

Irreversibility in quantum mechanics

B B Kadomtsev

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Abstract. From the Editorial Board. November 9, 2003 would have marked the seventy-fifth birthday of Boris Borisovich Kadomtsev, were he alive. An outstanding theoretical physicist, teacher, and enlightener, a prominent scientist in plasma physics and controlled nuclear fusion, Kadomtsev was also actively involved in science organization activities. In particular, from 1976 until his untimely death on August 19, 1998, Kadomtsev was the Editor-in-Chief of *Physics-Uspekhi*, and it is owing to his efforts that the journal improved notably during his tenure. Now, the Editorial Board, with gratitude and sorrow, would like to celebrate his birthday and to honor his blessed memory in these pages. There is, however, a rule — indeed an immutable tradition — in the journal that, except for the *Personalia* section, no anniversary can be marked in any way other than in a scientific publication. This rule was strictly observed under Kadomtsev, and certainly should not be violated now, even when honoring his memory. Fortunately, there is a video which remained of a lecture on modern problems of quantum physics that Kadomtsev delivered on May 12, 1997. Prepared for publication by M B Kadomtsev, the lecture allows the reader to revisit the heritage of B B Kadomtsev, to appreciate his logic in treating this very difficult area of physics, to hear his voice as it were, to recall Boris Borisovich Kadomtsev and to honor his memory.

1. Introduction.

The Einstein – Podolsky – Rosen Paradox. Discussion between Einstein and Bohr

With respect to quantum irreversible processes, there is, as Mikhail Gorbachev would put it, a plurality of opinions. This may sound strange to a physicist, but nevertheless it is true. Most physicists believe that a sufficiently reliable methodology for calculation of all kinds of irreversible processes is already available, and all one needs to do in order to solve a particular problem is to find the right textbook or the published solution (probably even not too recent). At the same time, there is a group of enthusiasts — which includes both theoreticians and experimentalists — who argue that irreversibility in quantum mechanics is a fundamental issue, a key point that is necessary for understanding the very foundations of quantum theory. It is this group of people, and the related theoretical and experimental studies, that will be the subject of my presentation today.

Every student starting to get acquainted with quantum mechanics immediately learns about wave-particle duality. It is this phenomenon, so often discussed in monographs and textbooks, that is a vivid example of irreversibility. In order to clarify this statement, let us consider the standard Young's experiment with two slits, incident on which are certain quantum particles (Fig. 1). They can be photons, electrons, neutrons, or atoms. If the wavelength of these particles is large enough for interference to occur, then we shall see the interference pattern on a screen. If, however, we try to measure this interference pattern, we find that the instrument detects a particle instead of a wave. And not only the instrument. Let us place a photographic plate here on the screen. In Fig. 2 you see the experimental data obtained by Tanamura (who gained fame as the first observer of the Aaronov–Bohm effect). In these pictures you see bright dots produced by the electrons hitting the screen. At first they are very few (Fig. 2a). As the time of exposure increases, there are more and more dots (Figs 2b and 2c), until finally we see a distinct interference pattern (Fig. 2d). Going back we see

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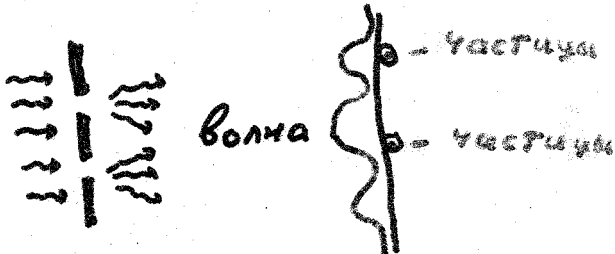
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Необратимость в квантовой теории (явление и роль).

Дуализм волна-частица:



Измерение - необратимый процесс.

Эволюция квантовой системы

Эволюция системы уравнения Шредингера $\psi \rightarrow P\psi$ "Измерение"

При измерении волновая функция "схлопывается" в одно из ортогональных состояний:

$$\psi \rightarrow P\psi \equiv \psi_i$$

P - проекционный оператор
Процесс измерения уравнением Шредингера не описывается

Чем же он описывается?

Что такое P с точки зрения физики?

(Или P - просто математический символ, не имеющий физического смысла?)

Figure 1.

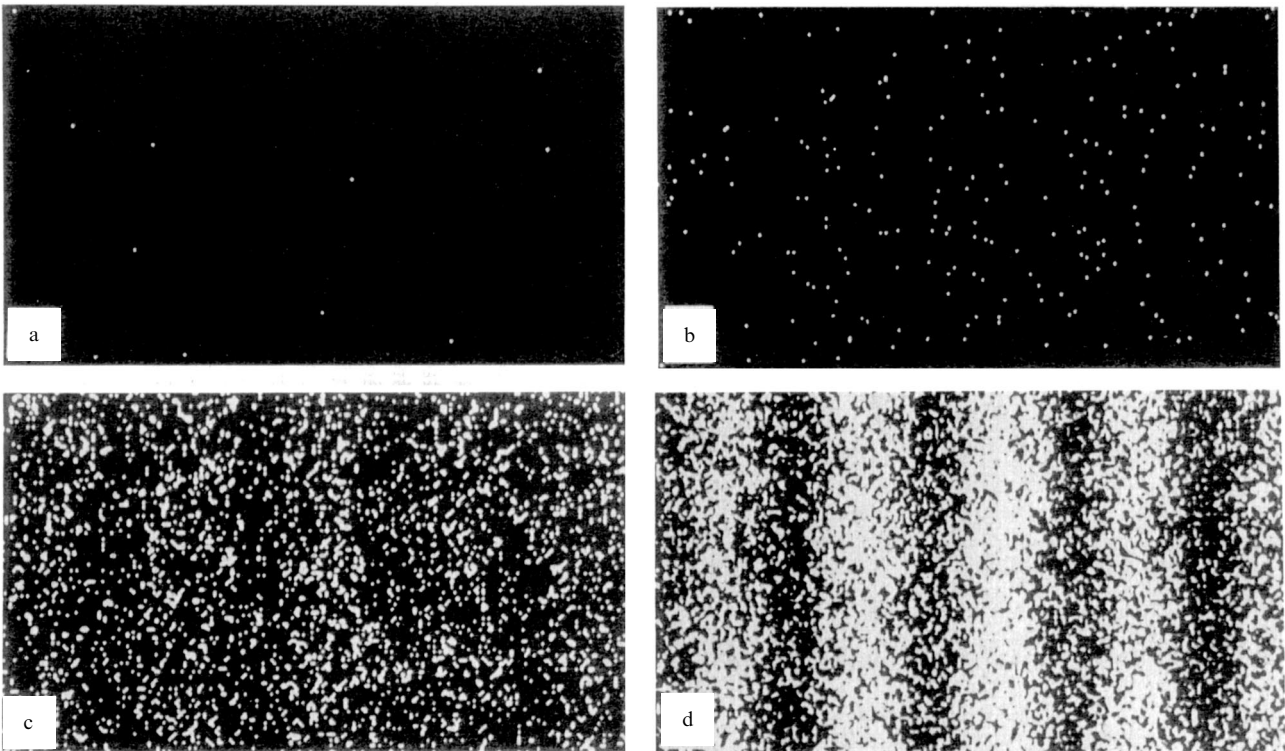


Figure 2.

that there was some hint of these bands in the first pictures, but a good interference pattern is only produced with a large number of electrons.

Obviously, the complete interference wave had been there before the dot was produced. Nevertheless, an individual electron had no way of manifesting its presence other than by producing a dot. It happens that out of the wave we suddenly get a particle. Students (and not only students) are always taken aback by this: how could this happen? Usually the following explanation is given: "This is the effect of the instrument that registers the particles (in our case the electrons) as individual dots". But, after all, the photographic plate is a perfectly ordinary physical object, and therefore the registration of particles is at the same time a physical process of some kind. This process is certainly irreversible, because all you get after the registration is one small dot, and you can forget about the big wave that had produced this dot.

How does this dot arise? The standard recipe (and please note that 'recipe' is exactly what this is called) is the following: the predictions of quantum theory are essentially probabilistic. Where there is wave motion we can use the Schrödinger equation, while the screen on which the registration of particles takes place is not described by the Schrödinger equation. The thing that causes this registration is called the measuring instrument, even though in this case it is simply a photographic plate known to most people from their school years — and certainly not regarded as an 'instrument'. Well, how does the measurement proceed according to this recipe? It is assumed that in the course of registration the wave

function experiences the action of the so-called projection operator, which (with a certain probability) transforms this huge and diffuse wave function (see Fig. 1) into a small tiny object. This is referred to as either reduction of the wave function or simply the measurement.

The first question that naturally arises in this connection is whether we can put up with such an abstract arrangement — the use of the projection operator as a kind of makeweight to the Schrödinger equation — instead of analyzing the actual irreversible physical process? If (indeed) the process of measurement is not described by the Schrödinger equation, then by what it is described? And a more profound question — where does the probability come from? In my presentation I will try to answer all these questions.

We shall need to start from something a little far away — from the famous Einstein–Podolsky–Rosen paradox. Einstein, Rosen, and Podolsky once expressed a wave function [1] with the following characteristics (Fig. 3). If you measure the coordinate of particle x_1 , then, since here we have the delta-function, this means that we have simultaneously measured the coordinate of particle x_2 . If instead we measure the momentum of particle p_1 , then, since here we have the delta-function of the difference $p_2 - p_1$, we have measured the momentum of the second particle p_2 . In other words, depending on what the first experimenter is doing, the second experimenter either gets a particle with fixed coordinate, or a particle with fixed momentum. This is a kind of long-range action, action at a distance. Einstein was outraged by such a possibility of long-range action, and he considered this a result of the incompleteness of quantum mechanics. Accord-

Парадокс Эйнштейна-Розена-Подольского.

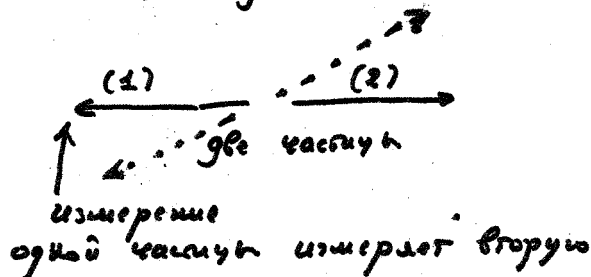
$$\Psi = \delta(x_1 - x_2 + L) \delta(p_1 + p_2)$$

Если измерить x_1 ,
то $x_2 = L + x_1$: $\frac{L}{x_1} \quad x_2$

Если измерить p_2 ,
то $p_2 = -p_1$ $\overleftarrow{p_1} \quad \overrightarrow{p_2}$

Измерение над одной
частицей меняет состояние
второй частицы, даже если
они удалены друг от
друга на большое рас-
стояние L .

Парадокс Эйнштейна-Розена-Подольского



Например: D-T реакция

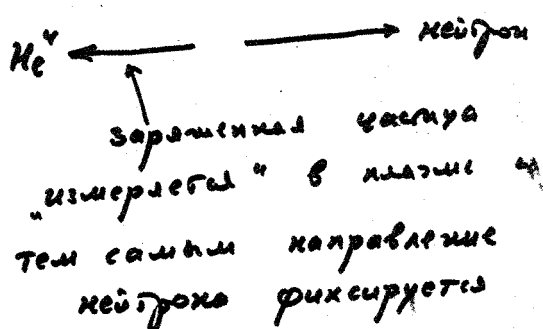


Figure 3.

ingly, the paper was entitled “Is quantum mechanics sufficiently complete?”

