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Some remarks on the radiation of charges and multipoles uniformly moving in a medium

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The classical theory of the Vavilov – Cherenkov (VC) effect discovered in 1934 was formulated by I E Tamm and I M Frank in 1937 [1]. For the energy radiated per unit time, they derived the formula

$$\frac{\mathrm{d}W}{\mathrm{d}t} = \frac{q^2 v}{c^2} \int\limits_{vn(\omega)/c \geqslant 1} \left[1 - \frac{c^2}{v^2 n^2(\omega)}\right] \omega \, \mathrm{d}\omega \,, \tag{1}$$

where q is the charge of a particle moving with a constant velocity $\mathbf{v}$ in a homogeneous, isotropic, nonmagnetic transparent medium with a refractive index $n(\omega)$ at a cyclic frequency ω; the integration in (1) is carried out over frequencies for which $vn(\omega)/c \geqslant 1$, that is, VC radiation exists that proceeds at an angle θ_{VC}, $\cos\theta_{\mathrm{VC}} = c/vn(\omega)$.

In paper [1] the radiated power is estimated using the expression

$$n^2(\omega) = 1 + \frac{A}{\omega_0^2 - \omega^2} \,. \tag{2}$$

Obviously, the frequency dispersion, i.e., the dependence of n on ω is involved in (1). This is certainly done correctly, and formula (1) can also be obtained by two other methods [2, 3]. In this connection, the statement, made recently in paper [4], that the theory of the VC effect in a dispersive medium 'was first derived' in papers by Tamm [5] and Fermi [6] is erroneous. In paper [1], the refractive index was supposedly believed to be frequency independent[1]. Clearly, the authors of Ref. [4] have never seen paper [1]. Such an error was obviously due to the paper by G Afanas'ev et al. [7], which the authors of Ref. [4] referred to. Indeed, paper [7] begins with the assertion that Tamm and Frank [1] "considered charge motion in a medium with constant electric permittivity", that is, without dispersion. This is not a slip of the pen, because in the paper by Afanas'ev [8] it is stated that "Tamm and Frank did not, however, formulate the dispersion mathematically". This did not prevent the authors of Ref. [7] from devoting all their paper to the analysis of precisely formula (1) making use of expression (2), presented in [1] (see formulas (4.2) and (1.1) in Ref. [7]). This analysis is more extensive than any other known to me from the literature, and it is useful, but why it should have been based on the above-mentioned erroneous characteristic given in Ref. [1] is beyond my understanding. The note [8], placed in a popular scientific journal, may on the whole only mislead the reader unacquainted with the theory of the VC effect and arouse surprise in those familiar with the theory. For instance, the author of Ref. [8] states that in paper [4] it is supposedly shown that VC radiation "can also come from electric dipoles moving slower than the velocity of light" and that a uniformly moving charge can generally "radiate irrespective of its velocity". Meanwhile, a common truth is the statement that in the case of the VC effect a uniformly moving charge can radiate only if its velocity exceeds the phase velocity of the considered waves in the medium. Only having read paper [7], to which there are no direct references in Ref. [8] (only paper [4] is referred to), could I understand what was meant.

