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METHODOLOGICAL NOTES

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Macroscopic representation of the magnetization vector field in a magnetic substance

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Contents

Abstract. **Expressions for the parameters of the macroscopic magnetization vector field are obtained based on a model of point magnetic moments. It is shown that the magnetization vector field consists of a vortex and a potential part. The form of the obtained expansion depends on the system of units chosen. The magnetic field of the magnetization vector and the electric field of the polarization vector are compared and shown to be equivalent. In relation to the problems discussed, the methodical aspects of teaching an "Electricity and Magnetism" section of a physics course are highlighted.**

1. Introduction

The "Electricity and Magnetism" section of the college physics course commonly contains the multitude of practical problems that, to a greater or lesser degree of success, are described by various idealized models. Macroscopic magnetodynamics, which studies the magnetic fields in a space filled by matter, occupies an important place among such problems.

The form of magnetodynamic equations and the meaning of the various quantities in these equations depend on the physical nature of the material medium. However, there is one equation that is valid for any fairly dense medium. If we adhere to the dipole model, this equation links the magnetization **M** (the magnetic moment density) to the induction vector **B** and the strength **H** of the magnetic field in the substance. In the Gauss and in the Heaviside – Lorentz (Kohn) systems of units, this equation has the respective forms

$$\mathbf{B} = \mathbf{H} + 4\pi\mathbf{M} \quad [1, \text{pp. } 26, 46, 415]\,, \qquad \text{(1a)}$$
$$\mathbf{B} = \mathbf{H} + \mathbf{M} \quad [2, \text{p. } 76]\,.$$

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In the SI system of units, this equation takes on different forms in different textbooks, viz.

$$\mathbf{B} = \mu_0(\mathbf{H} + \mathbf{M}) \quad [3, \text{p. } 269]\,,$$
$$\mathbf{B} = \mathbf{H} + \mu_0\mathbf{M} \quad [4, \text{p. } 142]\,, \qquad \text{(1b)}$$
$$\mathbf{B} = \mu_0\mathbf{H} + \mathbf{M} \quad [5, \text{p. } 260]\,,$$

where μ_0 is the magnetic constant (also known as the permeability of vacuum).

What makes all this a problem is that not only are there the recommendations of the International Union of Pure and Applied Physics (IUPAP) [6] to use a single form for all these equations but, and this is more important, that the physical quantities in equations (1) are defined ambiguously.

For instance, in his *microscopic substantiation* of the Maxwell equations in matter, Tamm states [7, p. 154] that "...the average value of the strength of the microscopic field, $\langle\mathbf{H}_{\text{micro}}\rangle$, is called magnetic induction and is denoted $\mathbf{B} = \langle\mathbf{H}_{\text{micro}}\rangle$", and, later, that "...the strength of the macroscopic field, **H**, in magnetic substances is defined by the relationship $\mathbf{H} = \mathbf{B} - 4\pi\mathbf{M}$ ...".

Similarly, on page 494 of the encyclopedic dictionary [28] it is said that "The magnetic induction vector **B** is the force characteristic of the magnetic field and amounts to the average value of the total strength of the microscopic magnetic fields generated by individual electrons and other elementary particles". The same is said on page 153 of Ref. [29]: "The average magnetic field strength is commonly known as magnetic induction and is denoted $\bar{\vec{\mathbf{h}}} = \mathbf{B}$". A similar definition is given on page 179 of Vol. 2 of the *Physics Encyclopedic Dictionary* [31] under the entry 'Induction'.

At the same time, according to Kalashnikov [8, p. 210]: "the magnetic induction in a magnetic substance is the volume average of the microscopic value of magnetic induction inside the magnet, $\mathbf{B} = \langle\mathbf{B}\rangle_{\text{micro}}$", and also, "...the average value of magnetic induction $\bar{\mathbf{B}}_{\text{M}}$ comprises the induction $\mu_0\mathbf{H}$ generated by the magnetizing coil and the induction generated by the surface currents of the magnetic substance, $\mu_0\mathbf{I}$ ...", i.e., $\mathbf{B} = \mu_0(\mathbf{H} + \mathbf{I})$.

