

REVIEWS OF TOPICAL PROBLEMS

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Kinetics of weakly collisional plasma

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Contents

Abstract. **Under conditions which are usually associated with collisionless plasma, and in which the mean free path of charged particles considerably exceeds the characteristic size of the spatial inhomogeneities involved, plasmas always contain slow particles whose mean free path proportional to the fourth power of their velocity is less than the inhomogeneity scale. Although relatively few in number, these subthermal particles play a dominant role in such 'weakly collisional' plasmas. In this paper, the results of the analytical kinetic theory of plasma are discussed, which highlight the determining role slow collisional particles play in such plasma phenomena as ion-acoustic wave damping and nonlinear electron-density perturbations due to the inhomogeneous intensity of the plasma-heating electromagnetic field. It is shown that by affecting these plasma properties the subthermal electrons correspondingly make an impact on parametric instabilities such as plasma radiation filamentation and stimulated Mandelstam – Brillouin scattering. Theoretical predictions are compared with numerical solutions of the Boltzmann equation. The concept of nonlocal plasma transfer processes, attracted to the interpretation of such solutions, is also discussed.**

1. Introduction

The development of the theory of parametric action of electromagnetic radiation on a fully ionized plasma has led to the concept of a weakly collisional plasma, for which new specific manifestations of charged particle collisions were discovered [1 – 30]. They were found to show up in the peculiar behavior of the nonlinear plasma density perturbation, which determines the development of electromagnetic pump field filamentation and stimulated Mandelstam – Brillouin scattering (SMBS) [1], as well as in the new collisional dependence of the damping of ion-acoustic plasma waves, which defines the SMBS threshold. First of all, there is a good reason to provide a qualitative definition of the notion of a weakly collisional plasma, which is governed by the Coulomb law of interaction of charged particles — electrons and ions in a fully ionized plasma. The notion of a weakly collisional plasma is essentially intermediate between the notion of a collisionless plasma and that of a strongly collisional or, as is often said, collisional plasma. While on the subject of a collisionless plasma limit, it is common to imply a situation when the characteristic scale of inhomogeneity $\lambda\!\!\!^{-}(\equiv k^{-1})$ proves to be much shorter than the mean free path $l_{\rm ei}[V_{\rm Te}]$ of a thermal [i.e. having a thermal velocity $V_{\rm Te} = (\kappa_{\rm B} T_{\rm e}/m_{\rm e})^{1/2}$] electron:

$$\lambda\!\!\!^{-} \equiv \frac{1}{k} \ll l_{\rm ei}[V_{\rm Te}] = \frac{V_{\rm Te}}{\nu_{\rm ei}[V_{\rm Te}]} = \frac{3m_{\rm e}^2 V_{\rm Te}^4}{\sum_{\rm i} 4\sqrt{2\pi}\, e^2 e_{\rm i}^2 n_{\rm i} \Lambda} \,. \qquad (1.1)$$

Here, e and $e_{\rm i}$ are the electric charges of ions and electrons, $m_{\rm e}$ is the electron mass, $T_{\rm e}$ is the electron temperature, $n_{\rm i}$ is the ion number density, and Λ is the Coulomb logarithm [31]; the summation is extended over plasma ions of all sorts. We can conveniently invoke the effective degree of ionization $Z_{\rm eff}$ defined by the relationship

$$Z_{\rm eff} = \frac{\sum_{\rm i} e_{\rm i}^2 n_{\rm i}}{e^2 n_{\rm e}} \,, \qquad (1.2)$$

[1] According to the tradition established in off-Russian scientific literature this latter scattering is termed a stimulated Brillouin scattering, viz. SBS. (*Editor's note.*)

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where n_e is the electron number density. In the analytical treatment being outlined below we consider the conditions when the effective degree of ionization is everywhere high:

$$Z_{\text{eff}} \gg 1 \,. \tag{1.3}$$

In this connection it is pertinent to note that the mean free path of a thermal electron for its collisions with ions, which appears in the right-hand side of inequality (1.1), proves to be short in comparison with the mean free path l_{ee} of a thermal electron for its collisions with thermal electrons:

$$l_{\text{ei}}[V_{\text{Te}}] \ll l_{\text{ee}}[V_{\text{Te}}] = \frac{3m_e^2 V_{\text{Te}}^4}{4\sqrt{2\pi}\, e^4 n_e \Lambda} \,. \tag{1.4}$$

On the other hand, a strongly collisional plasma is referred to when the following inequality is valid:

$$\lambda\!\!\!^{-} \gg l_{\text{ee}}[V_{\text{Te}}] \gg l_{\text{ei}}[V_{\text{Te}}] \,. \tag{1.5}$$

In this case, the plasma is described in the framework of hydrodynamic notions of viscosity, thermal conduction, and plasma conductivity. By contrast, in the collisionless limit (1.1) the plasma description is based on the collisionless kinetic equation with a self-consistent Vlasov field.

The present review is concerned with the theory of phenomena caused by charged particle collisions in the conditions where inequality (1.1) is satisfied and the plasma is commonly referred to as collisionless. On the face of it, this statement sounds paradoxical. It is reliant on the fact that the Rutherford scattering cross section of colliding charged plasma particles, defined by their Coulomb interaction, viz.

$$\frac{\mathrm{d}\sigma_{ab}}{\mathrm{d}o_{\mathbf{n}}} = \left(\frac{e_a e_b}{2\mu_{ab}}\right)^2 \frac{1}{V_{ab}^4 \sin^4(\Theta/2)} \,, \tag{1.6}$$

is inversely proportional to the fourth power of the relative velocity V_{ab} of the colliding particles. In formula (1.6), e_a and e_b are the electric charges, μ_{ab} is the reduced mass of the colliding particles, and Θ is the scattering angle. The velocity dependence of the cross section (1.6) has the effect that, for instance, the mean free path of an electron with a velocity V for its collisions with ions can be written as

$$l_{\text{ei}}[V] = \left(\frac{V}{V_{\text{Te}}}\right)^4 l_{\text{ei}}[V_{\text{Te}}] \,. \tag{1.7}$$

The direct proportionality of the mean free path to the fourth power of the electron velocity leads to the result that there always exist plasma electrons slow to the extent that the condition for frequent collisions is fulfilled for them:

$$\lambda\!\!\!^{-} \gg l_{\text{ei}}[V] \,. \tag{1.8}$$

The velocities of these electrons, which will be referred to as 'strongly collisional', satisfy the inequality

$$V < V_{\text{Te}} \left(\frac{\lambda\!\!\!^{-}}{l_{\text{ei}}[V_{\text{Te}}]}\right)^{1/4} \equiv V_{\bullet} \,. \tag{1.9}$$

When condition (1.1) holds good, the inequality

$$V_{\bullet} \ll V_{\text{Te}} \tag{1.10}$$

is valid. In our consideration this signifies the necessity not to forget about the collisions between the ions and the relatively small fraction of cold (or subthermal) collisional electrons having velocities that satisfy inequality (1.9). When the collisions of these subthermal electrons prove to be significant for one or other physical phenomenon in a plasma, in the subsequent discussion such a plasma will be referred to as weakly collisional.

When inequality (1.1) is satisfied, the division of electrons constituting a so-called collisionless plasma into thermal collisionless and subthermal collisional, which was done in Ref. [12], forms the physical basis which makes it possible to construct an analytical kinetic theory of phenomena in a weakly collisional plasma. This kinetic theory underlies the materials in the subsequent sections.

We emphasize that formula (1.9) is appropriate when the velocity of slow subthermal collisional electrons exceeds the thermal ion velocity $V_{\text{Ti}} = (\kappa_{\text{B}} T_{\text{i}}/M_{\text{i}})^{1/2}$. This requires that

$$1 \ll \frac{l_{\text{ei}}[V_{\text{Te}}]}{\lambda\!\!\!^{-}} \ll \frac{T_e^2 M_i^2}{T_i^2 m_e^2} \,. \tag{1.11}$$

It is evident that the velocity range defined by inequalities (1.11) is quite broad. In such a broad range of parameters, the electron–ion collision integral can be represented in the following form

$$J_{\text{ei}}[f] = \sum_{\text{i}} \frac{2\pi e^2 e_i^2 n_i \Lambda}{m_e^2 V^3} \frac{\partial}{\partial V_k} \left[(V^2 \delta_{kj} - V_k V_j) \frac{\partial f}{\partial V_j}\right], \tag{1.12}$$

where f is the electron distribution function. This formula describes the momentum transfer in electron–ion collisions. It is worth mentioning that formula (1.12) does not describe the small effect of the electron-to-ion energy transfer, which depends on the electron-to-ion mass ratio. This effect is insignificant for the subsequent consideration.

We use the notation

$$\nu_{\text{ei}}[V_{\text{Te}}] = \sum_{\text{i}} \frac{4\sqrt{2\pi}\, e^2 e_i^2 n_i \Lambda}{3m_e^2 V_{\text{Te}}^3} = \frac{4\sqrt{2\pi}\, e^4 n_e Z_{\text{eff}} \Lambda}{3m_e^2 V_{\text{Te}}^3} \tag{1.13}$$

for the electron–ion collision frequency to represent formula (1.12) in the form

$$J_{\text{ei}}[f] = \sqrt{\frac{9\pi}{8}} \frac{V_{\text{Te}}^3 \nu_{\text{ei}}[V_{\text{Te}}]}{V^3} \frac{\partial}{\partial V_k} \left[(V^2 \delta_{kj} - V_k V_j) \frac{\partial f}{\partial V_j}\right]. \tag{1.14}$$

Hence it is easily seen that the effective collision frequency increases according to the law

$$\left(\frac{V_{\text{Te}}}{V}\right)^3 \tag{1.15}$$

with a decrease in electron velocity.

We now address ourselves to the description of collisions between slow subthermal electrons and the bulk of thermal electrons, which is highly significant for a weakly collisional plasma. Since the relative velocity at the collision of a subthermal electron with a thermal one is of the order of V_{Te}, then on the strength of inequality (1.4), when the collisionlessness condition (1.1) is fulfilled, also fulfilled is the condition

$$\lambda\!\!\!^{-} \ll l_{\text{ee}}[V_{\text{Te}}] \,. \tag{1.16}$$

This seemingly allows one to neglect the electron–electron collisions. However, the situation is not that simple. The matter is that the collision integral describing the collisions of cold subthermal electrons with the thermal ones which form the bulk of the electron distribution in the conditions of our interest, may be written as

$$J_{\mathrm{ee}}[f] = \nu_{\mathrm{ee}}[V_{\mathrm{Te}}]\,\mathrm{div}_{\mathbf{V}}\,(V_{\mathrm{Te}}^2\,\mathrm{grad}_{\mathbf{V}}\,f + \mathbf{V}f)\,, \tag{1.17}$$

where f is the subthermal electron distribution function. The vectorial operations of divergence and gradient in formula (1.17) are performed in the electron velocity space, and the effective electron – electron collision frequency for thermal electrons is defined by the relationship

$$\nu_{\mathrm{ee}}[V_{\mathrm{Te}}] = \frac{4\sqrt{2\pi}\,e^4 n_{\mathrm{e}} \Lambda}{3m_{\mathrm{e}}^2 V_{\mathrm{Te}}^3} = \frac{\nu_{\mathrm{ei}}[V_{\mathrm{Te}}]}{Z_{\mathrm{eff}}}\,. \tag{1.18}$$

The senior term in the right-hand side of relationship (1.17) describes diffusion in the velocity space. The corresponding diffusion coefficient bearing the responsibility for the effect of collisions is determined by thermal electrons:

$$D = V_{\mathrm{Te}}^2 \nu_{\mathrm{ee}}[V_{\mathrm{Te}}]\,. \tag{1.19}$$

At the same time, the differential operator $\Delta_{\mathbf{V}}$ in the velocity region defined by inequality (1.9) has the effect that the effective electron – electron collision frequency for subthermal electrons proves to be significantly higher than in expression (1.18), namely

$$\nu_{\mathrm{ee,\,eff}}[V] \sim \frac{D}{V^2} > \frac{D}{V_\bullet^2} \sim \left(\frac{l_{\mathrm{ei}}[V_{\mathrm{Te}}]}{\lambda}\right)^{1/2} \nu_{\mathrm{ee}}[V_{\mathrm{Te}}]\,. \tag{1.20}$$

This is indicative of the possible increase in significance of the part played by electron – electron collisions of subthermal electrons of a weakly collisional plasma in comparison with the collisions of only thermal electrons between each other. An important note is in order. A comparison of the collision integrals (1.14) and (1.17) shows that under conditions where slow subthermal electrons are of interest and the electron collision integral (1.14) increases with decreasing their velocity in inverse proportion to the third power of velocity, the electron – electron collision integral increases, in accordance with expression (1.20), as the reciprocal of the velocity squared. That is why the electron – electron collision integral proves to be smaller than the electron – ion one to the extent of the smallness of subthermal-to-thermal electron velocity ratio (V/V_{Te}). Under the conditions of interest when inequality (1.3) is satisfied, the second cause for a similar smallness arises from the large value of the effective degree of ionization. This reasoning permits us to neglect electron – electron collisions when they compete with electron – ion collisions. The latter is the case when the electron distribution function is anisotropic in the velocity space. By contrast, an isotropic electron distribution function nullifies expression (1.12) for the electron – ion collision integral. Then, it is precisely the electron – electron collisions which determine the form of electron distribution function.

This is the reason for consistent accounting of electron – electron collisions in the theory being worked out. The relatively simple form of the linear differential operators, namely the collision integrals (1.14) and (1.17), makes it possible, as shown in the present review, to construct an analytical theory of kinetic phenomena in a weakly collisional plasma.

In the framework of ideas outlined here, the material of our review allows one to see the existence conditions for a weakly collisional plasma in which along with collisionless effects due to thermal electrons there occur collisional effects attributed to subthermal electrons. Collisional and collisionless processes thereby compete under the conditions (1.11) in which a plasma is usually assumed to be collisionless.

The development of the theory of a weakly collisional plasma involved matching the resultant data to the notions of the so-called nonlocal electron heat transfer in plasmas. In this connection we now dwell upon these notions here.

First of all we emphasize that the conventional Fourier – Fick law

$$\mathbf{q}(\mathbf{r}) = -\chi\,\mathrm{grad}\,T(\mathbf{r}) \tag{1.21}$$

relates the heat flux density $\mathbf{q}(\mathbf{r})$ to the temperature gradient $T(\mathbf{r})$ at the same point $\mathbf{r}$ in space. The latter fact, as is known from the Hilbert – Chapman – Enskog approach to the derivation of gas hydrodynamic equations from the kinetic Boltzmann theory, arises from the smallness of departure of the particle distribution from the local Maxwellian one, which is due to the smallness of the particle mean free path in comparison with the characteristic scale of spatial inhomogeneity of gas hydrodynamic quantities. That is why in the case of a weakly collisional plasma, when the mean free path of a thermal electron proves to be long in comparison with the scale of temperature nonuniformity, the local spatial relation between the electron heat flux density and the electron temperature inherent in the Fourier – Fick law does not appear to be natural.

On the other hand, even in 1974 when interpreting experimental data obtained in the execution of a laser fusion program at the Los Alamos Laboratory, R L Morse found the electron heat flux to be much smaller than that corresponding to the Fourier – Fick law (1.21) (see Ref. [32] on this issue). Notice that as early as 1973 Bickerton [33] regarded the ion-acoustic instability as a possible limitation for electron heat transfer. However, the last-named question falls outside the scope of our paper and deserves special consideration, despite the fact that it is also related to the notion of nonlocal heat transport. This notion, in particular, corresponds to the so-called Knudsen collisionless heat transfer mode, which in turn corresponds to D Bernoulli's ideas of a free collisionless motion of gas particles. These particles, which travel from a hot wall with a temperature T to a cold wall with a negligibly low temperature, produce, in particular, an electron heat flux density

$$q = \phi n_{\mathrm{e}} V_{\mathrm{Te}} \kappa_{\mathrm{B}} T\,, \tag{1.22}$$

where n_{e} is the particle number density, V_{Te} is the thermal velocity of particles, and ϕ is a numerical factor which appears to be of the order of unity for a Maxwellian particle distribution. This formula for the heat flux density at a given point is characterized by the temperature corresponding to the temperature of the hot wall (or, in the more general case, of a hot region). Since the heat flux density at the given point is defined by the temperature of the hot region an appreciable distance away, the law of heat transfer (1.22) is nonlocal. The heat flux (1.22) is, in accordance with the concept of the hot-region temperature, determined only by those particles which arrive from this hot region. We note here that this is merely one 'portion' of the Maxwellian particle distribution which comprises particles having a velocity component directed from the hot region to the region of heat transfer. In this sense the Knudsen collisionless transfer is characterized by a particle distribution strongly different from the Maxwellian one, which is one of the reasons why the laws (1.21) and (1.22) are qualitatively different.

However, an analysis of experiments on laser-produced plasmas revealed that the use of the law (1.22) in conditions where the Fourier – Fick law (1.21) is inapplicable leads to reasonable results only when the ϕ coefficient in formula (1.22) is small, i.e.

$$\phi \ll 1 \,. \tag{1.23}$$

This property of electron heat transfer in laser-produced plasma was given the name 'electron heat transport inhibition' [34]. In the pioneering work of Malone, McCrory, and Morse, the ϕ coefficient, which is referred to as the heat transport inhibition coefficient, was assumed to lie in the range between 0.01 and 0.03 [34].

Under the conditions of interest to us, when small, though nonlocal, i.e. varying sharply in space, deviations (for instance, δT_{e}) from spatially uniform equilibrium state are of significance for the description of parametric instabilities, the following interpolation formula is written down (see, for instance, Ref. [35]):

$$\mathbf{q} = \frac{-\chi \nabla \delta T_{\mathrm{e}}}{1 + \chi |\nabla \delta T_{\mathrm{e}}| / \phi n_{\mathrm{e}} V_{\mathrm{Te}} \kappa_{\mathrm{B}} T_{\mathrm{e}}} \,. \tag{1.24}$$

In the limiting case when the collisional transfer is very small due to the weakness of the spatial gradient, the denominator in the right-hand side of formula (1.24) differs little from unity. Then, formula (1.24) goes over into formula (1.21). By contrast, when the spatial gradient is strong, the denominator in formula (1.24) is large in comparison with unity. Accordingly, the heat flux density proves to be much smaller than the value given by the Fourier – Fick law. However, the existence of a small heat transport inhibition coefficient ϕ leads, in accordance with formula (1.24), to formula (1.22) which gives, in view of the condition (1.23), a heat flow which is much weaker than the conventional free molecular flow corresponding to $\phi \sim 1$. Such are the properties of electron heat transport inhibition characterized by formula (1.24).

For temperature perturbations with a spatial dependence $\delta T_{\mathrm{e}} \exp(\mathrm{i}\mathbf{kr})$, use is made, in lieu of interpolation formula (1.24), of the expression

$$\mathbf{q} = -\mathrm{i}\mathbf{k} \chi(k)\, \delta T \,, \tag{1.25}$$

where $\chi(k)$ is the nonlocal electron thermal conductivity coefficient. In the subsequent discussion we set $k = |\mathbf{k}|$. In the case of formula (1.24), one finds

$$\chi(k) = \frac{\chi_{\mathrm{SH}}}{1 + \chi_{\mathrm{SH}} k\, \delta T_{\mathrm{e}} / \phi n_{\mathrm{e}} \kappa_{\mathrm{B}}\, \delta T_{\mathrm{e}}\, V_{\mathrm{Te}}} = \frac{\chi_{\mathrm{SH}}}{1 + (128/3\pi\phi) k l_{\mathrm{ei}}[V_{\mathrm{Te}}]} \,. \tag{1.26}$$

From this point on, χ_{SH} is the conventional local electron thermal conductivity coefficient which, under the condition (1.3), according to Refs [36, 37] is given in the form

$$\chi_{\mathrm{SH}} = \frac{128}{3\pi} \frac{n_{\mathrm{e}} \kappa_{\mathrm{B}} V_{\mathrm{Te}}^2}{\nu_{\mathrm{ei}}[V_{\mathrm{Te}}]} \,. \tag{1.27}$$

The values of electron heat transport inhibition coefficient now are assumed to lie in the 0.1 – 0.03 range (see, for instance, Ref. [38]). In this case, the effective nonlocal electron thermal conductivity coefficient turns out to be significantly smaller than the local coefficient χ_{SH}, when the product $k l_{\mathrm{ei}}[V_{\mathrm{Te}}]$ runs into 0.0074 – 0.0022, respectively.

