

# Unusual acoustic birefringence phenomena in antiferromagnets

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**Abstract.** Experimenters and, to some extent solid-state electronics specialists, will find in this paper a comprehensive review and much original material in a relatively new area of physical magnetoacoustics — acoustic birefringence (circular and linear) in antiferromagnets — where effects quite extraordinary from the viewpoint of nonmagnetic (and even ferromagnetic) crystals are studied. The availability of the theory of these effects, the possibility of controlling them with magnetic (**B**) and electrical (**E**) fields, and the understanding of the role of such factors as the crystal and magnetic structure, orientation state, sample size, the direction and magnitude of **B** and **E**, etc., all this paves the way and indeed calls for intensive experimental work and holds promise of new and exciting discoveries in solid-state magnetoacoustics.

## 1. Introduction

In this article, we study a fairly new region of magnetoacoustics (antiferromagnetoacoustics, to be exact). Its main goal is to attract the attention of experimenters in physics to the possibility of discovering new effects in acoustics that have been predicted theoretically for antiferromagnetic (AF)

crystals of different types. We not only review the numerous data from the literature in this area of research but also discuss the prediction of several new effects that are of interest not only to physicists but also to specialists in solid-state electronics. In discussing the problems, we define or refine several concepts and terms characteristic of the acoustics of antiferromagnets (and, generally, of optics).

The very title of the review needs explaining, since we use the terms ‘acoustic birefringence,’ although it is known that three normal acoustic modes (for a given wave vector **k**) exist in crystalline bodies in the general case [1]. However, from the viewpoint of symmetry and the orientation of the antiferromagnetism vector **L**, in the simplest case one can always select a situation (the directions of the vectors **k** and **L** and the magnetic and electric field **B** and **E**) in which the elastic wave proves to be a superposition of only two modes. Thus the third mode becomes separated and can be ignored. The first two modes have different phase velocities corresponding to different polarizations of the elastic displacement **u**, so that their mixing results in effects that can be called, as in optics, birefringence (BR) effects. Hence the title of the review.

Limiting our study to particular cases not only simplifies the theory (as well as setting up an experiment). It also makes it possible to follow the direct analogy to the corresponding effects in optics and to establish the existence of effects that make, in this sense, acoustics different from optics [2–6].

Two macroscopic approaches can be used in describing the acoustic properties of AF crystals [4–7]. The first is a purely symmetry approach, which begins with the elastic constant tensor  $\hat{C} \equiv C_{ijkn}$ , written on the basis of the requirement of invariance with respect to the crystallochemical symmetry group  $G_F 1'$  ( $G_F$  is the space group, and  $1'$  is time inversion  $t \rightarrow -t$ ) with allowance for the contribution to  $\hat{C}$  provided by the magnetic variables, the vectors of

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antiferromagnetism  $\mathbf{L}$ , magnetization  $\mathbf{M}$ , and magnetic field  $\mathbf{B}$ . For centrally antisymmetric (CAS) antiferromagnets, one must also allow for the contributions related to the electric field  $\mathbf{E}$  and the terms linear in wave vectors  $\mathbf{k}$  and caused by spatial dispersion of AF origin.

Symmetry analysis enables the establishment (prediction) purely qualitatively of the very existence of the characteristic AF effects (which are of interest to us) in acoustics and, in particular, acoustic BR caused by antiferromagnetism. But within this approach we can say nothing about the size of these effects. We use it primarily in the symmetry classification of the effects (Section 2).

In the second approach, we examine magnetoelastic waves using the coupled equations of magnetoelastic dynamics: the dynamical equations of elasticity theory and the equations of motion of the spin (antiferromagnetic) subsystem. This approach is more complicated and cumbersome in the mathematical sense. However, it has the advantage of providing the possibility of actually building a quantitative theory of acoustic BR, since the equations contain constants (elastic, magnetic, and magnetoelastic) that can be found from other, independent, experiments. On the basis of the coupled equations of magnetoelastic dynamics we examine the effects in centrosymmetric antiferromagnets (Section 3). On the other hand, in describing acoustic BR in antiferromagnets with an antisymmetry center (Section 4), we use purely a symmetry approach because of insufficient experimental data on CAS antiferromagnets.

It would seem that, when dealing with the acoustics of crystals, we should begin with the general theoretical fundamentals of physical acoustics and introduce such concepts as the Christoffel equations and tensors, characteristic surfaces, etc. There are, however, many excellent monographs and even textbooks where all these concepts are discussed very thoroughly (e.g., see Refs [8–10]). We would like to mention the book by Sirotnin and Shaskol'skaya [8], where the topics are treated in full and yet simply. More than that, the area of new (or little-known) acoustic effects related to antiferromagnetism in some cases lies outside the scope of the general theoretical fundamentals. Hence, we chose a more direct and fast (from our viewpoint) way that leads to the result, a way that follows from the equations of dynamics and avoids the generally known concepts and mathematical tools. Actually, we simply give a brief description of the acoustic BR phenomena caused by antiferromagnetism, depending on the crystal system, the exchange magnetic structure (EMS)<sup>1</sup> and the orientation state (the direction of vector  $\mathbf{L}$ ) [7, 11]. Here, for the sake of convenience, we determine only some concepts that are often not defined precisely in acoustics (and in optics, for that matter). This is true of acoustic activity, gyrotropy, nonreciprocal phenomena, etc. Using these concepts, we classify the effects (see Table 1 below) and assign abbreviations.

In the concluding section of this review, we list some effects that are most interesting, we believe, from the viewpoint of setting up experiments, specifying where possible the substance and the conditions in which the effects are realized, and sometimes even the expected values.

<sup>1</sup> By exchange magnetic structure we, as is commonly done, mean the magnetic order (mutual orientation) that sets in only because of the exchange interaction of spins without allowance for magnetocrystalline anisotropy, which determines the orientation of these spins in relation to the crystallographic axes.

## 2. Symmetry classification of effects and the main definitions

### 2.1 Antiferromagnetic (AF) contributions to the elastic constant tensor

The symmetry approach is based on the invariant decomposition of the magnetic contribution  $\Delta\hat{C}$  to the elastic constant tensor  $\tilde{C}_{ijkn}$  along the vectors  $\mathbf{L}$ ,  $\mathbf{M}$ , and  $\mathbf{B}$ , and also, in the case of CAS antiferromagnets, along the vectors  $\mathbf{E}$  and  $\mathbf{k}$  [5, 7]. Here, only the spatial dispersion of  $\tilde{C}_{ijkn}$  that is linear in  $\mathbf{k}$  is taken into account. We separate this AF contribution  $\Delta\hat{C}$  from the nonmagnetic contribution  $\hat{C}$ , so that

$$\tilde{C}_{\alpha\beta} = C_{\alpha\beta} + \Delta C_{\alpha\beta}(\mathbf{L}, \mathbf{M}, \mathbf{B}, \mathbf{k}, \mathbf{E}). \quad (1)$$

Here we have introduced the standard notation for the pairs of indices  $ij \equiv \alpha$  and  $kn \equiv \beta$ :

$$\begin{aligned} xx \equiv 1, \quad yy \equiv 2, \quad zz \equiv 3, \quad yz = zy \equiv 4, \\ zx = xz \equiv 5, \quad xy = yx \equiv 6. \end{aligned} \quad (2)$$

We will employ, following Ref. [7], the Onsager relations

$$\Delta C_{\alpha\beta}(\mathbf{L}, \mathbf{M}, \mathbf{B}, \mathbf{k}, \mathbf{E}) = \Delta C_{\beta\alpha}(-\mathbf{L}, -\mathbf{M}, -\mathbf{B}, -\mathbf{k}, \mathbf{E}). \quad (3)$$