In this connection, I shall make several comments. In the narrow sense of the word, VC radiation is understood as optical radiation occurring in a medium traversed by charged particles moving at a velocity $v > c/n(\omega)$, where ω is the frequency of the considered radiation. But, as has always been emphasized (see, e.g., Refs [2, 3, 5, 9, 10]), in the broad sense the VC effect is the radiation of any waves occurring when any source moves uniformly with a velocity exceeding the phase velocity of the considered waves. In this sense, for instance, Mach waves represent the acoustic VC effect or, alternatively, an analogue of the optical VC effect in acoustics. Incidentally, this has already been stressed in paper [5]. Therefore, even at the initial stage of the development of the theory it was clear that the VC effect would take place if the source were not a charge, but a magnetic dipole [11], an electric dipole [12], or certainly any multipole. A source may also be a light (electromagnetic) pulse, and in this case the role of the charge velocity $\mathbf{v}$ is played by the group velocity $\mathbf{v}_{\mathrm{g}}$ with which the pulse moves. But, of course, by virtue of the superposition principle, the intensity of VC radiation from a light pulse is nonzero only if the nonlinearity of the medium is taken into account. The radiation of such pulses has rather long been observed (see, e.g., Ref. [13]). A new thing for me in paper [4] was the understanding of the possibility for multipoles and, in particular, electric dipoles, to be sources in the case of light pulses. Meanwhile, for all known particles (neutrons among them), the magnetic moment and, the more so, other multipole moments are so small that the study of their VC radiation is not considered to be really interesting (see also below).

[1] In paper [4], the angle θ_{VC} is erroneously assumed to be the angle not between $\mathbf{v}$ and $\mathbf{k}$ (the wave vector), but between $\mathbf{v}$ and the wave front.

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Multipoles and, in particular, electric dipoles of the pulse generate VC radiation with a frequency ω only if $v_g > c/n(\omega)$. However, when the dispersion is taken into account, this is in principle also possible provided that $v_g < c_{ph}(0) = c_0 = c/n(0)$. Here we borrowed the notation of Ref. [4] where, obviously, $c_0 = c/n(0)$ is the phase velocity of electromagnetic waves in the limit of arbitrarily low frequencies. The conditions under which the particle velocity v or the group velocity v_g is below c_0 is referred to by the authors as subluminal and the radiation for $v_g > c_0$ is called superluminal. This terminology seems to me to be extremely inappropriate and even misleading. But terminology is, of course, a matter of agreement. An important thing is that the statement of the existence of VC radiation of a dipole "when it moves at a velocity lower than the velocity of light" means that the dipole also radiates at $v_g < c_0$, but of course radiates only waves whose spectrum possesses frequencies ω satisfying the condition $v_g > c/n(\omega)$.

As concerns the assertion that VC radiation may occur at any particle velocity v [8], this merely rests on the use of expression (2), according to which the refractive index $n(\omega)$ near resonance may be arbitrarily large, and hence the velocity $c_{ph} = c/n(\omega)$ may be arbitrarily small. This is absolutely obvious, and it is not serious to note that Fermi, one of the most prominent physicists of the last century, may have misunderstood it: "although Fermi failed to reach this conclusion himself, it follows inevitably from his work". The point, certainly, is that sufficiently close to the resonance the absorption should be taken into account, and an expression of type (2) is inapplicable. The consequences that follow have always been clear to everybody (and, in any case, to Fermi).

Over 60 years have passed since the VC effect was discovered and explained, and it is perhaps for this reason that questions understood long ago have now become subjects of discussion again. In addition to what has been said, this also concerns the paper by A A Tyapkin [14], i.e., the microtheory of VC effect.

From the point of view of microtheory, the mechanism of VC radiation was understood at the very early stage of the development of the theory. So, in Tamm's paper [5] (p. 79 of the Russian edition) it was said: "From the point of view of the microscopic theory, the considered radiation is not emitted directly by an electron, but is due to electron-induced coherent oscillations of the medium". Citing this phrase in [14], A Tyapkin notices: "It seems that after such a concrete and deep direction of their chief, researchers of the Theoretical Department of the Lebedev Physics Institute should have set to calculating the vibration excitation mechanism in a molecule or at least should have adopted the categorical assertion of the chief that the "considered radiation is not directly emitted by an electron". But none of Tamm's followers even paid attention to such an important indication". But as a matter of fact, at the Mandelstam – Tamm school, to which I have the honor to belong, we knew very well, unlike A A Tyapkin, what the refractive index $n(\omega)$ is from the point of view of the microscopic theory and when one should or should not apply microelectrodynamics and when it suffices to use macroscopic electrodynamics (or, according to another terminology, electrodynamics of continua [15]).

