

- lish: *High-Temperature Superconductivity* (New York: Consultants Bureau, 1982)]
14. Abrikosov A A *Phys. Rev. B* **64** 104521 (2001)
  15. Gofron K et al. *Phys. Rev. Lett.* **73** 3302 (1994)
  16. Ding H et al. *Phys. Rev. Lett.* **78** 2628 (1997); Timusk T, Statt B *Rep. Prog. Phys.* **62** 61 (1999)
  17. Shen Z-X, Dessau D S *Phys. Rep.* **253** 1 (1995)
  18. Vobornik I et al. *Physica C* **317–318** 589 (1999)
  19. Markiewicz R S, Kusko C, Kidambi V *Phys. Rev. B* **60** 627 (1999)
  20. Tallon J L, Williams G V M, Loram J W *Physica C* **338** 9 (2000)
  21. Kleiner R, Müller P *Phys. Rev. B* **49** 1327 (1994); Schlenga K et al. *Phys. Rev. B* **57** 14518 (1998)
  22. Kleiner R, Müller P *Physica C* **293** 156 (1997)
  23. Heim S et al., cond-mat/0107463
  24. Yurgens A A *Supercond. Sci. Technol.* **13** R85 (2000)
  25. Krasnov V M et al. *Phys. Rev. B* **59** 8463 (1999)
  26. Yamashita T et al. *Physica C* **335** 219 (2000)
  27. Ponomarev Ya G et al. *Physica C* **315** 85 (1999)
  28. Ponomarev Ya G et al., in *5th Intern. Workshop "High-Temperature Superconductivity and Novel Inorganic Materials Engineering" (MSU-HTSC-V), Moscow, Russia, 1998* (Abstracts) S-58
  29. Kaneko S et al. *Surf. Sci.* **438** 353 (1999)
  30. Mitchell C E J et al. *Surf. Sci.* **433–435** 728 (1999)
  31. Wei J Y T et al. *Phys. Rev. B* **57** 3650 (1998)
  32. Suzuki M, Watanabe T, Matsuda A *Phys. Rev. Lett.* **82** 5361 (1999); Suzuki M, Watanabe T *Phys. Rev. Lett.* **85** 4787 (2000)
  33. Krasnov V M et al. *Phys. Rev. Lett.* **86** 2657 (2001); cond-mat/0002172; *Phys. Rev. Lett.* **84** 5860 (2000)
  34. BenDaniel D J, Duke C B *Phys. Rev.* **152** 683 (1966); **160** 679 (1967)
  35. Mironova G A, Ponomarev Ya G, Roshta L *Fiz. Tverd. Tela* (Leningrad) **17** 906 (1975)
  36. Sherman E Ya, Misochko O V *Phys. Rev. B* **59** 195 (1999)
  37. Ponomarev Ya G et al. *Solid State Commun.* **111** 513 (1999)
  38. Ponomarev Ya G et al., in *5th Intern. Workshop "High-Temperature Superconductivity and Novel Inorganic Materials Engineering" (MSU-HTSC-V), Moscow, Russia, 1998* (Abstracts) S-59
  39. Lorenz M A et al. *J. Low Temp. Phys.* **117** 527 (1999)
  40. Ponomarev Ya G et al., in *XXXI Soveshch. po Fizike Nizkikh Temperatur, Moscow, 1998* (Tez. Dokladov) (31st Workshop on Low-Temperature Physics (Abstracts)) p. 228
  41. Maksimov E G, Arseyev P I, Maslova N S *Solid State Commun.* **111** 391 (1999)
  42. Lanzara A et al., cond-mat/0102227; *Nature* **412** 510 (2001)
  43. Shen Z-X, Lanzara A, Nagaosa N, cond-mat/0102244 (v2)
  44. Zhao G M et al. *Nature* **385** 236 (1997)
  45. Franck J P et al. *Phys. Rev. B* **44** 5318 (1991)
  46. Vedenev S I et al. *Physica C* **235–240** 1851 (1994)
  47. Shimada D et al. *Physica C* **298** 195 (1998)
  48. Gonnelli R S, Ummerino G A, Stepanov V A *Physica C* **275** 162 (1997)
  49. Aminov B A et al., in *Superconducting Devices and Their Applications: Proc. of the 4th Intern. Conf. SQUID'91, Berlin, FRG, 1991* (Springer Proc. in Phys., Vol. 64, Eds H Koch, H Lübbig) (Berlin: Springer-Verlag, 1992) p. 45