The ‘minus’ in the arguments on the right-hand side indicates that the corresponding vectors change sign under the operation of time inversion  $I'$ . We denote such vectors by the letter  $\mathbf{I}$ . We see that only the vector  $\mathbf{E}$  does not obey this rule. In addition to magnetic vectors, the wave vector  $\mathbf{k}$  also belongs to the  $\mathbf{I}$  type; in this sense, this vector  $\mathbf{k}$  acts as momentum.<sup>2</sup> If we take equation (3) into account, the even powers of  $\mathbf{I}$  form a symmetric part ( $\Delta\hat{C}^s$ ) under the permutation of  $\alpha$  and  $\beta$ , while the odd powers of  $\mathbf{I}$  form an antisymmetric part ( $\Delta\hat{C}^a$ ) under such a permutation:  $\Delta C_{\alpha\beta}^s = \Delta C_{\beta\alpha}^s$ ,  $\Delta C_{\alpha\beta}^a = -\Delta C_{\beta\alpha}^a$  ( $\Delta C_{\alpha\beta} = \Delta C_{\alpha\beta}^s + \Delta C_{\alpha\beta}^a$ ). The sought decomposition can schematically be represented as follows:

$$\Delta\hat{C}^s = (LL) + (LB) + (BB) + (Lk) + \dots, \quad (4)$$

$$\Delta\hat{C}^a = (k) + (L) + (B) + (LBB) + (LE) + \dots \quad (5)$$

To the right-hand sides of these equations, we must add terms with  $\mathbf{M}$  similar to those with  $\mathbf{B}$ , since the transformation properties of the vectors  $\mathbf{M}$  and  $\mathbf{B}$  are identical. Thus, in the final formulas we must incorporate terms with  $\mathbf{M}$  replacing  $\mathbf{B}$ . Of the schematically written terms in (4) and (5), we ‘decode’ the following two terms:

$$(LB) \equiv f_{ijknpq} L_p B_q \quad (\text{from } \Delta C_{\alpha\beta}^s \equiv \Delta C_{ijkn}^s), \quad (6)$$

$$(LBB) \equiv i\beta_{ijknpqs} L_p B_q B_s \quad (\text{from } \Delta C_{\alpha\beta}^a \equiv \Delta C_{ijkn}^a). \quad (7)$$

<sup>2</sup> The Onsager relations (theorem or symmetry principle) were first derived for transport coefficients and then were generalized to the case of what is known as generalized susceptibilities (linear response functions), to which the elastic constants  $\hat{C}$  also belong. The first to carry out such generalization for magnetically polarized media was Vlasov [12]. (The principles of such generalization are discussed, e.g., in Landau and Lifshitz's book [13].) The Onsager relations follow from the invariance of the equations of motion of microparticles under time inversion  $t \rightarrow -t$  combined with a change in sign of the macroscopic parameters of the  $\mathbf{I}$  type determining the state of the system ( $\mathbf{M}$ ,  $\mathbf{B}$ ,  $\mathbf{L}$ , etc.) or processes ( $\mathbf{k}$ ). A more general notation and a thorough discussion of these relations can be found in the new book [14] written with the help of the authors of the present review.

(As usual, here and in what follows we adopt the common rule of summation over repeated indices.) In the above formulas,  $f_{ijknpq}$  and  $\beta_{ijknpqs}$  are real coefficients (generally speaking, dependent on  $\mathbf{L}^2$  and the frequency  $\omega$ ). Their explicit form can be found from the requirement that these relations (with a right-hand side presented in the parentheses) be invariant with respect to elements of spatial crystallochemical symmetry. It is sufficient to use the elements  $g(\pm)$  entering the code ( $\equiv$  the minimum set of generators of the crystallochemical symmetry group) of the corresponding EMS with allowance for their parity ('+' stands for even elements and '-', for odd elements:  $g(\pm)\mathbf{L} = \pm g\mathbf{L}$ , etc.) [11, 15]. For collinear and weakly (relativistically) noncollinear AF structures, which are the only structures discussed in this review, the parity of the element  $g$  precisely reflects its spatial nature: the type of permutation of atoms caused by it.

The use of *crystallochemical* symmetry (with explicitly specifying the 'agents'  $\mathbf{L}$ ,  $\mathbf{B}$ , etc. that break this symmetry) instead of magnetic symmetry, i.e., the true symmetry of a magnetic substance, is preferable in the sense that it makes it possible to analytically study the behavior of the phenomena of interest to us as depending not only on the EMS (its code) but also on the orientation state (the direction of  $\mathbf{B}$ , etc.).

The present review deals solely with centrosymmetric (CS), in the crystallochemical sense, antiferromagnets, which with allowance for AF order either remain centrally symmetric or become centrally antisymmetric. For the first, the symmetry center (spatial inversion) in the EMS code is an even symmetry element  $\bar{1}(+)$ , while for the second, it is an odd symmetry element  $\bar{1}(-)$ . For the CS case,  $\Delta\hat{C}^s$  does not contain  $(Lk)$  terms, while for the CAS case, there are no  $(LB)$  terms. These products transform in the same way with respect both to  $\bar{1}$  and to other crystallochemical symmetry elements, with the result that their tensor factors have the same form.

As for  $\Delta\hat{C}^a$ , the term  $(k)$  is absent from it both for centrosymmetric EMS and for centroantisymmetric EMS. This term appears only in crystals whose symmetry contains no spatial inversion  $\bar{1}$  and is listed in (5) (and in Table 1) only because it will be needed in our further discussion. Of the remaining terms in  $\Delta\hat{C}^a$ , for EMS with a symmetry center all terms in (5) except  $(LE)$  may exist, while for the centrally antisymmetric case only the terms  $(B)$  and  $(LE)$  remain. All terms that are linear (and, generally, odd) in  $\mathbf{L}$  except  $(LE)$  for such EMS disappear from  $\Delta\hat{C}^a$  (5) in view of the fact that, due to the condition  $\bar{1}(-)\mathbf{L} = -\mathbf{L}$ , they change sign, while the tensor  $\Delta C_{\alpha\beta}^a$  on the left-hand side remains invariant with respect to  $\bar{1}(-)$  (and to  $\bar{1}(+)$ ). The invariance of  $(LE)$  follows from the fact that since  $\mathbf{E}$  is a polar vector, it changes sign (just as  $\mathbf{L}$  does):  $\bar{1}(\pm)\mathbf{E} = -\mathbf{E}$ .

Below, we limit ourselves to acoustically transparent media, for which the tensor  $\Delta C_{\alpha\beta}$  is Hermitian [7]:

$$\Delta C_{\alpha\beta}^* = \Delta C_{\beta\alpha}. \quad (8)$$

Since these functions are Fourier transforms (components of the plane-wave expansion of  $\exp(i\mathbf{k} \cdot \mathbf{r} - i\omega t)$ ) of the real linear response function, they possess the following property [16]:

$$\Delta C_{\alpha\beta}^*(\omega, \mathbf{k}, \mathbf{L}, \dots) = \Delta C_{\alpha\beta}(-\omega, -\mathbf{k}, \mathbf{L}, \dots). \quad (9)$$

Finally, the Onsager relation (3) plays an important role in the analysis of  $\Delta C_{\alpha\beta}$ .

**Table 1.** Classification of effects in the crystal acoustics of acoustically transparent antiferromagnets:  $\Delta C_{\alpha\beta} = \Delta C_{\alpha\beta}^s + \Delta C_{\alpha\beta}^a = \Delta C_{\alpha\beta}' + i\Delta C_{\alpha\beta}''$ .

Symmetric (real) tensor $\Delta C_{\alpha\beta}^s \equiv \Delta C_{\alpha\beta}'$ (no gyrotropy)	Antisymmetric (imaginary) tensor $\Delta C_{\alpha\beta}^a \equiv i\Delta C_{\alpha\beta}''$ (gyrotropy)
Linear birefringence (LBR)	Circular birefringence (CBR)
Reciprocal LBR effects:	Reciprocal CBR effects:
CC ( $\mathbf{B} = \mathbf{M} = \mathbf{L} = 0$ ), elastic anisotropy;	Natural CBR of $(k)$ type — no CC CS
AF of $(LL)$ type {CS, CAS}; AF-M of $(LB)$ type {CS}	
Nonreciprocal LBR effects: AF of $(Lk)$ type {CAS}	Nonreciprocal CBR effects: M of types $(B)$ and $(M)$ {CS, CAS}; AF of $(L)$ type {CS}; AF-M <sup>2</sup> of $(LBB)$ type {CS}; AF-E of $(LE)$ type {CAS}

Notation: CC, crystallochemical; AF, antiferromagnetic; M, magnetic; AF-M, antiferromagnetic-magnetic; AF-M<sup>2</sup>, antiferromagnetic-doubly magnetic; AF-E, antiferromagnetic-electric. The abbreviations in braces indicate to what EMS, centrally symmetric (CS) or centrally antisymmetric (CAS), the corresponding effect belongs.

Equations (3), (8), and (9) make it possible to draw certain conclusions concerning the properties of the symmetric [Eqn (4)] and antisymmetric [Eqn (5)] parts of the elastic constant tensor  $\Delta C_{\alpha\beta}$ .

First, the hermiticity of (8) implies that the real and imaginary parts of the tensor for an acoustically transparent medium

$$\Delta C_{\alpha\beta} = \Delta C_{\alpha\beta}' + i\Delta C_{\alpha\beta}'' \quad (10)$$

coincide, respectively, with its symmetric and antisymmetric parts (see Table 1).