The introduction of the refractive index $n(\omega)$ has the sense of allowance for interference of the secondary waves scattered by all the atoms ('oscillators') of the medium and a primary wave incident on the medium. To be concrete, I mean here a microscopic calculation of wave refraction on a plane interface between, say, a vacuum and a medium with refractive index $n_2(\omega)$. The refraction law $\sin\theta_1 = n_2 \sin\theta_2$ is obtained (see [16, Sec. 74], where the original literature is cited; see also [17, Sec. 69]) as a result of 'damping' in the medium of a primary (incident) wave and its replacement by the refracted wave with phase velocity $c_{ph} = c/n_2(\omega)$; the reflected wave also appears. The calculation is rather cumbersome and certainly quite unnecessary after the question is clarified because the introduction of the refractive index $n(\omega)$ and of the corresponding macroscopic equations automatically involves everything. However, the understanding of the meaning and possibility of microscopic calculation is not superfluous, for it may turn out to be useful when some complications and specifications appear (for example, in an analysis of deviations from the Fresnel formulas; see [17, Sec. 70]). In the case of the VC effect, the calculation [1] is quite adequate to the problem and completely allows for the role of the medium in the approximation employed. Since for a charge no problems have ever been known, to me in particular, for the solution of which there is reason to doubt the results of the calculations [1], no microtheory of VC radiation has been developed to the best of my knowledge. Incidentally, the fact that the macrotheory of the VC effect for charges does not require specifications is not quite trivial. The point is that as was shown in Ref. [18], for charge motion not in a continuous medium, but in a fairly thin empty channel in this medium, the existence of the channel does not affect the radiation. From this it follows that for the VC effect for a charge, the immediate vicinity of the trajectory does not play any role, and therefore there is no reason to overstep the limits of the macrotheory. For multipoles, beginning with magnetic and electric dipoles, this is no longer the case because the radiation during the motion in channels is generally different from that in a continuous medium. Hence, to analyze VC radiation for multipoles in the general case, one has to make a special consideration of a region close to the source and thus to go beyond the scope of the macroscopic approach. The questions mentioned here are discussed in Ref. [3, Ch. 7]. The study of VC radiation of multipoles has not been completed because of the above-mentioned smallness of the VC effect for known particles with multipole moments. The use of light pulses [4, 13] offers the opportunity to examine VC radiation of multipoles[2], but obviously not microscopic ones, and the problems there are absolutely different (see, in particular, [3, Ch. 7]).

In paper [19], which is published in the present issue of *Physics-Uspekhi*, and in some other papers the question is discussed of whether VC radiation is the 'self-radiation' of a fast particle or whether it is the radiation of the medium, excited by this particle. The authors of Ref. [19] believe that this VC radiation is self-radiation of the particle. For the VC radiation of a particle to occur, the particle itself (the source of radiation) and the medium are both needed, of course. Therefore, the question of which one is more important is somewhat scholastic. However, I believe it physically more grounded, although not obligatory, to think of VC radiation as radiation of the medium. This is especially reasonable considering that the VC effect also occurs without any particle as a source but, for example, with a light pulse as a

[2] In Ref. [4] electric dipoles are discussed, but the use of an elliptically polarized field in a pulse probably allows the study of magnetic dipoles and multipoles as well.

source (see above). The same opinion is shared in Ref. [15, Sec. 115]).

