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A joint scientific session of the Physical Sciences Division of the Russian Academy of Sciences and the Joint Physical Society of the Russian Federation was held on 30 October 2002 at the P N Lebedev Physics Institute, RAS. The following reports were presented at the session:

(1) **Rubakov V A** (Institute for Nuclear Research, RAS, Moscow) "Multidimensional models of particle physics";

(2) **Vasil'ev M A** (P N Lebedev Physics Institute, RAS, Moscow) "Higher spin gauge theory".

An abridged version of the reports is given below.

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Multidimensional models of particle physics

V A Rubakov

1. Introduction

In this talk, using simple models with extra spatial dimensions and a 'brane world' as an example, we discuss possible exotic phenomena appearing both in particle physics at high energies and in classical physics at large distances.

Physical theories in four-dimensional space-time share some common properties that are very hard, if at all, to revise. These properties include:

— The existence of long-distance forces related to gauge fields (electrodynamics, chromodynamics, gravity) requires conservation of the corresponding charges. For example, one of the Maxwell equations in electrodynamics

$$\operatorname{div} \mathbf{E} = 4\pi\rho \,,$$

contains no derivatives of the electric field with respect to time, i.e., it is an 'instantaneous' equation. So the violation of electric charge conservation would lead to an instantaneous change of the electric field in the entire space and hence contradict the causality principle. In a similar way, the violation of energy conservation would lead to instantaneous change of the gravitational field everywhere in space.

— The geometrical nature of gravity requires the equivalence of the gravitational and inertial mass.

— The gravitational energy scale is characterized by the value $M_{\rm Pl} = 1/\sqrt{G} \sim 10^{19}$ GeV, where G is the Newton gravity constant (here and below we use units $\hbar = c = 1$). This scale, the Planck mass, is much larger than other known scales in particle physics, namely, the strong interaction scale, which is of the order of the proton mass $m_{\rm p} \sim 1$ GeV, and the electroweak scale, which is determined by the W-, Z-boson masses, $m_{\rm W,Z} \sim 100$ GeV. Thus, the scale hierarchy

$$m_{\rm Z} \ll M_{\rm Pl} \,,$$

holds in nature and requires an explanation.

— Due to such a large energy scale of gravitational interactions, gravity is described by general relativity and is weak down to the Planck scales

$$l_{\rm Pl} \sim \frac{1}{M_{\rm Pl}} \sim 10^{-33} \text{ cm} \,.$$

Only at distances $r \lesssim l_{\rm Pl}$ and correspondingly at energies (more precisely, transferred momenta) $E \gtrsim M_{\rm Pl}$ is gravitational interaction comparable in strength with other known interactions in particle physics. At the same time, gravitational interactions have been studied experimentally only at rather large (from the particle physics viewpoint) distances: the Newton gravity law is experimentally tested at distances above $r = 0.02$ cm [1], and it is unknown from experiments what gravitational interaction looks like at smaller scales. So the notion that general relativity remains valid down to Planckian scales is the 31-orders-of-magnitude extrapolation; nevertheless, within the framework of four-dimensional theories it is hardly possible to abandon this notion in a non-contradictory way.

— As for large distances, four-dimensional theories imply that both Coulomb's and Newton's laws (more precisely, classical electrodynamics and general relativity) are valid at arbitrarily large scales. There are, however, alternatives to this concept: in electrodynamics, it is possible to introduce a very small photon mass (and a weak violation of the electric charge conservation) by introducing hypothetical particles with the charge smaller than $10^{-3}e$, where e is the electron charge [2]; the graviton can also be made massive at the expense of abandoning the geometrical treatment of gravity (see, for example, Ref. [3]).

Until recently, all these general considerations have rarely been called into question. The situation essentially changed in connection with the detailed discussion of models with extra spatial dimensions based on the notion of the 'brane world'.

The discussion of multidimensional models is basically stimulated by superstring theory and its generalization, M-theory, which presently is the only quantum theory that unifies, at least in principle, all interactions (including gravity)

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and is presumably self-consistent at arbitrarily small scales (see, for example, Ref. [4]). Superstring theory and M-theory are most naturally formulated in space-time of $D = 10$ and 11 dimensions, respectively, and it is this property that points to the possibility of the existence of extra dimensions. Moreover, in superstring theory gauge fields and particles interacting with them are localized on D-branes – hypersurfaces of a, generally speaking, lower dimensionality embedded in the $(D-1)$-dimensional space. In this way the concept of the brane world emerges in superstring theory.

Of course, there is no (as yet?) experimental evidence for extra dimensions. From the phenomenological viewpoint, some rather weak motivation for considering multidimensional models comes from the above mentioned hierarchy $m_Z \ll M_{Pl}$ and observations [5] giving strong evidence for the presence of a non-zero cosmological Λ-term in nature (for a review, see, for example, Ref. [6]),

$$\Lambda \sim (10^{-3}\ \mathrm{eV})^4 \sim 10^{-48}\ \mathrm{GeV}^4 .$$

Such a small, but at the same time non-zero, value of the Λ-term is extremely difficult to explain within the framework of four-dimensional theories. It should be stressed that neither of these phenomenological arguments can in any way be considered as direct indication of the existence of extra spatial dimensions: the hierarchy $m_Z \ll M_{Pl}$ has a nice explanation in four-dimensional Grand Unified Theories (see [7] for a review), and no convincing solution of the Λ-term problem has been found so far in multidimensional theories either, though some interesting approaches to this problem appear in these theories (see below and also [8] and review [9]).