A very original definition is given in Ref. [30, p. 351]: "When a magnetic substance is magnetized, it generates an additional magnetic field strength $\mathbf{H}'$ which is added to the initial strength $\mathbf{H}$ of the magnetic field generated by the currents that flow in the wires. For historical reasons, the vector sum of these strengths is called magnetic induction vector $\mathbf{B} = \mathbf{H} + \mathbf{H}'$".

Purcell [9, p. 387] made some interesting remarks concerning magnetic induction. He stated that $\mathbf{B}$ must be considered a *fundamental quantity characterizing the magnetic field*, since the absence of any kind of magnetic charge implies that $\operatorname{div} \mathbf{B} = 0$. He went on to say that this condition suggests that the average macroscopic field inside matter is $\mathbf{B}$ rather than $\mathbf{H}$. He also said that in some old books $\mathbf{H}$ is interpreted as the *primary magnetic field*, while the quantity $\mathbf{B}$ is defined as $\mathbf{H} + 4\pi\mathbf{M}$ and is called magnetic induction. Purcell claimed that even some modern authors, who think of $\mathbf{B}$ as the primary field, feel obliged to call it magnetic induction. Purcell suggested that $\mathbf{B}$ be called the magnetic field as before, while $\mathbf{H}$ will be called field $\mathbf{H}$ or even magnetic field $\mathbf{H}$.

Similar remarks were made by Matveev [3, p. 269] and amount to the following: "...$\mathbf{H}$ is not a purely field quantity, since it incorporates the vector $\mathbf{M}$ characterizing the magnetization of the medium". At the same time, Sivukhin [10, p. 254] stated that "...$\mathbf{H}$ does not contain magnetizing currents, so that only conduction currents remain in it...". Zil'berman [22, p. 294] as well as Novozhilov and Yappa [12, p. 222] used equations (1) to define the magnetization vector. Finally, Pamyatnykh and Turov [32, p. 21] stated that "The induction of magnetic field is called the magnetic field strength".

There is no unanimity, either, on the *units of measurement* of $\mathbf{B}$ and $\mathbf{H}$ fields. For instance, Feynman et al. [4, pp. 142 and 143] believed that it is convenient to measure $\mathbf{H}$ in the same units as $\mathbf{B}$ and not in units of $\mathbf{M}$. They went on to say that actually this is the same unit, equal to 10^{-4} SI units. Similarly, Sivukhin [10, p. 254] said that "...there is absolutely no difference between a gauss and an oersted. These are different names of the same unit. Only one name should be left: either gauss or oersted". Purcell [9, p. 387] was also of this opinion and stated that there is no need for a different name for the unit of $\mathbf{H}$. He said that there are people who like to give names to things, with the result that the unit of $\mathbf{H}$ has its own name, oersted. However, Sommerfeld [11] and Novozhilov and Yappa [12] were of the opposite opinion. For instance, on page 222 of Ref. [12] it is stated that "...The difference in the dimensionalities of magnetic field strength and induction in the SI system of units is a reflection of the distinction in the physical meaning of these quantities, which is unimportant in the case of vacuum but is not taken into account in the Gauss system of units even when a material medium is involved and when this distinction should be constantly taken into account".

To stress the particular properties of $\mathbf{B}$ and $\mathbf{H}$ fields, the authors of many textbooks *compare them to the corresponding electrostatic fields*, the strength $\mathbf{E}$ and the electric induction $\mathbf{D}$ in a material medium. For example, Tamm [7, p. 291] says that "...when the equations of the electric and magnetic fields, viz.

$$\mathbf{D} = \mathbf{E} + 4\pi\mathbf{P}, \qquad \mathbf{B} = \mathbf{H} + 4\pi\mathbf{M},$$

are compared from the formal standpoint, there is the feeling that, on the one hand, $\mathbf{E}$ is similar to $\mathbf{H}$ and, on the other, $\mathbf{D}$ is similar to $\mathbf{B}$, while actually the magnetic induction vector $\mathbf{B}$ is the analog of the macroscopic electric field strength $\mathbf{E}$ and the macroscopic magnetic field strength $\mathbf{H}$ is the analog of the electric induction vector $\mathbf{D}$". This correspondence is written in compact form as follows

$$\mathbf{H} \leftrightarrow \mathbf{D}, \quad \mathbf{B} \leftrightarrow \mathbf{E}. \tag{2}$$