**Asymptotic behavior as  $\omega \rightarrow 0$  for a centrosymmetric EMS.** What is important is the asymptotic behavior of  $\Delta C_{\alpha\beta}^s(\omega, \mathbf{k}, \dots)$  and  $\Delta C_{\alpha\beta}^a(\omega, \mathbf{k}, \dots)$  as  $\omega \rightarrow 0$  and  $k \rightarrow 0$ . At first, we focus on CS antiferromagnets, which have no spatial dispersion [terms of the  $(Lk)$  type are absent from Eqns (4) and (5)]. We will discuss the asymptotic behavior of the CAS case later in Section 4. For the CS case, we use the property (9) and immediately find that

$$\Delta C_{\alpha\beta}^s(\omega, \mathbf{L}, \dots) = \Delta C_{\alpha\beta}^s(-\omega, \mathbf{L}, \dots), \quad (11)$$

$$\Delta C_{\alpha\beta}^a(\omega, \mathbf{L}, \dots) = -\Delta C_{\alpha\beta}^a(-\omega, \mathbf{L}, \dots). \quad (12)$$

Hence, the  $\Delta C_{\alpha\beta}^s$  are even functions of  $\omega$  and contribute to the static values  $\Delta C_{\alpha\beta}^s(0, \mathbf{L}, \dots)$ . But if we turn to  $\Delta C_{\alpha\beta}^a$ , we see that they are odd functions of  $\omega$ , and

$$\Delta C_{\alpha\beta}^a(\omega, \mathbf{L}, \dots) \rightarrow 0 \quad \text{as } \omega \rightarrow 0. \quad (13)$$

This is a very important property of dynamic elastic constants, which will be used later.

Using symmetry considerations, we can find the form (finite components or equal components) of the elastic constant tensor  $\tilde{C}_{\alpha\beta}$  including the AF contributions (4) and (5), in terms of which we can write the stress tensor

$$t_{ij} = \tilde{C}_{ijkn} e_{kn}, \quad (14)$$

where

$$e_{kn} = \frac{1}{2} \left( \frac{\partial u_k}{\partial x_n} + \frac{\partial u_n}{\partial x_k} \right) \quad (15)$$

is the elastic strain tensor ( $x_1 \equiv x$ ,  $x_2 \equiv y$ ,  $x_3 \equiv z$ ). Note that a symmetry-invariant expression for the tensor  $t_{ij}$  (14) can be derived directly, without finding the  $\tilde{C}_{\alpha\beta}$ .

To find the normal acoustic waves, one is forced to solve the elastic-dynamics system of equations [1]

$$\rho \ddot{u}_i = \frac{\partial t_{ij}}{\partial x_j} \quad (16)$$

(the dots stand for time derivatives). The very procedure of solving the system of linear equations (16) combined with (14) will not be considered here. Instead, we will focus on describing and discussing the results.

## 2.2 Main concepts

### and classification of birefringence (BR) effects

Now we will define several concepts necessary for describing AF effects in acoustics and set up a table based on them. Almost all of these concepts can be introduced in optics: one must only replace the tensor  $\tilde{C}_{\alpha\beta}$  with the permittivity tensor  $\tilde{\varepsilon}_{ij} = \varepsilon_{ij} + \Delta\varepsilon_{ij}(\mathbf{L}, \mathbf{M}, \mathbf{B}, \mathbf{k})$ . It must be stressed, however, that in optics, too, the definition of these concepts is not always sufficiently rigorous and unique, especially in connection with antiferromagnetism.

We begin with the concept of *gyrotropy*, or a *gyrotropic medium* [16–18]. The most lucid (but formal) definition is the following. In a gyrotropic medium the tensor  $\tilde{C}_{\alpha\beta}$  (in acoustics) or  $\tilde{\varepsilon}_{ij}$  (in optics) proves to be nonsymmetric:  $\tilde{C}_{\alpha\beta} \neq \tilde{C}_{\beta\alpha}$  (or  $\tilde{\varepsilon}_{ij} \neq \tilde{\varepsilon}_{ji}$ ), so that gyrotropy is determined by the presence in these tensors of antisymmetric terms of type (5) that are imaginary for a transparent medium. A characteristic feature of gyrotropic media is the presence of *acoustic* (or optical) *activity* in them. Actually, gyrotropy and activity may be regarded as synonyms.

Let us now examine specific physical phenomena. Note that gyrotropy is the reason for circular birefringence (CBR) of waves. What we have just said does not depend on the nature of gyrotropy, i.e., CBR may be related to any term in  $\Delta\hat{C}^a$  (5). In textbooks devoted to nonantiferromagnetic crystals, the natural gyrotropy (activity) that occurs in crystals without a symmetry center is usually stressed. (The term ( $k$ ) in  $\Delta\hat{C}^a$  (5) represents such gyrotropy.) In antiferromagnets, the gyrotropic terms are usually augmented by terms of the ( $L$ ) and ( $LBB$ ) types in the CS case and of the ( $LE$ ) type in the CAS case. The first of these is also the source of ‘natural’ (spontaneous, to be exact) gyrotropy, but already related to antiferromagnetism, while the terms ( $LBB$ ) and ( $LE$ ) bring to life entirely new gyrotropic AF effects induced by  $\mathbf{B}$  or  $\mathbf{E}$ .

The symmetric part of the elastic constants,  $\Delta C_{\alpha\beta}^s$ , is responsible for linear BR (or LBR), i.e., the difference in phase velocities of the natural waves with a given wave vector  $\mathbf{k}$  and different linear polarizations. Here, to the usual LBR effects occurring in a nonantiferromagnetic medium (both in acoustics and in optics) and, as a rule, quadratic in  $\mathbf{B}$  (or  $\mathbf{M}$ ), AF terms incorporating the vector  $\mathbf{L}$  are added. These are the terms ( $LB$ ) for EMS with a symmetry center and ( $Lk$ ) for the CAS case.

Below, we will see how specific and unusual the effects generated by the above AF terms in  $\Delta\hat{C}$  are.

Now let us define another concept, *nonreciprocity*, or *nonreciprocal effects*, first introduced for antiferromagnets in optics by Brown et al. [19]. These effects change their magnitude or even sign when the direction of the wave vector  $\mathbf{k}$  changes to the opposite. Here, it is assumed that all the other vectors ( $\mathbf{L}$ ,  $\mathbf{B}$ ,  $\mathbf{M}$ , and  $\mathbf{E}$ ) remain the same.

Gyrotropic effects corresponding to all terms in  $\Delta\hat{C}^a$  (5), with the exception of ( $k$ ), prove to be nonreciprocal. Among the terms in  $\Delta\hat{C}^s$  (4), however, only ( $Lk$ ) produces a nonreciprocal effect. Note that in this case the effect is nonreciprocal but nongyrotropic.

The above facts and the corresponding classification of BR effects are listed in Table 1. (We note once more that a similar table can be set up for the AF part of the permittivity tensor,  $\Delta\hat{\varepsilon} = \Delta\hat{\varepsilon}^s + \Delta\hat{\varepsilon}^a$ .) To a great extent, the effects and the corresponding concepts are introduced here in a formal manner. Their physical meaning will be revealed gradually, as we examine the specific effects of acoustic BR corresponding to the separate term in expansions (4) and (5). Here, we will only note that the presence in  $\Delta C_{\alpha\beta}$  of symmetric [from (4)] and antisymmetric [from (5)] terms leads to superposition of linear and circular BR. (The term superposition does not mean that the LBR and CBR effects are additive. Possibly, it would be more correct to speak of a mixing of the effects.) Here the nonmagnetic part  $C_{\alpha\beta}$  in (1), i.e.,  $\tilde{C}_{\alpha\beta}$  at  $\mathbf{L} = \mathbf{B} = \mathbf{M} = 0$ , which is totally symmetric (in crystals with a center of crystallochemical symmetry, which is the type of crystal considered here), may also lead to linear BR. Since this nonmagnetic contribution of linear BR is of a crystallochemical nature related to the anisotropy of the elastic constants  $C_{\alpha\beta}$ , it is usually much larger than the magnetic contributions mentioned earlier. And if these magnetic contributions are only additions to the first (instead of being a new independent effect of an antiferromagnetic nature), the magnetic part of linear BR is usually ignored in comparison to the crystallochemical part.

