_{0}+r_{H} \cos \left(\omega_{H} t+\varphi_{0}\right), \\
& y=-r_{H} \sin \left(\omega t+\varphi_{0}\right),  \tag{51}\\
& z=v_{\|} t .
\end{align*}
$$

Calculating $\phi$ from Eqn (51), and comparing the result with formula (48), we get the expression

$$
\begin{equation*}
L=\frac{1}{2} \omega_{H}\left(r_{0}^{2}-r_{H}^{2}\right)=\text { const }, \tag{52}
\end{equation*}
$$

which relates the momentum $L$ to the position $r_{0}$ of the center of the initial positron orbit.

Formulas (49) and (50) describe the two-dimensional motion of a positron in the $(r, z)$ half-plane $(r>0$, $-\infty<z<+\infty)$. At large values of the parameter $r_{0}$, namely

$$
\begin{equation*}
r_{0} \gg \rho_{0}, \quad \rho_{0}=z_{0}^{1 / 3}\left(\omega_{H}\right)^{-2 / 3} \sim 2000 \tag{53}
\end{equation*}
$$

the positron executes oscillatory - translatory motions in the Larmor orbit in the ( $r, z$ ) plane (see Figs 4 and 5).

Observe that $\rho_{0}=\left(r_{H}^{2} R_{\mathrm{T}}\right)^{1 / 3}$, and therefore

$$
\begin{equation*}
r_{H} \ll \rho_{0} \ll R_{\mathrm{T}} . \tag{54}
\end{equation*}
$$

In the conditions specified by Eqn (53), the positron drifts in the crossed $\mathbf{E}$ and $\mathbf{H}$ fields with the velocity [naturally, this follows from equations (48), (49)]

$$
\begin{equation*}
\mathbf{v}_{\mathrm{D}}=\frac{c(\mathbf{E} \times \mathbf{H})}{H^{2}}, \tag{55}
\end{equation*}
$$

which leads to the displacement of the center of the Larmor orbit by

$$
\begin{equation*}
\delta \mathbf{r}=\frac{2 z_{0}}{\omega_{H} v_{\|} r_{0}} \hat{\varphi}, \tag{56}
\end{equation*}
$$



Figure 4. Effective potential $u(r, z)$ vs. $r$ with fixed $z$. Curve $l$ corresponds to $r_{0} \gg \rho_{0}$, curve 2 to $r_{0} \sim \rho_{0}$.


Figure 5. Positron trajectories in the field of an antiproton in supermagnetized plasma. Curve $l$ corresponds to $r_{0} \gg \rho_{0}$, curve 2 to $r_{0} \sim \rho_{0}$.
where $\hat{\varphi}$ is the unit vector in the azimuthal direction. Notice that at the distant collisions (53), with the accuracy [32]

$$
\begin{equation*}
\sim \exp \left(-a \frac{r_{0}}{\rho_{0}}\right), \quad a \sim 1 \tag{57}
\end{equation*}
$$

the adiabatic invariant is conserved [32,79]:

$$
\begin{equation*}
I_{0}=\frac{v_{\perp}^{2}}{H} \approx \text { const } . \tag{58}
\end{equation*}
$$

In the experiments in question [3, 4, 21], the magnetic field within the confines of the positron cloud is highly homogeneous:

$$
\begin{equation*}
\frac{\Delta H}{H}<10^{-4} \tag{59}
\end{equation*}
$$

which is necessary for ensuring the stability of the positron cloud, and therefore from Eqn (58) it follows that

$$
\begin{equation*}
v_{\perp}^{2} \approx \text { const } \tag{60}
\end{equation*}
$$

In the case of

$$
\begin{equation*}
r_{0} \leqslant \rho_{0}, \tag{61}
\end{equation*}
$$

the collision pattern is changed dramatically [84]: upon reaching region ' b ' (Fig. 5), the positron motion becomes stochastic. At first, the positron moves in the region ' $a$ ',
executing Larmor motion. Getting into 'box b', the positron all but completely 'forgets' through which entrance ('a' or 'c') it had initially got there. It leaves 'box b' through 'openings' ' $a$ ' or ' $c$ ' with about equal probability $1 / 2$. The adiabatic invariant (58) is not conserved in this case, and therefore the collision alters dramatically both the velocity $v_{\perp}$ and the Larmor orbit radius $r_{H}$ :

$$
\begin{equation*}
\Delta v_{\perp} \sim v_{\perp} \sim v, \quad \Delta r_{H} \sim r_{H} \tag{62}
\end{equation*}
$$

Here we have taken into account the energy conservation:

$$
\begin{equation*}
\Delta\left(v^{2}\right)=0, \quad v^{2}=v_{\perp}^{2}+v_{\|}^{2} \tag{63}
\end{equation*}
$$

From the conservation of momentum (52) it follows that in the case (61) one has

$$
\begin{equation*}
r_{0} \Delta r_{0}=r_{H} \Delta r_{H}, \quad \Delta r_{0} \sim \frac{r_{H}^{2}}{\rho_{0}} . \tag{64}
\end{equation*}
$$

The motion of the positron inside 'box b' is stochastic. To get out of the box, according to relationships (64), the positron must hit the area element

$$
S_{\mathrm{e}} \sim \rho_{0} \Delta r_{0} \sim r_{H}^{2}
$$

on the surface of 'box b', the area of which is

$$
S_{\mathrm{b}} \sim \rho_{0}^{2}
$$

Accordingly, the positron performs inside 'box b' oscillations whose number is of order

$$
\begin{equation*}
N_{0} \sim \frac{S_{\mathrm{b}}}{S_{\mathrm{e}}} \sim\left(\frac{R_{\mathrm{T}}}{r_{H}}\right)^{2 / 3} \sim \frac{\left(z_{0} \omega_{H}\right)^{2 / 3}}{T} \tag{65}
\end{equation*}
$$

after which it escapes either through exit ' $c$ ' or through entrance 'a' with approximately the same probability $(\approx 1 / 2)$. The characteristic kinetic energy $K_{0}$ and velocity $v_{0}$ of the positron inside 'box b' are respectively

$$
\begin{equation*}
K_{0} \sim \frac{z_{0}}{\rho_{0}}, \quad v_{0} \sim \frac{z_{0}}{\rho_{0}^{1 / 2}} \sim\left(z_{0} \omega_{H}\right)^{1 / 3} . \tag{66}
\end{equation*}
$$

The particle residence time in the box approximates

$$
\begin{equation*}
\tau_{\mathrm{c}} \sim \frac{\rho_{0}}{v_{0}} N_{0} \sim \frac{z_{0}^{2 / 3}}{\omega_{H}^{1 / 3} T}, \tag{67}
\end{equation*}
$$

and the positron-antiproton recombination rate (the probability per unit time) is given by

$$
\begin{equation*}
\lambda_{\mathrm{G}} \equiv \alpha_{\mathrm{eff}} n_{\mathrm{e}} \sim \rho_{0}^{2} v_{\mathrm{T}} n_{\mathrm{e}} W_{\mathrm{R}} \tag{68}
\end{equation*}
$$

Here, $\alpha_{\text {eff }}$ is the recombination rate constant, $v_{\mathrm{T}}=\sqrt{T}$ is the thermal velocity of the positrons, the quantity

$$
\begin{equation*}
W_{\mathrm{R}} \sim \sigma_{\mathrm{R}}\left(v_{0}\right) v_{0} n_{\mathrm{eff}} \tau_{\mathrm{c}} \tag{69}
\end{equation*}
$$

is the recombination probability of the positron captured in 'box b', $n_{\text {eff }} \sim 1 / \rho_{0}^{3}$ is the effective concentration of positrons in the box, and $\sigma_{\mathrm{R}}\left(v_{0}\right)$ is the radiative recombination cross section at a positron velocity $v_{0}$. According to formula (39), one finds

$$
\sigma_{\mathrm{R}}\left(v_{0}\right)=\sigma_{\mathrm{R}}\left(v_{\mathrm{T}}\right) \frac{v_{\mathrm{T}}^{2}}{v_{0}^{2}}
$$

From this and estimates (65)-(69), we conclude that

$$
\begin{equation*}
\alpha_{\mathrm{eff}} \sim \alpha_{\mathrm{r}} \sim v_{\mathrm{T}} \sigma_{\mathrm{R}}\left(v_{\mathrm{T}}\right), \tag{70}
\end{equation*}
$$

where $\alpha_{\mathrm{r}}$ is defined by formula (40), and $\sigma_{\mathrm{R}}(v)$ by formula (39).

Thus we see that the positron confined in 'box b' has many opportunities ( $\sim N_{0}$ ) for radiative recombination. However, the positron here moves at a high velocity, and therefore the recombination cross section is small. As a result of the mutual compensation of these effects, in plasma with the parameters (44) we obtain

$$
\begin{equation*}
\frac{\alpha_{\mathrm{eff}}}{\alpha_{\mathrm{r}}} \sim 1 . \tag{71}
\end{equation*}
$$

Obviously, in conventional hot plasma (43) we have

$$
\begin{equation*}
\frac{\alpha_{\mathrm{eff}}}{\alpha_{\mathrm{r}}}=1 . \tag{72}
\end{equation*}
$$

In the general case, this ratio is a function of the dimensionless parameter

$$
\begin{equation*}
\frac{r_{H}}{R_{\mathrm{T}}}=\frac{T}{T_{1}}, \quad T_{1}=\left(z_{0} \omega_{H}\right)^{2 / 3} . \tag{73}
\end{equation*}
$$

This parameter is composed of two characteristic scales entering relations (43) and (44), which in turn characterize the classical motion of the positron in 'box b'.

So, in the general case one can write down the relationship

$$
\begin{equation*}
\frac{\alpha_{\mathrm{eff}}}{\alpha_{\mathrm{r}}}=F\left(\frac{T}{T_{1}}\right) . \tag{74}
\end{equation*}
$$

According to formulas (71) and (72), it is readily shown that

$$
F(0) \sim 1, \quad F(\infty)=1
$$

(to avoid a misunderstanding, please note that here we use the values of the argument defined in formula (74): for example, the zero argument corresponds to $H=\infty$ ). Dependence (74) agrees with the results of the experiment [85] concerned with the radiative recombination on bare ions in the systems of cooling the ion beams by electrons (coolers) in a strong magnetic field (it should be emphasized that the mechanism of dielectronic recombination prevails for ions that have electrons in their shells).

For ions $\mathrm{C}^{6+}\left(z_{0}=6\right)$ in the magnetic field $H=0.042 \mathrm{~T}$, from formula (73) we get $T_{1}=0.003 \mathrm{eV}$, which agrees with Fig. 1b from Ref. [85]. From this figure it also follows that

$$
\begin{equation*}
F(0) \approx 2, \quad F(\infty)=1 \tag{75}
\end{equation*}
$$

Thus, owing to the stochastic motion of a positron in the collisions with an antiproton with impact parameters (61), the recombination coefficient $\alpha_{\text {eff }}$ in experiments on the production of cold antihydrogen will be twice the value calculated by formula (40).

Of considerable interest is the theoretical calculation of the function $F$, which can be done by the Monte Carlo method.

## 4. Stimulated radiative recombination

Stimulated radiative recombination (SRR) is discussed in Refs [86-89], where it is proposed to raise the rate of radiative recombination using the effect of stimulated radiation:

$$
\begin{equation*}
\overline{\mathrm{p}}+\mathrm{e}^{+}+N \hbar \omega \rightarrow \overline{\mathrm{H}}_{n l}+(N+1) \hbar \omega, \tag{76}
\end{equation*}
$$

which takes place when a positron recombines with an antiproton in the field of a laser beam. In this section we discuss the current status of research on SRR, and present the details of the kinetics of this process. We also discuss the effect of diffusion ionization of the emerging atoms with the laser beam, which previously had been left out of the consideration.

The probability of SRR on one antiproton per unit time (the rate of SRR), according to the Fermi 'golden rule', is defined (see Appendix I) as

$$
\begin{equation*}
\lambda_{\mathrm{S}}(n l)=\frac{2 I v}{3 c} d^{2} n_{\mathrm{e}} f(v)(2 l+1) \tag{77}
\end{equation*}
$$

Here, $I$ is the intensity of the laser beam (the beam energy transferred across the unit area per unit time), $d^{2}$ is the dipole moment squared of the positron transition as defined by formula (26), $\mathbf{v}, \mathbf{p}=m_{\mathrm{e}} \mathbf{v}$ are the velocity and momentum of the positron in the initial state, respectively, and

$$
\begin{equation*}
f(v)=\frac{1}{(2 \pi T)^{3 / 2}} \exp \left(-\frac{v^{2}}{2 T}\right) \tag{78}
\end{equation*}
$$

is the Maxwellian distribution function of positrons over velocities. In Eqns (77) and (78), we assume the feasibility of the resonance condition (21), in which $\omega$ this time is the laser frequency.

The laser field leads not only to recombination, i.e. to the production of antihydrogen atoms, but also to their destruction as a result of running the inverse process with respect to Eqn (76), namely the ionization in the laser field (Fig. 6), which occurs at the rate $\lambda_{\mathrm{i}}(n l)$. According to the principle of detailed balance, this ionization rate is linked with the aboveintroduced rate as

$$
\begin{equation*}
\frac{\lambda_{\mathrm{s}}(n l)}{\lambda_{\mathrm{i}}(n l)}=n_{\mathrm{e}} f(v)(2 l+1) \tag{79}
\end{equation*}
$$

Summation of the SRR rate constant (77) with respect to $l$ using formulas (28) and (32) gives us the total rate of stimulated radiative recombination to the level of an antihydrogen atom with the principal quantum number $n$ :

$$
\begin{equation*}
\lambda_{\mathrm{S}}(n)=\frac{8 \pi \operatorname{In} \mathrm{e}_{\mathrm{e}} f(v)}{3 \sqrt{3} c \omega^{4} n^{3}} \tag{80}
\end{equation*}
$$

The corresponding recombination cross sections of the positron on the antiproton in the field of the laser beam are


Figure 6. Scheme of kinetics relevant to stimulated radiative recombination.
given by the expressions

$$
\begin{equation*}
\sigma_{\mathrm{S}}(n l)=\frac{\lambda_{\mathrm{S}}(n l)}{n_{\mathrm{e}} v}, \quad \sigma_{\mathrm{S}}(n)=\frac{\lambda_{\mathrm{S}}(n)}{n_{\mathrm{e}} v} . \tag{81}
\end{equation*}
$$

The ratio of recombination cross sections for stimulated radiative process (81) and conventional (spontaneous) radiative process (33), (34) is called the laser-enhanced recombination gain [87]

$$
\begin{equation*}
G=\frac{I c^{2} f(v) v}{2 \omega^{3}} . \tag{82}
\end{equation*}
$$

In experiment [90] with an electron cooler using a carbon dioxide laser, stimulated radiative recombination to the level $n=12$ of hydrogen atoms was studied, and the measured gain amounted to

$$
\begin{equation*}
G_{\text {expt }}=4790 \pm 2830 . \tag{83}
\end{equation*}
$$

The electron energy comprised

$$
\begin{equation*}
\bar{\varepsilon}_{\mathrm{k}}=23.3 \mathrm{meV} \equiv \bar{\varepsilon}, \tag{84}
\end{equation*}
$$

and the laser beam intensity equaled

$$
\begin{equation*}
I=500 \frac{\mathrm{~W}}{\mathrm{~cm}^{2}}=10^{-13} \text { a.u. } \tag{85}
\end{equation*}
$$

Function $f(v)$ is related to the probability

$$
\mathrm{d} W=f(\mathbf{v}) \mathrm{d}^{3} v, \quad \mathrm{~d}^{3} v=d v_{x} \mathrm{~d} v_{y} \mathrm{~d} v_{z}
$$

for the electron to have a velocity in the interval $\mathrm{d}^{3} v$. For the rectangular distribution of the electron energy $\varepsilon_{\mathrm{k}}$ in the form

$$
\begin{equation*}
\mathrm{d} W=\frac{1}{\Delta \varepsilon} \Theta\left(\bar{\varepsilon}+\frac{1}{2} \Delta \varepsilon-\varepsilon_{\mathrm{k}}\right) \Theta\left(\varepsilon_{\mathrm{k}}-\bar{\varepsilon}+\frac{1}{2} \Delta \varepsilon\right) \mathrm{d} \varepsilon_{\mathrm{k}} \tag{86}
\end{equation*}
$$

we ought to set

$$
\begin{equation*}
f(v) \rightarrow \frac{1}{4 \pi \Delta \varepsilon} \tag{87}
\end{equation*}
$$

in formula (82), after which it becomes

$$
\begin{equation*}
G=A \frac{\bar{\varepsilon}}{\Delta \varepsilon}, \quad A=\frac{I c^{2}}{8 \pi \bar{\varepsilon} \omega^{3}} . \tag{88}
\end{equation*}
$$

From formulas (84), (85) and (88) it follows that

$$
\begin{equation*}
A=2 . \tag{89}
\end{equation*}
$$

Assuming $\bar{\varepsilon} / \Delta \varepsilon \sim 2$ (this parameter was not introduced in Ref. [90]), from Eqn (88) we get

$$
\begin{equation*}
G_{\text {theor }} \sim 4 . \tag{90}
\end{equation*}
$$

In experiment [91], a dye laser was used for placing electrons on the level $n=2$ of a hydrogen atom, and the measured gain approximated

$$
\begin{equation*}
G_{\text {expt }} \sim 60, \tag{91}
\end{equation*}
$$

normalized to the value of $I=20 \mathrm{MW} \mathrm{cm}{ }^{-2}$, under the assumption that the recombination cross section is directly
proportional to the laser intensity [as in formulas (81)]. The temperatures of electron motions in the cooler in the transverse and longitudinal directions with respect to the magnetic field were equal to

$$
T_{\perp}=0.11 \mathrm{eV}, \quad T_{\|}=0.45 \mathrm{meV}
$$

respectively. For such a two-temperature electron distribution

$$
f(\mathbf{v})=\frac{1}{2 \pi T_{\perp} \sqrt{2 \pi T_{\|}}} \exp \left(-\frac{v_{\perp}^{2}}{2 T_{\perp}}-\frac{v_{\|}^{2}}{2 T_{\|}}\right)
$$

with $T_{\|} \ll T_{\perp}$ in Eqn (82) we should set

$$
f(v) v \rightarrow \frac{1}{2 \pi T_{\perp}} \exp \left(-\frac{v_{\perp}^{2}}{2 T_{\perp}}\right) \sim 100
$$

which results in the theoretical value of the gain

$$
\begin{equation*}
G_{\text {theor }} \sim 1 \tag{92}
\end{equation*}
$$

It should be emphasized that in these two experiments the condition (43) was satisfied, and therefore the effect of the magnetic field on the recombination was negligible.

According to Wolf [89], the large disagreement between the values (83), (90) and (91), (92) is rooted in the wrong assumption that the stimulated radiative recombination takes place from the states in the continuous spectrum of the electrons (or, in our case, positrons).