The first to make this assertion was Sommerfeld in his textbook [11], which was based on a course of lectures he delivered at the beginning of the 20th century. There he emphasized, among other things, the correspondence between $\mathbf{E}$ and $\mathbf{B}$, which he called *force* quantities, and the correspondence between $\mathbf{D}$ and $\mathbf{H}$, which he called *quantitative* quantities (the correspondence (2) can be found on page 69 of Ref. [11]). In the same textbook Sommerfeld also stated that the analogy (2) does not hold for an arbitrary case. In particular, in Section 13 he said that if we study the behavior of *arbitrary* bodies in an external magnetic field, we find that the laws governing this behavior are similar to those of electrostatics, and

$$\mathbf{H} \leftrightarrow \mathbf{E}, \quad \mathbf{B} \leftrightarrow \mathbf{D}. \tag{3}$$

A remark similar to that resulting in Eqn (3) was made by Tamm in his textbook [7]. Note that in textbooks written later the correspondence (3) in most cases is ignored, while analogies (2) are assumed to be of a general nature.

There are many more examples of such inconsistencies that abound in various textbooks. However, even what has been said above is capable of totally confusing the reader. Note that the material we have quoted, directly or indirectly, from various textbooks is physically sound and does not raise any doubts. However, the method of exposition in each case depends on the inner logic of each textbook, and this logic is not the same for other textbooks, which leads to such disastrous results. All these inconsistencies make teaching physics extremely difficult. They can be eliminated if a relationship of type (1) is derived on the basis of the theorem of decomposition of the magnetization vector field into its vortex and potential parts. The term 'decomposition of the magnetization vector field' corresponds to the terminology of vector analysis, where according to the Helmholtz theorem [27, 33], the field of any vector can be represented as a linear combination of its vortex part $\mathbf{a}_\mathrm{v}$ ($\operatorname{div} \mathbf{a}_\mathrm{v} = 0$) and its potential part $\mathbf{a}_\mathrm{p}$ ($\operatorname{rot} \mathbf{a}_\mathrm{p} = 0$), i.e., $\mathbf{a} = \mathbf{a}_\mathrm{v} + \mathbf{a}_\mathrm{p}$.

2. Decomposition of the magnetization vector field

By definition (see Ref. [15]), the magnetization of a magnetic substance, viz.

$$\mathbf{M} = \frac{\mathrm{d}\mathbf{m}}{\mathrm{d}\tau},$$

is a macroparameter (here, $\mathrm{d}\mathbf{m}$ is the total magnetic moment of the atoms of the magnetic substance in an ultimately small but macroscopical volume element $\mathrm{d}\tau$).

The common approach in describing the magnetization vector field of a magnetic substance occupying a volume τ in the dipole approximation is to introduce the Hertz magnetic potential $\mathbf{Z}_\mathrm{m}$ which, by definition (see Ref. [21]), is equal at point A (Fig. 1) to

$$\mathrm{Z}_\mathrm{m} = \frac{\mu_0}{4\pi} \int_\tau \frac{\mathbf{M}}{r} \, \mathrm{d}\tau. \tag{4}$$

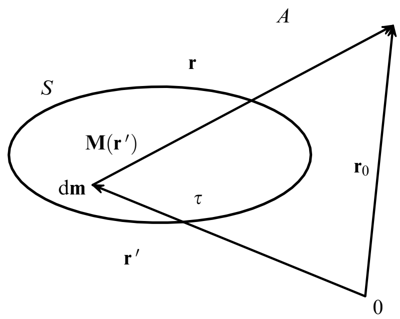

Figure 1. Defining the coordinates $\mathbf{r}$, $\mathbf{r}_0$, and $\mathbf{r}'$.

