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Neutrino masses, mixing and oscillations

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Contents

Abstract. **Neutrino mixing and the basics of neutrino oscillations in a vacuum are considered. Recent evidence in favor of neutrino oscillations, obtained in solar and atmospheric neutrino experiments, are discussed. Neutrino oscillations in the solar and atmospheric ranges of Δm^2 are considered in the framework of the minimal scheme with the mixing of three massive neutrinos. Experiments on the measurement of neutrino mass via investigation of the high-energy part of the β-spectrum of tritium and experiments on the search for neutrinoless double β-decay are also discussed.**

1. Introduction

There exists at present convincing evidence of neutrino oscillations, obtained in experiments with neutrinos from natural sources: in atmospheric [1 – 3] and in solar [4 – 10] neutrino experiments.

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The observation of neutrino oscillations gave us the first evidence of nonzero neutrino masses and neutrino mixing. From all existing data, including astrophysical data, it follows that neutrino masses are many orders of magnitude smaller than masses of other fundamental fermions — leptons and quarks. The smallness of neutrino masses is generally considered as a signature of a New Physics beyond the Standard Model.

In this review we will present the phenomenological theory of neutrino masses and mixing. Then, the theory of neutrino oscillations in a vacuum will be considered. Further we will discuss the results of the Super-Kamiokande atmospheric neutrino experiment [1], in which significant up – down asymmetry of high-energy muon events was discovered, and the recent results of the SNO solar neutrino experiment [8 – 10], in which direct evidence of the transition of the solar neutrino ν_e into ν_μ and ν_τ was obtained. We will also discuss the results of the long baseline CHOOZ [11] and Palo Verde [12] reactor experiments, in which no indications in favor of neutrino oscillations were found. These results are important for neutrino mixing. We will consider neutrino oscillations in the atmospheric and solar ranges of the neutrino mass-squared differences in the framework of three-neutrino mixing. It will be demonstrated that in the leading approximation neutrino oscillations in these two regions are decoupled and are described by the two-neutrino formulas, which are characterized by two oscillation parameters. In the last sections of the review we will discuss the results of experiments on the measurement of neutrino mass via the detailed investigation of the end-point part of the tritium β-spectrum and neutrinoless double β-decay.

The investigation of neutrino oscillations is based on two points:

(1) The assumption supported by all existing data, including the data of the very precise LEP experiments, that the *interaction of neutrinos with other particles is described by the Standard Model of the electroweak interaction*. The Standard Charged Current (CC) and Neutral Current (NC) Lagrangians are given by the following expressions

$$\mathcal{L}_I^{\rm CC} = -\frac{g}{2\sqrt{2}} j_\alpha^{\rm CC} W^\alpha + \text{h.c.}\,, \qquad \mathcal{L}_I^{\rm NC} = -\frac{g}{2\cos\theta_{\rm w}} j_\alpha^{\rm NC} Z^\alpha\,. \tag{1}$$

Here, g is the SU(2) gauge coupling constant, $\theta_{\rm w}$ is the weak angle, W^α and Z^α are the fields of charged ($W^\pm$) and neutral (Z^0) vector bosons, and the leptonic charged current $j_\alpha^{\rm CC}$ and neutrino neutral current $j_\alpha^{\rm NC}$ are given by

$$j_\alpha^{\rm CC} = \sum_l \bar\nu_{l,\rm L}\gamma_\alpha l_{\rm L}\,, \qquad j_\alpha^{\rm NC} = \sum_l \bar\nu_{l,\rm L}\gamma_\alpha \nu_{l,\rm L}\,. \tag{2}$$

(2) The fact, which was established in SLC and LEP experiments, that *three flavor neutrinos* ν_e, ν_μ, *and* ν_τ *exist in nature*.

From the LEP experiments on the measurement of the $Z \to \nu_l + \bar\nu_l$-decay width for the number of flavor neutrinos $n_{\nu_{\rm f}}$, the value [13]

$$n_{\nu_{\rm f}} = 3.00 \pm 0.06 \tag{3}$$

was obtained. From the global fit of the LEP data for $n_{\nu_{\rm f}}$, it was found that

$$n_{\nu_{\rm f}} = 2.984 \pm 0.008\,. \tag{4}$$

2. Neutrino mixing

The hypothesis of neutrino mixing is based on the assumption that *a neutrino mass term* enters into the total Lagrangian. Several mechanisms for generating the neutrino mass term have been proposed. Later we will discuss the most popular see-saw mechanism [14].

We will present first the phenomenological theory of neutrino masses and mixing. There are two types of possible neutrino mass terms (see Refs [15, 16]).

(1) The Dirac mass term

$$\mathcal{L}^{\rm D} = -\bar\nu'_{\rm R} M^{\rm D} \nu'_{\rm L} + \text{h.c.}, \tag{5}$$

where the notation is applied:

$$\nu'_{\rm L} = \begin{pmatrix} \nu_{e,\rm L} \\ \nu_{\mu,\rm L} \\ \nu_{\tau,\rm L} \\ \vdots \end{pmatrix}, \qquad \nu'_{\rm R} = \begin{pmatrix} \nu_{e,\rm R} \\ \nu_{\mu,\rm R} \\ \nu_{\tau,\rm R} \\ \vdots \end{pmatrix}, \tag{6}$$

and $M^{\rm D}$ is a complex nondiagonal matrix. We have assumed that in the $\nu'_{\rm L}$ column there can be not only left-handed flavor neutrino fields $\nu_{l,\rm L}$ but also so-called sterile fields, the fields that do not enter into the standard charged and neutral currents (2). After the diagonalization of the matrix $M^{\rm D}$ [15, 16], for the flavor fields $\nu_{l,\rm L}$ we have the relation

$$\nu_{l,\rm L} = \sum_i U_{li}\nu_{i,\rm L}\,, \tag{7}$$

where U is a unitary mixing matrix, and ν_i is the field of a neutrino with mass m_i.

The total Lagrangian with the mass term (5) is invariant under global gauge transformation

$$\nu'_{\rm L} \to \exp(i\alpha)\,\nu'_{\rm L}\,, \quad \nu'_{\rm R} \to \exp(i\alpha)\,\nu'_{\rm R}\,, \quad l \to \exp(i\alpha)\,l\,,$$

where α is an arbitrary constant phase. This invariance means that the total lepton number $L = \sum_l L_l$ is conserved, and ν_i is the field of the *Dirac neutrinos and antineutrinos* $[L(\nu_i) = 1, L(\bar\nu_i) = -1]$.

(2) The Majorana mass term

$$\mathcal{L}^{\rm Mj} = -\frac{1}{2}\,(\overline{\nu'_{\rm L}})^{\rm c}\, M^{\rm Mj}\nu'_{\rm L} + \text{h.c.} \tag{8}$$

Here, $M^{\rm Mj}$ is a complex nondiagonal *symmetrical* matrix, and $(\nu'_{\rm L})^{\rm c} = C\bar\nu'^{\rm T}_{\rm L}$, where C is the unitary matrix of the charge conjugation, which satisfies the conditions $C\gamma_\alpha^{\rm T}C^{-1} = -\gamma_\alpha$, $C^{\rm T} = -C$. After the diagonalization of the symmetrical matrix $M^{\rm Mj}$ [15, 16], for the flavor fields $\nu_{l,\rm L}$ we have

$$\nu_{l,\rm L} = \sum_i U_{li}\nu_{i,\rm L}\,, \tag{9}$$

where U is the unitary matrix, and ν_i is the field of a neutrino with mass m_i, which satisfies the condition

$$\nu_i = \nu_i^{\rm c} = C\bar\nu_i^{\rm T}\,. \tag{10}$$

The field ν_i is the field of the *Majorana neutrinos*.

In the case of the Majorana mass term, there is no global gauge invariance of the total Lagrangian. Hence, Majorana neutrinos are truly neutral particles: they carry not only electric charge but also lepton numbers which can allow us to distinguish neutrinos and antineutrinos.

If there are only flavor fields $\nu_{l,\rm L}$ in the column $\nu'_{\rm L}$, the number of massive neutrinos ν_i is equal to the number of flavor neutrinos (three), and U is a 3×3 unitary matrix.

If in the column $\nu'_{\rm L}$ there are also sterile fields $\nu_{s,\rm L}$, the number of massive neutrinos ν_i will be larger than three. In this case, the mixing relation takes the form

$$\nu_{l,\rm L} = \sum_{i=1}^{3+n_s} U_{li}\nu_{i,\rm L}\,, \qquad \nu_{s,\rm L} = \sum_{i=1}^{3+n_s} U_{si}\nu_{i,\rm L}\,, \tag{11}$$

where n_s is the number of sterile fields, U is a unitary $(3+n_s)\times(3+n_s)$ matrix, and ν_i is the field of a neutrino with mass m_i $(i = 1, 2, \ldots, 3+n_s)$.

Sterile fields can be right-handed neutrino fields, SUSY fields, etc. If more than three neutrino masses are small, transition of the flavor neutrinos ν_e, ν_μ, and ν_τ to sterile states becomes possible. [1]

The so-called Dirac and Majorana mass terms are of special interest. Let us assume that left-handed sterile fields are $(\nu_{l,\rm R})^{\rm c}$, where $\nu_{l,\rm R}$ are right-handed neutrino fields ($l = e, \mu, \tau$). The Majorana mass term can be presented in

[1] If the data of the LSND accelerator experiment [17] are confirmed, we will need to consider the mixing of four (or more) massive neutrinos (see discussion below).

this case in the form of the sum of the left-handed Majorana, Dirac, and right-handed Majorana mass terms:

$$\begin{aligned}\mathcal{L}^{\mathrm{D+Mj}} &= -\frac{1}{2}(\bar{\nu}_{\mathrm{L}})^{\mathrm{c}} M_{\mathrm{L}}^{\mathrm{Mj}} \nu_{\mathrm{L}} - \bar{\nu}_{\mathrm{R}} M^{\mathrm{D}} \nu_{\mathrm{L}} \\ &\quad -\frac{1}{2}\bar{\nu}_{\mathrm{R}} M_{\mathrm{R}}^{\mathrm{Mj}} (\nu_{\mathrm{R}})^{\mathrm{c}} + \text{h.c.} \\ &= -\frac{1}{2}(\bar{\nu}'_{\mathrm{L}})^{\mathrm{c}} M^{\mathrm{D+Mj}} \nu'_{\mathrm{L}} + \text{h.c.},\end{aligned} \tag{12}$$

where the notation is employed:

$$\nu_{\mathrm{L}} = \begin{pmatrix} \nu_{\mathrm{e,L}} \\ \nu_{\mu,\mathrm{L}} \\ \nu_{\tau,\mathrm{L}} \end{pmatrix}, \qquad \nu_{\mathrm{R}} = \begin{pmatrix} \nu_{\mathrm{e,R}} \\ \nu_{\mu,\mathrm{R}} \\ \nu_{\tau,\mathrm{R}} \end{pmatrix}, \tag{13}$$

$M_{\mathrm{L}}^{\mathrm{Mj}}$ and $M_{\mathrm{R}}^{\mathrm{Mj}}$ are complex nondiagonal symmetrical Majorana 3×3 matrices, and M^{D} is a complex nondiagonal Dirac 3×3 matrix. After the diagonalization of the mass term (12), we obtain

$$\nu_{l,\mathrm{L}} = \sum_{i=1}^{6} U_{li} \nu_{i,\mathrm{L}}\,, \qquad (\nu_{l,\mathrm{R}})^{\mathrm{c}} = \sum_{i=1}^{6} U_{\bar{l}i} \nu_{i,\mathrm{L}}\,, \tag{14}$$

where U is the unitary 6×6 mixing matrix, and ν_i is the field of the Majorana neutrino with mass m_i.