Beside the idealized example proposed by Einstein, Rosen, and Podolsky, in real physics there are many similar instances of quantum mechanical long-range action. For example, when two particles scatter from one point in different directions in a spherically symmetric way (see Fig. 3), the measurement of one particle will immediately register the direction taken by the other particle. In another realization of this experiment we could measure the particle here (dotted line in Fig. 3), and the second particle would then take a different path. Or, here is a typical example of thermonuclear reaction: a D-T reaction takes place in plasma, after which the neutron and the nucleus of helium-4 go in different directions. Now if this helium-4 is registered by somebody (later I will show that this registration must not necessarily be performed with a special instrument — it can be done by the medium itself), then the direction of motion of the neutron will also be known.

The discovery of this immanent quantum mechanical paradox gave rise to the argument between Einstein and Bohr. Einstein was the advocate of local realism. In his opinion, the paradoxicality of the laws of quantum mechanics is not real, but is related to the properties of the deeper unobservable subquantum world (this direction of research later became known as the theory of hidden variables). In other words, he deemed it possible to find hidden and previously unknown physical variables whose complex

dynamic behavior gives rise to stochasticity in the quantum world. You may ask me what ‘local realism’ has to do with this (Fig. 4). As a matter of fact, keeping within the realm of the conventional concepts of classical physics, it would be natural to expect that no manipulations with the first particle could ever affect the state of the faraway second particle. According to Einstein, quantum mechanical long-range action should be regarded as a paradox that has no place in any realistic physical theory.

In his response, Bohr, in the same year (1935) [2], formulated the principle of complementarity and indicated that the elements of physical reality (momentums or coordinates) are not objectively existing attributes of the microscopic world — instead, they directly depend on the instruments used in measurements. In this way, according to Bohr, the example proposed by Einstein, Podolsky, and Rosen (the EPR paradox) is not a demonstration of the deficiency of quantum theory, but rather signals the advent of the new reality that has come to replace the conventional notions of classical physics. In other words, long-range action and stochasticity, in spite of being seemingly paradoxical, are the most fundamental properties of our surrounding world.

So, more than 60 years ago two fundamentally different approaches to the understanding of quantum mechanical phenomena were established. One, supported by Einstein, was looking for purely classical processes acting at some submicroscopic level, which could be responsible for the probabilistic nature of quantum mechanics. By contrast, Bohr held that it was necessary to modify all our concepts taken from the compendium of classical physics so as to reconcile them with the more comprehensive quantum mechanical view of the world. In this confrontation of two outstanding thinkers, some physicists sided with Einstein, and put considerable effort into the development of a modified version of quantum theory with the classical hidden variables (in this connection we should recall the original approach of Bohm, who replaced the wave function with two interacting objects — the classical wave field and the point particle — the pilot wave theory), others were heart and soul with Bohr (the Copenhagen school, Fock in this country), while the more practically-minded scientists regarded this discussion as purely philosophical, since both sides in their opinion just wanted to validate the equations that worked well enough for all practical purposes. I think, however, that the overwhelming majority of physicists were uneasy with the radical novelty of quantum mechanical concepts, and together with Einstein they hoped that sooner or later a more ‘reasonable’ and complete physical theory would be created, which resolve all ‘oddities’ and paradoxes of quantum theory.

2. Bell’s inequalities. Experimental proof of the nonlocal nature of quantum theory

The situation changed radically when in 1964 John Bell [3] formulated his famous theorem, or the famous Bell’s inequalities, which implied that this argument can be resolved by purely experimental means. In other words, he showed that it is possible to invent such experiments that would give a straightforward answer as to who is right, Einstein or Bohr.

Bell’s inequalities consist of the following: assuming that hidden variables do exist, it is possible to derive certain simple inequalities for quantum operators or quantum physical quantities and see that these simple relations — inequalities

Вопрос второй - откуда берётся случайность?

Эйнштейн - локальный реализм: случайность обязана локальной статистичности „чего-то“ (скрытые параметры субквантового мира)

Бор - принцип дополненности: результат измерения зависит от выбранного прибора (нелокальность с принципиальной случайностью)

Figure 4.

— directly contradict those formulas that can be derived by the standard methods of quantum mechanics. I shall now explain how to construct Bell's inequalities in the most simple version, proposed somewhat later by Clauser, Horne, Shimony, and Holt [4]. We shall be concerned with the simplest quantum quantities — spins. Assume that we have spins correlated in pairs (so far, it is not important how they are correlated). One experimenter measures the direction of one spin, the other experimenter measures the direction of the second spin correlated with the first. The observers sit at different locations, and each can arbitrarily change the spatial orientation of his spin-measuring instrument.

Each spin (up to the coefficient $1/2$, which we drop) from the standpoint of mathematics is an operator that has only two eigenvalues, and from the standpoint of physics the spin can only have one out of two orientations in space: $+1$ along the axis, and -1 counter to the direction of the axis. Observer A can only register each spin that comes his way in one of two directions set by vectors a and c , and observer B in the directions set by vectors b and d . Assume now that there are hidden variables. The existence of hidden variables implies that the observed value of spin: $+1$ or -1 with respect to any spatial direction — is only determined by the local properties of the particle involved in the measurement, and does not depend on what happens with the other particle. If this is true, then a can be ± 1 , and c can also be ± 1 . If a and c have different signs, then $(a + c)$ is zero, and $(a - c)$ is either $+2$ or -2 . If a and c are of the same sign, then the opposite is true: $(a - c)$ is 0, and $(a + c)$ is either $+2$ or -2 , while b and d must be either $+1$ or -1 . It turns out that this expression at any value of a, b, c, d must give ± 2 (Fig. 5). Now where is the hidden variable in these calculations?

As a matter of fact, if we follow Bohr, the experimenter cannot simultaneously know the value of spin along two different directions (a and c in our case), because in changing the direction of measurement the experimenter must every time readjust his instrument. If, however, we admit the existence of hidden variables, then the rotation of the instrument does not help in any way: the local hidden variables would always yield ± 1 irrespective of where exactly you point your instrument at your particular location. Because of this, if hidden variables do exist, for each pair we have this identity, and if we perform averaging with respect to our hidden variables (in other words, perform measurements with a large number of pairs), we get the following inequality (Fig. 5). Indeed, since this combination is always less than or equal to 2, the mean value, which I have denoted by S in my drawing, cannot be greater than 2. And indeed, if we take real pairs — for example, singlets (pairs of particles with antiparallel spins), and use quantum mechanics for calculating these mean values, we find that S may be greater than 2. In particular, if we select the directions so that a and c , b and d are mutually orthogonal, and the angle between a and b is 135 degrees, S takes on the maximum value of $2\sqrt{2}$, and it is such geometry of the experiment that makes it especially easy to verify experimentally the validity of one theory or the other. Experiments with various quantum objects reveal that Bell's inequalities do not hold, in complete agreement with the quantum theory. I am not going to quote all results, but will only mention two papers [5, 6], which are especially helpful for understanding the basic features of quantum long-range action.

The first crucial experiment [5] was carried out by Aspect, Dalibard, and Roger in 1982. They studied polarization

Неравенства Белла (1964)
 - если допустить наличие скрытых параметров, то можно получить неравенства, противоречащие квантовой теории.
 Вариант Клаузера-Хорне-Шимони-Холта:
 a, b, c, d операторы $\sim \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$
 Измерение даёт ± 1 .
 Наблюдатель А измеряет a, c
 Наблюдатель В измеряет b, d
 $(a+c)b + (a-c)d = \pm 2$:
 если $a+c=0$, то $a-c=\pm 2$
 если $a-c=0$, то $a+c=\pm 2$
 $S = \langle ab \rangle + \langle bc \rangle + \langle cd \rangle - \langle ad \rangle$
 - среднее по скрытым параметрам.
 $|S| \leq 2$ - это неравенство.
 Противоречие КВТ.

Figure 5.

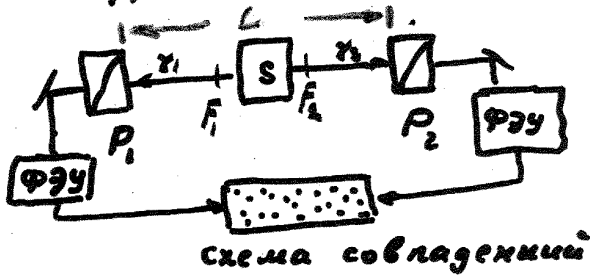
correlations of photon pairs using very fast-changing analyzers. The source of correlated pairs was the so-called S - P - S cascade on a calcium atom. It works like this. Assume that initially we have an atom in the excited S state, which means that its total momentum J is zero. Then it decays to an intermediate level, and, according to the known law of dipole radiation, its momentum becomes equal to 1; then it emits a second quantum, and the atom's angular momentum again becomes 0 (Fig. 6). Obviously, the result of this cascade transition from $J = 0$ to $J = 0$ will be absolutely spherically symmetrical. In addition to space symmetry, the polarizations of the two quanta will be concerted in such a way that their total polarization is zero. Then, at some sufficiently large distance from the source of photons, they placed polarizers that could change the direction of polarization faster than the time of transit of the photon from the source to the polarizer. This was done with an ultrasonic lattice that could be switched very quickly, so they had a polarizer that could be easily switched in a very short time.

Question from the audience: How short?

I do not know — questions perhaps later? In fact, the situation here was more complicated: these polarizers were rotating quickly, and made several revolutions in the mean time between two consecutive emissions. One could assume, therefore, that these polarizers simply change their polarization before the emitted photons hit them. Then there was the coincidence circuit, and this circuit produced appropriate counts, and it was found that these coincidence events,

Эксперимент Аспекта,
Далибарда, Роджерса (1982).

S-P-S каскад: $\begin{array}{c} \text{---} \gamma=0, S \\ \downarrow \gamma=1, P \\ \text{---} \gamma=0, S \end{array}$
Получаются
коррелированные фотоны.



Поларизаторы P_1, P_2
переклещивались за время,
значительно меньшее,
чем $L/c \rightarrow$
 \rightarrow сверхсветовая связь
между ФДУ?

Figure 6.

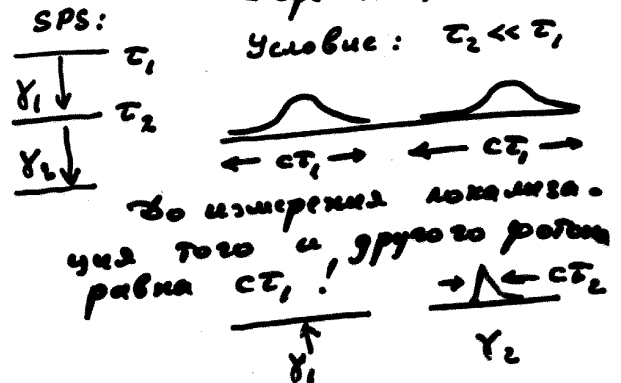
depending on the directions of the polarizers p_1 and p_2 , were in perfect agreement with quantum mechanics, and ‘Bell’s inequalities’ did not hold. Moreover, this experiment proved the nonlocal nature of quantum correlations — the correlations exist at any time, and are not transmitted by an electromagnetic signal of any kind.