In his paper [14] A Tyapkin also dwells on the quantum theory of VC radiation developed by me [11] (see also Refs [2, 3]). He declares that I "naturally obtained certainly erroneous quantum corrections, the absurdity of which has not yet been noticed by anyone up to now. The most surprising thing is that in his subsequent works Tamm, too, went on referring to this paper [23] (it is paper [11] in the list of references in the present note — *V L G*) and failed to notice that it had been constructed in definite contradiction with his correct statement about the secondary nature of the occurrence of photons of Cherenkov radiation". A Tyapkin did not find it necessary to show why the corrections obtained in [11] were 'certainly erroneous'. Truly, he did not demonstrate anything except that he failed to understand that in both paper [11] and paper [20] cited by him macroscopic and not microscopic electrodynamics was quantized in the medium. That is why, the 'photon momentum in a medium' is from the very beginning equal to

$$\hbar\mathbf{k} = \frac{\hbar\omega n(\omega)}{c}\,\frac{\mathbf{k}}{k}\,, \tag{3}$$

rather than $\hbar\omega/c$ as in a vacuum. It is of importance that it is precisely the momentum (3) that is obtained automatically [11, 20] in quantization of equations of macroscopic electrodynamics. At the same time, momentum (3) consists of two parts, of which one is connected with the field and the other with the medium [21]. I think that the consideration undertaken in Ref. [11] is correct, but its accuracy is *a priori* insufficiently clear. For this reason, it may be reasonable to give a certain microscopic substantiation of the employed approximation on the basis of microscopic quantum electrodynamics (I already mentioned this in a footnote to the proofreading of preprint [22]). The quantum corrections of the order of $\hbar\omega/mc^2$, etc. indicated in [11] and partially presented also in Refs [10, 23] are the best ones suggested up to now. However, these corrections are of no interest for they are very small in all known real conditions. The criticism expressed in [14] in this respect is absolutely groundless. By the way, it has already been said that the calculations [20] for the intensity of VC radiation were made on the same basis as in Ref. [11] and are, moreover, clearly incorrect. Indeed, the author of Ref. [20] presents only the expression [formula (3.14)] for the intensity in the classical limit (i.e., without any quantum corrections), but differing from (1) by the additional factor u_λ/U_λ (u_λ and U_λ are the phase and the group velocities of light). Since the result (1) is positively correct, it follows that in Ref. [20] there is an undoubted error at this point.

I note that in paper [24], which was kindly sent to me by G N Afanas'ev along with papers [7] and [25], there is a reference to the possibility, pointed out by A Tyapkin in 1993, of some additional radiation occurring for the particle velocity $v = c_{\rm ph} = c/\sqrt{\varepsilon\mu}$ (in the notation used in paper [24] $c_{\rm ph} = c_{\rm n}$; dispersion is neglected). So, we are speaking of the threshold of the VC effect. I, however, failed to find any threshold characteristic of the VC effect. If one proceeds from formula (1), there is no threshold effect, of course, and the radiation power tends to zero as $v \to c_{\rm ph}$. Paper [25] presents a more complete solution of the problem (which was first considered by Tamm [5]) of radiation of a charge moving uniformly only over a finite interval of time and, therefore, of path. As far as I can judge, the analysis performed there is correct and useful.