Formula (1.26) is the interpolation consequence of experimental indications. On the other hand, in works [38, 40] there arose a proposal to describe the nonlocal electron heat transfer by employing the results of collisional kinetic theory of gases, based on the Hilbert – Chapman – Enskog technique. To clarify the approach elaborated in papers [38 – 40] we point out that the inclusion of higher derivatives of the temperature in the determination of electron heat flux density gives [41]

$$\mathbf{q} = -13.6 \left(1 - \frac{4.20}{Z_{\mathrm{eff}}}\right) n_{\mathrm{e}} \kappa_{\mathrm{B}} V_{\mathrm{Te}} l_{\mathrm{ei}}[V_{\mathrm{Te}}] \times \left\{ \nabla T_{\mathrm{e}} + 264 (Z_{\mathrm{eff}} - 6.47) l_{\mathrm{ei}}^2 [V_{\mathrm{Te}}]\, \nabla (\Delta T_{\mathrm{e}}) \right\} . \tag{1.28}$$

In the limit $Z_{\mathrm{eff}} \gg 10$, this formula leads to the relationship

$$\chi(k) = \chi_{\mathrm{SH}} \left[1 - 264 Z_{\mathrm{eff}} \left(k l_{\mathrm{ei}}[V_{\mathrm{Te}}]\right)^2\right] . \tag{1.29}$$

In Refs [38, 39], the authors made use of the Padé – Borel approximation which suggests that recourse should be made to the formula

$$\chi(k) = \frac{\chi_{\mathrm{SH}}}{1 + 264 Z_{\mathrm{eff}} \left(k l_{\mathrm{ei}}[V_{\mathrm{Te}}]\right)^2} \tag{1.30}$$

in lieu of formula (1.29). The second term in the denominator of formula (1.30) becomes comparable to unity when $k l_{\mathrm{ei}}[V_{\mathrm{Te}}] \sim 0.06 Z_{\mathrm{eff}}^{-1/2}$. Like in the experimental case (1.26), the parameter $k l_{\mathrm{ei}}[V_{\mathrm{Te}}]$ results in the suppression of conventional collisional heat transfer even when its value is quite small. However, the similarity extends no further, since the functional dependences of formulas (1.26) and (1.30) on the argument $k l_{\mathrm{ei}}[V_{\mathrm{Te}}]$ prove to be significantly different. Note that the appearance of a large coefficient of the higher derivative in expression (1.28) is caused by the contribution made by electrons with high velocities V, which is due to their related long mean free paths. It is pertinent to note here that a conventional kinetic consideration in the context of the Hilbert – Chapman – Enskog method shows the contribution to heat transfer from higher-derivative terms to be made by particles with progressively higher velocities, i.e. progressively longer mean free paths. That is why, as shown by Gurevich and Istomin [42], the employment of the conventional Hilbert – Chapman – Enskog method becomes inadequate even for a moderately high temperature gradient. It is for rather high temperature gradients that the heat transfer is not determined by heat conduction, but becomes convective and kinetically described. Accordingly, the heat flux is defined by the particle temperature in the hot region, allowing the heat transfer in work [42] to be regarded as nonlocal.

The understanding of collisional phenomena on the basis of the notion of nonlocal heat conduction in the theory of parametric instabilities was proposed by Epperlein [1] in the course of theoretical investigation of such a parametric plasma instability as the filamentation instability. The difficulties of this approach will be discussed below.

On the other hand, the authors of Refs [43, 16] suggested the use of a system of moment equations to describe the phenomena in a highly tenuous collisionless and weakly collisional plasmas. In so doing, this system of equations, unlike that arising in the Grad moments method, contains expressions for flows describing nonlocal transfer and, therefore, integrally related to the thermodynamic forces that cause these flows. These equations bear, among other things, an integral relation between the electron heat flux density and the temperature gradient. This seemingly highly attractive possibility for plasma description, as will be seen from the material outlined below, does not prove to be consistent with the implications of the kinetic theory of a weakly collisional plasma. Moreover, this kind of description may lead to incorrect conclusions.

We dwell briefly on the contents of subsequent sections of the review. Section 2 states the conditions wherein the damping of ion-acoustic plasma waves occurs through a competition between the collisionless Landau damping due to the Cherenkov effect on thermal electrons and the collisional damping due to slow subthermal electrons. Section 3 is concerned with an issue principal to several parametric plasma instabilities — the theory of nonlinear electron density perturbation by the electromagnetic field of pump radiation. It is shown how in the course of formation of this nonlinear perturbation the competition occurs between the ponderomotive force effect defined by collisionless thermal electrons and the collisional effect of subthermal electrons. Section 4 explains how a relatively weak high-frequency electromagnetic field changes the velocity distribution of subthermal cold electrons. As shown in Section 5, the resultant nonlinear influence of the pump field on slow electrons gives rise to a nonlinear modification of the nonlinear density perturbation. First, this modification shows up as a peculiar, fractional-power dependence of the nonlinearity of electron density perturbation on the pump intensity. Second, this modification makes it possible to see how the effect of collisions is nonlinearly suppressed with increasing pump intensity and the effect of ponderomotive force becomes dominant. The effect of a nonlinear change of the electron distribution on the collisional absorption of ion-acoustic waves by electrons is considered in Section 6 on the basis of the analytical theory. Sections 7 and 8 serve to illustrate the application of the results of the analytical kinetic theory of a weakly collisional plasma to the description of such parametric instabilities as filamentation and stimulated Mandelstam–Brillouin scattering. Finally, the last Section 9 outlines conclusions.

2. Ion-acoustic wave damping

In this section we begin to concretize the notion of a weakly collisional plasma with a treatment of the phenomenon of ion-acoustic wave damping. We consider the competition between the collisionless Landau damping related to the Cherenkov interaction of thermal electrons with ion-acoustic waves, on the one hand, and the collisional damping caused by subthermal slow electrons, on the other hand. Our theoretical treatment will follow the analytical theory [10, 11] in the context of the ideas developed in Ref. [12] concerning the possibility of inclusion of the additive contributions from both the thermal collisionless electrons and collisional subthermal electrons. This consideration proceeds from the kinetic Boltzmann equation for the electron distribution function f:

$$\frac{\partial f}{\partial t} + \mathbf{V}\,\frac{\partial f}{\partial \mathbf{r}} - e\,\frac{\partial \varphi}{\partial \mathbf{r}}\,\frac{\partial f}{\partial \mathbf{p}} = J_{\mathrm{ei}}[f] + J_{\mathrm{ee}}[f, f]\,. \tag{2.1}$$

Here, φ is the electric field potential; the electron collision integral $J_{\mathrm{ei}}[f]$ is given by formula (1.14), and for the electron–electron collision integral we have a conventional expression in the Fokker–Planck–Landau form (see, for instance, monograph [31]):

$$J_{\mathrm{ee}}[f, f] = \frac{2\pi e^4 \Lambda}{m^2}\,\frac{\partial}{\partial V_k}\int \mathrm{d}\mathbf{V}'$$
$$\times\,\frac{(\mathbf{V}-\mathbf{V}')^2\delta_{kj} - (V_k - V'_k)(V_j - V'_j)}{|\mathbf{V}-\mathbf{V}'|^3}\left(\frac{\partial}{\partial \mathbf{p}} - \frac{\partial}{\partial \mathbf{p}'}\right)_j f(\mathbf{p})f(\mathbf{p}')\,. \tag{2.2}$$

The simplest is the situation wherein electrons, in the absence of an electric potential, obey the Maxwellian velocity distribution, which is independent of the time and the spatial coordinates. Then, assuming the plasma-perturbing potential to be of the form

$$\varphi(\mathbf{r}, t) = \varphi\exp\left(-\mathrm{i}\omega t + \mathrm{i}\mathbf{k}\mathbf{r}\right), \tag{2.3}$$

the perturbed electron distribution function can be represented as

$$f(\mathbf{p}, \mathbf{r}, t) = f_{\mathrm{M}}(p) + \delta f(\mathbf{p})\exp\left(-\mathrm{i}\omega t + \mathrm{i}\mathbf{k}\mathbf{r}\right). \tag{2.4}$$

We linearize Eqn (2.1), as applied to the problem of plasma waves, to obtain

$$-\,\mathrm{i}(\omega - \mathbf{k}\mathbf{V})\,\delta f - \mathrm{i}e\varphi\mathbf{k}\,\frac{\partial f_{\mathrm{M}}}{\partial \mathbf{p}} = J_{\mathrm{ei}}[\delta f] + J_{\mathrm{ee}}[f_{\mathrm{M}}, \delta f] + J_{\mathrm{ee}}[\delta f, f_{\mathrm{M}}]\,. \tag{2.5}$$

For our purposes, $\delta f(\mathbf{p})$ is conveniently represented in the following form

$$\delta f(\mathbf{p}) = -\frac{e\varphi}{\kappa_{\mathrm{B}} T_{\mathrm{e}}}\, f_{\mathrm{M}} + \delta\bar{f}. \tag{2.6}$$

The first term in the right-hand side of formula (2.6) makes the following contribution to the electron density perturbation caused by the perturbing electric potential:

$$\delta_1 n_{\mathrm{e}} = -\frac{e n_{\mathrm{e}}\varphi}{\kappa_{\mathrm{B}} T_{\mathrm{e}}}\,. \tag{2.7}$$

Accordingly, there arises the following static electron contribution to the longitudinal plasma permittivity:

$$\delta_1\varepsilon_{\mathrm{l,e}} = -\frac{4\pi e\delta_1 n_{\mathrm{e}}}{k^2\varphi} = \frac{4\pi e^2 n_{\mathrm{e}}}{\kappa_{\mathrm{B}} T_{\mathrm{e}} k^2} \equiv \frac{1}{k^2 r_{\mathrm{De}}^2}\,. \tag{2.8}$$

Since the first term in the right-hand side of formula (2.6) nullifies the right-hand side of Eqn (2.5), for the $\delta\bar{f}$ function we obtain the equation

$$-\,\mathrm{i}(\omega - \mathbf{k}\mathbf{V})\,\delta\bar{f} + \mathrm{i}\omega\,\frac{e\varphi}{\kappa_{\mathrm{B}} T_{\mathrm{e}}}\, f_{\mathrm{M}} = J_{\mathrm{ei}}[\delta\bar{f}] + J_{\mathrm{ee}}[f_{\mathrm{M}}, \delta\bar{f}] + J_{\mathrm{ee}}[\delta\bar{f}, f_{\mathrm{M}}]\,. \tag{2.9}$$

Furthermore, since the ion sound velocity is low in comparison with the thermal electron velocity, viz.

$$V_{\rm s} \ll V_{\rm Te}\,, \tag{2.10}$$

expression (2.8) represents, to a high accuracy, the total electron contribution to the real part of the longitudinal plasma permittivity. The corresponding ion contribution is [44]

$$\delta\varepsilon_{\rm l,\,i} = -\frac{\sum_{\rm i}\omega_{\rm Li}^2}{\omega^2} \equiv -\frac{\bar\omega_{\rm Li}^2}{\omega^2} \equiv -\frac{4\pi e^2 n_{\rm e} Z_{\rm eff}}{\omega^2 M_{\rm eff}}\,, \tag{2.11}$$

where $\omega_{\rm Li} = (4\pi e_{\rm i}^2 n_{\rm i}/M_{\rm i})^{1/2}$ is the ion Langmuir frequency. For ion-acoustic frequencies lower than the ion Langmuir frequency, from the equation

$$\varepsilon_{\rm l}(\omega,k) = 1 + \delta_{\rm l}\varepsilon_{\rm l,\,e} + \delta\varepsilon_{\rm l,\,i} = 0 \tag{2.12}$$

we obtain $\omega = kV_{\rm s} \equiv k\bar\omega_{\rm Li} r_{\rm De}$.

The ion-acoustic wave damping is described by Eqn (2.9). Neglecting collisions completely, from Eqn (2.9) we have the following conventional solution for thermal electrons:

$$\delta\bar f_{\rm T} = \left(\frac{P}{\omega - \mathbf{kV}} - {\rm i}\pi\delta(\omega - \mathbf{kV})\right)\frac{\omega e\varphi}{\kappa_{\rm B}T_{\rm e}}\,f_{\rm M}\,. \tag{2.13}$$

Here, division by zero corresponds to the result of solution of the initial problem, which verifies the fact of occurrence of the collisionless Landau damping. Formula (2.13), with the condition (2.10), permits one to write down the following expressions for the imaginary contributions to the electron density perturbation and the complex longitudinal plasma permittivity caused by thermal electrons:

$$\delta_{\rm T} n_{\rm e} = -{\rm i}\,\frac{e n_{\rm e}\varphi}{\kappa_{\rm B}T_{\rm e}}\sqrt{\frac{\pi}{2}}\,\frac{\omega}{kV_{\rm Te}}\,, \tag{2.14}$$

$$\delta_{\rm T}\varepsilon_{\rm l,\,e}(\omega,k) = \frac{\rm i}{k^2 r_{\rm De}^2}\sqrt{\frac{\pi}{2}}\,\frac{\omega}{kV_{\rm Te}}\,. \tag{2.15}$$

Formulas (2.8) and (2.15) describe the conventional collisionless electron contribution to the longitudinal permittivity (see, for instance, a set of lectures [44]).

It is now an appropriate time to turn to the consideration of collisional effects in conditions where inequality (1.1), which is commonly used to neglect collisions, is satisfied. Our treatment is reliant on a relatively small number of subthermal electrons whose collisions with the bulk of thermal electrons are described by the collision integral (1.17). Let $\delta\bar f_{\rm c}$ denote the contribution from cold electrons to the perturbation $\delta\bar f$ of the electron distribution function. Then, Eqn (2.9) may be written as follows

$$-{\rm i}(\omega - \mathbf{kV})\,\delta\bar f_{\rm c} + {\rm i}\omega\,\frac{e\varphi}{\kappa_{\rm B}T_{\rm e}}\,f_{\rm M} = J_{\rm ei}[\delta\bar f_{\rm c}] + J_{\rm ee}[\delta\bar f_{\rm c}]\,. \tag{2.16}$$

For subthermal electrons with velocities lower than $V_\bullet$, the collisions are the crucial factor. Moreover, collisions have the effect that $\delta\bar f_{\rm c}$ is only slightly different from the distribution isotropic in the velocity space. Under these conditions, the solution of the kinetic equation (2.16) can be written as the sum of a symmetric component and a small antisymmetric component:

$$\delta\bar f_{\rm c} = \delta\bar f_{\rm c,\,s} + \delta\bar f_{\rm c,\,a}\,. \tag{2.17}$$

In this case, the latter is defined by the condition that its average over the angles of the electron velocity vector goes to zero:

$$\langle\delta\bar f_{\rm c,\,a}\rangle = \int\frac{{\rm d}o_{\mathbf n}}{4\pi}\,\delta\bar f_{\rm c,\,a} = 0\,.$$

Furthermore, the significance of the part played by collisions is represented by the inequality

$$\delta\bar f_{\rm c,\,s} \gg \delta\bar f_{\rm c,\,a}\,. \tag{2.18}$$

Averaging Eqn (2.16) over the angles gives

$$-{\rm i}\omega\,\delta\bar f_{\rm c,\,s} + \langle{\rm i}\mathbf{kV}\,\delta\bar f_{\rm c,\,a}\rangle + {\rm i}\omega\,\frac{e\varphi}{\kappa_{\rm B}T_{\rm e}}\,f_{\rm M} = J_{\rm ee}[\delta\bar f_{\rm c,\,s}]\,. \tag{2.19}$$

We subtract this equation from Eqn (2.16) and invoke inequality (2.18) to find

$$-{\rm i}\omega\,\delta\bar f_{\rm c,\,a} + {\rm i}\mathbf{kV}\,\delta\bar f_{\rm c,\,s} = J_{\rm ei}[\delta\bar f_{\rm c,\,a}]\,. \tag{2.20}$$

In the right-hand side of Eqn (2.20) we neglected the contribution from electron – electron collisions in comparison with that of the electron – ion ones, because the inequality

$$\left(\frac{l_{\rm ei}[V_{\rm Te}]}{\lambdabar}\right)^{1/4} Z_{\rm eff} \gg 1 \tag{2.21}$$

is assumed to be satisfied.

The exact solution of Eqn (2.20) is written down in the following simple form

$$\delta\bar f_{\rm c,\,a} = -\frac{{\rm i}\mathbf{kV}\,\delta\bar f_{\rm c,\,s}(V)}{\nu_{\rm ei}[V_{\rm Te}]3\sqrt{\pi/2}\,(V_{\rm Te}^3/V^3) - {\rm i}\omega}\,. \tag{2.22}$$

On the strength of condition (1.10), which in our case (when $\lambdabar = 1/k$) is of the form

$$V < \frac{V_{\rm Te}}{(kl_{\rm ei}[V_{\rm Te}])^{1/4}} \equiv V_\bullet \ll V_{\rm Te}\,, \tag{2.23}$$

the frequency ω in the denominator in the right-hand side of expression (2.22) can be neglected provided that

$$kl_{\rm ei}[V_{\rm Te}] < \left(\frac{9\pi}{2}\right)^2\left(\frac{M_{\rm i}}{Zm_{\rm e}}\right)^2 \approx 7\times10^8\left(\frac{A_{\rm eff}}{Z_{\rm eff}}\right)^2, \tag{2.24}$$

where $A_{\rm eff} = M_{\rm eff}/M_{\rm H}$, and $M_{\rm H}$ is the proton mass. The last-named condition is easy to fulfill. Assuming it to be satisfied, it is readily seen that, in accordance with expression (2.22), condition (2.18) is met for subthermal particles with velocities lower than $V_\bullet$. This makes it possible to derive from Eqn (2.19) the following equation for the symmetric part of the distribution function perturbation:

$$\left(-{\rm i}\omega + \frac{k^2V^5 l_{\rm ei}[V_{\rm Te}]}{9\sqrt{\pi/2}\,V_{\rm Te}^4}\right)\delta\bar f_{\rm c,\,s} + {\rm i}\,\frac{\omega e\varphi}{\kappa_{\rm B}T_{\rm e}}\,f_{\rm M}(V)$$
$$= V_{\rm Te}^2\nu_{\rm ee}[V_{\rm Te}]\,\frac{1}{V^2}\frac{\rm d}{{\rm d}V}\left(V^2\,\frac{{\rm d}\delta\bar f_{\rm c,\,s}}{{\rm d}V}\right). \tag{2.25}$$

Being concerned with the effect of electron – ion collisions, we assume that the contribution from the frequency ω to the first term in the left-hand side of Eqn (2.25) is small. This imposes the following restriction on the velocity of subthermal electrons:

$$V > V_{\mathrm{Te}}\left(\frac{V_{\mathrm{s}}}{V_{\mathrm{Te}}}\,9\sqrt{\frac{\pi}{2}}\,\frac{1}{kl_{\mathrm{ei}}[V_{\mathrm{Te}}]}\right)^{1/5} \equiv V_{\omega}\,. \tag{2.26}$$

For the electrons with such velocities to be subthermal and collisional, the following inequality should be satisfied:

$$V_{\bullet} = V_{\mathrm{Te}}\,\frac{1}{\left(kl_{\mathrm{ei}}[V_{\mathrm{Te}}]\right)^{1/4}} \geqslant V_{\omega}\,. \tag{2.27}$$

This inequality reduces to the condition

$$\frac{1}{kl_{\mathrm{ei}}[V_{\mathrm{Te}}]} \geqslant 4.57\times10^{-3}\,\frac{Z_{\mathrm{eff}}^2}{A_{\mathrm{eff}}^2}\,. \tag{2.28}$$

Since the left-hand side of this inequality is, in accordance with inequality (1.1), small in comparison with unity, the condition (2.27) can be met for a small $Z_{\mathrm{eff}}/A_{\mathrm{eff}}$ ratio.

Assuming condition (2.23) to be fulfilled, we introduce the dimensionless variable

$$\xi = \frac{V^2}{2V_{\mathrm{Te}}^2}\,N^{2/7}\,, \quad \text{with} \quad N = \frac{4Z_{\mathrm{eff}}}{9\sqrt{\pi}}\left(kl_{\mathrm{ei}}[V_{\mathrm{Te}}]\right)^2, \tag{2.29}$$

and also make use of the relationship

$$\delta\bar{f}_{\mathrm{c,s}}(V) = f_{\mathrm{M}}(V)\,\frac{\mathrm{i}\omega}{\nu_{\mathrm{ei}}[V_{\mathrm{Te}}]}\,\frac{e\varphi}{\kappa_{\mathrm{B}}T_{\mathrm{e}}}\,\frac{9\sqrt{\pi}}{8k^2l_{\mathrm{ei}}^2[V_{\mathrm{Te}}]}\,N^{5/7}\Psi_{1/2}(\xi)\,. \tag{2.30}$$

Then, Eqn (2.25) reduces to

$$L_{\xi}\left[\Psi_{1/2}(\xi)\right] \equiv \frac{\mathrm{d}}{\mathrm{d}\xi}\left(\xi^{3/2}\,\frac{\mathrm{d}\Psi_{1/2}(\xi)}{\mathrm{d}\xi}\right) = \xi^3\Psi_{1/2}(\xi) + \sqrt{\xi}\,. \tag{2.31}$$

The latter equation has the following exact solution [10]

$$\Psi_{1/2}(\xi) = -\frac{4}{7\xi^{1/4}}\left[K_{1/7}\left(\frac{4}{7}\,\xi^{7/4}\right)\int_0^{\xi}\mathrm{d}\zeta\,\zeta^{1/4}I_{1/7}\left(\frac{4}{7}\,\zeta^{7/4}\right) + I_{1/7}\left(\frac{4}{7}\,\xi^{7/4}\right)\int_{\xi}^{\infty}\mathrm{d}\zeta\,\zeta^{1/4}K\left(\frac{4}{7}\,\zeta^{7/4}\right)\right]. \tag{2.32}$$

From this point on, $K_{\nu}(z)$ and $I_{\nu}(z)$ are the Bessel functions of an imaginary argument. This solution enables one to write down the following expression for the electron density perturbation due to subthermal collisional electrons, which is induced by the electric potential [10]:

$$\delta n_{\mathrm{e,c}} = -n_{\mathrm{e}}\,\frac{\mathrm{i}\omega}{\nu_{\mathrm{ei}}[V_{\mathrm{Te}}]}\,\frac{e\varphi}{\kappa_{\mathrm{B}}T_{\mathrm{e}}}\,\frac{Z_{\mathrm{eff}}C_{\varepsilon}}{N^{5/7}}\,, \tag{2.33}$$

where

$$C_{\varepsilon} = \frac{8}{7\sqrt{\pi}}\int_0^{\infty}\mathrm{d}\xi\,\xi^{1/4}K_{1/7}\left(\frac{4}{7}\,\xi^{7/4}\right) \times\int_0^{\xi}\mathrm{d}y\,y^{1/4}I_{1/7}\left(\frac{4}{7}\,y^{7/4}\right) = 0.8\,. \tag{2.34}$$

In accordance with formulas (2.29)–(2.33), the inequality (2.26) may be written as

$$V_{\mathrm{c}} = \frac{\sqrt{2}\,V_{\mathrm{Te}}}{N^{1/7}} > V_{\omega}\,, \tag{2.35}$$

which is equivalent to the following condition

$$\frac{1}{\left(kl_{\mathrm{ei}}[V_{\mathrm{Te}}]\right)^{1/4}} > 0.21\times Z_{\mathrm{eff}}^{5/12}\left(\frac{Z_{\mathrm{eff}}}{A_{\mathrm{eff}}}\right)^{7/24}. \tag{2.36}$$

The condition $V_{\mathrm{c}} < V_{\bullet}$ is reduced to the easily feasible inequality

$$\frac{1}{\left(kl_{\mathrm{ei}}[V_{\mathrm{Te}}]\right)^{1/4}} \leqslant \frac{Z_{\mathrm{eff}}}{45}\,. \tag{2.37}$$

The simultaneous fulfillment of the conditions (2.36) and (2.37) requires that the following inequality should be satisfied:

$$Z_{\mathrm{eff}}A_{\mathrm{eff}} \geqslant 2200\,. \tag{2.38}$$

In the special case that the plasma ions are ions of gold ($A_{\mathrm{eff}} = 197$), the last-mentioned condition reduces to the inequality $Z_{\mathrm{eff}} \geqslant 12$.