  50. Yurgens A et al. *Proc. SPIE* **2697** 433 (1996)
  51. Schlenga K et al. *Phys. Rev. Lett.* **76** 4943 (1996); Helm Ch et al. *Phys. Rev. Lett.* **79** 737 (1997); cond-mat/9909318
  52. Boekholt M, Hoffmann M, Güntherodt G *Physica C* **175** 127 (1991); Gasparov L, Güntherodt G, unpublished; Kendziora C, Kelley R J, Onellion M *Phys. Rev. Lett.* **77** 727 (1996)
  53. Zeyer R, Zwicknagl G Z. *Phys. B: Cond. Mat.* **78** 175 (1990)
  54. Karakozov A E, Maksimov E G *Zh. Eksp. Teor. Fiz.* **115** 1799 (1999) [*JETP* **88** 987 (1999)]
  55. Deutscher G *Nature* **397** 410 (1999)
  56. Choi H-Y, Bang Y, Campbell D K *Phys. Rev. B* **61** 9748 (2000)
  57. Dagan Y et al. *Phys. Rev. B* **61** 7012 (2000)
  58. Ozyuzer L et al. *Physica C* **341–348** 927 (2000)
  59. Miyakawa N et al. *Phys. Rev. Lett.* **83** 1018 (1999)
  60. Renner Ch et al. *Phys. Rev. Lett.* **80** 149 (1998)
  61. Oda M et al. *Int. J. Mod. Phys. B* **13** 3605 (1999)
  62. Ponomarev Ya G et al., in *Applied Superconductivity 1999: Proc. of EUCAS 1999, Sitges, Spain, 1999* (Inst. Phys. Conf. Ser., N 167, Eds X Obradors, F Sandiumenge, J Fontcuberta) (Philadelphia, P.A.: Institute of Physics Publ., 2000) p. 241
  63. Schmidt H et al., in *6th Intern. Conf. on Materials and Mechanisms of Superconductivity and High-Temperature Superconductors: M2S-HTSC-VI, February 20–25, 2000, Houston, Texas, USA* (Abstracts) 2C2.6, p. 170; <http://m2s-conf.uh.edu/abstracts/2C2.html>
  64. Timergaleev N Z, in *XXXII Vseross. Soveshch. po Fizike Nizkikh Temperatur, 3–6 Oktyabrya 2000* [Tez. Dokladov] (32nd National Workshop on Low-Temperature Physics, October 3–6, 2000 (Abstracts)] SCP30, p. 104
  65. Machida M, Koyama T, Tachiki M *Phys. Rev. Lett.* **83** 4618 (1999)
  66. Muller C J et al. *Physica C* **191** 485 (1992)
  67. Ponomarev Ya G et al., in *Applied Superconductivity 1999: Proc. of EUCAS 1999, Sitges, Spain, 1999* (Inst. Phys. Conf. Ser., N 167, Eds X Obradors, F Sandiumenge, J Fontcuberta) (Philadelphia, P.A.: Institute of Physics Publ., 2000) p. 245
  68. Tsai J S et al. *Physica C* **162–164** 1133 (1989)
  69. Moreland J et al. *Appl. Phys. Lett.* **55** 1463 (1989)
  70. Phillips J C *Phys. Rev. Lett.* **72** 3863 (1994)
  71. Nagamatsu J et al. *Nature* **410** 63 (2001)
  72. Bud'ko S L et al. *Phys. Rev. Lett.* **86** 1877 (2001)
  73. Liu A Y, Mazin I I, Kortus J, cond-mat/0103570; *Phys. Rev. Lett.* **87** 087005 (2001); Mazin I I et al., cond-mat/0204013
  74. Neaton J B, Perali A, cond-mat/0104098
  75. Choi H J et al., cond-mat/0111183
  76. Kuz'michev S A et al., in *Tez. Dokladov Mezhdunarodnoy Konf. Studentov i Aspirantov po Fundamental'nyim Naukam 'Lomonosov-2002', Moskva, 9–12 Aprelya 2002* (Abstracts of the 'Lomonosov-2002' Intern. Conf. of Students and Postgraduates in Fundamental Sciences, Moscow, April 9–12, 2002) (Moscow: Izd. MGU, 2002) p. 433