In the steady-state plasma, the electron density near the proton is higher than the mean density $n_{\mathrm{e}}$. The excess of density comes from the electrons residing the highly excited Rydberg states in the field of the proton. For example, in the case of a plasma, in which the dominating process of relaxation towards equilibrium state is the collisions, the calculation done by Mansbach and Keck [92] using the Monte Carlo method gave an additional factor related to such an increase in the electron density:

$$
\begin{equation*}
F_{\mathrm{d}} \sim 45 \tag{93}
\end{equation*}
$$

Multiplying the theoretical result (92) by this factor, we reconcile theory (92) and experiment (91). However, as is seen from results (83) and (90), theory still disagrees sharply with the data obtained by Yousif et al. [90]. It is the authors' opinion [59] that this can be explained by the fact that in the experiments of Ref. [90] the laser field caused electron diffusion over the Rydberg states, which alters the entire pattern of radiative recombination. This stochastic diffusive motion of the electron over the Rydberg levels occurs when the laser intensity exceeds the threshold value [93]

$$
\begin{equation*}
I>I_{\mathrm{D}}, \quad I_{\mathrm{D}}=\frac{2 \times 10^{10}}{n^{8}} \mathrm{~W} \mathrm{~cm}^{-2} \tag{94}
\end{equation*}
$$

For level $n=12$, we have

$$
I_{\mathrm{D}}=50 \mathrm{~W} \mathrm{~cm}^{-2},
$$

which is less than the laser intensity in the experiment of Ref. [90] [formula (85)]. The characteristic time of the diffusion ionization ( $\sim 30 \mathrm{~ns}$ ) of hydrogen atoms produced in the states with $n=12$ is greater than the length of laser pulses ( $\sim 0.1 \mathrm{~ns}$ ) in the experiment [90]. Accordingly, the
emerging atoms do not have time to be destroyed, but the flux of atoms to the level $n=12$ is large because of the fast diffusion of electrons from the continuum. Despite the high observed value of the recombination gain (83), the use of a pulsed laser for the production of antihydrogen atoms, as in the experiments of Ref. [90], is not suitable because most of the time the laser beam is turned off. If a cw laser is utilized and the intensity of the laser beam is kept at the level (85), the emerging atoms will be rapidly destroyed through the mechanism of diffusion ionization. As the intensity goes down to the threshold value $I_{\mathrm{D}}$ (and below), when the linear theory of ionization holds, as described above, according to Ref. [90] the gain falls to the value of $G<0.4$ which is less than unity, and hence the stimulated radiative recombination becomes less efficient than the conventional spontaneous recombination. The theory of stochastic ionization of atoms in a laser field is far from completion [94-96]. Accordingly, today we do not have even a qualitative understanding of the processes associated with stimulated radiative recombination to the Rydberg levels with high principal quantum numbers. Further we will confine our discussion to the subcritical range of laser intensities

$$
\begin{equation*}
I<I_{\mathrm{D}} . \tag{95}
\end{equation*}
$$

First of all we need to study the evolution of atomic states after laser-stimulated recombination into the initial excited state $(n l)$. The rate $\lambda_{\gamma}$ of the radiative transition (see Fig. 6)

$$
\overline{\mathrm{H}}_{n l} \rightarrow \overline{\mathrm{H}}_{n^{\prime} l^{\prime}}+\gamma
$$

into all possible states $n^{\prime} l^{\prime}$ is given by [71, 97]

$$
\begin{equation*}
\lambda_{\gamma}(n l)=\frac{4}{\pi \sqrt{3} c^{3} n^{3} l^{2}} . \tag{96}
\end{equation*}
$$

From Eqns (18) and (96) we conclude that the typical value of $\lambda_{\gamma}$ for atoms produced in the states $n \sim 12$ is of order

$$
\begin{equation*}
\lambda_{\gamma} \sim 5 \times 10^{5} \mathrm{~s}^{-1} . \tag{97}
\end{equation*}
$$

By virtue of the selection rule for dipole radiation, namely

$$
l^{\prime}=l \pm 1
$$

the main contribution to the radiative transition rate (96) comes from transitions to the states

$$
n^{\prime} \approx l^{\prime} \approx l \ll n .
$$

The time $\tau_{\text {esc }}$, within which the atom produced escapes from the positron plasma with the radius typical of experiments [3, 4]:

$$
\begin{equation*}
R \sim 0.5 \mathrm{~cm}, \tag{98}
\end{equation*}
$$

is approximately equal to

$$
\begin{equation*}
\tau_{\mathrm{esc}}=\frac{R}{v_{\overline{\mathrm{H}}}} \sim 0.5 \times 10^{-5} \mathrm{~s}, \tag{99}
\end{equation*}
$$

and therefore

$$
\begin{equation*}
\lambda_{\mathrm{esc}}=\frac{1}{\tau_{\mathrm{esc}}} \sim 2 \times 10^{5} \mathrm{~s}^{-1} \tag{100}
\end{equation*}
$$

Now we need to evaluate to what extent the orbital quantum number of the hydrogen atoms produced in the
states $n \sim 12$ may change during the time of radiative relaxation (99) as a result of Stark transitions

$$
\begin{equation*}
\overline{\mathrm{H}}_{n l}+\mathrm{e}^{+} \rightarrow \overline{\mathrm{H}}_{n l^{\prime}}+\mathrm{e}^{+} . \tag{101}
\end{equation*}
$$

In plasma with parameters (42) and at $n \sim 12$, the rate of Stark transitions is evaluated as [59, 98, 99]

$$
\begin{equation*}
\Gamma_{\mathrm{D}}=\frac{3 n^{2} n_{\mathrm{e}} \Lambda^{2}}{\Lambda_{1}} \sqrt{\frac{2 \pi}{T}} \sim 10^{5} \mathrm{~s}^{-1} \tag{102}
\end{equation*}
$$

Here, the notation was introduced:

$$
\begin{align*}
& \Lambda=\ln \left(r_{\mathrm{D}} \frac{v_{\mathrm{T}}}{n}\right), \quad \Lambda_{\mathrm{l}}=\ln n, \\
& r_{\mathrm{D}}=\sqrt{\frac{T}{4 \pi n_{\mathrm{e}} e^{2}}} \sim 0.03 \mathrm{~cm} \tag{103}
\end{align*}
$$

is the Debye plasma radius, and

$$
v_{\mathrm{T}}=\sqrt{T} \text { a.u. }=10^{6} \mathrm{~cm} \mathrm{~s}^{-1}
$$

is the thermal velocity of positrons. The physical meaning of $\Gamma_{\mathrm{D}}$ is that within the characteristic time $1 / \Gamma_{\mathrm{D}}$ any initial distribution of atoms with respect to orbital momenta relaxes to the equilibrium, or the so-called 'statistical', distribution

$$
\begin{equation*}
W_{\mathrm{st}}(l)=\frac{2 l+1}{n^{2}} \tag{104}
\end{equation*}
$$

From estimates (97), (100), (102) and the kinetic scheme presented in Fig. 6 we conclude that the initial distribution of atoms with respect to $l$ remains practically unchanged and, moreover, the radiative deexcitation is practically completed over the time taken by the atoms produced to escape from the plasma.

The flux of atoms from the plasma (the number of atoms escaping the plasma per unit time) is defined as follows

$$
J_{\overline{\mathrm{H}}}=\lambda_{\overline{\mathrm{H}}} N_{\overline{\mathrm{p}}},
$$

where $N_{\overline{\mathrm{p}}}$ is the stationary number of antiprotons in the plasma volume, and

$$
\begin{equation*}
\lambda_{\overline{\mathrm{H}}}=\sum_{l} \frac{\lambda_{\mathrm{S}}(n l) \lambda_{\gamma}(n l)}{\lambda_{\gamma}(n l)+\lambda_{\mathrm{i}}(n l)} \equiv \lambda_{\mathrm{SRR}} \tag{105}
\end{equation*}
$$

is the rate (the probability per unit time) of production of atoms on one antiproton. In formula (105), we took into account the probability

$$
W_{\mathrm{S}}=\frac{\lambda_{\gamma}}{\lambda_{\gamma}+\lambda_{\mathrm{i}}}
$$

of the atomic stabilization as a result of radiative deexcitation. With the probability $1-W_{\mathrm{S}}$, the emerging atom is ionized in the laser field.

At low laser intensities, we have

$$
\lambda_{\mathrm{i}} \ll \lambda_{\gamma}, \quad W_{\mathrm{S}} \approx 1
$$

and therefore it follows from Eqn (105) that

$$
\begin{equation*}
\lambda_{\mathrm{SRR}} \approx \lambda_{\mathrm{S}}(n) \tag{106}
\end{equation*}
$$

From this and relationship (80) we see that $\lambda_{\text {SRR }} \sim I$. In the transition range of intensities, when $\lambda_{\mathrm{i}} \sim \lambda_{\gamma}$, the recombination rate comes to saturation.

From the above formulas for $\lambda_{\mathrm{i}}$ and $\lambda_{\gamma}$ it is easy to verify that saturation occurs at $I \sim I_{\mathrm{D}}$. Finally, when $I \gtrdot I_{\mathrm{D}}$, we have $\lambda_{\mathrm{i}} \gg \lambda_{\gamma}$, and therefore formula (105), with due account for relationship (79), reduces to the expression

$$
\begin{equation*}
\lambda_{\mathrm{SRR}}=\frac{8 \ln (n)}{\pi \sqrt{3} c^{3} n^{3}} f(v) n_{\mathrm{e}} \tag{107}
\end{equation*}
$$

that is, as $I \rightarrow \infty$ the recombination rate tends to a constant (the saturation effect [89]). This, however, relates to the supercritical region (94), in which, as indicated above, the theoretical formulas linear with respect to laser intensity no longer hold [this also applies to Eqn (107)], and for which there is no reliable theory so far. It is clear, however, that owing to the fast diffusion ionization the rate $\lambda_{\text {SRR }}$ in the region (94) decreases with increasing laser intensity. At $I \sim I_{\mathrm{D}}$, the recombination rate $\lambda_{\mathrm{SRR}}$ reaches its maximum. However, as indicated earlier, the rate of stimulated radiative recombination even with the most advantageous stationary laser regime is less than the rate of spontaneous radiative recombination discussed in Section 2.

The theory of stimulated radiative recombination presented above is far from completion. Additional research is required here.

## 5. Three-particle recombination in supermagnetized plasma

In supermagnetized plasma (SMP) with the parameters (44), the magnetic field has a considerable effect on the process of three-particle recombination (TPR) of antiprotons with positrons [59, 98, 100, 101]. The physical cause of this effect is that under conditions (44) the positron motion transverse to the magnetic field is heavily suppressed, and therefore the recombination on the antiprotons is suppressed.

The first and the fastest stage of the three-particle recombination is the formation of the Thomson atom:

$$
\begin{equation*}
\mathrm{e}^{+}+\mathrm{e}^{+}+\overline{\mathrm{p}} \rightleftharpoons \overline{\mathrm{H}}_{\mathrm{T}}+\mathrm{e}^{+} \tag{108}
\end{equation*}
$$

with a positron binding energy $I_{n} \sim T$, and a radius of the positron orbit $\sim R_{\mathrm{T}}$. In reaction (108), we also showed the inverse process, namely the ionization of the Thomson atom by the positron. The direct and inverse processes are fast compared with all the subsequent stages, and therefore a quasi-stationary cloud gathers around each antiproton, with a size $\sim R_{\mathrm{T}}$ and the number of positrons in it reaching

$$
\begin{equation*}
N^{*} \sim \frac{4}{3} \pi R_{\mathrm{T}}^{3} n_{\mathrm{e}} \tag{109}
\end{equation*}
$$

If we take into account the electric field $E_{\perp}$ within the plasma, the concept of the Thomson atom is preserved, although the shape of the positron orbits becomes different. As demonstrated in Ref. [98], this electric field does not affect the resulting rate of the three-particle recombination. The reason is that this rate is determined by the slowest processes that take place at the levels $n \sim 30$ (see below). The effect of the electric field $E_{\perp}$ on these processes is insignificant.

The second step in the three-particle recombination is a further relaxation of the atom in the processes of the so-called replacement collisions [100]:

$$
\begin{equation*}
\mathrm{e}^{+}+\overline{\mathrm{H}}_{n} \rightarrow \mathrm{e}^{+}+\overline{\mathrm{H}}_{n^{\prime}}, \quad n^{\prime}<n \tag{110}
\end{equation*}
$$

The number $n$ here does not have the direct meaning of the principal quantum number, because the positron orbits are
much distorted by the magnetic field, and they bear little resemblance to Keplerian orbits. This number only serves to denote the binding energy of the atom that is related to it by formula (22). The number in question only acquires the proper meaning of the principal quantum number at the lower levels ( $n<30$ ), when the positron follows Keplerian orbits slightly distorted by the magnetic field. At the upper $\overline{\mathrm{H}}$ levels, the positron moves along the magnetic field line like a bead on a string, and resides in the bound state in the potential well of the antiproton, defined by formula (50). We should also add that the recombining positron spends most of the time in the states $n \gg 1$, and therefore the analysis of the recombination process can be performed by the methods of classical mechanics.

In reaction (110), the incident positron knocks out the positron that finds itself in the bound state characterized by the minimal distance $r$ to the antiproton [see formula (50)], and lands in the new bound state where the distance to the antiproton is less: $r^{\prime}<r$. In the course of the resulting diffusive motion along the coordinate $r$, the positron comes closer and closer to the antiproton. The characteristic value of $r$ varies according to the law

$$
\begin{equation*}
\dot{r}=-\lambda_{\mathrm{GN}} r, \quad \lambda_{\mathrm{GN}} \sim \pi r^{2} n_{\mathrm{e}} v_{\mathrm{T}} \tag{111}
\end{equation*}
$$

Hence we see that at low values of $r$ this process slows down and becomes inefficient. The reason is that, as $r$ decreases, the incident positron needs to 'hit' a circle of a smaller radius ( $\sim \pi r^{2}$ ), which is highly improbable. In this region, the main mechanism of recombination (in this case, the decrease of $r$ ) becomes the drift mechanism [98]. In the context of this mechanism, the incident positrons fly past at considerable distances $r^{\prime} \lesssim r_{\mathrm{D}}$ and affect the bound positron with their electric fields $E \sim 1 / r^{\prime 2}$. For the duration of collision, $\tau \sim r^{\prime} / v_{\mathrm{T}}$, the bound positron drifts with the velocity defined by formula (55) and is shifted in a direction transverse to the magnetic field to the distance defined by formula (56) in which we need to make the replacements $r_{0} \rightarrow r^{\prime}, z_{0} \rightarrow 1$. Since the displacement vectors $\delta \mathbf{r}$ of a positron are oriented at random and are small $(\delta r \ll r)$, the bound positron executes diffusive motion in the direction transverse to the magnetic field with the diffusion coefficient

$$
\begin{equation*}
D_{\perp}=\sqrt{\frac{\pi}{2 T}} \frac{n_{\mathrm{e}} c^{2} \Lambda_{0}^{2}}{H^{2}} \tag{112}
\end{equation*}
$$

where

$$
\Lambda_{0}=\ln \left(\frac{1}{f}\right) \sim 2
$$

is the Coulomb logarithm, and the parameter $f$ is defined by formula (46).

Observe that lateral diffusion of the positron occurs much slower than the relaxation of its longitudinal (with respect to the magnetic field) motion, and therefore practically all positrons that diffuse towards the antiproton find themselves near the bottom (at distance $\sim T$ ) of the potential well (50). Consequently, we may assume that $|z| \ll r$, and the force of attraction of the positron to the antiproton is directed practically perpendicular to the magnetic field, and equals $F=-1 / r^{2}$. Under the action of this force, the positron, simultaneously with diffusion, drifts towards the antiproton with the velocity

$$
\begin{equation*}
v_{\mathrm{D}}=b_{\perp} F, \quad b_{\perp}=\frac{D_{\perp}}{T} . \tag{113}
\end{equation*}
$$

Here $b_{\perp}$ is the transverse mobility. One may question the applicability of Einstein's relation (113) for the motion transverse to the magnetic field. However, it can easily be derived from the generalized Ohm law for plasma (see, for example, Ref. [80]). Physically this means that the plasma confined in the magnetic field does not reside in the state of thermodynamic equilibrium. However, as a result of lateral diffusion of its particles that stems from collisions, interactions with the photons emitted by the thermostat, etc., the plasma will eventually go into an equilibrium state in which the particle density is distributed over the transverse direction according to the Boltzmann formula, from which relation (113) follows directly. In this way, the validity of relationship (113) only requires the local equilibrium which is practically always featured by the plasma. The easiest way to derive it is to use the imaginary case of complete thermodynamic equilibrium, which is rarely realized in experiments.

Formula (112) was first derived by Belyaev and Budker [102]. It describes the lateral diffusion and the drift of the probe particle in plasma. From the law of conservation of momentum it follows that the diffusive flux for free electrons (or positrons) is zero. In our case, however, owing to the presence of the field of the antiproton, the positron momentum is not conserved, and therefore there is a nonzero diffusion flow of positrons towards the antiproton, which determines their recombination rate.

Given the replacement and the drift mechanisms, the distance between the antiproton and the positron captured by its field changes from the initial values $r \sim R_{\mathrm{T}}$ to small values of $r$ according to the law [101]:

$$
\begin{equation*}
\dot{r}=-\lambda_{\mathrm{GN}} r-\frac{b_{\perp}}{r^{2}} . \tag{114}
\end{equation*}
$$

Hence we get the following expression for the time of decrease of $r$ from $r \sim R_{\mathrm{T}}$ to the small values $r \ll R_{\mathrm{T}}$ :

$$
\begin{equation*}
\tau_{1}=\int_{0}^{R_{\mathrm{T}}} \frac{d r}{\beta n_{\mathrm{e}} v_{\mathrm{T}} r^{3}+b_{\perp} / r^{2}} \tag{115}
\end{equation*}
$$

where $\beta \sim 1$. Integration gives the result

$$
\begin{align*}
& \tau_{1} \sim \frac{\rho_{\mathrm{c}}^{3}}{b_{\perp}} \sim 10^{-6} \mathrm{~s},  \tag{116}\\
& \rho_{\mathrm{c}}=\left(\frac{b_{\perp}}{\beta n_{\mathrm{e}} v_{\mathrm{T}}}\right)^{1 / 5} \sim 10^{4} \text { a.u. } \tag{117}
\end{align*}
$$

Observe that the main contribution to $\tau_{1}$ comes from the distances $r \sim \rho_{\mathrm{c}}$. The reason is that at such $r$ the rate of variation of this quantity, $|\dot{r}|$, is the lowest, and therefore the motion across this range of $r$ is the slowest stage that determines the total recombination time. We shall call this region the first (or the collision) limiting stage of the recombination process.

There also is another limiting stage, which is even slower - and thus the most important. It sets in upon reaching the region of stochastic motion of the positron:

$$
\begin{equation*}
r \sim \rho_{0}, \quad n \sim n_{1}=\omega_{\mathrm{H}}^{-1 / 3} \sim 30, \tag{118}
\end{equation*}
$$

where the distance $\rho_{0}$ is given by formula (53). At such values of $r$, the typical size of the positron orbit, according to Eqn (23), is $R_{n} \sim \rho_{0} \sim r$, and the Coulomb force $F_{\mathrm{c}} \sim 1 / r^{2}$, acting from the side of the antiproton on the positron, is of the
order of the Lorentz force $F_{\mathrm{L}}=v_{0} H / c$, where the characteristic orbital velocity $v_{0}$ of the positron is defined by formula (66). The meaning of the characteristic size $\rho_{0}$ is that for $r>\rho_{0}$ the Larmor motion dominates, $F_{\mathrm{L}} \gg F_{\mathrm{c}}$ : the positron executes fast transverse rotation in a small circle around the magnetic force line, and slow oscillations in the longitudinal direction in the potential well (50) created by the Coulomb attraction to the antiproton. When $r \ll \rho_{0}$, the Keplerian motion dominates $\left(F_{\mathrm{c}} \gg F_{\mathrm{L}}\right)$ : the positron follows the Keplerian ellipse with the frequency $\omega_{\mathrm{K}} \sim 1 / n^{3} \sim 1$ :

$$
\omega_{\mathrm{K}} \sim \frac{1}{n^{3}} \sim R_{n}^{-3 / 2} .
$$

Under the action of the magnetic field this ellipse performs slow Larmor precession - it rotates as a whole about the axis parallel to $\mathbf{H}$ with an angular velocity [74]

$$
\begin{equation*}
\omega_{\mathrm{L}}=\frac{1}{2} \omega_{\mathrm{H}} \ll \omega_{\mathrm{K}} \tag{119}
\end{equation*}
$$

These two types of trajectories, two types of motion $\left(r \gg \rho_{0}\right)$ and $\left(r \ll \rho_{0}\right)$ are separated by the intermediate region $r \sim \rho_{0}$ of stochastic motion, in which $F_{\mathrm{c}} \sim F_{\mathrm{L}}$.