Another experiment — known as Franson’s experiment — was proposed much later, in 1989. Franson did not do that experiment himself; he proposed the scheme for producing and observing nonlocal interference [7], and this scheme was subsequently realized by three independent teams. I shall only speak about one of them [6], but they all got similar results [8, 9]. Franson’s experiment is a really wonderful one, it is a straightforward demonstration of the nonlocality of quantum theory. I will be speaking of the nonlocality of quantum theory — this is an established term — so as I speak, please get accustomed to the fact that quantum theory is a nonlocal theory.

So what is the idea of Franson’s experiment? We take the same S-P-S cascade, but in an unusual configuration. The atom is selected (Fig. 7) so that its upper S level undergoes a very long and slow decay over the characteristics time τ_1 , but as soon as the atom gets into the intermediate P level, it very quickly goes on into the lower S state, so that τ_2 — the lifetime of state P — is much shorter than τ_1 .

Now we apply quantum mechanical reasoning. Before the measurement, quantum mechanics cannot tell us at what time the second quantum will be emitted. This second quantum will wait for the time when the atom goes from the upper S level to the intermediate level. This time τ_1 is large. Because of this, before the measurement neither the first nor the second

“Эксперимент Франсона” (1989)
(неопределенность энергии -
времени)



Если измеряется положение
фотона γ_1 (срабатывает
счётчик фотонов), то
второй фотон локализуется
на пространственном
интервале $ct_2 \ll ct_1$.

Figure 7.

quantum is localized either in time or in space, so that the width of both packets in space is ct_1

Assume now that you place a photodetector on the path of this first quantum, and this device at some point in space suddenly detects our photon γ_1 . As soon as it has detected the photon γ_1 — and the photons are correlated, they move in different directions — the second photon is immediately localized at the same distance in the opposite direction. This means that the broad wave packet disappears, and there remains only a small portion whose width is on the order of ct_2 . So, when we measure the position of the photon γ_1 (the photon counter is activated), the second photon is localized on the space interval ct_2 , which is much smaller than ct_1 . In brief, the positions of the two photons are mutually correlated with an accuracy down to the width ct_2 , but before the measurement it is absolutely impossible to say in which part of the big packet of width ct_1 each photon occurs. Moreover, we can cut two non-overlapping packets of width ct_2 each out of the big packet of the first photon, and then in the second correlated packet a similar structure will be automatically cut out, consisting of two narrow packets. It will be easier for me to further explain Franson’s arguments using the experimental results obtained by one of the teams [6]. I already said that there were three teams [6, 8, 9]. This is one of the experiments (Fig. 8). This is how it works. Here we have the source of correlated photons. It is a nonlinear crystal that converts the incident photon with a very precisely defined frequency into two photons whose frequencies are defined with a much worse accuracy: one has (with an accuracy of $\approx 1/\tau_2$) the frequency ω_1 , the other, ω_2 . We can install filters that direct, for example, γ_1 to the right, and γ_2 to the left. Then

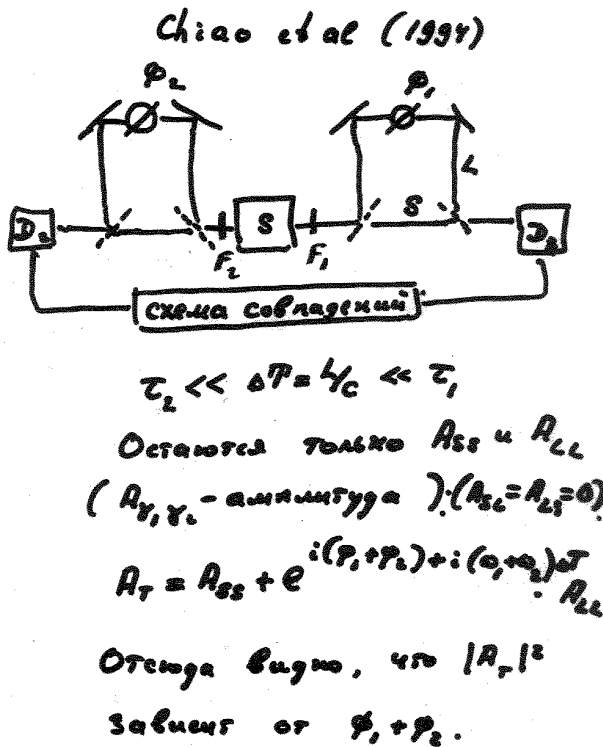


Figure 8.

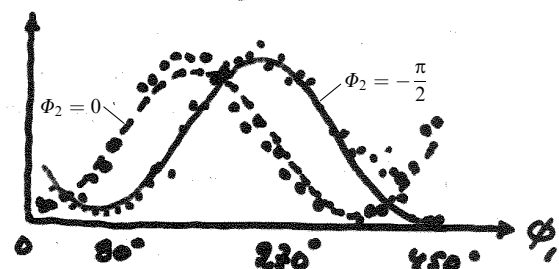
each photon goes into an interferometer of the Mach–Zehnder type: the semitransparent mirror splits the incident beam into two beams of half the intensity. Half of the intensity is passed through, and the other half takes a longer path — it is bounced off the mirror back to the point where we again have a semitransparent mirror — here the two halves meet and continue together. Thanks to this second semitransparent mirror it is possible to observe interference in the Mach–Zehnder interferometer. The fact is that the probability of registration of a photon by detector D (see Fig. 8) is $1/2$ only in the case of a noncoherent beam — that is, when the characteristic length of the wave packet of the photon is less than the path difference between the long and short arms of the interferometer. If, however, the frequency has a precisely defined value, the probability of detecting the photon oscillates depending on the phase difference between the two paths covered by the photon in the interferometer. In addition, one of the interferometers (the left one in Fig. 8) has a device that can change the length of the longer arm and produce a step change in the phase (the step was made equal to $\pi/2$); in the other interferometer (the right one in Fig. 8) the phase could be varied continuously with several π . The experimental configuration is such that τ_2 is much shorter than the time of transit of the photon over the longer arm. Because of this, no internal primary interference can be observed in either of the interferometers, because the uncertainty of the values of individual phases φ_1 and φ_2 is much greater than π . This means that detector D_1 and detector D_2 will each fire exactly one-half of the time after such quanta get into the first and second interferometers. On the other hand, this time of transit of photons is selected much shorter than τ_1 , and therefore both the total energy and the total phase $\varphi_1 + \varphi_2$ are defined with great accuracy. This allows us to observe interference under very strange circumstances when remote quanta, each of which gets into independent interferometers and which then follows the

short and the long paths in different interferometers, are registered by two independent detectors D_1 and D_2 . We get a striking situation — it seems that there can be no interference at all, because the first detector works independently, and the second detector works independently, but they are both connected to the coincidence circuit! What does this give us? Remember my drawing (Fig. 7): when we measure the position of one quantum at some distance from the source, the second quantum at the very same time is at the same distance from the source (naturally, to the other side from the source). In our present case (see Fig. 8), for the two detectors to fire simultaneously it is necessary that the two quanta should simultaneously take either the short or the long paths, each in its own interferometer. In order to find the probability of this process, it is necessary to take into account the interference between the short-short and the long-long amplitudes of propagation of the two photons. It turns out that one has to add up these two amplitudes, square the result, and this will give the probability of simultaneous actuation of the two detectors. The resulting interference pattern (Fig. 9) is very similar in appearance (only the intensity is somewhat lower because some photons miss the detectors and others are not detected simultaneously) to that which would have been produced by the primary quantum with the frequency $\omega_1 + \omega_2$ getting into one of the interferometers. Our interference, however, is essentially nonlocal, because it involves two remote particles at the same time.

Here are the results of experiments carried out by Chiao and colleagues [6] (there were also two other experiments [8,

Результаты эксперимента.

Число совпадений



Результаты для двух значений φ_2 хорошо совпадают с квантовой теорией.

$S = -2.63 \pm 0.08$ противоречит $|S| \leq 2$ - согласно квантовой теории скрытых параметров скранных параметров нет!

Figure 9.

9)). We show (see Fig. 9) two experimental curves for different values Φ_2, Φ_1 . The jump in the phase of the second curve is caused by activation of the device (see Fig. 8) that changes Φ_2 by $\pi/2$. What you see is a genuine interference pattern, which exhibits a sine-wave dependence on the total phase $\varphi_1 + \varphi_2$: the intensities may drop to zero, and may rise up to their maximum values. Franson [7] further proves that there is no logical explanation of such interference from the standpoint of local ‘elements of physical reality’. And, indeed, it is easy to demonstrate that any assumption of the existence of ‘objective’ (that is, prior to any measurement) characteristics of the particle, such as its energy or position in space, obviously contradicts the experimental findings. The quantity S (see definition in Fig. 5), whose absolute value in any theory with hidden variables cannot be greater than 2, is here -2.63 ± 0.08 (at $\Phi_2 = 0, -\pi/2$, and $\Phi_1 = \pi/4, 3\pi/4$).

3. Entangled states and Schmidt’s form

All these experiments show that the theory of hidden variables is refuted with gigantic accuracy. There are no hidden variables. Let me now give an illustration of what can be meant by hidden variables. It will be convenient to use the following simple analogy. Assume that we have two balls, a black ball and a white ball (Fig. 10). We put them into a box, shake the box, insert a divider not knowing the color of the ball in each of the two new boxes, and pull the boxes away from each other. In the classical case we have hidden variables. We do not know them, but they are there. One box contains the black ball, the other the white one. We do not know which box contains which color, but we know that this color exists and is preserved, therefore it is our hidden

variable. Now we open one box. If we see black color, we know immediately that the color in the other box is white. Of course, we cannot speak of long-range action or nonlocality — it is simply that the initial correlation leads to a situation when, as soon as we see one hidden variable (the black color of one ball), we know the hidden variable of the other. I think that everything here is quite obvious.

Quantum theory is in complete contradiction with this simple picture, and this ought to be understood very clearly. In quantum theory there are no hidden variables, and before the measurement the corresponding ball is neither white nor black. We can say that it has no color at all — that is, before the measurement we cannot speak of the color of any individual ball. We can only say that the two balls (two at once, and not each separately) occur in the superposition of white and black colors, and this superposition is such that if we measure one ball and find that it is black, the other ball is white. And conversely, if the first ball is white, then the second is black. Such closely correlated states of two quantum systems (of which I will be speaking further on) are known in quantum mechanics as entangled states. So, assume that these colors here are entangled in such way that if one ball turns black, the other immediately turns white. Now we have separated these balls, the ball has no color — neither white nor black. Open the box. If the color we see is black, this means that the other ball automatically assumes the white color. Its color appears at the instant the box is opened. Nothing happens before the box is opened, and only when one box is opened does the other assume (the opposite) color.