The see-saw mechanism of neutrino mass generation [14] is based on the assumption that the neutrino mass term is the Dirac and Majorana one. In order to explain the idea of the mechanism, we will consider the simplest case of one type of neutrino. Let us assume that the standard Higgs mechanism with one Higgs doublet, which bears the responsibility for generating the masses of quarks and leptons, also generates the Dirac neutrino mass term

$$\mathcal{L}^{\mathrm{D}} = -m\,\bar{\nu}_{\mathrm{R}} \nu_{\mathrm{L}} + \text{h.c.} \tag{15}$$

It is natural to expect that the mass m is of the same order of magnitude as masses of the corresponding leptons or quarks. We know, however, from experimental data that neutrino masses are much smaller than the masses of leptons and quarks. In order to 'suppress' the neutrino mass, we assume that there exists beyond the Standard Model a mechanism which violates the lepton number and generates the right-handed Majorana mass term

$$\mathcal{L}_{\mathrm{R}}^{\mathrm{Mj}} = -\frac{1}{2} M\,\bar{\nu}_{\mathrm{R}} (\nu_{\mathrm{R}})^{\mathrm{c}} + \text{h.c.} \tag{16}$$

with $M \gg m$ (usually it is assumed that $M \simeq M_{\mathrm{GUT}} \simeq 10^{15} - 10^{16}$ GeV).

The total mass term is the Dirac and Majorana one with

$$M^{\mathrm{D+Mj}} = \begin{pmatrix} 0 & m \\ m & M \end{pmatrix}, \qquad \nu'_{\mathrm{L}} = \begin{pmatrix} \nu_{\mathrm{L}} \\ (\nu_R)^{\mathrm{c}} \end{pmatrix}. \tag{17}$$

After the diagonalization of the mass term, we find

$$\begin{aligned}\nu_{\mathrm{L}} &= \mathrm{i}\nu_{1,\mathrm{L}} \cos\theta + \nu_{2,\mathrm{L}} \sin\theta\,, \\ (\nu_{\mathrm{R}})^{\mathrm{c}} &= -\mathrm{i}\nu_{1,\mathrm{L}} \sin\theta + \nu_{2,\mathrm{L}} \cos\theta\,,\end{aligned} \tag{18}$$

where ν_1 and ν_2 are the fields of the Majorana particles with masses

$$\begin{aligned}m_1 &= -\frac{1}{2} M + \frac{1}{2}(M^2 + 4m^2)^{1/2} \simeq \frac{m^2}{M} \ll m\,, \\ m_2 &= \frac{1}{2} M + \frac{1}{2}(M^2 + 4m^2)^{1/2} \simeq M\,.\end{aligned} \tag{19}$$

The mixing angle θ is given by the relationship

$$\tan 2\theta = \frac{2m}{M} \ll 1\,. \tag{20}$$

Thus, the see-saw mechanism is based on the assumption that, in addition to the standard Higgs mechanism for generating the Dirac mass term, a mechanism exists beyond the SM for generating[2] the right-handed Majorana mass term, which changes the lepton number by two and is characterized by a mass $M \gg m$. The Dirac mass term mixes the left-handed field ν_{L}, the doublet component, and the singlet field $(\nu_{\mathrm{R}})^{\mathrm{c}}$. As a result of this mixing, the neutrino acquires the Majorana mass, which is much smaller than the masses of leptons or quarks.

In the general case of three generations for neutrino masses, we have

$$m_i \simeq \frac{(m_i^{\mathrm{f}})^2}{M_i} \ll m_i^{\mathrm{f}}\,. \tag{21}$$

Here m_i^{f} is the mass of the quark or lepton in the ith family.

Let us stress that if neutrino masses are of the see-saw origin, then

- neutrinos with definite masses are Majorana particles;
- there are three light neutrinos;
- the heavy Majorana particles must exist.

The existence of the heavy Majorana particles, appearing as see-saw partners of neutrinos, could be a source of the barion asymmetry of the universe (see Ref. [18]).

3. Neutrino oscillations in a vacuum

We will discuss here the phenomenon of neutrino oscillations [19, 16]. If there occurs neutrino mixing, the total Lagrangian does not conserve flavor lepton numbers L_{e}, L_{μ}, and L_{τ}. The flavor neutrinos ν_{e}, ν_{μ}, and ν_{τ} are particles that take part in the standard weak interaction. For example, a neutrino produced together with μ^+ in the decay $\pi^+ \to \mu^+ + \nu_{\mu}$ is the muon neutrino ν_{μ}, the electron antineutrino $\bar{\nu}_{\mathrm{e}}$ produces e^+ in the process $\bar{\nu}_{\mathrm{e}} + \mathrm{p} \to \mathrm{e}^+ + \mathrm{n}$, etc.

If there is neutrino mixing

$$\nu_{\alpha,\mathrm{L}} = \sum_i U_{\alpha i} \nu_{i,\mathrm{L}}\,, \tag{22}$$

then the vector of the state of the flavor neutrino is given by the expression

$$|\nu_{\alpha}\rangle = \sum_i U_{\alpha i}^* |\nu_i\rangle\,, \tag{23}$$

where $|\nu_i\rangle$ is the vector of the state of a neutrino with mass m_i.

[2] It is obvious that such a mechanism does not exist for charged particles.

The probability of the transition $\nu_\alpha \to \nu_{\alpha'}$ (indices α and α' take the values $e, \mu, \tau, s_1, \ldots$, where s denotes a sterile neutrino) can be presented in the form[3]

$$P(\nu_\alpha \to \nu_{\alpha'}) = \left| \delta_{\alpha'\alpha} + \sum_i U_{\alpha' i} U^*_{\alpha i} \left[\exp\left(-\mathrm{i}\Delta m^2_{i1} \frac{L}{2E} \right) - 1 \right] \right|^2, \tag{24}$$

where $L \simeq t$ is the distance between the neutrino source and the neutrino detector, E is the neutrino energy, and $\Delta m^2_{i1} = m^2_i - m^2_1$.

Analogously, for the probability of the transition $\bar\nu_\alpha \to \bar\nu_{\alpha'}$, we have

$$P(\bar\nu_\alpha \to \bar\nu_{\alpha'}) = \left| \delta_{\alpha'\alpha} + \sum_i U^*_{\alpha' i} U_{\alpha i} \left[\exp\left(-\mathrm{i}\Delta m^2_{i1} \frac{L}{2E} \right) - 1 \right] \right|^2. \tag{25}$$

Let us mention the following general properties of the transition probabilities:

- Transition probabilities depend on L/E;
- Neutrino oscillations can be observed, if the condition

$$\Delta m^2_{i1} \frac{L}{E} \gtrsim 1$$

is satisfied for at least one value of i;

- From the comparison of Eqns (24) and (25) we conclude that the following relation

$$P(\nu_\alpha \to \nu_{\alpha'}) = P(\bar\nu_{\alpha'} \to \bar\nu_\alpha)$$

holds true. This relation is the consequence of the CPT invariance intrinsic in any local quantum field theory;

- CP conservation in the lepton sector means that the mixing matrix U is real in the case of Dirac neutrinos. For the Majorana neutrinos, the mixing matrix satisfies the condition [20]

$$U_{\alpha i} = U^*_{\alpha i} \eta_i, \tag{26}$$

where $\eta_i = \pm \mathrm{i}$ is the CP parity of the Majorana neutrino ν_i. From expressions (24)–(26) we conclude that in the case of CP invariance in the lepton sector we have the following relationship

$$P(\nu_\alpha \to \nu_{\alpha'}) = P(\bar\nu_\alpha \to \bar\nu_{\alpha'}).$$

We have considered neutrino oscillations in a vacuum. The amplitudes of the elastic $\nu_e - e$ and $\nu_{\mu,\tau} - e$ scattering in matter are different. Therefore, refraction indices for ν_e and $\nu_{\mu,\tau}$ in matter are dissimilar. The strong (exponential) dependence of the electron density in the Sun on the radius leads to important matter effects for solar neutrinos [21, 22]. For matter effects see reviews [23, 16, 24].

4. Oscillations between two types of neutrinos in a vacuum

We will consider here the simplest case of transitions between two types of neutrinos ($\nu_\mu \to \nu_\tau$ or $\nu_\mu \to \nu_e$, and so forth). In this case, the index i in Eqn (24) takes only one value ($i = 2$) and for the transition probability we have

$$P(\nu_\alpha \to \nu_{\alpha'}) = \left| \delta_{\alpha'\alpha} + U_{\alpha' 2} U^*_{\alpha 2} \left[\exp\left(-\mathrm{i}\Delta m^2 \frac{L}{2E} \right) - 1 \right] \right|^2, \tag{27}$$

where $\Delta m^2 = m^2_2 - m^2_1$.

From Eqn (27), for the neutrino appearance probability ($\alpha' \neq \alpha$), we obtain the expression

$$P(\nu_\alpha \to \nu_{\alpha'}) = \frac{1}{2} A_{\alpha',\alpha} \left[1 - \cos\left(\Delta m^2 \frac{L}{2E} \right) \right], \tag{28}$$

where the amplitude $A_{\alpha',\alpha}$ is given by the relationship

$$A_{\alpha',\alpha} = 4 |U_{\alpha' 2}|^2 |U_{\alpha 2}|^2 = A_{\alpha,\alpha'}.$$

Let us introduce the mixing angle θ:

$$|U_{\alpha 2}|^2 = \sin^2\theta, \qquad |U_{\alpha' 2}|^2 = 1 - |U_{\alpha 2}|^2 = \cos^2\theta.$$

For the amplitude $A_{\alpha',\alpha}$, we obtain

$$A_{\alpha',\alpha} = \sin^2 2\theta.$$

Hence, the two-neutrino transition probability takes the standard form

$$P(\nu_\alpha \to \nu_{\alpha'}) = \frac{1}{2} \sin^2 2\theta \left[1 - \cos\left(\Delta m^2 \frac{L}{2E} \right) \right], \quad \alpha' \neq \alpha. \tag{29}$$

It is obvious that in the two-neutrino case the following relations are valid:

$$P(\nu_\alpha \to \nu_{\alpha'}) = P(\nu_{\alpha'} \to \nu_\alpha) = P(\bar\nu_\alpha \to \bar\nu_{\alpha'}). \tag{30}$$

Thus, the CP violation in the lepton sector cannot be revealed in the transitions between two types of neutrinos.

The survival probability $P(\nu_\alpha \to \nu_\alpha)$ is determined by the condition of the conservation of the probability. We then obtain

$$\begin{aligned} P(\nu_\alpha \to \nu_\alpha) &= 1 - P(\nu_\alpha \to \nu_{\alpha'}) \\ &= 1 - \frac{1}{2} \sin^2 2\theta \left[1 - \cos\left(\Delta m^2 \frac{L}{2E} \right) \right]. \end{aligned} \tag{31}$$

From Eqns (30) and (31) it follows that the two-neutrino survival probabilities satisfy the following relation

$$P(\nu_\alpha \to \nu_\alpha) = P(\nu_{\alpha'} \to \nu_{\alpha'}). \tag{32}$$

Thus, in the case of the transition between two types of neutrinos all transition probabilities are characterized by the two oscillation parameters: $\sin^2 2\theta$ and Δm^2.

Expressions (29) and (31) describe periodical transitions between two types of neutrinos (neutrino oscillations). They are widely used in the analysis of experimental data[4]. The expression (29) for the two-neutrino transition probability

[3] We label neutrino masses in such a way that $m_1 < m_2 < m_3 < \ldots$

[4] As we will see later, in the leading approximation neutrino oscillations are described by the two-neutrino formulas in the case of three-neutrino mixing.

can be presented in the form

$$P(\nu_\alpha \to \nu_{\alpha'}) = \frac{1}{2}\sin^2 2\theta\left[1 - \cos\left(2\pi\,\frac{L}{L_0}\right)\right], \qquad (33)$$

where

$$L_0 = 4\pi\,\frac{E}{\Delta m^2} \qquad (34)$$

is the oscillation length.

Finally, the two-neutrino transition probability and the oscillation length can be written down as

$$P(\nu_\alpha \to \nu_{\alpha'}) = \frac{1}{2}\sin^2 2\theta\left[1 - \cos\left(2.53\Delta m^2\,\frac{L}{E}\right)\right], \qquad (35)$$

$$L_0 \simeq 2.48\,\frac{E}{\Delta m^2}, \qquad (36)$$

where E is the neutrino energy in MeV, L is the distance in m, and Δm^2 is the neutrino mass-squared difference in eV^2.