In the region $r \lesssim \rho_{0}$, the drift approximation (55) becomes invalid, and therefore the earlier discussed drift of positrons to antiproton, described by formulas (112) and (113), at $r<\rho_{0}$ slows down, and for $r \ll \rho_{0}$ stops. In this region, a third mechanism is 'switched on' - the radiative transitions of atoms $\overline{\mathrm{H}}_{n}$ to the lower levels ( $n \sim 1$ ).

When the atom reaches the states $n \sim n_{1}$ [see formula (118)], the Stark transitions (101) and (102) whose characteristic rate $\Gamma_{\mathrm{D}} \sim 10^{6} \mathrm{~s}^{-1}$ is high compared with the rates of other processes, are first 'switched on'. Stark's 'mixing' of states results in the statistical distribution (104) of atoms with respect to the orbital states. Further relaxation of atoms takes place as a result of radiative transitions to the lower $(n \sim 1)$ quantum states, which, according to Eqns (96) and (104), come about with the rate

$$
\begin{equation*}
\Gamma_{\gamma}(n)=\sum_{l} W_{\mathrm{st}}(l) \lambda_{\gamma}(n l)=\frac{8 \ln n}{\pi \sqrt{3}}\left(c^{3} n^{5}\right)^{-1} . \tag{120}
\end{equation*}
$$

The characteristic time of this last radiative stage of atomic relaxation is of the order of

$$
\begin{equation*}
\tau_{2}=\frac{1}{\Gamma_{\gamma}\left(n_{1}\right)} \sim 2 \times 10^{-4} \mathrm{~s} \tag{121}
\end{equation*}
$$

The total time of positron recombination (from the Thomson atom, $r \sim R_{\mathrm{T}}$, to the lower states $n \sim 1$ ) is defined as follows

$$
\begin{equation*}
\tau_{\mathrm{R}}=\tau_{1}+\tau_{2} \approx \tau_{2} \tag{122}
\end{equation*}
$$

In this way, the limiting stage of the process of complete recombination (to the states $n \sim 1$ ) is the radiative transitions

$$
\begin{equation*}
n \sim 30 \rightarrow n \sim 1 \tag{123}
\end{equation*}
$$

The number of antihydrogen atoms that form per unit time on one antiproton (the three-particle recombination rate) is equal to

$$
\begin{equation*}
\lambda_{\mathrm{TBR}}=\frac{N^{*}}{\tau_{\mathrm{R}}} \approx \frac{N^{*}}{\tau_{2}}=\frac{4}{3} \pi \frac{n_{\mathrm{e}}}{T^{3} \tau_{2}} . \tag{124}
\end{equation*}
$$

An unusual feature is the linear dependence of this rate on the positron number density. The reason is that under typical
experimental conditions (2) and (42), the processes that proceed at the upper levels of antihydrogen atoms and exhibit quadratic dependence on the density, are very fast. While the overall recombination rate, as remarked above, is limited by the most prolonged stage - the radiative transitions, whose rates do not depend on the positron density.

## 6. Comparison of the efficiency of recombination mechanisms

In the previous sections we obtained the formulas for the recombination rates characterizing the most important mechanisms of production of antihydrogen atoms in cold plasma: radiative (spontaneous) $\lambda_{\mathrm{R}}$, stimulated radiative $\lambda_{\text {SRR }}$, and, finally, three-particle $\lambda_{\text {TBR }}$. In spite of the approximate nature of some of these formulas, their accuracy is quite sufficient for analyzing and planning experiments with antihydrogen atoms. In this section we shall present simple practical formulas, which can be used for calculating the rates of such processes.

The number of atoms $J_{\overline{\mathrm{H}}}$ produced in plasma per unit time has been defined at the end of Section 4; this time, however, $\lambda_{\overline{\mathrm{H}}}$ is the total recombination rate:

$$
\begin{equation*}
\lambda_{\overline{\mathrm{H}}}=\lambda_{\mathrm{R}}+\lambda_{\mathrm{SBR}}+\lambda_{\mathrm{TBR}} . \tag{125}
\end{equation*}
$$

According to formulas (39) and (40), and applying coefficient 2 derived in Section 3, we have

$$
\begin{equation*}
\lambda_{\mathrm{R}}=4 \times 10^{-3} \frac{\bar{n}}{\sqrt{\bar{T}}} \mathrm{~s}^{-1} . \tag{126}
\end{equation*}
$$

By this mechanism, the antihydrogen atoms are produced mainly in the lower states $(n \sim 1)$. Hereinafter we shall sometimes use the reduced parameters

$$
\begin{align*}
& \bar{n}=\frac{n_{\mathrm{e}}}{n_{0}}, \quad \bar{T}=\frac{T}{T_{0}}, \\
& \bar{H}=\frac{H}{H_{0}}, \quad \bar{R}=\frac{R}{R_{0}}, \\
& \bar{L}=\frac{L}{L_{0}}, \quad \bar{I}=\frac{I}{I_{\mathrm{D}}},  \tag{127}\\
& \bar{P}=\frac{P}{P_{0}},
\end{align*}
$$

where $R$ and $L$ are the radius and the length of the positron cloud, $I$ is the intensity of the carbon dioxide laser beam (in the case of stimulated radiative recombination to levels $n=11$ or 12 ), and $P$ is the heat release power supplied to the positron plasma by the decelerating antiprotons. Formulas (127) involve the following parameters typical for experiments dealing with the production of antihydrogen atoms:

$$
\begin{align*}
& n_{0}=10^{8} \mathrm{~cm}^{-3}, \quad T_{0}=10 \mathrm{~K} \\
& H_{0}=3 \mathrm{~T}, \quad R_{0}=0.5 \mathrm{~cm}, \quad L_{0}=10 \mathrm{~cm}  \tag{128}\\
& I_{\mathrm{D}}=50 \mathrm{~W} \mathrm{~cm}^{-2}, \quad P_{0}=3 \times 10^{-11} \mathrm{~W}
\end{align*}
$$

Now let us discuss stimulated radiative recombination to levels $n \approx 12$, effected using the $\mathrm{cw} \mathrm{CO}_{2}$ laser. According to the analysis performed in Section 4, one has

$$
\begin{equation*}
\lambda_{\mathrm{SRR}} \sim 2 \times 10^{-2} \frac{\bar{n}}{\bar{T}^{3 / 2}} g(I) \mathrm{s}^{-1}, \tag{129}
\end{equation*}
$$

where $g(I)=\bar{I} /(1+\bar{I})$.

Here we have taken into account the additional factor 2 (see Section 3), and the coefficient from formula (93). It must be kept in mind that where $\bar{I}>1$, as explained at the end of Section 4, formula (129) gives an overestimate.

According to relationship (124), we have

$$
\begin{equation*}
\lambda_{\mathrm{TBR}}^{(\mathrm{g})} \sim \frac{5 \bar{n}}{\bar{T}^{3}} \frac{1}{(1+0.01 / \bar{n})} \mathrm{s}^{-1} \tag{130}
\end{equation*}
$$

where the superscript g indicates that we are dealing with atoms in the ground state. The last term in Eqn (130) takes into account the circumstance (discussed in Section 5) that for

$$
\bar{n}<0.01 \quad\left(n_{\mathrm{e}}<10^{6} \mathrm{~cm}^{-3}\right)
$$

the relation $\tau_{2}>\tau_{1}$ is reversed $\left(\tau_{2}<\tau_{1}\right)$, and therefore the limiting stage of recombination is the collision stage corresponding to the proton diffusion over the region $r \sim \rho_{0}$.

During the characteristic time (121) of the radiative transition (123), the excited atoms (in the states $n \sim 30$ ) cover the distance $l \sim 20 \mathrm{~cm}$. Since $l \gg R$, it is the atoms of antihydrogen in the state $n \sim 30$ that are escaping the plasma. In this case, according to estimate (38), they are not destroyed by the electric field. These atoms must be confined in the Ioffe-Pritchard trap and cooled, as discussed in the Introduction. Accordingly, the estimate (130) is not relevant, and the actual rate of the three-particle formation of atoms is described by formula (124) in which we must replace $\tau_{\mathrm{R}}$ not with $\tau_{2}$, but rather with $\tau_{1}$ [see expression (116)]. In the end we get

$$
\begin{equation*}
\lambda_{\mathrm{TBR}} \approx \frac{N^{*}}{\tau_{1}} \sim 500 \frac{\bar{n}^{2}}{\bar{H}^{0.8} \bar{T}^{3.3}} \mathrm{~s}^{-1} \tag{131}
\end{equation*}
$$

From comparison of the rates (126), (129), and (131) we see that the three-particle recombination dominates at $T<100 \mathrm{~K}$.

## 7. Deceleration and stopping of antiprotons in positron supermagnetized plasma

The nested Penning trap is currently utilized in the ATRAP experiment (see Fig. 3). When the depth of the potential wells A and C is decreased, the antiprotons penetrate into the positron cloud and stop. The Penning-Malmberg trap is used in the ATHENA experiment with the profile of the potential along the axis shown in Fig. 7. Antiprotons accumulate in zone A . When the depth $\varphi_{\mathrm{A}}-\varphi_{\mathrm{B}}$ of the potential well is reduced, the antiprotons roll down the potential 'hill' B and get into the positron cloud C. After each passage across the cloud they lose some of their energy, and eventually come to a stop in the cloud. The height $\varphi_{\mathrm{C}}-\varphi_{\mathrm{B}}$ of the potential 'hill' in the experiments can be varied.

Before embarking on analyzing the process of antiproton stopping, let us note that the properties of the positron cloud in many ways are similar to the properties of the conventional quasi-neutral plasma, namely plasma where the positive and negative charges balance out. The one-component (or unipolar [103]) plasma is also known as the nonneutral plasma [35, 36, 63, 104-106]. Such plasma is confined with the external magnetic and electric fields. The total field in plasma is the combination of the external and intrinsic fields. The latter results from the redistribution of charges under the


Figure 7. Potential profile along the axis of the trap in the ATHENA experiment: $\varphi_{\mathrm{B}} \sim-20 \mathrm{~V}, \varphi_{\mathrm{C}} \sim-1 \mathrm{~V}$ (each of the two potentials can be varied).
action of the external electric field. In a strong magnetic field, the charges only move along the magnetic field lines. For this reason, the longitudinal electric field in the plasma becomes zero $\left(E_{\|}=0\right)$. It is nonzero only at the end portions of the plasma with thickness of the order of the Debye radius. If in some region inside the one-component plasma the charge distribution is disturbed, this gives rise to a nonzero electric field ( $E_{\|} \neq 0$ ), which causes the Landau-damped Langmuir (plasma) oscillations with the conventional plasma frequency

$$
\omega_{\mathrm{p}}=\sqrt{\frac{4 \pi n_{\mathrm{e}} e^{2}}{m_{\mathrm{e}}}}
$$

If a foreign charge is introduced into the one-component plasma, then the plasma charges will redistribute themselves along the magnetic field lines and cause the conventional Debye shielding of the foreign charge. These properties show similarity between the one-component and the quasi-neutral plasmas.

In our case of a strong magnetic field, the transverse motion of charges is hampered, and therefore the resulting transverse electric field is, generally speaking, nonzero ( $E_{\perp} \neq 0$ ), which follows from the fact that plasma as a whole is charged. In this respect, one-component plasma is different from conventional plasma. There are numerous other features that distinguish one-component supermagnetized plasma from conventional plasma - for example, in the mechanisms of the lateral transfer of particles, momentum and energy (see below).

The description of the process of deceleration and stopping of antiprotons requires knowing the magnitude of the force of friction $\mathbf{F}$ that acts on the antiprotons as they travel in the positron plasma. For supermagnetized plasma (44) this force has been calculated and discussed in a number of papers [ $81-83,107-110$ ] in connection with the development of methods of beam cooling by electrons [83]. Since there are certain discrepancies in these papers both regarding the expressions for the force of friction and their physical interpretation, we shall reproduce the pertinent details of these calculations and give certain explanations that allow establishing the rigorous formulas.

The main assumptions made in calculations [81-83, 107-110] will be easier to understand if we start with the case of a zero magnetic field, $H=0$. The electric potential of
an antiproton at rest (in this section $\mathbf{r}$ is the radius vector measured from the antiproton) is defined as

$$
\begin{equation*}
\varphi(r)=-\frac{1}{r} \exp \left(-\frac{r}{r_{\mathrm{D}}}\right)=\varphi_{0}+\varphi_{1} \tag{132}
\end{equation*}
$$

Here

$$
\begin{equation*}
\varphi_{0}=-\frac{1}{r}, \quad \varphi_{1}=\frac{1}{r}\left[1-\exp \left(-\frac{r}{r_{\mathrm{D}}}\right)\right] \tag{133}
\end{equation*}
$$

are the potentials of the antiproton's self-field and the surrounding positron cloud, respectively. It should be noted that by virtue of condition (46) one has

$$
\begin{equation*}
r_{\mathrm{D}} \sim f^{-3 / 2} R_{\mathrm{T}} \gg R_{\mathrm{T}} \tag{134}
\end{equation*}
$$

which supplements inequalities (44). In the ideal plasma satisfying relation (46), fluctuations of the potential (132) are small, because it is created by a large number of positrons

$$
\begin{equation*}
N_{\mathrm{D}} \sim n_{\mathrm{e}} r_{\mathrm{D}}^{3} \sim f^{-3 / 2} \gtrdot 1 \tag{135}
\end{equation*}
$$

Naturally, the electric field strength

$$
\begin{equation*}
\mathbf{E}_{1}=-\nabla \varphi_{1} \tag{136}
\end{equation*}
$$

of the positron cloud at the location of the antiproton $(\mathbf{r}=\mathbf{0})$ is zero, and therefore there is no force acting upon it: $\mathbf{F}_{\mathrm{c}}=0$, where

$$
\begin{equation*}
\mathbf{F}_{\mathrm{c}}=-\mathbf{E}_{1}(0) \tag{137}
\end{equation*}
$$

When the antiproton starts moving with velocity u (further on $u \ll c$ ), the positron cloud displaces slightly back and becomes a 'tail' traveling together with the antiproton. In this case $\mathbf{E}_{1}(0) \neq 0$, and therefore there appears a nonzero force of friction $\mathbf{F}_{\mathrm{c}}$, directed oppositely to $\mathbf{u}$.

The total force of friction acting upon the antiproton

$$
\begin{equation*}
\mathbf{F}=\mathbf{F}_{\mathrm{c}}+\mathbf{F}_{\mathrm{p}} \tag{138}
\end{equation*}
$$

consists of the two terms [111-113]: the collective force $\mathbf{F}_{c}$ discussed above, and the term $\mathbf{F}_{\mathrm{p}}$ that is due to binary collisions of positrons with antiprotons. These collisions occur at distances $r<\bar{r}=n_{\mathrm{e}}^{-1 / 3}$, at which the collective approximation (136) does not hold. In other words, the fluctuations of positron potential $\varphi_{1}$ become important:

$$
\left|\Delta \varphi_{1}\right| \sim \frac{1}{R_{\mathrm{T}}} \gg \varphi(\bar{r}) \sim \frac{1}{r_{\mathrm{D}}}
$$

In the conventional plasma with parameters (43), the two parts of $F$ are comparable:

$$
\begin{aligned}
& F_{\mathrm{p}} \sim \ln \left(\frac{\bar{r}}{R_{\mathrm{T}}}\right), \quad F_{\mathrm{c}} \sim \ln \left(\frac{r_{\mathrm{D}}}{\bar{r}}\right) \\
& F=F_{\mathrm{p}}+F_{\mathrm{c}} \sim \ln \left(\frac{r_{\mathrm{D}}}{R_{\mathrm{T}}}\right)
\end{aligned}
$$

In the limiting case (44) that is the most important for us, and when the antiproton moves along the magnetic field:
$\mathbf{u} \| \mathbf{H}, \quad u_{\perp}=0, \quad u_{\|}=u$,
the collision term $F_{\mathrm{p}}$ vanishes, and friction is completely due to the collective effects. This is because of the 'freezing' of the transverse motion of the positron in the case (44) - that is, the practically rigorous conservation of the adiabatic invariants (58) and (60). Owing to this 'freezing', the motion is in fact one-dimensional, and only the longitudinal part of the positron energy $\varepsilon_{\|}=v_{\|}^{2} / 2$ can vary. Indeed, let us consider the binary collision of a positron with an antiproton whose velocity possesses the components

$$
\mathbf{u}=\left(u_{\|}, \mathbf{u}_{\perp}\right)
$$

in the reference system moving along the magnetic field $\mathbf{H}$ ( $z$-axis) with the velocity $u_{\|}$. Axis $x$ is directed along the vector $\mathbf{u}_{\perp}$ (Fig. 8). The equation of motion of the positron is written down as

$$
\begin{equation*}
\ddot{z}=-\frac{z}{\left[\left(x_{0}-u_{\perp} t\right)^{2}+y_{0}^{2}+z^{2}\right]^{3 / 2}} \tag{140}
\end{equation*}
$$

where $x_{0}, y_{0}$ are the transverse coordinates of the positron, which may be considered fixed because the Larmor radius is small. At $u_{\perp}=0$, the right-hand side of equation (140) does not depend on time, and therefore the total energy of the positron is conserved. Hence, we conclude that its kinetic energy after the collision is the same as before the collision, the transfer of energy and momentum to the antiproton is zero, and therefore the time-average force acting on the antiproton from the side of the positrons also goes to zero.

In the general case, when $u_{\perp} \sim\left|u_{\|}\right|$, the collision and the collective parts of the force of friction are comparable, and the force of friction itself is given by [107, 108, 110]

$$
\begin{align*}
& \mathbf{F}=-\frac{\sqrt{2} n_{\mathrm{e}} \Lambda u}{\sqrt{\pi} v_{\mathrm{T}}^{3}} \mathbf{R},  \tag{141}\\
& \mathbf{R}=\int \mathrm{d} \Omega_{\mathbf{k}} \frac{\hat{k}(\hat{k} \hat{u})}{\left|\hat{k_{z}}\right|} \exp \left[-\frac{(\hat{k} \mathbf{u})^{2}}{2 T \hat{k}_{z}^{2}}\right], \tag{142}
\end{align*}
$$

where $v_{\mathrm{T}}=\sqrt{T}, \hat{u}=\mathbf{u} / u$, and $\Lambda=\ln \left(r_{\mathrm{D}} / R_{\mathrm{T}}\right) \sim 2$.
In expression (142), integration is performed with respect to the directions of the unit vector $\hat{k}=\mathbf{k} / k$.