You may say that, well, quantum mechanics works that way — there are projection operators that randomly project the quantum state into one of the colors, and in this particular case it turns out to be white. Now the question is: what is a projection operator? What is the real irreversible physical process that corresponds to this purely formal procedure? To be able to discuss this question we need to go back to the problem of quantum correlations. Assume that we have two quantum systems: system A and system B . We do not know yet what they are — they are simply quantum systems with a certain set of variables. If these systems are not correlated with each other, then their joint wave function is expressed as the direct product of the wave function of one quantum system by the wave function of the other quantum system. Accordingly, the Schrödinger equation splits up, the variables separate, and it becomes two Schrödinger equations: one in variables A , and the other in variables B . You can write two Schrödinger equations, they have no connection with each other. We may assume, for example, that B is the external world, and A is an isolated system. Here, you have isolated the system and say: there are no links between this system and the surrounding world, therefore A evolves according to the Schrödinger equation, and we do not care about the evolution of the external world because we know that the variables are completely separated. The situation is entirely different if $\psi(A, B)$ is a joint wave function which is not the direct product of the two. This means that the wave functions — or the two systems — are mutually correlated, and their wave function is indivisible. What does this mean? Let us first take the simplest example proposed by Bohm. Assume that one quantum system is the spin of one particle, the other is the spin of the second particle, and their joint function is the so-called singlet state (Fig. 11). This is the superposition of two possibilities: one with the spin of particle a directed upwards and the spin of particle b directed downwards, and the

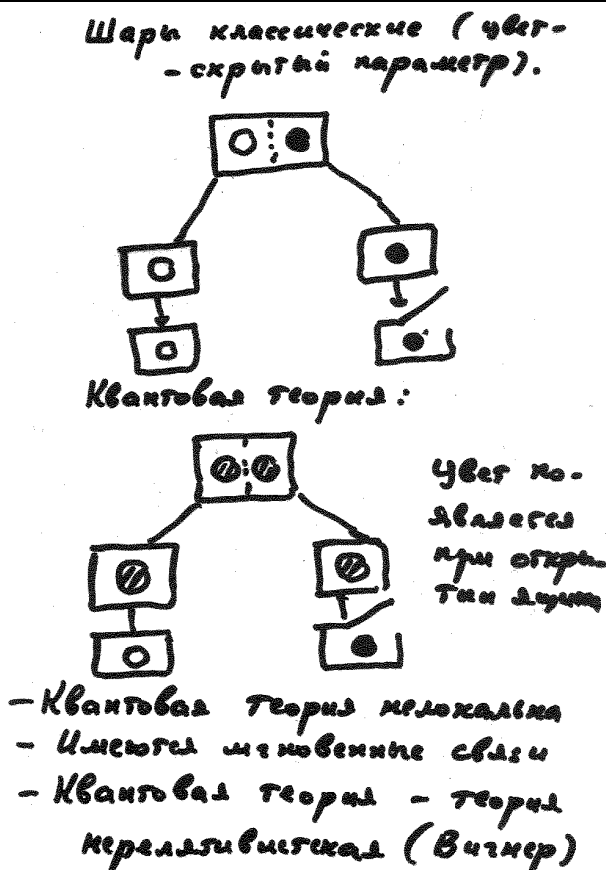


Figure 10.

Квантовые корреляции

A B Если $\Psi_{AB} = \Psi_A \Psi_B$,

то квантовые системы A, B
не коррелированы

Если $\Psi_{AB} \neq \Psi_A \Psi_B$, то возникает
запутывание (entanglement).

↑ A - синглет - простейший пример
↓ B $\Psi_{AB} = \frac{1}{\sqrt{2}} \{ |\uparrow\rangle_A |\downarrow\rangle_B - |\downarrow\rangle_A |\uparrow\rangle_B \}$.

Это состояние максимально
запутано.

Пусть $\Psi_{AB} = \cos\theta |\uparrow\rangle_A |\downarrow\rangle_B - \sin\theta |\downarrow\rangle_A |\uparrow\rangle_B$.

При $\theta = 0$ и $\theta = \pi/2$ запутывания нет.

При $\theta = \pi/4$ - максимальное запутывание

Ψ_{AB} - квантовый элемент
памяти.

Figure 11.

opposite with the spin of particle *a* directed downwards and the spin of particle *b* directed upwards. This is Bohm's singlet. Even if these particles fly far apart, this singlet state remains, and if we measure one spin in the singlet, the other spin will become known at once — it will be directed the other way. Spins in a singlet are always directed oppositely. This is an entangled state. For two spins this is the state of maximum entanglement, which I will explain shortly. How could we use the same arrows for constructing other entangled states? Very simple. Here I take Bohm's wave function, it is written in Dirac's notation (which in our present case has a rather clear meaning), and write in place of the numeric coefficient $1/\sqrt{2}$ the amplitude $\cos\theta$, and here the amplitude $\sin\theta$. Now look (see Fig. 11): if I take $\theta = 0$, then the first term is multiplied by 1, and the second by 0, so my wave function becomes the product of one wave function times the other, and the state is not an entangled one. If we take $\theta = \pi/2$, we again get a non-entangled state. That is, by varying the angle θ we can vary the degree of entanglement, and the highest entanglement occurs at $\theta = \pi/4$.

Now I shall tell you in what sense this state may be regarded as the simplest variant of a quantum memory element. To get the feel of the quantum memory element and the degree of entanglement, it will be useful to provide here the general definition of the measure of an entangled state which was quite recently — just a year ago — proposed

by Bennett, Bernstein, Popescu, and Schumacher [10]. The measure of entanglement is denoted by the symbol *E* and is defined as follows. Assume that we have two quantum systems, *A* and *B*. It turns out that for two quantum systems *A* and *B* it is always possible to select the presentation of their wave function in a certain particular form which is known as Schmidt's form. Schmidt's form can be obtained in the following manner. Assume that we have system *A*, whose wave function is defined on a certain basis of unit vectors as a superposition of different unit vectors with certain coefficients, and system *B*, which is also defined in terms of other unit vectors with some coefficients. We denote these basic functions by the letters α and β . You have great freedom in the selection of a particular basis: any unitary transformation gives you a new system of basic vectors as good as your old one. However, in order to obtain Schmidt's form for a given wave function ψ of the two systems *A* and *B*, these vectors must be selected in a very special way. Namely, we take a scalar product of ψ with any unit vector defined on a certain basis α , then with a vector defined on β , and then rotate these α and β until we get the maximum of this scalar product. Having found the maximum of this scalar product c_1 , we write the first element of Schmidt's form $c_1 |\alpha_1 \beta_1\rangle$, and fix the first basic function $|\alpha_1 \beta_1\rangle$. Then we go into space orthogonal to the first basic function and perform this procedure once again. We find the next term of the expansion with a smaller coefficient c_2 , and the second basic function $|\alpha_2 \beta_2\rangle$. This is repeated over and over until we come to zero. Assume that

Беннетт, Шушманер -
мера запутывания *E*.

Общий вид запутанного
состояния:

$$\Psi(A, B) = \sum_{i=1}^d c_i |\alpha_i\rangle |\beta_i\rangle -$$

- форма Шмидта.

c_i - действительные числа,
 d - размерность запутывания

$$E = - \sum_{i=1}^d c_i^2 \log_2 c_i^2 - \text{мера запутывания.}$$

У синглета $E = 1 \text{ бит}$
(1 запут)

N синглетов имеют

N запут (запутанной

информации)

Унитарные преобразования (т.е.
согласно уравнению Шрудинера)
не меняют *E*.

Figure 12.

this happens at step number d , so that the coefficients c_i are numbered from 1 to d . Number d is referred to as the dimension of the space, and the representation of ψ in terms of the basic functions $|\alpha_i\beta_i\rangle$ is known as Schmidt's form (Fig. 12). Since the unit vectors are defined up to an arbitrary phase, it is always possible to make all these coefficients c_i real-valued. This done, we can define the measure of entanglement as $E = -\sum c_i^2 \log_2 c_i^2$.

Why do we need here this logarithm to base 2? In order to express the result in bits. Let us find the measure of entanglement E for the maximally entangled singlet state, when $\theta = \pi/4$. Then the coefficient by each term is $1/\sqrt{2}$, squared $1/2$, logarithm of 2 to base 2 is 1. When I add up all this, $(1/2 \times 1) + (1/2 \times 1)$, I get 1. This means that the measure of entanglement of the singlet is 1 bit. Note that since the measure of entanglement does not have a straightforward classical analogy, for avoidance of confusion this measure is usually expressed in special units called ebits (from 'entangled bits', of course). Similarly, for a completely factored-out state at $\theta = 0, \pi/2$, there is no entanglement, and its measure E becomes equal to zero. Please note that the measurement of the direction of the spin of one particle (one is enough — quantum mechanics is nonlocal!) in the singlet immediately reduces the entangled state ($\theta = \pi/4$) to a factored-out state ($\theta = 0$ or $\theta = \pi/2$), and thus destroys one bit (ebit) of quantum memory.

4. Quantum irreversibility and probability

A singlet is the simplest example of an entangled system; however, similar irreversible transitions also occur in more complex systems, which we are going to discuss now. Assume that we have two arbitrary quantum systems, A and B . What happens with these systems after they have interacted with each other? According to the general approach, after the interaction the wave function of the system is no longer equal to the product of the wave function that only involves the variables of system A and the wave function that only involves the variables of system B . This means that we can write their joint wave function in the form of the so-called Schmidt decomposition, and find the number of bits that describes the degree of entanglement of the two subsystems. In the experiments of Yuri Sokolov [11], this number is slightly less than one, but is not too small. It turns out that no unitary transformations applied to either of the two systems, or involving a third system, can change this entanglement. If the system evolves according to the Schrödinger equation without any irreversibility, then this entanglement stays forever, and no trick will help you to get the direct non-entangled product of the two wave functions. It so happens that, in the absence of irreversible processes, the information that from the beginning has rested in the interacting systems would remain there forever, and then all our world would be in terrible confusion, because it would be built up of completely entangled states, always remembering all information about one another.

This, however, is not the case with our world, and entanglement disappears with time, so that the erasing of information — reciprocal information of one quantum system with respect to another — is the simplest example of an irreversible process, possibly the most fundamental irreversible process in quantum mechanics that one could imagine. What, then, is this irreversibility that erases the quantum memory?

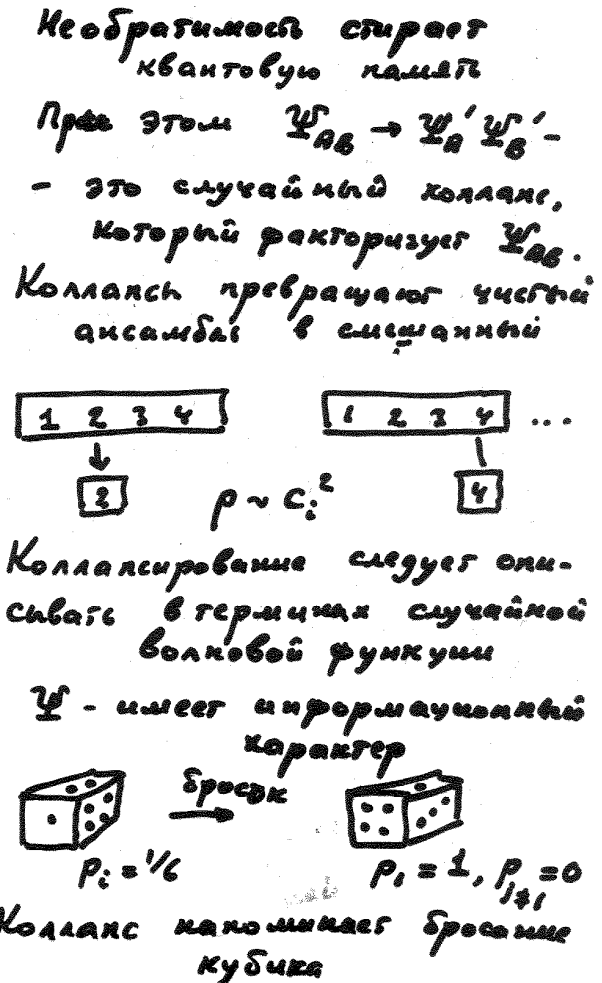


Figure 13.