5. Neutrino oscillation data

5.1 Evidence of oscillations of atmospheric neutrinos

Atmospheric neutrinos are produced mainly in the decays of pions and muons:

$$\pi \to \mu + \nu_\mu, \qquad \mu \to e + \nu_\mu + \nu_e. \qquad (37)$$

Pions themselves are produced in the processes of the interaction of cosmic rays in the atmosphere. In the Super-Kamiokande (S-K) experiment [1], neutrinos are detected via the observation of the Cherenkov radiation emitted by electrons and muons in a large water Cherenkov detector (50 kt of H_2O).

At energies smaller than about 1 GeV, practically all muons decay in the atmosphere and from reactions (37) it follows that $R_{\mu/e} \simeq 2$, where $R_{\mu/e}$ is the ratio between the numbers of muon and electron events. At higher energies, the ratio $R_{\mu/e}$ is larger than two. It can be predicted, however, with an accuracy of better than 5%.

The ratio $R_{\mu/e}$ measured in the S-K [1] and Soudan 2 [2] atmospheric neutrino experiments is significantly smaller than the predicted ratio $(R_{\mu/e})_{MC}$. In the S-K experiment, the following ratios between measured and predicted $R_{\mu/e}$ were obtained in the Sub-GeV ($E_{vis} \leqslant 1.33$ GeV) and multi-GeV ($E_{vis} > 1.33$ GeV) regions, respectively:

$$\frac{(R_{\mu/e})_{meas}}{(R_{\mu/e})_{MC}} = 0.638 \pm 0.016 \pm 0.050,$$

$$\frac{(R_{\mu/e})_{meas}}{(R_{\mu/e})_{MC}} = 0.658 \pm 0.030 \pm 0.078.$$

The fact that the ratio $(R_{\mu/e})_{meas}$ is significantly smaller than the predicted ratio was known from the results of the previous atmospheric neutrino experiments Kamiokande [25] and IMB [26]. For many years this 'atmospheric neutrino anomaly' was considered an indication in favor of neutrino oscillations.

Compelling evidence in favor of neutrino oscillations was obtained recently by the S-K collaboration [1] from observation of the large up–down asymmetry of high-energy atmospheric muon events.

If there are no neutrino oscillations, the following relation is valid for the number of electron (muon) events:

$$N_l(\cos\theta_z) = N_l(-\cos\theta_z), \qquad l = e, \mu, \qquad (38)$$

where θ_z is the azimuth angle. For electron events, S-K data are in good agreement with this relation.

For multi-GeV muon events, the significant violation of relation (38) was observed. For the ratio of the number of the upward muons U_μ ($-1 \leqslant \cos\theta_z \leqslant -0.2$) to the number of the downward muons D_μ ($0.2 \leqslant \cos\theta_z \leqslant 1$), the value of

$$\frac{U_\mu}{D_\mu} = 0.54 \pm 0.04 \pm 0.01$$

was found. At high energies, leptons are emitted practically in the direction of neutrinos. Thus, upward muons are produced by neutrinos which travel distances from $\simeq 500$ to $\simeq 13{,}000$ km, and downward muons are produced by neutrinos which travel distances from $\simeq 20$ to $\simeq 500$ km. The observation of the up–down asymmetry clearly demonstrates the dependence of the number of muon neutrinos on the distance which they travel from the production point in the atmosphere to the detector.

The S-K data [1] and data from other atmospheric neutrino experiments (Soudan 2 [2], MACRO [3]) are well described, if we assume that two-neutrino $\nu_\mu \to \nu_\tau$ oscillations take place. From the analysis of the S-K data, it was found [1] that at 90%C.L. two-neutrino oscillation parameters Δm^2_{atm} and $\sin^2 2\theta_{atm}$ [see Eqn (29)] lie in the ranges

$$1.6 \times 10^{-3}\ eV^2 \leqslant \Delta m^2_{atm} \leqslant 3.9 \times 10^{-3}\ eV^2,$$
$$\sin^2 2\theta_{atm} > 0.92.$$

The best-fit values of the parameters are equal to

$$\Delta m^2_{atm} = 2.5 \times 10^{-3}\ eV^2,$$
$$\sin^2 2\theta_{atm} = 1.0, \qquad \chi^2_{min} = 163.2/170\,\text{d.o.f.} \qquad (39)$$

5.2 Evidence of transitions of solar ν_e into $\nu_{\mu,\tau}$

The energy of the Sun is released in the reactions of the thermonuclear pp and CNO cycles, in which protons and electrons are converted into helium atoms and electron neutrinos:

$$4p + 2e^- \to {}^4He + 2\nu_e.$$

The most important reactions for the solar neutrino experiments are listed in Table 1.

Table 1. The main sources of the solar neutrinos. The maximum neutrino energies and SSM BP00 neutrino fluxes are also given.

Reaction	Neutrino energy	Flux ν_e (SSM BP00)
$pp \to de^+\nu_e$	$\leqslant 0.42$ MeV	5.95×10^{10} cm^{-2} s^{-1}
$e^- + {}^7Be \to \nu_e {}^7Li$	0.86 MeV	4.77×10^{9} cm^{-2} s^{-1}
${}^8B \to {}^8Be^* e^+\nu_e$	$\leqslant 15$ MeV	5.05×10^{6} cm^{-2} s^{-1}

As seen from the table, low-energy pp-neutrinos make up the main fraction of solar neutrino flux. According to the SSM BP00 [27], medium-energy monoenergetic ^{7}Be-neutrinos constitute about 10% of the total flux. High-energy ^{8}B-neutrinos comprise only about 10^{-2}% of the total flux.

However, in the S-K [7] and SNO [8–10] experiments, due to high-energy thresholds, practically only neutrinos from ^{8}B-decay can be detected[5]. ^{8}B-neutrinos make the dominant contribution to the event rate, measured in the Chlorine Homestake experiment [4].

The event rates measured in all solar neutrino experiments turned out to be significantly smaller than the event rates predicted by the Standard Solar models. For the ratio R between the rates observed in the Homestake [4], GALLEX–GNO [5], SAGE [6], and S-K [7] experiments and those predicted by SSM BP00 [27], the following values were obtained[6]:

$$
\begin{aligned}
&R = 0.34 \pm 0.03 \quad \text{(Homestake)}\\
&\nu_e + {}^{37}\mathrm{Cl} \to e^- + {}^{37}\mathrm{Ar}\,,\\
&R = 0.58 \pm 0.05 \quad \text{(GALLEX–GNO)}\\
&\nu_e + {}^{71}\mathrm{Ga} \to e^- + {}^{71}\mathrm{Ge}\,,\\
&R = 0.60 \pm 0.05 \quad \text{(SAGE)}\\
&\nu_e + {}^{71}\mathrm{Ga} \to e^- + {}^{71}\mathrm{Ge}\,,\\
&R = 0.465 \pm 0.018 \quad \text{(S-K)}\\
&\nu_x + e \to \nu_x + e\,.
\end{aligned}
$$

If there occurs neutrino mixing, original solar ν_e's due to neutrino oscillations or matter MSW transitions are transferred into another type of neutrino, which cannot be detected in the radiochemical Homestake, GALLEX–GNO, or SAGE experiments. In the S-K experiment, mainly ν_e are detected: the sensitivity of the experiment to ν_μ and ν_τ is about six times smaller than the sensitivity to ν_e. Thus, neutrino oscillations or MSW transitions in matter provide a natural explanation of the depletion of the solar ν_e fluxes.

Recently, strong model-independent evidence in favor of the transition of the solar ν_e into ν_μ and ν_τ has been produced in the SNO experiment [8–10]. The neutrino detector in the SNO experiment is a heavy water Cherenkov detector (1 kt of D_2O). Neutrinos from the Sun are detected via the observation of the following three reactions[7]

(1) CC reaction

$$\nu_e + d \to e^- + p + p\,, \tag{40}$$

(2) NC reaction

$$\nu_x + d \to \nu_x + n + p\,, \tag{41}$$

(3) neutrino–electron elastic scattering (ES)

$$\nu_x + e \to \nu_x + e\,. \tag{42}$$

Over the course of 306.4 days, $1967^{+61.9}_{-60.9}$ CC events, $576.5^{+49.5}_{-48.9}$ NC events, and $263.6^{+26.4}_{-25.6}$ ES events were recorded in the SNO experiment. The kinetic energy threshold for the detection of the electrons is equal to 5 MeV. The NC threshold is equal to 2.2 MeV. Thus, practically only neutrinos from ^{8}B-decay were detected in the SNO experiment. It is important for the interpretation of the results of the experiment that the initial spectrum of electron neutrinos from the decay $^8\mathrm{B} \to {}^8\mathrm{Be} + e^+ + \nu_e$ is known [28].

The total CC event rate is given by the relationship

$$R^{CC}_{\nu_e} = \langle \sigma^{CC}_{\nu_e d} \rangle \Phi^{CC}_{\nu_e}\,, \tag{43}$$

where $\langle \sigma^{CC}_{\nu_e d} \rangle$ is the cross section of the CC process (40), averaged over the known initial spectrum of ^{8}B-neutrinos, and $\Phi^{CC}_{\nu_e}$ is the flux of ν_e on the Earth. The flux $\Phi^{CC}_{\nu_e}$ is given by the relation

$$\Phi^{CC}_{\nu_e} = \big\langle P(\nu_e \to \nu_e) \big\rangle_{CC} \Phi^0_{\nu_e}\,, \tag{44}$$

where $\Phi^0_{\nu_e}$ is the total initial flux of ν_e, and $\big\langle P(\nu_e \to \nu_e) \big\rangle_{CC}$ is the averaged ν_e survival probability.

All flavor neutrinos ν_e, ν_μ, and ν_τ are recorded via the detection of the NC process (41). Taking into account $\nu_e - \nu_\mu - \nu_\tau$ universality of the NC, for the total NC event rate we obtain

$$R^{NC}_{\nu} = \langle \sigma^{NC}_{\nu d} \rangle \Phi^{NC}_{\nu}\,, \tag{45}$$

where $\langle \sigma^{NC}_{\nu d} \rangle$ is the cross section of the process (41), averaged over the initial spectrum of the ^{8}B-neutrinos, and Φ^{NC}_{ν} is the total flux of all flavor neutrinos on the Earth. We have for the latter quantity:

$$\Phi^{NC}_{\nu} = \sum_{l = e, \mu, \tau} \Phi^{NC}_{\nu_l}\,. \tag{46}$$

Here, the flux $\Phi^{NC}_{\nu_l}$ is given by the relation

$$\Phi^{NC}_{\nu_l} = \big\langle P(\nu_e \to \nu_l) \big\rangle_{NC} \Phi^0_{\nu_e}\,, \tag{47}$$

where $\big\langle P(\nu_e \to \nu_l) \big\rangle_{NC}$ is the averaged probability of the transition $\nu_e \to \nu_l$.

All flavor neutrinos are also detected via the observation of the ES process (42). However, the cross section of the (NC) $\nu_{\mu,\tau} + e \to \nu_{\mu,\tau} + e$ scattering is about six times smaller than the cross section of the (CC + NC) $\nu_e + e \to \nu_e + e$ scattering. The total ES event rate can be presented in the form

$$R^{ES}_{\nu} = \langle \sigma_{\nu_e e} \rangle \Phi^{ES}_{\nu}\,. \tag{48}$$

Here, $\langle \sigma_{\nu_e e} \rangle$ is the cross section of the process $\nu_e e \to \nu_e e$, averaged over the initial spectrum of the ^{8}B-neutrinos, and

$$\Phi^{ES}_{\nu} = \Phi^{ES}_{\nu_e} + \frac{\langle \sigma_{\nu_\mu e} \rangle}{\langle \sigma_{\nu_e e} \rangle} \Phi^{ES}_{\nu_{\mu,\tau}}\,, \tag{49}$$

where $\Phi^{ES}_{\nu_e}$ is the flux of ν_e, $\Phi^{ES}_{\nu_{\mu,\tau}}$ is the flux of ν_μ and ν_τ, and

$$\frac{\langle \sigma_{\nu_\mu e} \rangle}{\langle \sigma_{\nu_e e} \rangle} \simeq 0.154\,, \tag{50}$$

$$\Phi^{ES}_{\nu_l} = \big\langle P(\nu_e \to \nu_l) \big\rangle_{ES} \Phi^0_{\nu_e}\,, \tag{51}$$

$\big\langle P(\nu_e \to \nu_l) \big\rangle_{ES}$ is the averaged probability of the transition $\nu_e \to \nu_l$.