In order to analyze deceleration of antiprotons in the cloud of positrons in the conditions of experiments discussed


Figure 8. Geometry of the binary collision of the positron with an antiproton. AB is the trajectory of the positron. The antiproton travels along the $x$-axis.
in this review, it will suffice to consider the case

$$
u \ll v_{\mathrm{T}},
$$

for which from Eqns (141) and (142) we get $(\mathbf{h}=\mathbf{H} / H)$

$$
\begin{align*}
& \mathbf{F}=F_{\|} \mathbf{h}+\mathbf{F}_{\perp}, \\
& F_{\|}=-\frac{u_{\|}}{b_{\|}}, \quad b_{\|}=\frac{v_{\mathrm{T}}^{3}}{2 \sqrt{2 \pi} n_{\mathrm{e}} \Lambda}, \\
& \mathbf{F}_{\perp}=-\frac{2 \sqrt{2 \pi} n_{\mathrm{e}} \Lambda}{v_{\mathrm{T}}^{2}} \Lambda_{\mathrm{l}} \mathbf{u}_{\perp} \tag{143}
\end{align*}
$$

Here, $\Lambda_{1}=\ln \left(v_{\mathrm{T}} / u\right)$, and $\mathbf{u}_{\perp}=\mathbf{u}-\mathbf{h}(\mathbf{h u})$ is the component of the antiproton velocity perpendicular to the magnetic field.

According to expressions (143), deceleration of an antiproton in plasma is described by the equation

$$
\begin{equation*}
M \frac{\mathrm{~d} u}{\mathrm{~d} t}=-\frac{2 \sqrt{2 \pi} n_{\mathrm{e}} \Lambda}{v_{\mathrm{T}}^{3}} u \tag{144}
\end{equation*}
$$

where $M$ is the proton mass. Hence it follows that the stopping range of an antiproton in plasma equals

$$
\begin{equation*}
L_{\mathrm{s}}=\frac{M v_{\mathrm{T}}^{3} u_{0}}{2 \sqrt{2 \pi} \Lambda n_{\mathrm{e}}}=500 \frac{\bar{T}^{3 / 2} \bar{E}_{\overline{\mathrm{p}}}^{1 / 2}}{\bar{n}} \mathrm{~cm} \tag{145}
\end{equation*}
$$

Here, $u_{0}$ is the initial velocity of the antiproton, $E_{\overline{\mathrm{p}}}=M u_{0}^{2} / 2$ is its initial energy, $\bar{E}_{\overline{\mathrm{p}}}=E_{\overline{\mathrm{p}}} / E_{0}$, and $E_{0}=20 \mathrm{eV}$ is the typical initial energy of antiprotons injected into the positron plasma. From equation (144) it also follows that the typical deceleration time of the antiprotons is about 10 s [for the temperature we took the estimates (233) and (241) obtained in Section 14].

Later on, we shall use these results in the analysis of specific experiments.

## 8. Longitudinal diffusion of stopped antiprotons. Their confinement time in the positron cloud

As already said in Section 7, inside the positron cloud we have $E_{\|} \approx 0$, namely, $\varphi \approx$ const. Exceptions are the regions at the ends of plasma of thickness $\sim r_{\mathrm{D}}$, where $E_{\|} \neq 0$.

Inside the plasma there is no force acting on the antiproton, and it executes Brownian motion along the magnetic field with the diffusion coefficient

$$
\begin{equation*}
D_{\|}=\frac{v_{\mathrm{T}}^{5}}{2 \sqrt{2 \pi} n_{\mathrm{e}} \Lambda} . \tag{146}
\end{equation*}
$$

Formula (146) follows from the Einstein relation

$$
D_{\|}=b_{\|} T
$$

and expression (143) defining the longitudinal mobility of the antiproton.

Let us calculate the confinement (residence) time of an antiproton in plasma. Inside the plasma it stops with approximately the same probability in any portion of the trajectory:

$$
w(z, t=0)= \begin{cases}\frac{1}{L}, & |z|<\frac{L}{2}  \tag{147}\\ 0, & |z|>\frac{L}{2}\end{cases}
$$

where $t=0$ is the time of stopping, and $\mathrm{d} W=w(z, t) \mathrm{d} z$ is the probability of occurrence of an antiproton on the interval
$(z, z+\mathrm{d} z)$ at some instant of time $t$. The probability density $w(z, t)$ obeys the equation

$$
\begin{equation*}
\frac{\partial w}{\partial t}=D_{\|} \frac{\partial^{2} w}{\partial z^{2}} \tag{148}
\end{equation*}
$$

Reaching the ends of the plasma, where $E_{\|} \neq 0$, the antiproton is ejected, which gives us the boundary condition

$$
\begin{equation*}
w\left(z= \pm \frac{L}{2}, t\right)=0 . \tag{149}
\end{equation*}
$$

Ejection of the negative antiproton from the positively charged positron cloud may seem strange at first sight. The reason becomes clear if we recall that the positron plasma is confined by the external electric field. Figures 3 and 7 show the profiles of the total electric potential along the axes of the traps (the sum of the external potential and the self-potential created by the plasma charges). At the ends of the positron cloud these particles experience the force that returns them to the cloud - otherwise the cloud would not be confined by the trap. This means that the antiproton that gets into the end region whose thickness is of the order of the Debye radius will be ejected from the positron cloud (to regions A and C in Fig. 3).

From Eqns (147) - (149) we get

$$
\begin{aligned}
& w(z, t)=\frac{4}{L} \sum_{n=0}^{\infty} \exp \left(-D_{\|} K_{n}^{2} t\right) \varphi_{n}(z), \\
& \varphi_{n}(z)=(-1)^{n} \frac{\cos \left(K_{n} z\right)}{K_{n}}, \\
& K_{n}=\frac{\pi(2 n+1)}{L} .
\end{aligned}
$$

The probability of occurrence of an antiproton in plasma at the instant of time $t$ is defined as

$$
\begin{align*}
& n(t)=\int_{-L / 2}^{L / 2} w(z, t) \mathrm{d} z=\exp \left(-\frac{t}{\tau}\right),  \tag{150}\\
& \tau=\frac{L^{2}}{\pi^{2} D_{\|}}=2 \times 10^{-4} \frac{\bar{L}^{2} \bar{n}}{\bar{T}^{5 / 2}} \mathrm{~s} . \tag{151}
\end{align*}
$$

From equation (150) we conclude that if the flux of antiprotons into the positron plasma (i.e., the number of antiprotons stopped in the plasma per unit time) is $J_{\overline{\mathrm{p}}}(t)$, then the number of antiprotons $N_{\overline{\mathrm{p}}}(t)$ of the plasma at the point in time $t$ will be given by the equation

$$
\begin{equation*}
\frac{\mathrm{d} N_{\overline{\mathrm{p}}}}{\mathrm{~d} t}=-\frac{1}{\tau} N_{\overline{\mathrm{p}}}+J_{\overline{\mathrm{p}}}(t) . \tag{152}
\end{equation*}
$$

## 9. Lateral diffusion of antiprotons.

## Time of evolution of antiproton distribution in plasma. Effects of drift rotation of a positron cloud

The antiproton is a 'probe' particle in the positron cloud. The radius of the Larmor orbit of an antiproton is small, viz.

$$
r_{\mathrm{H} \mathrm{\bar{p}}} \sim 5 \times 10^{-4} \mathrm{~cm} \ll r_{\mathrm{D}},
$$

and therefore its diffusion in the transverse direction is described by the theory developed in paper [102] and formula
(112). In Section 7 it was demonstrated that the longitudinal electric field $E_{\|}$in plasma is negligibly small, but there is the radial field $\left(E_{\perp} \neq 0\right)$ created both by the positrons and by the external electrodes of the confining trap. For the sake of simplicity we consider the case

$$
\begin{equation*}
n_{\mathrm{e}}=\text { const }, \tag{153}
\end{equation*}
$$

for which

$$
\begin{equation*}
E_{\perp}=2 \pi n_{\mathrm{e}} r, \tag{154}
\end{equation*}
$$

where for the purposes of this section $r$ is the distance to the axis of the trap. As a result of the drift motion (55), the plasma at distance $r$ from the axis of the trap rotates with an angular velocity

$$
\begin{equation*}
\Omega=\frac{v_{\mathrm{D}}}{r}=\frac{2 \pi n_{\mathrm{e}} c}{H}=\text { const } \tag{155}
\end{equation*}
$$

that is, like a solid. By the action of friction (143), the antiproton tends to stop in the plasma, but the centrifugal force

$$
f=M \Omega^{2} r
$$

acting upon the antiproton, causes the radial drift of the latter. The evolution of the number density of antiprotons $n(r, t)$ is described by the equations

$$
\begin{align*}
& \frac{\partial n}{\partial t}=-\frac{1}{r} \frac{\partial}{\partial r}(r j), \\
& j=\frac{D_{\perp}}{T} f n-D_{\perp} \frac{\partial n}{\partial r} . \tag{156}
\end{align*}
$$

Equilibrium distribution of antiprotons $n_{0}(r)$ is derived from the equation $j=0$ and is given by

$$
\begin{equation*}
n_{0}(r)=n(0) \exp \left(-\frac{M \Omega^{2} r^{2}}{2 T}\right) \tag{157}
\end{equation*}
$$

Antiprotons drift in the radial direction with the velocity

$$
\dot{r}=\frac{D_{\perp}}{T} f=\frac{D_{\perp} M \Omega^{2}}{T} r .
$$

Hence it follows that the relaxation time of any initial distribution to the equilibrium distribution (157) is equal to

$$
\begin{equation*}
\tau_{r}=\frac{T}{D_{\perp} M \Omega^{2}} \sim 50 \mathrm{~s} . \tag{158}
\end{equation*}
$$

From comparison of the characteristic times of longitudinal (151) and lateral (158) diffusion we conclude that the radial drift of antiprotons stopped in the plasma can be neglected.

## 10. Processes accompanying the stopping of antiprotons in the positron cloud. Transverse and longitudinal heat transfer in supermagnetized plasma

Let us consider for definiteness some typical experiments in the framework of the ATHENA project [3]. Here, the bunches of antiprotons with $10^{5}$ particles in each are injected into a positron cloud (see Fig. 7) with a repetition rate of 100 Hz .

The typical initial kinetic energy of antiprotons entering the positron plasma is equal to

$$
E_{\overline{\mathrm{p}}}=e\left(\varphi_{\mathrm{C}}-\varphi_{\mathrm{B}}\right) \sim 20 \mathrm{eV}
$$

Having performed a few oscillations (see Fig. 7), all antiprotons stop in the plasma. Thus, the flux of antiprotons into the plasma is estimated as

$$
\begin{equation*}
J_{\overline{\mathrm{p}}}=10^{7} \mathrm{~s}^{-1}=2 \times 10^{-10} \text { a.u. } \tag{159}
\end{equation*}
$$

and the typical heat release power approximates

$$
\begin{equation*}
P=E_{\overline{\mathrm{p}}} J_{\overline{\mathrm{p}}}=3 \times 10^{-11} \mathrm{~W}=1.5 \times 10^{-10} \text { a.u. } \tag{160}
\end{equation*}
$$

Distribution of the plasma temperature $T(\mathbf{r}, t)$ is described by the equation

$$
\begin{equation*}
\frac{\partial T}{\partial t}=\frac{\partial}{\partial z}\left(\chi_{\|} \frac{\partial T}{\partial z}\right)+\frac{1}{r} \frac{\partial}{\partial r}\left(\chi_{\perp} r \frac{\partial T}{\partial r}\right)+\frac{q-q_{\mathrm{c}}}{n_{\mathrm{e}} C_{\mathrm{p}}} . \tag{161}
\end{equation*}
$$

Here, $c_{\mathrm{p}}=5 / 2$ is the heat capacity per positron, $q(\mathbf{r}, t)=P / V$ is the specific heat release power, $V=\pi R^{2} L$ is the volume of the positron plasma, and $q_{\mathrm{c}}$ is the specific power of energy loss by the plasma through the cyclotron radiation. Equation (161) also includes the thermal diffusivities of the plasma:

$$
\begin{align*}
& \chi_{\|}=\frac{x_{\|}}{n_{\mathrm{e}} c_{\mathrm{p}}} \sim 3 \times 10^{-5} \frac{\bar{T}^{5 / 2}}{\bar{n}} \text { a.u. }, \\
& \chi_{\perp}=\frac{\chi_{\perp}}{n_{\mathrm{e}} c_{\mathrm{p}}} \tag{162}
\end{align*}
$$

here $\chi_{\|}, \chi_{\perp}$ are the longitudinal and transverse plasma heat conductivity coefficients. The first of these does not depend on the magnetic field and coincides with the heat conductivity coefficient of plasma in the absence of the magnetic field [79, 80, 113]. From Eqns (161) and (162) it follows that the relaxation time of the temperature in the longitudinal direction is of order

$$
\begin{equation*}
\tau_{\|} \sim \frac{L^{2}}{4 \chi_{\|}} \sim 10^{-4} \frac{\bar{L}^{2} \bar{n}}{\bar{T}^{5 / 2}} \mathrm{~s} . \tag{163}
\end{equation*}
$$

The transverse heat transfer in supermagnetized plasma (SMP) with parameters (44), (46) and (134) was calculated by Dubin and O'Neil [104]. These calculations are rather complicated, so we shall give a qualitative description of the physical mechanism of transverse heat conduction in SMP (see also Appendix II).

Consider the collision of two positrons in SMP, which is described by the equations

$$
\begin{align*}
& \ddot{\boldsymbol{p}}_{1}=\omega_{\mathrm{H}}\left(\dot{\boldsymbol{p}}_{1} \times \mathbf{h}\right)+\frac{\boldsymbol{\rho}}{\rho^{3}}, \\
& \ddot{\boldsymbol{p}}_{2}=\omega_{\mathrm{H}}\left(\dot{\boldsymbol{p}}_{2} \times \mathbf{h}\right)-\frac{\boldsymbol{\rho}}{\rho^{3}}, \tag{164}
\end{align*}
$$

where $\boldsymbol{\rho}_{1}$ and $\boldsymbol{\rho}_{2}$ are the radius vectors of the positrons, and $\boldsymbol{\rho}=\boldsymbol{\rho}_{1}-\boldsymbol{\rho}_{2}$. After separation of the motion of the center of mass $\mathbf{R}=\left(\boldsymbol{\rho}_{1}+\boldsymbol{\rho}_{2}\right) / 2$, we arrive at the set of equations

$$
\begin{equation*}
\ddot{\boldsymbol{\rho}}=\omega_{\mathrm{H}}(\dot{\boldsymbol{\rho}} \times \mathbf{h})+\frac{2 \boldsymbol{p}}{\rho^{3}}, \quad \ddot{\mathbf{R}}=\omega_{\mathrm{H}}(\dot{\mathbf{R}} \times \mathbf{h}) . \tag{165}
\end{equation*}
$$

In case (44) we have $r_{\mathrm{H}} \ll R_{\mathrm{T}}$, and therefore from equations (165) it follows that to a good accuracy we may consider the motion of positrons as one-dimensional:

$$
\begin{align*}
& \ddot{z}=\frac{2 z}{\rho^{3}}, \quad \rho=\left(r^{2}+z^{2}\right)^{1 / 2},  \tag{166}\\
& z=\rho_{1 z}-\rho_{2 z}, \quad \dot{z}_{0}=\mathrm{const},
\end{align*}
$$

where $r=\left|\boldsymbol{\rho}_{1 \perp}-\boldsymbol{\rho}_{2 \perp}\right| \approx$ const is the impact parameter of the collision, and $z_{0}=\left(\rho_{1 z}+\rho_{2 z}\right) / 2$. Integrating the first equation in (166) we get the conservation law for the longitudinal part of the positron energy

$$
\begin{equation*}
\varepsilon_{\|}=\frac{1}{4} \dot{z}^{2}+\frac{1}{\sqrt{r^{2}+z^{2}}}=\text { const } . \tag{167}
\end{equation*}
$$

Hence it follows that in collisions with the impact parameters

$$
\begin{equation*}
r<\frac{1}{\sqrt{\varepsilon_{\|}}} \tag{168}
\end{equation*}
$$

the positrons exchange their kinetic energies like billiard balls in head-on collisions:

$$
\left(\varepsilon_{1}, \varepsilon_{2}\right) \rightarrow\left(\varepsilon_{1}^{\prime}=\varepsilon_{2}, \varepsilon_{2}^{\prime}=\varepsilon_{1}\right),
$$

where $\varepsilon_{1}, \varepsilon_{2}, \varepsilon_{1}^{\prime}, \varepsilon_{2}^{\prime}$ are the initial and the final kinetic energies of positrons in the laboratory system of coordinates. In distant collisions $r>1 / \sqrt{\varepsilon_{\|}}$, however, the particle energies do not change and one has

$$
\left(\varepsilon_{1}, \varepsilon_{2}\right) \rightarrow\left(\varepsilon_{1}^{\prime}=\varepsilon_{1}, \varepsilon_{2}^{\prime}=\varepsilon_{2}\right) .
$$

Hence we conclude that if the transverse heat transport is governed by the binary collisions, then from inequality (168) it follows that

$$
\begin{equation*}
x_{\perp} \sim n_{\mathrm{e}}^{2} R_{\mathrm{T}}^{4} v_{\mathrm{T}} . \tag{169}
\end{equation*}
$$

Relation (169) does not agree with the experiments (see review [105]): the observed values of transverse heat conductivity prove to be much higher.

This contradiction is rooted in the wrong assumption regarding the prevalence of binary collisions in heat transport: in supermagnetized plasma the transverse transfer of energy (heat conductivity) and momentum (viscosity) are almost entirely determined by the collective effects. The two colliding positrons carry their Debye clouds in their wake (see Section 7). Their interaction extends to distances $\sim r_{\mathrm{D}}$ and lasts $\tau_{\mathrm{D}} \sim r_{\mathrm{D}} / v_{\mathrm{T}} \sim 1 / \omega_{\mathrm{p}}$, where $\omega_{\mathrm{p}}=\sqrt{4 \pi n_{\mathrm{e}} e^{2} / m_{\mathrm{e}}}$ is the plasma frequency.

Now let the temperature change only along the transverse coordinate $x$. The energy flux density through the plane $x=0$ equals

$$
\begin{equation*}
q=q_{+}-q_{-}, \tag{170}
\end{equation*}
$$

where $q_{+}$is the energy flux density from the region $x<0$ to the region $x>0$.

In due course the heat $q_{-}$is carried from top to bottom. The above qualitative description implies that $q_{+}$and $q_{-}$are mainly determined by the interaction of the two plasma layers located in the coordinate ranges

$$
\begin{aligned}
& 0<x<r_{\mathrm{D}} \text { (upper layer) } \\
& -r_{\mathrm{D}}<x<0 \text { (lower layer) } .
\end{aligned}
$$

Let the temperatures of these layers be $T_{+}$and $T_{-}$, respectively, and the thermal positron velocities within these layers be $v_{+}=\sqrt{T_{+}}$and $v_{-}=\sqrt{T_{-}}$. Obviously, one finds

$$
\begin{equation*}
\Delta T=T_{+}-T_{-} \sim r_{\mathrm{D}} \frac{\mathrm{~d} T}{\mathrm{~d} x} . \tag{171}
\end{equation*}
$$

The number of positrons in each layer amounts to

$$
\begin{equation*}
N_{\mathrm{p}}=S r_{\mathrm{D}} n_{\mathrm{e}} \tag{172}
\end{equation*}
$$

where $S$ is the area of the surface $x=0$ that separates these layers. The positrons are magnetized and move in each layer parallel to the plane $x=0$ (along the magnetic field $\mathbf{H}$ ).