If a medium is homogeneous and stationary [i.e., its electromagnetic characteristics, e.g. $n(\omega)$, do not change with time], it is only VC radiation that may appear (of course, for $v > c_{\rm ph}$ only) for uniform source (e.g., charge) motion. If the medium is nonuniform and (or) nonstationary, then transition radiation generally occurs for uniform source motion (see [26] and also Refs [2, 3, 9, 10, 15, 23, 27–29]). Transition radiation may of course coexist and interfere with VC radiation. A somewhat formal, but the most general explanation of transition radiation is as follows. Restricting our example to charge motion, we note that in the case of a vacuum the parameter so-to-say determining the radiation is the ratio $\varkappa = v(t)/c$ of the charge velocity $v(t)$ to the velocity of light c in a vacuum. From electrodynamics it follows that for a constant velocity, that is, for $v = \text{const}$ over the entire interval $-\infty < t < \infty$ the radiation is absent. For its occurrence it is necessary that $\varkappa$ should change, i.e., that acceleration should exist in the present or in the past. In the presence of a medium, the determining parameter for radiation is the ratio $\varkappa = v/c_{\rm ph}$, where $c_{\rm ph}$ is the phase velocity of electromagnetic waves (generally speaking, instead of $c_{\rm ph}$ another characteristic of the medium with the dimensionality of velocity can be chosen). Considering for simplicity a transparent medium, we have $c_{\rm ph} = c/n(\omega)$, and again for $\varkappa < 1$ radiation exists only if the parameter $\varkappa$ changes in time in the place where the charge (source) is located or in its neighborhood in which the field is formed. Obviously, such a change of $\varkappa$ is now possible not only in the presence of acceleration (i.e., for a time dependence of v), but also for a time dependence of the velocity $c_{\rm ph}$ in the place where the charge is located. When the charge crosses the interface, $\varkappa$ changes and transition radiation occurs which was considered earlier than other types of such radiation [26]. The parameter $\varkappa$ also changes when the charge moves at a constant velocity through a medium periodically changing in space [28]. Such radiation is sometimes called simply resonance radiation, but in my opinion the term resonance transition radiation is more relevant. Transition radiation occurring when a charge moves uniformly near inhomogeneities (screens, grating, etc.) is often referred to as diffraction radiation. And again, the term transition diffraction radiation seems to be more appropriate. Transition bremsstrahlung radiation (see, for instance, Refs [2, 27]) is named polarization bremsstrahlung in paper [19]. And in this case, too, the term 'transition bremsstrahlung' seems preferable to me. Of course, the choice of terminology is largely a matter of taste, and I shall not continue the discussion. I shall only note that it seems tempting to term 'transition radiation' any radiation of a source uniformly moving in a medium inhomogeneous in space and (or) in time. However, as was noticed by B M Bolotovskiĭ, such a definition would also embrace VC radiation in channels or near a medium. That is why it is obviously more accurate to refer to any radiation of a source uniformly moving in or near a medium, except VC radiation, as transition radiation.

In connection with terminology, I recall that the term 'transition radiation' arose in quite a natural way, because the first problem of this type dealt with precisely the radiation occurring when a charge passed over from one medium to another [26]. But what would the radiation be if a charge crossed not one, but several interfaces, as is the case with a pile of plates, a medium with chaotic inhomogeneities, etc.? Obviously, the radiation preserves some features typical of radiation generated when one interface is crossed, but the

waves emitted by different interfaces interfere. In the limit of an infinite inhomogeneous (but homogeneous 'on average') medium, and physically speaking, for a rather large such medium, the intensity of occurring radiation is proportional to the path covered by the charge. In such cases there is no transition process in the proper sense of the word. Therefore, in his review of the present note B M Bolotovskiĭ expressed the opinion that it would be more correct to speak simply of resonance radiation rather than transition resonance radiation (we mean the radiation of a uniformly moving source in a periodically inhomogeneous medium [28]). I however prefer the term 'transition resonance radiation' because it seems to be more informative. Similarly, in the case of a charge radiating during a uniform motion above a diffraction grating, I regard the term 'transition diffraction radiation' to be preferable to simply 'diffraction radiation'. In both cases the adjective 'transition' immediately clarifies that we are dealing with a uniformly moving source (charge) because it resembles the initial, so-to-say, transition effect [26]. What should be understood by resonance or diffraction radiation is unclear without corresponding specifications. But such arguments are surely not convincing, and in any case the choice of terminology is a matter of taste provided that the essence of the issue or the character of the problem is well explained. Incidentally, the same refers to the Vavilov–Cherenkov effect because as mentioned at the beginning of the present note this term applies to both the characteristic optical radiation in a transparent isotropic medium and to a more general case of radiation of various waves upon uniform motion of their source at a velocity exceeding the phase velocity of these waves in a given medium. Such an extended interpretation, for instance, in the case of radiation of longitudinal (plasma) waves in an isotropic plasma may sometimes lead to misunderstanding. And again, with a more precise formulation of the problem and terminology everything becomes clear.

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