Physics – Uspekhi **45** (5) 553 – 559 (2002)

METHODOLOGICAL NOTES PACS numbers: 41.20.Jb, 42.25.Fx

Dipole-wave theory of electromagnetic diffraction

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DOI: 10.1070/PU2002v045n05ABEH001091

Contents

Abstract. **A theory of diffraction is presented which systematically employs the wave approach used in the Kirchhoff method and, unlike the Kirchhoff integral, does not have severe applicability limitations. The diffraction problem is solved by using the Hertz vector instead of the field vector used in the Kirchhoff integral. The basic diffraction problems for linearly, radially, and azimuthally polarized radiation are solved analytically. The key qualitative feature of the solutions is the presence of 'poles' — zero field points within the diffraction pattern of light and dark fringes. The poles lie along the direction of the electric field vector. The solutions obtained satisfy the Maxwell equations and the reciprocity principle.**

1. Introduction

The most general and rigorous approach to diffraction problems is to solve Maxwell's vector equations for appropriate boundary conditions. This approach has been described many times (see, e.g., [1 – 5]). However, the mathematical difficulties it involves put strong limits on the practical application of the solutions it yields. Thus, even a formally rigorous solution for diffraction from a circular hole [6, 7] is practically impossible to use because of the poor convergence of the series that represent the solution [1, 3, 8]. In solving specific real-life problems, physical simplifications and approximate calculations are usually employed.

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Received 24 October 2001
Uspekhi Fizicheskikh Nauk **172** (5) 601 – 607 (2002)
Translated by E G Strel'chenko; edited by S N Gorin

There are many methods available by which diffraction problems can be solved approximately based on Huygens' principle. These methods generally combine the ideas of geometrical optics with the wave approach and are exemplified by the classic Fresnel zone method. Among other similar methods, the Debye method for sharply focused fields should be noted [4]. The semi-empirical approaches underlying these methods strictly limit their range of applicability.

The Kirchhoff method for describing the electromagnetic field employs the pure wave approach to the diffraction problem. The scalar Kirchhoff integral is derived from the wave equations and strict mathematical logic [4, 9 – 11]. However, as is well known, the domain of applicability of the scalar Kirchhoff integral is also very narrow — for purely physical reasons relating to the way in which the problem is formulated.

The scalar wave equation for a field in free space — the equation employed in deriving the Kirchhoff integral — *a priori* gives no information on the way in which the field changes its direction. As for a scalar equation for the Green's function, this yields just a simple spherical wave from a point source with a uniformly directed field. However, since the electromagnetic field is transverse, no such wave can exist. Formally, the field the wave equation yields is not equivalent to that given by Maxwell's equations, and an 'independent' component-for-component scalar approach is invalid for a vector problem. Hence, a solution based on the scalar Kirchhoff integral does not satisfy the Maxwell equation $\operatorname{div} \mathbf{E} = 0$, and the approximate diffraction pattern so obtained is only correct in a small solid angle. There is only one, very special case (discussed in Section 3), where the scalar Kirchhoff integral can be used without the restrictions mentioned above.

An extension of the Kirchhoff method to vector fields is the so-called Kirchhoff – Kottler integral [4, 9, 11, 12], whose idea is generally that the scalar Kirchhoff integral is applied to the components of the field and the solutions obtained are then added as vectors. Because this approach is empirical, the authors of the classic monograph [9] point out, it admits of no physical interpretation and leads to solutions inconsistent with the Maxwell equation $\operatorname{div} \mathbf{E} = 0$. The shortcomings of the scalar method that are mentioned above all go over automatically to the Kirchhoff–Kottler integral, so the solutions it yields are approximate and are only valid over a limited range of problem parameters and over a narrow region of the diffraction pattern. Formal aspects of the correct methods for the 'scalarization' of vector electrodynamic equations are discussed, in particular, in Ref [13].

Kottler (see Refs [14–16]) introduced an additional contour integral (along the edges of the hole) into the solution procedure — to account for the electrical and magnetic charges distributed along the hole's contour. The reader is referred to Ref. [4] for a logical explanation of this idea. A frequently employed approach to diffraction problems is to take the unperturbed field of the incident wave as a prescribed field on the aperture. The region of applicability of the method is then $kr \gg 1$, $ka \gg 1$, where k is the wave vector, a is the characteristic size of the aperture, and r the distance from the edge of the aperture to the point of observation. Employing the contour integral also enables one to calculate the diffraction field at small distances from the diaphragm edge — the region where field distortions cannot be ignored. Still, introducing the contour integral does not make up for the Kirchhoff method's inherent shortcomings, namely a scalar integral and an vector extension.

The purpose of this paper is twofold. The first is to develop a physically justified and mathematically correct vector theory of diffraction, which relies on the logic of the Kirchhoff method and has no stringent applicability limits inherent to the Kirchhoff integral. The second purpose is to use this theory to obtain analytical solutions for a number of key diffraction problems.

2. General approach

The fundamental physical restrictions inherent in the Kirchhoff method can be avoided by using the polarization potential or the Hertz vector $\mathbf{Z}$ [11] instead of the fields — analogous to the theory of antennas, for example.