When quantum memory is erased, the wave function must transform from this entangled state (Fig. 13) into the direct product of two wave functions. We see that this erasing only takes place because of the irreversible interaction with the outside world. And this outside world erases the information in such a way that each of the quantum systems gets into one of the orthogonal states.

Assume that we have a system A that has only four quantum states (see Fig. 13). As a result of interaction with system B , which may have a very large number of states, we get an entangled state as a sum of only four terms, each term containing the product of the wave functions of the system with a certain real-valued coefficient c_i . As a matter of fact, what I am drawing here is a multiple repetition of one and the same system — known as the quantum ensemble. If we have a quantum ensemble, then even after the interaction we have the ensemble of pure quantum states — albeit entangled and showing dependence on the variables of two systems at once. It is only the contact of the system B with a third system — and not simply a third system, but with an irreversible external environment — that leads to the loss of quantum information. After that the pure (but entangled) wave function collapses into a single state with the probability determined by the coefficient c_i^2 . If we repeat the experiment once again, the wave function may collapse at random into a different state. In other words, the initial ensemble of totally identical quantum states after the encounter with the irreversible environment transforms into a set of different

(in our case four) quantum states. After each experiment with a new member of the ensemble we get a system that has collapsed into one of the states; eventually — having conducted a sufficiently large number of experiments — we get the statistics enabling us to determine the probability of each particular finite state.

Let us go back once again to the picture that I showed you at the beginning (see Fig. 2). What we see here is the result of interaction of the electron with the molecules of silver bromide present in the sensitive layer of the photographic plate. Therefore (simplifying a little but not departing from the logic) we may assume that the electron hitting the photosensitive layer gives rise to a multicomponent highly entangled wave function spread over the entire plate and composed of the compact wave functions of the electron itself and the silver atom (produced in the chemical reaction of the decomposition of silver bromide). Owing to the irreversibility of the environment, this entanglement is erased, and the smeared wave function collapses each time into a dot on the photographic plate. It is exactly in this way that the irreversible process of registration of one electron takes place in the photographic plate; if we repeat the experiment over and over again, we will see the interference pattern. This means that the pure ensemble, as they say in statistical mechanics, becomes a mixed ensemble. In fact, however, each member of this mixed ensemble may be regarded as a new quantum system. It can also be regarded as a 'pure' system, but it will evolve differently after the registration. I will talk about this a little later.

So we see that the erasing of quantum information leads automatically to collapses of wave functions. A collapse leaves only a single component of the wave function; all

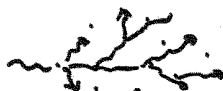
other components are either erased or destroyed. Reasoning in this way, we can give the answer to the question asked at the beginning of the lecture — how do probabilities arise in quantum mechanics? — without resorting to the non-native formalized method of projection operators.

5. Quantum and classical chaos of atomic gas

To better understand the physics of the collapse, it is convenient to analyze certain particular systems. With my son Mikhail, I have been looking into this matter for a rather long time, and have found quite a number of quantum systems that exhibit such processes of collapsing. The most convenient system turns out to be atomic gas at room temperature. That is, atomic gas at a relatively high temperature, when at first glance quantum effects might not seem to be important. For the sake of clarity, we may even take air under normal conditions. Air is as good as anything, because the rotational degrees of freedom of molecules are not important for our subsequent analysis. From the standpoint of classical physics, gas is an awfully stochastic system, and a small perturbation leads to exponentially fast deviation of the new path from the initial one. The stochastic behavior of gas atoms in quantum mechanics, even though it may look similar, is due to entirely different reasons.

For the sake of convenience, let us first consider some test particle — it can be a particle with almost the same properties, like some isomer or isotope, as long as the isotopic effects are not important. This particle flies and collides with atoms. It collides and scatters, as we know from classical mechanics, at the mean free path length λ . From the standpoint of classical mechanics, the atom will experience one collision on this free


Коллапс необратим.
Атомарный газ — простейшая
необратимая система.
Как выглядят волновые
функции атомов газа?
Пробная частица (изотоп).
 m — масса, n — плотность,
 $\lambda = 1/\sigma n$ — длина пробега,
 $v_T = \sqrt{p/m}$ — тепловая скорость,
 $\lambda_0 = h/mv_T$ — ср. длина волн де Бройля



рассеяние волн

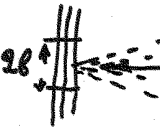
В изолированной системе эволюция обратима: $t \rightarrow -t$ ничем не меняется. Это означает:
рассеяние (расходящиеся) +
+ сходящиеся волны

При контакте с окружением
(термостатом) остаются только
расходящиеся волны
Обратим время: $t \rightarrow -t$



сходящиеся волны

Волновой пакет



Частица 1

Частица 2

сам частица 2

коллапсирует пакет, то
волновая функция частицы 1
Тоже испытывает коллапс
(парадокс Эйнштейна —
Розена — Подольского)

Δx — размер 1-ой зоны Френеля:
 $\Delta x \sim \sqrt{\lambda \lambda_0}$

Figure 14.

path length, and will take a different path. From the standpoint of quantum mechanics, however, everything goes on differently, because the wave function of the atom is always more or less smeared in space, and therefore the particle moving in the gas — even as a small wave packet — gives rise to a lot of scattered waves, quite a lot of scattered waves.

Question from the audience: But with small amplitudes?

Yes, with small amplitudes. So we may ask what happens with these scattered waves. Let us try to understand step by step what is going on. First we consider a reversible system. In other words, assume that we take a small lump of the gas and isolate it completely from the outside world. After that the atoms behave in complete compliance with the Schrödinger equation (and hence in a completely reversible way), and the transformation of the wave function of the system from one point in time to another is a unitary transformation. This is so from the standpoint of mathematics. From the standpoint of physics, complete reversibility means the following. If at some point we turn the time backwards, all these arrows must be reversed (Fig. 14). Accordingly, all these diverging waves start to converge, and eventually they ought to merge into the initial wave packet with which we started. In this way, complete reversibility implies that we must simultaneously have both converging waves and diverging waves.

Assume now that our gas is in a weak contact with the environment. In my opinion, it is simply impossible to imagine that all the converging waves could be phased so perfectly that they would be able to merge back into that small packet. We must say that if there is any external perturbation, then all the converging waves must be crossed out in the same way that it is done in electrodynamics, and we only have the diverging waves. This means that a weak contact with the outside world leads to a situation when only the diverging waves remain. Let us now estimate the characteristic size of the wave packets. Our reasoning is as follows. Take a cube with the side equal to the path length. Obviously, all memory — quantum memory or any other memory — must disappear on the scale of the free path length. The particles forget with what and how they have collided. If so, then we already understand the scale of wave packets that could correspond to the gas atoms. These wave packets cannot be larger than the free path length. In fact, however, they are even smaller.

This can be proved in the following manner. For this we need to reason again in the spirit of the Einstein–Podolsky–Rosen paradox — that is, in the spirit of correlated wave functions. Assume that we have two particles (see Fig. 14): having experienced a collision, they travel far away from each other. If at some distance — a distance on the order of λ — the second particle experiences a certain irreversible process, like a collapse into some region of size b , then the first particle also

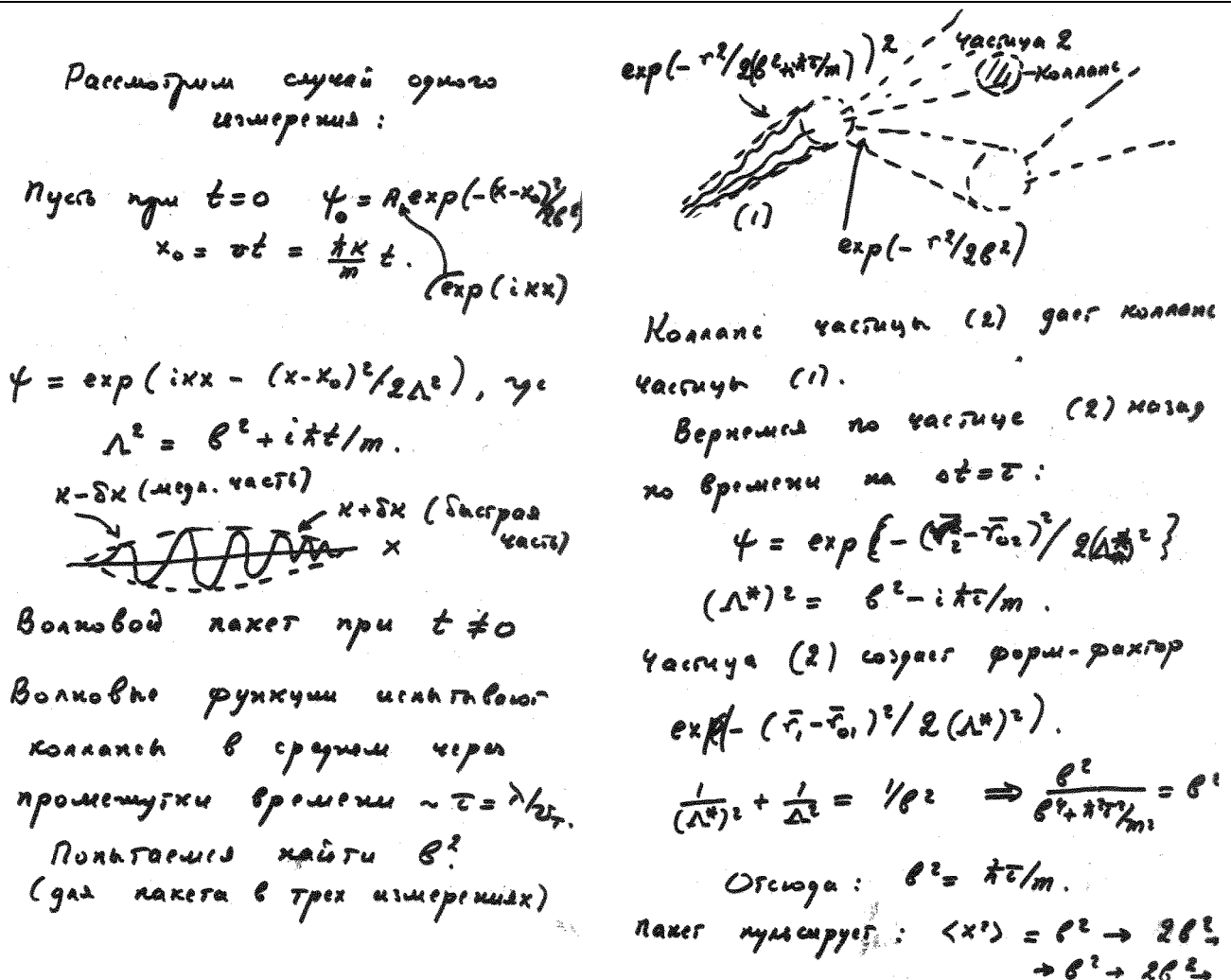


Figure 15.

undergoes a collapse, since the particles in correlated quantum systems cannot experience a collapse separately. Then — either from theoretical constructions or from simple considerations like Fresnel zones — we conclude that if the second particle experiences a collapse, then, since both particles were emitted from the same source, the first particle is localized on the scale of the first Fresnel zone. From these simple considerations we can find the size of the wave packet b of atoms of the gas. It turns out to be the geometric mean, that is, the square root of the product, of the de Broglie's wavelength λ_B and the free path λ , and is thus considerably smaller than the free path length.