2. Brane world

The 'brane world' models assume that all particles, with the exception of the graviton, are localized on a three-dimensional hypersurface (brane) embedded in the N-dimensional space. In the simplest case, this hypersurface is flat and physics on the brane possesses the four-dimensional Lorentz-invariance. The 'brane world' notion was proposed rather long ago [10], however intense discussions of this possibility began only recently, primarily due to the appearance of the concept of D-branes in superstring theory. There are a number of field theory and string mechanisms of particle localization on the brane, but anyhow there is some kind of a potential well in the directions perpendicular to the brane that localizes the wave functions of particles (Fig. 1). Somewhat simplifying the situation, we can write down the equation for the wave function of a particle in the form:

$$\left[\Box^{(N+1)} + V(\mathbf{y})\right] \Psi(x^\mu, \mathbf{y}) = 0\,, \tag{1}$$

where x^0 is the time coordinate, $x^i = (x^1, x^2, x^3)$ are spatial coordinates on the brane, $\mu = 0, 1, 2, 3$; $\mathbf{y} = (x^4, \ldots, x^N)$ is the radius-vector in the transverse direction, $V(\mathbf{y})$ is the potential, and $\Box^{(N+1)}$ is the wave operator in the $(N+1)$-dimensional space-time,

$$\Box^{(N+1)} = \frac{\partial^2}{\partial (x^0)^2} - \Delta^{(N)}\,,$$

$\Delta^{(N)}$ is the Laplacian in the N-dimensional space. Solutions to equation (1) are linear combinations of wave functions of

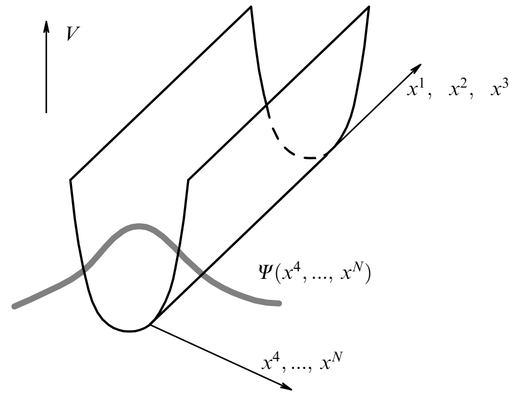

Figure 1.

the form

$$\Psi(x^\mu, \mathbf{y}) = \exp\,(\mathrm{i}\omega t - \mathrm{i}\, p_i x^i)\, \Psi_M(\mathbf{y})\,, \tag{2}$$

with the square of the four-momentum being

$${}^{(4)}p^2 \equiv \omega^2 - {}^{(3)}\mathbf{p}^2 = M^2\,, \tag{3}$$

and Ψ_M and M being eigenfunctions and eigenvalues of the transverse equation

$$\left[\Delta_y^{(N-3)} + V(\mathbf{y})\right] \Psi_M(\mathbf{y}) = M^2 \Psi_M(y)\,. \tag{4}$$

Such a model is phenomenologically acceptable if (a) the lowest level in the well has $M^2 \approx 0$: in accordance with (2) and (3), particles occupying this level propagate along the brane and from the four-dimensional viewpoint have low masses — they correspond to known, relatively light particles; (b) excited levels have $M \gtrsim$ TeV, and they would correspond to heavy analogs of the known particles (heavy electrons, quarks, etc.).

A situation similar to the 'brane world' is well known in condensed matter physics; the quantum well, for example, is the direct analog to the brane world.

Depending on the model, either the potential well can have infinitely high walls or the situations shown in Fig. 2a and b, can be realized. The continuum spectrum in the latter case corresponds to particles propagating in the entire N-dimensional space. In the case shown in Fig. 2a, such particles can be created at high energies; for us, the observers made of particles localized on the brane, this means that at high energies processes like

$$e^+e^- \to \text{nothing}\,, \tag{5}$$

become possible, where 'nothing' denotes particles that leave the brane and are not detected by the observer located on the brane. In the situation shown in Fig. 2b, even light particles have a finite probability to leave the brane, i.e., processes like

$$n \to \text{nothing} \tag{6}$$

become possible, where n denotes a neutral particle (neutron, neutrino, Z-boson), or even electron decay

$$e^- \to \text{nothing} \tag{7}$$

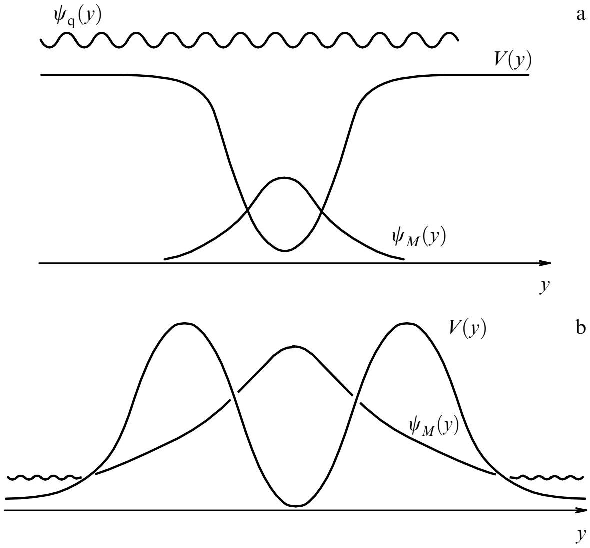

Figure 2.