It is known that $\mathbf{Z}(\mathbf{r}, t)$, like electric and magnetic fields, satisfies the wave equation $\Box \mathbf{Z} = 0$ and is related to the fields $\mathbf{E}$ and $\mathbf{H}$ by the equations

$$\mathbf{E} = \operatorname{rot}\operatorname{rot}\mathbf{Z}\,, \quad \mathbf{H} = \frac{1}{c}\operatorname{rot}\frac{\partial \mathbf{Z}}{\partial t}\,. \tag{1}$$

There is an important point to make here about the wave equation solution obtained using the Hertz vector. The fields found from Eqns (1) automatically satisfy the Maxwell equation $\operatorname{div}\mathbf{E} = 0$ [5]. The inconsistency inherent in the conventional Kirchhoff method is absent in this approach.

An expression for the electromagnetic field of a radiating dipole $\mathbf{p} = \mathbf{p}_0 \exp(-\mathrm{i}\omega t)$ is obtained from the solution of the wave equation with a dipole source [11]

$$\mathbf{Z} = \mathbf{p}_0\,\frac{\exp(-\mathrm{i}\omega t + \mathrm{i}\mathbf{kr})}{r}\,. \tag{2}$$

The polarization potential of a dipole wave is parallel to $\mathbf{p}_0$ and is transferred by a spherical wave (Fig. 1). Of the two scalar wave equations employed in the Kirchhoff method, one — the inhomogeneous equation for the Green's function — may be given a quite certain physical interpretation when written for Z: it describes the field produced by a real, point source of electromagnetic radiation. The known expressions for the electric and magnetic fields of a radiating dipole are obtained by substituting Eqn (2) into Eqn (1); the dipole wave is illustrated in Fig. 1.

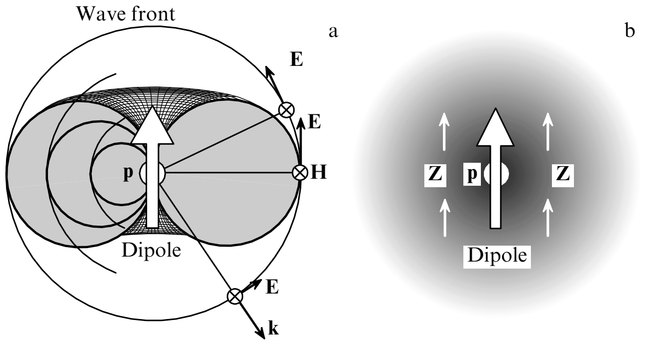

Figure 1. Radiating dipole: (a) electric and magnetic field distributions; (b) Hertz vector distribution.

Noting that the polarization potential of a dipole wave retains its direction at various points in space, we see that the case of linearly polarized radiation incident on the aperture can be treated by applying the scalar equations to Z. The scalar Kirchhoff integral written for Z then contains vector information about the diffraction pattern field.

This makes it possible to write down the integral for $\mathbf{Z}$ without any approximations related to the inhomogeneity of the vector function, and to obtain the fields $\mathbf{E}$ and $\mathbf{H}$ by simple differentiation. This approach is extended in a natural way to the general case with the field direction distributed arbitrarily over the aperture.

Thus, the vector diffraction problem for linearly polarized radiation is solved based on the scalar Kirchhoff integral [11] written for the Hertz vector,

$$Z(\mathbf{r}) = \int_{S'} \left[G(\mathbf{n}\nabla)\,Z_0 - Z_0(\mathbf{n}\nabla G)\right]\,\mathrm{d}S'\,. \tag{3}$$

Here, $Z_0(\mathbf{r}')$ is the Hertz vector distribution over the surface under the consideration S'; $Z(\mathbf{r})$ is the Hertz vector at the point of observation; $\mathbf{n}$ is the unit normal to the surface of a given field at the aperture; and $G(\mathbf{r}, \mathbf{r}') = \exp\left[\mathrm{i}\,\mathbf{k}(\mathbf{k} - \mathbf{r}')\right]/|\mathbf{r} - \mathbf{r}'|$ is the Green's function of the scalar wave equation.

Let us consider the important and practical case of a plane linearly polarized wave diffracted by the aperture. In this case $\mathbf{Z}_0(\mathbf{r}')$ and $\mathbf{E}_0(\mathbf{r}')$ at the surface S' are related by the simple equation $\mathbf{E}_0(\mathbf{r}') = -k^2\mathbf{Z}_0(\mathbf{r}')$. Noting this, Eqn (3) becomes

$$\mathbf{Z}(\mathbf{r}) = -\mathbf{e}_0\,\frac{1}{k^2}\int_{S'}\left[G\,\frac{\mathrm{d}}{\mathrm{d}n}\,E_0 - E_0\,\frac{\mathrm{d}}{\mathrm{d}n}\,G\right]\mathrm{d}S'\,, \tag{4}$$

where $\mathbf{e}_0$ is a unit vector in the field direction, and $\mathrm{d}/\mathrm{d}n$ is the derivative in the direction $\mathbf{n}$, $\mathrm{d}/\mathrm{d}n = \mathbf{n}\nabla$.

To calculate the diffraction pattern, one must now evaluate the integral in expression (4), write $\mathbf{Z}$ in vector form with the same unit vector as in the filed $\mathbf{E}_0$, and, finally, calculate the field itself from formulas (1). The solution obtained for the plane incident wave, Eqn (4), makes it possible to solve the problem in the general case as well.