The expression is the solution of the equation

$$\nabla^2 \mathbf{Z}_\mathrm{m} = -\mu_0 \mathbf{M} \,. \tag{5}$$

The meaning of the Hertz potential is that its vortices and sources determine, respectively, the vector ($\mathbf{A}_\mathrm{m}$) and scalar (φ_m) magnetic potentials [13, 24] of the magnetization vector:

$$\mathbf{A}_\mathrm{m} = \operatorname{rot} \mathbf{Z}_\mathrm{m} \,, \qquad \varphi_\mathrm{m} = -\frac{1}{\mu_0} \operatorname{div} \mathbf{Z}_\mathrm{m} \,. \tag{6}$$

In turn, the magnetic induction vector $\mathbf{B}_\mathrm{m}$ and strength vector $\mathbf{H}_\mathrm{m}$ of the field generated by the distribution of the magnetization vector are defined as

$$\begin{gathered} \mathbf{B}_\mathrm{m} = \operatorname{rot} \mathbf{A}_\mathrm{m} = \operatorname{rot}(\operatorname{rot} \mathbf{Z}_\mathrm{m}) \,, \\ \mathbf{H}_\mathrm{m} = -\operatorname{grad} \varphi_\mathrm{m} = \frac{1}{\mu_0} \operatorname{grad}(\operatorname{div} \mathbf{Z}_\mathrm{m}) \,. \end{gathered} \tag{7}$$

Using the well-known identity

$$\nabla^2 \mathbf{Z}_\mathrm{m} = \operatorname{grad}(\operatorname{div} \mathbf{Z}_\mathrm{m}) - \operatorname{rot}(\operatorname{rot} \mathbf{Z}_\mathrm{m}) \,, \tag{8}$$

we arrive at the following relations

$$\begin{gathered} \mathbf{M}_\mathrm{v} = \frac{1}{\mu_0} \mathbf{B}_\mathrm{m} \,, \quad \mathbf{M}_\mathrm{p} = -\mathbf{H}_\mathrm{m} \,, \quad \mathbf{M} = \frac{1}{\mu_0} \mathbf{B}_\mathrm{m} - \mathbf{H}_\mathrm{m} \,, \\ \mathbf{Z}_\mathrm{m} = \frac{\mu_0}{4\pi} \int_\tau \frac{\mathbf{M}}{r} \, \mathrm{d}\tau \end{gathered} \tag{9}$$

in the SI system of units; then

$$\begin{gathered} \mathbf{M}_\mathrm{v} = \frac{1}{4\pi} \mathbf{B}_\mathrm{m} \,, \quad \mathbf{M}_\mathrm{p} = -\frac{1}{4\pi} \mathbf{H}_\mathrm{m} \,, \quad \mathbf{M} = \frac{1}{4\pi} \mathbf{B}_\mathrm{m} - \frac{1}{4\pi} \mathbf{H}_\mathrm{m} \,, \\ \mathbf{Z}_\mathrm{m} = \int_\tau \frac{\mathbf{M}}{r} \, \mathrm{d}\tau \end{gathered} \tag{10}$$

in the CGS electromagnetic system of units, and

$$\begin{gathered} \mathbf{M}_\mathrm{v} = \mathbf{B}_\mathrm{m} \,, \quad \mathbf{M}_\mathrm{p} = -\mathbf{H}_\mathrm{m} \,, \quad \mathbf{M} = \mathbf{B}_\mathrm{m} - \mathbf{H}_\mathrm{m} \,, \\ \mathbf{Z}_\mathrm{m} = \frac{1}{4\pi} \int_\tau \frac{\mathbf{M}}{r} \, \mathrm{d}\tau \end{gathered} \tag{11}$$

in the Heaviside – Lorentz (Kohn) system of units [10, p. 372; 34]. In equations (9) – (11), $\mathbf{M}_\mathrm{v}$ and $\mathbf{M}_\mathrm{p}$ stand, respectively, for the vortex and potential parts of the magnetization vector field:

$$\mathbf{M} = \mathbf{M}_\mathrm{v} + \mathbf{M}_\mathrm{p} \,.$$

3. Analysis and discussion

(1) Equations (9) – (11) are written usually in a form where, in addition to the constituents of the magnetization vector field (the induction vector $\mathbf{B}_\mathrm{m}$ and the strength vector $\mathbf{H}_\mathrm{m}$), there are magnetic fields generated by external (extraneous) vortices, e.g., 'free' electric currents of density $\mathbf{j}_\mathrm{m}$ or displacement currents $\mathbf{j}_\mathrm{dc} = \varepsilon_0 \, \partial\mathbf{E}/\partial t$ or polarization currents $\mathbf{j}_\mathrm{p} = \partial\mathbf{P}/\partial t$ [25, p. 13; 14, 26]. Then, in accordance with the Maxwell equation written in d'Alembert's form [21, p. 133], we have

$$\operatorname{rot} \mathbf{H}_0 = \mathbf{j}_0 + \mathbf{j}_\mathrm{dc} + \mathbf{j}_\mathrm{p} \,,$$

$$\operatorname{rot} \mathbf{B}_0 = \mu_0 (\mathbf{j}_0 + \mathbf{j}_\mathrm{dc} + \mathbf{j}_\mathrm{p}) \,.$$