can occur. For all these processes, the observer on the brane would discover apparent non-conservation of energy, and in the last case — apparent non-conservation of electric charge. It is relevant to ask whether such a possibility is consistent with the four-dimensional character of gravity on the brane (and in the case of the electric charge non-conservation — with the four-dimensional character of electrodynamics on the brane). In other words, can the argument given in the Introduction requiring charge and energy conservation become invalid in multidimensional models? We consider this question in Section 4, while now discuss a simple model illustrating the possibility of the new approach to the hierarchy problem $m_Z \ll M_{\rm Pl}$.

3. Large extra dimensions

Until now, we have set aside the question how gravity for particles on the brane becomes effectively four-dimensional. There are several possible answers to this question [11–14]. One simple possibility [11] is that extra dimensions are compact[1] and are characterized by size R. Figure 3 provides an example; extra dimensions in this example are flat and represent circles of radius R. Omitting tensor structure, the linearized Einstein equations in the last case can be written in the schematic form

$$\left[\Box^{(4)} - \frac{\partial^2}{\partial(x^4)^2} - \ldots - \frac{\partial^2}{\partial(x^N)^2}\right] h = 0\,,$$

where h is the deviation of metrics from that of flat space. Solutions to this equation are superpositions of waves

$$h_n = \exp\left(\mathrm{i}\omega t - \mathrm{i}\,p_i x^i\right)\exp\left(-\mathrm{i}\,\frac{x^4}{R}\,n_4\right)\ldots\exp\left(-\mathrm{i}\,\frac{x^N}{R}\,n_N\right),$$

where $n_4, \ldots, n_N$ are integer numbers (angular momenta along compact dimensions) and the four-momentum squared is

$$^{(4)}p^2 \equiv \omega^2 - {}^{(3)}\mathbf{p}^2 = \frac{\mathbf{n}^2}{R^2}\,. \qquad (8)$$

The state with $\mathbf{n} = 0$ has zero four-dimensional mass, and it is the usual graviton. Massive gravitons with $\mathbf{n} \neq 0$ do not contribute to gravitational interactions at large distances since they lead to a Yukawa-type potential, falling off exponentially at $r \gg R$.

Thus gravity in this model is four-dimensional at $r \gg R$ but is no longer such at $r \sim R$.

For $r \ll R$, the N-dimensional Newton's law is valid:

$$V(r) = G_* \frac{m_1 m_2}{r^{1+d}}\,, \qquad (9)$$

where G_* is the fundamental gravity constant of the theory in the $(N+1)$-dimensional space-time, and $d = N - 3$ is the number of extra dimensions. At $r \gg R$ the four-dimensional Newton's law holds,

$$V(r) = G\,\frac{m_1 m_2}{r}\,, \qquad (10)$$

where G is the usual Newton gravity constant. Matching of potentials (9) and (10) at $r \sim R$ yields

$$\frac{G_*}{R^d} \sim G\,. \qquad (11)$$

Introducing the fundamental mass M_* related to G_* as

$$G_* = \frac{1}{M_*^{2+d}}$$

(on dimensional grounds), from (11) we obtain

$$(RM_*)^d = \frac{M_{\rm Pl}^2}{M_*^2}\,. \qquad (12)$$

Therefore, the four-dimensional gravity constant G and Planck mass $M_{\rm Pl}$ are effective quantities in this model and $M_{\rm Pl}$ may not coincide with the fundamental mass M_*. This enables us to approach the hierarchy problem $m_Z \ll M_{\rm Pl}$ from an unexpected side. It is possible to assume that the fundamental scale M_* coincides with the electroweak scale to within an order of magnitude, i.e., to choose $M_* \sim$ TeV. Then relation (12) will determine the size R of extra dimensions. For example, $d = 1$ yields an unacceptably high value $R \sim 10^{15}$ cm; for $d = 2$ $R \sim 0.1$ cm, which is interesting

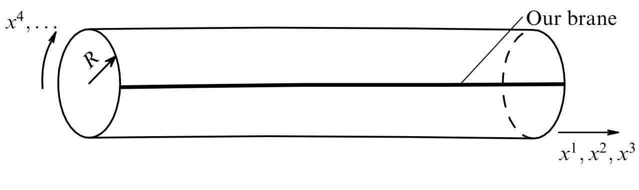

Figure 3.