Relation (2.33) leads us to the following contribution from subthermal electrons to the imaginary part of the longitudinal plasma permittivity [10]:

$$\delta\varepsilon_{\mathrm{l,ec}}(\omega,k) = \frac{\mathrm{i}\omega}{\nu_{\mathrm{ei}}[V_{\mathrm{Te}}]}\,\frac{1}{k^2r_{\mathrm{De}}^2}\,\frac{Z_{\mathrm{eff}}C_{\varepsilon}}{N^{5/7}}\,. \tag{2.39}$$

We sum up expressions (2.8), (2.15), and (2.38) to obtain the following total electron contribution to the longitudinal plasma permittivity [11]:

$$\delta\varepsilon_{\mathrm{l,e}}(\omega,k) = \frac{1}{k^2r_{\mathrm{De}}^2} \times\left\{1 + \mathrm{i}\,\frac{\omega}{kV_{\mathrm{Te}}}\left[\sqrt{\frac{\pi}{2}} + 2.17\left(\frac{Z_{\mathrm{eff}}^2}{k^3l_{\mathrm{ei}}^3[V_{\mathrm{Te}}]}\right)^{1/7}\right]\right\}. \tag{2.40}$$

Employing formulas (2.11), (2.12), and (2.40), we arrive at the following expression for the ion-acoustic damping constant due to electrons [11]:

$$\gamma_{\mathrm{e}} = \frac{kV_{\mathrm{s}}^2}{V_{\mathrm{Te}}}\left[\sqrt{\frac{\pi}{8}} + 1.1\left(\frac{Z_{\mathrm{eff}}^2}{k^3l_{\mathrm{ei}}^3[V_{\mathrm{Te}}]}\right)^{1/7}\right]. \tag{2.41}$$

The first term in square brackets on the right-hand side of formula (2.41) proves to be no greater than the second collisional term provided that

$$1 > \frac{1}{\left(kl_{\mathrm{ei}}[V_{\mathrm{Te}}]\right)} > \frac{0.27}{Z_{\mathrm{eff}}^{2/3}}\,. \tag{2.42}$$

In particular, for $Z_{\mathrm{eff}} > 27$, the right-hand side of inequality (2.42) turns out to be less than 0.03. This signifies that the collisional and collisionless effects may well compete in the course of determination of the strength of ion-acoustic wave damping in the conditions of the so-called collisionless plasma (1.1), when the wavelength of ion-sound waves is short in comparison with the mean free path of a thermal electron. The analytical results of this section will be compared with numerical data below.

3. Nonlinear electron density perturbation by a nonuniform pump field intensity. Weak-field case

In this section we turn to the theoretical treatment of the phenomena caused by the action of a high-frequency electromagnetic field on a plasma. In doing so, it is assumed that the frequency ω_0 of this field is not only much higher than the effective collision frequency of thermal electrons with ions, as is commonly assumed (see, for instance, books [31, 44]), namely

$$\omega_0 \gg \nu_{\mathrm{ei}}[V_{\mathrm{Te}}] ,$$

but is also much higher than the corresponding collision frequency of subthermal electrons having a characteristic velocity V_{c}:

$$\omega_0 \gg \nu_{\mathrm{ei}}[V_{\mathrm{Te}}]\left(\frac{V_{\mathrm{Te}}}{V_{\mathrm{c}}}\right)^3 \sim \nu_{\mathrm{ei}}[V_{\mathrm{Te}}]\big(kl_{\mathrm{ee}}[V_{\mathrm{Te}}]kl_{\mathrm{ei}}[V_{\mathrm{Te}}]\big)^{3/7} . \tag{3.1}$$

Under conditions of interest to us, the right-hand side of relationship (3.1) is considerably greater than $\nu_{\mathrm{ei}}[V_{\mathrm{Te}}]$. In this case, the theory of action of a high-frequency electromagnetic field on a fully ionized plasma allows us to represent the electron distribution function as the sum

$$f+\tilde{f},$$

where f is a component of the distribution function varying only slightly over the period $2\pi/\omega_0$, and $\tilde{f}$ is the high-frequency component.

We write down the high-frequency electric field intensity as

$$\mathbf{E} = \frac{1}{2}\left[\mathbf{E}(\mathbf{r},t)\exp(-\mathrm{i}\omega_0 t) + \mathbf{E}^*(\mathbf{r},t)\exp(\mathrm{i}\omega_0 t)\right] , \tag{3.2}$$

where $\mathbf{E}(\mathbf{r},t)$ varies slowly in time in comparison with $\exp(-\mathrm{i}\omega_0 t)$. Let $V_E = |eE/m\omega_0|$ denote the characteristic amplitude of the electron quiver velocity in the electromagnetic field. In this case, we shall assume that the high-frequency field is relatively weak to satisfy the inequality

$$V_E \ll V_{\mathrm{Te}} . \tag{3.3}$$

Then, to describe the distribution function f varying only slightly over the period of a high-frequency field, it is possible to resort to the approach elaborated in papers [45, 41] to obtain the following kinetic equation

$$\begin{aligned}
&\frac{\partial f}{\partial t} + \mathbf{V}\frac{\partial f}{\partial \mathbf{r}} + e\mathbf{E}_0\frac{\partial f}{\partial \mathbf{p}} - J_{\mathrm{ei}}[f] - J_{\mathrm{ee}}[f,f] = \frac{e^2}{4\omega_0^2}\left(\frac{1}{m}\frac{\partial|\mathbf{E}|^2}{\partial \mathbf{r}}\frac{\partial f}{\partial \mathbf{p}}\right.\\
&\quad + \frac{1}{2}\frac{\partial^2 f}{\partial p_k\,\partial p_j}\left(\frac{\partial}{\partial t} + \mathbf{V}\frac{\partial}{\partial \mathbf{r}}\right)(E_kE_j^* + E_k^*E_j)\\
&\quad + (E_kE_j^* + E_k^*E_j)\left\{\frac{1}{m}\frac{\partial^2 f}{\partial r_k\,\partial p_j} + \left(\frac{\partial}{\partial t} + \mathbf{V}\frac{\partial}{\partial \mathbf{r}}\right)\frac{\partial^2 f}{\partial p_k\,\partial p_j}\right.\\
&\quad \left.\left. - \frac{\partial}{\partial p_k}J_{\mathrm{ei}}\left[\frac{\partial f}{\partial p_j}\right] - J_{\mathrm{ee}}\left[\frac{\partial f}{\partial p_k},\frac{\partial f}{\partial p_j}\right]\right\}\right) ,
\end{aligned} \tag{3.4}$$

where $\mathbf{E}_0$ is a slowly varying (low-frequency) electric field.

Equation (3.4) is somewhat more complicated than the kinetic equation (2.1) which underlay our consideration in Section 2. Accordingly, some more interesting consequences arise from Eqn (3.4). In this section we restrict ourselves to the case where $\mathbf{E}_0 = 0$ and consider that nonlinear effect of the high-frequency field on the electron distribution which distinguishes this distribution from the coordinate- and time-independent Maxwellian distribution in the approximation quadratic in intensity of the high-frequency field, when one has

$$\begin{aligned}
&f(\mathbf{p},\mathbf{r},t) = f_{\mathrm{M}}(p) + \delta f(\mathbf{p})\exp(\mathrm{i}\mathbf{kr}) ,\\
&E_kE_j^* \to E_kE_j^* + \delta(E_kE_j^*)_{\mathbf{k}}\exp(\mathrm{i}\mathbf{kr}) .
\end{aligned} \tag{3.5}$$

Under these conditions, we neglect the variation of E_{i} and $\delta f(\mathbf{p})$ with time. Then, from Eqn (3.4) follows the equation

$$\begin{aligned}
&\mathrm{i}\mathbf{kV}\,\delta f - J_{\mathrm{ei}}[\delta f] - J_{\mathrm{ee}}[f_{\mathrm{M}},\delta f] - J_{\mathrm{ee}}]\delta f, f_{\mathrm{M}}]\\
&= \frac{e^2}{4\omega_0^2}\left(-\frac{\mathrm{i}\mathbf{kV}}{m\kappa_{\mathrm{B}}T_{\mathrm{e}}}f_{\mathrm{M}}\,\delta|\mathbf{E}|^2_{\mathbf{k}} + \delta(E_kE_j^* + E_k^*E_j)_{\mathbf{k}}\right.\\
&\quad\left.\times\left\{\frac{1}{2}\,\mathrm{i}\mathbf{kV}\left[-\delta_{kj}\frac{1}{m\kappa_{\mathrm{B}}T_{\mathrm{e}}} + \frac{V_kV_j}{(\kappa_{\mathrm{B}}T_{\mathrm{e}})^2}\right]f_{\mathrm{M}} + \frac{\partial}{\partial p_k}J_{\mathrm{ei}}\left[\frac{V_jf_{\mathrm{M}}}{\kappa_{\mathrm{B}}T_{\mathrm{e}}}\right]\right\}\right) .
\end{aligned} \tag{3.6}$$

Here, we took into account the fact that

$$J_{\mathrm{ee}}\left[\frac{\partial f_{\mathrm{M}}}{\partial p_k},\frac{\partial f_{\mathrm{M}}}{\partial p_j}\right] = 0 .$$

We first consider the contribution of thermal electrons with velocities $\sim V_{\mathrm{Te}}$ to the perturbation of electron density, like we did in Section 2. Bearing in mind the condition (1.1) of the so-called collisionlessness, we neglect in Eqn (3.6) all the terms containing collision integrals. Then, Eqn (3.6) reduces to the relationship

$$\begin{aligned}
\delta_{\mathrm{T}}f = \frac{e^2f_{\mathrm{M}}}{4\omega_0^2m\kappa_{\mathrm{B}}T_{\mathrm{e}}}&\left[-\delta|\mathbf{E}|^2_{\mathbf{k}} + \frac{1}{2}\left(-\delta_{rs} + \frac{mV_rV_s}{\kappa_{\mathrm{B}}T_{\mathrm{e}}}\right)\right.\\
&\left.\times\,\delta(E_rE_s^* + E_r^*E_s)_{\mathbf{k}}\right] .
\end{aligned} \tag{3.7}$$

This relationship enables one to write down the formula

$$\delta n_{\mathrm{e,T}} = -\frac{e^2n_{\mathrm{e}}\,\delta|\mathbf{E}|^2_{\mathbf{k}}}{4\omega_0^2V_{\mathrm{Te}}^2} \tag{3.8}$$

describing the nonlinear perturbation of electron number density, caused by thermal electrons. Formula (3.8) defines the nonlinear perturbation caused by the ponderomotive force, or the Miller force [46].

We now address ourselves to the consideration of the corresponding contribution caused by subthermal electrons with velocities of the order of V_{c} (2.35). In this case, first, it is possible to neglect all the terms on the right-hand side of Eqn (3.6), with the exception of the term containing the contribution from electron – ion collisions. Retention of this term corresponds, in particular, to the description of electron heating due to the inverse bremsstrahlung absorption of the high-frequency field. Second, it would suffice to use the electron – electron collision integral in the form of expression (1.17) to describe subthermal electrons in Eqn (3.6). We then obtain

$$\mathbf{ikV}\,\delta f_{\mathrm{c}} - J_{\mathrm{ei}}[\delta f_{\mathrm{c}}] - J_{\mathrm{ee}}[\delta f_{\mathrm{c}}] = \frac{e^2}{4\omega_0^2 \kappa_{\mathrm{B}} T_{\mathrm{e}}}\,\delta(E_r E_s^* + E_r^* E_s)_{\mathbf{k}}\,\frac{\partial}{\partial p_r}\,J_{\mathrm{ei}}[V_s f_{\mathrm{M}}]\,. \tag{3.9}$$

In the solution of Eqn (3.9), first, we once again assume the condition (1.3) to be fulfilled, like we did in solving Eqn (2.19). Second, representing the distribution function perturbation as the sum of symmetric and antisymmetric parts

$$\delta f_{\mathrm{c}} = \delta f_{\mathrm{c,\,s}} + \delta f_{\mathrm{c,\,a}}\,, \tag{3.10}$$

we suppose that

$$\delta f_{\mathrm{c,\,s}} \gg \delta f_{\mathrm{c,\,a}}\,. \tag{3.11}$$

Then, from Eqn (3.9) follows

$$\mathrm{i}\langle \mathbf{kV}\,\delta f_{\mathrm{c,\,a}}\rangle - \nu_{\mathrm{ee}}[V_{\mathrm{Te}}]V_{\mathrm{Te}}^2\,\frac{1}{V^2}\,\frac{\mathrm{d}}{\mathrm{d}V}\left(V^2\,\frac{\mathrm{d}\delta f_{\mathrm{c,\,s}}}{\mathrm{d}V}\right) = -\sqrt{2\pi}\,\nu_{\mathrm{ei}}[V_{\mathrm{Te}}]\,\frac{e^2 V_{\mathrm{Te}}\,\delta|E|^2_{\mathbf{k}}}{4m^2\omega_0^2}\,f_{\mathrm{M}} 4\pi\delta(\mathbf{V})\,, \tag{3.12}$$

$$\mathbf{ikV}\,\delta f_{\mathrm{c,\,s}} - J_{\mathrm{ei}}[\delta f_{\mathrm{c,\,a}}] = 3\sqrt{\frac{\pi}{2}}\,\nu_{\mathrm{ei}}[V_{\mathrm{Te}}]\left(\frac{\delta_{rs}}{V^3} - \frac{3V_r V_s}{V^3}\right) \times \frac{e^2 f_{\mathrm{M}}}{4m^2\omega_0^2 V_{\mathrm{Te}}^2}\,\delta\left(E_r E_s^* + E_r^* E_s - \frac{2}{3}\,\delta_{rs}|E|^2\right)_{\mathbf{k}}\,. \tag{3.13}$$

Equation (3.13) is immediately solved, yielding

$$\delta f_{\mathrm{c,\,a}} = -\frac{\mathbf{ikV}\,\delta f_{\mathrm{c,\,s}}(V)}{\nu_{\mathrm{ei}}(V_{\mathrm{Te}})3\sqrt{\pi/2}\,(V_{\mathrm{Te}}^3/V^3)} + \frac{1}{3}\left(\delta_{rs} - 3\,\frac{V_r V_s}{V^2}\right)\frac{e^2 f_{\mathrm{M}}}{4m^2\omega_0^2 V_{\mathrm{Te}}^2} \times \delta\left(E_r E_s^* + E_r^* E_s - \frac{2}{3}\,|E|^2\delta_{rs}\right)_{\mathbf{k}}\,. \tag{3.14}$$

The first term on the right-hand side of expression (3.14) is small in comparison with $\delta f_{\mathrm{c,\,s}}$. The smallness of the second term will be analyzed later. We only note that this term does not make a contribution to Eqn (3.12), which can now be written in the following form

$$\left(\frac{k^2 V^5 l_{\mathrm{ei}}[V_{\mathrm{Te}}]}{9\sqrt{\pi/2}\,V_{\mathrm{Te}}^4}\right)\delta f_{\mathrm{c,\,s}} - V_{\mathrm{Te}}^2\nu_{\mathrm{ee}}[V_{\mathrm{Te}}]\,\frac{1}{V^2}\,\frac{\mathrm{d}}{\mathrm{d}V}\left(V^2\,\frac{\mathrm{d}\delta f_{\mathrm{c,\,s}}}{\mathrm{d}V}\right) = -\sqrt{2\pi}\,\nu_{\mathrm{ei}}[V_{\mathrm{Te}}]\,\frac{e^2\,\delta|E|^2_{\mathbf{k}}}{4m^2\omega_0^2}\,f_{\mathrm{M}}\,\frac{V_{\mathrm{Te}}}{V^2}\,\delta(V)\,. \tag{3.15}$$

This equation differs from Eqn (2.25), in particular, in its inhomogeneous part. Like the solution of Eqn (2.25), the solution of Eqn (3.15) is therefore represented as

$$\delta f_{\mathrm{c,\,s}}(V) = f_{\mathrm{M}}(V)\,\frac{9\sqrt{\pi}}{8k^2 l_{\mathrm{ei}}^2[V_{\mathrm{Te}}]}\,\frac{e^2\,\delta|E|^2_{\mathbf{k}}}{4m^2\omega_0^2 V_{\mathrm{Te}}^2}\,F_\delta\left(\frac{V^2}{2V_{\mathrm{Te}}^2}\right). \tag{3.16}$$

We are led to the following relationship defining the electron density perturbation:

$$\delta n_{\mathrm{c}} = n_{\mathrm{e}}\,\frac{Z_{\mathrm{eff}}}{N}\,\frac{e^2\,\delta|E|^2_{\mathbf{k}}}{4m^2\omega_0^2 V_{\mathrm{Te}}^2}\int_0^\infty \mathrm{d}x\,\sqrt{x}\,F_\delta(x)\,. \tag{3.17}$$

Accordingly, for the function $F_\delta(x)$ from Eqn (3.15) follows the equation

$$\frac{1}{N}\,L_x\big[F_\delta(x)\big] - x^3 F_\delta(x) = \sqrt{\pi}\,\delta(x)\,. \tag{3.18}$$

We take advantage of a change of variable (2.29) and avail ourselves of the substitution

$$F_\delta(x) = N^{8/7}\Psi_\delta(\xi)\,, \tag{3.19}$$

to come up with

$$L_\xi\big[\Psi_\delta(\xi)\big] - \xi^3\Psi_\delta(\xi) = \sqrt{\pi}\,\delta(\xi)\,. \tag{3.20}$$

The second term in formula (3.14) can now be compared with $\delta f_{\mathrm{c,\,s}}(V)$. As is easily seen, from expressions (3.16) and (3.19) it follows that $\delta f_{\mathrm{c,s}}(V)$ turns out to be greater than $Z_{\mathrm{eff}}N^{1/7} \gg 1$. This substantiates the correctness of equation (3.15) and, hence, of expression (3.20).

The right-hand side of Eqn (3.20) corresponds to the boundary condition

$$\xi^{3/2}\,\frac{\mathrm{d}\Psi_\delta(\xi)}{\mathrm{d}\xi}\bigg|_{\xi\to 0} = \sqrt{\pi}\,. \tag{3.21}$$

This boundary condition is satisfied with the following solution bounded at infinity:

$$\Psi_\delta(\xi) = -\frac{4\sqrt{\pi}}{\Gamma(1/7)\xi^{1/4}}\left(\frac{2}{7}\right)^{1/7} K_{1/7}\left(\frac{4}{7}\,\xi^{7/4}\right). \tag{3.22}$$

The latter expression, if formulas (3.19) and (3.16) are accounted for, finally makes it possible to find the sought-for electron density perturbation due to subthermal electrons [7]:

$$\delta n_{\mathrm{c}} = -\frac{e^2 n_{\mathrm{e}}\,\delta|E|^2_{\mathbf{k}}}{4\omega_0^2 m^2 V_{\mathrm{Te}}^2}\,\frac{Z_{\mathrm{eff}}}{N^{2/7}}\left(\frac{1024}{343}\right)^{1/7}\frac{\Gamma(2/7)\Gamma(3/7)}{\Gamma(1/7)}\,. \tag{3.23}$$

Summing up the contributions from thermal (3.8) and subthermal (3.23) electrons we arrive at the following expression for the total nonlinear electron density perturbation [7]:

$$\delta n_{\mathrm{e}} = -\frac{e^2 n_{\mathrm{e}}\,\delta|E|^2_{\mathbf{k}}}{4\omega_0^2 m^2 V_{\mathrm{Te}}^2}\left[1 + \frac{Z_{\mathrm{eff}}C}{\big(kl_{\mathrm{ei}}[V_{\mathrm{Te}}]\big)^{4/7}}\right], \tag{3.24}$$

where

$$C = \left(\frac{5184\pi}{343}\right)^{1/7}\frac{\Gamma(2/7)\Gamma(3/7)}{\Gamma(1/7)} \cong 1.73\,. \tag{3.25}$$

The first term on the right-hand side of formula (3.24) corresponds to the ponderomotive pushing out of electrons by the electromagnetic field, caused by the Miller force [46]. This collisionless contribution proves to be smaller than the collisional contribution due to inverse bremsstrahlung absorption when

$$1 \ll kl_{\mathrm{ei}}[V_{\mathrm{Te}}] < 2.6Z_{\mathrm{eff}}^{5/4}\,. \tag{3.26}$$

This relation is valid over a wide range under conditions specified by inequality (1.3). It is thereby shown that electron collisions can well compete with the action of the Miller force on electrons when determining the nonlinear electron perturbation by a nonuniform electromagnetic field under conditions specified by the inequality (1.1), whereby the plasma is conventionally assumed to be collisionless.