I am not going into too many details, because all this has been published, and those interested can refer to paper [12]. To complete the picture, however, I will tell you how one can estimate this value. Consider first the case of one measurement (Fig. 15). If at the initial time we postulate the wave packet as a plane wave delimited by some Gaussian envelope, then we can use the nonstationary Schrödinger equation to see how this packet evolves in time. It broadens little by little, the short-wave portion running faster forward, and the longer-wave portion lagging behind. This is the result of dispersion. In my lectures for the students of the Physico-Technical Institute I describe in detail how this happens. But I think it is anyway obvious that because of the dispersion this is what takes place in quantum mechanics. Now that we know this, it is easy to understand that the three-dimensional packet will spread out in exactly the same way in each dimension. However, it does not occur in an empty space, so on the free path length it will necessarily collide with one of the gas atoms. We assume that the wave function of the atom is a similar Gaussian packet. A diverging wave is created after the collision, and the wave function of the system (test particle + gas atom) after the collision becomes highly entangled. The next collision (let us assume that it will first happen with the atomic partner) will lead to factorization (or collapsing) — that is, the quantum memory of the previous collision will be instantly erased, and the wave function of the test particle will have exactly the same functional form as before the first interaction, which is easily proved by straightforward calculation. This will take place if we select the initial size of packet b equal to the square root of the quantity $\hbar\tau/m$ (where \hbar is Planck's constant, τ is the time between the consecutive interactions, and m is the mass of the atom). From Fig. 15 we see that before each collision the imaginary and the real parts in the Gaussian exponent of the packet are exactly the same. For the sake of clarity (using the most straightforward algebra), the average size of packet b can be expressed as the square root of the free path times the de Broglie wavelength. For a typical molecule of the surrounding air we find $b \cong 20 \text{ \AA}$, which is about an order of magnitude greater than the size of the molecule.

So, in the quantum case, the motion of the test particle (and, for that matter, of any gas atom) looks like the following. The initial wave packet of the atom moves and creates numerous scattered waves (with small amplitudes — about 1/10 for air, as follows from the estimate developed above). This packet gradually spreads out, then suddenly one of these scattered waves (only one!) survives, while all the rest are destroyed, and a collapse occurs. The wave packet (see Fig. 15) contracts a little (by a factor of $\sqrt{2}$) — in other words, it pulsates from one collision to the next. This pulsation, however, can be neglected, and then we come to the model of continuous collapsing.

6. Model of continuous collapse. Gas as a measuring instrument

For describing the process of collapse in terms of the one-particle equation, it is necessary to extend the Schrödinger equation so as to make it explicitly take into account the irreversible processes of interaction with the surrounding world — in our case, with atomic gas particles. Upon collision, the two wave packets approach each other and exchange momentums (and, of course, energies), and since each of them has some amount of uncertainty as regards its momentum, the exchange of momentums is associated with a kind of diffusion with respect to momentums. After colliding, each of the wave packets exhibits additional broadening with respect to momentum. Since each packet occupies a cell in the phase space, there is some contraction with respect to x . This diffusion with respect to momentums can be described with a simple diffusion equation (Fig. 16), and after the transition to the configuration space we get the Schrödinger equation with a new term that leads to the contraction of the packet. There is also an added term that accounts for the normalization of the wave function. We get a kind of equation for the oscillator, but this is a special kind of oscillator, with purely imaginary stiffness. This is the model of continuous collapsing. In this model all atoms may be regarded as wave packets of steady shape traveling along classical paths, and when these packets pass through one another, they either scatter with some small probability, or (with overwhelming likelihood) continue their

Модель непрерывного коллапсирования

Неопределенность энергии

$$\delta E \sim \hbar/\tau = \hbar^2 x^2/2m \Rightarrow x^2 = m/\hbar\tau$$

За счет столкновений пакеты неопределенность по x возрастает:

$$\frac{\partial \psi}{\partial t} = \frac{\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2} \dots$$

$\frac{\partial^2}{\partial x^2} \rightarrow -(x-x_0)^2$ в координатном представлении

$$i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2} - i \frac{m(x-x_0)^2}{2\tau} \psi + i\hbar \gamma \psi$$

Стационарное решение выглядит как

$$\exp(-i\omega t + i\kappa x - (x-x_0)^2/2\Delta^2),$$

где $|\Delta|^2 = \hbar\tau/m$

волновой пакет

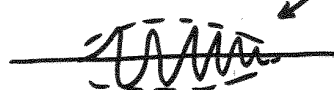


Figure 16.

motion, as though they have not noticed the collision. Of course, the formulas for the differential cross sections of scattering (if any) of the particles are taken from quantum mechanics, but in the model under consideration these packets have essentially become classical particles. Yes, they are wave formations of rather large size — the square root of the free path times the de Broglie wavelength, but still they are analogs of the particles. The kinetic equation describing the dynamic behavior of such quasi-particles will be constructed as a purely classical equation. In this way, this ongoing internal self-measurement leads to continuous collapsing of the wave functions, and at the end the same circumstance gives us the opportunity to describe the gas with standard methods based on the kinetic equations for purely classical distribution functions. If we know the mechanism of irreversibility in the gas, we can now understand the mechanism of the irreversibility of the measuring act of which I spoke at the very beginning. What is it from the microscopic point of view? What can we see here?

Now you can often hear the argument (reproduced in many publications) that everything we see in measurements and instruments is in fact the natural consequence of irreversible processes, whereas evolution according to the Schrödinger equation is typical of closed systems. If so, we must learn to describe the irreversibility — especially the type of irreversible processes that lead to the contraction of the wave function to the size of a little spot (see Fig. 2) on the photographic plate — usually referred to as the measurement. Now I will show a scheme that can be used instead of registration of an electron with a photographic plate. Assume that we have a gas layer (Fig. 17) confined between walls. These walls contain the atoms of the gas, but freely let through some particle with some wave. Assume that this gas layer is entered by electrons that have passed through two slits — the scheme is the same as in the very first picture (see Fig. 1). The electrons after the two slits have the same distribution $|\psi|^2$, but now they fall on the gas layer (see Fig. 17) rather than on the photographic plate. Now we ask what kind of irreversible process is taking place in the gas. In the gas the atoms collide with one another all the time, and their wave packets are sustained continuously owing to the continuous collapsing of their wave functions. As soon as the wave function of our particle gets in, it becomes entangled with the atoms of the gas. In other words, having entered the gas, our particle becomes entangled with all atoms (a great number of atoms) that happen to be near the wall at that time, and there is also additional entanglement with each atom due to the fact that the wave functions of the atom and the particle are not representable as the direct product of one with the other, but are expressed as the Schmidt form of which I spoke earlier (see Fig. 12). In Fig. 17 I tried to show each elementary collision of a particle with an atom as a little dot. I thought it better not to draw such tiny details as the diverging waves, and so let us believe that each point symbolizes one of the basic orthogonal states of the wave functions of the atom that have to be multiplied by certain similarly basic and orthogonal wave functions of the particles, so as to get eventually the Schmidt representation of the complete wave function. We may also assume — and this will always be true if it is an electron — that the incident particle initially has a very large velocity (as compared with the mean thermal velocity), and then these points will represent the sites of emission of secondary waves. What is the amplitude c_i for each point, with which it enters the Schmidt form? It is of the order of

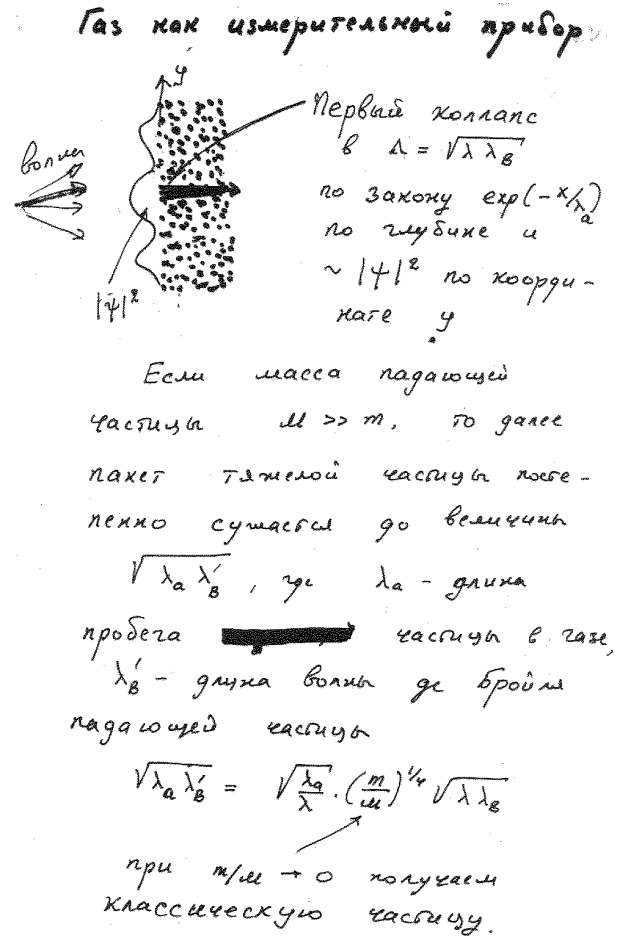


Figure 17.

$\sim 1/\sqrt{N}$, where N is the total number of atoms in the subsurface layer; in addition, we need to take into account variation of this quantity (see Fig. 17) both along the y -axis (owing to the interference pattern) and into the depth of the gas, because of the damping of the incident particle. Such an immensely complicated wave function has no chance of surviving in the gas, and, as I have already noted, over the characteristic time between collisions τ only one atom happens to have a collision with the incident particle, and the wave function, smeared until now over the entire gas layer, turns into a quite small object with a characteristic size equal to the square root of the de Broglie wavelength times the free path time. The situation will be slightly different if the mass of the incident particle is much smaller than the atom's mass [15]. It turns out that when the particle is light, each separate collision with an atom does not result in the complete measurement of the position of the light particle, and in the limit of a very lightweight particle, its passage through the gas will be similar to the passage of light through a turbid medium — that is, the atoms of the gas will no longer act as a measuring instrument for this particle. In this way, the wave function of the electron after several collisions, when its energy becomes equal to the thermal energy, will look much more swelled [15] than the wave functions of the atoms. However, as follows from simple estimates, the location of the first collision, when the electron has not yet lost its entrance energy, is determined with this degree of precision.

We see that for the electron, the layer of gas is a measuring instrument as efficient as the photographic plate — what

happens in both cases is a very fast collapse of the initially very extensive wave function into a wave packet of very small size, in both cases following the $|\psi|^2$ law. The only difference is that this locus of collapse remains on the plate as a latent image (which can later be developed) and is remembered for years to come, while in the gas the memory of this event is lost almost at once — within the time τ between two collisions of atoms.