Since we are currently talking about the contribution to $x_{\perp}$ from the collective effects, we first need to group the positrons into such collectives. Each collective is a Debye cloud with a large number of positrons, which is defined by formula (135). The size of such clouds is approximately equal to $r_{\mathrm{D}}$, and therefore their concentration is of order

$$
\begin{equation*}
n_{\mathrm{c}} \sim \frac{1}{r_{\mathrm{D}}^{3}} \tag{173}
\end{equation*}
$$

Each positron from the lower layer experiences per unit time a certain number of interactions with the Debye clouds of the upper layer:

$$
\begin{equation*}
v_{+} \sim n_{\mathrm{c}} v_{+} r_{\mathrm{D}}^{2} \tag{174}
\end{equation*}
$$

Inside each cloud there is an electric field [see formulas (133)] $E_{\mathrm{D}} \sim 1 / r_{\mathrm{D}}^{2}$. In the interaction with one cloud, the positron from the lower layer receives energy

$$
\begin{equation*}
\Delta \varepsilon \sim E_{\mathrm{D}} r_{\mathrm{D}} \sim \frac{1}{r_{\mathrm{D}}} \tag{175}
\end{equation*}
$$

From this and Eqns (173) and (174) we get the relation

$$
\begin{equation*}
S q_{-} \sim v_{+} \Delta \varepsilon N_{\mathrm{p}} . \tag{176}
\end{equation*}
$$

Its physical meaning is that the power transported across the surface $x=0$ from above is used for changing the energy of the particles in the lower layer.

From relations (170) - (176), taking also account of

$$
q=-\chi_{\perp} \frac{\mathrm{d} T}{\mathrm{~d} x}
$$

we get estimates for the heat conductivity coefficient

$$
\begin{equation*}
\chi_{\perp} \sim n_{\mathrm{e}} v_{\mathrm{T}} R_{\mathrm{T}} \tag{177}
\end{equation*}
$$

and for the thermal diffusivity defined in Eqn (162):

$$
\begin{equation*}
\chi_{\perp} \sim v_{\mathrm{T}} R_{\mathrm{T}} \sim v_{\mathrm{e}} r_{\mathrm{D}}^{2} \tag{178}
\end{equation*}
$$

where $v_{\mathrm{e}}=n_{\mathrm{e}} v_{\mathrm{T}} R_{\mathrm{T}}^{2}$ is the frequency of positron collisions. Comparing the 'collective' (177) and the 'one-particle' (169) results, we conclude that the former relates to the latter as

$$
\sim \frac{1}{f^{3}} \gg 1
$$

According to estimate (178), the characteristic time of transverse relaxation of the temperature is of the order of

$$
\begin{equation*}
\tau_{\perp} \sim \frac{R^{2}}{4 \chi_{\perp}} \sim 2 \times 10^{-4} \bar{R}^{2} \sqrt{\bar{T}} \mathrm{~s} . \tag{179}
\end{equation*}
$$

Estimates (163) and (179) allow us to infer that the heat transfer in the positron cloud occurs practically instantaneously, and therefore the temperature at every point may be considered the same after any external action on the cloud:

$$
\begin{equation*}
T(\mathbf{r}, t) \approx T(t) . \tag{180}
\end{equation*}
$$

## 11. Perturbation of positron density distribution

In this section we shall estimate the perturbation of density of the positron cloud, caused by the antiproton bunches stopped in the cloud. Observe first of all that at temperatures of about 10 K the gas parameter amounts to

$$
n_{\mathrm{e}} R_{\mathrm{T}}^{3} \sim 10^{-4}
$$

being even smaller at higher temperatures. This means that the approximation of binary collisions is valid with a good margin. Let us consider an individual collision of positrons.

From the second equation in set (165) we see that the motion of the center of mass of the pair of colliding positrons does not change in the course of the collision, and it is not affected by the interaction between the positrons. For this reason, the usual Fick's law for the transport of mass does not work for plasma that only contains particles of the same kind. Mass transport only takes place when the plasma does not rotate as a solid [105]:

$$
\begin{equation*}
\frac{\partial \Omega}{\partial r} \neq 0 \tag{181}
\end{equation*}
$$

that is, we are dealing with differential rotation. From the first equation in set (165) we see that the colliding positrons engage in drift motion with velocity (55) in the azimuthal direction. In the presence of differential rotation, the resulting viscous friction gives rise to drift that leads to the mass transfer in the radial direction: the radial density of positron flux is defined as

$$
\begin{equation*}
j_{r}=\frac{c}{e H r^{2}} \frac{\partial}{\partial r}\left(r^{3} \eta \frac{\partial \Omega}{\partial r}\right) \tag{182}
\end{equation*}
$$

where $\eta$ is the viscosity coefficient of the positron plasma. This coefficient is determined by the collective effects and is evaluated in a way similar to the preceding section (see Appendix II):

$$
\begin{equation*}
v \equiv \frac{\eta}{n_{\mathrm{e}} m_{\mathrm{e}}} \sim \chi_{\perp} \sim v_{\mathrm{e}} r_{\mathrm{D}}^{2}, \tag{183}
\end{equation*}
$$

where $v$ is the kinematic viscosity.
The entry of the antiproton bunch into the positron cloud first causes perturbation of the temperature field, which results in a slight variation of the positron density. Then there is a change in the electric field $E_{r}$, and, in accordance with formula (155), the differential rotation appears. All these effects are described by a closed set of equations: Eqn (161) for the temperature, for the positron number density we have

$$
\begin{equation*}
\frac{\partial n_{\mathrm{e}}}{\partial t}=-\frac{1}{r} \frac{\partial}{\partial r}\left(r j_{r}\right) \tag{184}
\end{equation*}
$$

for the angular velocity of plasma drift [105] one obtains

$$
\begin{equation*}
\Omega(r, t)=\frac{c}{r H} \frac{\partial \phi}{\partial r}+\frac{1}{m_{\mathrm{e}} n_{\mathrm{e}} \omega_{\mathrm{H}} r} \frac{\partial}{\partial r}\left(n_{\mathrm{e}} T\right) \tag{185}
\end{equation*}
$$

and the Poisson equation for the electric potential $\phi(r, t)$ of the cloud:

$$
\begin{equation*}
\frac{1}{r} \frac{\partial}{\partial r}\left(r \frac{\partial \phi}{\partial r}\right)=-4 \pi e n_{\mathrm{e}} . \tag{186}
\end{equation*}
$$

The first term on the right-hand side of equation (161) can be dropped, since $L \gtrdot R$. We shall demonstrate below that the relaxation time of the temperature due to cyclotron radiation is much larger than other characteristic times for example, the time specified in Eqn (179). Because of this, in the description of processes that immediately follow the entry of antiprotons into the positron plasma, we may also drop the term with the cyclotron radiation in equation (161).

The period of oscillations of the antiproton bunch decelerating in the positron cloud (see Fig. 3 and Fig. 7) measures approximately 5 microseconds. From comparison of this time with the characteristic times of temperature relaxation in Eqns (163) and (179) we conclude that for the deceleration of a single bunch, which is common to both the ATRAP experiment and the so-called 'cold mixture procedure' in the latest version of the ATHENA experiment (see Appendix at the end of this review), the quasi-stationary approximation is suitable, according to which the left-hand side of equation (161) may be considered equal to zero. In the design regime of the ATHENA experiment [3] about 100 antiproton bunches per second are injected into the positron plasma (see the beginning of Section 10), and therefore the quasi-stationary approximation holds even better.

From equation (161) it follows that after the time (179) of entry of the first bunch of antiprotons into the plasma, the temperature in the middle $(r=0)$ of the positron cloud increases relative to the temperature on the boundary of the plasma $(r=R)$ by the quantity

$$
\begin{equation*}
\Delta T \sim \frac{q R^{2}}{\chi_{\perp}} . \tag{187}
\end{equation*}
$$

The resulting pressure difference of the positron plasma, which appears in the last term in formula (185), causes a difference in angular velocities along the radius of the plasma

$$
\begin{equation*}
\Delta \Omega \sim \frac{\Delta T}{m_{\mathrm{e}} \omega_{\mathrm{H}} R^{2}} \sim \frac{q}{m_{\mathrm{e}} \omega_{\mathrm{H}} \varkappa_{\perp}} . \tag{188}
\end{equation*}
$$

The resulting differential rotation (181) gives rise to the azimuthal viscous friction between the separate layers of plasma rotating about the axis of the trap. Joint action of these forces of friction and the magnetic field brings into existence the radial drift of the positrons, which is described by equations (182) and (184):

$$
\begin{equation*}
j_{r} \sim \frac{\eta \Delta \Omega}{m_{\mathrm{e}} \omega_{\mathrm{H}}} \sim \frac{\eta q}{\left(m_{\mathrm{e}} \omega_{\mathrm{H}}\right)^{2} \varkappa_{\perp}} . \tag{189}
\end{equation*}
$$

A slight redistribution of the positron number density $n_{\mathrm{e}}$, in accordance with Eqn (186), causes a change in the first term on the right-hand side of equation (185). This redistribution occurs over the time

$$
\begin{equation*}
\tau_{n} \sim \frac{H^{2} R^{2}}{c^{2} \eta} \sim 10^{3} \frac{\bar{H}^{2} \bar{R}^{2}}{\bar{T}} \mathrm{~s} . \tag{190}
\end{equation*}
$$

Such a redistribution results in the establishment of a new equilibrium in 'solid-state' plasma:

$$
\begin{equation*}
\Omega(r)=\text { const } . \tag{191}
\end{equation*}
$$

From Eqns (191), (185), and (186) we get the equation

$$
\begin{equation*}
2 \Omega=-\frac{4 \pi c e n_{\mathrm{e}}}{H}+\frac{c}{e H r} \frac{\mathrm{~d}}{\mathrm{~d} r}\left(\frac{r}{n_{\mathrm{e}}} \frac{\mathrm{~d}}{\mathrm{~d} r}\left(n_{\mathrm{e}} T\right)\right), \tag{192}
\end{equation*}
$$

which we linearize by setting

$$
n_{\mathrm{e}}=n_{0}+n_{1}, \quad T=T_{0}+T_{1}
$$

Assuming that

$$
\left|n_{1}\right| \ll n_{0}, \quad\left|T_{1}\right| \ll T_{0}
$$

where

$$
n_{0}=-\frac{\Omega H}{2 \pi c e}
$$

we arrive at the equation for the perturbation $\Delta n_{\mathrm{e}} \equiv n_{1}$ of the positron number density:

$$
\begin{equation*}
\frac{1}{r} \frac{\mathrm{~d}}{\mathrm{~d} r}\left(r \frac{\mathrm{~d} n_{1}}{\mathrm{~d} r}\right)-\frac{1}{r_{\mathrm{D}}^{2}} n_{1}=\frac{q}{c_{\mathrm{p}} T_{0} \chi_{\perp}} . \tag{193}
\end{equation*}
$$

Equation (193) allows us to find the sought-for perturbation of the number density of positrons:

$$
\begin{equation*}
\frac{\Delta n_{\mathrm{e}}}{n_{\mathrm{e}}} \sim \frac{q}{\varkappa_{\perp} e^{2} n_{\mathrm{e}}} \sim 5 \times 10^{-3} \frac{\bar{P} \sqrt{\bar{T}}}{\bar{n}^{2} \bar{L} \bar{R}^{2}} . \tag{194}
\end{equation*}
$$

The values of the variables were defined by formulas (127).
The estimate (194) indicates that the perturbation of positron density in the cloud is negligibly small.

## 12. Cooling of supermagnetized plasma as a result of cyclotron radiation

A positron rotating in a magnetic field emits electromagnetic cyclotron waves. The task now is to understand the extent of the influence of the plasma environment of the positron on the resulting radiation, which is the purpose of this section. The Larmor frequencies of individual positrons are practically in resonance (the difference comes up when the relativistic effects are taken into account [74]), and therefore in collisionless plasma (45) the effect of shielding of cyclotron radiation may appear to be important, and can considerably reduce it [114].

In our case the positrons are nonrelativistic, and therefore the main contribution to the radiation comes from the 'simple' cyclotron resonance $n=1$ [79, 80, 113, 115, 116]. In the range of frequencies $\omega$ nearest to this resonance, the nonzero components $\varepsilon_{\alpha \beta}$ of the permittivity tensor are given by

$$
\begin{align*}
\varepsilon_{x x} & -1=\varepsilon_{y y}-1=\mathrm{i} \varepsilon_{x y}=-\mathrm{i} \varepsilon_{y z} \\
& =\frac{\omega_{\mathrm{p}}^{2}}{2 \omega\left(\omega-\omega_{\mathrm{H}}\right)} F\left(\frac{\omega-\omega_{\mathrm{H}}}{\sqrt{2} v_{\mathrm{T}}\left|K_{z}\right|}\right),  \tag{195}\\
\varepsilon_{z z} & =1
\end{align*}
$$

Here, $\mathbf{K}$ is the propagation vector of the electromagnetic wave traveling in the plasma. Function $F$ in Eqn (195) describes the Doppler frequency shift of the cyclotron waves emitted by positrons moving along the magnetic field lines ( $z$-axis) with the Maxwellian distribution with respect to $v_{\|}$:

$$
\begin{equation*}
F(x)=\frac{x}{\sqrt{\pi}} \int_{-\infty}^{\infty} \frac{e^{-z^{2}} \mathrm{~d} z}{z-x-\mathrm{i} 0} . \tag{196}
\end{equation*}
$$

From formulas (195) and (196) we conclude that the width of the cyclotron resonance is of the order of

$$
\begin{equation*}
\left|\omega-\omega_{\mathrm{H}}\right| \sim v_{\mathrm{T}} K \sim \frac{v_{\mathrm{T}} \omega_{\mathrm{H}}}{c}, \tag{197}
\end{equation*}
$$

where we have used the relation

$$
\begin{equation*}
K \approx \frac{\omega}{c} \approx \frac{\omega_{\mathrm{H}}}{c} . \tag{198}
\end{equation*}
$$

From expression (195) it is clear that

$$
\begin{equation*}
\left|\varepsilon_{x x}-1\right| \sim \frac{n_{\mathrm{e}} m_{\mathrm{e}} c^{2}}{H^{2}} \frac{c}{v_{\mathrm{T}}} \sim 0.01 \frac{\bar{n}}{\bar{H}^{2} \sqrt{\bar{T}}} . \tag{199}
\end{equation*}
$$

Hence it follows that to a good accuracy

$$
\varepsilon_{\alpha \beta} \approx \delta_{\alpha \beta},
$$

which means that the cyclotron waves travel almost like electromagnetic waves in a vacuum. This serves to confirm relation (198), which indicates that the wavelength in the plasma is $\lambda=0.3 \mathrm{~cm}$. Since

$$
\lambda \gg n_{\mathrm{e}}^{-1 / 3}
$$

the approximation of the continuous medium, used in the derivation of formula (195), is valid.

The specific power of cyclotron radiation is given by

$$
\begin{equation*}
q_{\mathrm{c}}=I_{0} n_{\mathrm{e}}, \tag{200}
\end{equation*}
$$

where

$$
\begin{equation*}
I_{0}=\frac{4 e^{2} \omega_{\mathrm{H}}^{2} T_{\perp}}{3 m_{\mathrm{e}} c^{3}} \tag{201}
\end{equation*}
$$

is the intensity of the cyclotron radiation of the free positron [74], and

$$
T_{\perp}=\frac{m_{\mathrm{e}}\left\langle v_{\perp}^{2}\right\rangle}{2} .
$$

For example, if there is no energy exchange between longitudinal and transverse (with respect to the magnetic field) motions of the positron, then the plasma will cool down according to the law

$$
\begin{align*}
& \frac{\mathrm{d} T_{\perp}}{\mathrm{d} t}=-\frac{1}{\tau_{0}} T_{\perp}, \\
& \tau_{0}=\frac{3 m_{\mathrm{e}} c^{3}}{4 e^{2} \omega_{\mathrm{H}}^{2}}=\frac{0.3}{\bar{H}^{2}} \mathrm{~s} . \tag{202}
\end{align*}
$$

Thus, in the experiments under consideration a single positron emits radiation in the same way as in a vacuum, and the collective effects are not important. However, as
follows from estimate (199), the range of plasma parameters where these effects are significant is not far from the experimental range. It is achieved when the temperature is reduced, and/or the plasma density is increased. According to Ref. [114], in such a case the shielding effect occurs and it reduces the intensity of cyclotron radiation.

## 13. Two-temperature model of supermagnetized plasma (SMP). Longitudinal - transverse relaxation in SMP

The rate (reciprocal time) of relaxation of the distribution function to the Maxwellian distribution in conventional plasma (43) is defined as $[79,80]$

$$
\begin{align*}
\lambda_{0} & =\frac{n_{\mathrm{e}} e^{4} \Lambda_{\mathrm{c}}}{m_{\mathrm{e}}^{1 / 2} T^{3 / 2}}=1.5 \times 10^{-10} \frac{\bar{n}}{\bar{T}^{3 / 2}} \text { a.u. } \\
& =7 \times 10^{6} \frac{\bar{n}}{\bar{T}^{3 / 2}} \mathrm{~s}, \tag{203}
\end{align*}
$$

where $\Lambda_{\mathrm{c}}=\ln \left(r_{\mathrm{D}} / R_{\mathrm{T}}\right)$ is the Coulomb logarithm. In supermagnetized plasma (44), one has $R_{\mathrm{T}} \gg r_{\mathrm{H}}$. In the limiting case $\xi \rightarrow 0$, where

$$
\begin{equation*}
\xi=\frac{r_{\mathrm{H}}}{R_{\mathrm{T}}}, \tag{204}
\end{equation*}
$$

the Larmor circle may be considered as a point compared with the typical distance of closest approach $\left(\sim R_{\mathrm{T}}\right)$ to which the positrons may come together. This means that with $\xi \rightarrow 0$ the energy exchange between the longitudinal and transverse motion of positrons in SMP discontinues, and the corresponding temperatures $T_{\|}$and $T_{\perp}$ become independent, they 'deviate' from one another like it often takes place with the electron and the ion temperatures in conventional plasma (43). At finite but small values of the parameter (204) (and it is this case that holds the interest for the experiments discussed in this review), this energy exchange will be slow because of the exponentially low degree of nonconservation of the adiabatic invariants during the slow variation of external conditions [32] [in our case it is the quantities (58) and (60)].