PACS numbers: 74.20.Mn, 74.20.Rp, 74.62.Dh  
DOI: 10.1070/PU2002v045n06ABEH001196

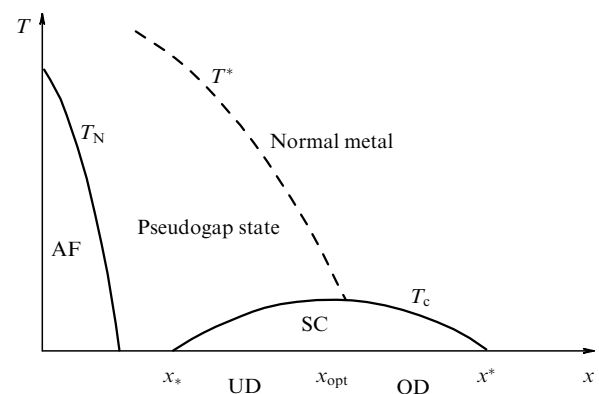
## High-temperature superconductivity models

Yu V Kopaev

### 1. Introduction

Cuprate-based high-temperature superconductors exhibit special properties in both normal and superconducting states, namely,

- (1) a high superconducting transition temperature  $T_c$ ;
- (2) d-type symmetry of the superconducting order parameter  $\Delta$  and low sensitivity to scattering on nonmagnetic impurities;



**Figure 1.** Phase diagram typical of hole-doped high-temperature cuprates (temperature vs. doping level  $x$ ). UD and OD stand for the underdoped and overdoped regions, respectively.

(3) the following features of the phase diagram: proximity of the antiferromagnetic and superconducting states, the existence of the latter within a limited (on both sides) interval of carrier concentrations (Fig. 1), and the existence of a pseudogap state at  $T > T_c$  in the underdoped (UD) region;

(4) ‘violation’ of the optical sum rule;

(5) the presence of a peak–dip–hump structure in angle-resolved photoemission spectra (ARPES) and in the tunneling characteristics;

(6) the static and dynamic structure of stripes and its relation to superconductivity;

(7) characteristic features of the inelastic neutron scattering spectra at  $T < T_c$ ;

(8) a difference between the concentration of the superconducting component and the total carrier concentration;

(9) a large ratio  $2\Delta(T=0)/T_c$  in the low-doping region; and

(10) anomalous temperature and frequency dependence of the diagonal and Hall resistivities.

It is understood that most of these features are related to electronic correlations not only of a superconducting nature, a fact indicated, in particular, by the proximity of superconducting (SC) and antiferromagnetic (AF) ordering in the phase diagram (see Fig. 1) and by the manifestation of short-range AF order in the SC region.

## 2. The Fermi contour of hole cuprates

As most solids, cuprates have no small parameter that could be used to describe them consistently by theoretical means, since the kinetic and potential energies of the particle interaction are of the same order of magnitude.

Hence, a consistent approach from the opposite ‘sides’ is desirable, i.e. from the strong-coupling ‘side’ (when the energy of the interaction of two particles on a single center is higher than the width of the band, or the Hubbard model) and from the weak-coupling ‘side’ (the band description as the zeroth approximation).

An antiferromagnetic state in the case of weak coupling (spin density waves, or SDW) is possible only if there is nesting of the Fermi contour (FC), i.e. matching of some sections of the FC that occurs on displacement by a certain momentum  $Q$ . Approximately such a shape of the contour (a square with rounded apexes) is obtained from ARPES data for all hole-doped cuprates [1]. The possibility of a substantial rise in  $T_c$  due to the mutual effect of dielectric [antiferromagnetic in the case of SDW or structural in the case of charge density waves (CDW)] and superconducting correlations was studied long before the discovery of superconducting cuprates [2].

In the Hubbard limit (which excludes the possibility of two particles existing on one center), the  $t$ – $J$  model is very popular. The hole FC and excitation spectrum are determined, as in the band model, by the integrals of hopping to neighboring centers, the only difference being that these integrals are multiplied by the hole concentration in the final state. The observed shape of the Fermi contour (a square with rounded apexes — almost nesting) is theoretically obtained when only the nearest-neighbor electron hopping is taken into account. However, the Fermi contour obtained in this manner proves to be rotated by  $\pi/4$  with respect to that observed in experiments. Apparently, allowing for hopping not only to the nearest neighbors generally violates the nesting condition, but for certain ratios of the nearest-neighbor to the next-nearest-neighbor

hopping integrals there is good agreement with the experimental data.

This rotation, which is a characteristic feature of hole cuprates, leads to two important consequences: (1) nesting is preserved within a broad interval of hole doping and rapidly disappears under electron doping; (2) as the hole concentration increases, the Fermi contour moves closer to the region of extended saddle points.

From the viewpoint of Cooper pairing with zero total momentum [3], this may be the reason for a higher critical temperature and a number of special features in the physical characteristics (due to the emergence of singularities in the density of states) [4]. These two properties manifest themselves more vividly in the formation of hole pairs with large total momenta (see below).

The same property of the Fourier contour is present in the generalized  $t$ – $J$  model.