Turning now back to the question of scalarization of the vector electrodynamic equations [13], we conclude that the above treatment of the diffraction problem is yet another 'physical' scalarization approach. In the general case of a three- dimensional vector field, the vector diffraction problem for linearly polarized radiation should be solved for each of the components separately. As indicated above, this problem reduces to a scalar integral using the Hertz vector. The solutions of the problem are valid throughout the entire space and satisfy Maxwell's equations; therefore, the diffraction fields from each of the components can be added as vectors.

Let us now discuss the validity limits of the method. Since the field specified on the aperture is taken to be edge-unperturbed, the hole size should obey the condition $ka \gg 1$ typical of such situations. A good agreement between such calculations and their exact counterparts is usually achieved for $ka \geqslant 5$ [10]. As for the distances r to the point of observation, the wave zone approximation $kr \gg 1$ is usually employed. Farther away from the edge of the hole, field distortions on the diaphragm edges cannot be ignored whatever the size of the hole. In all of the problems discussed below we adopt a more restrictive assumption $r \gg a$, which enables analytical formulas to be obtained.

In what follows, several examples of the diffraction of light from holes of different shape in a nontransparent screen are given to illustrate the method proposed. In all cases, the field on the aperture is assumed to be prescribed, and the surface S' covering the aperture is taken to be flat, the vector $\mathbf{n}$ in formulas (3) and (4) being directed along the z axis. The plane xz is the incidence plane of radiation in all cases. For diffraction from a slit and from rectangular and circular holes, two possible field directions are considered, in and normal to the incidence plane. On this basis, the solution for any field direction can be constructed.

3. Diffraction of radiation from an infinite slit

Let us consider an ideally conducting screen lying in the xy plane, with a y-aligned slit of width $2\Delta x$ in it. Incident on the slit is a linearly polarized plane wave traveling in the xz plane at an angle θ_0 with respect to the z axis,

$$\begin{aligned}\mathbf{E}_0 &= \mathbf{e}_0 E_0 \exp\left[\mathrm{i}(k_x x' + k_z z')\right] \\ &= \mathbf{e}_0 E_0 \exp(\mathrm{i}kx' \sin\theta_0)\exp(\mathrm{i}kz' \cos\theta_0)\,.\end{aligned}$$

The time factor $\exp(-\mathrm{i}\omega t)$ is omitted for brevity, $\mathbf{e}_0$ is a unit vector in the field direction.

The vector $\mathbf{Z}$, Eqn (4), has the same direction, namely that of $\mathbf{E}_0$, at all points. The scalar integral (4) is evaluated by a standard procedure. In the wave zone $k\rho \gg 1$, the integral for the Hertz vector has the form

$$\begin{aligned}Z = -\frac{\mathrm{i}}{k} E_0 \int_{-\Delta x}^{\Delta x} \exp(\mathrm{i}kx' \sin\theta_0)\, \frac{\exp\left(\mathrm{i}k(\rho-\rho')\right)}{\rho-\rho'} \\ \times \left(\cos\theta_0 + \frac{z}{\rho-\rho'}\right) \mathrm{d}x'\,. \qquad (5)\end{aligned}$$

Here, $\rho - \rho' = \sqrt{(x-x')^2 + z^2}$, ρ is the radius vector to the point under consideration, and Δx is the slit width. The approximation $\Delta x \ll \rho$ leads to the analytical solution

$$\begin{aligned}\bar{Z} &= -\mathrm{i}\bar{S}\, \frac{\exp(\mathrm{i}\bar{\rho})}{\rho}\,(\cos\theta_0 + \cos\theta)\,\frac{\sin\chi}{\chi}\,; \\ \chi &= \overline{\Delta x}\,(\sin\theta - \sin\theta_0)\,. \qquad (6)\end{aligned}$$

Here, we have transformed to the coordinates ρ and θ (θ being measured from the z axis) and introduced the dimensionless parameters

$$\bar{Z} = Z\,\frac{k^2}{E_0}\,, \quad \bar{S} = 2k\,\Delta x\,, \quad \overline{\Delta x} = k\,\Delta x\,, \quad \bar{\rho} = k\rho\,.$$

This approximation differs markedly from that of Kirchhoff in that it does not put a limit on the angle and potentially contains information on how the local direction of the diffraction field depends on the direction of the initial field.

If the field $\mathbf{E}_0$ is perpendicular to the plane of incidence ($\mathbf{E}_0 \| y$), then in vector form we have $\mathbf{Z} = Z(x,z)\mathbf{e}_y$. The calculation of the field from the formula (1) was carried out in Cartesian coordinates. The procedure of taking the rotor is simplified considerably by considering the order of magnitude of the derivatives of the multipliers in Eqn (6). Because of the factor $k = 2\pi/\lambda$, the derivative of the exponential with respect to the coordinate is maximum on the order of magnitude. The same factor also appears in the derivative of χ — but this time with a coefficient of the type $\Delta x/\rho$, which is small because of the approximations we have adopted. The expression for $\mathbf{E}$ in this case takes the simple form

$$\mathbf{E} = E_0\,\bar{Z}\,\mathbf{e}_y\,, \qquad (7)$$

with $\bar{Z}$ defined by Eqn (6). Expression (7), quite understandably, is identical to that obtained from the usual scalar Kirchhoff integral. Only in this special and unique case does the usual method give a solution satisfying the equation $\operatorname{div}\mathbf{E} = 0$.