4. Variation of the subthermal plasma electron distribution in a weak heating electromagnetic field, caused by inverse bremsstrahlung absorption

This section is concerned with that departure from the electron Maxwellian distribution function which is exhibited by slow subthermal electrons exposed to the electromagnetic field that heats the plasma. Our prime interest here is the conditions wherein the intensity of radiation heating the plasma is moderate to the extent that, apart from condition (3.3), the condition

$$Z_{\mathrm{eff}} V_E^2 \ll V_{\mathrm{Te}}^2 \tag{4.1}$$

is also fulfilled. When the inequality (3.3) and the condition opposite to inequality (4.1) are met, the velocity distribution of electrons heated due to inverse bremsstrahlung absorption is known to be qualitatively different from the Maxwellian distribution over almost all the velocity phase space. Then, the Langdon distribution [47 – 50] is realized in lieu of the Maxwell one. In the limit (4.1) of the sufficiently weak heating field under consideration, as will be seen below, only a relatively small group of subthermal electrons departs from the Maxwellian distribution. However, this exerts, as one might guess from the material of the previous sections, a significant effect on those plasma properties which are determined by slow subthermal electrons.

In this section our main concern is with the plasmas exposed to a heating high-frequency field with a spatially uniform intensity, when Eqn (3.4) subject to the conditions (1.3) and (3.3) in the absence of the field $\mathbf{E}_0$ allows the use of the following equation

$$\frac{\partial f}{\partial t} - J_{\mathrm{ei}}[f] - J_{\mathrm{ee}}[f, f] = -\frac{e^2}{4m^2\omega_0^2}\left(E_k E_j^* + E_k^* E_j\right) \frac{\partial}{\partial p_k} J_{\mathrm{ei}}\left[\frac{\partial f}{\partial p_j}\right]. \tag{4.2}$$

The nonstationary and velocity-anisotropic electron distribution described by equation (4.2) becomes isotropic in a time $\sim \nu_{\mathrm{ei}}^{-1}[V_{\mathrm{Te}}]$ for thermal electrons. The isotropization of the distribution of subthermal electrons (significant for our consideration) with a velocity V sets in an even shorter time $(V/V_{\mathrm{Te}})^3 \nu_{\mathrm{ei}}^{-1}[V_{\mathrm{Te}}]$. The isotropic distribution arising in this case obeys the equation

$$\frac{\partial f_{0\mathrm{s}}}{\partial t} - J_{\mathrm{ee}}[f_{0\mathrm{s}}, f_{0\mathrm{s}}] = \frac{V_E^2}{3V^2}\frac{\partial}{\partial V}\left(V^2 \nu[V]\frac{\partial f_{0\mathrm{s}}}{\partial V}\right), \tag{4.3}$$

where

$$\nu[V] = 3\sqrt{\frac{\pi}{8}}\,\nu_{\mathrm{ei}}[V_{\mathrm{Te}}]\,\frac{V_{\mathrm{Te}}^3}{V^3}. \tag{4.4}$$

Equation (4.3) results from averaging Eqn (4.2) over the angles of the electron velocity vector.

In the context of a weak pump field (4.1) and assuming a slow electron heating, Eqn (4.3) reduces to the equation

$$J_{\mathrm{ee}}[f_{0\mathrm{s}}, f_{0\mathrm{s}}] = 0,$$

to which the Maxwellian electron velocity distribution corresponds. Assuming this distribution to be realized for at least the thermal electrons, for the temporal variation of the electron thermal velocity (and hence the temperature) we obtain from Eqn (4.3) the following equation

$$V_{\mathrm{Te}}\frac{\mathrm{d}V_{\mathrm{Te}}}{\mathrm{d}t} = \frac{1}{6}\,\nu_{\mathrm{ei}}[V_{\mathrm{Te}}]V_E^2. \tag{4.5}$$

Hence it follows, in particular, that the characteristic heating time for thermal electrons, viz.

$$t_{\mathrm{HT}} = \frac{V_{\mathrm{Te}}^2}{V_E^2}\left(\nu_{\mathrm{ei}}[V_{\mathrm{Te}}]\right)^{-1}, \tag{4.6}$$

is substantially longer than their isotropization time $\nu_{\mathrm{ei}}^{-1}[V_{\mathrm{Te}}]$ in conditions when inequality (3.3) is satisfied. Of special interest to us is the velocity distribution of a relatively small fraction of subthermal electrons which experience a pronounced action of the pump field owing to their low velocity and whose distribution proves to be different from Maxwellian. In this case, the electron – electron collision integral describes, according to expression (1.17), the collisions of subthermal electrons with the bulk of thermal particles. Then, for the cold-electron distribution function from Eqn (4.3) we have [27]

$$\frac{\partial f_{0,\mathrm{sc}}}{\partial t} - \nu_{\mathrm{ee}}[V_{\mathrm{Te}}]V_{\mathrm{Te}}^2\frac{1}{V^2}\frac{\partial}{\partial V}\left[V^3\left(\frac{1}{V}\frac{\partial f_{0,\mathrm{sc}}}{\partial V} + \frac{f_{0,\mathrm{sc}}}{V_{\mathrm{Te}}^2}\right)\right] = \sqrt{\frac{\pi}{3}}\,V_E^2\nu_{\mathrm{ei}}[V_{\mathrm{Te}}]\frac{V_{\mathrm{Te}}^3}{V^2}\frac{\partial}{\partial V}\left(\frac{1}{V}\frac{\partial f_{0,\mathrm{sc}}}{\partial V}\right). \tag{4.7}$$

When the temporal variation of the electron distribution function is determined by electron heating with a characteristic time (4.6), the time derivative of the distribution function in Eqn (4.7) is small in comparison with the second term. With the inequality (4.1) being satisfied, the contribution from the right-hand side of Eqn (4.7) to the temporal variation is also small. Accordingly, our consideration reduces to a quasi-stationary treatment, in which case from Eqn (4.7) we have the following ordinary differential equation

$$\frac{1}{V^2}\frac{\mathrm{d}}{\mathrm{d}V}\left[V^3\left(\frac{1}{V}\frac{\mathrm{d}f_{0,\mathrm{sc}}}{\mathrm{d}V} + \frac{f_{0,\mathrm{sc}}}{V_{\mathrm{Te}}^2}\right) + \frac{V_{\mathrm{L}}^3}{V}\frac{\mathrm{d}f_{0,\mathrm{sc}}}{\mathrm{d}V}\right] = 0, \tag{4.8}$$

where

$$V_{\mathrm{L}} = \left(\sqrt{\frac{\pi}{8}}\,Z_{\mathrm{eff}}V_E^2 V_{\mathrm{Te}}\right)^{1/3} \tag{4.9}$$

is referred to as the Langdon velocity [47]. The condition that there is no electron source at the zero velocity corresponds to the boundary condition

$$\frac{1}{V}\frac{\mathrm{d}f_{0,\mathrm{sc}}}{\mathrm{d}V} = 0.$$

The solution of Eqn (4.8), complying with the last boundary condition, takes the form

$$f_{0,\mathrm{sc}}(V) = f_{0,\mathrm{sc}}(0)\exp\left(-\frac{1}{V_{\mathrm{Te}}^2}\int_0^V \frac{u^4\,\mathrm{d}u}{u^3 + V_{\mathrm{L}}^3}\right). \tag{4.10}$$

Considering that, according to condition (4.1), the inequality

$$V_{\mathrm{L}}^2 \ll V_{\mathrm{Te}}^2 \tag{4.11}$$

is satisfied, it is possible to take advantage of the expression [25–27]

$$f_{0,\mathrm{sc}}(V) = \frac{n_{\mathrm{e}}}{(2\pi)^{3/2} V_{\mathrm{Te}}^3} \exp\left(-\frac{1}{V_{\mathrm{Te}}^2}\int_0^V \frac{u^4\,\mathrm{d}u}{u^3 + V_{\mathrm{L}}^3}\right) \tag{4.12}$$

as that electron velocity distribution function which corresponds to the Maxwellian distribution for velocities much higher than the Langdon velocity (4.9), and describes the distribution

$$f_{0,\mathrm{sc}}(V) = \frac{n_{\mathrm{e}}}{(2\pi)^{3/2} V_{\mathrm{Te}}^3}\left(1 - \frac{V^5}{5 V_{\mathrm{Te}}^2 V_{\mathrm{L}}^3}\right) \tag{4.13}$$

for velocities lower than the Langdon one. Below we also resort to a somewhat more complicated velocity distribution for subthermal electrons when, according to expression (4.12), we have

$$f_{0,\mathrm{sc}}(V) = \frac{n_{\mathrm{e}}}{(2\pi)^{3/2} V_{\mathrm{Te}}^3}\left(1 - \frac{1}{V_{\mathrm{Te}}^2}\int_0^V \frac{u^4\,\mathrm{d}u}{u^3 + V_{\mathrm{L}}^3}\right). \tag{4.14}$$

One can see directly from formulas (4.13) and (4.14) that they apply to a narrow domain of the subthermal-electron distribution.

The material outlined in this section allows us to make an important remark concerning the scope of the results of Sections 2 and 3, as applied to the description of a plasma exposed to the heating high-frequency field. The characteristic velocity of the subthermal electrons responsible for the weakly collisional effects considered in the previous sections is given by

$$V_{\mathrm{c}} = V_{\mathrm{Te}}\left[\frac{9\sqrt{8\pi}}{Z_{\mathrm{eff}}\big(k l_{\mathrm{ei}}[V_{\mathrm{Te}}]\big)^2}\right]^{1/7}. \tag{4.15}$$

In our treatment this velocity is much lower than the thermal velocity. Meanwhile, one can see from the distribution (4.14) that, with the heating radiation, subthermal electrons obey the Maxwellian electron velocity distribution employed in Sections 2 and 3 only for velocities much higher than the Langdon one. In other words, the results of these sections are valid, naturally, in the absence of a field heating the plasma and, with this field, only for relatively low intensities of the radiation field which heats the plasma, when the velocity (4.15) can be supposed to far exceed the Langdon one. This corresponds to the following inequality which restricts the heating field intensity:

$$\frac{V_E^2}{V_{\mathrm{Te}}^2} \ll \frac{8}{Z_{\mathrm{eff}}^{10/7}\big(k l_{\mathrm{ei}}[V_{\mathrm{Te}}]\big)^{6/7}} \ll 1. \tag{4.16}$$

The smallness of the right-hand side of the last inequality suggests that new nonlinear effects can show up in a plasma for very weak heating fields, when the condition (4.16) is violated, i.e. when the difference of the distribution (4.14) from the Maxwellian one should be taken into account and when the Langdon velocity is not negligibly low, namely

$$V_{\mathrm{L}} > V_{\mathrm{c}}. \tag{4.17}$$

The next two sections contain the consideration appropriate in this case, which extends the theory of Sections 2 and 3 to the case of such a heating field that the modification of subthermal-electron velocity distribution due to heating through the inverse bremsstrahlung absorption is significant.

5. Nonlinear electron density perturbation by a nonuniform pump field intensity. Nonlinear case of not-too-weak fields

The subthermal-electron distribution (4.13) obtained in the previous section will now be employed to analyze the corresponding variation of the nonlinear electron density perturbation caused by a spatially nonuniform perturbation of the high-frequency field. In addition to the spatially uniform perturbation considered in the previous section we therefore include a spatially nonuniform component

$$E_k E_j^* + E_k^* E_j \to E_k E_j^* + E_k^* E_j + \delta(E_k E_j^* + E_k^* E_j)_{\mathbf{k}} \exp(\mathrm{i}\mathbf{kr}). \tag{5.1}$$

Accordingly, for the electron distribution function we have [see formula (3.5)]

$$f(\mathbf{p}, \mathbf{r}, t) = f_{0\mathrm{s}}(p) + \delta f(\mathbf{p}) \exp(\mathrm{i}\mathbf{kr}). \tag{5.2}$$

By analogy with Eqn (3.6) we now can write

$$\begin{aligned}\mathrm{i}\mathbf{kV}\,\delta f - J_{\mathrm{ei}}[\delta f] - J_{\mathrm{ee}}[\delta f] = \frac{e^2}{4m^2\omega_0^2}\Bigg\{&\mathrm{i}\mathbf{k}\,\frac{\partial f_{0\mathrm{s}}}{\partial \mathbf{V}}\,\delta|E|_{\mathbf{k}}^2 \\ &+ \delta(E_k E_j^* + E_k^* E_j)_{\mathbf{k}}\left(\frac{1}{2}\,\mathrm{i}\mathbf{kV}\,\frac{\partial^2 f_{0\mathrm{s}}}{\partial V_k\,\partial V_j} - \frac{\partial}{\partial V_k}\,J_{\mathrm{ei}}\left[\frac{\partial f_{0\mathrm{s}}}{\partial V_j}\right]\right) \\ &- (E_k E_j^* + E_k^* E_j)\,\frac{\partial}{\partial V_k}\,J_{\mathrm{ei}}\left[\frac{\partial \delta f}{\partial V_j}\right]\Bigg\}.\end{aligned} \tag{5.3}$$

Since the equilibrium distribution function for thermal electrons is hardly different from the Maxwellian distribution, there follows relation (3.7) from expression (5.1) and, accordingly, formula (3.8) for the electron density perturbation.

For subthermal electrons whose velocity is assumed to be small in comparison with the Langdon one, viz.

$$V \ll V_{\mathrm{L}}, \tag{5.4}$$

advantage can be taken of the distribution function (4.13). This gives, in particular, the equation

$$\begin{aligned}&\frac{e^2}{4m^2\omega_0^2}\,\delta(E_k E_j^* + E_k^* E_j)_{\mathbf{k}}\,\frac{\partial}{\partial V_k}\,J_{\mathrm{ei}}\left[\frac{\partial f_{0\mathrm{s}}}{\partial V_j}\right] \\ &\quad = \frac{e^2\,\delta|E|_{\mathbf{k}}^2}{4m^2\omega_0^2}\,\frac{n_{\mathrm{e}}\nu_{\mathrm{ei}}[V_{\mathrm{Te}}]}{V_{\mathrm{Te}}^2 V_{\mathrm{L}}^3}\,\frac{3}{2\pi}.\end{aligned} \tag{5.5}$$

With the proviso that inequality (5.4) is satisfied, the electron–electron collision integral on the left-hand side of Eqn (5.3) turns out to be small in comparison with the last term in the right-hand side of this equation. For collisional subthermal electrons, it therefore follows from equation (5.3) that

$$\begin{aligned}&\mathrm{i}\mathbf{kV}\,\delta f_{\mathrm{c}} - J_{\mathrm{ei}}[\delta f_{\mathrm{c}}] + \frac{e^2}{4m^2\omega_0^2}(E_k E_j^* + E_k^* E_j)\,\frac{\partial}{\partial V_k}\,J_{\mathrm{ei}}\left[\frac{\partial \delta f}{\partial V_j}\right] \\ &\quad = -\frac{e^2\,\delta|E|_{\mathbf{k}}^2}{4m^2\omega_0^2}\,\frac{n_{\mathrm{e}}\nu_{\mathrm{ei}}[V_{\mathrm{Te}}]}{V_{\mathrm{Te}}^2 V_{\mathrm{L}}^3}\,\frac{3}{2\pi}.\end{aligned} \tag{5.6}$$

We substitute δf_{c} as a sum of symmetric and asymmetric functions [see formula (3.10)] and also invoke the assumption of smallness of the asymmetric function to derive the following system of two equations from Eqn (5.6):

$$\mathrm{i}\langle\mathbf{kV}\,\delta f_{\mathrm{c,a}}\rangle - \nu_{\mathrm{ee}}[V_{\mathrm{Te}}]\sqrt{\frac{\pi}{8}}\,V_{\mathrm{Te}}^3 V_E^2\left(\frac{1}{V^3}\,\frac{\mathrm{d}^2\delta f_{\mathrm{c,s}}}{\mathrm{d}V^2} - \frac{1}{V^4}\,\frac{\mathrm{d}\delta f_{\mathrm{c,s}}}{\mathrm{d}V}\right)$$
$$= -\frac{e^2\,\delta|E|_{\mathbf{k}}^2}{4m^2\omega_0^2}\,\frac{3n_{\mathrm{e}}\nu_{\mathrm{ei}}[V_{\mathrm{Te}}]}{2\pi V_{\mathrm{Te}}^2 V_{\mathrm{L}}^3}\,, \tag{5.7}$$

$$\mathrm{i}\mathbf{kV}\,\delta f_{\mathrm{c,s}} - J_{\mathrm{ei}}[\delta f_{\mathrm{c,a}}] = 3\sqrt{\frac{\pi}{2}}\,\nu_{\mathrm{ei}}[V_{\mathrm{Te}}]\left(V_r V_s - \frac{1}{3}\,\delta_{rs}V^2\right)\frac{V_{\mathrm{Te}}^3}{V}$$
$$\times\frac{\mathrm{d}}{\mathrm{d}V}\left(\frac{1}{V^4}\,\frac{\mathrm{d}\delta f_{\mathrm{c,s}}}{\mathrm{d}V}\right)\frac{e^2}{4m^2\omega_0^2}\left(E_r E_s^* + E_r^* E_s - \frac{2}{3}\,\delta_{rs}|E|^2\right). \tag{5.8}$$

The solution of the last equation takes the form

$$\delta f_{\mathrm{c,a}} = -\frac{\mathrm{i}\mathbf{kV}\,\delta f_{\mathrm{c,s}}(V)}{\nu_{\mathrm{ei}}(V_{\mathrm{Te}})3\sqrt{\pi/2}\,(V_{\mathrm{Te}}^3/V^3)} + \frac{V^2}{3}\left(\delta_{rs} - 3\,\frac{V_r V_s}{V^2}\right)$$
$$\times\frac{\mathrm{d}}{\mathrm{d}V}\left(\frac{1}{V^4}\,\frac{\mathrm{d}\delta f_{\mathrm{c,s}}}{\mathrm{d}V}\right)\frac{e^2}{4m^2\omega_0^2}\left(E_r E_s^* + E_r^* E_s - \frac{2}{3}\,|E|^2\delta_{rs}\right). \tag{5.9}$$

The second term on the right-hand side of formula (5.9) proves to be small in comparison with $\delta f_{\mathrm{c,s}}$, provided that

$$V_E < V\,. \tag{5.10}$$

This term makes no contribution to equation (5.7). As a result, Eqn (5.7) assumes the form (compare with Ref. [27])

$$\left(\frac{k^2 V^5 l_{\mathrm{ei}}^2[V_{\mathrm{Te}}]}{9\sqrt{\pi/2}\,V_{\mathrm{Te}}^5}\right)\delta f_{\mathrm{c,s}}$$
$$-\sqrt{\frac{\pi}{8}}\,V_{\mathrm{Te}}^3 V_E^2\left(\frac{1}{V^3}\,\frac{\mathrm{d}^2\delta f_{\mathrm{c,s}}}{\mathrm{d}V^2} - \frac{1}{V^4}\,\frac{\mathrm{d}\delta f_{\mathrm{c,s}}}{\mathrm{d}V}\right) = \frac{e^2\,\delta|E|_{\mathbf{k}}^2}{4m^2\omega_0^2}\,\frac{n_{\mathrm{e}}}{V_{\mathrm{Te}}^2 V_{\mathrm{L}}^3}\,. \tag{5.11}$$

On changing the variables

$$V = V_2 x^{1/5} = V_{\mathrm{Te}}\left[\frac{V_E}{V_{\mathrm{Te}}}\,\frac{1}{kl_{\mathrm{ei}}[V_{\mathrm{Te}}]}\,\frac{15\sqrt{\pi}}{2}\right]^{1/5} x^{1/5}\,,$$
$$\delta f_{\mathrm{c,s}}(V) = x^{1/5}\Phi^{(1)}(x)\,\frac{9n_{\mathrm{e}}\,\delta|V_E|_{\mathbf{k}}^2}{10\pi^{3/2}kl_{\mathrm{ei}}[V_{\mathrm{Te}}]V_E^3 V_{\mathrm{Te}}^2}\,, \tag{5.12}$$

we obtain the following simple differential equation [27]:

$$x^2\Phi_{xx}^{(1)\,\prime\prime} + x\Phi_x^{(1)\,\prime} - \left(\frac{1}{25} + x^2\right)\Phi^{(1)}(x) = x^{4/5}\,. \tag{5.13}$$

The regular at infinity solution to this equation is of the form

$$\Phi^{(1)}(x) = C_1 K_{1/5}(x) - I_{1/5}(x)\int_x^\infty \frac{\mathrm{d}z}{z^{1/5}}\,K_{1/5}(z)$$
$$- K_{1/5}(x)\int_0^x \frac{\mathrm{d}z}{z^{1/5}}\,I_{1/5}(z)\,. \tag{5.14}$$

The boundary condition $V^{-1}\,\mathrm{d}\delta f_{\mathrm{c,s}}/\mathrm{d}V = 0$, which corresponds to the absence of a particle flux source in the velocity space at $V = 0$, enables determination of the constant C_1 for which the following expression is obtained:

$$C_1 = -\frac{1}{2^{1/5}\sqrt{\pi}}\,\Gamma\left(\frac{3}{10}\right)\sin\frac{\pi}{5}\,. \tag{5.15}$$

Formulas (5.12)–(5.15) allow one to find the electron density perturbation which is due to cold subthermal particles:

$$\delta n_{\mathrm{c}} = -\frac{\delta|V_E|_{\mathbf{k}}^2}{4V_{\mathrm{Te}}^2}\left(\frac{V_{\mathrm{Te}}}{V_E}\right)^{12/5}\frac{C_0}{kl_{\mathrm{ee}}[V_{\mathrm{Te}}]\left(kl_{\mathrm{ei}}[V_{\mathrm{Te}}]\right)^{3/5}}\,, \tag{5.16}$$

where

$$C_0 = -\frac{36}{5}\left(\frac{108}{25\pi}\right)^{1/5}\int_0^\infty \frac{\mathrm{d}x}{x^{1/5}}\,\Phi^{(1)}(x) = 44\,.$$

It is now time to define more exactly the conditions wherein the resultant solution (5.14) of Eqn (5.11) and formula (5.16) are valid. We first address ourselves to the assumption that the asymmetric part of the perturbation of subthermal-electron distribution function is small in comparison with the symmetric one. In this case, advantage is taken of the fact that the characteristic electron velocities which appear in expression (5.16) prove to be close to V_2. Then, the condition (5.10) is written as

$$\frac{V_E^2}{V_{\mathrm{Te}}^2} \ll \left(\frac{15\sqrt{\pi}}{2}\right)^{1/2}\frac{1}{\left(kl_{\mathrm{ei}}[V_{\mathrm{Te}}]\right)^{1/2}} \approx \frac{3{,}6}{\left(kl_{\mathrm{ei}}[V_{\mathrm{Te}}]\right)^{1/2}}\,. \tag{5.17}$$

The fulfilment of this condition underlies the possibility of considering the second term in Eqn (5.9) to be small in comparison with the symmetric part $\delta f_{\mathrm{c,s}}$. The first term in the right-hand side of expression (5.9) is relatively small, viz.

$$\sim\frac{kV^4}{3\sqrt{\pi/2}\,\nu_{\mathrm{ei}}[V_{\mathrm{Te}}]V_{\mathrm{Te}}^3}\,,$$

at $V \approx V_2$, when the following condition is fulfilled:

$$\frac{V_E^2}{V_{\mathrm{Te}}^2} \ll \frac{0.05}{\left(kl_{\mathrm{ei}}[V_{\mathrm{Te}}]\right)^{1/2}}\,. \tag{5.18}$$

The latter condition is more rigorous as compared to the condition (5.17). It is the meeting of precisely the condition (5.18) that should be borne in mind, for the fulfilment of this condition automatically implies the satisfaction of the inequality (5.17).