## 2.3 Nonreciprocal effects

The concept of nonreciprocity is comparatively new and, to our knowledge, so far has not been discussed in monographs (and the more so in textbooks) on acoustics or optics. Hence, before we even begin to discuss the AF effects of interest, it would be wise to clarify it via well-known examples (it does not matter whether they are from optics or acoustics) by comparing CBR related to natural gyrotropy [the term of type ( $k$ ) in (5)] to that related to magnetic gyrotropy [the term of type ( $B$ ) in (5)]. The first effect is reciprocal in the sense that from the viewpoint of an observer *looking along vector*  $\mathbf{k}$  the sign of the angle  $\Theta$  of rotation of the polarization plane is not reversed when the direction of  $\mathbf{k}$  is changed to the opposite (a reciprocal effect). At the same time, from the viewpoint of the laboratory coordinate system, the reversal of the sign of  $\mathbf{k}$  changes the sign of the rotation angle  $\Theta$  to the opposite. What we have is that the resulting rotation angle for a wave that has traveled in the direct and reverse directions is zero:  $\Theta(\mathbf{k}) + \Theta(-\mathbf{k}) = 0$ .

The situation is different for magnetic BR (the Faraday effect). Here, the sign of  $\Theta$  is determined by the sign of the field  $\mathbf{B}$ , so that when  $\mathbf{k}$  is reversed without reversal of  $\mathbf{B}$ , the sign of  $\Theta$  in the of coordinate system linked to  $\mathbf{k}$  is reversed (nonreciprocal effect), while remaining unchanged in the laboratory coordinates system. Hence, with a wave traveling there and back, the Faraday angle doubles:  $\Theta(\mathbf{k}) + \Theta(-\mathbf{k}) = 2\Theta$  [18].

This constitutes the main difference between natural CBR (a reciprocal effect) and magnetic CBR (a nonreciprocal effect). Reasoning along similar lines, we see that the other magnetic contributions to CBR [of types (*L*), (*LBB*), and (*LE*) in (5)] of AF origin prove to be nonreciprocal.

Here, we should mention the term of type (*Lk*) in the symmetric part of  $\Delta\hat{C}$  [Eqn (4)], which is the only one generating a nonreciprocal LBR effect. We repeat once more: nonreciprocal but nongyrotropic (it belongs to the symmetric part of  $\Delta\hat{C}$ ). The nonreciprocity here consists in the following: the reversal of the sign of  $\mathbf{k}$  changes the contribution of this term to the refractive index (in optics) or to the elastic constants (in acoustics) and hence to the phase velocity of the waves. We stress this fact because Brown et al. [19], who were the first to point out the possibility of such effects occurring in antiferromagnets, for a reason unknown to us called them ‘gyrotropic or nonreciprocal.’ (Possibly the first, incorrect, term emerged because of the linear spatial dispersion of the tensor  $\Delta\epsilon_{ij}$ .) Following the authors of Ref. [19], this term (gyrotropic or nonreciprocal) has been automatically repeated in many papers devoted to optical properties of CAS antiferromagnets. One of the authors (E A T) of the present review was also unable to avoid this ‘sin’ in his book [7]. Not having the space to list the numerous papers that would corroborate what we have just said, we would like to mention (in addition to Ref. [7]) two reviews, one by Gehring [20] and the other by Pisarev [21], where the necessary citations are given.

We compiled Table 1 with the view of removing this misunderstanding and to clarify the use of other terms and concepts as applied to antiferromagnets.

First, we note that in the papers just mentioned (Refs [19–21]), which deal with optics, the study of the (*Lk*) effects (and also (*LE*) effects) was done on the basis of magnetic groups, due to which the vector  $\mathbf{L}$  did not appear in them explicitly. To our knowledge, a sufficiently clear and consistent exposition of how this is achieved does not exist in the literature.<sup>3</sup> Neither will we study this problem in this review; instead, we limit our approach to that based on crystallochemical symmetry. Not only is the latter sufficient for our purposes, but it is more informative than the magnetic approach (as mentioned earlier). Hence, we base our reasoning on the crystallochemical approach and on the acoustics of antiferromagnets (which is the topic of the present review). This is all the more necessary because, despite its advantages, it is insufficiently utilized, especially in the Western publications.

## 2.4 Spontaneous AF effect of CBR in crystals with different EMS

We begin our study of specific AF effects with antiferromagnets with a CS EMS. The first is the spontaneous AF effect of CBR linear in *L*. According to Table 1, this is a gyrotropic nonreciprocal effect. What the nonreciprocity amounts to was discussed in Section 2.3 in connection with the ordinary Faraday effect [term (*B*) in equation (5)]. Only here there is the vector  $\mathbf{L}$  instead of  $\mathbf{B}$ . We study consecutively orthorhombic, tetragonal, and rhombohedral (trigonal)

crystals and other uniaxial antiferromagnets with an even principal symmetry axis  $N_z \equiv N_z(+)$ .

The condition (common for all crystal systems) for the existence of a term of type (*L*) in  $\Delta C_{\alpha\beta}^a$  (5) states that the corresponding EMS and the orientation state must allow for weak Dzyaloshinski ferromagnetism, defined as follows [11, 15, 22]:

$$M_i = d_{ij}L_j, \quad (17)$$

where the coefficients  $d_{ij}$  are found from the requirement that (17) be invariant with respect to the symmetry elements that enter into the code of the corresponding EMS. The requirement that the EMS be centrally symmetric ( $\bar{1} \equiv \bar{1}(+)$ ) is the principal condition among all of these conditions.

**2.4.1 Orthorhombic antiferromagnet with an EMS  $\bar{1}(+)2_x(-)2_y(+)$ .** Orthoferrites and orthochromites with the space group  $D_{2h}^{16} \equiv Pbnm$  usually have such an EMS [15, 23]. An example well known among specialists in magnetic materials is the orthoferrite  $\text{YFeO}_3$  (with the Néel point  $T_N = 643 \text{ K}$ ). Here, we limit ourselves to a single orientation state that realizes itself in orthoferrites,

$$\mathbf{L} \parallel 2_x(-) \parallel X \quad \text{and} \quad \mathbf{M} \parallel \mathbf{B} \parallel 2_z(-) \parallel Z, \quad (18)$$

for two directions of the wave vector: (a)  $\mathbf{k} \parallel Z$  and (b)  $\mathbf{k} \parallel X$ .

(a)  $\mathbf{L} \parallel X$  and  $\mathbf{k} \parallel \mathbf{M} \parallel \mathbf{B} \parallel Z$ . A characteristic feature of orthorhombic crystals is that even in the absence of magnetism ( $\mathbf{L} = \mathbf{M} = \mathbf{B} = 0$ ) LBR of crystallochemical origin can arise in them. And we are forced to study this LBR first, since the CBR of antiferromagnetic origin we are interested in manifests itself against the background of this LBR. For a wave propagating along the *Z* axis, Eqn (16) gives two transverse acoustic modes with polarizations (directions of the elastic displacement vector  $\mathbf{u}$ ), wave numbers *k*, and phase velocities specified by the following formulas [5, 7]:

$$\begin{aligned} \mathbf{u}_1 \parallel X, \quad k_1 &= \frac{\omega}{v_1} = \omega \left( \frac{\rho}{C_{55}} \right)^{1/2}, \\ \mathbf{u}_2 \parallel Y, \quad k_2 &= \frac{\omega}{v_2} = \omega \left( \frac{\rho}{C_{44}} \right)^{1/2}. \end{aligned} \quad (19)$$

There is also, of course, a longitudinal wave

$$\mathbf{u}_3 \parallel \mathbf{k} \parallel Z, \quad k_3 = \frac{\omega}{v_3} = \omega \left( \frac{\rho}{C_{33}} \right)^{1/2}. \quad (20)$$

The LBR effect considered here amounts to the following. If we take any two of these three modes, we see that the waves in such a pair have different polarizations and, hence, different wave numbers and velocities. The effect manifests itself in experiments in the following. If we introduce into the sample a wave with a displacement  $\mathbf{u}$  that does not coincide with any of the above normal modes but can be represented as the sum of two of these modes, say,  $\mathbf{u} = \mathbf{u}_1 + \mathbf{u}_2$ , then at the exit the component waves will have different phases (due to the different velocities). Consequently, the polarization of the resulting wave will generally be elliptic (since  $\mathbf{u}_1 \perp \mathbf{u}_2$ ), which means that the tip of the vector  $\mathbf{u}$  describes an ellipse. The ellipticity (the axial ratio *a/b*) is determined, as we will see below, by the product  $\Delta kd$  (the phase difference), where  $\Delta k = k_1 - k_2$ , and *d* is the distance traveled by the wave, and

<sup>3</sup> Some material referring to this problem can be found in the above-cited monograph by Sirotnin and Shaskol'skaya [8] (Section 76). In particular, they state that the ordinary Onsager relations of type (3), which we use here, may prove to be invalid when applied to magnetically ordered crystals. Note, however, that such a statement is based on the approach that uses magnetic symmetry.

by the boundary conditions for the impinging wave. Under certain conditions the polarization may even be circular ( $a/b = 1$ ). This phenomenon has an analog in optics, where it is known as the Cotton–Mouton (or Voigt) effect (see Ref. [24], vol. 2).