Accordingly, for the field $\mathbf{E}_0$ in the plane of incidence ($\mathbf{E}_0 \perp y$), the polarization potential has the form $\mathbf{Z} = Z(x,z)(\mathbf{e}_x\cos\theta_0 - \mathbf{e}_z\sin\theta_0)$, and Eqn (1) yields the following expression for the electric field:

$$\mathbf{E} = E_0\cos(\theta-\theta_0)\,\bar{Z}\,\mathbf{e}_\theta\,. \qquad (8)$$

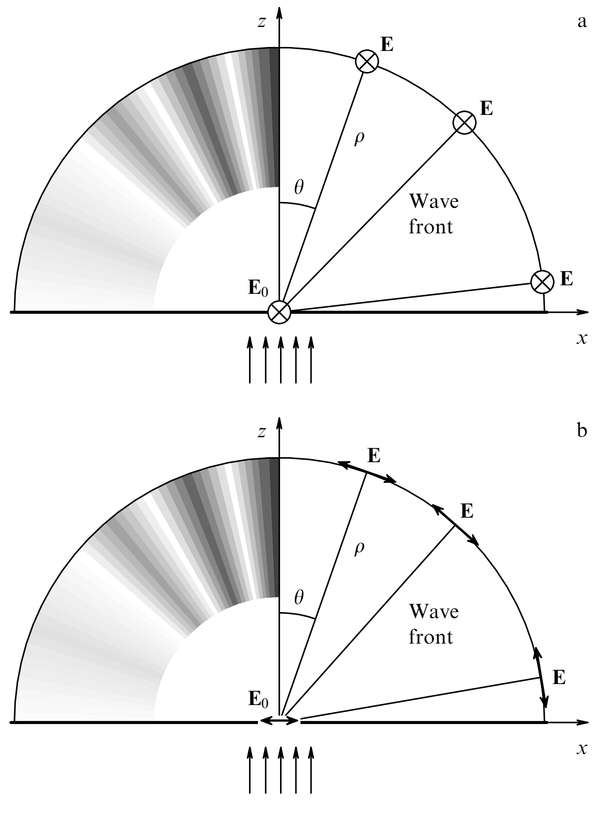

Figure 2. Diffraction of linearly polarized light from an infinite slit: arrangement, notation, and the computationally synthesized diffraction pattern. The direction of the diffraction field is shown for various field directions at the slit. The angle of incidence is zero.

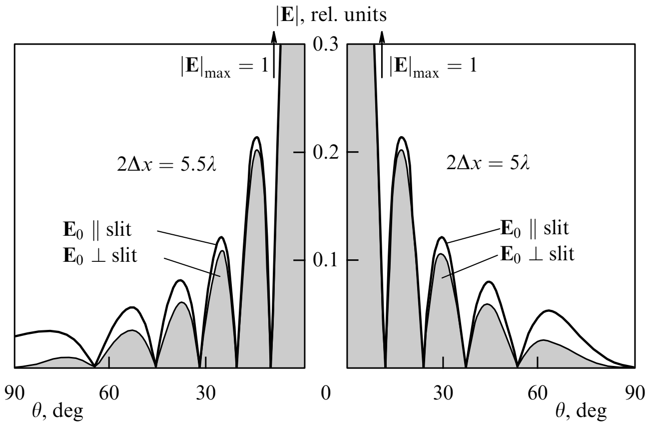

Figure 3. Diffraction of linearly polarized field from an infinite slit for two polarization directions relative to the slit and for various slit widths. The angle of incidence is zero.

The direction of the diffracted field given by Eqns (7) and (8) is illustrated in Fig. 2, and the field amplitude distributions calculated from Eqns (7) and (8) are shown for zero angle of incidence in Fig. 3. The figures clearly show the following qualitative feature of the results obtained. In the case where $\mathbf{E}_0 \perp y$, the diffraction field at $\theta = \pi/2$, i.e., along the screen, is zero for any slit width. There is no radiation in the direction of the initial field. The amplitude of the diffraction field for $\theta = \pi/2$ depends on the slit width. If the width is a multiple of the wavelength, the field vanishes; if the width equals an odd number of half-widths, the field along the screen is nonzero.

Figure 4 illustrates the calculations for an angle of incidence of 72°. A characteristic feature of the case $\mathbf{E}_0 \perp y$ is the presence of an additional zero-field point at $\theta = \theta_0 - \pi/2$ [see Eqn (8)]. This 'pole' is located along the direction of the field.

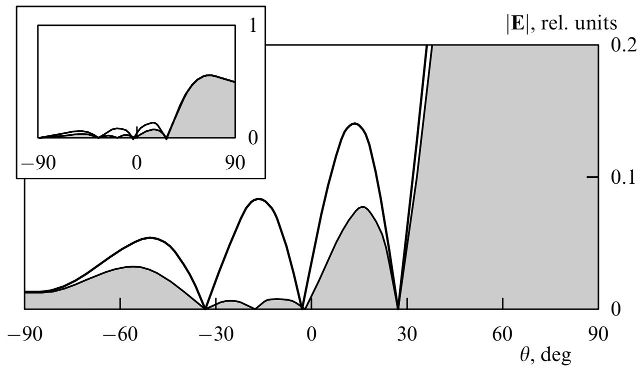

Figure 4. Diffraction of linearly polarized light from an infinite slit for two polarization directions relative to the slit and for a slit width $2\Delta x = 2\lambda$. The angle of incidence is 72°.