The purpose of this section consists in calculating the rate (reciprocal time) $\lambda_{\mathrm{r}}$ of the longitudinal - transverse relaxation (LTR) with an exponential accuracy. For this it will suffice to calculate the mean value

$$
\left\langle\left(\Delta \varepsilon_{\perp}\right)^{2}\right\rangle
$$

of the variation of energy of the transverse motion:

$$
\varepsilon_{\perp}=m_{\mathrm{e}} \frac{v_{\perp}^{2}}{2}
$$

for two positrons moving along the same magnetic field line and experiencing a head-on collision. From formulas (165) follow the equations

$$
\left\{\begin{array}{l}
\dot{v}_{+}-\mathrm{i} \omega_{\mathrm{H}} v_{+}=F_{+},  \tag{205}\\
\dot{v}_{-}+\mathrm{i} \omega_{\mathrm{H}} v_{-}=F_{-},
\end{array}\right.
$$

where $v_{ \pm}=v_{x} \pm \mathrm{i} v_{y}, v_{x} \equiv \dot{\rho}_{x}, v_{y} \equiv \dot{\rho}_{y}$, and

$$
\begin{equation*}
F_{ \pm}=F_{x} \pm \mathrm{i} F_{y} \approx 2 \frac{\left(r_{x} \pm \mathrm{i} r_{y}\right)}{\rho^{3}(t)}, \tag{206}
\end{equation*}
$$

$$
\rho(t)=\left(r^{2}+z^{2}(t)\right)^{1 / 2} .
$$

To solve the problem with an exponential accuracy, we only need to drop in formula (206) the constant factor $r_{x} \pm \mathrm{i} r_{y}$ related to the plane of the collision parameters. For a head-on collision, the impact parameter is zero: $r=0$. With these simplifications, the function $z(t)$ is calculated from the equation

$$
\begin{equation*}
\frac{1}{4} \dot{z}^{2}+\frac{1}{|z|}=\frac{1}{4} v^{2}, \tag{207}
\end{equation*}
$$

where $v \equiv v_{\|}$is the relative velocity of positrons at infinity $(|z| \rightarrow \infty)$.

Dropping now all constant factors, we find

$$
\begin{equation*}
\left(\Delta \varepsilon_{\perp}\right)^{2} \sim Q^{2} \tag{208}
\end{equation*}
$$

$$
\begin{equation*}
Q=\int_{-\infty}^{\infty} \frac{\mathrm{d} t}{z^{3}(t)} \exp \left(\mathrm{i} \omega_{\mathrm{H}} t\right) . \tag{209}
\end{equation*}
$$

Solving equation (207) in the parametric form [32]

$$
\begin{aligned}
z & =\frac{2}{v^{2}}(\cosh x+1), \\
t & =\frac{2}{v^{3}}(\sinh x+x), \\
& -\infty<x<+\infty
\end{aligned}
$$

we get

$$
\begin{equation*}
Q \sim \int_{-\infty}^{\infty} \frac{\mathrm{d} x}{\cosh (x / 2)} \exp \left[\mathrm{i} \beta_{0}(\sinh (x)+x)\right], \tag{210}
\end{equation*}
$$

where

$$
\begin{equation*}
\beta_{0}=\frac{2 \omega_{\mathrm{H}}}{v^{3}} \gg 1 \tag{211}
\end{equation*}
$$

The main contribution to this integral of the quickly oscillating function comes from the singularity of the integrand, which is nearest to the real $x$-axis [73]. In this case it is a pole $x=\mathrm{i} \pi$, which simultaneously is a point of the stationary phase:

$$
\begin{equation*}
Q \sim \exp \left(-2 \pi \beta_{0}\right) . \tag{212}
\end{equation*}
$$

Owing to the smallness of typical values of $\Delta \varepsilon_{\perp}$, the process of longitudinal-transverse relaxation bears the nature of diffusion with respect to energy $\varepsilon_{\perp}$, with the diffusion coefficient [80]

$$
\begin{equation*}
D_{\varepsilon}=\frac{1}{2}\left\langle\frac{\left(\Delta \varepsilon_{\perp}\right)^{2}}{\tau}\right\rangle=\frac{1}{2} \int_{0}^{\infty} 2 \pi r \mathrm{~d} r \int_{-\infty}^{\infty} \mathrm{d} v f(v)\left(\Delta \varepsilon_{\perp}\right)^{2} v n_{\mathrm{e}} \tag{213}
\end{equation*}
$$

where

$$
\begin{equation*}
f(v)=\frac{1}{\sqrt{4 \pi T_{\|}}} \exp \left(-\frac{v^{2}}{4 T_{\|}}\right) . \tag{214}
\end{equation*}
$$

To analyze the experiments (see below), the most important is the case when

$$
\begin{equation*}
T_{\|} \gg T_{\perp} . \tag{215}
\end{equation*}
$$

In this case, the relaxation rate $\lambda_{\mathrm{r}}$ is of the order of the reciprocal time $\tau_{\mathrm{r}}$ of diffusive motion over the scale of energies $\varepsilon_{\perp}$ to the characteristic distance $\sim T_{\|}:$

$$
\begin{equation*}
\lambda_{\mathrm{r}} \sim \frac{1}{\tau_{\mathrm{r}}} \sim \frac{D_{\varepsilon}}{T_{\|}^{2}} \sim\left\langle Q^{2}\right\rangle, \tag{216}
\end{equation*}
$$

where we carry out averaging over the relative velocity $v$ of the longitudinal motion with the distribution function (214).

Evaluating the integral (216), viz.

$$
\left\langle Q^{2}\right\rangle \sim \int_{0}^{\infty} \mathrm{d} v \exp \left(-\frac{4 \pi \omega_{\mathrm{H}}}{v^{3}}-\frac{v^{2}}{4 T_{\|}}\right),
$$

by the steepest descent method, we get

$$
\begin{equation*}
\lambda_{\mathrm{r}} \sim \lambda_{0} \exp (-\beta), \tag{217}
\end{equation*}
$$

where the preexponential factor $\lambda_{0}$ is defined by formula (203) in which, with due account for inequality (215), we must make the replacement $T \rightarrow T_{\|}$, and

$$
\beta=\frac{5}{12}\left(\frac{24 \pi e^{3} H}{m_{\mathrm{e}}^{1 / 2} c T_{\|}^{3 / 2}}\right)^{2 / 5} \approx 13(\bar{H})^{2 / 5}\left(\bar{T}_{\|}\right)^{-3 / 5} .
$$

Here the notation was introduced: $\bar{T}_{\| \mid}=T_{\|} / T_{0}$.
It should be recognized that

$$
\beta \sim\left(\frac{R_{\mathrm{T}}}{r_{\mathrm{H}}}\right)^{3 / 2} \gg 1
$$

where for the temperature we must take $T_{\|}$. It is interesting that the main contribution to $\lambda_{\mathrm{r}}$ comes from the remote tail of the Maxwellian distribution (214):

$$
v \sim v_{0}=\left(\frac{12}{5} \beta\right)^{1 / 2} v_{\mathrm{T}} \gg v_{\mathrm{T}}
$$

## 14. Kinetic model of antiproton stopping in the positron plasma with regard to longitudinal - transverse relaxation. Analysis of recent experiments

Let us summarize the results discussed in the previous sections.

The temperature balance of supermagnetized plasma is described by the equations

$$
\begin{align*}
& \frac{\mathrm{d} T_{\|}}{\mathrm{d} t}=q_{0}-\lambda_{\mathrm{r}}\left(T_{\|}-T_{\perp}\right),  \tag{218}\\
& \frac{\mathrm{d} T_{\perp}}{\mathrm{d} t}=\lambda_{\mathrm{r}}\left(T_{\|}-T_{\perp}\right)-\frac{1}{\mathrm{C}_{\perp} \tau_{0}} T_{\perp} . \tag{219}
\end{align*}
$$

Here, the heat release power density

$$
q_{0}=\frac{P}{n_{\mathrm{e}} \pi R^{2} L C_{\|}},
$$

where $P$ is the heat release power in the plasma [expression (160)], $C_{\| \mid}=3 / 2, C_{\perp}=2$ are the heat capacities at constant pressure for the one-dimensional and two-dimensional gases,
respectively (which corresponds to the longitudinal and transverse motions of positrons), and $\lambda_{\mathrm{r}}$ is given by formula (217). In developing equations (218) and (219) we have taken into consideration that the cyclotron radiation is produced by the motion of positrons that is transverse but not longitudinal relative to the direction of the magnetic field. In addition, we have taken into account that because of the long-range nature of Coulomb forces, the energy of decelerated antiprotons is transferred mainly to the longitudinal motion of the positrons (see Section 7): owing to the conservation of adiabatic invariants (58) and (60) in the distant collisions which play the dominant role, the transverse positron energy is not changed.

The number of antiprotons $N_{\overline{\mathrm{p}}}(t)$ in the positron cloud at the point in time $t$ obeys equation (152) corrected to account for the formation of antihydrogen atoms:

$$
\begin{equation*}
\frac{\mathrm{d} N_{\overline{\mathrm{p}}}}{\mathrm{~d} y}=-\left(\frac{1}{\tau}+\lambda_{\overline{\mathrm{H}}}\right) N_{\overline{\mathrm{p}}}+J_{\overline{\mathrm{p}}}(t) . \tag{220}
\end{equation*}
$$

Finally, the yield of atoms per unit time is given by equations (105) and (125).

Let us discuss next the results of experiments [23, 26].
The former was concerned with the stopping of protons in the electron cloud, and the latter with the stopping of antiprotons in the positron cloud. The parameters of the Penning traps in both the experiments are rather similar, as is the number of particles:

$$
N_{\mathrm{e}} \sim 3 \times 10^{5}, \quad N_{\mathrm{p}} \sim 10^{4}
$$

and the energies of decelerated particles are of order

$$
E_{\mathrm{p}} \sim 10-50 \mathrm{eV}
$$

From this we can make an estimate for the plasma temperature established in the process of deceleration:

$$
T \sim N_{\mathrm{p}} \frac{E_{\mathrm{p}}}{N_{\mathrm{e}}} \sim 0.5-1 \mathrm{eV}
$$

which will be rigorously substantiated below.
It will be important and interesting to discuss here the method of measuring the number of particles in the cloud confined in the trap, which was developed in works [25, 117].

The external potential on the axis of the Penning trap near point B (see Fig. 3) takes the form

$$
\varphi_{0} \approx-k \frac{z^{2}}{2}
$$

where $z$ is the coordinate counted along the axis of the trap. Consequently, a single electron (for the sake of definiteness we shall speak of the electrons) oscillates along the $z$-axis with the frequency

$$
\omega_{\mathrm{e}}=\left(\frac{e k}{m_{\mathrm{e}}}\right)^{1 / 2}
$$

Since the mass and the charge of the cloud are proportional to the number of particles, the cloud will oscillate with the same frequency. To the electrodes B and C we connect the external RLC circuit and tune it in resonance with these axial oscillations of the cloud. In this way, we have two
'oscillators' with the same natural frequencies. As the cloud oscillates, the image charges are induced on electrodes B and C, which create additional potential difference proportional to the charge of the cloud, thus establishing coupling between the two oscillators. As known from mechanics, if two oscillators with coinciding eigenfrequencies are coupled, we get two new nearby frequencies $\omega_{1}$ and $\omega_{2}$ characteristic of the system as a whole. In the case of the Penning trap, the magnitude of frequency splitting

$$
\Delta \omega=\omega_{1}-\omega_{2}
$$

depends on the charge of the cloud, and hence on the number of particles therein (splitting is proportional to the square root of the number of particles). For measuring the number of particles in the cloud we have to measure the spectrum of noise in the RLC circuit near the resonance frequency. In the absence of a cloud this spectrum exhibits the Lorentz profile. In the presence of the cloud, the spectrum appears as two resonances with a frequency splitting as described above. The comparison of theoretical and observed profiles allows us to deduce the absolute number of particles in the cloud.

As explained earlier, inside the plasma we have $E_{\|}=0-$ that is, the external potential $\varphi_{0}$ inside the plasma is compensated for by the plasma potential $\varphi_{1}$. For our estimates we represent the electron cloud as a sphere of radius $R$, then from the relations

$$
\varphi_{1}=\frac{e N_{\mathrm{e}}}{R}, \quad\left|\varphi_{1}\right|=\left|\varphi_{0}(z=R)\right|
$$

follow the estimates

$$
R \sim\left(\frac{2 e N_{\mathrm{e}}}{k}\right)^{1 / 3}, \quad n_{\mathrm{e}} \sim \frac{k}{8 e}
$$

In this way, the density of the cloud does not depend on the number of particles and is determined by the parameters of the installation. According to Gabrielse et al. [26], one has

$$
\begin{equation*}
n_{\mathrm{e}}=7 \times 10^{6} \mathrm{~cm}^{-3} \tag{221}
\end{equation*}
$$

which means that a close value of the number density was also realized in the experiment [23]. At this density, the radius of the cloud is about 2 mm , and the time of proton deceleration in the cloud (see Section 7) approximates

$$
\begin{equation*}
\tau_{\mathrm{d}} \sim 10 \mathrm{~s} \tag{222}
\end{equation*}
$$

which agrees with the observations [23]. The calculation of the deceleration time was based on the above estimate of the electron plasma temperature established in the course of deceleration of protons. In the experiment with antiprotons [26], the deceleration time is about 1 s .

The heat release power and the heat release power density in Ref. [23] were (in a.u.)

$$
\begin{equation*}
P=\frac{N_{\mathrm{p}} E_{\mathrm{p}}}{\tau_{\mathrm{d}}}=1 \times 10^{-13}, \quad q_{0}=2 \times 10^{-19} \tag{223}
\end{equation*}
$$

Towards the end of the proton deceleration, the value of $q_{0}$ increases to $5 \times 10^{-18} \mathrm{a} . \mathrm{u}$. The time $\tau_{1}=\mathrm{C}_{\perp} \tau_{0}=0.2 \mathrm{~s}$ is small compared with $\tau_{\mathrm{d}}$, and therefore the quasi-stationary approximation $\mathrm{d} T_{\perp} / \mathrm{d} t \approx 0$ holds true for equation (219),
which gives us the linkage between the temperatures

$$
\begin{align*}
& T_{\perp}=\frac{\lambda_{\mathrm{r}}}{\lambda_{\mathrm{r}}+1 / \tau_{1}} T_{\|},  \tag{224}\\
& T_{\|}-T_{\perp}=\frac{T_{\|}}{1+\lambda_{\mathrm{r}} \tau_{1}} . \tag{225}
\end{align*}
$$

The physical meaning of the result (225) is clear: when the rate $\lambda_{\mathrm{r}}$ is high $\left(\lambda_{\mathrm{r}} \tau_{1} \gg 1\right)$, the temperature deviation is negligibly small, namely

$$
\frac{T_{\|}-T_{\perp}}{T_{\|}} \ll 1 .
$$

Such a deviation of temperatures becomes considerable at low rates of longitudinal - transverse relaxation. Substituting formula (224) into equation (218), we arrive at the equation for $T_{| |}$alone:

$$
\begin{align*}
& \frac{\mathrm{d} T_{\|}}{\mathrm{d} t}=q_{0}-\Lambda T_{\|},  \tag{226}\\
& \Lambda=\frac{\lambda_{\mathrm{r}}}{1+\lambda_{\mathrm{r}} \tau_{1}} . \tag{227}
\end{align*}
$$

From formula (227) it is clear that

$$
\begin{equation*}
\Lambda<\min \left(\lambda_{r} ; \frac{1}{\tau_{1}}\right) . \tag{228}
\end{equation*}
$$

The rate $\lambda_{\mathrm{r}}\left(T_{\|}\right)$at the magnetic field strength $H=6 \mathrm{~T}$, as in the experiment of Ref. [23], has a maximum at $T_{\|}=300 \mathrm{~K}$ (Fig. 9). The second term on the right-hand side of equation (226) has a maximum

$$
\begin{equation*}
\left(\Lambda T_{\|}\right)_{\max }=0.5 \times 10^{-19} \text { a.u. } \tag{229}
\end{equation*}
$$

at temperature $T_{| |}=400 \mathrm{~K}$. From estimates (223) and (229) we conclude that in the experiment of Ref. [23] one can recognize that

$$
\begin{equation*}
q_{0} \gg \Lambda T_{\|} \tag{230}
\end{equation*}
$$

over the entire time of proton deceleration:

$$
0<t<\tau_{\mathrm{d}}
$$



Figure 9. Temperature dependence of the rate of longitudinal - transverse relaxation, $H_{2}>H_{1}=6 \mathrm{~T}$.
when the heat release takes place $\left(q_{0} \neq 0\right)$. Physically, relation (230) implies that practically all the energy of the protons is transferred to the longitudinal motion of the electrons.

At the end of proton deceleration $\left(t=\tau_{\mathrm{d}}\right)$, as follows from the law of conservation of energy and formula (224), the parameters of the plasma account for

$$
\begin{equation*}
T_{\|}=2 \mathrm{eV}, \quad T_{\perp}=0.05 \mathrm{eV} \tag{231}
\end{equation*}
$$

To measure the energy of the protons, the electrostatic barrier in the region D (see Fig. 7) was reduced to a certain value $E_{1}$. The protons with energies $E>E_{1}$ were no longer retained by the trap; they escaped in the axial direction and were absorbed by the detector that performed their counting. In the experiment of Ref. [23], the microchannel plates (MCP) were used as detectors. The estimate (231) agrees with the thus measured value of the temperature of the electron cloud, $T=2 \mathrm{eV}$, which follows from the analysis of the proton spectrum reproduced in Fig. 3 from paper [23].

For $t>\tau_{\mathrm{d}}$, the heat release stops, $P=0$, and $q_{0}=0$. At $T_{\|}=2 \mathrm{eV}$, the parameter $\Lambda$ measures

$$
\Lambda=0.5 \times 10^{-17} \text { a.u. }=0.2 \mathrm{~s}^{-1}
$$

From this and equation (226) we see that for $t>\tau_{\mathrm{d}}$ the temperature $T_{\|}$slowly (with the characteristic time $\sim 5 \mathrm{~s}$ ) falls to almost zero ( $\sim 20 \mathrm{~K}$ ) over the time of approximately $10-15 \mathrm{~s}$. The last stage of the temperature relaxation

$$
T_{\|} \rightarrow T_{\perp}, \quad T_{\|}=20 \mathrm{~K} \rightarrow T_{\|}=4 \mathrm{~K}
$$

proceeds very slowly because of the effect of slow-down of longitudinal - transverse relaxation at such temperatures (see Fig. 9), which agrees with Refs [23, 26]. Production of antihydrogen atoms was not observed in Ref. [26] (nor in Refs [24, 25]). The materials of this review disentangle this puzzle. The first reason is the very short confinement time of antiprotons in the positron cloud, as follows from formula (151). The second reason is the low rate of production of atoms because of the high temperature $T_{\|}$[formula (231)], as follows from the results presented in Section 6.

Now let us discuss the ATHENA experiment in the design regime (see Fig. 7), and specify the optimal conditions for the conduction of the experiment, ensuring the highest yield of antihydrogen atoms.