Furthermore, when considering the conditions wherein expression (5.16) is valid, it is well to also bear in mind that the characteristic velocity of the electron distribution should be lower than the Langdon velocity, which permits the use of the distribution (4.14). Accordingly, from the condition

$$V_2 < V_{\mathrm{L}} \tag{5.19}$$

follows that

$$\frac{V_E^2}{V_{\mathrm{Te}}^2} \gg \frac{18}{Z_{\mathrm{eff}}^{10/7}\left(kl_{\mathrm{ei}}[V_{\mathrm{Te}}]\right)^{6/7}}\,. \tag{5.20}$$

Thus, the two conditions (5.18) and (5.20) restrict on either side those values of the intensity of the plasma-heating field at which the relation (5.16) holds true. Meanwhile, the simultaneous fulfilment of these two conditions is possible only when the following relationship is recognized:

$$Z_{\text{eff}} > \frac{62}{\left(kl_{\text{ei}}[V_{\text{Te}}]\right)^{1/4}} . \tag{5.21}$$

In other words, for expression (5.16) to be realized the condition (1.1) specifying the collisionlessness of thermal electrons should be fulfilled and, in addition, the ion charge should be high enough.

The total contribution to the electron density perturbation brought about by a spatially nonuniform high-frequency electromagnetic field, which is due to the ponderomotive contribution from thermal electrons (3.8) and the collisional contribution from subthermal electrons considered in this section, is given by the following formula [25–27]

$$\delta n_{\text{e}} = -n_{\text{e}} \frac{e^2 \, \delta|E|^2_{\mathbf{k}}}{4m^2\omega_0^2 V_{\text{Te}}^2} \times \left\{ 1 + \frac{44}{kl_{\text{ee}}[V_{\text{Te}}]\left(kl_{\text{ei}}[V_{\text{Te}}]\right)^{3/5}} \left(\frac{V_{\text{Te}}}{V_E}\right)^{12/5} \right\} . \tag{5.22}$$

Second, the term in braces in the right-hand side of this equation, which is due to subthermal collisional electrons, proves to be greater than unity (the case corresponding to the ponderomotive effect) only for a relatively weak heating field, when the following relation is met:

$$\frac{V_E^2}{V_{\text{Te}}^2} < \frac{V_{E,\text{d}}^2}{V_{\text{Te}}^2} \equiv \frac{23}{Z_{\text{eff}}^{5/6}\left(kl_{\text{ei}}[V_{\text{Te}}]\right)^{4/3}} . \tag{5.23}$$

Bearing in mind the restriction (5.20) on the intensity of plasma-heating radiation from below, it is easily seen that the condition (5.23), which defines the possibility of neglecting the ponderomotive effect, is possible only when

$$Z_{\text{eff}} > 0.7\left(kl_{\text{ei}}[V_{\text{Te}}]\right)^{4/5} . \tag{5.24}$$

This inequality is, naturally, more rigorous than the constraint (5.21).

We now give the formula for the nonlinear electron density perturbation by a nonuniform high-frequency field, which unifies the result of our consideration in this section with that in Section 3:

$$\delta n_{\text{e}} = -n_{\text{e}} \frac{e^2 \, \delta|E|^2_{\mathbf{k}}}{4m^2\omega_0^2 V_{\text{Te}}^2} \times \left\{ 1 + \frac{1.73 Z_{\text{eff}}^{5/7} / \left(kl_{\text{ei}}[V_{\text{Te}}]\right)^{4/7}}{1 + 0.04 Z_{\text{eff}}^{12/7} (V_E/V_{\text{Te}})^{12/5} \left(kl_{\text{ei}}[V_{\text{Te}}]\right)^{36/35}} \right\} . \tag{5.25}$$

According to this formula, the nonlinear modification of the electron distribution function of low-velocity electrons being heated, which was considered in Section 4 and is due to inverse bremsstrahlung absorption of a spatially uniform radiation field, defines the nonlinear electron density perturbation brought about by a spatially nonuniform radiation field, when the inequalities (5.23) and (5.24) are satisfied.

It is evident from formula (5.25) that strengthening of the weak heating radiation field reduces (suppresses) the collisional influence on the nonlinear electron density perturbation in plasma. In this case, when the heating field intensity attains a value defined by the relation $V_E = V_{E,\text{d}}$ [see formula (5.23)], the collisional influence becomes weaker than the ponderomotive one. With this heating field, the characteristic velocity V_2 is given by

$$(V_2)_{\text{d}} = \frac{2.3}{Z_{\text{eff}}^{1/12}\left(kl_{\text{ei}}[V_{\text{Te}}]\right)^{1/3}} V_{\text{Te}} . \tag{5.26}$$

This value is significantly higher than the amplitude of the electron quiver velocity in the heating field, because

$$\frac{(V_2)_{\text{d}}}{V_{E,\text{d}}} \cong 0.5 Z^{1/3} \left(kl_{\text{ei}}[V_{\text{Te}}]\right)^{1/3} \gg 1 . \tag{5.27}$$

That is why the complete suppression of the contribution from subthermal collisional electrons to the nonlinear electron density perturbation, described by relationship (5.25), takes place in conditions wherein the expansion of the collisional Boltzmann integral in a power series of the high-frequency pump field intensity is appropriate. In other words, this suppression occurs when Eqns (4.1) and (5.3), which underlie our consideration, are applicable.

6. Effect of a heating high-frequency field on the weakly collisional dissipation of ion-acoustic waves

In the previous section we saw how a relatively weak high-frequency heating electromagnetic field in the conditions (5.19)–(5.21) brings about a nonlinear modification of the collisional contribution to the electron density perturbation, which arises due to the nonuniform pump field intensity. This effect, which is caused by collisional subthermal electrons, emerges under conditions (1.1) which commonly permit the plasma to be spoken of as collisionless. In our consideration, subthermal electrons are collisional and the low-intensity heating field is, in accordance with formula (4.14), responsible for a nonlinear modification of the functional dependence of the subthermal-electron velocity distribution. The influence of this nonlinear modification of the electron distribution on the collisional absorption of ion-acoustic waves by electrons is considered in this section on the basis of an analytical theory [25–27].

Since the effect discussed in this section is only caused by subthermal electrons, the contribution to the longitudinal plasma permittivity made by thermal collisionless electrons, whose distribution is hardly influenced by the weak heating field, does not experience a nonlinear modification. The contribution from thermal electrons will therefore be described by formulas (2.10) and (2.15).

For subthermal electrons, from Eqn (3.3) with $\mathbf{E}_0 = -\text{grad}\,\varphi$ we have

$$-\,\mathrm{i}(\omega - \mathbf{kV})\delta f_{\text{c}} - J_{\text{ee}}[\delta f_{\text{c}}] - J_{\text{ei}}[\delta f_{\text{c}}] = -\frac{e^2}{4m^2\omega_0^2}\left(E_k E_j^* + E_k^* E_j\right)\frac{\partial}{\partial V_k} J_{\text{ei}}\left[\frac{\partial \delta f}{\partial V_j}\right] + \frac{\mathrm{i}e\varphi\mathbf{kV}}{mV}\frac{\mathrm{d}f_{0\text{s}}(V)}{\mathrm{d}V} . \tag{6.1}$$

Here, $f_{0\text{s}}$ is described by formula (4.14). Furthermore, the right-hand side of Eqn (6.1) contains a term nonlinear in the electric field intensity of the radiation that heats the plasma.

To describe the subthermal collisional electrons concerned, we resort to the expansion

$$\delta f_{\text{c}} = \delta f_{\text{c,s}} + \delta f_{\text{c,a}} \tag{6.2}$$

of the distribution function perturbation into symmetric and antisymmetric components, which is routinely used in our treatment. In this case, it will be assumed that

$$\delta f_{\mathrm{a,s}} \ll \delta f_{\mathrm{c,s}} - \frac{e\varphi}{mV}\frac{\mathrm{d}f_{\mathrm{s,c}}}{\mathrm{d}V}\,. \tag{6.3}$$

Then we obtain from Eqn (6.1) that

$$-\mathrm{i}\omega\,\delta f_{\mathrm{c,s}} + \mathrm{i}\langle \mathbf{kV}\,\delta f_{\mathrm{c,a}}\rangle - J_{\mathrm{ee}}[\delta f_{\mathrm{c,s}}] = \nu_{\mathrm{ee}}[V_{\mathrm{Te}}]\sqrt{\frac{\pi}{8}}\,V_{\mathrm{Te}}^3 V_E^2\left(\frac{1}{V^3}\frac{\mathrm{d}^2\delta f_{\mathrm{c,s}}}{\mathrm{d}V^2} - \frac{1}{V^4}\frac{\mathrm{d}\delta f_{\mathrm{c,s}}}{\mathrm{d}V}\right), \tag{6.4}$$

$$-\mathrm{i}\omega\,\delta f_{\mathrm{c,a}} - J_{\mathrm{ei}}[\delta f_{\mathrm{c,a}}] - \frac{3\sqrt{2\pi}}{4}\,\nu_{\mathrm{ei}}[V_{\mathrm{Te}}]\left(|\mathbf{VV}_E|^2 - \frac{1}{3}\,V^2 V_E^2\right) \times \frac{V_{\mathrm{Te}}^3}{V}\frac{\mathrm{d}}{\mathrm{d}V}\left(\frac{1}{V^4}\frac{\mathrm{d}\delta f_{\mathrm{c,a}}}{\mathrm{d}V}\right) = -\mathrm{i}\mathbf{kV}\left[\delta f_{\mathrm{c,s}} - \frac{e\varphi}{mV}\frac{\mathrm{d}f_{0\mathrm{s}}(V)}{\mathrm{d}V}\right]. \tag{6.5}$$

The solution of Eqn (6.5) can be written directly as

$$\delta f_{\mathrm{c,a}} = -\frac{\mathrm{i}\mathbf{kV}\big[\delta f_{\mathrm{c,s}}(V) - (e\varphi/mV)\,\mathrm{d}f_{\mathrm{c,s}}/\mathrm{d}V\big]}{\nu_{\mathrm{ei}}[V_{\mathrm{Te}}]3\sqrt{\pi/2}\,(V_{\mathrm{Te}}^3/V^3) - \mathrm{i}\omega} + \frac{3\sqrt{\pi/8}\,V_{\mathrm{Te}}^3\nu_{\mathrm{ei}}[V_{\mathrm{Te}}]\big(|\mathbf{VV}_E|^2 - (1/3)V^2|V_E|^2\big)}{9\sqrt{\pi/2}\,\nu_{\mathrm{ei}}[V_{\mathrm{Te}}](V_{\mathrm{Te}}/V^3) - \mathrm{i}\omega} \times \frac{1}{V}\frac{\mathrm{d}}{\mathrm{d}V}\left(\frac{1}{V^4}\frac{\mathrm{d}\delta f_{\mathrm{c,s}}}{\mathrm{d}V}\right). \tag{6.6}$$

The second term in formula (6.6), first, is small in comparison with $\delta f_{\mathrm{c,s}}$ due to the smallness of (V_E^2/V^2) and, second, makes no contribution to Eqn (6.4). Therefore, from Eqn (6.4) we now have

$$-\mathrm{i}\omega\,\delta f_{\mathrm{c,s}} + \frac{k^2V^2\big[\delta f_{\mathrm{c,s}} - (e\varphi/mV)\,\mathrm{d}f_{0\mathrm{s}}/\mathrm{d}V\big]}{3\big\{\nu_{\mathrm{ei}}[V_{\mathrm{Te}}]3\sqrt{\pi/2}\,(V_{\mathrm{Te}}^3/V^3) - \mathrm{i}\omega\big\}} - \sqrt{\frac{\pi}{8}}\,\nu_{\mathrm{ei}}[V_{\mathrm{Te}}]V_E^2\left(\frac{V_{\mathrm{Te}}^3}{V^3}\frac{\mathrm{d}^2\delta f_{\mathrm{c,s}}}{\mathrm{d}V^2} - \frac{V_{\mathrm{Te}}^3}{V^4}\frac{\mathrm{d}\delta f_{\mathrm{c,s}}}{\mathrm{d}V}\right) - \nu_{\mathrm{ee}}[V_{\mathrm{Te}}]\,\frac{V_{\mathrm{Te}}^2}{V^2}\frac{\mathrm{d}}{\mathrm{d}V}\left(V^2\,\frac{\mathrm{d}\delta f_{\mathrm{c,s}}}{\mathrm{d}V}\right) = 0\,. \tag{6.7}$$

To reveal, with the aid of Eqn (6.7), the role played by subthermal electrons with velocities of the order of and smaller than the Langdon velocity, we change the variable:

$$\zeta = \frac{V}{V_{\mathrm{L}}}\,, \qquad \delta f_{\mathrm{c,s}}(V) = F_{\mathrm{L}}(\zeta)\,. \tag{6.8}$$

Then, Eqn (6.7) can be rewritten as

$$-\mathrm{i}\omega\,\frac{9\sqrt{\pi/2}\,\nu_{\mathrm{ei}}[V_{\mathrm{Te}}]}{k^2V^2}\left(\frac{V_{\mathrm{Te}}}{V}\right)^5 F_{\mathrm{L}}(\zeta) + \zeta^5\left[F_{\mathrm{L}}(\zeta) + \frac{en_{\mathrm{e}}\varphi}{\kappa_{\mathrm{B}}T_{\mathrm{e}}}\frac{1}{(2\pi)^{3/2}V_{\mathrm{Te}}^3}\frac{\zeta^3}{\zeta^3+1}\right] \times \left[1 - \mathrm{i}\,\frac{\omega\zeta^3}{\nu_{\mathrm{ei}}[V_{\mathrm{Te}}]3\sqrt{\pi/2}}\left(\frac{V_{\mathrm{L}}}{V_{\mathrm{Te}}}\right)^3\right]^{-1} - \mu^{7/3}\left[\frac{1}{\zeta^2}\frac{\mathrm{d}}{\mathrm{d}\zeta}\left(\zeta^2\,\frac{\mathrm{d}F_{\mathrm{L}}}{\mathrm{d}\zeta}\right) + \frac{1}{\zeta^3}\frac{\mathrm{d}^2F_{\mathrm{L}}}{\mathrm{d}\zeta^2} - \frac{1}{\zeta^4}\frac{\mathrm{d}F_{\mathrm{L}}}{\mathrm{d}\zeta}\right] = 0\,, \tag{6.9}$$

where

$$\mu \cong \frac{5}{Z_{\mathrm{eff}}^{10/7}\big(kl_{\mathrm{ei}}[V_{\mathrm{Te}}]\big)^{6/7}}\frac{V_{\mathrm{Te}}^2}{V_E^2}\,. \tag{6.10}$$

When seeking an approximate solution to Eqn (6.9), we first of all assume that

$$1 \gg \frac{\omega}{\nu_{\mathrm{ei}}[V_{\mathrm{Te}}]3\sqrt{\pi/2}}\left(\frac{V_{\mathrm{L}}}{V_{\mathrm{Te}}}\right) \approx 4\times10^{-3}\,\frac{Z_{\mathrm{eff}}^{3/2}}{A_{\mathrm{eff}}^{1/2}}\,kl_{\mathrm{ei}}[V_{\mathrm{Te}}]\,\frac{V_E^2}{V_{\mathrm{Te}}^2}\,. \tag{6.11}$$

This permits us to neglect the expression proportional to the frequency ω in the denominator of the second term of Eqn (6.9). Furthermore, it will be recalled that the effect of inverse bremsstrahlung absorption on the distribution modification under consideration can, according to the condition (4.17), manifest itself only when the left-hand side of the inequality (4.16) is violated, which corresponds to $\mu \gg 1$. That is why, endeavoring to consider the opposite limit, we suppose that

$$\mu \ll 1\,. \tag{6.12}$$

In this limit, the differential operator in Eqn (6.9) can be neglected, making it possible to immediately write down the solution of the elementary equation arising in this case. The solution sought has the following simple form

$$F_{\mathrm{L}}(\zeta) = -\frac{e\varphi n_{\mathrm{e}}}{\kappa_{\mathrm{B}}T_{\mathrm{e}}}\frac{1}{(2\pi)^{3/2}V_{\mathrm{Te}}^3} \times \left\{\frac{\zeta^3}{1+\zeta^3} + \frac{1}{\zeta^2(1+\zeta^3)}\frac{\mathrm{i}\omega}{kV_{\mathrm{Te}}}\frac{9\sqrt{\pi/2}}{kl_{\mathrm{ei}}[V_{\mathrm{Te}}]}\left(\frac{V_{\mathrm{Te}}}{V_{\mathrm{L}}}\right)^5\right\} \tag{6.13}$$

subject to the inequality

$$\frac{\omega}{kV_{\mathrm{Te}}}\frac{9\sqrt{\pi/2}}{kl_{\mathrm{ei}}[V_{\mathrm{Te}}]}\left(\frac{V_{\mathrm{Te}}}{V_{\mathrm{L}}}\right)^5 \ll 1\,. \tag{6.14}$$

Upon integration of the real part of formula (6.13) over the subthermal-electron velocity region, it makes a contribution to the real part of the density perturbation, which is negligible in comparison with the collisionless contribution (2.7) of thermal electrons. By contrast, the imaginary part of formula (6.13) gives the collisional dissipative contribution

$$\delta n_{\mathrm{e,c}} = -\frac{e\varphi n_{\mathrm{e}}}{\kappa_{\mathrm{B}}T_{\mathrm{e}}}\frac{\mathrm{i}\omega}{kV_{\mathrm{Te}}}\frac{4\sqrt{3}\,\pi^{2/3}}{kl_{\mathrm{ei}}[V_{\mathrm{Te}}]}\frac{1}{Z_{\mathrm{eff}}^{2/3}}\left(\frac{V_{\mathrm{Te}}}{V_E}\right)^{4/3} \tag{6.15}$$

which is due to subthermal electrons and is of interest to us. It describes the competition between the effect of collisions and the Cherenkov effect, the latter corresponding to the collisionless contribution to the density perturbation (2.14).