The difference from optics is that in optics both waves in the pair are usually transverse waves (in any case, in a medium without spatial dispersion), while in acoustics one component can be transverse (e.g.,  $\mathbf{u}_1$ ) and the other, longitudinal ( $\mathbf{u}_3$ ). (Of course, in acoustics there may be a case where the impinging wave is a superposition of all three modes, but we have agreed not to examine such complicated cases.)

One must bear in mind that in an antiferromagnet terms of AF origin of the types ( $LL$ ) and ( $LB$ ) may also contribute to LBR; these terms are small compared to the differences of the diagonal components of the elastic constant tensor (e.g.,  $C_{44} - C_{55}$ ), which determine the crystallochemical LBR, and we agreed to ignore this AF contribution, allowing only for the terms ( $L$ ) in (5) that introduce a new quality, i.e., gyrotropy and the corresponding CBR.

For the situation considered here, these antisymmetric terms invariant with respect to the symmetry elements  $\bar{1}(+)$ ,  $2_x(-)$ , and  $2_y(+)$  that determine the EMS code have the form

$$\Delta C_{54}^a = -\Delta C_{45}^a \equiv i\Delta C = i(r_L L_x + r_M M_z + r_B B_z). \quad (21)$$

Here, for the sake of completeness, we also allowed for purely magnetic terms with  $M$  and  $B$ . According to (13), the constants  $r...$  must vanish as  $\omega \rightarrow 0$ . The total components of the stress tensor (14), which determine the waves with  $\mathbf{k} \parallel Z$  (here, one of the indices on  $t_{ij}$  and  $e_{ij}$  must be  $z$ ) are

$$t_{xz} = 2C_{55}e_{xz} + 2C_{54}^a e_{yz}, \quad t_{yz} = 2C_{44}e_{yz} + 2C_{45}^a e_{xz}. \quad (22)$$

Thus, if we allow for (22), the waves (19) mix and produce a different pair of normal modes. Here, the natural wave numbers  $k_1$  and  $k_2$  change (and so do the velocities  $v_1$  and  $v_2$ ), but these changes are small (quadratic in parameter  $r$ ) and we again ignore them. The main effect consists in the change in the polarization of the new normal modes. Instead of  $\mathbf{u}_1 \parallel X$  and  $\mathbf{u}_2 \parallel Y$ , we now have

$$\left( \frac{u_x}{u_y} \right)_2 = \left( \frac{u_y}{u_x} \right)_1 = i \frac{\Delta C}{C_{44} - C_{55}} \equiv iA. \quad (23)$$

This expression was arrived at by solving equations (16) with allowance for (22) and (21) and for the fact that  $A$  is small:

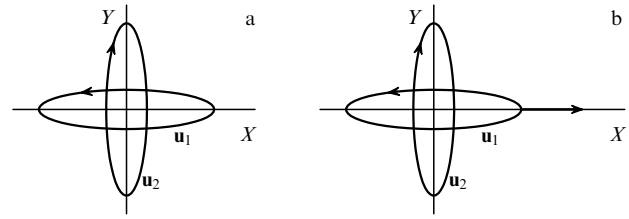
$$|A| \ll 1. \quad (24)$$

Here, both modes are transverse, but for one of them,  $u_x$  is ahead of  $u_y$  in phase by  $\pi/2$  while for the other,  $u_x$  lags behind  $u_y$  in phase by  $\pi/2$ . The linear polarization becomes elliptical, with the tip of the vector  $\mathbf{u}$  traversing these ellipses in opposite directions (Fig. 1a).

If we assume that the boundary condition is  $\mathbf{u}(0) \parallel X$ , then, using the polarization relations (23), we easily find that the polarization vector of the linearly polarized wave impinging on the sample undergoes a rotation (more precisely, the major axis of the ellipse undergoes that rotation) by an angle

$$\Theta(d) \approx A \sin(\Delta k d) \quad (25)$$

[approximately, with allowance for condition (24)], which means that we have CBR against the background of LBR



**Figure 1.** Normal elliptical acoustic modes corresponding to gyrotropic mixing: (a) two transverse modes with  $\mathbf{u}_1 \parallel X$  and  $\mathbf{u}_2 \parallel Y$  for  $\mathbf{k} \parallel Z$  (normal to the plane of the figure); and (b) a longitudinal mode with  $\mathbf{u}_1 \parallel X$  and a transverse mode with  $\mathbf{u}_2 \parallel Y$  for  $\mathbf{k} \parallel X$ .

( $\Delta k = k_1 - k_2$ ). The ellipticity in this case is

$$\frac{a}{b} \approx 2A \sin^2 \left( \frac{\Delta k d}{2} \right), \quad (26)$$

where  $A$  is taken from (23), and  $\Delta C$  is given by (21).

Note that in contrast to the case of purely circular acoustic activity (see below), where the angle  $\Theta$  increases linearly with  $d$ , here this angle is an oscillating function of the distance  $d$  the wave traveled through the sample. The same can be said of the ellipticity. The angle  $\Theta$  is at its maximum at  $\Delta k d = (2p + 1)(\pi/2)$  (with  $p$  an integer), while the ellipticity is at its maximum at  $\Delta k d = (2p + 1)\pi$ . In Eqns (25) and (26) the first factors ( $A$ ) are related to acoustic activity of AF origin, while the second factors are related to the LBR of crystallochemical origin.

(b)  $\mathbf{k} \parallel L \parallel X$  and  $\mathbf{M} \parallel B \parallel Z$ . In this case, the acoustic activity is related to antisymmetric elastic constants of the form

$$\Delta C_{16}^a = -\Delta C_{61}^a = i(r_L L_x + r_M M_z + r_B B_z) \equiv i\Delta C. \quad (27)$$

[Generally speaking, here the constants  $r$  differ from those in (21).] These constants mix the former transverse wave with

$$\mathbf{u}_2 \parallel Y, \quad k_2 = \frac{\omega}{v_2} = \omega \left( \frac{\rho}{C_{66}} \right)^{1/2} \quad (28)$$

and the former longitudinal wave with

$$\mathbf{u}_1 \parallel \mathbf{k} \parallel X, \quad k_1 = \frac{\omega}{v_1} = \omega \left( \frac{\rho}{C_{11}} \right)^{1/2}, \quad (29)$$

producing two elliptical modes whose ellipse planes contain the vector  $\mathbf{k} \parallel L \parallel X$  and are perpendicular to the field  $\mathbf{B} \parallel Z$  (Fig. 1b). The polarization relations again have the form (23), only now  $A$  must be replaced by

$$A = \frac{\Delta C}{C_{66} - C_{11}} \quad (30)$$

with  $\Delta C$  taken from (27). As in the previous case, the polarization vector of the linearly polarized wave impinging on the sample undergoes a rotation and the wave acquires ellipticity. However, now the plane of rotation and the ellipse lie in the same plane  $XY$  (the planar effect of CBR and ellipticity).

Suppose that at the entrance to the sample ( $x = 0$ ) the wave had a longitudinal linear polarization, so that its amplitude  $u_x(0) = u_0$ . Then the rotation angle  $\Theta$  and the ellipticity  $\Theta$  at the point of exit ( $x = d$ ) are again given by

formulas of the form (25) and (26), where one must use  $A$  given by (30) instead of (23) and replace  $\Delta k$  with  $\Delta k = k_1 - k_2$  from (28) and (29).

The above implies that after the wave has traveled a distance  $x = d$  in the sample, the elastic displacements acquire a transverse component,

$$u_y(d, t) = 2u_0 A \sin\left(\frac{\Delta k d}{2}\right) \cos\left(\frac{k_1 + k_2}{2} - \omega t\right). \quad (31)$$

A longitudinal wave impinging on the plate partially transforms into a transverse wave. It can also be demonstrated that an impinging wave that is transverse with  $\mathbf{u} \parallel Y$  acquires, due to AF acoustic activity, a longitudinal displacement with  $\mathbf{u} \parallel X$ .

**2.4.2 Tetragonal antiferromagnet with an EMS  $\bar{1}(+)4_z(-)2_d(+)$ .** The fluorides of transition metals  $\text{CoF}_2$ ,  $\text{MnF}_2$ ,  $\text{FeF}_2$ , and  $\text{NiF}_2$  have such an EMS. Their Néel points are, respectively  $T_N = 37.7$ ,  $72.0$ ,  $78.0$ , and  $73.2$  K. These compounds have a crystallochemical structure of the rutile type [7] with the  $D_{4h}^{14}$  symmetry, and the magnetic ions occupy the positions  $2a$  [25]. The first three compounds are easy-axis (EA) with  $\mathbf{L} \parallel 4_z \parallel Z$ , and the fourth one is easy-plane (EP) with  $\mathbf{L} \perp Z$  [15, 26].