For $\theta_0 = 0$ and $\theta \to 0$, formulas (7) and (8) go over to classical ones.

Based on Eqns (7) and (8) we have obtained, it is possible to revise the formula for the simplest diffraction grating formed by parallel slits cut in a nontransparent screen. It is known that the formula for the light that has passed through such a grating has two terms: a term for single slit diffraction, and an expression related to the collective effect of diffraction from many slits. The formula for this grating will then take into account the polarization of the radiation.

4. Diffraction of linearly polarized radiation from rectangular and circular holes

The orientation of the screen with respect to the coordinate axes is the same as for the slit case. The holes are centered at the origin. The dimensions of the rectangular hole are $2a$ and $2b$ along the axes x and y, respectively, and the radius of the circular hole is r_0. The formulas both for $\mathbf{Z}$ and $\mathbf{E}$ for diffraction from the rectangular holes are derived similarly to those above, giving

$$\mathbf{E} = E_0 \bar{Z}(r, \varphi, \theta) \mathbf{q}(\varphi, \theta) ,$$

$$\bar{Z}(r, \varphi, \theta) = -\mathrm{i}\, \bar{S}\, \frac{\exp(\mathrm{i}\bar{r})}{\bar{r}} (\cos\theta_0 + \cos\theta) \frac{\sin\chi_a}{\chi_a} \frac{\sin\chi_b}{\chi_b} , \tag{9a}$$

$$\chi_a = \bar{a}(\sin\theta\cos\varphi - \sin\theta_0) , \quad \chi_b = \bar{b}\sin\theta\sin\varphi ,$$
$$\bar{S} = 4\bar{a}\bar{b} , \quad \bar{a} = ka , \quad \bar{b} = kb , \quad \bar{r} = kr .$$

The form of the vector function $\mathbf{q}(\varphi, \theta)$ depends on the field direction at the hole and is given in the table.

Table.

Field $\mathbf{E}_0$	Vector $\mathbf{Z}$	Vector $\mathbf{q}\,(\varphi, \theta)$
$\mathbf{E}_0 \| y$	$Z \cdot \mathbf{e}_y$	$\mathbf{e}_\theta \cos\theta\sin\varphi + \mathbf{e}_\varphi \cos\varphi$
$\mathbf{E}_0 \perp y$	$Z \cdot (\mathbf{e}_x \cos\theta_0 - \mathbf{e}_z \sin\theta_0)$	$\mathbf{e}_\theta(\sin\theta\sin\theta_0 + \cos\theta\cos\varphi\cos\theta_0) - \mathbf{e}_\varphi \sin\varphi\cos\theta_0$

As for the integral of type (5) for a circular hole, it reduces to

$$Z = -\frac{\mathrm{i}}{k} E_0 (\cos\theta_0 + \cos\theta) \frac{\exp(\mathrm{i}kr)}{r}$$
$$\times \int_0^{r_0} \int_0^{2\pi} \exp\left\{\mathrm{i}kr'\big(\sin\theta_0\cos\varphi' - \sin\theta\cos(\varphi - \varphi')\big)\right\}$$
$$\times\, r'\, \mathrm{d}\varphi'\, \mathrm{d}r' .$$

The expression in round brackets in the integrand can be written in the form $M\cos(\varphi + \varphi')$, where

$$M = \sqrt{(\sin\theta_0 - \sin\theta\cos\varphi)^2 + (\sin\theta\sin\varphi)^2} ,$$

$$\cos\varphi = \frac{\sin\theta_0 - \sin\theta\cos\varphi}{M} , \quad \sin\varphi = \frac{\sin\theta\sin\varphi}{M} .$$

The calculation then proceeds as before, giving the following expressions for the Hertz vector $\mathbf{Z}$ and the electric field $\mathbf{E}$:

$$\mathbf{E} = E_0 \bar{Z}(r, \varphi, \theta)\, \mathbf{q}(\varphi, \theta) ,$$

$$\bar{Z}(r, \varphi, \theta) = -2\mathrm{i}\, \bar{S}\, \frac{\exp(\mathrm{i}\bar{r})}{\bar{r}} (\cos\theta_0 + \cos\theta) \frac{J_1(\bar{r}_0 M)}{\bar{r}_0 M} , \tag{9b}$$

$$M = \sqrt{\sin^2\theta - 2\sin\theta\sin\theta_0\cos\varphi + \sin^2\theta_0} ,$$

$$\bar{Z} = Z\frac{k^2}{E_0} , \quad \bar{S}_0 = \pi\bar{r}_0^2 , \quad \bar{r} = kr , \quad \bar{r}_0 = kr_0 .$$