## 3. Leading mechanisms of superconducting pairing

The main models that have been developed to explain the properties of superconducting cuprates can be classified according to the following features:

1. Singlet Cooper pairing with the d-symmetry of the order parameter is accepted as an unquestionable fact; various mechanisms of attraction responsible for such pairing have been proposed, namely,

(a) the phonon mechanism, typical of conventional superconductors [2, 5]. The main difficulty with this mechanism is not so much obtaining fairly large values of  $T_c$  as justifying d-type pairing. It is expected that the renormalization of electron–phonon coupling caused by strong electronic correlations may lead to what is known as preferential forward scattering, which ensures the stability of the d-type pairing [6];

(b) various electronic mechanisms of attraction, including the excitonic mechanism [2].

2. Cooper pairing of the d-type due to repulsive interaction. Such a pairing mechanism with finite angular momentum was first examined (in 1959) by Akhiezer and Pomeranchuk [7]. At present, the most sophisticated theory here is the theory of pairing due to spin wave exchange. This interaction is repulsive, and pairing is caused by the fact that, due to the proximity of the system to antiferromagnetic ordering, the interaction of this type is at its maximum when the transferred momentum corresponds to such ordering. It is this momentum that corresponds to scattering between the FC parts in which the superconducting order parameter has opposite signs for the d-type pairing [8].

3. Pairing occurs at both attractive and repulsive interaction if the gain in energy obtained as a result of the superconducting transition is due primarily to kinetic energy. Note that in the BCS model [3] the change in the kinetic energy corresponds to the loss in energy.

There are several semi-phenomenological approaches of this type:

(a) in the model proposed by Hirsch [9] the effective mass decreases (the gain in kinetic energy) as a result of the transition to the superconducting state. A graphic example of this is the correlated motion of two holes in the antiferromagnetic state. When only one hole is moving, spin order becomes violated, and the size of this violation is proportional to the length of the hole’s trajectory. The second hole restores AF order. Usually, this approach

ignores the loss in the kinetic energy of the relative motion of the two holes;

(b) as a result of analysis of ARPES data and the frequency dependence of resistivity, Norman et al. [10] proposed a self-energy part for the superconducting state that corresponds to the gain in the kinetic energy. A more thorough comparison of the photoemission spectra according to the electron momenta for a fixed energy and according to the energy distribution for a fixed momentum done by Norman et al. [11] made it possible to substantiate more rigorously the self-energy part selected in Ref. [10].

(c) the gain in the kinetic energy occurs as a result of tunneling of a Cooper pair (the intrinsic Josephson effect) in the stripes, between their metallic parts separated by AF parts [12]. Such a structure is a characteristic feature of hole cuprates, both in the static and the dynamic regimes.

In the Josephson effect the probability of a pair of particles tunneling is equal to the probability of a single particle tunneling (and not to the square of such a probability, as would be the case for uncorrelated motion of two particles). As a result, there is a gain in the kinetic energy of motion between the stripes when the system goes into the superconducting state. This gain is the reason for the superconducting transition.

4. Strong electronic correlations responsible for the AF state under light hole doping may lead to other nontrivial states when doping increases in the direction of the superconducting region. Such states may be the reason for a number of special features of cuprates. In this connection the staggered current state corresponding to orbital anti-ferromagnetism is being widely discussed [13].

Earlier such a state was studied within the band approach with FC nesting by Halperin and Rice [14], Volkov et al. [15], and Ginzburg et al. [16], and within the Hubbard model by Affleck and Marston [17].

Another avenue of research is related to studies of the effect of electron–electron correlations on the separation of charge and spin [18]. Here the superconducting state is formed on the basis of new quasi-particles: charged spinless bosons (holons) and neutral fermions (spinons). Corresponding to the superconducting state is the simultaneous Bose condensation of bosons and an analog of Cooper pairs consisting of spinons. A rigorous proof of such separation of charge and spin exists only for the one-dimensional case. It is believed that, in the Hubbard case of strong correlations and at the absence of a small parameter in the theory, after the Hamiltonian has been transformed in a way corresponding to one of the methods of separating charge and spin considered below, the residual interaction will be weak. The boson–boson interaction corresponds to repulsion, needed for the stability of the Bose condensate. Cooper pairing between spinons is ensured by one of the variants of Cooper pairing between holes. The residual interaction between holons and spinons must be weak so that no reverse coupling between them is possible. In this approach all the special features of the Fermi contour mentioned earlier are related to spinons.