[1] Models with compact extra dimensions appeared long before the brane world concept. These are Kaluza–Klein type models [15] in which the compactness of the extra dimensions is fully responsible for physics being four-dimensional at not too high energies. The absence of heavy partners of ordinary particles in the studied mass range, $m \lesssim$ TeV, is explained in the Kaluza–Klein models by the small size of the extra dimensions: it is necessary to suppose that $R \lesssim (\mathrm{TeV})^{-1} \sim 10^{-17}$ cm [see formula (8)].

from the viewpoint of testing Newton's law at small distances (in the model under discussion it is violated at $r \sim R$, while sub-millimeter scales are within the reach of modern experiments: as mentioned above, Newton's gravity law is checked down to 0.02 cm). For $d = 3$ we get $R \sim 10^{-7}$ cm and even smaller values of R for $d > 3$ so in those cases deviations from Newton's law at distances of the order of R are extremely difficult to discover, if possible at all.

Clearly, such an approach does not provide a solution to the hierarchy problem but rather suggests its reformulation: the problem becomes to explain why the size of extra dimensions is large compared to the fundamental scale $l_* \sim M_*^{-1} \sim 10^{-17}$ cm. Nevertheless this approach seems interesting, especially because in other models with compact extra dimensions the hierarchy between the fundamental scale l_* and the size of extra dimensions may be not so significant [12].

The characteristic feature of the above approach to the hierarchy problem is that gravitational interactions become strong not at the Planckian energy scale $M_{\rm Pl}$ but at the fundamental scale $M_* \sim$ TeV. Such a possibility can be tested in future collider experiments (first at the proton-proton collider LHC under construction at CERN), which will study the TeV energy range. For example, gravitational interactions at $M_* \sim$ TeV can show up in the following processes

$$q\bar{q} \to g + G\,, \tag{13}$$

where q, q̄, g, and G denote quark, antiquark, gluon, and graviton, respectively, and at the e^+e^--collider —

$$e^+e^- \to \gamma + G\,. \tag{14}$$

The graviton in these processes is not detected and is manifested as 'missing energy'. As an example, Fig. 4 presents the cross section [16] of process (14) as a function of energy in the center-of-mass frame for the discussed model with $M_* = 2.5$ TeV and the cross section of the background process $e^+e^- \to \gamma + \nu\bar{\nu}$ (accounting for necessary cuts on energy and emission angle of the photon). The

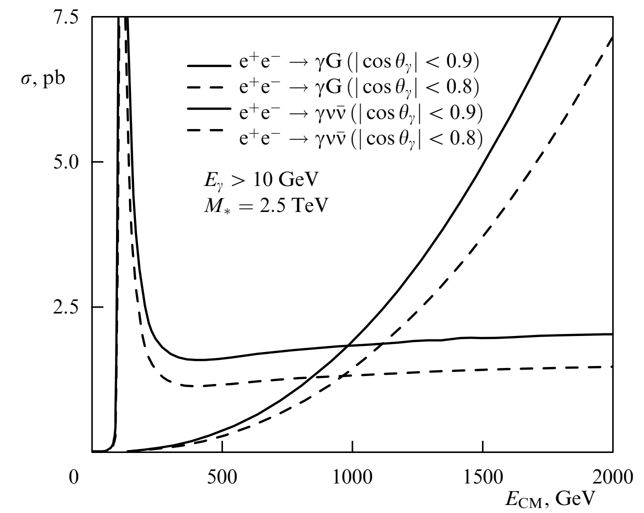

Figure 4. The cross section of the process $e^+e^- \to \gamma + G$ compared to that of the background process $e^+e^- \to \gamma + \nu\bar{\nu}$ for two possible cuts on the photon emission angle.

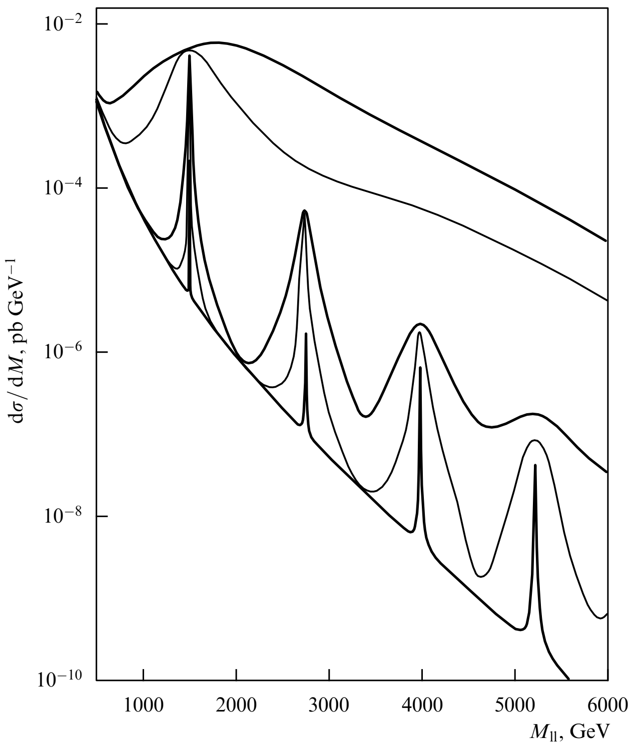

Figure 5. Lepton pair production in proton – proton collisions at the LHC energy for different model parameters.

cross section of process (14) increases, as it should for processes involving gravitons, and notably exceeds that of the background process at sufficiently high energy of the colliding particles.