We begin with the EP state, which exhibits weak ferromagnetism (17) and, hence, linear terms of the ( $L$ ) type in  $\Delta C^a$ , which we are discussing in the current section. Assume that

$$\mathbf{L} \parallel Y \parallel 2_y(-), \quad \mathbf{M} \parallel \mathbf{B} \parallel X \parallel 2_x(-). \quad (32)$$

Here,  $2_x(-)$  and  $2_y(-)$  are odd twofold axes directed along the edges of the basis square. Their odd character follows from the well-known equality  $4_z(-) \cdot 2_d(+) = 2(-)$ . With  $\mathbf{L} \parallel Y$ , the weakly ferromagnetic moment (17) would be directed along the  $X$  axis [15], while with  $\mathbf{L} \parallel X$ , the vector  $\mathbf{M}$  would be parallel to the  $Y$  axis. Both orientation states are of equal status, so we may consider one of them. As in the previous case, we first consider two particular cases for the direction of the wave vector  $\mathbf{k}$ : (a)  $\mathbf{k} \parallel \mathbf{M} \parallel \mathbf{B}$ , and (b)  $\mathbf{k} \parallel \mathbf{L}$ . One must bear in mind, however, that here the coordinate axes are chosen differently: comparison of (18) and (32) shows that the coordinate system for the second case is obtained from that of the first through a cyclic permutation

$$x \rightarrow y \rightarrow z \rightarrow x. \quad (33)$$

After we have established this fact, we can easily understand how the results of the present case can be obtained from those of the previous (Section 2.4.1), provided that we allow for (33). The elastic constants also change correspondingly:

$$\Delta C_{54} \equiv \Delta C_{xzyz} \rightarrow \Delta C_{yxzx} \equiv \Delta C_{65}$$

(and, of course,  $L_x \rightarrow L_y$ , etc.). One must also bear in mind that, for a tetragonal crystal, we have  $C_{55} = C_{44}$ .

The final results are as follows:

(a)  $\mathbf{k} \parallel \mathbf{B} \parallel X$ . Two transverse waves (with  $\mathbf{u}_1 \parallel Y$  and  $\mathbf{u}_2 \parallel Z$ ) mix and produce two elliptically polarized waves; the ellipses are elongated along the given axes and their plane is normal to  $\mathbf{k} \parallel X$  (cf. Fig. 1a) [with allowance for (33)]. By changing the coordinate axes via (33), we also obtain polarization relations from (23), the rotation angle  $\Theta$  is obtained from (25), and the ellipticity from (26) (provided, of course, that the boundary conditions  $\mathbf{u}(0) \parallel X$  are replaced by  $\mathbf{u}(0) \parallel Y$ ).

(b)  $\mathbf{k} \parallel \mathbf{L} \parallel Y$ . In this case, the transverse wave with  $\mathbf{u}_1 \parallel \mathbf{k} \parallel Y$  mixes with the longitudinal wave with  $\mathbf{u}_2 \parallel Z$ , similar to the result of Section 2.4.1(b) (cf. Fig. 1b). The reader can easily write the corresponding formulas.

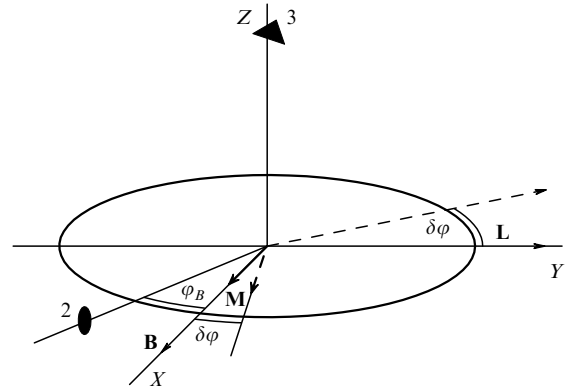
**2.4.3 Uniaxial EP antiferromagnets with an even principal symmetry axis with an EMS  $\bar{1}(+)N_z(+)2_x(-)$  ( $N=3, 4, 6$ ).** For all uniaxial antiferromagnets with an even principal axis  $N_z = 3, 4, 6$ , the components  $t_{zz}$ ,  $t_{zx}$ , and  $t_{yz}$  of the tensor  $t_{ij}$ , which determine the acoustic waves with  $\mathbf{k} \parallel Z$  considered here (having the state with  $\mathbf{L} \perp Z$  in mind), assume the following form [7]:

$$\begin{aligned} t_{zz} &= C_{33}e_{zz} + 2ir_L(L_x e_{zx} + L_y e_{yz}), \\ t_{zx} &= 2C_{44}e_{zx} - ir_L L_x e_{zz}, \quad t_{yz} = 2C_{44}e_{yz} - ir_L L_y e_{zz}. \end{aligned} \quad (34)$$

Indeed, these expressions are invariant not only under rotations through angles corresponding to the specified axes  $N_z$ ; there is also isotropy in the basal plane for a rotation through any angle about the  $Z$  axis (this is still true when we allow in (34) for the terms with  $\mathbf{M}$ , which, incidentally, are not written down here for the sake of brevity). And, of course, there is invariance under the axis  $2_x(-)$  common for all three cases. What we have makes it possible to select (without loss of generality) the  $X$  and  $Z$  axes in such a way that  $X \parallel \mathbf{B}$  and  $Y \parallel \mathbf{L}$  (Fig. 2). Here, we assume that  $\mathbf{L} \perp \mathbf{B}$  for all directions of  $\mathbf{B}$  in the  $XY$  plane (any angle  $\varphi_B$ ). In this system of coordinates, the antisymmetric constants

$$\Delta C_{34}^a = -\Delta C_{43}^a \equiv i\Delta C = i(r_L L + r_M M + r_B B)$$

that we will need (with  $\mathbf{k} \parallel Z$ ) are also independent of  $\varphi_B$ .



**Figure 2.** Selection of the coordinate axes in the  $XY$  plane: the  $X$  axis is parallel to the field  $\mathbf{B}$ , which makes an angle  $\varphi_B$  with the twofold symmetry axis  $2(-)$ ;  $\delta\varphi$  is the angle of rotation of the vectors  $\mathbf{L}$  and  $\mathbf{M}$  under elastic strain (see footnote 6 in this review).

As a result, equation (16) yields three normal modes:

$$k_1 = \frac{\omega}{v_1} = \omega \left( \frac{\rho}{C_{44}} \right)^{1/2} \equiv k_t, \quad \mathbf{u}_1 \parallel X, \quad (35)$$

$$k_2 \approx k_t, \quad (u_x)_2 = 0, \quad \left( \frac{u_z}{u_y} \right)_2 = i \frac{\Delta C}{C_{44} - C_{33}} \equiv iA,$$

$$k_3 = \frac{\omega}{v_3} = \omega \left( \frac{\rho}{C_{33}} \right)^{1/2} \equiv k_l, \quad (u_x)_3 = 0, \quad \left( \frac{u_y}{u_z} \right)_2 = iA. \quad (36)$$

The first wave is transverse; it splits off from the other two waves, and the vector  $\mathbf{L}$  has no effect on the velocity of this wave. Nevertheless, the wave has well-defined polarization, since always  $\mathbf{u}_1 \parallel \mathbf{X} \perp \mathbf{L}$ , and the direction of  $\mathbf{u}_1$  changes with the variation of the direction of  $\mathbf{L}$  caused by  $\mathbf{B}$ .

The second wave, which coincides with the first when  $\mathbf{L} = 0$ , mixes with the former longitudinal wave, and the two produce two elliptically polarized waves, a quasi-transverse and a quasi-longitudinal, similar to those depicted in Fig. 1b. The variations in  $k_2$  and  $k_3$  (and the corresponding velocities) caused by the presence of antiferromagnetism are small quantities proportional to  $A^2$  and are ignored in (36).

Similar to the previous cases [for orthorhombic crystals and for tetragonal crystals with an odd axis  $4_z(-)$ ], in which the normal waves are polarized along ellipses that contain the vector  $\mathbf{k}$ , here also there are planar effects of ellipticity and rotation of the polarization vector of the wave that was linearly polarized before entering the sample. And again these effects are approximately described by formulas (25) and (26). What distinguishes the last case with  $N_z(+)$  in the EMS code is the presence of the above-mentioned isotropy: all the above formulas are valid for an arbitrary direction of  $\mathbf{L} \perp \mathbf{B}$  in the basal plane. In orthorhombic and tetragonal (with  $4_z(-)$ ) antiferromagnets, the corresponding formulas referred to well-defined states. The fact that there is basic anisotropy substantially complicates the problem of analyzing more general orientation states. More than that, the results strongly depend on the adopted model (e.g., if we impose the conditions  $\mathbf{M} \cdot \mathbf{L} = 0$  and  $M^2 + L^2 = \text{const}$  or  $L^2 = \text{const}$  [15, 27]); the anisotropy of the  $g$  factor may also play an important role [28]. For the sake of simplicity and brevity, we do not examine these cases here.