Here we have allowed for the fact that $\mathbf{B}_0 = \mu_0 \mathbf{H}_0$. Combining this with equations (9), we get

$$\mathbf{B} = \mu_0 (\mathbf{H} + \mathbf{M}) \,,$$

where $\mathbf{B} = \mathbf{B}_0 + \mathbf{B}_\mathrm{m}$, and $\mathbf{H} = \mathbf{H}_0 + \mathbf{H}_\mathrm{m}$ (in the Heaviside – Lorentz system of units or the CGS system of units, $\mathbf{B}_0 = \mathbf{H}_0$).

A more logical name for the quantity $\mathbf{H}_0$ is the induction of the magnetic field of macroscopic currents (rather than strength), since by its very meaning $\mathbf{H}_0$ is a redesignated vector $\mathbf{B}_0$ or (as is the case in the SI system of units) a renormalized vector $\mathbf{B}_0$. However, the use of the term 'strength' for $\mathbf{H}_0$ in describing the vortex field of currents also has historical roots [7, p. 204], and the term is widely used in textbooks.

At the same time, only a potential vector can serve as a 'true' strength vector, as is the case in 'free charge' electrostatics, where the potential vector $\mathbf{E}_0$ is referred to as the strength of the electric field of macroscopically 'free' charges.

Thus, in a region of space where *there is a field generated by macroscopic currents* that is characterized by a vector $\mathbf{B}_0$ [or, in the SI system of units, $\mathbf{H}_0 = (1/\mu_0)\mathbf{B}_0$] and, at the same time, *there is a field generated by magnetized magnetic materials* that consists of the vortex ($\mathbf{B}_\mathrm{m}$) and potential ($\mathbf{H}_\mathrm{m}$) parts, in such a region the field

$$\mathbf{B} = \mathbf{B}_0 + \mathbf{B}_\mathrm{m}$$

is always a vortex field, while the field

$$\mathbf{H} = \frac{1}{\mu_0} \mathbf{B}_0 + \mathbf{H}_\mathrm{m} = \mathbf{H}_0 + \mathbf{H}_\mathrm{m}$$

(or $\mathbf{H} = \mathbf{B}_0 + \mathbf{H}_\mathrm{m}$ in the CGS system of units) is a composite one.

All attempts to assign a different meaning to the field $\mathbf{B}$ or its constituents $\mathbf{B}_0$ and $\mathbf{B}_\mathrm{m}$ (as well as to the field $\mathbf{H}$ or its constituents $\mathbf{H}_0$ and $\mathbf{H}_\mathrm{m}$) are groundless and only confuse the reader.

(2) Relationships similar to those in Eqn (9) exist in the electrostatics of insulators, where the field of the polarization vector $\mathbf{P}$ is represented as a linear combination of its constituents, i.e., the induction vector $\mathbf{D}_\mathrm{e}$ and the strength vector $\mathbf{E}_\mathrm{e}$ [18]:

$$\mathbf{P} = \mathbf{D}_\mathrm{e} - \varepsilon_0 \mathbf{E}_\mathrm{e} \,. \tag{12}$$