Since the pulse repetition rate $f_{0}$ is large, $f_{0} \tau_{1} \gg 1$, we may assume with high accuracy that the antiproton current $J_{\overline{\mathrm{p}}}$ is constant, and consider the stationary conditions

$$
\frac{\mathrm{d} T_{\|}}{\mathrm{d} t}=0, \quad \frac{\mathrm{~d} T_{\perp}}{\mathrm{d} t}=0
$$

In general, the stationary approximation holds true when $f_{0}>5 \mathrm{~Hz}$. The typical parameters of the experiment are as follows

$$
\begin{equation*}
H=3 \mathrm{~T}, \quad E_{\overline{\mathrm{p}}}=e\left(\varphi_{\mathrm{C}}-\varphi_{\mathrm{B}}\right) \sim 20 \mathrm{eV} \tag{232}
\end{equation*}
$$

Let us introduce the notation for the reduced antiproton current:

$$
\begin{equation*}
\bar{J}_{\overline{\mathrm{p}}}=\frac{J_{\overline{\mathrm{p}}}}{J_{0}}, \quad J_{0}=10^{4} \mathrm{~s}^{-1} \tag{233}
\end{equation*}
$$

then

$$
\begin{equation*}
q_{0}=5 \times 10^{-19} \bar{J}_{\overline{\mathrm{p}}} \text { a.u. } \tag{234}
\end{equation*}
$$

From equation (226) at $\mathrm{d} T_{\|} / \mathrm{d} t=0$ we get the equation for the longitudinal temperature $T_{\|}$:

$$
\begin{equation*}
\frac{\lambda_{\mathrm{r}} T_{\|}}{1+\lambda_{\mathrm{r}} \tau_{1}}=q_{0} . \tag{235}
\end{equation*}
$$

From Eqn (235) it follows that for

$$
J_{\overline{\mathrm{p}}}>3 \times 10^{4} \mathrm{~s}^{-1}
$$

one obtains

$$
\begin{align*}
& T_{\|} \approx 6 \bar{J}_{\overline{\mathrm{p}}} \mathrm{~K} \\
& J_{\overline{\mathrm{H}}}=\frac{1.5 \times 10^{4}}{\left(\bar{J}_{\overline{\mathrm{p}}}\right)^{4.8}} \mathrm{~s}^{-1} . \tag{236}
\end{align*}
$$

For $J_{\overline{\mathrm{p}}}<10^{4} \mathrm{~s}^{-1}$, the temperature $T_{\|}$weakly depends on the magnitude of $J_{\overline{\mathrm{p}}}$ :

$$
\begin{align*}
& T_{\|} \sim 10 \mathrm{~K}, \\
& J_{\overline{\mathrm{H}}} \sim 5 \times 10^{2} \bar{J}_{\overline{\mathrm{p}}} \mathrm{~s}^{-1} . \tag{237}
\end{align*}
$$

The plasma temperature as a function of the antiproton current $J_{\overline{\mathrm{p}}}$ is shown in Fig. 10, and the achievable yields $J_{\overline{\mathrm{H}}}$ (per unit time) of antihydrogen atoms are shown in Fig. 11. The highest value of $J_{\overline{\mathrm{H}}}=400 \mathrm{~s}^{-1}$ is reached at the current $J_{\overline{\mathrm{p}}}=2 \times 10^{4} \mathrm{~s}^{-1}$. The temperature of the plasma then is $T_{\|}=15 \mathrm{~K}$. Observe that in the region near the maximum yield $J_{\mathrm{H}}$, we have

$$
T_{\perp} \approx T_{\| \mid} \equiv T
$$

and therefore in Fig. 10 the temperature is depicted without any subscripts.

It is interesting to note that the initial design [3] assumed the following conditions:

$$
\begin{equation*}
J_{\overline{\mathrm{p}}}=10^{7} \mathrm{~s}^{-1}, \quad n_{\mathrm{e}}=10^{8} \mathrm{~cm}^{-3} . \tag{238}
\end{equation*}
$$



Figure 10. Temperature of positron plasma in the steady-state regime of antiproton injection.


Figure 11. Yield of antihydrogen atoms vs. the number of antiprotons injected per unit time.

Under these conditions, the magnitude of specific heat release $q_{0} \sim 1 \times 10^{-18}$ a.u. is close to the value given in Eqn (223), characteristic of the experiments by Gabrielse et al. [23, 26]. However, the positron density number $n_{\mathrm{e}}$ is much higher, and therefore the temperatures differ from the values given in Eqn (231):

$$
\begin{equation*}
T_{\|} \sim 1 \mathrm{eV}, \quad T_{\perp} \sim 0.9 \mathrm{eV} \tag{239}
\end{equation*}
$$

According to Fig. 11, owing to the high values of temperature (a short confinement time of antiprotons in the plasma, low rate of atomic production), there ought to be no atoms observed under conditions (238).

## 15. Conclusions

Currently, the recombination processes are the key issues in the production and study of cold antihydrogen atoms. The rate of radiative (spontaneous) recombination decreases rather slowly with increasing temperature. In spite of the relatively low characteristic values of this rate, this circumstance allows us to use this process for the production of antihydrogen in plasma with a temperature of about 100 K . The rate of this process is considerably affected by the magnetic field: at $H \gtrsim 1 \mathrm{~T}$, the rate of radiative recombination increases approximately twofold compared with the case of $H=0$.

The situation with stimulated (laser-assisted) radiative recombination is not yet completely clear. The construction of the theory of this process is currently impeded by the deficiencies in the theory of stochastic ionization of Rydberg atoms. The available data were obtained with pulsed lasers. For this reason, these results are not relevant to the continuous-wave laser mode required for the production of antihydrogen atoms: in the pulsed mode, the emerging atoms do not have time to be ionized with the laser field, whereas the cw laser produces fast stochastic ionization.

Rather high yields of antihydrogen atoms are anticipated if the temperature of the positron plasma can be reduced to $T \leqq 100 \mathrm{~K}$. At such temperatures, the main mechanism of the production of atoms is the three-particle recombination. As a result of this process, excited antihydrogen atoms will be ejected from the positron plasma in the states with $n \sim 30$. The problem of capture and confinement of these atoms calls for special research. According to preliminary estimates, this problem can be best solved using the Ioffe-Pritchard traps. It is also very interesting to study the idea of the possible cooling of excited antihydrogen atoms by collisions with atoms of
noble gases. The best candidate for such a cooling is probably neon.

Another key process that determines the yield of antihydrogen atoms is the longitudinal diffusion of antiprotons (along the magnetic field), which allows the latter to escape through the ends of the positron plasma. A typical residence time of antiprotons in plasma is approximately equal to $10^{-3}-10^{-4} \mathrm{~s}$. The loss of antiprotons due to their lateral diffusion in the field can be neglected.

Injection of antiproton bunches into the positron plasma is accompanied by a number of heat and particle transport processes. In supermagnetized positron (or electron) plasma, these processes are determined by the collective effects, and are therefore fast. Because of this, the temperature and density of positron plasma after injection of antiprotons remain practically unchanged in the bulk of the plasma.

In cold magnetized plasma it would be natural to expect manifestations of the shielding of cyclotron radiation, leading to a considerable suppression of this radiation. Our analysis indicates, however, that this effect is negligibly small in the typical conditions of the ATHENA and ATRAP experiments.

In low-temperature supermagnetized plasma, the energy transfer from longitudinal (along the magnetic field) to transverse motion of positrons is suppressed. This effect leads to the 'disengagement' of the longitudinal temperature of positrons from their transverse temperature in the course of injection of antiprotons. This two-temperature model explains the results of recent experiments with stopping of protons (antiprotons) in cold electron (positron) plasmas.

Under conditions (232) and an antiproton current $J_{\overline{\mathrm{p}}} \sim 10^{4} \mathrm{~s}^{-1}$, this effect of 'disengagement' of temperatures is not significant. According to the theory presented here, under these conditions the production of antihydrogen atoms will be efficient.

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Explanatory note 1 added in proofs. This paper was received by Phys. Usp. Editorial Board in May 2002. In August 2002, the first antihydrogen atoms were registered in the ATHENA experiments [118]. Recombination of positrons and antiprotons was accomplished in a special regime selected on the basis of the material presented in this review. In early November 2002, antihydrogen atoms were registered in ATRAP [119]. As the situation in this area of research is changing fast (in particular, new interesting theoretical results have been produced), the authors decided not to make any major updates of this review (which would increase its size and delay publication), and only introduced minor corrections as required.

Explanatory note 2 added in proofs. Looking through the literature cited in Ref. [81], we found that the idea of the effect of LTR suppression, considered in Section 13 of our review, was actually contained in item [41] (Ref. [120] in this review) in the bibliography of Ref. [81]. In that paper this effect was first established in the experiment staged in the context of studying the method of beam cooling by electrons. According to Ref. [120], the LTR is suppressed in a sufficiently strong magnetic field, when

$$
r_{H}<r_{1}=\min \left(R_{\mathrm{T}}, \bar{R}\right),
$$

where $R_{\mathrm{T}}=e^{2} / T_{\|}$, and $\bar{R}=n_{\mathrm{e}}^{-1 / 3}$. By virtue of the effect of kinematic cooling of the accelerated beam [121], the achiev-
able longitudinal temperature of the electron beam is low $\left(T_{\|} \ll T_{\perp}\right)$ and, as a rule, close to its limiting value

$$
T_{\|} \sim e^{2} n_{\mathrm{e}}^{-1 / 3}
$$

at which $R_{\mathrm{T}} \sim \bar{R}$. The physical meaning of this relation is that the kinetic energy of Larmor circles moving along the magnetic field is of the order of the potential energy of their interaction. Calculation of the effect of LTR suppression has not been done in Ref. [120] (and is hardly feasible given the nonideality of plasma). In our case, in the cloud of positrons

$$
T_{\|} \gg T_{\perp},
$$

the plasma is ideal, and so the calculation was done relatively straightforward.

## 16. Appendices

## I. Rate of stimulated radiative recombination.

## Derivation of formula (77)

The Hamiltonian of the interaction of an atom with a linearly polarized (along the $z$-axis) laser field $E=E_{0} \cos (\omega t)$ takes the form

$$
\begin{equation*}
V=d_{z} E_{0} \cos (\omega t) \tag{I.1}
\end{equation*}
$$

where $d_{z}$ is the projection of the operator of the atomic dipole moment onto the $z$-axis, and $E_{0}$ is the amplitude of laser field. In the first order of perturbation theory, the probability amplitude of the atomic transition

$$
|i\rangle \equiv|\mathbf{p}\rangle \rightarrow|f\rangle \equiv|n \operatorname{lm}\rangle
$$

is given by

$$
C_{f i}(t)=\int_{0}^{t}\left(d_{z}\right)_{f i} E_{0} \cos \left(\omega t^{\prime}\right) \exp \left[-\mathrm{i}\left(\frac{p^{2}}{2}+I_{n}\right) t^{\prime}\right] \mathrm{d} t^{\prime}
$$

As $t \rightarrow+\infty$, the transition probability is equal to

$$
\begin{equation*}
W_{f i}=\left|C_{f i}\right|^{2} \rightarrow \frac{\pi}{2} t E_{0}^{2}\left|\left(d_{z}\right)_{f i}\right|^{2} \delta\left(\frac{p^{2}}{2}+I_{n}-\omega\right) \tag{I.2}
\end{equation*}
$$

The time-averaged laser intensity is given by

$$
\begin{equation*}
I=\frac{E_{0}^{2}}{8 \pi} c \tag{I.3}
\end{equation*}
$$

The wave function of the positron continuous spectrum has the asymptotic form

$$
\Psi_{\mathbf{p}}(\mathbf{r}) \rightarrow \exp (\mathrm{ipr}),
$$

and is thus normalized to unity in a unit volume. The number of positrons in this volume with velocities from the interval $\mathrm{d}^{3} v$ equals

$$
\mathrm{d} N_{\mathrm{e}}=n_{\mathrm{e}} f(\mathbf{v}) \mathrm{d}^{3} v, \quad \int f(\mathbf{v}) \mathrm{d}^{3} v=1
$$

where $f(\mathbf{v})$ is the positron distribution function.

The rate of recombination on one antiproton (probability per unit time) is defined as

$$
\lambda_{\mathbf{S}}(n l)=\sum_{m=-l}^{l} \int n_{\mathrm{e}} f(\mathbf{v}) \mathrm{d}^{3} v \frac{W_{f i}}{t} .
$$

After integration with respect to velocity, the delta-function in formula (I.2) vanishes, which gives us result (77). Numerical factor $1 / 3$ comes from the relation

$$
\sum_{m}\left|\left(d_{z}\right)_{f i}\right|^{2}=\frac{1}{3} \sum_{m}\left|(\mathbf{d})_{f i}\right|^{2} .
$$

## II. Transverse transport of energy and momentum by plasmons

In supermagnetized plasma $r_{\mathrm{H}} \ll R_{\mathrm{T}}$, the transverse motion of electrons (for definiteness, in this section we shall be speaking of the electron plasma) is suppressed by the magnetic field, and therefore, as indicated in Section 10, the transverse transport of energy and momentum is performed in collective processes. To be more specific, let us demonstrate that this transport is performed by plasmons. To a reasonable accuracy we may disregard the transverse motion of electrons, and assume that they only move along the magnetic field lines ( $z$-axis). In this approximation (and in the hydrodynamic approximation) the equations that describe plasmons are written down as

$$
\begin{align*}
& \Delta \varphi=4 \pi e n_{\mathrm{e}}  \tag{II.1}\\
& m_{\mathrm{e}} \frac{\mathrm{~d} v}{\mathrm{~d} t}=e \frac{\partial \varphi}{\partial z}  \tag{II.2}\\
& \frac{\partial n_{\mathrm{e}}}{\partial t}+\frac{\partial}{\partial z}\left(n_{\mathrm{e}} v\right)=0 . \tag{II.3}
\end{align*}
$$

Here, $n_{\mathrm{e}}$ is the number density of electrons, $\varphi$ is the electric potential, and $v \equiv v_{z}$ is the longitudinal velocity of the electrons. Now we linearize the equations:

$$
n_{\mathrm{e}}=n_{0}+n_{1}(x, y, z, t), \quad \varphi=\varphi_{0}+\varphi_{1},
$$

where $n_{0}$ and $\varphi_{0}$ are the unperturbed quantities that satisfy the equation

$$
\Delta \varphi_{0}=4 \pi e n_{0}
$$

and $n_{1}$ and $\varphi_{1}$ are the perturbations that arise in plasma oscillations:

$$
n_{1}=\bar{n}_{1} \exp (\mathrm{i} \psi), \quad \varphi_{1}=\bar{\varphi}_{1} \exp (\mathrm{i} \psi), \quad \psi=\mathbf{k r}-\omega t .
$$

From the linearized equations

$$
\begin{aligned}
& \Delta \varphi_{1}=4 \pi e n_{1}, \\
& m_{\mathrm{e}} \frac{\partial v}{\partial t}=e \frac{\partial \varphi_{1}}{\partial z}, \\
& \frac{\partial n_{1}}{\partial t}+n_{0} \frac{\partial v}{\partial z}=0
\end{aligned}
$$

follows the dispersion relation for plasmons in SMP:

$$
\begin{equation*}
\omega=\omega_{\mathrm{p}}|\cos \alpha|, \tag{II.4}
\end{equation*}
$$

where $\alpha$ is the angle between the vectors $\mathbf{k}$ and $\mathbf{H}$. Relation (II.4) holds if we can neglect the collisions between electrons, as seen from equation (II.2). The mean free path of electrons is of order

$$
\lambda_{\mathrm{e}} \sim\left(10 R_{\mathrm{T}}^{2} n_{\mathrm{e}}\right)^{-1} \sim 0.05 \mathrm{~cm}
$$

and is small compared with the size of the plasma. This means that, along with the Coulomb long-range interaction which is described by the potential $\varphi$ in formula (II.2), we must also take into account the interaction between the particles at small distances ( $\sim R_{\mathrm{T}}$ ), which, owing to the conservation of momentum, is transferred to the large scales in the form of pressure, and thus affects the collective phenomena. These effects are taken into account by introducing into the righthand side of equation (II.2) the term $-\partial p / \partial z$ with the pressure $p=n_{\mathrm{e}} T$ of the electron gas, where $T \equiv T_{\|}$. Of practical importance is the case

$$
\frac{\omega_{\mathrm{p}}}{\lambda_{\mathrm{r}}} \gg 1,
$$

when the relaxation of temperatures $T_{\perp}$ and $T_{\|}$does not have time to complete. In this case we have the formula

$$
T \sim n_{\mathrm{e}}^{2}
$$

which characterizes the adiabatic compression of one-dimensional gas, from which follows the relation

$$
\frac{\partial p}{\partial z}=3 T \frac{\partial n_{\mathrm{e}}}{\partial z}
$$

Given this, the dispersion law (II.4) is changed:

$$
\begin{equation*}
\omega=|\cos \alpha|\left(\omega_{\mathrm{p}}^{2}+3 v_{\mathrm{T}}^{2} k^{2}\right)^{1 / 2} \tag{II.5}
\end{equation*}
$$

where $v_{\mathrm{T}}=\left(T / m_{\mathrm{e}}\right)^{1 / 2}$. In this way, owing to thermal motion, the plasmons exhibit dispersion - that is, their frequency shows dependence on the wave vector $\mathbf{k}$. In addition, we have Landau damping [79, 80] whose decrement $\gamma$ is small when $k r_{\mathrm{D}} \ll 1$, and becomes large, viz.

$$
\begin{equation*}
\gamma \sim \omega_{\mathrm{p}}, \tag{II.6}
\end{equation*}
$$

at $k r_{\mathrm{D}} \sim 1$.
For typical plasmons one finds

$$
\begin{equation*}
k \sim \frac{1}{r_{\mathrm{D}}}, \quad \omega \sim \omega_{\mathrm{p}} \tag{II.7}
\end{equation*}
$$

and the group velocity is of the order of $v_{\mathrm{T}}$. Given that $T \gg \omega_{\mathrm{p}}$, from the Planck distribution for the concentration of plasmons we get the estimate ( $\hbar=m_{\mathrm{e}}=e=1$ ):

$$
\begin{equation*}
n_{\mathrm{p}} \sim \frac{T}{r_{\mathrm{D}}^{3} \omega_{\mathrm{p}}} \tag{II.8}
\end{equation*}
$$

Assume now there is a transverse temperature gradient directed along the $x$-axis. Then the heat flux density carried by the plasmons traveling from the region $x<0$ to the region $x>0$ is of the order of

$$
\begin{equation*}
q_{+} \sim \omega_{\mathrm{p}} n_{\mathrm{p}} v_{\mathrm{T}} \tag{II.9}
\end{equation*}
$$

From estimate (II.6) we conclude that the mean free path of the plasmon is about $r_{\mathrm{D}}$. The variation of temperature on this
length approximates

$$
\begin{equation*}
\Delta T \sim r_{\mathrm{D}}\left|\frac{\mathrm{~d} T}{\mathrm{~d} x}\right| \tag{II.10}
\end{equation*}
$$

and the change of thermal velocity is of order

$$
\Delta v_{\mathrm{T}} \sim v_{\mathrm{T}} \frac{\Delta T}{T}
$$

and so the resulting heat flux along the $x$-axis is given by

$$
q=q_{+}-q_{-} \sim \omega_{\mathrm{p}} n_{\mathrm{p}} \Delta v_{\mathrm{T}} \sim \frac{v_{\mathrm{T}}}{r_{\mathrm{D}}^{2}}\left|\frac{\mathrm{~d} T}{\mathrm{~d} x}\right| .
$$

Hence we get the following estimates for the heat conductivity coefficient and thermal diffusivity:

$$
\begin{equation*}
\chi_{\perp} \sim \frac{v_{\mathrm{T}}}{r_{\mathrm{D}}^{2}}, \quad \chi_{\perp} \sim \frac{\chi_{\perp}}{n_{\mathrm{e}}} \sim v_{\mathrm{T}} R_{\mathrm{T}} \tag{II.11}
\end{equation*}
$$

For evaluation of the transverse viscosity let us consider the case when plasma is flowing along the magnetic field (z-axis) with a velocity

$$
u=u(x) .
$$

The plasmon dispersion law (II.5) holds true in the plasma's frame of rest. The formula for the laboratory system of coordinates is derived from relation (II.5) through the replacement

$$
\omega \rightarrow \omega-k_{z} u .
$$

Hence, we arrive at

$$
\begin{equation*}
\omega-k_{z} u=|\cos \alpha|\left(\omega_{\mathrm{p}}^{2}+3 v_{\mathrm{T}}^{2} k^{2}\right)^{1 / 2} . \tag{II.12}
\end{equation*}
$$

This simple procedure is also suitable for the quasi-classical case of a plasmon propagation in inhomogeneous plasma:

$$
k_{\perp} L \gtrdot 1,
$$

where $L=u|\mathrm{~d} u / \mathrm{d} x|^{-1}$. Given the estimates (II.7), this means that $L \gg r_{\mathrm{D}}$.