Formula (6.15) in combination with formulas (2.7) and (2.14) now allows us to write down the following expression for the electron contribution from thermal and subthermal electrons to the longitudinal plasma permittivity [25, 27]:

$$\delta\varepsilon_{\mathrm{l,e}}(\omega,k) = \frac{1}{k^2r_{\mathrm{De}}^2}\left(1 + \frac{\mathrm{i}\omega}{kV_{\mathrm{Te}}}\left\{\sqrt{\frac{\pi}{2}} + \frac{4\sqrt{3}\,\pi^{2/3}}{kl_{\mathrm{ei}}[V_{\mathrm{Te}}]}\frac{1}{Z_{\mathrm{eff}}^{2/3}}\left(\frac{V_{\mathrm{Te}}}{V_E}\right)^{4/3}\right\}\right). \tag{6.16}$$

The collisional dissipation proves to be more significant than the Cherenkov dissipation only at moderate field intensities of the radiation that heats the plasma, when the inequality

$$\frac{V_E^2}{V_{\mathrm{Te}}^2} < \frac{41}{Z_{\mathrm{eff}}\big(kl_{\mathrm{ei}}[V_{\mathrm{Te}}]\big)^{3/2}} \tag{6.17}$$

is satisfied. Since the right-hand side of this inequality is quite small, the collisional dissipation becomes insignificant when the plasma is exposed to the heating field with a very low intensity.

When inequality (6.17) is fulfilled, the right-hand side of Eqn (6.11) proves to be smaller than

$$0.016\left(\frac{Z_{\mathrm{eff}}}{A_{\mathrm{eff}}}\right)^{1/2}\frac{1}{\sqrt{kl_{\mathrm{ei}}[V_{\mathrm{Te}}]}}\,,$$

making the inequality (6.11) always satisfiable in the conditions specified by Eqn (1.1).

We must ascertain the feasibility of the above assumption (6.14). To this end we note that a comparison of formulas (6.16) and (2.40) testifies to the fact that the nonlinear collisional contribution in formula (6.16) overrides the linear collisional contribution in formula (2.40), i.e. advantage should be taken of formula (6.16) in lieu of formula (2.40) when the heating field intensity becomes high enough:

$$\frac{V_E^2}{V_{\mathrm{Te}}^2} > \frac{18}{Z_{\mathrm{eff}}^{10/7}\big(kl_{\mathrm{ei}}[V_{\mathrm{Te}}]\big)^{6/7}}\,. \tag{6.18}$$

Then, the left-hand side of inequality (6.14) proves to be smaller than

$$5\times10^{-3}\sqrt{\frac{Z_{\mathrm{eff}}}{A_{\mathrm{eff}}}}\,Z_{\mathrm{eff}}^{10/7}\big(kl_{\mathrm{ei}}[V_{\mathrm{Te}}]\big)^{3/7}\,. \tag{6.19}$$

On the other hand, the simultaneous satisfaction of inequalities (6.17) and (6.18) imposes the condition

$$kl_{\mathrm{ei}}[V_{\mathrm{Te}}] \leqslant 3.6Z_{\mathrm{eff}}^{2/3}\,.$$

Then, expression (6.19) and hence the left-hand side of inequality (6.14) prove to be much smaller than the following expression:

$$0.0086\sqrt{\frac{Z_{\mathrm{eff}}}{A_{\mathrm{eff}}}}\,Z_{\mathrm{eff}} < \frac{Z_{\mathrm{eff}}}{166}\,.$$

This condition ensures the fulfilment of the condition (6.14) and substantiates the solution (6.13), and therefore the validity of the relations (6.15) and (6.16).

We employ formulas (2.11), (2.12), and (6.16) to obtain the following expression for the electron contribution to the damping decrement of ion-acoustic waves [25, 26]:

$$\gamma = \frac{kV_{\mathrm{s}}^2}{V_{\mathrm{Te}}}\left\{\sqrt{\frac{\pi}{8}} + \frac{2\sqrt{3}\,\pi^{2/3}}{kl_{\mathrm{ei}}[V_{\mathrm{Te}}]}\frac{1}{Z_{\mathrm{eff}}^{2/3}}\left(\frac{V_{\mathrm{Te}}}{V_E}\right)^{4/3}\right\}. \tag{6.20}$$

The qualitative distinction between this expression and formula (2.41) arises from its nonlinear dependence on the heating field.

To summarize this section we give the formula conclusive for Sections 2 and 6, which describes the weakly collisional electron contribution to the longitudinal plasma permittivity for $\omega \ll kV_{\mathrm{Te}}$:

$$\begin{aligned}\delta\varepsilon_{\mathrm{l,e}}(\omega,k) &= \frac{1}{k^2r_{\mathrm{De}}^2}\left[1 + \mathrm{i}\,\frac{\omega}{kV_{\mathrm{Te}}}\left(\sqrt{\frac{\pi}{2}} + 2{,}17\left(\frac{Z_{\mathrm{eff}}^2}{k^3l_{\mathrm{ei}}^3[V_{\mathrm{Te}}]}\right)^{1/7}\right.\right.\\ &\times\left\{1 + 2.17\left(\frac{Z_{\mathrm{eff}}^2}{k^3l_{\mathrm{ei}}^3[V_{\mathrm{Te}}]}\right)^{1/7}\right.\\ &\times\left.\left.\left.\left[\frac{4\sqrt{3}\,\pi^{2/3}}{Z_{\mathrm{eff}}^{2/3}kl_{\mathrm{ei}}[V_{\mathrm{Te}}]}\left(\frac{V_{\mathrm{Te}}}{V_E}\right)^{4/3}\right]^{-1}\right\}^{-1}\right)\right]. \end{aligned} \tag{6.21}$$

For very low intensities of the heating electromagnetic field, formula (6.21) goes over into formula (2.40). When the intensity of the heating field deliberately satisfies the condition (6.18), formula (6.21) goes over into formula (6.16).

7. Filamentation instability of a weakly collisional plasma

This section is concerned with the filamentation instability of electromagnetic radiation in fully ionized, weakly collisional plasma. The material of Sections 3 and 5 underlies the presentation of the theory of this instability. When considering this instability by way of illustration in a spatially uniform plasma, our concern will be with the instability of the plasma-heating radiation, to which we put in correspondence in formula (3.2) the following dependence

$$\mathbf{E}(\mathbf{r},t) \to \mathbf{E}\exp(\mathrm{i}\mathbf{k}_0\mathbf{r})\,. \tag{7.1}$$

In this case, the heating field frequency ω_0 and the wave vector $\mathbf{k}_0$ are related as

$$\omega_0^2 = \omega_{\mathrm{Le}}^2 + c^2k_0^2\,, \tag{7.2}$$

where $\omega_{\mathrm{Le}} = \sqrt{4\pi e^2 n_{\mathrm{e0}}/m_{\mathrm{e}}}$ is the Langmuir electron frequency, and c is the velocity of light.

The filamentation instability corresponds to the occurrence, along with the field (7.1) heating the plasma, of the perturbation field corresponding to

$$\begin{aligned}\mathbf{E}(\mathbf{r},t) \to{}& \mathbf{E}\exp(\mathrm{i}\mathbf{k}_0\mathbf{r}) + \big[\delta\mathbf{E}_+(\mathbf{r})\exp(\mathrm{i}\mathbf{k}\mathbf{r})\\ &+ \delta\mathbf{E}_-(\mathbf{r})\exp(-\mathrm{i}\mathbf{k}\mathbf{r})\big]\exp(\mathrm{i}\mathbf{k}_0\mathbf{r})\,.\end{aligned} \tag{7.3}$$

The dependence of $\delta\mathbf{E}_\pm$ on the coordinates only relates to the stationary formulation of the problem on filamentation instability, when the instability development is considered in space. The instability corresponds to the growth of filamentary perturbations in space. We retain only the terms linear in the perturbation field to obtain

$$\begin{aligned}\big|\mathbf{E}(\mathbf{r},t)\big|^2 \to{}& |\mathbf{E}_0|^2 + \delta|\mathbf{E}|_{\mathbf{k}}^2\exp(\mathrm{i}\mathbf{k}\mathbf{r}) + \delta|\mathbf{E}|_{-\mathbf{k}}^2\exp(-\mathrm{i}\mathbf{k}\mathbf{r})\\ \equiv{}& |\mathbf{E}_0|^2 + (\mathbf{E}_0^*\,\delta\mathbf{E}_+ + \mathbf{E}_0\,\delta\mathbf{E}_-^*)\exp(\mathrm{i}\mathbf{k}\mathbf{r})\\ &+ (\mathbf{E}_0\,\delta\mathbf{E}_+^* + \mathbf{E}_0^*\,\delta\mathbf{E}_-)\exp(-\mathrm{i}\mathbf{k}\mathbf{r})\,.\end{aligned} \tag{7.4}$$

Bearing in mind the consequence of the Maxwell equations

$$\Delta\mathbf{E} + \frac{\omega_0^2}{c^2}\left\{1 - \frac{\omega_{\mathrm{Le}}^2}{\omega_0^2}\left[1 + \frac{\delta n(\mathbf{r})}{n_{\mathrm{e0}}}\right]\right\}\mathbf{E} = 0\,, \tag{7.5}$$

where

$$\delta n(\mathbf{r}) = \delta n_{\mathbf{k}}\exp(\mathrm{i}\mathbf{k}\mathbf{r}) + \delta n_{-\mathbf{k}}\exp(-\mathrm{i}\mathbf{k}\mathbf{r})\,,$$

we next assume that the vector $\mathbf{k}_0$ is aligned with the z-axis and the vector $\mathbf{k}$ is orthogonal to this direction. Then, supposing the variation of $\delta\mathbf{E}_\pm$ over a distance $\lambdabar_0 = 1/k_0$ to be small, we obtain from Eqn (7.5) the following reduced equations

$$\left(2\mathrm{i}k_0\,\frac{\mathrm{d}}{\mathrm{d}z} - k^2\right)\delta\mathbf{E}_\pm = \frac{\omega_{\mathrm{Le}}^2\,\delta n_{\pm\mathbf{k}}}{c^2 n_{\mathrm{e}0}}\,\mathbf{E}\,. \tag{7.6}$$

In accordance with the results of Sections 3 and 5, we have

$$\frac{\delta n_{\pm\mathbf{k}}}{n_{\mathrm{e}0}} = -\frac{e^2\,\delta|\mathbf{E}|_{\pm\mathbf{k}}^2}{4m_0^2\omega_0^2 V_{\mathrm{Te}}^2}\left[1 + F(k)\right], \tag{7.7}$$

where

$$F(k) = \frac{1.73 Z_{\mathrm{eff}}^{5/7}}{\left(kl_{\mathrm{ei}}[V_{\mathrm{Te}}]\right)^{4/7}\left[1 + 0.04 Z_{\mathrm{eff}}^{12/7}(V_E/V_{\mathrm{Te}})^{12/5}\left(kl_{\mathrm{ei}}[V_{\mathrm{Te}}]\right)^{36/35}\right]}\,. \tag{7.8}$$

Employing formula (7.4), we obtain from Eqn (7.6) a system of two equations

$$\begin{aligned}
&\left(2\mathrm{i}k_0\,\frac{\mathrm{d}}{\mathrm{d}z} - k^2\right)(\mathbf{E}^*\,\delta\mathbf{E}_+) \\
&\qquad = -\frac{\omega_{\mathrm{Le}}^2 V_E^2(1+F)}{4c^2 V_{\mathrm{Te}}^2}(\mathbf{E}^*\,\delta\mathbf{E}_+ + \mathbf{E}\,\delta\mathbf{E}_-^*)\,, \\
&\left(-2\mathrm{i}k_0\,\frac{\mathrm{d}}{\mathrm{d}z} - k^2\right)(\mathbf{E}\,\delta\mathbf{E}_-^*) \\
&\qquad = -\frac{\omega_{\mathrm{Le}}^2 V_E^2(1+F)}{4c^2 V_{\mathrm{Te}}^2}(\mathbf{E}^*\,\delta\mathbf{E}_+ + \mathbf{E}\,\delta\mathbf{E}_-^*)\,,
\end{aligned} \tag{7.9}$$

where V_E is the amplitude modulus of the electron quiver velocity in the pump field. The solution of this system bears the following functional dependence on the coordinate:

$$\exp Gz\,. \tag{7.10}$$

Here, the spatial gain coefficient G is defined by the relation

$$G^2 = \frac{1}{4k_0^2}\left\{-k^4 + \frac{\omega_{\mathrm{Le}}^2 V_E^2 k^2}{2c^2 V_{\mathrm{Te}}^2}\left[1 + F(k)\right]\right\}. \tag{7.11}$$

It is advantageous to rewrite formula (7.11) using a somewhat different notation, which will simplify its comparison with the theoretical results obtained in several papers. However, prior to doing this, there is a good reason to violate the sequence of our exposition. We are reminded that earlier it was general practice to consider two qualitatively different reasons for the emergence of radiation filamentation in a fully ionized plasma [51, 52]. One of them lies with the ponderomotive force (or the Miller force), which forces out the plasma electrons from the domain of strong electromagnetic field, resulting in the increase in field intensity in the rarefaction domain originated. The other reason is identified with the thermal mechanism and consists in the following. Owing to the inverse bremsstrahlung absorption of radiation, the plasma temperature increases. In circumstances where the pressure is approximately constant, this is responsible for a lowering of electron density, with a consequent increase in field intensity once again. In this case, in the context of the thermal mechanism, the rise in temperature of the electrons being heated is proportional to the high-frequency conductivity

$$\sigma_{\mathrm{hf}} = \frac{e^2 n_{\mathrm{e}}}{m_{\mathrm{e}}\omega_0^2}\,\nu_{\mathrm{ei}}[V_{\mathrm{Te}}] \tag{7.12}$$

and inversely proportional to the electron thermal conductivity coefficient, which assumes the form (1.27) in the limit $Z_{\mathrm{eff}} \gg 1$. The analogue of formula (7.11), which arises when the ponderomotive and thermal mechanisms are included, is written down as

$$G^2 = \frac{1}{4k_0^2}\left[-k^4 + \frac{\omega_{\mathrm{Le}}^2}{c^2}\left(\frac{k^2 V_E^2}{2V_{\mathrm{Te}}^2} + \frac{\sigma_{\mathrm{hf}}|\mathbf{E}|^2}{\chi_{\mathrm{SH}} T_{\mathrm{e}}}\right)\right]. \tag{7.13}$$

For several years, the studies of the filamentation theory in conditions (1.1) were pursued in the expectation of describing the macroscopic motion in a weakly collisional plasma on the basis of the notion of nonlocal heat transfer, with the aid of which the generalized Fourier – Fick law for the spatial dependence of temperature perturbation $\sim \delta T \exp(\mathrm{i}\mathbf{kr})$ is written in the form of expression (1.25). In accord with this formula we arrive at the following generalization of the relationship (7.13):

$$G^2 = \frac{1}{4k_0^2}\left\{-k^4 + \frac{\omega_{\mathrm{Le}}^2}{c^2}\left[\frac{k^2 V_E^2}{2V_{\mathrm{Te}}^2} + \frac{\sigma_{\mathrm{hf}}|\mathbf{E}|^2}{\chi(k) T_{\mathrm{e}}}\right]\right\}. \tag{7.14}$$

Relating the result (7.11) to the notion of nonlocal heat transfer, one can rewrite expression (7.11) in the form (7.14) if the electron thermal conductivity coefficient is taken as

$$\chi(k) = \chi_{\mathrm{SH}}\,\frac{3\pi}{64k^2\left(l_{\mathrm{ei}}[V_{\mathrm{Te}}]\right)^2 F(k)}\,. \tag{7.15}$$

The generalization of the formulas (1.21) and (7.15), which bears the interpolation dependence on the magnitude of the wave vector k when passing from the weakly collisional conditions (1.1) to the strongly collisional conditions $kl_{\mathrm{ei}}[V_{\mathrm{Te}}] \ll 1$, is the relationship

$$\chi(k) = \frac{\chi_{\mathrm{SH}}}{\Xi(k)}\,, \tag{7.16}$$

where

$$\begin{aligned}
\Xi(k) &= 1 + \frac{64}{3\pi}\left(kl_{\mathrm{ei}}[V_{\mathrm{Te}}]\right)^2 F(k) \\
&\cong 1 + \frac{12\left(kl_{\mathrm{ei}}[V_{\mathrm{Te}}]\right)^{10/7}}{1 + 0.04 Z_{\mathrm{eff}}^{12/7}(V_E/V_{\mathrm{Te}})^{12/5}\left(kl_{\mathrm{ei}}[V_{\mathrm{Te}}]\right)^{36/35}}\,.
\end{aligned} \tag{7.17}$$

Prior to the work [7] concerned with an analytical theory of radiation filamentation in a weakly collisional plasma, a series of foreign theoretical studies in this field had been known, which were based on the numerical solution of the kinetic Boltzmann equation. These papers considered only the linear theory, in which the nonlinear dependence (7.17) on the intensity of the heating field subject to filamentation was not revealed. In other words, the results of the foreign papers only apply to low-intensity plasma-heating radiation, when we adopt

$$Z_{\mathrm{eff}}\,\frac{V_E^2}{V_{\mathrm{Te}}^2} < \frac{15}{Z_{\mathrm{eff}}^{3/7}(kl_{\mathrm{ei}})^{6/7}} \ll 1\,. \tag{7.18}$$

For such weak fields being the case, formula (7.17) assumes the form

$$\Xi_0(k) = 1 + a\left(\sqrt{Z_{\mathrm{eff}}}\, kl_{\mathrm{ei}}[V_{\mathrm{Te}}]\right)^{\alpha}. \tag{7.19}$$

Then, according to formula (7.17), the following result of the analytical theory holds: $a \cong 12$, $\alpha = 10/7 \cong 1.43$ [7].

In the foreign papers proceeded from the solution of the Boltzmann equation, different values of the α index of a power were given. The first value of two, given in Ref. [38], is related to the Padé–Borel approximation (1.30) which corresponds to the emergence of corrections to the thermal conductivity coefficient of order $\left(kl_{\mathrm{ei}}[V_{\mathrm{Te}}]\right)^2$, when the Hilbert–Chapman–Enskog method is taken advantage of (see, for instance, Refs [38, 39]). Furthermore, here we give the values of the α power derived in the special studies on the numerical solution of the Boltzmann equation: $\alpha = 4/3$ [1], $\alpha = 1$ [2], $\alpha = 1.148$ [15], $\alpha = 1.15$ [16], and, finally, $\alpha = 1.44$ and $a \cong 12$ in Ref. [6]. The numerical finding of Ref. [6] is closest to the analytical result of Ref. [7]. After the advent of the analytical theory of filamentation instability [7], the work on numerical simulation of the solutions of the Boltzmann equation, as applied to this problem, no longer appeared to be quite indispensable. This, in particular, accounts for the absence of numerical investigations for plasma-heating radiation intensities violating the inequality (7.18), i.e. in the case when the Langdon velocity is small in comparison with the electron thermal velocity. In this sense we can state with assurance that the analogues of the nonlinear dependence (7.8) which describes the suppression of collisional influence on the filamentation phenomenon due to relatively weak pump fields cannot be found in the world scientific literature.