In the SNO experiment, the value of

$$(\Phi^{ES}_{\nu})_{SNO} = \big[2.39^{+0.24}_{-0.23}\ \text{(stat.)} \pm 0.12\ \text{(syst.)}\big] \times 10^6\ \text{cm}^{-2}\,\text{s}^{-1} \tag{52}$$

was obtained [10]. This value is in good agreement with the S-K finding. In the S-K experiment [7], solar neutrinos were

[5] According to the SSM BP00, the flux of high-energy *hep*-neutrinos produced in the reaction $^3\mathrm{He} + p \to {}^4\mathrm{He} + e^+ + \nu_e$ is about three orders of magnitude smaller than the flux of the ^{8}B-neutrinos.

[6] The neutrino detection reactions are also given.

[7] ν_x stands for *any* flavor neutrino.

detected via the observation of the ES process $\nu_x e \to \nu_x e$. Over a period of 1496 days, a large number (22,400 $\pm$ 800) of solar neutrino events with a recoil total energy threshold of 5 MeV were recorded.

From the data of the S-K experiment it was obtained that

$$(\Phi^{\rm ES}_{\nu_e})_{\text{S-K}} = \left[2.35 \pm 0.02\,(\text{stat.}) \pm 0.08\,(\text{syst.})\right] \times 10^6\ \text{cm}^{-2}\,\text{s}^{-1}. \tag{53}$$

In the S-K experiment, the spectrum of the recoil electrons was measured. No sizable distortion of the spectrum in comparison to the expected spectrum was observed. The spectrum of electrons produced in the CC process (40) was measured in the SNO experiment [10]. No distortion of the electron spectrum was observed in this experiment either [8].

Thus, the data of the S-K and SNO experiments are consistent with the assumption that in the high-energy ^{8}B-neutrino region the probability of the solar neutrino surviving is a constant:

$$P(\nu_e \to \nu_e) \simeq \text{const}\,. \tag{54}$$

From expression (54) it follows that

$$\left\langle P(\nu_e \to \nu_e)\right\rangle_{\rm CC} \simeq \left\langle P(\nu_e \to \nu_e)\right\rangle_{\rm NC} \simeq \left\langle P(\nu_e \to \nu_e)\right\rangle_{\rm ES}\,.$$

Taking into account these relations, from Eqns (44), (47), and (51) it follows that in the high-energy ^{8}B-neutrino region the fluxes of electron neutrinos, detected via the observation of CC, NC and ES processes, are the same:

$$\Phi^{\rm CC}_{\nu_e} \simeq \Phi^{\rm NC}_{\nu_e} \simeq \Phi^{\rm ES}_{\nu_e}\,. \tag{55}$$

From the data of the SNO experiment [10], it was obtained that the flux of ν_e on the Earth is equal to

$$(\Phi^{\rm CC}_{\nu_e})_{\rm SNO} = \left[1.76^{+0.06}_{-0.05}\,(\text{stat.})^{+0.09}_{-0.09}\,(\text{syst.})\right] \times 10^6\ \text{cm}^{-2}\,\text{s}^{-1}\,. \tag{56}$$

For the flux of all flavor neutrinos $\Phi^{\rm NC}_{\nu}$, the value

$$(\Phi^{\rm NC}_{\nu})_{\rm SNO} = \left[5.09^{+0.44}_{-0.43}\,(\text{stat.})^{+0.46}_{-0.43}\,(\text{syst.})\right] \times 10^6\ \text{cm}^{-2}\,\text{s}^{-1} \tag{57}$$

was found, which is about three times larger than the value of the flux of electron neutrinos. It is obvious that the NC flux $\Phi^{\rm NC}_{\nu}$ is given by

$$\Phi^{\rm NC}_{\nu} = \Phi^{\rm NC}_{\nu_e} + \Phi^{\rm NC}_{\nu_{\mu,\tau}}\,, \tag{58}$$

where $\Phi^{\rm NC}_{\nu_e}$ is the flux of electron neutrinos ν_e, and $\Phi^{\rm NC}_{\nu_{\mu,\tau}}$ is the flux of ν_μ and ν_τ.

Combining CC and NC fluxes and using the relation (55), we can determine now the flux $\Phi^{\rm NC}_{\nu_{\mu,\tau}}$. Taking into account also the value (52) of the ES flux, for the flux of ν_μ and ν_τ on the Earth, the value

$$(\Phi_{\nu_{\mu,\tau}})_{\rm SNO} = \left[3.41^{+0.45}_{-0.45}\,(\text{stat.})^{+0.48}_{-0.45}\,(\text{syst.})\right] \times 10^6\ \text{cm}^{-2}\,\text{s}^{-1} \tag{59}$$

was obtained in Refs [10, 9]. Thus, detection of solar neutrinos via the simultaneous observation of CC, NC, and ES processes allowed the SNO collaboration to obtain *the direct model-independent evidence of the presence of* ν_μ *and* ν_τ *in the flux of the solar neutrinos on the Earth at the level of* 5.3σ.

The total flux of the ^{8}B-neutrinos, predicted by SSM BP00 [27], is equal to

$$(\Phi^0_{\nu_e})_{\rm SSM\,BP00} = (5.05^{+1.01}_{-0.81}) \times 10^6\ \text{cm}^{-2}\,\text{s}^{-1}\,. \tag{60}$$

This flux is consistent with the total flux of all flavor neutrinos (57), measured in the SNO experiment.

The flux of ν_μ and ν_τ on the Earth can also be obtained from the SNO CC data and the S-K ES data. In the first SNO publication [8], the value

$$(\Phi_{\nu_{\mu,\tau}})_{\text{S-K, SNO}} = (3.69 \pm 1.13) \times 10^6\ \text{cm}^{-2}\,\text{s}^{-1} \tag{61}$$

was found, which is in a good agreement with the value (59).

The data of all solar neutrino experiments can be described if we assume that there are transitions of the solar ν_e neutrinos into $\nu_{\mu,\tau}$ and ν_e survival probability is given by the two-neutrino expression, which is characterized by the two oscillation parameters $\Delta m^2_{\rm sol}$ and $\tan^2\theta_{\rm sol}$. From the global χ^2 fit of the total event rates measured in all solar neutrino experiments, several allowed regions in the plane of the oscillation parameters were obtained (see, for example, Ref. [29]): large mixing angle MSW LMA and LOW regions, small mixing angle MSW SMA region, vacuum oscillations VO region, and others. The situation changed after the day and night recoil electron spectra were measured in the S-K experiment [7], and SNO results [8–10] were obtained. From all analyses of the existing solar neutrino data it is evident that the most plausible allowed region is the MSW LMA region (see Ref. [30] and references cited therein).

In Ref. [10], $\Delta m^2_{\rm sol}$, $\tan^2\theta_{\rm sol}$, and the initial flux $\Phi^0_{\nu_e}$ of the ^{8}B-neutrinos were considered as free variable parameters. From the analysis of all solar neutrino data, the following best-fit values of the parameters were found ($\chi^2_{\rm min} = 57/72$ d.o.f.):

$$\Delta m^2_{\rm sol} = 5 \times 10^{-5}\ \text{eV}^2\,, \quad \tan^2\theta_{\rm sol} = 0.34\,, \qquad \Phi^0_{\nu_e} = 5.89 \times 10^6\ \text{cm}^{-2}\,\text{s}^{-1}\,. \tag{62}$$

If neutrino oscillation parameters lie in the LMA region, neutrino oscillations in the solar range of Δm^2can be explored in experiments with reactor $\bar\nu_e$ if the distance between the reactor and detector is about 100 km. In the KamLAND experiment [31], which started in January 2002, $\bar\nu_e$ from several Japanese reactors are recorded by a large liquid scintillator detector (1 kt of liquid scintillator). The distance between reactors and the detector equals 175 ± 35 km. The average energy of $\bar\nu_e$ from a reactor is about 3 MeV. Thus, at large mixing angles the KamLAND experiment is sensitive to the solar LMA range of the neutrino mass-squared difference: $\Delta m^2 \simeq E/L \simeq 2 \times 10^{-5}$ eV^2.

5.3 Reactor experiments CHOOZ and Palo Verde

The results of the long baseline reactor experiments CHOOZ [11] and Palo Verde [12] are very important for studying the neutrino mixing. In these experiments, the disappearance of the reactor $\bar\nu_e$ in the atmospheric range of Δm^2 was searched for.

[8] Expected spectra were calculated under the assumption that the shape of the spectrum of ν_e on the Earth is given by the known initial ^{8}B-neutrino spectrum.

In the CHOOZ experiment, $\bar{\nu}_e$ from two reactors located at a distance of about 1 km from the detector were detected via the observation of the process

$$\bar{\nu}_e + p \to e^+ + n\,.$$

No indication in favor of the disappearance of $\bar{\nu}_e$ was found in the experiment. For the ratio R of the total number of detected $\bar{\nu}_e$ events to the expected number, the following value

$$R = 1.01 \pm 2.8\%\,(\text{stat.}) \pm 2.7\%\,(\text{syst.})\ (\text{CHOOZ})$$

was found. In the similar Palo Verde experiment, it was discovered that

$$R = 1.01 \pm 2.4\%\,(\text{stat.}) \pm 5.3\%\,(\text{syst.})\ (\text{Palo Verde})\,.$$

The data from the above experiments were analyzed in Refs [11, 12] in the framework of two-neutrino oscillations, and exclusion plots in the plane of the oscillation parameters Δm^2 and $\sin^2 2\theta$ were obtained. From the CHOOZ exclusion plot at $\Delta m^2 = 2.5 \times 10^{-3}$ eV^2 (the S-K best-fit value), we have the following upper bound:

$$\sin^2 2\theta \lesssim 1.5 \times 10^{-1}\,.$$

6. Neutrino oscillations in the framework of three-neutrino mixing

6.1 Neutrino oscillations in the atmospheric range of the neutrino mass-squared difference

We have discussed satisfactory evidence in favor of neutrino oscillations that was obtained in the solar and atmospheric neutrino experiments. There also exists at present an indication in favor of the transitions $\bar{\nu}_\mu \to \bar{\nu}_e$, which was given in the single accelerator experiment LSND [17]. The LSND data can be explained by neutrino oscillations. From the analysis of the data for the values of the oscillation parameters, the following ranges

$$2 \times 10^{-1}\ \text{eV}^2 \lesssim \Delta m^2 \lesssim 1\ \text{eV}^2\,,$$
$$3 \times 10^{-3} \lesssim \sin^2 2\theta \lesssim 4 \times 10^{-2}$$

were obtained.

In order to describe the data of the solar, atmospheric, and LSND experiments, which require three different values of neutrino mass-squared differences, it is necessary to assume that (at least) four massive and mixed neutrinos exist (see, for example, Ref. [16]). However, the result of the LSND experiment requires confirmation. The MiniBooNE experiment at Fermilab [32], which started in 2002, aims to check the LSND result.

We will consider here the *minimal* scheme of three-neutrino mixing

$$\nu_{\alpha,\mathrm{L}} = \sum_{i=1}^{3} U_{\alpha i} \nu_{i,\mathrm{L}}\,, \tag{63}$$

where U is the unitary 3×3 PMNS mixing matrix [33, 34]. This scheme provides two independent Δm^2's and allows us to describe solar and atmospheric neutrino oscillation data.