If the mass of the particle entering the gas was very large, then further behavior of the particle (after the loss of its initial energy, which can be greater than the thermal energy by several orders of magnitude) takes on the appearance of irregular Brownian motion. Right now, together with my son, I am publishing a paper in *Physics Letters* [14] where we consider the Brownian motion of this particle and show what happens with the wave function of this Brownian particle as it further evolves in the gas. It turns out that little by little it narrows down because of subsequent interactions, and if the mass of this Brownian particle is large, it actually turns into a classical particle. In other words, if you throw a heavy particle into the gas — not an electron, as we did here, but a heavy particle — and then look at what happens with the track of the wave function (not of the particle but of the wave function), you will see that this wave function contracts, and the particle becomes classical (see Fig. 17). This shows in a simple way how quantum objects develop classical properties. The heavier the particle, the more macroscopic it is, and the narrower is the wave packet produced in the course of its interaction with, say, a gas or some other irreversible macroscopic system.

7. General method of description of collapses. Microworld and macroworld

So what is the general approach to the description of irreversible processes in quantum mechanics? In a way it is a combination of all the things that I have described in the course of this lecture. In other words, the general method consists of the following: instead of describing the measurements with an abstract mathematical procedure using the projection operators of dubious origin, we say that each irreversible quantum system has its own irreversible process that looks like collapsing — that is, like the destruction of the most part of the wave functions, so that only a small packet remains. As a matter of fact, this looks like some kind of projection on to a small region, but this is a physical mechanism that arises because of irreversibility. It is conveniently described in the following manner. We take the conventional Schrödinger equation and add two new terms. First is the term that describes regular washout of the initial wave function because of the interaction with the atoms of the gas (if it is a gas). Indeed, when you pass a particle through a turbid medium, its initial wave function gradually disappears. But this is not all. In this gas there is a continuous process that is one of internal self-measurements, and this process produces collapses. These collapses occur at random, and therefore this equation is the equation for a random wave function (Fig. 18). It is very similar in form to the Langevin equation. The Langevin equation is written for the Brownian motion of a classical particle and is constructed like this. Following Langevin, we assume that the Brownian particle is under the action of forces of two types: one is the slow deceleration of a particle because of viscosity, and the second is the incessant impacts from the surrounding atoms. So, if

you take such regular viscous damping, and add impacts of atoms, you can use the equation in the coordinates for describing the random motion of a Brownian particle.

Alternatively, instead of the equation for the random coordinate we can define the distribution function, and write the Fokker–Planck equation for the distribution function. However, the equation for the distribution function has nothing to do with the underlying physical process — it simply is a convenient method of describing a random process. In our case, we employ essentially the same logic. This is the equation (see Fig. 18) for a random wave function, and it extends the Schrödinger equation to the case of irreversible processes. Here τ is the mean damping time and K is an operator similar to the projection operator, which destroys the random wave function in the largest part of the Hilbert space and projects it on to one of the orthogonal states, just as happens when quantum information is erased. Quantum information from previous correlations is erased, and a structured wave function is created — this is forgetting. Forgetting of quantum information means that our wave functions become simpler and simpler. In other words, from a highly entangled state they go to a factorized state; then, owing to the interaction between the particles, a new entangled state is born, irreversibility then erases quantum information, and so this process continues ever and ever again. If you wish, we can average this random process over the ensemble, and get the conventional kinetic equation for the density matrix. But again, we must keep in mind that the kinetic equation for the density matrix is not the original physical picture, but a mathematical method used for describing something that lies in the foundation of everything — that is, the evolution that leads to the random wave function.

Then, from this approach we see what the macroworld is — the macroscopic world, or the classical world in which we live, to which we are accustomed, as opposed to the quantum world that we can only observe with our instruments. The classical world is the world of informationally open systems. As a matter of fact, we can only live because we continuously exchange not only entropy, not only energy with the surrounding world, but also information, quantum information.

Imagine, for example, that we have a macroscopic body, like a soccer ball. In principle, for such a body of macroscopic size we could imagine any kind of a wave function, including the function localized, for example, within the confines of this room. We immediately see the eccentricity of such a situation, but we may be slow to understand why this is not possible. This is not possible because this macroscopic body is under continuous monitoring from the side of the environment. Light quanta, atoms fall on this body, bounce off, and fly away. What happens, then, with these atoms and quanta of light? These quanta of light find their way into the irreversible world and are measured. If photons bounce off this screen, they are measured not only with your eyes — I mean the eyes of those who have not yet fallen asleep — but with any objects that are in this room. They are damped and thus measured, and atoms are also measured because the processes of internal measurements are under way all the time in the air. Accordingly, the wave function of a macroscopic body cannot be highly delocalized. It is easy to calculate that irrespective of how it is initially defined, it very soon contracts into a small wave packet, and the size of this

Общий метод описания коллапсов (т.е. необратимости)

$$i\hbar \frac{\partial \psi}{\partial t} = H\psi - \frac{1}{2\tau} \psi + K\psi.$$

τ – среднее время загрузки,

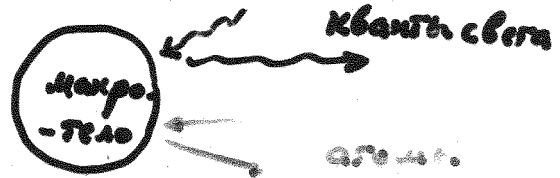
K – случайный оператор коллапсирования (аналог P)
 K – уничтожает ψ в большей области Гильбертова пространства

Никакого „сжатия ψ “ нет!

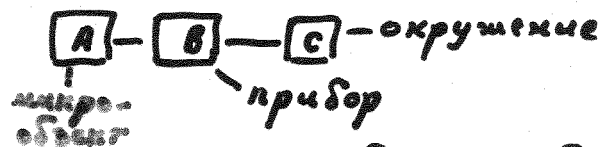
Квантовая информация о корреляции сгущается, рождается факторизованная ψ .

Усреднение случайного процесса по ансамблю даёт кинетическое уравнение для матрицы плотности –
 – это описание, а не первоприты

Макромир (классический).
 – мир информации открытой систем.



Во внешнем окружении происходит „самозмерение“ квантов света и атомов – волновая функция тела превращается в узко локализованный пакет – тело становится классическим.



Измерение: прибор „запутывается“ с микрообъектом, окружение коллапсирует их совместную волновую функцию.

Figure 18.

wave packet is much smaller than the actual dimensions of this macroscopic body — the body becomes a classical object.

Now what is a measurement? Measurement is the interaction of two quantum systems. One system is a microscopic object that we can keep for some time in an isolated state, and then bring it into contact with another object called the instrument. The instrument itself is being permanently measured by the environment. The surrounding world keeps monitoring the instrument; this linkage between the instrument and the environment can be considered very weak. It is convenient to view it simply as the information link, quantum information link. However weak, it still exists, and therefore the wave function of the instrument must always be packetized, and it is the surrounding world that transforms the joint wave function of the entangled state of the microscopic object and the instrument into a simple product, and this is why we get the statistical results of measurements when the instrument is brought into contact with the microscopic object.

It follows that today we must admit that the nonlocality of quantum theory has been confirmed experimentally. It has been proved that there are internal correlations that link together quantum objects at large distances and in all collapse-like processes. The correlation is such that one object will collapse simultaneously with the other, irrespec-

tive of the distance between these objects. We may ask now whether this effect can be used for building the quantum telegraph. What is a quantum telegraph? This is a conventional name for a device that can send information from one quantum object to another quantum object using only the quantum correlation links. Assume that we have a certain joint system AB (Fig. 19); it is entangled because if the wave functions had not been entangled they would simply be independent from one another. So we assume that they are entangled, and we carry A and B away to a large distance from one another. Then we perform measurements on system B , or some other irreversible manipulations. If we do something reversible in system B , or bring it into contact with another reversible system C , then nothing happens with the reversibility. This theorem was proved by Bennett, Bernstein, Popescu, and Schumacher [10, 16].

If, however, system B touches the external environment — well, let us say something is measured in this system — then system A immediately responds to that. This is an experimental fact, there is no room for doubt. The question is whether we could use this correlation for sending a signal from system B to system A without using any material carriers: no light, no waves, no anything, only purely quantum correlation links. The answer to this question is the following. If the principle that the probability is proportional to the corresponding $|\psi|^2$, that is, the probability of the

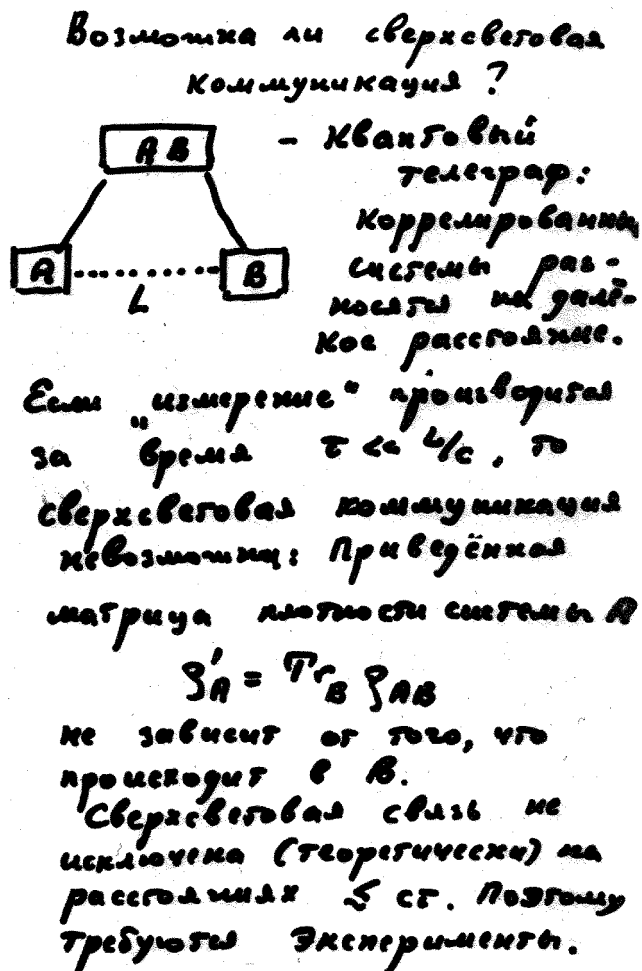


Figure 19.

corresponding collapse when quantum information is erased is proportional to c_l^2 , is true, then in the case where this distance is large it is not possible to transmit quantum information. To be more precise, if this distance L is much greater than ct , where τ is the characteristic relaxation time in system B , then quantum information certainly cannot be transmitted. This statement can be proved using the standard mathematical methods of quantum mechanics. If we take the density matrix of system A , i.e., perform complete averaging of the joint wave function with respect to the base vectors of system B , then it turns out that ρ_A absolutely does not depend on whatever is taking place in system B . The same idea can be conveyed in the following words: in spite of the existence of a quantum correlation bond, in spite of the nonlocality of quantum theory (quantum physics is nonlocal indeed), this effect cannot be used for transmitting information, because in order to transmit information it is necessary to accumulate the signal over a large number of single events. In order to see in system A that system B is sending signals, we need to perform averaging over a large number of events. Going back again to our picture (see Fig. 2), in order to see that yes, this is a quantum system, and quantum information is being transmitted, we need to have a very large number of events. But you can say nothing looking at individual points, and you cannot transmit information. If the information is transmitted over a large number of events, then we need to perform statistical averaging. In quantum mechanics it is a known procedure. It is based on the same principle as taking

the trace of the density matrix — Tr , as mathematicians write it. In other words, it is the averaging of the overall density matrix with respect to the variables of system B . Then we find that communication faster than light is not possible at a large distance. So the discussion reduces to the issue of what happens on a time scale smaller than τ , and space scales smaller than ct , when there is no real separation of the system into system A and system B , and the system AB continues in fact to relax as a joint system. In this case, the possibility of transmitting quantum information from one system to the other is open, and it is not possible to resolve it from purely theoretical considerations. These distances are very small — these links (correlation links) exist within one and the same complex quantum system, and, therefore, it is an experimental problem. We need experiments to tell us in a straightforward manner whether it is possible, or whether it is not possible.