The picture can be somewhat different in other models with fundamental gravitational scale $M_* \sim$ TeV. For example, in model [12] heavy gravitons with a mass of order of M_* appear. In the collider experiments they will show up, for example, as resonances in the scattering:

$$gg \to G \to e^+e^-,\ \mu^+\mu^-\,.$$

As an example, Fig. 5 shows the dependence [17] of the production cross section of a lepton pair at the LHC energies in model [12] on the invariant lepton pair mass in the case $M_* = 1.5$ TeV. Peaks in the cross section correspond to heavy gravitons; they can be reliably detected if the fundamental mass M_* is actually that small.

There are other possibilities of searching for gravitational effects at the colliders, but we will not consider them here (see review [18]). The general conclusion is that models with $M_* \lesssim 2-5$ TeV (the precise value depends on the model) allow for experimental study at the colliders.

4. Induced gravity

Extra dimensions can be not only large but infinite in size [13, 14]. Consider, for example, model [14, 19], in which terms in the effective gravitational action induced by the matter on a brane play a significant role. We will assume that the initial gravitational action in the $(N+1)$-dimensional space-time

$$S_{\rm bulk} = \int d^{\,N+1}x\ \mathcal{L}(g_{AB}) \tag{15}$$

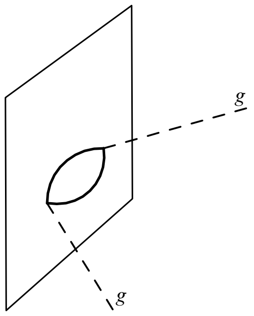

Figure 6.

is characterized by the fundamental parameter M_* which is small compared to $M_{\rm Pl}$. In particular, at energies below M_* this part of the action takes the form of the Einstein–Gilbert action

$$S_{\rm bulk} = M_*^{N+2} \int {\rm d}^{N+1}x \sqrt{^{(N+1)}g}\ ^{(N+1)}R + \ldots\,,$$

where the ellipsis denotes terms with higher derivatives. The matter is again assumed to be localized on a three-dimensional brane. In full analogy with well-known papers on the induced gravity [20], radiation corrections due to this matter are expected to induce additional contributions into the effective gravitational action (Fig. 6). These contributions depend only on the value of the metrics $g_{\mu\nu}$ on the brane and are characterized by another energy scale — the effective cutoff in theory of matter fields on the brane. This scale can be identified with the Planck mass, and from symmetry considerations[2] these contributions to the effective action can be written in the form:

$$S_{\rm brane} = M_{\rm Pl}^2 \int_{\rm brane} {\rm d}^4x \sqrt{^{(4)}g}\ ^{(4)}R \qquad (16)$$

(possible terms with higher derivatives are unimportant). So the total effective action has the form:

$$S^{\rm eff} = S_{\rm bulk} + S_{\rm brane}\,. \qquad (17)$$

The idea is that at $M_{\rm Pl} \gg M_*$ the induced term $S_{\rm brane}$ dominates for sources on the brane, and gravity is effectively four-dimensional.

The result of solving linearized field equations corresponding to action (17) is rather unexpected (see the Appendix for the corresponding calculations).

First, the four-dimensional Newton's law for masses on the brane is valid only in the restricted distance range:

$$r_{\rm min} \ll r \ll r_{\rm max}\,, \qquad (18)$$

where [19]

$$r_{\rm max} \sim \frac{M_{\rm Pl}}{M_*^2}\,.$$

A cosmologically acceptable value $r_{\rm max} \gtrsim 10^{28}$ cm is obtained at $M_* \lesssim 10^{-3}$ eV. Thus the model is viable only if the fundamental gravity energy scale is very small. As for $r_{\rm min}$, the value of this parameter strongly depends on other parameters of the model and can be of the order of M_*^{-1} or much smaller. In the first case the validity of Newton's law at distances above 0.02 cm together with the above restriction yields

$$M_* \sim 10^{-3}\ {\rm eV}\,.$$

The same estimate follows from astrophysical and collider bounds [21] irrespectively of the value of $r_{\rm min}$.

The fact that the four-dimensional Newton's law is no longer valid at ultralarge distances can be explained in the following way. The induced term in the action is insignificant far away from the brane and there exist gravitons that propagate in the whole multidimensional space and have an arbitrarily small energy. From the four-dimensional viewpoint, these are states in the continuum spectrum. A four-dimensional graviton propagating along the brane represents a resonance in this continuum spectrum, i.e., there is a finite probability for it to escape from the brane. The situation here is somewhat analogous to what is shown in Fig. 2b. At ultralong times and, correspondingly, at ultralarge distances gravity on the brane is no longer four-dimensional. Note that such a possibility also emerged in earlier models [22].

The second result is that the four-dimensional gravitational interaction of masses on a brane at intermediate distances (18) depends, generally speaking, on the form of the transverse wave functions of these particles (the authors of this model did not discuss this property assuming fixed wave functions [19]), or in other words, on the mass density distribution in the direction transverse to the brane. Consequently, in this model, generally speaking, the principle of equivalence of gravitational and inertial mass is violated. The scale of this violation is of the order of $M_*^2 \Delta^2$, where Δ is the brane thickness, and at $M_* \lesssim 10^{-3}$ eV and $\Delta \lesssim 10^{-17}$ cm it is too small to be discovered experimentally. Nevertheless, the very possibility of the equivalence principle violation in a geometrical theory of gravity is very interesting.