Let us start with consideration of the neutrino oscillations in the atmospheric range of Δm^2, which can be explored in atmospheric and long baseline accelerator (reactor) neutrino experiments. In the framework of the three-neutrino mixing (63) with $m_1 < m_2 < m_3$ and $\Delta m^2_{ik} = m_i^2 - m_k^2$ there are two possibilities:

I. Hierarchy of neutrino mass-squared differences:

$$\Delta m^2_{21} \simeq \Delta m^2_{\text{sol}}\,,\quad \Delta m^2_{32} \simeq \Delta m^2_{\text{atm}}\,,\quad \Delta m^2_{21} \ll \Delta m^2_{32}\,; \tag{64}$$

II. Inverted hierarchy of neutrino mass-squared differences:

$$\Delta m^2_{32} \simeq \Delta m^2_{\text{sol}}\,,\quad \Delta m^2_{21} \simeq \Delta m^2_{\text{atm}}\,,\quad \Delta m^2_{32} \ll \Delta m^2_{21}\,. \tag{65}$$

We will first assume that the neutrino mass spectrum is of the type I. For the values of L/E relevant for neutrino oscillations in the atmospheric range of neutrino mass-squared difference ($\Delta m^2_{32} L/E \gtrsim 1$), we have the inequality

$$\Delta m^2_{21}\,\frac{L}{E} \ll 1\,.$$

We can disregard the contribution of Δm^2_{21} to the transition probability (24). For the probability of the transition $\nu_\alpha \to \nu_{\alpha'}$, we obtain in this case the following expression

$$P(\nu_\alpha \to \nu_{\alpha'}) \simeq \left|\delta_{\alpha'\alpha} + U_{\alpha'3} U^*_{\alpha3}\left[\exp\left(-\mathrm{i}\Delta m^2_{32}\,\frac{L}{2E}\right) - 1\right]\right|^2\,. \tag{66}$$

Hence, in the leading approximation the transition probabilities in the atmospheric range of Δm^2 are determined by the largest neutrino mass-squared difference Δm^2_{32} and the elements of the third column of the neutrino mixing matrix, which connect flavor neutrino fields $\nu_{\alpha\mathrm{L}}$ with the field of the heaviest neutrino $\nu_{3\mathrm{L}}$.

For the neutrino appearance probability, from formula (66) we obtain

$$P(\nu_\alpha \to \nu_{\alpha'}) = \frac{1}{2}\,A_{\alpha',\alpha}\left[1 - \cos\left(\Delta m^2_{32}\,\frac{L}{2E}\right)\right],\quad \alpha \neq \alpha'\,, \tag{67}$$

where the oscillation amplitude is given by the expression

$$A_{\alpha',\alpha} = 4|U_{\alpha'3}|^2 |U_{\alpha3}|^2\,. \tag{68}$$

The survival probability can be obtained from the condition of the conservation of probability and Eqn (67). Following this procedure we arrive at

$$P(\nu_\alpha \to \nu_\alpha) = 1 - \sum_{\alpha' \neq \alpha} P(\nu_\alpha \to \nu_{\alpha'})$$
$$= 1 - \frac{1}{2}\,B_{\alpha,\alpha}\left[1 - \cos\left(\Delta m^2_{32}\,\frac{L}{2E}\right)\right]. \tag{69}$$

Taking into account the unitarity of the mixing matrix, for the oscillation amplitude $B_{\alpha,\alpha}$ we have

$$B_{\alpha,\alpha} = \sum_{\alpha' \neq \alpha} A_{\alpha',\alpha} = 4|U_{\alpha3}|^2\left(1 - |U_{\alpha3}|^2\right)\,. \tag{70}$$

Let us note that in the case of the inverted hierarchy of the neutrino mass-squared differences, transition probabilities can be obtained from Eqns (67)–(70) by the changes $\Delta m^2_{32} \to \Delta m^2_{21}$ and $|U_{\alpha3}|^2 \to |U_{\alpha1}|^2$.

Transition probabilities (67) and (69) depend only on $|U_{\alpha 3}|^2$ and Δm_{32}^2. The CP phase does not enter into expressions for the transition probabilities. This means that in the leading approximation the relation

$$P(\nu_\alpha \to \nu_{\alpha'}) = P(\bar{\nu}_\alpha \to \bar{\nu}_{\alpha'}) \qquad (71)$$

is automatically satisfied. Thus, investigation of effects of the CP violation in the lepton sector in future long baseline neutrino oscillation experiments will be a difficult problem: possible effects are suppressed due to the smallness of the parameter $\Delta m_{12}^2/\Delta m_{32}^2$. High precision experiments on the search for effects of the CP violation in the lepton sector are planned for future Neutrino Superbeam facilities [35] and neutrino factories [36, 37].

Transition probabilities (67) and (69) take a *two-neutrino form* in every channel. This is the obvious consequence of the fact that only the largest mass-squared difference Δm_{32}^2 contributes to the transition probabilities.

The elements $|U_{\alpha 3}|^2$, which determine the oscillation amplitudes, satisfy the unitarity condition

$$\sum_\alpha |U_{\alpha 3}|^2 = 1\,.$$

Hence, in the leading approximation transition probabilities are characterized by three parameters. In the standard parametrization of the neutrino mixing matrix (see Ref. [13]), we have

$$\begin{aligned} U_{\mu 3} &= \left(1 - |U_{e3}|^2\right)^{1/2} \sin\theta_{23}\,, \\ U_{\tau 3} &= \left(1 - |U_{e3}|^2\right)^{1/2} \cos\theta_{23}\,, \end{aligned} \qquad (72)$$

where θ_{23} is the mixing angle.

From formulas (68) and (72), we will obtain for the respective amplitudes of the transitions $\nu_\mu \to \nu_\tau$ and $\nu_\mu \to \nu_e$:

$$\begin{aligned} A_{\tau,\mu} &= \left(1 - |U_{e3}|^2\right)^2 \sin^2 2\theta_{23}\,, \\ A_{e,\mu} &= 4|U_{e3}|^2\left(1 - |U_{e3}|^2\right) \sin^2\theta_{23}\,. \end{aligned} \qquad (73)$$

For the amplitude $B_{e,e}$, we have[9]

$$B_{e,e} = 4|U_{e3}|^2\left(1 - |U_{e3}|^2\right). \qquad (74)$$

In the S-K atmospheric neutrino experiment [1], no indications in favor of $\nu_\mu \to \nu_e$ transitions were obtained. The data of the experiment are well described under the assumption $|U_{e3}|^2 \simeq 0$. In this approximation, oscillations in the atmospheric range of Δm^2 are pure two-neutrino $\nu_\mu \to \nu_\tau$ oscillations. The best-fit values of the two-neutrino oscillation parameters $\Delta m_{\rm atm}^2 \simeq \Delta m_{32}^2$ and $\sin^2\theta_{\rm atm} \simeq \sin^2\theta_{23}$, obtained from the analysis of the S-K data, are given in Eqn (39).

6.2 Oscillations in the solar range of neutrino mass-squared difference

Let us now consider, in the framework of the three-neutrino mixing approach, neutrino oscillations in the solar range of Δm^2. The ν_e survival probability in a vacuum can be written in the form

$$\begin{aligned} P(\nu_e \to \nu_e) = \Bigg| \sum_{i=1,2} |U_{ei}|^2 \exp\left(-i\Delta m_{i1}^2 \frac{L}{2E}\right) \\ + |U_{e3}|^2 \exp\left(-i\Delta m_{31}^2 \frac{L}{2E}\right)\Bigg|^2. \end{aligned} \qquad (75)$$

We are interested in the survival probability averaged over the region where neutrinos are produced, over the neutrino energy spectrum, etc. Because of the hierarchy $\Delta m_{32}^2 \gg \Delta m_{21}^2$, the interference between the first and the second terms in the expression (75) for the averaged survival probability disappears. In this case, the averaged survival probability can be presented in the form

$$P(\nu_e \to \nu_e) = |U_{e3}|^4 + \left(1 - |U_{e3}|^2\right)^2 P^{(1,2)}(\nu_e \to \nu_e)\,, \qquad (76)$$

where $P^{(1,2)}(\nu_e \to \nu_e)$ is given by the expression

$$P^{(1,2)}(\nu_e \to \nu_e) = 1 - \frac{1}{2} A^{(1,2)}\left[1 - \cos\left(\Delta m_{21}^2 \frac{L}{2E}\right)\right]. \qquad (77)$$

For the amplitude $A^{(1,2)}$, we have the relationship

$$A^{(1,2)} = 4\,\frac{|U_{e1}|^2|U_{e2}|^2}{\left(1 - |U_{e3}|^2\right)^2}\,. \qquad (78)$$

In the standard parametrization of the neutrino mixing matrix, the elements $U_{e1,2}$ are given by formulas

$$U_{e1} = \left(1 - |U_{e3}|^2\right)^{1/2} \cos\theta_{12}\,, \quad U_{e2} = \left(1 - |U_{e3}|^2\right)^{1/2} \sin\theta_{12}\,, \qquad (79)$$

where θ_{12} is the mixing angle. From Eqns (78) and (79), we obtain the following expression for the amplitude $A^{(1,2)}$:

$$A^{(1,2)} = \sin^2 2\theta_{12}\,. \qquad (80)$$

Thus, the probability $P^{(1,2)}(\nu_e \to \nu_e)$ is characterized by two parameters and has the standard two-neutrino form.

Expression (76) is also valid in the presence of matter [38, 16]. In this case, $P^{(1,2)}(\nu_e \to \nu_e)$ is the two-neutrino ν_e survival probability in matter, calculated under the condition that the electron density $\rho_e(x)$ in the effective Hamiltonian of the interaction of a neutrino with matter is changed by $\left(1 - |U_{e3}|^2\right)\rho_e(x)$.

As we will see in the next section, from the data of the reactor CHOOZ and Palo Verde experiments it follows that the element $|U_{e3}|^2$ is small. If we neglect $|U_{e3}|^2$ in Eqn (76), we come to the conclusion that in the framework of three-neutrino mixing the ν_e survival probability in the solar range of neutrino mass-squared differences takes the two-neutrino form

$$P(\nu_e \to \nu_e) \simeq P^{(1,2)}(\nu_e \to \nu_e)\,. \qquad (81)$$

The values of the parameters $\Delta m_{\rm sol}^2 \simeq \Delta m_{21}^2$ and $\tan^2\theta_{\rm sol} \simeq \tan^2\theta_{12}$, obtained from the analysis of the solar neutrino data, are given in Eqn (62).

Thus, due to the smallness of the parameter $|U_{e3}|^2$and the hierarchy $\Delta m_{12}^2 \ll \Delta m_{32}^2$ of neutrino mass-squared differences, neutrino oscillations in the atmospheric and solar ranges of neutrino mass-squared differences are decoupled

[9] Notice the following relation between the oscillation amplitudes: $A_{e,\mu} = B_{e,e}\sin^2\theta_{23}$.

in the leading approximation [39] and are described by two-neutrino formulas, which are characterized by the oscillation parameters Δm^2_{32}, $\sin^2 2\theta_{23}$ and Δm^2_{21}, $\tan^2\theta_{12}$, respectively.

6.3 Upper bound on $|U_{e3}|^2$ parameter from the data of the CHOOZ experiment

The reactor long baseline CHOOZ [11] and Palo Verde [12] experiments are sensitive to the atmospheric range of Δm^2. No indication in favor of the disappearance of reactor $\bar{\nu}_e$ was obtained in these experiments. From the analysis of the data of the CHOOZ and Palo Verde experiments, the stringent restriction on the parameter $|U_{e3}|^2$ can be obtained.

In the framework of three-neutrino mixing, the probability of $\bar{\nu}_e$ surviving is given by the expression

$$P(\bar{\nu}_e \to \bar{\nu}_e) = 1 - \frac{1}{2}\, B_{e,e}\left[1 - \cos\left(\Delta m^2_{32}\,\frac{L}{2E}\right)\right], \tag{82}$$

where the amplitude $B_{e,e}$ is given by Eqn (74).