## 2.5 Acoustic AF activity quadratic in magnetic field

Note that in  $\Delta\hat{C}^a$  (5), in addition to  $(L)$ , there are terms of the  $(LBB)$  type related to antiferromagnetism and also leading to CBR. The corresponding effect is unusual if only for the fact that it is quadratic in  $\mathbf{B}$ . Incidentally, for symmetry reasons, this effect also exists if  $\mathbf{B}$  is replaced by an electric field  $\mathbf{E}$ .

Here, we study the example of the EMS  $\bar{1}(+)4_z(-)2_d(+)$  ( $\text{CoF}_2$  and the like), the orientation state  $\mathbf{L} \parallel \mathbf{Z}$  (EA), and the field  $\mathbf{B} \perp \mathbf{Z}$ . The wave vector  $k$  is parallel to the  $Z$  axis. This situation is especially favorable for observing the  $(LBB)$  [or  $(LEE)$ ] effect, since here it manifests itself in pure form, without any admixture of the AF mechanism of CBR, the  $(L)$  effect, and LBR effects.<sup>4</sup>

In the case at hand ( $\mathbf{B} \perp \mathbf{L} \parallel \mathbf{Z}$  and  $\mathbf{k} \parallel \mathbf{Z}$ ) the  $(LBB)$  effect is due to the following contribution to the elastic constants:

$$\Delta C_{54}^a = -\Delta C_{45}^a \equiv i\Delta C = 2i\beta L_z B_x B_y. \quad (37)$$

This contribution leads to circularly polarized normal modes with elastic displacements

$$u_{\pm} = u_x \pm iu_y \quad (38)$$

and wave numbers

$$k_{\pm} = \frac{\omega}{v_{\perp}} \left( 1 \mp \frac{\beta}{C_{44}} L_z B_x B_y \right) \quad \left( v_{\perp}^2 = \frac{C_{44}}{\rho} \right). \quad (39)$$

<sup>4</sup> Note, however, that if  $\mathbf{B} \perp \mathbf{L} \parallel \mathbf{Z}$ , in some cases (both for the EMS in question and for  $\bar{1}(+)3_z(+)2_x(-)$ ) there is a phase transition consisting in a gradual rotation of  $\mathbf{L}$  with the emergence of an  $\mathbf{L}_{\perp}$  component [29, 30], which, of course, may violate the purity of the effects of interest to us if  $\mathbf{B}_{\perp}$  is sufficiently large.

For the angle of quasi-Faraday rotation over a path of length  $d$ , we have the following:

$$\Theta(d) = \frac{1}{2}(k_- - k_+)d = \frac{1}{2} \left( \frac{\beta}{C_{44}} \right) \frac{\omega}{v_{\perp}} L_z B_{\perp}^2 d \sin 2\varphi_B. \quad (40)$$

As in optics, where a similar effect has been discovered by Kharchenko et al. [31], it depends on the direction of  $\mathbf{B}_{\perp}$  in the basal plane. (Incidentally, note that at  $\varphi_B = 45^\circ$ , when the effect is at its maximum, the phase transition mentioned in footnote 4 is absent, which means that in this case all disturbances associated with this transition that reduce the purity of the effect are also absent.)

We would like to note the uniqueness of the situation: in addition to the fact that the effect is quadratic in  $\mathbf{B}$ , it is important that this field is not parallel to  $\mathbf{k}$  (as is the case with ordinary magneto-optic BR) but perpendicular to it,  $\mathbf{B} \perp \mathbf{k}$ .

As mentioned earlier, the field  $\mathbf{B}$  can be replaced in this case by an electric field,  $\mathbf{E}$ . More than that, symmetry allows for another possibility, namely, the replacement of the product  $B_x B_y$  by the shear strain  $e_{xy}$ . It is such a strain that causes piezomagnetism (e.g., see Appendix 1 in Ref. [7]).

In other situations with both the  $\bar{1}(+)4_z(-)2_d(+)$  structure and with EMS in other antiferromagnets with a symmetry center, the  $(LBB)$  effect is supplemented by the  $(L)$  effect or by LBR of crystallochemical (or AF) origin or by both effects simultaneously. For example, take the (simpler) case where there are no terms of the  $(L)$  type but there is LBR caused by the crystallochemical anisotropy of the elastic constants. We examine the EMS  $\bar{1}(+)2_x(-)2_y(+)$  with the orientation state  $\mathbf{L} \parallel \mathbf{Y}$  in which there is no weak antiferromagnetism (17) and no terms of the  $(L)$  type in  $\Delta\hat{C}^a$ . Suppose that again  $\mathbf{k} \parallel \mathbf{L} \parallel \mathbf{Y}$  and  $\mathbf{B} \perp \mathbf{Y}$ . In this case, the components  $t_{ij}$  present in the wave equations (16) are

$$t_{xy} = 2C_{66}e_{xy} + 2\Delta C_{64}^a e_{yz}, \quad t_{yz} = 2C_{44}e_{yz} + 2\Delta C_{46}^a e_{xy},$$

where  $\Delta C_{46}^a = -\Delta C_{64}^a = i\Delta C = i\beta L_y B_x B_z$ . Then, the wave solutions of equations (16) produce two specific modes with

$$k_1 \approx \omega \left( \frac{\rho}{C_{66}} \right)^{1/2}, \quad k_2 \approx \omega \left( \frac{\rho}{C_{44}} \right)^{1/2} \quad (41)$$

and

$$\left( \frac{u_z}{u_x} \right)_1 = \left( \frac{u_x}{u_z} \right)_2 = i \frac{\Delta C}{C_{66} - C_{44}} \equiv iA. \quad (42)$$

In the expressions for  $k_{1,2}$ , we again ignore, due to their smallness, the terms proportional to  $\Delta C^2$ .

Formulas (41) and (42) are the standard expressions that determine elliptical BR; they lead to formulas of the form (25) and (26) for the rotation angle  $\Theta$  and the ellipticity  $a/b$ . What makes them so special is the constant  $A$  in (42), which is now proportional to  $LB^2$ . Here, we have CBR (quadratic in  $B$ ) against the background of LBR of crystallochemical origin.

## 2.6 Linear BR: AF contributions of the types $(LL)$ and $(LB)$ in the easy-axis state

Below we discuss effects related to the symmetric terms in  $\Delta C_{\alpha\beta}^s$  (4) that are proportional to the products  $L_i L_k$  and  $L_i B_k$ . Generally speaking, from the symmetry viewpoint, the terms of the  $L_i L_k$  type do not differ from the terms of type  $M_i M_k$  or  $B_i B_k$  and in this sense they are not specifically antiferromag-

netic terms. Nevertheless, as in optics (see Ref. [7]), these terms must be allowed for together with  $L_i B_k$  if only for the fact that the contributions of all these terms to BR have proved to be nonadditive.

In relation to uniaxial crystals (tetrahedral, rhombohedral, and hexagonal), the most interesting is transverse sound propagating along the principal symmetry axis, which is usually adopted as the  $Z$  axis.

What is important in this situation for the case where  $\mathbf{k} \parallel Z$ , which is examined below, is that the LBR is completely of AF origin, while in other cases it is only a correction to crystallochemical BR caused by the anisotropy of the elastic properties of the crystal (a factor mentioned earlier in the present review). Here, we will deal with LBR in longitudinal ( $\mathbf{B} \parallel \mathbf{k}$ ) and transverse ( $\mathbf{B} \perp \mathbf{k}$ ) fields.