Another feature of the model is that the apparent nonconservation of energy on the brane (at the expense of particles leaving the brane) is quite admissible and does not contradict the four-dimensional character of gravitational interactions on the brane (at intermediate distances). Many models with infinite extra dimensions share this property which is due to the four-dimensional description of gravity being no longer valid for masses outside the brane and the situation becoming intrinsically multidimensional (see [23] for detailed discussion of this point in model [13]). Note in this connection that in a similar way electric charge on a brane might not be conserved; see [24] for the corresponding models.

Next, gravity outside the brane becomes strong at low energy scale M_*. But gravity on the brane remains weak up to much higher energies, which is caused by gravitational fluctuations on the brane being suppressed due to the induced contribution (16) in the action. Nonetheless an intermediate scale $\sqrt{M_* M_{\rm Pl}}$ emerges in the model at which, presumably, one can expect gravitational effects to arise; these effects are testable in experiments at high energies [21]. Interestingly, at $M_* \sim 10^{-3}$ eV this scale is of the order of several TeV, i.e., again falls within the energy range of future colliders.

As noted above, at small distances $r \lesssim r_{\rm min}$ the four-dimensional Newton's law is no longer valid. It is interesting

[2] No cosmological constant is assumed to emerge, either on the brane or outside it.

to note that the corresponding corrections to Newton's law depend, generally speaking, on the mass density distribution in the directions transverse to the brane, i.e., on the wave functions of particles on the brane. Therefore, short-distance contributions depend on the type of particles and are analogs of the 'fifth force'.

Finally, the cosmological expansion in this model literally corresponds to the motion of the brane in the embedding flat space [25]. Since the four-dimensional description of gravitational interactions on the brane is applicable at distances $r > r_{\min}$, the expansion of the Universe is described by the standard Friedmann equations for sufficiently small values of the Hubble parameter, $H \lesssim r_{\min}^{-1}$. At the hot stage of the evolution of the Universe the Hubble parameter is related to temperature, $H = \text{const} \cdot T^2/M_{\text{Pl}}$, so the standard theory of the 'Big Bang' is applicable at

$$T \lesssim \sqrt{M_{\text{Pl}}\, r_{\min}^{-1}} < 3\ \text{TeV}\,.$$

Here we have taken into account that $r_{\min} < 0.01$ cm from experiments testing Newton's law. At temperatures above $(M_{\text{Pl}}\, r_{\min}^{-1})^{1/2}$, the standard theory of a hot Universe is no longer valid, but this does not contradict any observational data. However, there remain questions like how does the inflationary stage proceed, how the primordial density fluctuations are created, etc. Note that the notion of the Universe as a brane moving in a fixed embedding space also appeared in earlier models of the 'brane world' [26, 27], and in some of them the questions on the inflationary stage and the generation of primordial density fluctuations are positively solved, see for example [28] and references therein.

5. Conclusion

As we have seen, traditional concepts are no longer valid in multidimensional theories with the 'brane world'. We return to the list of these notions discussed in the Introduction. Using models from Sections 3 and 4 as examples we now arrive at the following conclusions.

— The four-dimensional character of long-distance forces between particles on the brane does not exclude the possibility of apparent non-conservation of energy, electric charge and, perhaps, color in models with infinite extra dimensions. Roughly speaking, the four-dimensional equation $\operatorname{div} \mathbf{E} = 4\pi\rho$ does not hold for particles escaping from the brane and for this reason charge non-conservation does not contradict the causality principle.

—The geometric nature of gravity is consistent with the violation of the equivalence principle for gravitational and inertial masses. Literally in the model of Section 4, this violation is too small to be experimentally detected, but it is not excluded that in other models it can be appreciable.

— The fundamental gravitational energy scale M_* can be much smaller than M_{Pl}. This can give rise to phenomena detectable at high-energy colliders.

— Deviations from the four-dimensional Newton's law are possible at fairly large distances comparable to the experimental limit $r_{\min} = 0.02$ cm.

— Newton's law and the general relativity can become invalid at ultralarge distances. In principle, changing gravity at cosmological scales could be an alternative to the cosmological Λ-term.

Experimental discovery of any of the enumerated properties would be the most serious argument in favor of the existence of extra spatial dimensions. The future will show whether nature has given us such a possibility or multidimensional models remain purely speculative concepts.