In Refs [11, 12], exclusion plots in the plane of the parameters $\Delta m^2 \equiv \Delta m^2_{32}$ and $\sin^2 2\theta \equiv B_{e,e}$ were found. From these exclusion plots we have

$$B_{e,e} \leqslant B^0_{e,e}\,, \tag{83}$$

where the upper bound $B^0_{e,e}$ depends on Δm^2_{32}. For the S-K allowed values of Δm^2_{32}, from the CHOOZ exclusion plot we find the restrictions

$$1\times 10^{-1} \leqslant B^0_{e,e} \leqslant 2.4\times 10^{-1}\,. \tag{84}$$

Using formulas (74) and (83), for the parameter $|U_{e3}|^2$ we set the bounds

$$|U_{e3}|^2 \leqslant \frac{1}{2}\left[1 - (1 - B^0_{e,e})^{1/2}\right] \lesssim \frac{1}{4}\, B^0_{e,e} \tag{85}$$

or

$$|U_{e3}|^2 \gtrsim \frac{1}{2}\left[1 + (1 - B^0_{e,e})^{1/2}\right] \geqslant 1 - \frac{1}{4}\, B^0_{e,e}\,. \tag{86}$$

Thus, parameter $|U_{e3}|^2$ can be small or large (close to one). The latter possibility is excluded by the solar neutrino data. Indeed, if $|U_{e3}|^2$ is large, from Eqn (76) it follows that in the whole range of the solar neutrino energies the probability of ν_e surviving is close to unity in obvious contradiction to the solar neutrino data. Hence, the upper bound on the parameter $|U_{e3}|^2$ is given by Eqn (85). For the S-K best-fit value of $\Delta m^2_{32} = 2.5\times 10^{-3}$ eV^2, we have

$$|U_{e3}|^2 \leqslant 4\times 10^{-2}\ (95\%\text{C.L.})\,. \tag{87}$$

7. Neutrino mass from the measurement of the high-energy part of the tritium β-spectrum

The method of measuring the neutrino mass via the detailed investigation of the high-energy part of the β-spectrum was proposed by E Fermi [40] and F Perrin [41] in 1933.

The standard process, which is investigated with the aim of measuring the neutrino mass, is the tritium decay

$$^3\text{H} \to {}^3\text{He} + e^- + \bar{\nu}_e\,. \tag{88}$$

This decay has several advantages. It shows up as the superallowed decay. Thus, the nuclear matrix element is a constant and the electron spectrum is determined by the phase space. The decay (88) exhibits a small energy release ($E_0 \simeq 18.6$ keV), a convenient lifetime ($T_{1/2} \simeq 12.3$ years), and so forth.

Let us consider the decay (88) in the case of neutrino mixing. The effective Hamiltonian of the process is given by

$$\mathcal{H}_I^{CC} = \frac{G_F}{\sqrt{2}}\, 2\bar{e}_L\gamma_\alpha\nu_{e,L}\, j^\alpha + \text{h.c.} \tag{89}$$

Here j^α is the hadronic charged current. Neutrino field $\nu_{e,L}$ is represented by

$$\nu_{e,L} = \sum_i U_{ei}\nu_{i,L}\,, \tag{90}$$

where ν_i is the field of a neutrino with mass m_i, and U is the unitary mixing matrix.

Neglecting the recoil of the final nucleus, we obtain the following expression for the spectrum of electrons from Eqns (89) and (90):

$$\frac{d\Gamma}{dE} = \sum_i |U_{ei}|^2\,\frac{d\Gamma_i}{dE}\,. \tag{91}$$

Here, the notation is adopted:

$$\frac{d\Gamma_i}{dE} = Cp(E + m_e)(E_0 - E)\left((E_0 - E)^2 - m_i^2\right)^{1/2} \times F(E)\,\theta(E_0 - E - m_i)\,, \tag{92}$$

where E is the kinetic energy of the electron, p is the electron momentum, E_0 is the energy released in the decay, m_e is the electron mass, and the Fermi function $F(E)$ describes the Coulomb interaction of the final particles. The constant C is given by the expression

$$C = G_F^2\,\frac{m_e^5}{2\pi^3}\cos^2\theta_C |M|^2\,,$$

where G_F is the Fermi constant, θ_C is the Cabibbo angle, and M is the nuclear matrix element (a constant).

It should be emphasized that neutrino mass enters into expression (92) through the neutrino momentum $|\mathbf{p}_i| = \left((E_0 - E)^2 - m_i^2\right)^{1/2}$, and the step function $\theta(E_0 - E - m_i)$ provides the fulfilment of the condition $E \leqslant E_0 - m_i$.

Two experiments on the measurement of the neutrino mass by the tritium method are presently being carried out (Troitsk [42] and Mainz [43]). The sensitivity of these experiments to neutrino mass amounts to 2–3 eV. The sensitivity to the neutrino mass of the future experiment KATRIN [44] is expected to be 0.35 eV. We will now consider tritium β-decay, keeping in mind these sensitivity levels.

As is seen from Eqn (92), the largest distortion of the β-spectrum due to neutrino mass can be observed in the region

$$E_0 - E \simeq m_i\,. \tag{93}$$

However, at $m_i \simeq 1$ eV only a very small part (about 10^{-13}) of the decays of the tritium make a contribution to the region (93). For this reason, the data relevant to a relatively large part of the β-spectrum must be analyzed. (For example, in the Mainz experiment, the data regarding last 70 eV of the spectrum are used for analysis.) Taking this into account, we can present the tritium β-spectrum in the form [43, 45]

$$\frac{d\Gamma}{dE} = Cp(E + m_e)(E_0 - E)\left((E_0 - E)^2 - m_\beta^2\right)^{1/2} F(E)\,, \tag{94}$$

where the effective mass m_β is given by the expression

$$m_\beta^2 = \sum_i |U_{ei}|^2 m_i^2 \,. \tag{95}$$

We will dwell on the case of three massive and mixed neutrinos. Neutrino oscillation experiments allow us to determine neutrino mass-squared differences Δm_{21}^2 and Δm_{32}^2. For neutrino masses m_2 and m_3, we have the equalities

$$m_2 = (m_1^2 + \Delta m_{21}^2)^{1/2} \,, \quad m_3 = (m_1^2 + \Delta m_{32}^2 + \Delta m_{21}^2)^{1/2} \,. \tag{96}$$

The minimal neutrino mass m_1 and the character of neutrino mass spectrum are at present unknown.

We will consider three typical neutrino mass spectra.

(1) Neutrino mass hierarchy

$$m_1 \ll m_2 \ll m_3$$

results in the effective neutrino mass m_β estimated as

$$m_\beta \simeq (\sin^2\theta_{\rm sol}\,\Delta m_{\rm sol}^2 + |U_{e3}|^2 \Delta m_{\rm atm}^2)^{1/2} \,. \tag{97}$$

Using the best-fit values of the oscillation parameters (39) and (62) and the CHOOZ bound (87), from relation (97) we find the upper bound

$$m_\beta \lesssim 1.1 \times 10^{-2}\ \text{eV} \,. \tag{98}$$

This bound is smaller significantly than the expected sensitivity of the future KATRIN experiment [44].

(2) Inverted neutrino mass hierarchy implies that

$$m_1 \ll m_2 < m_3 \,.$$

In this case, the following estimates are valid:

$$m_2 \simeq m_3 \simeq \sqrt{\Delta m_{\rm atm}^2} \simeq 5 \times 10^{-2}\ \text{eV} \,, \quad m_1 \ll \sqrt{\Delta m_{\rm atm}^2} \,. \tag{99}$$

Taking into account that $|U_{e1}|^2 \ll 1$, from the unitarity of the mixing matrix we find the following value for the effective neutrino mass m_β:

$$m_\beta \simeq \sqrt{\Delta m_{\rm atm}^2} \simeq 5 \times 10^{-2}\ \text{eV} \,. \tag{100}$$

This value is also much smaller than the expected sensitivity of the KATRIN experiment.

(3) For practically degenerate neutrino mass spectrum, it can be inferred that the following inequality is satisfied:

$$m_1 \gg \sqrt{\Delta m_{\rm atm}^2} \,. \tag{101}$$

In this case we have

$$m_{3,2} - m_1 \leqslant \frac{\Delta m_{\rm atm}^2}{2 m_1} \ll m_1 \,. \tag{102}$$

Taking into account the unitarity of the mixing matrix, we arrive at the following estimate

$$m_\beta \simeq m_1 \,. \tag{103}$$

Thus, the positive effect of neutrino masses can be observed in future β-decay experiments in the case of the practically degenerate neutrino mass spectrum, with minimal neutrino mass m_1 being not smaller than the sensitivity of the experiment ($\simeq 0.35$ eV).

Let us discuss now the results of the tritium experiments. In the Mainz experiment [43], the target is the frozen molecular tritium condensed on the graphite substrate. The electron energy spectrum is measured by the integral MAC-E-Filter spectrometer (Magnetic Adiabatic Collimator with a retarding Electrostatic filter). This spectrometer combines high luminosity with high resolution. The resolution of the spectrometer is equal to 4.8 eV. In the analysis of the experimental data, four free variable parameters are used: the normalization C, the background B, the released energy E_0, and the neutrino mass squared m_β^2. From the fit of the data it was found that $E_0 = 18{,}575$ keV. In the last 70 eV of the spectrum, the combined statistical and systematical errors were minimal.

From the analysis of 1998, 1999, and 2001 data it was found that

$$m_\beta^2 = (-1.2 \pm 2.2 \pm 2.1)\ \text{eV}^2 \,. \tag{104}$$

This value corresponds to the upper bound

$$m_\beta < 2.2\ \text{eV}\ (95\%\text{C.L.}) \,. \tag{105}$$

In the Troitsk neutrino experiment [42], as in the Mainz experiment, the integral electrostatic spectrometer with a strong inhomogeneous magnetic field focusing the electrons is used. The resolution of the spectrometer ranges 3.5 – 4 eV. An important difference between the experiments is that in the Troitsk experiment the tritium source is a gaseous molecular source.

From the four-parameter fit of the Troitsk data, large negative values falling in the range $-(10-20)$ eV^2 were obtained for the parameter m_β^2. The investigation of the character of the measured spectrum suggests that the effect of the negative m_β^2 is due to a step function superimposed on the integral continuous spectrum. The step function in the integral spectrum corresponds to the narrow peak in the differential spectrum. In order to describe the experimental data, the authors of the Troitsk experiment added to the theoretical integral spectrum a step function with two additional variable parameters (the position of the step $E_{\rm step}$, and the height of the step).

From the six-parameter fit of the data it was obtained that

$$m_\beta^2 = -2.3 \pm 2.5 \pm 2.0\ \text{eV}^2 \,. \tag{106}$$

This value corresponds to the upper bound

$$m_\beta < 2.2\ \text{eV}\ (95\%\text{C.L.}) \,. \tag{107}$$

The position of the step $E_0 - E_{\rm step}$ is periodically changed within the interval 5 – 15 eV, and the average height of the step is about 6×10^{-11}. The existence of this anomaly was not confirmed by the Mainz experiment [46].

We have discussed up to now tritium β-decay experiments on the measurement of neutrino mass. Research groups in Genoa [47] and Milan [48] are developing low-temperature cryogenic detectors for the measurement of the β-decay spectrum of ^{187}Re. This element possesses the lowest known energy release ($E_0 \simeq 2.5$ keV). The relative fraction of the high-energy part in the energy spectrum is proportional to E_0^{-3}. Thus, decays with low E_0 values are very suitable for the calorimetric experiments, in which the full spectrum is measured. The present limit for the neutrino mass that was obtained in Ref. [47] amounts to

$$m_\beta < 26 \text{ eV}. \tag{108}$$

Later on, a sensitivity $\simeq 10$ eV is expected to be reached.

In the future KATRIN tritium experiment [44], two tritium sources will be used: a gaseous molecular source, as in the Troitsk experiment, and a frozen tritium source, as in the Mainz experiment. The integral MAC-E-Filter spectrometer will have two parts: the prespectrometer, which will select electrons in the last $\simeq 100$ eV of the spectrum, and the main spectrometer. This spectrometer will have a resolution $\simeq 1$ eV. It is planned that the KATRIN experiment will start to collect data in 2007. After three years of running it is expected that the accuracy of the measurement of the parameter m_β^2 will amount to 0.08 eV^2. This will allow us to reach the sensitivity 0.35 eV in the determination of the effective neutrino mass m_β.

In the KATRIN experiment, it is planned to measure not only the integral spectrum, but the differential spectrum either. These measurements will enable the KATRIN collaboration to clarify the problem of the Troitsk anomaly in a direct way.

8. Neutrinoless double β-decay

The search for neutrinoless double β-decay

$$(A, Z) \to (A, Z+2) + \mathrm{e}^- + \mathrm{e}^- \tag{109}$$

of some even – even nuclei is the most sensitive and direct way of investigating *the nature* of neutrinos with definite (Majorana or Dirac) masses. In the process (109), the total lepton number is violated and the decay is only allowed if the massive neutrinos ν_i are Majorana particles.