a

b

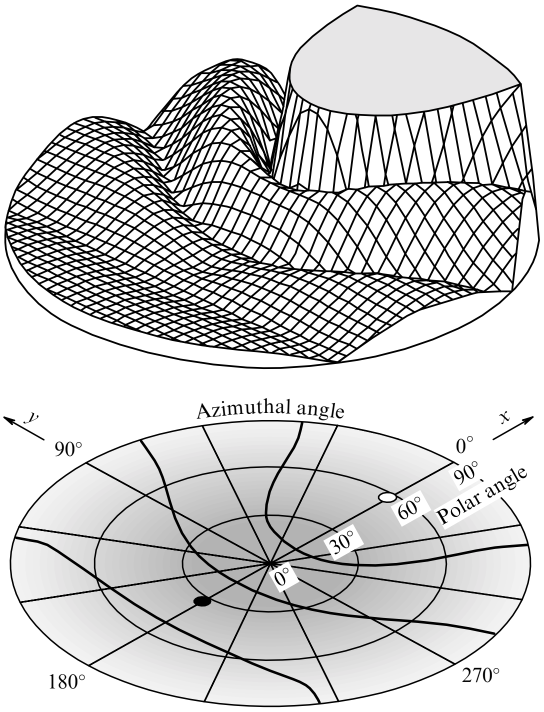

Figure 5. Diffraction of linearly polarized field from a circular hole for hole radius $r_0 = \lambda$ and $\mathbf{E}_0 \perp y$: (a) field amplitude distribution, (b) topology of the zero-field fringes and the pole (solid circle). The open circle is the position of the field maximum.

a

b

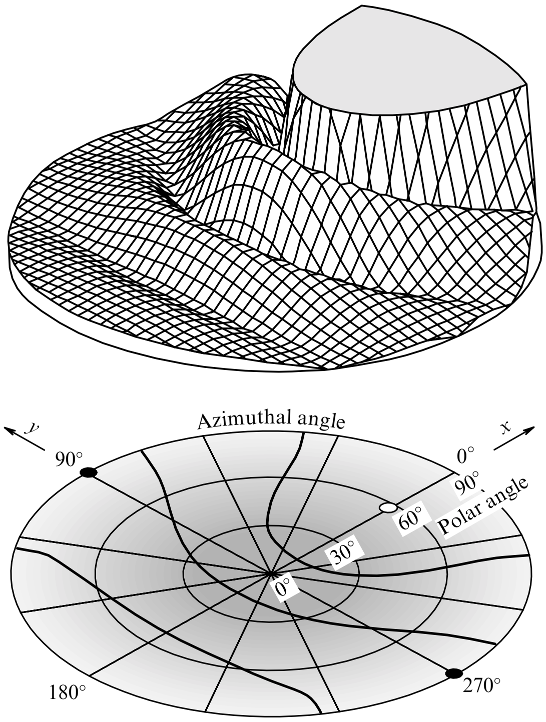

Figure 6. Diffraction of linearly polarized field from a circular hole for hole radius $r_0 = \lambda$ and $\mathbf{E}_0 || y$: (a) field amplitude distribution, (b) topology of the zero-field fringes and the poles. The open circle is the position of the field maximum.

The form of the vector function $\mathbf{q}(\varphi, \theta)$ is the same as for the rectangular hole and is determined by the formulas listed in the table.

A notable feature of the above results is the presence in the diffraction pattern of so-called 'poles', zero-field points located along the direction of $\mathbf{E}_0$. For $\mathbf{E}_0 || y$, there are two such points, with coordinates $\varphi = \pm\pi/2$, $\theta = \pi/2$. For $\mathbf{E}_0 \perp y$, there is only one such point on the hemisphere of observation, $\varphi = \pi$, $\theta = \pi/2 - \theta_0$.

Let us illustrate the results obtained from Eqns (9a) and (9b). The formation of the poles is the responsibility of the expression for $\mathbf{q}(\theta, \varphi)$, and the form of this latter is independent of the shape of the hole. In Fig. 5a the distribution of the field amplitude over the hemisphere θ, φ for diffraction from a circular hole is shown [Eqn (9b)], for an angle of incidence of 57°, and $\mathbf{E}_0 \perp y$. Figure 5b shows the fringes and zero field points. Figure 6 shows, for comparison, the results of similar calculations for $\mathbf{E}_0 || y$, all other conditions being equal. In terms of 'poles,' the diffraction pattern from a rectangular hole, Eqn (9a), is similar, but the topology of dark and bright fringes is, naturally, different from that for the circular case (Fig. 7).

a

b

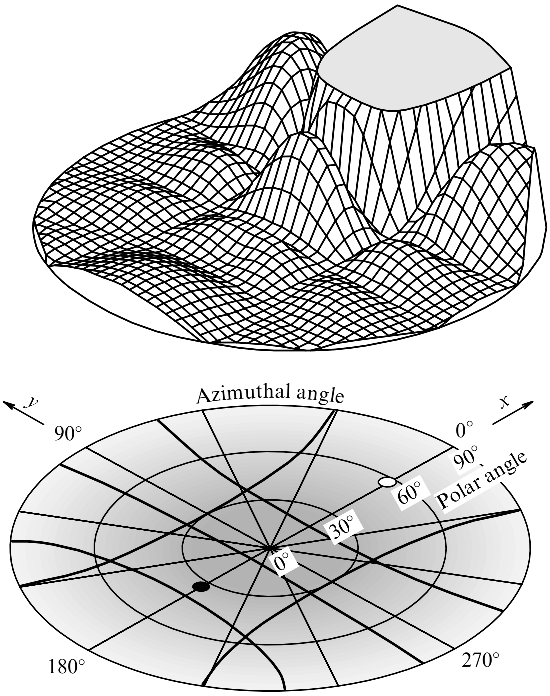

Figure 7. Diffraction of linearly polarized field from a rectangular hole with dimensions $a = b = \lambda$ for $\mathbf{E}_0 \perp y$: (a) field amplitude distribution, (b) the topology of the zero-field fringes and the pole (solid circle). The open circle is the position of the field maximum.