For an external (extraneous) field, which is usually the field of 'free' electric charges, one finds

$$\mathbf{D}_0 = \varepsilon_0 \mathbf{E}_0 \,, \tag{13}$$

where ε_0 is the electric constant (also known as permittivity of empty space). The induction $\mathbf{D}_0$ (or the strength $\mathbf{E}_0$) of the extraneous field is a purely potential vector:

$$\operatorname{rot}\mathbf{E}_0 = \operatorname{rot}\mathbf{D}_0 = 0\,.$$

Hence, in the equation

$$\mathbf{P} = \mathbf{D} - \varepsilon_0\mathbf{E}\,,$$

where $\mathbf{D} = \mathbf{D}_0 + \mathbf{D}_e$ and $\mathbf{E} = \mathbf{E}_0 + \mathbf{E}_e$, the field of vector $\mathbf{D}$ is a composite one, consisting of the vortex ($\mathbf{D}_e$) and potential ($\mathbf{D}_0$) parts, while the field of vector $\mathbf{E}$ is always a potential one, except for the case of electromagnetic induction, where $\operatorname{rot}\mathbf{E} = -\partial\mathbf{B}/\partial t$.

The above conditions imposed on the vectors $\mathbf{B}$, $\mathbf{H}$ and $\mathbf{D}$, $\mathbf{E}$ show that it is impossible to compare them (i.e., to establish their equivalence) in the form (2) or (3) in view of the fact that they have different constituents. For instance, the fields of $\mathbf{B} \leftrightarrow \mathbf{E}$ cannot be compared because the field $\mathbf{B}$ is a purely vortex one, while the field of $\mathbf{E}$ is a purely potential one. We can talk about the equivalence of fields comprising the polarization vector, $\mathbf{D}_e$ and $\mathbf{E}_e$, and the magnetization vector, $\mathbf{B}_m$ and $\mathbf{H}_m$, only if $\mathbf{P} \leftrightarrow \mathbf{M}$, and

$$\mathbf{D}_e \leftrightarrow \frac{1}{\mu_0}\,\mathbf{B}_m\,, \quad \varepsilon_0\mathbf{E}_e \leftrightarrow \mathbf{H}_m\,, \tag{14}$$

since they are the solutions of equivalent equations [18]

$$\mathbf{D}_e = \varepsilon_0 \operatorname{rot}(\operatorname{rot}\mathbf{Z}_e)\,, \quad \mathbf{E}_e = \operatorname{grad}(\operatorname{div}\mathbf{Z}_e)\,,$$

$$\mathbf{Z}_e = \frac{1}{4\pi\varepsilon_0}\int_\tau \frac{\mathbf{P}}{r}\,\mathrm{d}\tau\,;$$

$$\mathbf{B}_m = \operatorname{rot}(\operatorname{rot}\mathbf{Z}_m)\,, \quad \mathbf{H}_m = \frac{1}{\mu_0}\operatorname{grad}(\operatorname{div}\mathbf{Z}_m)\,,$$

$$\mathbf{Z}_m = \frac{\mu_0}{4\pi}\int_\tau \frac{\mathbf{M}}{r}\,\mathrm{d}\tau\,.$$

Here $\mathbf{Z}_e$ is the Hertz electric potential.

(3) The introduction of the fields $\mathbf{B}$ and $\mathbf{H}$ has meaning only if we are describing the magnetization vector field in a medium where $\mathbf{M} \neq 0$, since if in a certain region $\mathbf{M} = 0$, the fields of $\mathbf{B}$ and $\mathbf{H}$ are equivalent in this region. It can also be shown that in the analysis of properties of magnetic substances the fields of $\mathbf{B}$ and $\mathbf{H}$ have an alternative meaning [19, 20], which makes it possible to use either the field of vector $\mathbf{B}$ or the field of vector $\mathbf{H}$ in such a description.

(4) When using the theorem on the decomposition of the magnetization vector field into the vortex and potential parts in teaching physics, it is convenient to examine the particular case of such a decomposition, where a homogeneous magnetization of the magnetic substance takes place. Since $\mathbf{M} = \text{const}$, we have according to Ref. [21] that

$$\begin{gathered}\mathbf{Z}_m = \mu_0\mathbf{M}\Psi\,, \quad \mathbf{A}_m = \mu_0[\mathbf{M},\mathbf{h}]\,, \quad \varphi_m = (\mathbf{M},\mathbf{h})\,, \\ \mathbf{B}_m = \mu_0\operatorname{rot}[\mathbf{M},\mathbf{h}]\,, \quad \mathbf{H}_m = -(\mathbf{M},\nabla)\mathbf{h}\,,\end{gathered} \tag{15}$$

where

$$\Psi = \frac{1}{4\pi}\int_\tau \frac{1}{r}\,\mathrm{d}\tau\,,$$

$$\mathbf{h} = -\operatorname{grad}\Psi = \frac{1}{4\pi}\int_\tau \frac{\mathbf{r}}{r^3}\,\mathrm{d}\tau\,.$$

Allowing for the fact that $\operatorname{rot}[\mathbf{M},\mathbf{h}] = -(\mathbf{M},\nabla)\mathbf{h} + \mathbf{M}\operatorname{div}\mathbf{h}$ and that $\operatorname{div}\mathbf{h} = 1$, we arrive at Eqn (9).