We will assume that the Hamiltonian of the process (109) takes the standard form (89), and the flavor field $\nu_{\mathrm{e,L}}$ is given by the relation

$$\nu_{\mathrm{e,L}} = \sum_i U_{\mathrm{e}i} \nu_{i,\mathrm{L}}, \tag{110}$$

where ν_i are *Majorana fields*.

The neutrinoless double β-decay [$(\beta\beta)_{0\nu}$-decay] is the second-order process in the Fermi constant G_F with the participation of virtual neutrinos. The neutrino propagator is given by the expression

$$\begin{aligned} &\langle 0 | T\left(\nu_{\mathrm{e,L}}(x_1) \nu^{\mathrm{T}}_{\mathrm{e,L}}(x_2)\right) | 0 \rangle \\ &\simeq \langle m \rangle \frac{\mathrm{i}}{(2\pi)^4} \int \mathrm{d}^4 p \, \exp\left[-\mathrm{i}p(x_1 - x_2)\right] \frac{1}{p^2} \frac{1 - \gamma_5}{2} C. \end{aligned} \tag{111}$$

Here, the notation was introduced:

$$\langle m \rangle = \sum_i U_{\mathrm{e}i}^2 m_i. \tag{112}$$

Notice that we took into account in Eqn (111) that neutrino masses are much smaller than the binding energies of nucleons in nuclei.

The matrix element of the neutrinoless double β-decay is proportional to the product of the effective Majorana mass $\langle m \rangle$, which depends on neutrino masses m_i and elements $U_{\mathrm{e}i}^2$, and a nuclear matrix element, which does not depend on neutrino masses.

The elements $U_{\mathrm{e}i}$ of the neutrino mixing matrix are complex quantities. In the case of the CP invariance in the lepton sector, the elements $U_{\mathrm{e}i}$ satisfy the condition [20]

$$U_{\mathrm{e}i}^* = \eta_i^* U_{\mathrm{e}i}, \tag{113}$$

where $\eta_i = \mathrm{i}\rho_i$ is the CP parity of a neutrino with mass m_i ($\rho_i = \pm 1$). Let us write down $U_{\mathrm{e}i}$ in the form

$$U_{\mathrm{e}i} = |U_{\mathrm{e}i}| \exp(\mathrm{i}\alpha_i).$$

From formula (113) it follows that

$$2\alpha_i = \frac{\pi}{2} \rho_i.$$

The results of many experiments on the search for $(\beta\beta)_{0\nu}$-decay are available at present (see Refs [49, 50]). No evidence of $(\beta\beta)_{0\nu}$-decay has been obtained up to now[10]. The most stringent lower bounds on the $(\beta\beta)_{0\nu}$-decay lifetime were obtained in the Heidelberg – Moscow [54] and IGEX [55] ^{76}Ge experiments:

$$T_{1/2}^{0\nu} \geqslant 1.9 \times 10^{25} \text{ years (90\%C.L.) (Heidelberg – Moscow)},$$
$$T_{1/2}^{0\nu} \geqslant 1.57 \times 10^{25} \text{ years (90\%C.L.) (IGEX)}.$$

Taking into account different calculations of the nuclear matrix element, the following upper bounds were obtained from these results for the effective Majorana mass:

$$\begin{aligned} &|\langle m \rangle| \leqslant 0.35 - 1.24 \text{ eV (Heidelberg – Moscow)}, \\ &|\langle m \rangle| \leqslant 0.33 - 1.35 \text{ eV (IGEX)}. \end{aligned} \tag{114}$$

Many new experiments on the search for the neutrinoless double β-decay are in preparation at present (see Ref. [49]). In these experiments, the sensitivities at the level of

$$|\langle m \rangle| \simeq (1.5 - 8.6) \times 10^{-2} \text{ eV} \tag{115}$$

are expected to be achieved[11].

In the cryogenic CUORICINO (later on CUORE) experiment [57] with ^{130}Te, the lower bound $T_{1/2}^{0\nu} \geqslant 1.5 \times 10^{25}$ y ($T_{1/2}^{0\nu} \geqslant 7 \times 10^{26}$ y) is expected to be reached. This bound corresponds to a sensitivity $|\langle m \rangle| \simeq 1.9 \times 10^{-1}$ eV ($|\langle m \rangle| \simeq 2.7 \times 10^{-2}$ eV).

In the TPC EXO experiment [58] (^{136}Xe), it is expected to obtain the bound $T_{1/2}^{0\nu} \geqslant 8 \times 10^{26}$ y, which corresponds to the sensitivity $|\langle m \rangle| \simeq 5.2 \times 10^{-2}$ eV. In the GENIUS ^{76}Ge experiment [59], the lower bound $T_{1/2}^{0\nu} \geqslant 1 \times 10^{28}$ y is planned to be achieved. This bound corresponds to the sensitivity $|\langle m \rangle| \simeq 1.5 \times 10^{-2}$ eV. In the MOON experiment [60] (^{100}Mo), it is planned to reach the lower bound $T_{1/2}^{0\nu} \geqslant 1 \times 10^{27}$ y and the sensitivity $|\langle m \rangle| \simeq 3.6 \times 10^{-2}$ eV.

Evidence of neutrinoless double β-decay would be proof that neutrinos with definite masses ν_i are Majorana particles and that neutrino masses are of an origin beyond the Standard Model. *The value of the effective Majorana mass* $|\langle m \rangle|$, combined with the values of the neutrino oscillation parameters obtained from the results of neutrino oscillation

[10] The recent claim [51] for evidence of the $(\beta\beta)_{0\nu}$-decay, established from a reanalysis of the data of the Heidelberg – Moscow experiment, was strongly criticized in Refs [52, 53].

[11] In the calculations of the sensitivities, nuclear matrix elements given in Ref. [56] were used.

experiments would allow us to obtain important information about the character of the neutrino mass spectrum, minimal neutrino mass m_1, and the Majorana CP phase (see Ref. [61] and references cited therein).

We will dwell on the same three neutrino mass spectra in the three-neutrino case as in the previous section.

(1) The hierarchy of neutrino masses implies that

$$m_1 \ll m_2 \ll m_3 .$$

For the effective Majorana mass $|\langle m\rangle|$, we have in this case the following upper bound

$$|\langle m\rangle| \leqslant \sin^2\theta_{\rm sol}\sqrt{\Delta m^2_{\rm sol}} + |U_{e3}|^2\sqrt{\Delta m^2_{\rm atm}} . \tag{116}$$

Using the best-fit values of the oscillation parameters and the CHOOZ upper bound on $|U_{e3}|^2$ [see formulas (39), (62) and (87)], we arrive at

$$|\langle m\rangle| \leqslant 3.8\times10^{-3}\ {\rm eV} . \tag{117}$$

The upper bound (117) is significantly smaller than the expected sensitivities (115) of the future $(\beta\beta)_{0\nu}$-experiments.

The hierarchy of neutrino masses is a natural consequence of the see-saw mechanism of neutrino mass generation. Hence, the observation of the $(\beta\beta)_{0\nu}$-decay in experiments of the next generation could pose a problem for the standard see-saw mechanism of neutrino mass generation [12].

(2) Inverted hierarchy of neutrino masses results in

$$m_1 \ll m_2 < m_3 .$$

The effective Majorana mass is given in this case by the expression

$$|\langle m\rangle| \simeq (1-\sin^2 2\theta_{\rm sol}\sin^2\alpha)^{1/2}\sqrt{\Delta m^2_{\rm atm}} , \tag{118}$$

where $\alpha = \alpha_3 - \alpha_2$ is the difference of the Majorana CP phases. From this expression it follows that

$$\sqrt{\Delta m^2_{\rm atm}}\,|\cos 2\theta_{\rm sol}| \lesssim |\langle m\rangle| \lesssim \sqrt{\Delta m^2_{\rm atm}} , \tag{119}$$

where the upper and lower bounds correspond to the CP conservation with the equal and opposite CP parities of ν_3 and ν_2.

Using the best-fit value of the parameter $\tan^2\theta_{\rm sol}$ [see Eqn (62)], we have

$$\frac{1}{2}\sqrt{\Delta m^2_{\rm atm}} \lesssim |\langle m\rangle| \lesssim \sqrt{\Delta m^2_{\rm atm}} . \tag{120}$$

Thus, in the case of inverted mass hierarchy, the scale of $|\langle m\rangle|$ is determined by $\sqrt{\Delta m^2_{\rm atm}}$. If the value of $|\langle m\rangle|$ lies in the range (120), which can be reached in future experiments on the search for $(\beta\beta)_{0\nu}$-decay, it would be an argument in favor of inverted neutrino mass hierarchy.

The measurement of the effective Majorana mass $|\langle m\rangle|$ could allow us to gather information about the CP phase α [63, 64]. Indeed, from formula (118) we have

$$\sin^2\alpha \simeq \left(1-\frac{|\langle m\rangle|^2}{\Delta m^2_{\rm atm}}\right)\frac{1}{\sin^2 2\theta_{\rm sol}} . \tag{121}$$

(3) For practically degenerate neutrino mass spectrum, it can be assumed that the following inequality is valid:

$$m_1 \gg \sqrt{\Delta m^2_{\rm atm}} .$$

Then one finds

$$m_2 \simeq m_3 \simeq m_1 .$$

For the effective Majorana mass, independent of the character of the mass spectrum, we have

$$|\langle m\rangle| \simeq m_1\left|\sum_{i=1}^{3} U^2_{ei}\right| . \tag{122}$$

Neglecting the small contribution from $|U_{e3}|^2$ (or $|U_{e1}|^2$ in the case of the inverted hierarchy), we obtain for $|\langle m\rangle|$ the relations (118)–(120), in which $\sqrt{\Delta m^2_{\rm atm}}$ must be changed by m_1.

Thus, it would be a signature of the degenerate neutrino mass spectrum if it occurs that $|\langle m\rangle| \gg \sqrt{\Delta m^2_{\rm atm}}$. For neutrino mass m_1 we have in this case the range

$$|\langle m\rangle| \leqslant m_1 \leqslant \frac{|\langle m\rangle|}{|\cos 2\theta_{\rm sol}|} \lesssim 2|\langle m\rangle| . \tag{123}$$

The parameter $\sin^2\alpha$, which characterizes the violation of the CP invariance in the lepton sector, is given in this case by the expression [61, 64]

$$\sin^2\alpha \simeq \left(1-\frac{|\langle m\rangle|^2}{m^2_\beta}\right)\frac{1}{\sin^2 2\theta_{\rm sol}} . \tag{124}$$

If the mass m_1 is measured in future β-decay experiments and the value of the parameter $\sin^2 2\theta_{\rm sol}$ is determined in the solar, KamLAND [31], BOREXINO [65], and other neutrino experiments, then information on the Majorana CP phase can be inferred from the results of future $(\beta\beta)_{0\nu}$-experiments.

All previous conclusions were based on the assumption that the value of the effective Majorana mass $|\langle m\rangle|$ can be obtained from the measurement of the $(\beta\beta)_{0\nu}$-decay lifetime. There is, however, a serious theoretical problem in determining $|\langle m\rangle|$ from experimental data. It is connected with nuclear matrix elements.

In the framework of Majorana neutrino mixing, the total probability of $(\beta\beta)_{0\nu}$-decay takes the following general form [66]

$$\Gamma^{0\nu}(A,Z) = |\langle m\rangle|^2\left|M(A,Z)\right|^2 G^{0\nu}(E_0,Z) , \tag{125}$$

where $M(A,Z)$ is the nuclear matrix element, and $G^{0\nu}(E_0,Z)$ is the known phase-space factor (E_0 is the energy release). Thus, in order to determine $|\langle m\rangle|$ from the experimental data, we need to calculate the nuclear matrix element $M(A,Z)$.