Several models are used to describe the separation of charge and spin, as well as the various types of ordered states and phase transitions between the states:

(a) the simplest U(1) symmetry model of Ding et al. [19], in which the electron annihilation operator  $C_{xi}$  on the center  $i$  with spin  $\alpha$  is expressed as  $C_{xi} = f_{xi} b_i^+$ , the product of the fermion (spinon) annihilation operator  $f_{xi}$  and boson creation operator  $b_i^+$ . Unfortunately, the new state is character-

ized by broken translational symmetry, and the d-type of pairing for spinons proves to be unstable;

(b) the SU(2) symmetry model of Lee et al. [20] eliminates the above-noted difficulties of the U(1) model. It introduces two types 1 and 2 of spinons and holons, and the operator  $C_{xi}$  is expressed as

$$C_{xi} = \frac{1}{\sqrt{2}} (b_{1i}^+ f_{1i} + b_{2i}^+ f_{2i}^+).$$

In the d-type superconducting state,  $\langle b_1 \rangle \neq 0$ ,  $\langle b_2 \rangle = 0$ , and  $\langle f_{1i} f_{2j} - f_{2i} f_{1j} \rangle \neq 0$ ;

(c) the  $Z_2$  gauge model of Senthil and Fisher [21]; if on site  $j$  we introduce the phase  $\varphi_j$  of a Cooper pair, the boson creation operator  $b_j^+ \equiv \exp(i\varphi_j/2)$  corresponds to ‘one-half’ of the Cooper pair; the spinon creation operator  $f_{j\alpha}^+$  is expressed as

$$f_{j\alpha}^+ = b_j c_{j\alpha}^+.$$

Note that in standard BCS theory the excitations, Bogolyubov quasi-particles  $b_k$  with momentum  $k$ , are linear combinations of an electron with spin  $\sigma$  and momentum  $k$  and a hole with spin  $-\sigma$  and momentum  $-k$  [22]:

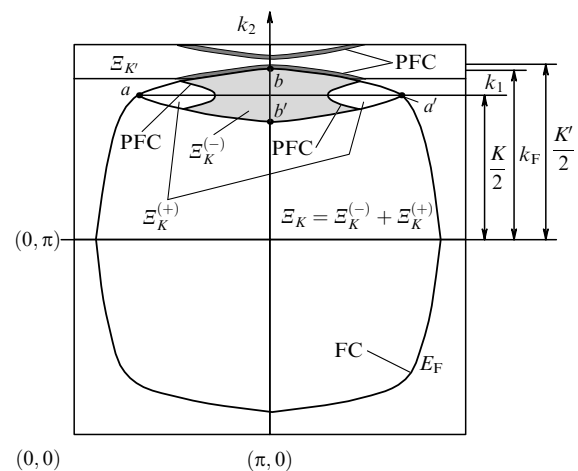
$$b_k = u(k)c_{k\sigma}^+ - v(k)c_{-k-\sigma}.$$

For the Fermi momentum  $k_F$  the coefficients  $u(k_F)$  and  $v(k_F)$  are equal to  $1/\sqrt{2}$ , i.e. the quasiparticle charge is zero. This calls for a number of special features of conventional superconductors, for instance, what is known as Andreev reflection, corresponding to transformation of an electron into a hole in backward scattering at the boundary between a normal metal and a superconductor.

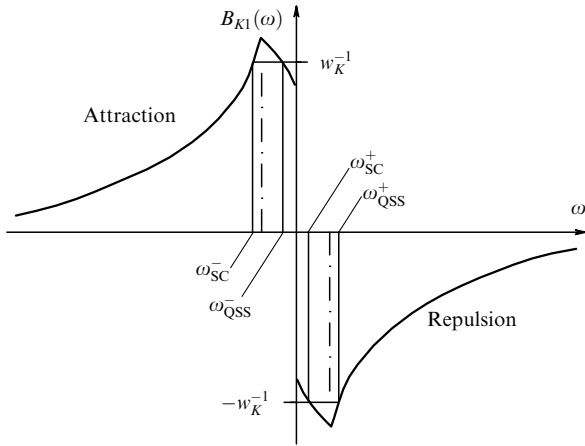
5. Pairing with large total momentum.

With the above-noted features of FC of hole cuprates, the hole–hole scattering amplitude has, in addition to the singularity at a zero total momentum of the hole pair, a singularity when the total momentum  $K$  is of the order of double the Fermi momentum,  $2k_F$  [23].

Here the region  $\mathcal{E}_K$  inside the Fermi contour corresponds to momenta  $K < 2k_F$  and the region  $\mathcal{E}_{K'}$  outside FC, to momenta  $K' > 2k_F$  (Fig. 2). Thus, there is no electron–hole



**Figure 2.** Fermi contour (FC) typical of hole cuprates: a square with rounded apices with the centered at point  $(\pi, \pi)$  of the Brillouin zone ( $E_F$  stands for the Fermi energy); the lines separating the regions of positive and negative energies of relative motion form a pair Fermi contour (PFC).