The diffraction formulas for rectangular and circular holes, developed based on the electrodynamic formulation of Huygens' principle (i.e., on ED theory) [10], fail to describe the 'poles' associated with the direction of the field $\mathbf{E}_0$. ED-theory solutions to some diffraction problems are inconsistent with the reciprocity principle. This fact was commented upon as follows in Ref [10]: "This inconsistency is characteristic of a number of problems in which surface currents are given by the formula $\mathbf{j} = c/(2\pi) \cdot [\mathbf{n}\mathbf{H}_0]$." As for the present method, all the solutions above are consistent with the reciprocity principle. For $\varphi = 0$ and $\varphi = \pi$, the angles θ and θ_0 are interchangeable.

In this section we have discussed in detail some aspects of the physics involved in the appearance in the diffraction pattern of zero-field points depending on the direction of the field at the aperture. But besides correctly accounting for this phenomenon, our formulas contain other useful information. They differ significantly from known ones in that they not only describe the qualitative features of the diffraction pattern (zero-field points) but also offer much better quantitative results for the distribution of field in the pattern. Accurate quantitative information is very important in this case: for example, the direction of the maximum diffraction field $\theta_{\max}$ generally does not coincide with θ_0; the difference $\theta_0 - \theta_{\max}$ depends on the angle of incidence and may be as high as 10°.

5. Diffraction of azimuthally polarized radiation from a circular slit

Noting that the azimuthal and radial polarization modes are well known in the theory of waveguides and open resonators, let us consider the diffraction of light with this polarization from a circular slit of radius r_0 and width Δr. The angle of incidence is assumed to be zero, $\theta_0 = 0$.

Assuming that $\mathbf{Z}_0 = Z_0 \exp(\mathrm{i}kz)\,\mathbf{e}_\varphi$, it is readily shown by directly substituting $\mathbf{Z}_0$ into Eqn (1) that the relation $\mathbf{E}_0(\mathbf{r}') = k^2\mathbf{Z}_0(\mathbf{r}')$ holds here too. When taking the rotor in cylindrical coordinates, it should be noted that there is in this case only one — φ — nonzero vector component, and that this latter depends (through the exponential) on the coordinate z alone. Following the already familiar pattern and assuming $\Delta r_0 \ll r_0 \ll r$ we arrive at

$$\mathbf{Z} = \frac{E_0}{k^2}\,(k^2 2\pi r_0 \Delta r)(1+\cos\theta)\,\frac{\exp(\mathrm{i}kr)}{kr}\,J_1(kr_0\sin\theta)\,\mathbf{e}_\varphi\,,$$

where the well-known transformations [17]

$$\int_0^{2\pi} \sin(n\beta)\exp\big(\mathrm{i}t\cos(\beta-\gamma)\big)\,\mathrm{d}\beta = 2\pi\,\mathrm{i}^n J_n(t)\sin(n\gamma)\,,$$
$$\int_0^{2\pi} \cos(n\beta)\exp\big(\mathrm{i}t\cos(\beta-\gamma)\big)\,\mathrm{d}\beta = 2\pi\,\mathrm{i}^n J_n(t)\cos(n\gamma)\,,$$

have been used. In this case the field determined by Eqn (1) is conveniently calculated in spherical coordinates, giving

$$\mathbf{E} = E_0\bar{S}_0\,\frac{\exp(\mathrm{i}\bar{r})}{\bar{r}}\,(1+\cos\theta)\,J_1(\bar{r}_0\sin\theta)\,\mathbf{e}_\varphi\,, \tag{10}$$

where $\bar{S}_0 = 2\pi k^2 r_0\Delta r$, $\bar{r} = kr$, $\bar{r}_0 = kr_0$. Along the polar axis, for $\theta = 0$, the field is zero. The magnitude of the field along the screen, for $\theta = \pi/2$, depends on the slit radius and may or may not be zero (Fig. 8).

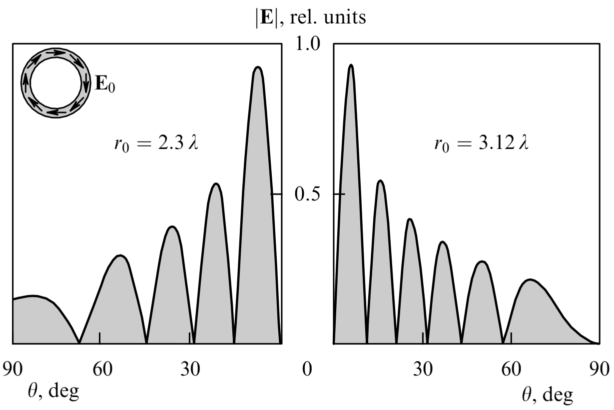

Figure 8. Diffraction of azimuthally polarized field from a circular slit for two slit radii.