There exist at present large uncertainties in the calculations of the nuclear matrix elements of the $(\beta\beta)_{0\nu}$-decay (see, for example, Refs [67–69]). Two basic approaches to the calculation are used: quasi-particle random phase approximation, and the nuclear shell model. Various calculations of

[12] Notice that at the next stage of the GENIUS experiment with 10 tons of enriched ^{76}Ge, the sensitivity to the effective Majorana mass $|\langle m\rangle|$ of order a few 10^{-3} eV is planned to be reached [62].

the $(\beta\beta)_{0\nu}$-decay lifetime differ by about one order of magnitude. For example, for the lifetime of the $(\beta\beta)_{0\nu}$-decay of ^{76}Ge, the following range [69] [13]

$$6.8 \times 10^{26} \text{ years} \leqslant T_{1/2}^{0\nu}(^{76}\text{Ge}) \leqslant 70.8 \times 10^{26} \text{ years} \qquad (126)$$

was obtained.

The problem of the calculation of the nuclear matrix elements of the neutrinoless double β-decay is a real theoretical challenge. It is obvious that without a solution to this problem the effective Majorana neutrino mass $|\langle m \rangle|$ cannot be determined from the experimental data with reliable accuracy (see discussion in Refs [70, 64]).

In Ref. [71], a method was proposed which allows us to check the calculated results of the nuclear matrix elements for the $(\beta\beta)_{0\nu}$-decay of different nuclei in a model-independent way. Let us take into account the following points:

(1) for small neutrino masses ($m_i \lesssim 10$ MeV), the nuclear matrix elements do not depend on neutrino masses [66];

(2) the sensitivity to determining $|\langle m \rangle|$ at the level of a few 10^{-2} eV is planned to be reached in experiments on the search for neutrinoless double β-decay of different nuclei.

Using Eqn (125) we obtain the ratio between the lifetimes of two nuclei:

$$R(A, Z/A', Z') = \frac{T_{1/2}^{0\nu}(A, Z)}{T_{1/2}^{0\nu}(A', Z')} = \frac{|M(A', Z')|^2 \, G^{0\nu}(E_0', Z')}{|M(A, Z)|^2 \, G^{0\nu}(E_0, Z)} \,. \qquad (127)$$

Thus, if the neutrinoless double β-decay of *different nuclei* is observed, the calculated ratios of the corresponding nuclear matrix elements squared can be compared with the experimental values.

In Table 2, the ratios of the $(\beta\beta)_{0\nu}$-decay lifetimes of several nuclei, calculated in six different models, are presented. The values of the lifetimes, given in Ref. [69], were utilized in the computation. As is seen from the table, the calculated ratios vary within about one order of magnitude.

Table 2. The calculated results for the ratios of the $(\beta\beta)_{0\nu}$-decay lifetimes of several nuclei in six different models.

Lifetime ratio	[72]	[73]	[74]	[56]	[75]	[76]
$R(^{76}\text{Ge}/^{130}\text{Te})$	11.3	3	20	4.6	3.6	4.2
$R(^{76}\text{Ge}/^{136}\text{Xe})$		1.5	4.2	1.1	0.6	2
$R(^{76}\text{Ge}/^{100}\text{Mo})$			14	1.8	10.7	0.9

Referring to Table 2, the ratio $R(^{76}\text{Ge}/^{130}\text{Te})$, calculated in Refs [56] and [75], is equal, correspondingly, to 4.6 and 3.6. It is clear that it will be difficult to distinguish models [56] and [75] by the observation of the neutrinoless double β-decay of ^{76}Ge and ^{130}Te. However, there will be no problem to distinguish the corresponding models via the observation of the $(\beta\beta)_{0\nu}$-decay of ^{76}Ge and ^{100}Mo (the corresponding ratios are equal to 1.8 and 10.7). This example illustrates the importance of the investigation of the $(\beta\beta)_{0\nu}$-decay of more than two nuclei.

The nuclear part of the matrix element governing the $(\beta\beta)_{0\nu}$-decay is determined by the matrix element of the T-product of two hadronic charged currents connected by the propagator of a massless boson (see, for example, Refs [66, 15]). This matrix element cannot be connected with the matrix element of any observable process. The method proposed in Ref. [71] is based only on the smallness of neutrino masses and on factorization of neutrino and nuclear parts of the matrix element for $(\beta\beta)_{0\nu}$-decay. Therefore, observation of the $(\beta\beta)_{0\nu}$-decay of different nuclei is required.

[13] The values given in Eqn (126) were calculated under the assumption that $|\langle m \rangle| = 5 \times 10^{-2}$ eV.

9. Conclusions

Compelling evidence in favor of neutrino oscillations, envisaged by B Pontecorvo more than thirty years ago [33], has been obtained in recent years in the S-K [1], SNO [8 – 10], and other atmospheric and solar neutrino experiments. These findings have opened a new field of research in particle physics and astrophysics: the *physics of massive and mixed neutrinos*.

The available experimental results suggest that neutrino masses are many orders of magnitude smaller than the masses of other fundamental fermions (leptons and quarks). There is a general consensus that tiny neutrino masses are of an origin beyond the Standard Model.

There are many unsolved problems in the physics of massive and mixed neutrinos. In the near future, LMA solution of the solar neutrino problem will be tested by the KamLAND [31] and BOREXINO [65] experiments. If neutrino oscillation parameters Δm^2_{sol} and $\tan^2\theta_{\text{sol}}$ lie in the LMA region, neutrino oscillations in the solar range of Δm^2 can be studied in detail in reactor experiments with the well-known initial antineutrino spectrum.

Another problem which will probably be solved in the near future is that of LSND anomaly [17]. If the LSND results are confirmed by the MiniBOONE experiment [32], it would mean that the number of light neutrinos is more than three and, in addition to the three flavor neutrinos, sterile neutrinos must exist. If the LSND results are refuted, the minimal scheme with three massive and mixed neutrinos will become a very plausible possibility.

The problem of the (*Dirac or Majorana*) *nature of massive neutrinos* is a most fundamental one. This problem can be solved by experiments on the search for neutrinoless double β-decay. From the existing data, the following upper bound on the effective Majorana mass was found:

$$|\langle m \rangle| \leqslant 0.3 - 1.3 \text{ eV} \,.$$

In future experiments, the sensitivities at the level of

$$|\langle m \rangle| \simeq 1.5 \times 10^{-2} - 1 \times 10^{-1} \text{ eV}$$

are expected to be achieved.

One of the very important problems of neutrino mixing is the problem of $|U_{e3}|^2$. In order to see the effects of three-neutrino mixing in future long baseline neutrino experiments and, in particular, the effects of CP violation in the lepton sector, it is necessary that the parameter Δm^2_{21} be in the LMA region and the parameter $|U_{e3}|^2$ be larger than $10^{-4} - 10^{-5}$ (see Ref. [36]). The best restrictions on $|U_{e3}|^2$ were found from the data of the reactor experiment CHOOZ [11]. New information on $|U_{e3}|^2$ will be obtained in the near future in the MINOS [77], ICARUS [78], and JHF [79] experiments.

Neutrino physics was born with the seminal paper of B Pontecorvo [80], in which he proposed the first method for detecting neutrinos (the radiochemical method, now used in the Homestake, GALLEX – GNO, and SAGE solar neutrino experiments) and for the first time paid attention to the fact that the Sun and reactors are the intensive sources of neutrinos. Now, a new phenomenon, neutrino oscillations, is observed in neutrino experiments. This finding gives us the first evidence of a new physics beyond the SM. We would like to note that it is not for the first time that a breakthrough to a new physics is connected with neutrinos. The first Fermi theory of the β-decay was based on the Pauli hypothesis of the neutrino. The phenomenological $(V-A)$ theory of weak interactions started with the two-component neutrino theory of Landau, Lee and Young, and Salam. The first evidence for the Glashow, Weinberg, and Salam Standard Model of the electroweak interaction was obtained in a neutrino beam (discovery of the NCs).

10. Note added in proof

After this review was finished and presented to the Editorial Board, important initial results of the long baseline reactor KamLAND experiment were published [81]. The initial results of the long baseline accelerator experiments K2K also appeared [82]. Below we will discuss these new results.

10.1 Evidence of neutrino oscillations obtained in the KamLAND experiment

In the KamLAND experiment [81], electron antineutrinos from many reactors in Japan and Korea were detected via the observation of the classical process

$$\bar{\nu}_e + p \to e^+ + n\,.$$

The threshold of this process amounts to about 1.8 MeV. About 80% of the total number of expected events is due to $\bar{\nu}_e$ from 26 reactors located within the distances of 138–214 km.

The 1-kt liquid scintillator detector of the KamLAND experiment is located in the Kamioka mine at a depth of about 1 km. Both prompt photons from the annihilation of e^+ in the scintillator and 2.2-MeV delayed photons from the neutron capture $n + p \to d + \gamma$ are detected. The mean neutron capture time is 188 ± 23 μs. In order to avoid the background, mainly from the decay of ^{238}U and ^{232}Th in the Earth, the cut $E_{\text{prompt}} > 2.6$ MeV was applied.

Over 145.1 days of the experiment 54 $\bar{\nu}_e$-events were observed. The number of events expected when there are no neutrino oscillations is equal to 86.8 ± 5.6. For the ratio between the observed and expected $\bar{\nu}_e$-events, the value

$$\frac{N_{\text{obs}} - N_{\text{BG}}}{N_{\text{exp}}} = 0.611 \pm 0.085\,(\text{stat.}) \pm 0.041\,(\text{syst.})$$

was obtained.

In the KamLAND experiment, the prompt energy spectrum was also measured. Prompt photon energy is connected with the energy of $\bar{\nu}_e$ by the relation

$$E_{\text{prompt}} = E_{\bar{\nu}_e} - 0.8\ \text{MeV} - \bar{E}_n\,,$$

($\bar{E}_n$ is the average energy of the neutron). Based on the two-neutrino analysis of the KamLAND data, the following best-fit values of the oscillation parameters were obtained:

$$(\Delta m^2)_{\text{KamLAND}} = 6.9 \times 10^{-5}\ \text{eV}^2\,, \quad (\sin^2 2\theta)_{\text{KamLAND}} = 1\,.$$

These values are consistent with the values of the oscillation parameters in the solar neutrino LMA region.

The KamLAND results provide strong evidence for neutrino masses and oscillations, obtained for the first time in experiment with terrestrial antineutrinos with the expected flux well under control. It allows us to exclude SMA, LOW, and VAC regions of neutrino oscillation parameters. The only viable solution of the solar neutrino problem reduces to the LMA solution.

10.2 Indications of neutrino oscillations obtained in the K2K experiment

Neutrino oscillations in the atmospheric range of Δm^2 were investigated in the first long baseline accelerator experiment K2K [82]. In this experiment, neutrinos mainly from the decays of pions produced at the 12-GeV KEK accelerator in Japan are recorded by the S-K detector at a distance of about 250 km from the accelerator. The average neutrino energy is $\simeq 1.3$ GeV.

Two near detectors at a distance of about 300 m from the beam-dump target are also used in the K2K experiment: a 1-kt water Cherenkov detector and a fine-grained detector. The total number and spectrum of muon neutrinos, observed in the S-K detector, are compared with the total number and spectrum calculated on the basis of the results of the near detectors under the assumption of the absence of neutrino oscillations. For measuring the energy of neutrinos in the S-K detector, quasi-elastic one-ring events $\nu_\mu + n \to \mu^- + p$ are selected. The total number of muon events observed in the S-K detector was equal to 56. The expected number of events is equal to $80.1^{+6.2}_{-5.4}$. The observed number of one-ring muon events that was used for the calculation of the neutrino spectrum amounts to 29. The expected number of one-ring events equals 44.

Thus, in the long baseline accelerator K2K experiment, evidence in favor of the disappearance of the accelerator ν_μ was obtained. From the maximum likelihood two-neutrino analysis of the data, the following best-fit values of the oscillation parameters were found:

$$(\sin^2 2\theta)_{\text{K2K}} = 1\,, \quad (\Delta m^2)_{\text{K2K}} = 2.8 \times 10^{-3}\ \text{eV}^2\,.$$

These findings are in agreement with the values of the oscillation parameters that were found from the analysis of the S-K atmospheric neutrino data. The first K2K results were obtained with 4.8×10^{19} protons on target (POT). It is planned that 10^{20} POT will be utilized in the experiment.

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