6. Appendix

Let us introduce into the model with action (17) a source of a gravitational field localized on a brane with the localization function $g(\mathbf{y})$, where $\mathbf{y} = (x^4, \ldots, x^N)$ are coordinates transverse to the brane,

$$T_\mu^{\ \nu} \propto g(\mathbf{y})\, \delta(x^\mu)\,.$$

Then the linearized equation of the gravitational field in this model can be schematically written as

$$\left[D_{\text{bulk}} + \delta(y)\, M_{\text{Pl}}^2\, \Box^{(4)}\right] h(x,y) = g(y)\, \delta(x)\,, \tag{19}$$

where D_{bulk} is some differential operator in $(N+1)$-dimensional space-time arising from linearized action (15); it contains the energy scale M_* and does not contain the parameter M_{Pl}. In the low energy limit $D_{\text{bulk}} = M_*^{N+2}\, \Box^{(N+1)}$. It is convenient to search for the solution of equation (19) in the momentum representation in the direction along the brane (including time), i.e., to calculate $h(p,y)$. Let $D_*(p, y-y')$ be the Green function of the operator D_{bulk} in this representation (depending on the parameter M_* and not on M_{Pl}). Let us introduce the quantities

$$\begin{aligned} D_g(p,y) &= \int \mathrm{d}^N y'\, D_*(p, y-y')\, g(y')\,, \\ D_0(p) &= D_*(p, y-y'=0)\,. \end{aligned} \tag{20}$$

It is assumed that $D_0(p)$ is finite [19]. Then the solution of equation (19) will have the form

$$\begin{aligned} h(p,y) &= \frac{D_g(p,y)}{1 + M_{\text{Pl}}^2 p^2 D_0(p)} \\ &\quad + \frac{M_{\text{Pl}}^2 p^2}{1 + M_{\text{Pl}}^2 p^2 D_0(p)} \left[D_0(p) D_g(p,y) - D_g(p,0) D_*(p,y)\right]. \end{aligned} \tag{21}$$

If $D_0(p)$ is also finite at $p \to 0$, then at small momenta

$$p \ll M_* \tag{22}$$

we have

$$D_0 = \text{const} \sim \frac{1}{M_*^4}\,. \tag{23}$$

In addition, if momenta are relatively large, namely,

$$p \gg \frac{M_*^2}{M_{\text{Pl}}}\,, \tag{24}$$

the unity in the denominator of (21) can be neglected and the gravitational field takes the form

$$\begin{aligned} h(p,y) &= \frac{1}{M_{\text{Pl}}^2 p^2}\, \frac{D_g(p,y)}{D_0(p)} \\ &\quad + \left[D_g(p,y) - \frac{D_g(p,0)\, D_*(p,y)}{D_0(p)}\right]. \end{aligned} \tag{25}$$

The interaction of sources with mass distributions near the brane characterized by functions $g(y)$ and $f(y)$ is determined by the convolution of solution (25) with function $f(y)$:

$$G_{gf}(p) = \frac{1}{M_{\rm Pl}^2 p^2}\,\frac{D_{gf}(p)}{D_0(p)} + \left[D_{gf}(p) - \frac{D_g(p,0)\,D_f(p,0)}{D_0(p)}\right], \qquad (26)$$

where

$$D_{gf}(p) = \int f(y)\,D_*(p, y-y')\,g(y')\,\mathrm{d}y'\,.$$

Note that for distributions $f(y)$ and $g(y)$ characterized by the brane width Δ, the last convolution has the form

$$D_{gf} = \left(1 + M_*^2 \Delta_{gf}^2\right) D_0\,, \qquad (27)$$

where $\Delta_{gf}^2 \sim \Delta^2$ and the parameter M_*^2 appears on dimensional grounds. So the first term in (26) at momenta satisfying (26) and (24) is equal to

$$\frac{1}{M_{\rm Pl}^2 p^2}\left(1 + M_*^2 \Delta_{gf}^2\right).$$

This term coincides with the Green function of the four-dimensional wave operator and corresponds to the ordinary Newton's law for particles on the brane. The correction $M_*^2 \Delta_{gf}^2$ depends on the mass distribution in direction transverse to the brane and leads to a weak violation of the equivalence principle.

The second term in (26) describes short-distance forces depending on the mass distributions $g(y)$ and $f(y)$. Taking into account (27) this term has, generally speaking, the form

$$C\,D_0(p)\,,$$

where $C \sim M_*^2 \Delta^2$. At distances $r \sim M_*^{-1}$ this contribution gives rise to a correction to the Newton potential of the order of

$$\frac{1}{M_{\rm Pl}^2 r}\,M_{\rm Pl}^2 \Delta^2\,.$$

It is necessary, however, to note that in this term some cancelations are possible: for example, for a point-like distribution $f(y) = g(y) = \delta(y)$ the second term in (26) identically turns to zero.

Expression (26) and, hence, the usual Newton's law is valid only at relatively large momenta (24), i.e., at distances $r < r_{\max} \sim M_{\rm Pl}/M_*^2$. At ultralarge distances the term with $M_{\rm Pl}^2$ in the denominators of solution (21) can be neglected and the dominating term has the form

$$h(p,y) = D_g(p,y) = \int D_*(p, y-y')\,g(y')\,\mathrm{d}y'\,.$$

The formula obtained is the expression for the gravitational field of the source $g(y)\,\delta(x)$ in the $(N+1)$-dimensional space-time. Gravity at ultralarge scales becomes multi-dimensional.

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Higher spin gauge theory

M A Vasil'ev

1. Standard gauge theories

The aim of the present talk is to present the key ideas and results of the higher spin gauge theory without delving into technical details of the setup. In essence, we shall consider the construction of a field theory model with maximally high gauge symmetry. It is expected that such theories allow a new vision of superstring theory which is presently thought to be the main candidate for the theory of fundamental interactions.