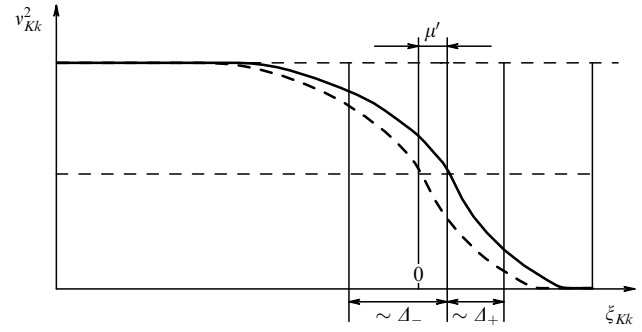


**Figure 3.** Real part  $B_{K1}(\omega)$  of the amplitude of scattering of a pair of holes with total momentum  $K$  as a function of the energy  $\omega$  for the cases of attraction (upper part of the diagram) and repulsion (lower part) between the holes; the points of intersection of the curve  $B_{K1}$  with the horizontal lines indicating the reciprocal of the effective interaction,  $w_K^{-1}$ , correspond to quasistationary states ( $\omega_{QSS}^\pm$ ) and superconducting instability ( $\omega_{SC}^\pm$ ).

symmetry (in the case of Cooper pairing the total zero momentum has corresponding to it states both outside and inside the Fermi contour).

The scattering amplitude has singularities both for an attractive potential and for a repulsive potential (Fig. 3). Here the effective interaction is proportional to the corresponding areas of  $\Xi_K$  and  $\Xi_{K'}$ . The observed singularities of hole cuprates correspond to the case of repulsive interaction. One of the poles in Fig. 3 ( $\omega_{QSS}^+$ ) corresponds to a quasistationary state and qualitatively describes the pseudogap region in the phase diagram (see Fig. 1). The second pole ( $\omega_{SC}^+$ ) has an imaginary part corresponding to superconducting instability [24]. Qualitatively, the frequency behavior of the scattering amplitude (see Fig. 3) coincides with that for the self-energy part phenomenologically introduced in Ref. [10]. What is important for the emergence of these solutions, in addition to the conditions of approximate nesting (whose presence guarantees the existence of finite allowed regions  $\Xi_K$  and  $\Xi_{K'}$ ), is the proximity of the Fermi contour to the extended saddle points. Then the kinetic energy of relative motion of the particles ( $k_x^2/2m_x - k_y^2/2|m_y|$ ) in the pair with momentum components  $k_x$  and  $k_y$  vanishes along some lines (hyperbolic metrics).

Pairing of the d-symmetry is determined through proper summation over the equivalent  $K$  or  $K'$  states [23]. To remove the above-noted instability, the ground state must be transformed by employing the appropriate Bogolyubov transformation. Here one must ensure that pairs with given momentum  $K$  can scatter from occupied states into free states with the same total momentum. For this to be true a fraction of the hole states must be moved from region  $\Xi_K$  into region  $\Xi_{K'}$  (outside the Fermi contour). If such a redistribution in momentum space corresponds to a redistribution in ordinary space (the emergence of static or dynamic stripes), the loss in kinetic energy may be compensated by the gain in the AF part due to the decrease of the hole concentration in this part [25, 26]. Superconducting pairing occurs along boundaries that emerge in this manner in the regions  $\Xi_K$  and  $\Xi_{K'}$ , boundaries that separate the states occupied by holes or electrons from the corresponding empty states. As a result there appear lines



**Figure 4.** Hole distribution function  $v_{Kk}^2$  in a superconducting state near a paired Fermi contour vs. the energy of relative motion  $\xi_{Kk}$ ; the solid curve represents the behavior with allowance for the shift  $\mu'$  of the Fermi level, and the dashed curve represent the behavior when the shift is ignored.

of a ‘pair Fermi contour’ (PFC) (see Fig. 2) that divides the regions  $\Xi_K$  and  $\Xi_{K'}$  into parts with positive  $\Xi_K^{(+)}$  and  $\Xi_{K'}^{(+)}$  and negative  $\Xi_K^{(-)}$  and  $\Xi_{K'}^{(-)}$  energies of relative motion. In the event of repulsive interaction there is no homogeneous (i.e. independent of the relative momentum) solution for the superconducting order parameter  $\Delta(K) = \text{const}$ . However, there is a solution for the parameter  $\Delta$  that varies both in magnitude and sign from  $\Delta_+$  to  $\Delta_-$  in the passage through the pair Fermi contour. Such a solution exists because the intensity of scattering of a pair with a given momentum  $K$  from the region  $\Xi_K^{(+)}$  to the region  $\Xi_K^{(-)}$  is proportional to the area of

$$\Xi_K = \Xi_K^{(+)} + \Xi_K^{(-)},$$

and the intensity of scattering inside  $\Xi_K^{(+)}$  and  $\Xi_K^{(-)}$  is proportional to the respective areas.