8. Conclusion.

There is no complete determinism in nature

So, I am coming to the conclusion. Irreversible processes in quantum mechanics, as I tried to demonstrate with all these examples, are closely associated with the collapses of wave functions, even those processes to which we are accustomed — for example, gas kinetics, which we have learned to describe with quantum or classical kinetic equations. As a matter of fact, averaging has been performed in advance, and we do not look at what is taking place behind the curtain, the collapses of the wave functions. If we follow this more carefully, as I attempted to describe, then we see that all kinetic phenomena are closely associated with the collapses of the wave functions. Collapsing is erasing of quantum information through factorization of the wave functions. The world continuously simplifies its wave functions, it erases the mutual memory of one system with respect to the other and turns them into independent systems. This means that their joint wave functions become independent products of one another. Moreover, quantum mechanics is nonlocal, there are irreversible processes of superluminal erasing of information. This fact has been established by now with absolute certainty, and not only in those experiments that I referenced here [5–9]. We can name a large number of experiments — certainly more than a dozen, maybe twenty or thirty — which give a straightforward demonstration of the nonlocality of quantum theory and quantum mechanics. From this it follows that the argument between Einstein and Bohr has been resolved in Bohr's favor. There are no hidden variables in quantum mechanics, and they cannot be used for constructing the random processes of quantum mechanics. Quantum mechanics must be accepted as it is — it can be understood if we get accustomed to the fact that the wave function ψ has informational nature, and is different from all other wave fields to which we are accustomed, which always obey certain conservation laws, for example, conservation of energy or charge (we multiply something, integrate, and see that something is conserved). For the wave function, these conservation laws only work in the case of reversible evolution, in the framework of the Schrödinger equation. If the system is irreversible, the wave function evolves as a probability. I drew a picture of a die somewhere — here, imagine that you roll a die (Fig. 13). Before the die is rolled, the probability of getting any of its six sides on top is $1/6$. As soon as the die is rolled, and some

Заключение.

- Необратимые процессы в квантовой механике тесно связаны с коллапсами волновых функций.
- Коллапс – стирание квантовой информации путем факторизации волновой функции.
- Квантовая механика не-локальна – имеются необратимые процессы сверхсветовой скорости стирания информации.
- Спор между Эйнштейном и Бором решился в пользу Бора: механика скрытых параметров в квантовой теории нет.
- Волновая функция ψ имеет информационный характер, связанный с вероятностью.
- Необратимая эволюция квантовых систем описывается не уравнением Шрёдингера, а уравнением типа Ланжевена для случайной ψ -функции.
- Классические тела – следствие мониторинга макротел внешней средой.
- Измерение осуществляется при взаимодействии объектов микромира с приборами, т.е. макротелами, «измеряемыми» внешними окружениями.
- Сверхсветовая коммуникация невозможна на расстояниях, значительно больших ct (т – время релаксации).
- Может ли существовать возможность сверхсветовой связи в процессе „измерения“ (релаксации квантовой системы) – вопрос чисто экспериментальный.
- У природы нет полного детерминизма – наше счастье жить в таком мире.

Figure 20.

number comes up — for example, one — this means that the probability of getting number one is one, and all the other probabilities turn to zero.

Speaking of probabilities, we are not embarrassed by the fact that for each given realization of the random process all other possibilities are immediately destroyed. When we speak of the wave function, this sounds a little bizarre, but this is simply because we are not accustomed to it. If we assume, however, that all these projection operators that are defined as indispensable attributes of the complete interpretation of quantum mechanics are, in fact, some kind of a physical process that exists along with evolution in accordance with the Schrödinger equation, we can assimilate this idea. This is something that does not entirely correspond to probability, but there is some semblance. It may be some analog of complex probability, so that $|\psi|^2$ becomes a probability after the collapse.

Evolution of quantum systems surrounded by the irreversible world is described not by the Schrödinger equation, but by the Langevin-type equation for the random ψ function. We must assume that the ψ function is a random function; in this way we introduce collapses into our theory, and then also the equation that describes this random process becomes similar to the Langevin equation.

Classical bodies arise because of the ongoing monitoring of macroscopic bodies by the environment. The measurement occurs through the interaction of the objects of the micro-world with instruments — that is, with macroscopic bodies — and these macroscopic bodies in turn are measured by their environment.

Superluminal communication is not possible at distances greater than ct , where τ is the characteristic time of relaxation processes like irreversible measurements. Is there a possibility of — here it is written (see Fig. 20) — superluminal communication? We should probably emphasize not the word ‘superluminal’, or faster than light, but simply the possibility of quantum-correlation communication, because this is the most important thing. Whether the quantum-correlation link can exist in the course of measurement —

this issue is purely experimental, and must be studied experimentally.

And the last point. As we see, there is no complete determinism in nature, it is neither entirely classical nor completely quantum; random processes are hidden inside. We are lucky to live in the non-deterministic ever-developing world, and not in a world that could have been predicted with equations of some kind.

Discussion

Question from the audience: I seem to have missed the main point, and I want to make it clear. Assume that we have a closed system containing 1000 atoms, which are isolated from the environment but can collide with one another. My argument is that in this case there is no irreversibility in quantum mechanics; the irreversibility is, so to say, effective, because we cannot calculate anything, if I take not packets, which by themselves are non-stationary states, and then I get time dependence. Each collision of two packets is even more non-stationary problem, even in the pure quantum mechanics of two particles. If I take a large number of particles and know the state at the initial time, then, irrespective of their collisions, I can predict all the future in principle — now we are only discussing the matter in principle — and therefore there is no need for introducing an irreversible term into the Schrödinger equation. If we consider ordinary kinetic phenomena, they arise because the nondiagonal elements of the density matrix or the correlations between different states are very sensitive to the external action, and can get lost, but this is something different. I take some part of the system, which, in spite of everything, interacts with another system, some information flows into the other system and, of course, I get some irreversibility, but really it is not there, like what you call self-measurement, unless I interfere with my external system.

Answer: You are saying practically the same things as I did, only in different words. I say the same. If there is a closed quantum system, its evolution is reversible. It evolves according to the Schrödinger equation, and the recalculation from one state to another is performed with an elementary transformation. There is no irreversibility. If we take 1000 atoms, they will naturally obey the Schrödinger equation, and there cannot be any irreversibility. What is more, there are publications by Sergei Yakovlenko and colleagues [19], he used to work at our institute for some time, and he argues that effects of this type occur even in classical mechanics, and he received the Krylov Award for this. They take 1024 atoms (2^{10}) interacting according to Coulomb's law, and argue that if these atoms are contained between mirror walls, then (or it can be stars with gravity) the processes of recombination ought to in principle be

taking place there: some of the oppositely charged particles ought to come closer and closer to each other, classically without limit. They say that if you take a good computer and do the calculation, there is no irreversibility. For me it is not quite clear what is behind that; it is likely that the high symmetry of the Coulombian interaction prevents homogeneous occupation of the phase space.

Question from the audience: Is the energy fixed?

Answer: It does not matter. The phase space for the recombining particles is immense, everything could roll down there. The energy would go to those particles that do not recombine. All cannot recombine, but a small part can — and will give the energy to the non-recombining ones. It seems that for describing the processes of recombination we need to postulate some linkage with the environment. But in quantum mechanics it is very straightforward: if the system is not linked with the environment, then it is absolutely reversible. But further on, the more complicated the system (I am repeating what you have said), the more complicated the system, the smaller the linkage with the environment that is sufficient for erasing the quantum information, which can be expressed either in terms of disappearance of the nondiagonal elements of the density matrix, or in terms of the collapse of wave functions in a separate constituent — that is, in the formation of wave packets. So you and I are saying the same thing, only in different words.

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References

- [doi>](#) 1. Einstein A, Podolsky B, Rosen N *Phys. Rev.* **47** 777 (1935); *Usp. Fiz. Nauk* **16** 440 (1936)
- [doi>](#) 2. Bohr N *Phys. Rev.* **48** 696 (1935); *Usp. Fiz. Nauk* **16** 446 (1936)
3. Bell J S *Physics* **1** 195 (1964)
- [doi>](#) 4. Clauser J F, Horne M A, Shimony A, Holt R A *Phys. Rev. Lett.* **23** 880 (1969)
- [doi>](#) 5. Aspect A, Dalibard J, Roger G *Phys. Rev. Lett.* **49** 1804 (1982)
- [doi>](#) 6. Kwiat P G, Steinberg A M, Chiao R Y *Phys. Rev. A* **47** R2472 (1993)
- [doi>](#) 7. Franson J D *Phys. Rev. Lett.* **62** 2205 (1989)
8. Brendel J, Mohler E, Martienssen W *Europhys. Lett.* **20** 575 (1992)
- [doi>](#) 9. Tapster P R, Rarity J G, Owens P C M *Phys. Rev. Lett.* **73** 1923 (1994)
- [doi>](#) 10. Bennett C H, Bernstein H J, Popescu S, Schumacher B *Phys. Rev. A* **53** 2046 (1996)
11. Kadomtsev B B, Kadomtsev M B, Kucheryaev Yu A, Podogov Yu L, Sokolov Yu L *Phys. Scripta* **54** 156 (1996)
- [doi>](#) 12. Kadomtsev B B, Kadomtsev M B *Chaos* **6** 399 (1996)
- [doi>](#) 13. Namiki M, Pascasio S *Phys. Rep.* **232** 301 (1993)
14. Kadomtsev B B, Kadomtsev M B *Dokl. Ross. Akad. Nauk* **358** 325 (1998) [*Dokl. Phys.* **43** 17 (1998)]
- [doi>](#) 15. Kadomtsev B B, Kadomtsev M B *Phys. Lett. A* **231** 52 (1997)
16. Bennett C H *Phys. Today* **48** (10) 24 (1995)
17. Kadomtsev B B *Dinamika i Informatsiya* (Dynamics and Information) (Moscow: Red. Zh. “Uspekhi Fizicheskikh Nauk”, 1997)
- [doi>](#) 18. Kadomtsev B B *Usp. Fiz. Nauk* **165** 967 (1995) [*Phys. Usp.* **38** 923 (1995)]
- [doi>](#) 19. Maĭorov S A, Tkachev A N, Yakovlenko S I *Usp. Fiz. Nauk* **164** 297 (1994) [*Phys. Usp.* **37** 279 (1994)]