As usually, the gauge symmetries are those whose parameters are arbitrary functions of space-time coordinates x^ν. Historically the first gauge theory was that of electromagnetism suggested by Maxwell. In this case the gauge field is identified with the vector potential A_ν that generates the field strength

$$F_{\nu\mu} = \partial_\nu A_\mu - \partial_\mu A_\nu , \qquad \partial_\nu = \frac{\partial}{\partial x^\nu} , \qquad \nu = 0, 1, 2, 3 , \tag{1}$$

invariant under the gauge (gradient) transformations

$$\delta A_\nu = \partial_\nu \varepsilon \tag{2}$$

with an arbitrary gauge parameter $\varepsilon(x)$. The gauge-invariant Maxwell's action

$$S = -\frac{1}{4}\int d^4x \, F_{\nu\mu} F^{\nu\mu} , \qquad \delta S = 0 \tag{3}$$

is known to describe massless particles of spin 1, the photons.

Maxwell theory can be generalized to Yang – Mills theory by introducing a system of mutually charged spin 1 particles described by the matrix-valued potential $A^{j}_{\nu i}$ which takes values in some Lie algebra h. The corresponding strengths, gauge transformations, and action have the forms

$$G_{\nu\mu} = \partial_\nu A_\mu - \partial_\mu A_\nu + g\,[A_\nu, A_\mu] , \tag{4}$$

$$\delta A_\nu = \partial_\nu \varepsilon + g\,[A_\nu, \varepsilon] , \tag{5}$$

$$S = -\frac{1}{4}\int d^4x \, \mathrm{tr}(G_{\nu\mu} G^{\nu\mu}) , \tag{6}$$

respectively. The Yang – Mills theory can be understood as the theory of interaction of massless spin 1 particles. Indeed, by imposing natural conditions bounding the orders of derivatives, the gauge symmetry principle fixes interactions of spin 1 fields unambiguously to within an arbitrary choice of the gauge group.

At first glance, the pure Yang – Mills theory seems poorly adapted to describe the real physics since quanta of the Yang – Mills fields are massless, at least perturbatively. At one time, it was this point that prevented Pauli from publishing the results he obtained where essentially the Yang – Mills theory was discovered. Later on, owing to discovery of the Higgs phenomenon in the phase with spontaneously broken symmetry, this difficulty was found to be apparent. From the symmetry viewpoint, the characteristic feature of this phenomenon is the appearance of the Higgs field Φ^A with the gauge transformation law in the form

$$\delta\Phi^A = \varepsilon^A(X) + \ldots , \tag{7}$$

where $\varepsilon^A(X)$ are some combinations of the gauge parameters $\varepsilon(X)$ and the ellipsis denotes higher-order terms. Such a transformation law of the Higgs field allows a partial fixation of the gauge freedom by choosing the gauge $\Phi^A = 0$. The remaining freedom is generated by those gauge parameters $\varepsilon(X)$ that do not contribute to $\varepsilon^A(X)$. Presently, the Yang – Mills theory is the foundation for the theory of strong and electroweak interactions. In particular, introducing Yang – Mills fields for the gauge group $SU(2) \times U(1)$ as carriers of the electroweak interaction has enabled one to surmount difficulties of Fermi theory of weak interactions.

The next textbook example of the gauge theory is general relativity. Here the metric tensor $g_{\mu\nu}$ plays the role of the gauge field and gauge transformations are identified with the coordinate ones

$$\delta g_{\nu\mu} = \partial_\nu(\varepsilon^\rho) g_{\rho\mu} + \partial_\mu(\varepsilon^\rho) g_{\rho\nu} + \varepsilon^\rho \partial_\rho (g_{\nu\mu}) , \tag{8}$$

where $\varepsilon^\rho(x)$ are infinitesimal parameters. The gauge invariance principle is identified with Einstein's equivalence principle. The invariant Einstein – Gilbert action

$$S = -\frac{1}{4\varkappa^2}\int \sqrt{-\det|g|}\,(R + \Lambda) \tag{9}$$

contains two independent coupling constants: the gravitational constant $\varkappa$ and the cosmological constant Λ. To interpret this theory in terms of particles, one should make the expansion $g_{\nu\mu} = \eta_{\nu\mu} + \varkappa h_{\nu\mu}$ in some fixed background metrics $\eta_{\nu\mu}$ (flat for $\Lambda = 0$ or (anti) de Sitter for $\Lambda \neq 0$) where $h_{\nu\mu}$ describes dynamical fluctuations. For the flat space ($\Lambda = 0$) Fierz and Pauli showed that the linearized action S describes free massless particles of spin 2, the gravitons. Again, by imposing some natural conditions the Einstein – Gilbert action is found to be the only consistent (gauge invariant) action for a self-interacting massless field of spin 2.

In four dimensions, the only non-trivial modification of the gauge theory of spin 1 and spin 2 is supergravity — the theory in which, in addition to the spin 1 and spin 2 gauge fields, a massless gauge field of spin 3/2 appears, the gravitino, which is responsible for local supersymmetry transformations with the spinor gauge parameters $\varepsilon_\alpha(x)$. A new feature of supergravity is that it unifies particles carrying different spins,