For Fermi contours with different signs of curvature on different parts of the contours or for multiply connected contours, the regions  $\Xi_K$  and  $\Xi_{K'}$  can simultaneously contain filled and empty parts even without forming stripes. Then PFC lines will represent a part of the Fermi contour. Such a FC structure was recently observed by Bogdanov et al. [27].

A feature that sets this type of pairing apart from the BCS model [3] is the presence in the chemical potential and in the expression for the condensation energy of a term linear in the parameter  $\Delta$  [25, 26]. The reason for this shift is that the smearing of the Fermi step on PFC when unequal parameters  $\Delta_+$  and  $|\Delta_-|$  emerge requires a shift in the chemical potential, so that the number of particles leaving the region inside the Fermi contour be equal to the number of places being vacated (Fig. 4). This linear shift is a manifestation of pairing in a wide region (compared to  $\Delta$ , in contrast to the BCS model [3]) and explains the observed ‘violation’ of the optical sum rule [28]. This is the reason for the gain in condensation energy at the expense of kinetic energy.

#### 4. Symmetry description of the phase diagram

For a better understanding of the nature of the superconducting state in cuprates it is often advisable to describe their properties from the symmetry point of view, which is not model-dependent.

Zhang [29] proposed an SO(5) symmetry theory that introduces five components of the order parameter: two components (the real and imaginary parts) of the super-

conducting order parameter and the three components of the antiferromagnetic order parameter. The breaking of  $SO(5)$  symmetry corresponds to phase transitions into the superconducting, antiferromagnetic, and mixed states. What is new here is the introduction of a dynamic degree of freedom associated with a collective triplet mode (the  $\pi$ -mode) that mixes the superconducting and antiferromagnetic components. The resonance in the inelastic scattering of neutrons observed in a number of hole cuprates is explained in Ref. [30] through the softening of this  $\pi$ -mode.

The above model with large total momentum of the pair in the singlet state corresponds to softening to zero at momentum  $K$  of the singlet  $\pi$ -mode at point  $T_c$ . It is assumed that the frequency of the triplet  $\pi$ -mode remains finite. The existence of the superconducting order parameter discussed earlier and not included in the  $SO(5)$  group is allowed for in the more general  $SU(4)$  group [31].

Yang [32] has pointed out that solutions with large total momentum of the pair may exist in the Hubbard model. Japaridze et al. [33] have studied numerically the possibility of realizing such a state as the ground state for the one-dimensional case with allowance for hopping between the centers of a pair of carriers.