The component $k_{z}$ of the wave vector of the plasmon moving along the $x$-axis is changed, which physically implies the transfer of the momentum component

$$
p_{z}=\hbar k_{z}
$$

from plasmon to plasma: the plasmon is the carrier of momentum from one region in plasma to another. Unlike the wave vector $\mathbf{k}$, the frequency of plasmon is preserved as it travels. From relation (II.12), with due account for estimates (II.7), for the most important case of the slow motion of plasma

$$
u \ll v_{\mathrm{T}},
$$

we get

$$
\begin{equation*}
\left|\Delta p_{z}\right| \sim \frac{\hbar}{r_{\mathrm{D}}} \frac{\Delta u}{v_{\mathrm{T}}} \tag{II.13}
\end{equation*}
$$

where $\Delta p_{z}$ is the change of the $z$-component of the plasmon momentum on the length $r_{\mathrm{D}}$, and $\Delta u$ is the same for the
velocity $u$ :

$$
\begin{equation*}
\Delta u \sim r_{\mathrm{D}}\left|\frac{\mathrm{~d} u}{\mathrm{~d} x}\right| \tag{II.14}
\end{equation*}
$$

The viscous force acting on the unit area of surface $x=0$ is equal to the flux density of the $z$-component of the momentum transferred across this plane (in atomic units):

$$
f \sim \Delta p_{z} n_{\mathrm{p}} v_{\mathrm{T}} \sim n_{\mathrm{p}}\left|\frac{\mathrm{~d} u}{\mathrm{~d} x}\right| .
$$

Accordingly, the transverse viscosity is approximately equal to

$$
\begin{equation*}
\eta_{\perp} \sim n_{\mathrm{p}} \tag{II.15}
\end{equation*}
$$

The density of electron momentum is

$$
q_{\mathrm{e}} \sim n_{\mathrm{e}} m_{\mathrm{e}} u
$$

The similar quantity for plasmons equals

$$
q_{\mathrm{p}} \sim n_{\mathrm{p}} \frac{\hbar}{r_{\mathrm{D}}} \frac{u}{v_{\mathrm{T}}} .
$$

Hence, we conclude that

$$
\frac{q_{\mathrm{p}}}{q_{\mathrm{e}}} \sim \frac{R_{\mathrm{T}}}{r_{\mathrm{D}}} \sim \xi^{-3 / 2} \ll 1
$$

where $\xi$ is the parameter of ideality of plasma [79, 80]. Consequently, the momentum of plasma consists practically of the momentum of electrons, but it is transported in the transverse direction by plasmons. The effective kinematic viscosity is given by

$$
v_{\perp}=\frac{\eta_{\perp}}{n_{\mathrm{e}} m_{\mathrm{e}}} \sim v_{\mathrm{T}} R_{\mathrm{T}}
$$

and coincides by an order of magnitude with the thermal diffusivity as defined by formula (II.11). This is not surprising, since the mechanisms of transport of energy and momentum are the same.

## References

\&oi> 1. Baur G et al. Phys. Lett. B 368251 (1996)
doi> 2. Blanford G et al. Phys. Rev. Lett. 803037 (1998)
do> 3. Holzscheiter M H et al. Nucl. Phys. B Proc. Suppl. 56A 336 (1997); http://varming.home.cern.ch/varming/Athena/ATHENA-Reading-Room/articles/ATHENA/athena-npb56.pdf
4. Gabrielse G et al. (ATRAP Collab.) "The production and study of cold antihydrogen", SPSC 97-8/P306 (Genève: CERN, 1997); http://hussle.harvard.edu/ atrap/Goals/Proposal.pdf
5. Budker G I, Skrinskiĭ A N Usp. Fiz. Nauk 124561 (1978) [Sov. Phys. Usp. 21277 (1978)]
6. Luders G Kgl. Danske Vidensk. Selsk. Mat. Fys. Medd. 28 (5) 1 (1954)
7. Pauli W, in Niels Bohr and the Development of Physics (Eds W Pauli, L Rosenfeld, V Weisskopf) (London: Pergamon Press, 1955) p. 30
ㅇoz 8. Rubakov V A Usp. Fiz. Nauk 1691299 (1999) [Phys. Usp. 421193 (1999)]
$10 \geq 1$ 9. Shabalin E P Usp. Fiz. Nauk 171951 (2001) [Phys. Usp. 44895 (2001)]
coiz10. Hughes R J, Deutch B I Phys. Rev. Lett. 69578 (1992)
ब0®1 1. Okun L B, hep-ph/0210052; in Conf. XIVth Rencontres de Blois "Matter-Antimatter Asymmetry", Château de Blois, France, June 16-22, 2002
$\qquad$






dō＞12．Carosi R et al．Phys．Lett．B 237303 （1990）
doi＞13．Adler R et al．（CPLEAR Collab．）Nucl．Phys．B Proc．Suppl．56A 361 （1997）
14．Lifshitz E M，Pitaevskiǐ L P Relyativistskaya Kvantovaya Teoriya （Relativistic Quantum Theory）Pt． 2 （Moscow：Nauka，1971） $00>61$ ［Translated into English（Oxford：Pergamon Press，1974）］
15．Lipkin H J Quantum Mechanics：New Approaches to Selected Topics （Amsterdam：North－Holland Publ．Co．，1973）［Translated into Russian（Moscow：Mir，1977）］
16．Hughes R J Hyperfine Interact． 763 （1993）
doi＞17．Meshkov I，Skrinsky A Nucl．Instrum．Meth．A 37941 （1996） Part．Nucl． 28198 （1997）］
19．Parkhomchuk V V Hyperfine Interact． 44315 （1988）
बoi＞20．Sokolov Yu L Usp．Fiz．Nauk 169559 （1999）［Phys．Usp． 42481 （1999）］
doiz21．Holzscheiter M H，Charlton M Rep．Prog．Phys． 621 （1999）
山oi＞22．Eades J，Hartmann F J Rev．Mod．Phys． 71373 （1999）
■0i＞23．Hall D S，Gabrielse G Phys．Rev．Lett． 771962 （1996）
dioi＞24．Brown B et al．Nucl．Phys．B Proc．Suppl．56A 326 （1997）
©oiz25．Gabrielse G et al．Phys．Lett．B 455311 （1999）
đoi＞26．Gabrielse G et al．Phys．Lett．B507 1 （2001）
27．Penning F M Physica 3873 （1936）
๔oi＞28．Paul W Rev．Mod．Phys． 62531 （1990）；Usp．Fiz．Nauk 160 （12） 109 （1990）
■0i＞29．Gabrielse G et al．Phys．Rev．Lett． 631360 （1989）
๗0i＞30．Malmberg J H，deGrassie J S Phys．Rev．Lett． 35577 （1975）
©0i＞31．Walz J et al．Phys．Rev．Lett． 753257 （1995）
32．Landau L D，Lifshitz E M Mekhanika（Mechanics）（Moscow： Nauka，1973）［Translated into English（Oxford：Pergamon Press， 1976）］
doi＞33．Dehmelt H G，Schwinberg P B，Van Dyck R S（Jr）Int．J．Mass Spectrom．Ion Phys． 26107 （1978）
©oi＞34．Schwinberg P B，Van Dyck R S（Jr），Dehmelt H G Phys．Lett．A 81 119 （1981）
』0i＞35．Surko C M et al．Phys．Rev．Lett． 611831 （1988）
■10i＞36．Greaves R G，Surko C M Phys．Rev．Lett． 753846 （1995）
37．Wineland D J et al．Hyperfine Interact． 76115 （1993）
बoiz38．Haarsma L H，Abdullah K，Gabrielse G Phys．Rev．Lett． 75806 （1995）
U0＞39．Estrada J et al．Phys．Rev．Lett． 84859 （2000）
40．Oshima N et al．，RIKEN Review，No． 31 ［Japan：RIKEN（The Institute of Physical and Chemical Res．），2000］p． 65
■0＞＞41．Mills A P（Jr），Gullikson E M Appl．Phys．Lett． 491121 （1986）
doi＞42．Canter K F，Mills A P（Jr），Berko S Phys．Rev．Lett． 34177 （1975）
Hoi＞43．Lynn K G Phys．Rev．Lett． 441330 （1980）
44．Walraven J T M Hyperfine Interact． 76205 （1993）
45．Hänsch T W，Zimmermann C Hyperfine Interact． 7647 （1993）
46．Vasilenko L S，Chebotaev V P，Shishaev A V Pis＇ma Zh．Eksp．Teor． Fiz． 12161 （1970）［JETP Lett． 12113 （1970）］
47．Letokhov V S，Chebotaev V P Printsipy Nelineĭnǒ̆ Lazernǒ̆ Spektroskopii（Principles of Nonlinear Laser Spectroscopy）（Mos－ cow：Nauka，1975）；Letokhov V S，Chebotayev V P Nonlinear Laser Spectroscopy（Springer Ser．in Optical Sci．，Vol．4）（Berlin：Springer－ Verlag，1977）
■0i＞48．Setija I D et al．Phys．Rev．Lett． 702257 （1993）
di＞ $\mathbf{4}$ 49．Weitz M et al．Phys．Rev．Lett． 72328 （1994）
doi＞50．Schmidt－Kaler F et al．Phys．Rev．A 512789 （1995）
๗0i＞51．Udem Th et al．Phys．Rev．Lett． 792646 （1997）
๔oiv52．Cesar C L et al．Phys．Rev．Lett． 77255 （1996）
53．Berestetskiĭ V B，Lifshitz E M，Pitaevskiĭ L P Relyativistskaya Kvantovaya Teoriya（Relativistic Quantum Theory）Pt． 1 （Mos－ cow：Nauka，1968）［Translated into English（Oxford：Pergamon Press，1971）］
』0i＞54．Morgan D L（Jr），Hughes V W Phys．Rev．D 21389 （1970）
55．Morgan D L（Jr）Hyperfine Interact． 44399 （1988）
56．Carbonell J，Protasov K Hyperfine Interact． 76327 （1993）
57．Men＇shikov L I，Preprint IAE－6036／3（Moscow：Russian Research đoiz Centre＇Kurchatov Institute＇，1997）

65．Gabrielse G Hyperfine Interact． 44349 （1988）
66．Kasevich M，Chu S Appl．Phys．B 54321 （1992）
67．Poggiani R Hyperfine Interact． 76371 （1993）
di＞68．Phillips T J Hyperfine Interact． 109357 （1997）
69．Kadomtsev B B Dinamika i Informatsiya（Dynamics and Informa－ tion）（Moscow：Red．Zhurn．＂Uspekhi Fizicheskikh Nauk＂，1997）
70．Kramers H A Philos．Mag． 46836 （1923）
doi＞71．Kogan V I，Kukushkin A B，Lisitsa V S Phys．Rep． 2131 （1992）
72．Bureeva L A，Lisitsa V S Vozmushchennyı̆ Atom（Disturbed Atom） （Moscow：IzdAT，1997）
73．Landau L D，Lifshitz E M Kvantovaya Mekhanika：Nerelyativist－ skaya Teoriya（Quantum Mechanics：Non－Relativistic Theory） （Moscow：Nauka，1974）［Translated into English（Oxford：Perga－ mon Press，1977）］
74．Landau L D，Lifshitz E M Teoriya Polya（Field Theory）（Moscow： Nauka，1988）［Translated into English：The Classical Theory of Fields（Oxford：Pergamon Press，1983）］
75．Abramowitz M，Stegun I A（Eds）Handbook of Mathematical Functions with Formulas，Graphs，and Mathematical Tables（New York：Dover Publ．，1965）［Translated into Russian（Moscow： Nauka，1979）］
76．Bethe H A，Salpeter E E Quantum Mechanics of One－and Two－ Electron Atoms（Berlin：Springer－Verlag，1957）［Translated into Russian（Moscow：Fizmatgiz，1960）］
77．Stobbe M Ann．Phys．（Leipzig） 7661 （1930）
78．Katkov V M，Strakhovenko V M Zh．Eksp．Teor．Fiz． 751269 （1978） ［Sov．Phys．JETP 48639 （1978）］
79．Artsimovich L A Upravlyaemye Termoyadernye Reaktsii（Con－ trolled Thermonuclear Reactions）2nd ed．（Moscow：Fizmatgiz， 1963）［Translated into English（New York：Gordon and Breach Sci． Publ．，1964）］
80．Lifshitz E M，Pitaevskiĭ L P Fizicheskaya Kinetika（Physical Kinetics）（Moscow：Nauka，1979）［Translated into English（Ox－ ford：Pergamon Press，1981）］
81．Meshkov I N Fiz．Elem．Chastits At．Yadra 251487 （1994）［Phys． Part．Nucl． 25631 （1994）］
82．Bosser J，Preprint CERN－PS／96－06（BD）（Genèva：CERN，1996）
doi＞83．Parkhomchuk V V，Skrinskiĭ A N Usp．Fiz．Nauk 170473 （2000） ［Phys．Usp． 43433 （2000）］
84．Monteiro T S Contemp．Phys． 35311 （1994）
oi＞85．Gwinner G et al．Phys．Rev．Lett． 844822 （2000）
86．Neumann R et al．Z．Phys．A：Hadrons Nucl． 313253 （1983）
87．Gabrielse G et al．Hyperfine Interact． 44287 （1988）
88．Blatt P Hyperfine Interact． 44295 （1988）
89．Wolf A Hyperfine Interact． 76189 （1993）
■0；＞90．Yousif F B et al．Phys．Rev．Lett． 6726 （1991）
■oi＞91．Schramm U et al．Phys．Rev．Lett． 6722 （1991）
92．Mansbach P，Keck J Phys．Rev． 181275 （1969）
93．Delone N B，Kraĭnov V P，Shepelyanskiĭ D L Usp．Fiz．Nauk 140355 （1983）［Sov．Phys．Usp． 26551 （1983）］
oì94．Casati G et al．Phys．Rep． 15477 （1987）
95．Jensen R V，in Atomic Physics 10：Proc．of the 10th Intern．Conf．on Atomic Physics，ICAP－X，Tokyo，Japan，Aug．25－29， 1986 （Eds H Narumi，I Shimamura）（Amsterdam：North－Holland，1987） p． 319
6．Benenti G，Casati G，Shepelyansky D L Eur．Phys．J．D 5311 （1999）
97．Sobel＇man I I Vvedenie v Teoriyu Atomnykh Spektrov（Introduction to the Theory of Atomic Spectra）（Moscow：Fizmatgiz，1963）
98．Mranslated into English（Oxford：Pergamon Press，1972）］
doi＞59．Men＇shikov L I，Eseev M K Usp．Fiz．Nauk 171149 （2001）［Phys． Usp． 44135 （2001）］

60．Radzig A A，Smirnov B M Spravochnik po Atomnoй i Molekulyarnoŭ Fizike（Handbook on Atomic and Molecular Physics）（Moscow： Atomizdat，1980）［Translated into English：Reference Data on Atoms，Molecules，and Ions（Berlin：Springer－Verlag，1985）］
Briggs J S，Greenland P T，Solov＇ev E A Hyperfine Interact． 119235 （1999）
62．Kottmann F，in Fundamental Interactions in Low－Energy Systems （Ettore Majorana Intern．Sci．Series，Physical Sciences，Vol．23，Eds P Dalpiaz，G Fiorentini，G Torelli）（New York：Plenum Press，1985） p． 481
O’Neil T M Phys．Fluids 232216 （1980）
Squires T M，Yesley P，Gabrielse G Phys．Rev．Lett． 865266 （2001）

 ［JETP 8178 （1995）］

๔0ㅍ99. Men'shikov L I Yad. Fiz. 63920 (2000) [Phys. Atom. Nucl. 63850 (2000)]

H0>>100. Glinsky M E, O’Neil T M Phys. Fluids B 31279 (1991)
div101. Fedichev P O Phys. Lett. A 226289 (1997)
102. Belyaev S T, Budker G I, in Fizika Plazmy i Problema Upravlyaemykh Termoyadernykh Reaktsiü (Plasma Physics and the Problem of Controlled Thermonuclear Reactions) (Ed. M A Leontovich) Vol. 3 (Moscow: Izd. AN SSSR, 1958) p. 41 [Translated into English (New York: Pergamon Press, 1959)]
103. Smirnov B M Problema Sharovoı̆ Molnii (Problem of Ball Lightning) (Moscow: Nauka, 1988)
doi>104. Dubin D H E, O’Neil T M Phys. Rev. Lett. 783868 (1997)
doi>105. Dubin D H E Phys. Plasmas 51688 (1998)
106. Davidson R C Physics of Nonneutral Plasmas (Reading, MA: Addison-Wesley, 1990)
107. Derbenev Ya S, Skrinsky A N Part. Accel. 81 (1977); 8235 (1978)
108. Derbenev Ya S, Skrinskiĭ A N Fiz. Plazmy 4492 (1978) [Sov. J. Plasma Phys. 4273 (1978)]
๗oi>109. Sørensen A H, Bonderup E Nucl. Instrum. Methods 21527 (1983)
doi>110. Pestrikov D V Nucl. Instrum. Meth. A 274435 (1989)
111. Belyaev S T, Budker G I Dokl. Akad. Nauk SSSR 107807 (1956)
112. Trubnikov B A, in Voprosy Teorii Plazmy (Topics in Plasma Theory) (Ed. M A Leontovich) Vyp. 1 (Moscow: Gosatomizdat, 1963) p. 98 [Translated into English as Reviews of Plasma Physics (Ed. M A Leontovich) Vol. 1 (New York: Consultants Bureau, 1965) p. 105]
113. Trubnikov B A Teoriya Plazmy (Plasma Theory) (Moscow: Energoatomizdat, 1996)
đoi>114. Men’shikov L I Usp. Fiz. Nauk 169113 (1999) [Phys. Usp. 42107 (1999)]
115. Shafranov V D, in Voprosy Teorii Plazmy (Topics in Plasma Theory) (Ed. M A Leontovich) Vyp. 3 (Moscow: Gosatomizdat, 1963) p. 3 [Translated into English as Reviews of Plasma Physics (Ed. M A Leontovich) Vol. 3 (New York: Consultants Bureau, 1967)]
116. Timofeev A V Rezonansnye Yavleniya v Kolebaniyakh Plazmy (Resonance Phenomena in Plasma Oscillations) (Moscow: Fizmatlit, 2000)
doi>117. Wineland D J, Dehmelt H G J. Appl. Phys. 46919 (1975)
div118. Amoretti M et al. Nature 419456 (2002)
119. CERN Courier 42 (9) (2002)
120. Kudelaĭnen V I et al. Zh. Eksp. Teor. Fiz. 832056 (1982) [Sov. Phys. JETP 561191 (1982)]
121. Budker G I, Bulyshev A F, Dikanskiĭ N S, in Trudy V Vsesoyuznogo Soveshchaniya po Uskoritelyam Zaryazhennykh Chastits (Proc. of the 5th All-Union Conf. on Accelerators of Charged Particles) Vol. 1 (Moscow: Nauka, 1977) p. 236; Preprint No. 76-92 (Novosibirsk: G I Budker Inst. for Nuclear Physics, Siberian Branch of the USSR Acad. of Sci., 1976)


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