We next consider the regularities for those values of the pump field intensity, which correspond to the filamentation instability threshold. Let L denote the dimension of the plasma domain in which filamentary perturbations build up along the z-axis. As usual we define the threshold sought by the condition

$$G_{\max} L = 2\pi. \tag{7.20}$$

For comparison with a conventional theory we first consider the implications of the approach which does not take into account the weakly collisional effects of interest, when relation (7.13) holds true. Then, for the wave vector at which the spatial gain coefficient is the highest we have

$$k^2_{\max} = \frac{\omega^2_{\mathrm{Le}} V^2_E}{4c^2 V^2_{\mathrm{Te}}}, \tag{7.21}$$

and the peak gain coefficient takes the form

$$G^2_{\max} = \frac{\omega^2_{\mathrm{Le}} V^2_{\mathrm{Te}}}{64 c^2 k_0^2 V^2_{\mathrm{Te}}} \left\{ \frac{\omega^2_{\mathrm{Le}} V^2_E}{c^2 V^2_{\mathrm{Te}}} + \frac{3\pi}{8\left(l_{\mathrm{ei}}[V_{\mathrm{Te}}]\right)^2} \right\}. \tag{7.22}$$

In accordance with formula (7.22), the thermal mechanism in a strongly collisional plasma is commonly assumed to be the decisive one when

$$\frac{3\pi}{8}\frac{c^2}{\omega^2_{\mathrm{Le}}} \gg \frac{V^2_E}{V^2_{\mathrm{Te}}}\left(l_{\mathrm{ei}}[V_{\mathrm{Te}}]\right)^2 \tag{7.23}$$

for sufficiently short mean free paths. In this case, according to formula (7.20), for the filamentation instability threshold we have

$$\frac{V^2_{E,\,\mathrm{th}}}{V^2_{\mathrm{Te}}} = \frac{2048\pi c^2 k_0^2 \left(l_{\mathrm{ei}}[V_{\mathrm{Te}}]\right)^2}{3\omega^2_{\mathrm{Le}} L^2}. \tag{7.24}$$

In this case, the inequality (7.23) takes on the form

$$L > \frac{128}{3} k_0 \left(l_{\mathrm{ei}}[V_{\mathrm{Te}}]\right)^2. \tag{7.25}$$

In the limit opposite to the inequality (7.23), when radiation filamentation is believed to be determined by the ponderomotive mechanism, for the filamentation instability threshold we find

$$\frac{V^2_{E,\,\mathrm{th}}}{V^2_{\mathrm{Te}}} = \frac{16\pi c^2 k_0^2}{\omega^2_{\mathrm{Le}}(Lk_0)}. \tag{7.26}$$

We now address ourselves to those implications of the theory of a weakly collisional plasma, which correspond to the radiation filamentation effect and permit us to describe the passage from the ponderomotive mechanism to the thermal one with greater consistency in comparison with relationship (7.14) which indicates only the two corresponding limits. To this end we employ formulas (7.14)–(7.17) which allow us to write down the following relation for the spatial gain coefficient of filamentation instability:

$$G^2 = \frac{1}{4k_0^2}\left\{ -k^4 + \frac{\omega^2_{\mathrm{Le}} V^2_E k^2}{2c^2 V^2_{\mathrm{Te}}}\left[1 + F(k) + \frac{3\pi}{64\left(kl_{\mathrm{ei}}[V_{\mathrm{Te}}]\right)^2}\right]\right\}. \tag{7.27}$$

In the limit of a low-intensity pump field (7.18) and with the condition (1.1), i.e. the weakly collisional plasma condition, formula (7.27) transforms to

$$G^2 = \frac{1}{4k_0^2}\left\{ -k^4 + \frac{\omega^2_{\mathrm{Le}} V^2_E k^2}{2c^2 V^2_{\mathrm{Te}}}\left[1 + \frac{1{,}73 Z_{\mathrm{eff}}^{5/7}}{\left(kl_{\mathrm{ei}}[V_{\mathrm{Te}}]\right)^{4/7}}\right]\right\}. \tag{7.28}$$

The weakly collisional effect under description prevails over the ponderomotive one (as well as over the conventional thermal effect) provided that

$$2.6 Z_{\mathrm{eff}}^{5/4} > kl_{\mathrm{ei}}[V_{\mathrm{Te}}] \gg 1. \tag{7.29}$$

Then, the increment is at its maximum for

$$k_{\max} = \frac{0.66 Z_{\mathrm{eff}}^{5/18}}{\left(l_{\mathrm{ei}}[V_{\mathrm{Te}}]\right)^{2/9}} \left(\frac{\omega^2_{\mathrm{Le}} V^2_E}{c^2 V^2_{\mathrm{Te}}}\right)^{7/18}, \tag{7.30}$$

when

$$G^2_{\max} = \frac{9k^4_{\max}}{k_0^2} = \frac{0.085 Z_{\mathrm{eff}}^{10/9}}{k_0^2\left(l_{\mathrm{ei}}[V_{\mathrm{Te}}]\right)^{8/9}} \left(\frac{\omega^2_{\mathrm{Le}} V^2_E}{c^2 V^2_{\mathrm{Te}}}\right)^{14/9}. \tag{7.31}$$

In accord with formulas (7.31) and (7.20), for the filamentation instability threshold in the linear weak-field mode (7.18) we obtain

$$\frac{V^2_{E,\,\mathrm{th}}}{V^2_{\mathrm{Te}}} \cong \frac{50 c^2 k_0^2 \left(l_{\mathrm{ei}}[V_{\mathrm{Te}}]/L\right)^{4/7}}{Z_{\mathrm{eff}}^{5/7}\,\omega^2_{\mathrm{Le}}(Lk_0)^{5/7}}. \tag{7.32}$$

A comparison of formulas (7.26), (7.24), and (7.32) shows that a low filamentation threshold in the limit, when the ponderomotive mechanism is prevalent, is ensured by the large length L of the filament amplification region, as compared to the pump field wavelength $\lambda\!\!\!^{-}_0 = 1/k_0$. In the opposite limit, when the thermal mechanism is the principal one, a low filamentation threshold is ensured by the large length L of the filament amplification region, as compared to the mean free path $l_{\text{ei}}[V_{\text{Te}}]$ of a thermal electron,. In the intermediate range, to which formula (7.32) corresponds and where the weakly collisional mechanism manifests itself, the filament amplification region L may be small in comparison with the mean free path. However, in this case the filamentation threshold proves to be low anew due to the large effective ion charge and the large, in comparison with the pump field wavelength, length of the filament amplification region.

We now consider, unlike the case (7.28), the nonlinear effect of the pump field on the weakly collisional thermal mechanism of filamentation. This effect manifests itself in the suppression of this thermal mechanism in conditions of a relatively weak pump field, when the following condition is fulfilled:

$$1 > Z_{\text{eff}} \frac{V_E^2}{V_{\text{Te}}^2} > \frac{15}{Z_{\text{eff}}^{3/7} (kl_{\text{ei}})^{6/7}} \,. \tag{7.33}$$

In this case we have

$$F(k) \cong \frac{44}{Z_{\text{eff}} \left(kl_{\text{ei}}[V_{\text{Te}}]\right)^{8/5}} \left(\frac{V_{\text{Te}}}{V_E}\right)^{12/5}, \tag{7.34}$$

and the function $\Xi(k)$ characterizing the nonlocal electron heat conduction assumes the form [29]

$$\Xi_1(k) \cong 1 + \frac{2816 \left(kl_{\text{ei}}[V_{\text{Te}}]\right)^{2/5}}{3\pi Z_{\text{eff}}} \left(\frac{V_{\text{Te}}}{V_E}\right)^{12/5}. \tag{7.35}$$

Accordingly, the filament spatial gain increment is defined by the formula

$$G^2(k) = \frac{1}{4k_0^2} \left\{ -k^4 + \frac{\omega_{\text{Le}}^2 V_E^2 k^2}{2c^2 V_{\text{Te}}^2} \times \left[1 + \frac{44}{Z_{\text{eff}} \left(kl_{\text{ei}}[V_{\text{Te}}]\right)^{8/5}} \left(\frac{V_{\text{Te}}}{V_E}\right)^{12/5} \right] \right\}. \tag{7.36}$$

This formula describes the competition between the ponderomotive filamentation mechanism and the weakly collisional thermal mechanism, when the latter endures suppression by the pump field in the conditions specified by Eqn (7.33). However, when the inequality

$$\frac{10.6}{Z_{\text{eff}}^{5/8}} \left(\frac{V_{\text{Te}}}{V_E}\right)^{3/2} \gg kl_{\text{ei}}[V_{\text{Te}}] \gg 1 \tag{7.37}$$

is satisfied, the weakly collisional thermal mechanism would remain more significant than the ponderomotive one. In this case, the filamentation increment peaks for [29]

$$k = k_{\max} = 1.24 \left(\frac{\omega_{\text{Le}}^2}{c^2 Z_{\text{eff}}}\right)^{5/18} \frac{1}{\left(l_{\text{ei}}[V_{\text{Te}}]\right)^{4/9}} \left(\frac{V_{\text{Te}}}{V_E}\right)^{1/9}. \tag{7.38}$$

When the ponderomotive contribution is neglected, formula (7.36) can be represented as

$$G^2(k) = \frac{k_{\max}^4}{4k_0^2} \left[10 \frac{k^{2/5}}{k_{\max}^{2/5}} - \frac{k^4}{k_{\max}^4} \right]. \tag{7.39}$$

On appropriate substitution, one arrives at [29]

$$G^2(k_{\max}) = \frac{9k_{\max}^4}{4k_0^2} \cong 5.3 \left(\frac{\omega_{\text{Le}}^2}{c^2 Z_{\text{eff}}}\right)^{10/9} \frac{1}{k_0^2 \left(l_{\text{ei}}[V_{\text{Te}}]\right)^{16/9}} \left(\frac{V_{\text{Te}}}{V_E}\right)^{4/9}. \tag{7.40}$$

The last formula and relation (7.22) lead to the following expression for the filamentation instability threshold:

$$\frac{V_{E,\text{th}}^2}{V_{\text{Te}}^2} \cong \frac{1.2 \times 10^{-4} L k_0}{Z_{\text{eff}}^5} \left(\frac{L}{l_{\text{ei}}[V_{\text{Te}}]}\right)^8 \left(\frac{\omega_{\text{Le}}^2}{c^2 k_0^2}\right)^5. \tag{7.41}$$

On the instability threshold, the weakly collisional mechanism is more significant than the ponderomotive one provided that

$$4Z_{\text{eff}}^{1/2} \geqslant \left(\frac{\omega_{\text{Le}}^2}{c^2 k_0^2}\right)^{3/5} \frac{Lk_0}{\left(k_0 l_{\text{ei}}[V_{\text{Te}}]\right)^{4/5}} \,.$$

8. Stimulated Mandelstam – Brillouin scattering in a weakly collisional plasma

For simplicity of presentation we shall consider a spatially uniform plasma. The plasma-heating radiation of the pump field described, like in the previous section, by formulas (7.1) and (7.2) propagates through the medium under discussion. Unlike the description of the filamentation instability perturbation field with the aid of expression (7.3), in the case of SMBS we consider, along with the plasma-heating field, a perturbation with the frequency shift

$$\omega_1 = \omega_0 - \omega \,. \tag{8.1}$$

Such a situation would occur if we adopt in formula (3.2) that

$$\mathbf{E}(\mathbf{r}, t) \to \mathbf{E} \exp(\mathrm{i}\mathbf{kr}) + \delta\mathbf{E}(\mathbf{r}, t) \exp\left[-\mathrm{i}(\omega_0 - \omega)t + \mathrm{i}(\mathbf{k}_0 - \mathbf{k})\mathbf{r}\right]. \tag{8.2}$$

This signifies that the perturbed field of the scattered radiation has a frequency (8.1) and a wave vector

$$\mathbf{k}_1 = \mathbf{k}_0 - \mathbf{k} \,. \tag{8.3}$$

The perturbation field amplitude varies slowly in time and space.

In the linear theory of the SMBS instability we proceed from the following linear approximation

$$\left|\mathbf{E}(\mathbf{r}, t)\right|^2 \to |\mathbf{E}|^2 + \mathbf{E}^* \delta\mathbf{E} \exp\left[\mathrm{i}(\omega t - \mathbf{kr})\right] + \mathbf{E}\, \delta\mathbf{E}^* \exp\left[-\mathrm{i}(\omega t - \mathbf{kr})\right]. \tag{8.4}$$

This formula allows us to take advantage of the results of the nonlinear theory of electron density perturbation outlined in Sections 3 and 5, in which the following relationships are valid:

$$\delta|\mathbf{E}|_{\mathbf{k}}^2 = \mathbf{E}\, \delta\mathbf{E}^* \exp\left[-\mathrm{i}(\omega t - \mathbf{kr})\right], \quad \delta|\mathbf{E}|_{-\mathbf{k}}^2 = \mathbf{E}^* \delta\mathbf{E} \exp\left[\mathrm{i}(\omega t - \mathbf{kr})\right]. \tag{8.5}$$

The frequency shift ω and the wave vector shift $\mathbf{k}$, which occur in the SMBS in the scattered wave field, is due to the transverse pump wave decomposition into the transverse scattered wave and the low-frequency longitudinal ion-acoustic wave:

$$t \to t' + l\,. \tag{8.6}$$

The longitudinal wave field is potential, viz. $\mathbf{E} = -\mathrm{grad}\,\varphi = -\mathrm{i}\mathbf{k}\varphi_{\mathbf{k}} \exp\left[-\mathrm{i}(\omega t - \mathbf{kr})\right]$, and obeys the equation

$$-k^2 \varepsilon_{\mathrm{l}}(\omega, k)\varphi_{\mathrm{k}} = 4\pi e\,\delta n_{\mathbf{k}}\,, \tag{8.7}$$

where the right-hand side is the electron density perturbation by the perturbed nonuniform pump field, which according to expressions (8.5) and (5.25) is defined by the perturbation $\delta|\mathbf{E}|^2_{\mathbf{k}}$. In other words, relationships (5.25), (8.5), and (8.7) give one equation which describes how the forcing action of the pump field in the presence of the scattered field gives rise to a low-frequency plasma perturbation — the induced ion-acoustic wave.

The other equation, which couples the pump and scattered wave fields with the low-frequency sound wave, owes its origin to the fact that the scattered wave field emerges from the pump field due to that electron density perturbation which is defined by the sound-wave potential field. This corresponds to the fact that the following expression for the electron density in the Maxwell equations should be taken into consideration:

$$n_{\mathrm{e}} \to n_{\mathrm{e}0} + \delta n = n_{\mathrm{e}0} + \frac{k^2\,\delta\varepsilon_{\mathrm{l,e}}(\omega, k)}{4\pi e}\,\varphi_{-\mathbf{k}} \exp\left[\mathrm{i}(\omega t - \mathbf{kr})\right]. \tag{8.8}$$

Accordingly, from the Maxwell equations we obtain

$$\begin{aligned} &c^2\left\{-(\mathbf{k}_0 - \mathbf{k})^2 + 2\mathrm{i}(\mathbf{k}_0 - \mathbf{k})\boldsymbol{\nabla} + \Delta\right\}\delta\mathbf{E} \\ &\quad + \left\{(\omega_0 - \omega)^2 - \omega_{\mathrm{Le}}^2\left(1 - \mathrm{i}\,\frac{\nu_{\mathrm{ei}}[V_{\mathrm{Te}}]}{\omega_0 - \omega}\right)\right\}\delta\mathbf{E} \\ &\quad = \omega_{\mathrm{Le}}^2\,\frac{\delta n_{-\mathbf{k}}}{n_{\mathrm{e}0}}\,\mathbf{E} = -\frac{ek^2\delta\varepsilon_{\mathrm{l,e}}(\omega, k)}{m_{\mathrm{e}}}\,\varphi_{-\mathbf{k}}\mathbf{E}\,. \end{aligned} \tag{8.9}$$

Here, unlike Eqn (7.5), account was taken of the weak collisional absorption $\sim \nu_{\mathrm{ei}}[V_{\mathrm{Te}}]$ of the scattered wave. Taking advantage of the fact that the amplitude of the scattered wave changes only slightly over a distance equal to its wavelength, we neglect the second derivative in Eqn (8.9). This allows the following reduced equation to be obtained:

$$\begin{aligned} &c^2\left\{-(\mathbf{k}_0 - \mathbf{k})^2 + 2\mathrm{i}(\mathbf{k}_0 - \mathbf{k})\boldsymbol{\nabla}\right\}\delta\mathbf{E} \\ &\quad + \left\{(\omega_0 - \omega)^2 - \omega_{\mathrm{Le}}^2\left(1 - \mathrm{i}\,\frac{\nu_{\mathrm{ei}}[V_{\mathrm{Te}}]}{\omega_0 - \omega}\right)\right\}\delta\mathbf{E} \\ &\quad = -\frac{ek^2\,\delta\varepsilon_{\mathrm{l,e}}(\omega, k)}{m_{\mathrm{e}}}\,\varphi_{-\mathbf{k}}\mathbf{E}\,. \end{aligned} \tag{8.10}$$

When writing down the right-hand side of Eqn (8.10), account was taken of relation (8.9). Formula (7.7), the second relationship in Eqn (8.5), as well as relationship (7.2) with allowance for

$$\omega \ll \omega_0 \tag{8.11}$$

permit the representation of the reduced field equation (8.10) in the form

$$\begin{aligned} &c^2\left\{(2\mathbf{k}\mathbf{k}_0 - k^2) + 2\mathrm{i}(\mathbf{k}_0 - \mathbf{k})\boldsymbol{\nabla}\right. \\ &\quad \left. - \frac{1}{c^2}\left[2\omega\omega_0 - \mathrm{i}\,\frac{\omega_{\mathrm{Le}}^2 \nu_{\mathrm{ei}}[V_{\mathrm{Te}}]}{\omega_0}\right]\right\}\delta\mathbf{E} \\ &\quad = -\frac{e^2\omega_{\mathrm{Le}}^2\,\delta\varepsilon_{\mathrm{l,e}}(\omega, k)}{4m_{\mathrm{e}}^2\omega_0^2 V_{\mathrm{Te}}^2 \varepsilon_{\mathrm{l}}(\omega, k)}\left[1 + F(k)\right](\mathbf{E}^*\,\delta\mathbf{E})\mathbf{E}\,. \end{aligned} \tag{8.12}$$

The last equation for the amplitude of the scattered wave makes it possible to consider the spatial amplification of the scattered field, arising from the SMBS instability. For simplicity of presentation we assume the scattered field and pump field polarizations to coincide, and will consider the variation of the scattered field along the direction of the vector $\mathbf{k}_0 - \mathbf{k}$, the z-axis being aligned with this vector. Then, equation (8.12) can be represented as

$$\begin{aligned} &\left\{c^2(2\mathbf{k}_0\mathbf{k} - k^2) - 2\omega_0\omega + \mathrm{i}\omega_{\mathrm{Le}}^2\,\frac{\nu_{\mathrm{ei}}[V_{\mathrm{Te}}]}{\omega_0}\right\}\delta\mathbf{E} \\ &\quad + 2\mathrm{i}|\mathbf{k}_0 - \mathbf{k}|\,\frac{\mathrm{d}\delta\mathbf{E}}{\mathrm{d}z} = -\frac{\omega_{\mathrm{Le}}^2|V_E|^2}{4V_{\mathrm{Te}}^2}\,\frac{\delta\varepsilon_{\mathrm{l,e}}(\omega, k)}{\varepsilon_{\mathrm{l}}(\omega, k)}\left[1 + F(k)\right]\delta\mathbf{E}\,. \end{aligned} \tag{8.13}$$

The solution of this equation is proportional to

$$\exp gz\,, \tag{8.14}$$

where

$$\begin{aligned} g = \frac{-\mathrm{i}}{2|\mathbf{k}_0 - \mathbf{k}|c^2}&\left\{-\left[c^2(2\mathbf{k}_0\mathbf{k} - k^2) - 2\omega_0\omega + \mathrm{i}\omega_{\mathrm{Le}}^2\,\frac{\nu_{\mathrm{ei}}[V_{\mathrm{Te}}]}{\omega_0}\right]\right. \\ &\left. - \frac{\omega_{\mathrm{Le}}^2|\mathbf{V}_E|^2}{4V_{\mathrm{Te}}^2}\,\frac{\delta\varepsilon_{\mathrm{l,e}}(\omega, k)}{\varepsilon_{\mathrm{l}}(\omega, k)}\left[1 + F(k)\right]\right\}. \end{aligned} \tag{8.15}$$

We consider the implications of Eqn (8.15) in circumstances when the following resonance relation holds good:

$$c^2(2\mathbf{k}_0\mathbf{k} - k^2) - 2\omega_0\omega = 0\,. \tag{8.16}$$

At the same time, the low-frequency wave is assumed to relate to the ion sound, when $\omega = kV_{\mathrm{s}}$. This, according to expressions (2.8) and (2.11), corresponds to the condition

$$\mathrm{Re}\,\varepsilon_{\mathrm{l}}(\omega, k) = (kr_{\mathrm{De}})^{-2} - \frac{\bar{\omega}_{\mathrm{Li}}^2}{\omega^2} = 0\,. \tag{8.17}$$

Then, in the denominator of the last term on the right-hand side of formula (8.15) there remains only a small imaginary part which is defined by the imaginary part of the longitudinal plasma permittivity:

$$\mathrm{Im}\,(\delta\varepsilon_{\mathrm{l,e}} + \delta\varepsilon_{\mathrm{l,i}})\,.$$

In the foregoing we were concerned with the electron contribution to the imaginary part and omitted the ion contribution. For our purposes it would suffice to employ the relationship

$$\mathrm{Im}\,(\delta\varepsilon_{\mathrm{l,e}} + \delta\varepsilon_{\mathrm{l,i}}) = \frac{1}{k^2 r_{\mathrm{De}}^2}\,\frac{2(\gamma_{\mathrm{e}} + \gamma_{\mathrm{i}})}{kV_{\mathrm{s}}}\,, \tag{8.18}$$

where γ_{i} is the ion contribution to the ion-acoustic damping decrement, and γ_{e} is the corresponding electron contribution. The latter, according to formula (6.21), is expressible as

$$\gamma_{\mathrm{e}} = \frac{kV_{\mathrm{s}}^2}{2V_{\mathrm{Te}}}\left[\sqrt{\frac{\pi}{2}} + F_1(k)\right]. \tag{8.19}$$

Here, the following notation is adopted:

$$\begin{aligned} F_1(k) = 2.17&\left(\frac{Z_{\mathrm{eff}}^2}{k^3 l_{\mathrm{ei}}^3[V_{\mathrm{Te}}]}\right)^{1/7} \\ \times&\left\{1 + 2.17\left(\frac{Z_{\mathrm{eff}}^2}{k^3 l_{\mathrm{ei}}^3[V_{\mathrm{Te}}]}\right)^{1/7}\left[\frac{4\sqrt{3}\,\pi^{2/3}}{kl_{\mathrm{ei}}[V_{\mathrm{Te}}]}\,\frac{1}{Z_{\mathrm{eff}}^{2/3}}\left(\frac{V_{\mathrm{Te}}}{V_E}\right)^{4/3}\right]^{-1}\right\}^{-1}. \end{aligned} \tag{8.20}$$

All this permits us to write down the following expression for the SMBS spatial gain coefficient

$$g = \frac{\omega_{\mathrm{Le}}^2}{2\omega_0 c^2 k_0} \left\{ \frac{|V_E|^2}{8V_{\mathrm{Te}}^2} \frac{\left[1 + F(k)\right]\omega_0 k V_{\mathrm{s}}}{\gamma_{\mathrm{e}} + \gamma_{\mathrm{i}}} - \nu_{\mathrm{ei}} \right\}. \qquad (8.21)$$

To the threshold of an absolute SMBS instability there corresponds the vanishing of the right-hand side of expression (8.21), when one has

$$\frac{|V_E|_{\mathrm{abs.\,th}}^2}{V_{\mathrm{Te}}^2} = \frac{8\nu_{\mathrm{ei}}[V_{\mathrm{Te}}]}{\omega_0\left[1 + F(k)\right]} \left\{ \frac{V_{\mathrm{s}}}{2V_{\mathrm{Te}}} \left[\sqrt{\frac{\pi}{2}} + F_1(k) \right] + \frac{\gamma_{\mathrm{i}}}{kV_{\mathrm{s}}} \right\}. \qquad (8.22)$$

The threshold of convective SMBS plasma instability in a layer of thickness L corresponds, similarly to expression (7.22), to the condition

$$gL = 2\pi\,. \qquad (8.23)$$

Accordingly, the convective SMBS instability threshold is given by the relation

$$\frac{|V_E|_{\mathrm{conv.\,th}}^2}{V_{\mathrm{Te}}^2} = \frac{8(\gamma_{\mathrm{e}} + \gamma_{\mathrm{i}})\nu_{\mathrm{ei}}[V_{\mathrm{Te}}]}{\omega_0 k V_{\mathrm{s}}\left[1 + F(k)\right]} \left\{ 1 + \frac{4\pi c^2}{L\omega_{\mathrm{Le}}^2} \frac{\omega_0}{\nu_{\mathrm{ei}}[V_{\mathrm{Te}}]} \right\}. \qquad (8.24)$$

Hence it follows that the convective SMBS instability threshold is as much higher than the absolute SMBS instability threshold as the last term (inversely proportional to the SMBS region dimension L) in the braces in the right-hand side of formula (8.24) is greater than unity. Since the spatial attenuation of the pump field intensity due to inverse bremsstrahlung absorption is described by the exponential relationship

$$\exp\left(-z \, \frac{\omega_{\mathrm{Le}}^2 \nu_{\mathrm{ei}}[V_{\mathrm{Te}}]}{c^2 k_0 \omega_0} \right), \qquad (8.25)$$

it is evident that the convective threshold is significantly higher than the absolute SMBS instability threshold whenever the absorption of the pump radiation is relatively small:

$$L \, \frac{\omega_{\mathrm{Le}}^2 \nu_{\mathrm{ei}}[V_{\mathrm{Te}}]}{c^2 k_0 \omega_0} \ll 4\pi\,. \qquad (8.26)$$

Then, from formula (8.24) follows that

$$\frac{|V_E|_{\mathrm{conv.\,th}}^2}{V_{\mathrm{Te}}^2} = \frac{32\pi(\gamma_{\mathrm{e}} + \gamma_{\mathrm{i}})c^2 k_0}{\omega_{\mathrm{Le}}^2 L k V_{\mathrm{s}}\left[1 + F(k)\right]} \gg \frac{|V_E|_{\mathrm{abs.\,th}}^2}{V_{\mathrm{Te}}^2}\,. \qquad (8.27)$$

The above-determined dependences of the functions $F_1(k)$ and $F(k)$ on the wave vector allow us to specify the conditions in which the weakly collisional electron contribution to the SMBS instability increment is significant. In the simplest case of backscattering, one has $k \cong 2k_0$.