## References

- Shen Z X et al. *Science* **267** 343 (1995)
- Ginzburg V L, Kirzhnits D A (Eds) *Problema Vysokotemperaturnoi Sverkhprovodimosti* (Moscow: Nauka, 1977) [Translated into English: *High-Temperature Superconductivity* (New York: Consultants Bureau, 1982)]
- Bardeen J, Cooper L N, Schrieffer J R *Phys. Rev.* **108** 1175 (1957)
- Abrikosov A A *Physica C* **341–348** 97 (2000)
- Maksimov E G *Usp. Fiz. Nauk* **170** 1033 (2000) [*Phys. Usp.* **43** 965 (2000)]
- Varelogiannis G *Phys. Rev. B* **57** 13743 (1998)
- Akhiezer A I, Pomeranchuk I L *Zh. Eksp. Teor. Fiz.* **36** 859 (1959) [*Sov. Phys. JETP* **9** 605 (1959)]
- Monthoux P, Pines D *Phys. Rev. B* **47** 6069 (1993)
- Hirsch J E *Phys. Rev. B* **62** 14487, 14498 (2000)
- Norman M R et al. *Phys. Rev. B* **61** 14742 (2000)
- Norman M R et al. *Phys. Rev. B* **64** 184508 (2001)
- Emery V J, Kivelson S A *Nature* **374** 434 (1995)
- Chakravarty S et al. *Phys. Rev. B* **63** 094503 (2001)
- Halperin B I, Rice T M, in *Solid State Physics* Vol. 21 (Eds F Seitz, D Turnbull, H Ehrenreich) (New York: Academic Press, 1968) p. 115
- Volkov B A et al. *Zh. Eksp. Teor. Fiz.* **81** 726 (1981) [*Sov. Phys. JETP* **54** 388 (1981)]
- Ginzburg V L et al. *Solid State Commun.* **50** 339 (1984)
- Affleck I, Marston J B *Phys. Rev. B* **37** 3774 (1988)
- Anderson P W *The Theory of Superconductivity in the High- $T_c$  Cuprates* (Princeton, N.J.: Princeton Univ. Press, 1997)
- Ding H et al. *Nature* **382** 51 (1996)
- Lee P A et al. *Phys. Rev. B* **57** 6003 (1998)
- Senthil T, Fisher M P A *Phys. Rev. B* **62** 7850 (2000)
- Bogolyubov N N *Zh. Eksp. Teor. Fiz.* **34** 58 (1958) [*Sov. Phys. JETP* **7** 41 (1958)]
- Belyavskii V I, Kopaev V V, Kopaev Yu V *Zh. Eksp. Teor. Fiz.* **118** 941 (2000) [*JETP* **91** 817 (2000)]; Belyavsky V I, Kopaev V V, Kopaev Yu V *Physica C* **341–348** 185 (2000)
- Belyavskii V I, Kopaev Yu V *Pis'ma Zh. Eksp. Teor. Fiz.* **72** 734 (2000) [*JETP Lett.* **72** 511 (2000)]
- Belyavsky V I, Kopaev Yu V *Phys. Lett. A* **287** 152 (2001)
- Belyavsky V I, Kopaev Yu V *Zh. Eksp. Teor. Fiz.* **121** 175 (2002) [*JETP* **94** 149 (2002)]
- Bogdanov P V et al. *Phys. Rev. B* **64** 180505 (2001)
- Basov D N et al. *Science* **283** 49 (1999)
- Zhang S-C *Science* **275** 1089 (1997)
- Hu J-P, Zhang S-C *Phys. Rev. B* **64** 100502 (2001)
- Guidry M et al. *Phys. Rev. B* **63** 134516 (2001)
- Yang C N *Phys. Rev. Lett.* **63** 2144 (1989)
- Japaridze G I et al. *Phys. Rev. B* **65** 014518 (2002)

PACS numbers: **74.72.-h**, **84.70.+p**, 85.25.Kx  
DOI: 10.1070/PU2002v045n06ABEH001198

## State of the art in applied high-current superconductivity

N A Chernoplekov

### 1. Introduction

The name ‘applied high-current superconductivity’ (AHCS) covers the practical applications of unique current-carrying capabilities of what is known as hard (type II) superconductors at temperatures and magnetic fields below their critical values. In superconducting devices, compared to those commonly manufactured of Cu and Al, the current density in the winding is 10 to 100 times higher than in that of traditional devices, there are no Joule heat losses in the DC mode, while in the AC mode at commercial frequencies the losses become as small as  $10^{-4}$  of those in resistive windings. Moreover, the use of AHCS makes it possible to develop devices that are cannot be manufactured by the conventional electrical engineering technologies, e.g. magnetic systems with almost ideally persistent (frozen) current, etc. With the cost of modern industrial low-temperature superconducting (LTSC) wires and cables being about US\$1 to US\$15 for 1 kA m and that of copper wire about US\$15 for 1 kA m, the question of expedience of using a superconducting device is determined by the acceptability of the costs of constructing a cryostatting system for the device and of the operation costs [1].

As is known, AHCS research began 50 years after the discovery of superconductivity phenomenon. Today it has a 40-year history and two areas of superconducting applications have developed. The first area deals with applications impossible without superconducting devices, such as modern accelerators and detectors for high-energy physics, facilities for thermonuclear research with magnetic confinement of hot plasma, magnets of unique precision, stability, and uniformity for magnetic resonance tomography, maximum-field magnets for NMR spectroscopy, and magnets used in research in physics, chemistry, and biology. The other area deals with the use of superconducting devices in ordinary industries, primarily electrical power engineering, transportation systems, mining, and other energy-intensive industries. The brave attempts of the 1970s (especially in the USSR and the US) to incorporate AHCS into ordinary industries after successful R&D of prototypes of various superconducting electrical devices proved to be unsuccessful due to the high costs involved and the low reliability of the superconducting devices of those times in comparison with traditional electrical devices. The main reason here was the high cost and low reliability of the cryogenic equipment operating at liquid-helium temperatures, and in the USSR especially the high cost of the coolant, helium.

Since then (the 1970s) much has improved in the LTSC technologies (the quality of superconducting materials, the possibilities of liquid-helium technology, and the very technology of magnetic systems). At the same time, after the discovery in 1986 of what are now known as high-tempera-