(5) Combining identity transformations with the curl and divergence theorems, we can show (see Ref. [21]) that the constituents of the magnetization vector field, i.e., the vectors $\mathbf{B}_m$ and $\mathbf{H}_m$ (or the vectors $\mathbf{M}_v = (1/\mu_0)\mathbf{B}_m$ and $\mathbf{M}_p = \mathbf{H}_m$) can be represented in a *self-consistent* way:

$$\begin{gathered}\mathbf{B}_m = \frac{\mu_0}{4\pi}\int_\tau \frac{[\operatorname{rot}\mathbf{M},\mathbf{r}]}{r^3}\,\mathrm{d}\tau + \frac{\mu_0}{4\pi}\int_S \frac{[\mathbf{r},[\mathbf{n},\mathbf{M}]]}{r^3}\,\mathrm{d}S\,, \\ \mathbf{H}_m = \frac{1}{4\pi}\int_\tau (-\operatorname{div}\mathbf{M})\,\frac{\mathbf{r}}{r^3}\,\mathrm{d}\tau + \frac{\mu_0}{4\pi}\int_S (\mathbf{n},\mathbf{M})\,\frac{\mathbf{r}}{r^3}\,\mathrm{d}S\,.\end{gathered} \tag{16}$$

Here, the derivatives are taken with respect to the source point (the coordinate $\mathbf{r}'$; see Fig. 1), and $\mathbf{n}$ is the unit positive vector normal to the surface S that confines the volume τ. The relations (16) are often used as the definitions of the fields $\mathbf{B}_m$ and $\mathbf{H}_m$. In this case the values of the derivatives $\operatorname{rot}\mathbf{M}$ and $-\operatorname{div}\mathbf{M}$ and the quantities $-[\mathbf{n},\mathbf{M}]$ and $(\mathbf{n},\mathbf{M})$ are postulated in the same way as is done for, say, the case of homogeneous magnetization [21], which leads to the system of equations (15).

(6) Note that it is advisable to measure the constituents of the field of the same vector $\mathbf{M}$, i.e., $\mathbf{B}_m$ and $\mathbf{H}_m$, by units of the same dimensionality, while the names of these quantities may be different, just as they are in the Gauss system of units. In the SI system of units, which we use here, the dimensionality of the magnetic potential $\mathbf{Z}_m$ is not defined (see the IUPAP recommendations in Ref. [6]). This fact, and also the introduction of a dimensional magnetic constant μ_0 into formula (8), leads to arbitrariness in the choice of the units of measurement of $\mathbf{Z}_m$, φ_m, $\mathbf{A}_m$, $\mathbf{H}_m$, and $\mathbf{B}_m$, as noted in formulas (1). A similar situation exists in the description of the field of a polarized insulator. Thus, the use of the SI system of units in teaching electricity and magnetism in the college physics course limits the logical comprehension of physical models by the students, compared to the Gauss and Heaviside – Lorentz systems.

Further discussions of the above scientific and methodical problems can be found in Refs [21, 18].

4. Conclusions

Today there is an urgent need for a new textbook devoted to the "Electricity and Magnetism" section in physics course. Such a textbook should eliminate all the methodical faults, including those that emerged for historical reasons. Faults of this kind abound, especially when we turn to the electrodynamics of continuous media, and there is no need to mention them here.

At the same time, a modern textbook must be simple enough for students just beginning their study of physics at college to be able to understand the material. This requirement should form the guidelines for the authors of such a textbook. Finally, the entire exposition should be based on physical systems of units rather than on technical ones. The remarks made in the Introduction to this article should be sufficient for such a decision.

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