6. Diffraction of radially polarized radiation from a circular slit

Let us assume that $\mathbf{Z}_0 = Z_0\exp(\mathrm{i}kz)\,\mathbf{e}_\rho$ within the circular slit; substitution into Eqn (1) then leads again to the relation $\mathbf{E}_0(\mathbf{r}') = k^2\mathbf{Z}_0(\mathbf{r}')$. The rotor is conveniently taken in cylindrical coordinates because we have only one nonzero vector component, ρ (which depends only on the coordinate z through the exponential). The solution for the Hertz vector in this case has a similar form:

$$\mathbf{Z} = \frac{E_0}{k^2}\,(k^2 2\pi r_0\Delta r)\left(1+\frac{z}{\sqrt{\rho^2+z^2}}\right)$$
$$\times\,\frac{\exp\big(\mathrm{i}k\sqrt{\rho^2+z^2}\,\big)}{k\sqrt{\rho^2+z^2}}\,J_1\!\left(kr_0\,\frac{\rho}{\sqrt{\rho^2+z^2}}\right)\mathbf{e}_\rho\,.$$

The above form has been adapted to the calculation of the electric field from Eqn (1) in the cylindrical coordinate system, giving

$$\mathbf{E} = E_0\,\bar{S}_0\,\frac{\exp(\mathrm{i}\bar{r})}{\bar{r}}\,(1+\cos\theta)\cos\theta\,J_1(\bar{r}_0\sin\theta)$$
$$\times\left\{\cos\theta\,\mathbf{e}_\rho + \left[\frac{\mathrm{i}}{\bar{r}\sin\theta} - \sin\theta\right]\mathbf{e}_z\right\}.$$

For convenience we have here introduced natural notation in terms of $r = \sqrt{\rho^2+z^2}$ and the polar angle θ measured from the vertical axis. It is convenient to decompose the above expression into two parts, a field in the meridional direction and a field along the z axis. The latter has a phase shift of $\pi/2$:

$$\mathbf{E}_\theta = E_0\,\bar{S}_0\,\frac{\exp(\mathrm{i}\bar{r})}{\bar{r}}\,(1+\cos\theta)\cos\theta\,J_1(\vartheta)\,\mathbf{e}_\theta\,, \tag{11}$$

$$\mathbf{E}_z = \mathrm{i}\,\frac{\bar{r}_0}{\bar{r}}\,E_0\,\bar{S}_0\,\frac{\exp(\mathrm{i}\bar{r})}{\bar{r}}\,(1+\cos\theta)\cos\theta\,\frac{J_1(\vartheta)}{\vartheta}\,\mathbf{e}_z\,. \tag{12}$$

Here, the dimensionless quantities $\vartheta = \bar{r}_0\sin\theta$, $\bar{r} = kr$, $\bar{r}_0 = kr_0$ have been introduced. The field distribution from formulas (11) and (12) is given in Fig. 9. The purely

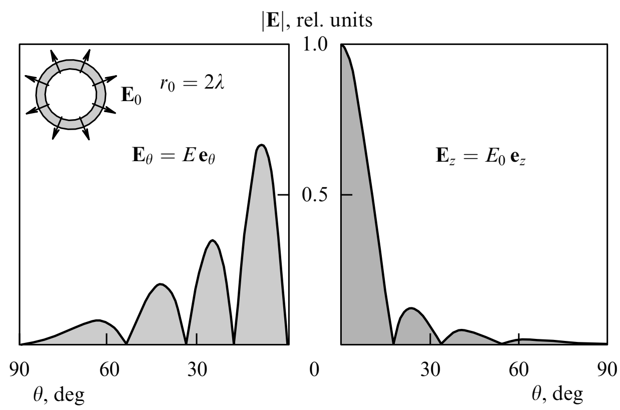

Figure 9. Diffraction of a radially polarized field from a circular slit. On the left is the component of the field $\mathbf{E}_\theta$ directed along the meridian. On the right is the longitudinal component of the field $\mathbf{E}_z$, directed along the z axis. The meridional and longitudinal field components differ in phase by $\pi/2$. The relative scale of the two curves is arbitrary.

longitudinal component of the field, Eqn (12), is small in magnitude (due to the factor r_0/r), but the maximum of the field is on the axis, where the meridional component, Eqn (11), is zero.

7. Conclusion

We have developed a method for solving diffraction problems based on the use of the Hertz vector in the Kirchhoff integral. With this approach, there are no stringent applicability limits inherent to the Kirchhoff integral. For the case of a plane wave incident on the aperture, the task of finding the solution is considerably simplified. Over the entire range of polar angles from 0 to 90° and for large distances from the aperture, analytical solutions have been obtained for basic diffraction problems, namely, the diffraction of linearly polarized radiation from an infinite slit and rectangular and circular holes for any angle of incidence and any polarization direction, and the diffraction of radiation with azimuthal and radial polarization from a circular slit. It is shown that a qualitative feature of the diffraction pattern obtained using the vector approach is the presence of 'poles' — zero-field points against the usual diffraction pattern of bright and dark fringes. The solutions satisfy Maxwell's equations and the reciprocity principle.

The author is grateful to A V Nesterov for fruitful discussions.

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