Here we note that the theory of SMBS instability in a weakly collisional plasma was outlined in Refs [3, 4, 6, 14, 17, 28]. In paper [37], the SMBS-determining nonlinearity corresponding to the electron density perturbation was considered in the context of the notion of nonlocal heat transfer. In this case, in accord with relationship (7.19), it was believed that the values $\alpha = 1.44$ and $a \cong 12$ are admissible for using. The employment of these parameter values in Ref. [3] was subject to criticism in work [4], which stated that it would be appropriate to employ the $\alpha = 4/3$ and $a \cong 16$ values. Furthermore, the parameters determining the nonlinearity of nonlocal thermal conduction were taken to be $\alpha = 1.44$ and $a \cong 12$ [compare with relationship (7.19)], which is closest to those parameter values which correspond to the analytical theory of a weakly collisional plasma. The ion-acoustic wave damping was considered in Ref. [6] without including weakly collisional effects. Furthermore, in Ref. [14] the effect of weak collisions on the absorption of ion-acoustic waves was not taken into account either, while the function $F(k)$ was treated in the linear approximation which neglects the nonlinear modification of the subthermal-electron distribution. In Ref. [17], the SMBS instability was treated in the same linear approximation, but with the inclusion of the effect of weak collisions on ion-acoustic wave damping. From the viewpoint of an analytical theory, all these results refer to a linear treatment, i.e. to the case when the pump field is so weak that condition (4.16) is fulfilled. Finally, the theory of SMBS in a weakly collisional plasma was considered with the inclusion of the nonlinear redistribution of subthermal electrons [28], when increasing the pump intensity is accompanied by a reduction of $F(k)$ and a decrease of the SMBS instability increment.

9. Conclusions

The previous sections were devoted to the consideration of the effect of collisions of slow subthermal electrons in conditions when a fully ionized plasma is commonly assumed to be collisionless on the strength of inequality (1.1). In this case, the subthermal electrons with velocities satisfying inequalities (1.9) and (1.1) should be thought to experience frequent collisions. In the foregoing we verified that there exist conditions wherein the contributions from frequent subthermal-electron collisions in so-called collisionless plasmas prove to be competitive or even exceeding the collisionless contributions to the quantities like the ion-acoustic damping decrement and the nonlinear electron density perturbation by a spatially nonuniform electromagnetic pump field. The perturbation mentioned determines that nonlinearity which characterizes parametric instabilities like the plasma radiation filamentation and SMBS. Apart from the analytical theory considered in detail in the foregoing, the problems considered in our review are also treated in the literature employing the numerical solution of the Boltzmann kinetic equation with the collision integral in the Fokker – Planck – Landau form, as discussed above.

It is believed that the numerical investigation of the phenomena related to nonlinear electron density perturbation was carried out most carefully. With reference to the discussion in the text following formula (7.19) as well as at the end of Section 8, the results of such a numerical solution to the Boltzmann equation are rather close to the results of the analytical theory concerned with the description of nonlinear density perturbation. However, all this pertains only to the case of relatively low intensity of the plasma-heating electromagnetic radiation of the pump field, when condition (4.16) is fulfilled. This condition is violated for a moderate pump intensity, when the following inequality is satisfied:

$$\bar{q} \equiv \left(\frac{q}{10^{14}\ \mathrm{W\,cm^{-2}}} \right) \left(\frac{\lambda}{1\ \mu\mathrm{m}} \right)^2 \left(\frac{T_{\mathrm{e}}}{1\ \mathrm{eV}} \right)^{-1} > \frac{0.2}{Z_{\mathrm{eff}}^{10/7} \left(k l_{\mathrm{ei}}[V_{\mathrm{Te}}] \right)^{6/7}} \ll 1\,. \qquad (9.1)$$

It is noteworthy that the idea of nonlinear influence of the plasma-heating radiation on the nonlinear density perturbation was advanced in Ref. [3], and in the comment to it [4]. It was precisely the authors of work [3] who were prone to believe that the exponent value $\alpha = 1$ in formula (7.19) is preferred to $\alpha = 4/3$, the datum of Ref. [1]. The authors of Ref. [4] indicated that the numerical investigation of the Boltzmann kinetic equation was performed under conditions wherein the Langdon velocity was low in comparison with the electron thermal velocity. According to contemporary views, this should not have resulted in a nonlinear variation of the index of a power in formula (7.19).

Following the discussion [3, 4], paper [21] made its appearance. It outlined the results of numerical solution of the Boltzmann equation in the conditions wherein the Langdon velocity was not assumed to be low in comparison with the electron thermal velocity. To this end, instead of the Maxwell electron velocity distribution function advantage was taken of interpolation distribution functions from Ref. [53], which transformed from the Maxwell distribution to the Langdon distribution with increasing intensity of the heating field. Work [21] revealed a significant modification of nonlocal heat conduction which characterizes the nonlinear electron density perturbation. The numerical results of Ref. [21] were followed by the analytical solution of the Boltzmann kinetic equation in Ref. [22], which established the fact of suppressing the electron heat transport inhibition by a relatively strong pump field. In Refs [21, 22], however, this suppression was considered for pump intensities much higher than those corresponding to a small right-hand side of inequality (9.1).

The step made in papers [25–27] and reflected in the fourth and subsequent sections of our review proved to be more interesting for the weakly collisional plasma kinetic theory nonlinear in the pump intensity, which we consider. In these papers the phenomenon of nonlinear suppression of weakly collisional effects was revealed at a significantly lower level of heating radiation intensity than required in Refs [21, 22], specifically, at the intensity level characterized by inequality (9.1). In this case, first, the α power assumed a value of $2/5$ in lieu of $10/7$, as follows from formula (7.35). Furthermore, the parameter a in formula (7.19) became a decreasing function of the plasma-heating radiation intensity. This corresponds to suppression of the weakly collisional thermal mechanism of filamentation instability with increasing pump intensity. However, in accord with inequality (7.37) the thermal mechanism still remains more significant than the ponderomotive one for a plasma-heating radiation intensity satisfying the inequality

$$\bar{q} \equiv \left(\frac{q}{10^{14}\ \mathrm{W\,cm^{-2}}}\right)\left(\frac{\lambda}{1\ \mu\mathrm{m}}\right)^{2}\left(\frac{T_{\mathrm{e}}}{1\ \mathrm{eV}}\right)^{-1} < \frac{0.62}{Z_{\mathrm{eff}}^{5/6}\left(kl_{\mathrm{ei}}[V_{\mathrm{Te}}]\right)^{4/3}} . \qquad (9.2)$$

Inequalities (9.1) and (9.2) are simultaneously satisfied with the proviso that

$$12 Z_{\mathrm{eff}}^{5/4} \gg kl_{\mathrm{ei}}[V_{\mathrm{Te}}] , \qquad (9.3)$$

which is realized over a wide range of parameters for a high degree of ionization.

We now address ourselves to a comparison of the above-outlined results with the data of numerical investigations into the weakly collisional dissipation of ion-sound waves. Here, the numerical results admitting of a comparison with analytical regularities are significantly fewer. As noted above, the weakly collisional dissipation of ion-sound waves in the theory of SMBS was taken into account only in an analytical approach. Nevertheless, Ref. [18] was dedicated specifically to the numerical solution of the Boltzmann equation to determine the ion-acoustic wave damping. When discussing Ref. [18], it might be well to recall that the absorption of ion-sound waves under conditions of frequent collisions is determined by electron heat conductivity. In particular, the electron contribution to the longitudinal plasma permittivity may be represented in the form (see, for instance, Ref. [11])

$$\delta\varepsilon_{\mathrm{e}}(\omega, k) = \frac{1}{k^2 r_{\mathrm{De}}^2}\left\{1 + \frac{\mathrm{i}\omega}{kV_{\mathrm{Te}}}\left[\frac{n_{\mathrm{e}}\kappa_{\mathrm{B}}V_{\mathrm{Te}}}{k\chi(k)}\right]\right\} . \qquad (9.4)$$

Here, the dissipation corresponding to the imaginary term in formula (9.4) is described by conventional electron heat conductivity, when $\chi = \chi_{\mathrm{SH}}$. Formula (9.4) was written under the assumption that the dissipation of ion-sound waves in the case of weakly collisional plasma can also be described by the electron heat conductivity coefficient which now takes into account the nonlocal character of heat transfer. This assumption arose in the approach made in Refs [42, 16], when the use of a system of moment equations, in which thermodynamic flows were nonlocally related to thermodynamic forces, was proposed to describe the processes in a fully ionized plasma. In particular, use was made of nonlocal heat conductivity. Among the possible applications of this, allegedly general, approach, according to Ref. [16] it was suggested to employ the ion sound description equivalent to formula (9.4). This approach was subject to criticism in Ref. [19] which was devoted to the comparison of the then known different nonlocal electron heat conductivities, derived in Refs [1, 2, 7, 15, 16, 37], with the result of the analytical theory [10]. In this case, at least two different heat conductivities were established to actually emerge in the approach to the nonlocal description of plasma processes, advanced in Refs [42, 16].

The above digression was necessary for two reasons. First, Epperlein [18] made a similar statement that nonlocal heat conductivity cannot be used for the description of ion-acoustic wave dissipation. This corresponds to the viewpoint of Ref. [19]. Second, it is not seen that the functional dependence of paper [18], which is employed in the context of the approach to nonlocal electron heat conductivity [43, 16], coincides with the analytical result. Indeed, according to Refs [10, 11, 17, 19] this nonlocal heat conductivity in the asymptotic limit of large wave numbers behaves as $\sim k^{-4/7}$. In Ref. [18], contrastingly, the nonlocal heat conductivity was found to be $\sim k^{-1}$ in the same limit. Here, we note that the latter result corresponds, in accord with expression (1.26), to the formula for electron heat flow of the form (1.22) with a small heat transport inhibition coefficient. By contrast, recourse to the effective conductivity corresponding to the results of Refs [10, 11, 17, 19] concerning the dissipation of ion-sound waves does not lead to a relationship of the form (1.22) (compare with Refs [9, 17]). We note that the effective heat conductivity of the nonlocal approach to ion-acoustic wave damping assumes an entirely local form, when the pump intensity satisfies the inequality (9.1):

$$\frac{\chi_{\mathrm{SH}}}{1+200(Z_{\mathrm{eff}}V_E^2/V_{\mathrm{Te}}^2)^{2/3}} .$$

The following should be evident from the above. Not only does the theory of a weakly collisional plasma provide an analytical description of ion-acoustic wave damping, plasma nonlinearity corresponding to the nonlinear electron density perturbation by a spatially nonuniform intensity of electromagnetic field, SMBS, and radiation filamentation, but it also allows one to draw a general conclusion concerning the rationality of nonlocal plasma hydrodynamics presented in Refs [43, 16]. It is precisely the necessity of employing different nonlocal heat conductivity coefficients for various processes, which poses the general question of whether unknown processes can be described by nonlocal hydrodynamics, when a demand to introduce new nonlocal heat conductivity coefficients will arise. The pressing demand for gas hydrodynamic equations or their analogues has led to the proposal [20] to take advantage of that system of moment equations which, in particular, comprises two sets of quantities of different genesis: two temperatures, two heat flows, two stress tensors, etc. So far, this line of investigation has not gathered force.

It remains to emphasize some formal originality of weakly collisional plasma kinetics. In the case of a strongly collisional plasma, in accordance with the Hilbert – Chapman – Enskog method of solution of the Boltzmann equation, the occurrence of corrections corresponds to terms quadratic in powers of k. This, in particular, manifested itself in formula (1.30) which is a partial summation of higher-order Hilbert – Chapman – Enskog approximations. Characteristic of the case of a weakly collisional plasma is an expansion like the Laurent series in inverse powers of k. In the case of a weakly collisional plasma, we witnessed the occurrence of fractional-power dependences whose asymptotic forms (2.40), (3.24), and (5.16) were possible to determine with the aid of exactly solvable differential equations. These exact analytical corollaries emerged in our treatment owing to the fact that the use of linear equations proved to be sufficient for the description of the problems involved. However, a demand would arise to go over to a nonlinear description in the course of development of the field of investigation, for instance, the description of radiation filamentation and SMBS, and in this case one would anticipate even more peculiar and interesting results. The material of our review, brought to the reader's notice, is merely a preliminary step toward this future approach to the nonlinear theory of a weakly collisional plasma.

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References

1. Epperlein E M *Phys. Rev. Lett.* **65** 2145 (1990)
2. Epperlein E M, Short R W *Phys. Fluids B* **3** 3092 (1991)
3. Rose H A, DuBois D F *Phys. Fluids B* **4** 1394 (1992)
4. Epperlein E M, Short R W *Phys. Fluids B* **4** 4190 (1992)
5. Epperlein E M, Short R W *Phys. Fluids B* **4** 2211 (1992)
6. Short R W, Epperlein E M *Phys. Rev. Lett.* **68** 3307 (1992)
7. Maksimov A V, Silin V P *Zh. Eksp. Teor. Fiz.* **103** 73 (1993) [*JETP* **76** 39 (1993)]
8. Maximov A V, Silin V P *Phys. Lett. A* **173** 33 (1993)
9. Maksimov A V, Silin V P *Pis'ma Zh. Eksp. Teor. Fiz.* **58** 264 (1993) [*JETP Lett.* **58** 271 (1993)]
10. Maksimov A V, Silin V P *Zh. Eksp. Teor. Fiz.* **105** 1242 (1994) [*JETP* **78** 669 (1994)]
11. Maksimov A V, Silin V P *Pis'ma Zh. Eksp. Teor. Fiz.* **59** 507 (1994) [*JETP Lett.* **59** 534 (1994)]
12. Silin V P *Zh. Eksp. Teor. Fiz.* **106** 1398 (1994) [*JETP* **79** 756 (1994)]
13. Silin V P *Pis'ma Zh. Eksp. Teor. Fiz.* **60** 766 (1994) [*JETP Lett.* **60** 775 (1994)]
14. Shukla P K *Phys. Fluids B* **5** 4253 (1993)
15. Berger R L et al. *Phys. Fluids B* **5** 2243 (1993)
16. Kaiser T B et al. *Phys. Plasmas* **1** 1287 (1994)
17. Maximov A V, Silin V P *Phys. Lett. A* **192** 57 (1994)
18. Epperlein E M *Phys. Plasmas* **1** 109 (1994)
19. Maximov A V, Silin V P *Phys. Plasmas* **2** 1355 (1995)
20. Epperlein E M, Short R W *Phys. Plasmas* **1** 3003 (1994)
21. Epperlein E M, Short R W *Phys. Rev. E* **50** 1697 (1994)
22. Silin V P *Zh. Eksp. Teor. Fiz.* **108** 193 (1995) [*JETP* **81** 103 (1995)]
23. Silin V P *Phys. Scripta* **T63** 148 (1996)
24. Ovchinnikov K N, Silin V P *Fiz. Plazmy* **22** 436 (1996) [*Plasma Phys. Rep.* **22** 395 (1996)]
25. Maximov A V et al. *Phys. Lett. A* **237** 53 (1997)
26. Maksimov A V et al. *Dokl. Ross. Akad. Nauk* **358** 618 (1998) [*Dokl. Phys.* **43** 88 (1998)]
27. Maksimov A V et al. *Zh. Eksp. Teor. Fiz.* **113** 1299 (1998) [*JETP* **86** 710 (1998)]
28. Maksimov A V et al. *Fiz. Plazmy* **25** 779 (1999) [*Plasma Phys. Rep.* **25** 715 (1999)]
29. Maksimov A V et al. *Fiz. Plazmy* **25** 448 (1999) [*Plasma Phys. Rep.* **25** 404 (1999)]
30. Ovchinnikov K N, Silin V P, Uryupin S A *Kratk. Soobshch. Fiz.* (1) 36 (2000) [*Bull. Lebedev Phys. Inst.* (1) 31 (2000)]
31. Silin V P *Vvedenie v Kineticheskuyu Teoriyu Gazov* (Introduction to the Kinetic Theory of Gases) (Moscow: Nauka, 1971); 2nd ed. (Moscow: FIAN, 1998)
32. McCall G H *Plasma Phys.* **25** 237 (1983)
33. Bickerton R J *Nucl. Fusion* **13** 457 (1973)
34. Malone R C, McCrory R L, Morse R L *Phys. Rev. Lett.* **34** 721 (1975)
35. Schmitt A J *Phys. Fluids* **31** 3079 (1988)
36. Spitzer L (Jr), Härm R *Phys. Rev.* **89** 977 (1953)
37. Landshoff R *Phys. Rev.* **76** 904 (1949)
38. Mora P, in *Proc. of the Intern. School of Plasma Physics "Piero Caldirola" — ISPP-4: Inertial Confinement Fusion* (*Course and Workshop*), *Varenna, Italy, September 6 – 16, 1988* (Eds A Caruso, E Sindoni) (Bologna: Società Italiana di Fisica, 1988) p. 237
39. Luciani J F, Mora P *J. Stat. Phys.* **43** 281 (1986)
40. Luciani J F, Mora P *Phys. Lett. A* **116** 237 (1986)
41. Maksimov A V, Silin V P, Chegotov M V *Fiz. Plazmy* **16** 575 (1990) [*Sov. J. Plasma Phys.* **16** 331 (1990)]
42. Gurevich A V, Istomin Ya N *Zh. Eksp. Teor. Fiz.* **77** 933 (1979) [*Sov. Phys. JETP* **50** 472 (1979)]
43. Hammett G W, Perkins F W *Phys. Rev. Lett.* **64** 3019 (1990)
44. Aleksandrov A F, Rukhadze A A *Lektsii po Elektrodinamike Plazmopodobnykh Sred* (Lectures on the Electrodynamics of Plasma-Like Media) (Moscow: Izd. MGU, 1999)
45. Perel' V I, Pinskiĭ Ya M *Zh. Eksp. Teor. Fiz.* **54** 1889 (1968) [*Sov. Phys. JETP* **27** 1014 (1968)]
46. Gaponov A V, Miller M A *Zh. Eksp. Teor. Fiz.* **34** 242 (1958) [*Sov. Phys. JETP* **7** 168 (1958)]
47. Langdon A B *Phys. Rev. Lett.* **44** 575 (1980)
48. Jones R D, Lee K *Phys. Fluids* **25** 2307 (1982)
49. Balescu R *J. Plasma Phys.* **27** 553 (1982)
50. Chichkov B N, Shumsky S A, Uryupin S A *Phys. Rev. A* **45** 7475 (1992)
51. Litvak A G *Izv. Vyssh. Uchebn. Zaved. Radiofiz.* **11** 1433 (1968)
52. Kruer W L *Comm. Plasma Phys. Contr. Fusion* **9** 63 (1985)
53. Matte J P et al. *Plasma Phys. Contr. Fusion* **30** 1665 (1988)