**2.6.1 The  $\bar{1}(+)4_z(-)2_d(+)$  structure and  $\mathbf{k} \parallel \mathbf{B} \parallel \mathbf{L} \parallel \mathbf{Z}$ .** In this case, to which  $\text{CoF}_2$  and other EA fluorides belong, the elastic constants (1) acquire the following AF terms:

$$\Delta C_{44}^s = \Delta C_{55}^s = f L_z^2, \quad \Delta C_{45}^s = \Delta C_{54}^s = \beta L_z B_z. \quad (43)$$

Combining (43) with equations (16), we arrive at two transverse normal modes whose wave numbers and phase velocities are

$$k_{1,2} = \frac{\omega}{v_{1,2}} \simeq k \left( 1 \mp \frac{\beta L_z B_z}{2C'_{44}} \right),$$

$$k = \omega \left( \frac{\rho}{C'_{44}} \right)^{1/2}, \quad C'_{44} = C_{44} + f L_z^2, \quad (44)$$

with the polarization vectors directed along the diagonals of the basal square:

$$\mathbf{u}_1 \parallel [110], \quad \mathbf{u}_2 \parallel [\bar{1}10] \quad \left( \left( \frac{u_x}{u_y} \right)_1 = - \left( \frac{u_y}{u_x} \right)_2 = 1 \right). \quad (45)$$

The linear BR is characterized by the difference of the velocities or wave vectors of these two modes:

$$\frac{\Delta v}{v} = - \frac{\Delta k}{k} = \frac{\beta}{C'_{44}} L_z B_z. \quad (46)$$

A transverse acoustic wave impinging on the medium with an arbitrary polarization of the vibrations  $\mathbf{u}$  can be decomposed into two normal modes, (44) and (45). Due to the difference in their velocities, the phases of these component waves at the exit from the medium prove to be different, which transforms the resulting wave into an elliptically polarized wave. The phase difference depends on the length of the path traveled by the waves,  $z = d$ , and on the magnetic field  $B_z$  (in terms of  $\Delta k$ ). Hence, the amplitude at the exit with selection of the projection of vibrations  $\mathbf{u}$  on a fixed direction proves to be an oscillating function of the distance  $d$  or the field  $B_z$  (a periodic function if  $\beta$  and  $L_z$  are independent of  $B_z$ ).

For instance, let  $\mathbf{u}(0) \parallel [100]$ . Then at the exit the amplitude of the vibrations in this direction is

$$u(d, B_z) = u(0) \left| \cos \left( \frac{\Delta k d}{2} \right) \right|, \quad (47)$$

where  $\Delta k$  is defined by formula (46). (A more general case is discussed in Ref. [6].)

This effect linearly depends on  $L_z$  ( $\Delta k$  is proportional to  $L_z$ ), which, apparently, opens the possibility of direct ‘audiovision’ of AF domains that differ in the sign of  $L_z$ . The latter is of special interest for researchers studying the domain structure of antiferromagnets that are opaque to light, when the similar effect in optics cannot be used.

**2.6.2 Linear BR in hematite at  $T < T_M$ .** Consider the EA state with  $L \parallel Z$  in hematite ( $\alpha\text{-Fe}_2\text{O}_3$ ). This state occurs at  $T < T_M = 260$  K (the Morin point). We assume that again  $\mathbf{k} \parallel Z$ . In this case, the AF effects of CBR of the ( $L$ ) and ( $LBB$ ) types are absent. However, there are LBR effects of the types ( $LL$ ) and ( $LB$ ) similar to those for the  $\bar{1}(+)4_z(-)2_d(+)$  structure (Section 2.6.1). In contrast to that case ( $\mathbf{B} \parallel Z$ ), the configuration with the field  $\mathbf{B} \perp Z$  proves to be more interesting. Here [7],

$$\Delta C_{44} = \alpha_1 L_z^2 + \beta_1 B_x L_z,$$

$$\Delta C_{55} = \alpha_1 L_z^2 - \beta_1 B_x L_z,$$

$$\Delta C_{45}^s = \Delta C_{54}^s = \beta_1 B_y L_z \quad (48)$$

and, instead of (44) and (45), we have

$$k_{1,2} = \frac{\omega}{v_{1,2}} \simeq k \left( 1 \mp \beta_1 \frac{L_z B_{\perp}}{2C'_{44}} \right), \quad (49)$$

$$\left( \frac{u_x}{u_y} \right)_1 = - \left( \frac{u_y}{u_x} \right)_2 = \frac{B_y}{B_{\perp} + B_x} = \tan \frac{\varphi_B}{2}, \quad (50)$$

where again  $\varphi_B$  is the azimuthal angle of  $\mathbf{B}_{\perp}$  measured from the axis  $X \parallel 2(-)$ , and  $C'_{44} = C_{44} + \alpha_1 L_z^2$ . Note that here we ignored the small admixture of longitudinal vibrations  $u_z$ , which manifest themselves in the approximation quadratic in  $\beta_1$ .

In the absence of  $\mathbf{L}$  (and also at  $\mathbf{B}_{\perp} = 0$ ), there would be transverse waves with  $\mathbf{u} \perp Z$  degenerate in polarization. The field  $\mathbf{B}_{\perp}$  (in the presence of  $L_z \neq 0$ ) lifts this degeneracy. As a result, two linearly polarized waves with different phase velocities emerge. Depending on the value of  $\varphi_B$ , the polarization vectors  $\mathbf{u}_1 \perp \mathbf{u}_2$  may point in all possible directions. What this means is that a linearly polarized wave impinging on the sample becomes an elliptically polarized wave at the exit, with a varying ellipticity — from straight lines to a circle. Below, we discuss this problem in greater detail for the two structures mentioned earlier,  $\bar{1}(+)4_z(-)2_d(+)$  and  $\bar{1}(+)3_z(+ )2_x(-)$ .

**2.6.3 Ellipticity and rotation of the polarization vector caused by LBR.** Here, we determine the ellipticity  $a/b$  and the rotation of the polarization vector  $\Theta$  caused by AF effects of the types ( $LL$ ) and ( $LB$ ) when these two phenomena are not masked by other effects, as is the case when  $\mathbf{k} \parallel \mathbf{L} \parallel Z$  in EA states corresponding to formulas (44), (45) and (49), (50) for the  $\bar{1}(+)4_z(-)2_d(+)$  and  $\bar{1}(+)3_z(+ )2_x(-)$  EMS, provided that  $\mathbf{B} \parallel Z$  and  $\mathbf{B} \perp Z$ , respectively. The first case is realizable in  $\text{CoF}_2$  and other fluorides; and the second, in hematite at temperatures below the Morin point ( $T < T_M(\mathbf{L} \parallel Z)$ ).

To this end, we begin by describing in greater detail the normal vibrations

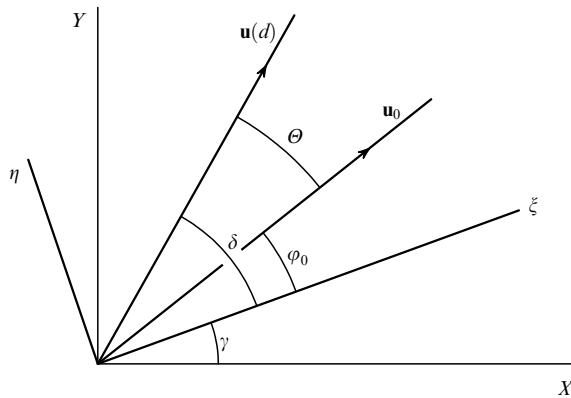
$$u_{\xi}(z, t) = u_{\xi}(0) \cos(k_1 z - \omega t),$$

$$u_{\eta}(z, t) = u_{\eta}(0) \cos(k_2 z - \omega t) \quad (51)$$

corresponding to the natural wave numbers  $k_{1,2}$  for  $\bar{1}(+)4_z(-)2_d(+)$  (44) and for  $\bar{1}(+)3_z(+)-2_x(-)$  (49). These normal vibrations are related to the components  $u_x$  and  $u_y$  (when the axis  $X \parallel 2_x(-)$ ) through the linear relationships

$$u_\xi = u_x \cos \gamma + u_y \sin \gamma, \quad u_\eta = -u_x \sin \gamma + u_y \cos \gamma, \quad (52)$$

which can be found by solving the system of equations (16) for the cases in question. Here,  $\gamma = \pi/4$  for the tetragonal case and  $\gamma = (n\pi - \varphi_B)/2$  (where  $n$  assumes integral values) for the trigonal case. The new axes  $\xi$  and  $\eta$  (along which the elastic displacements for the normal modes are polarized) are rotated (about the  $Z$  axis) with respect to the axes  $X \parallel 2$  and  $Y \perp 2$  through the above angles  $\gamma$  (Fig. 3). We note once more that here we are dealing with an EA state ( $\mathbf{L} \parallel Z$ ).



**Figure 3.** Selection of the coordinate axes in the  $XY$  in the description of LBR of the types  $(LL)$  and  $(LB)$ :  $X \parallel 2_x(-)$ ,  $Y \parallel 2_y(-)$ , the  $\xi$  and  $\eta$  axes are directed along the displacements  $\mathbf{u}_\xi$  and  $\mathbf{u}_\eta$  for normal acoustic modes;  $\mathbf{u}_0 \equiv \mathbf{u}(0)$  specifies the direction at the entrance ( $z = 0$ ), which makes an angle  $\varphi_0$  with the  $\xi$  axis;  $\mathbf{u}(d)$  is the displacement at the exit ( $z = d$ ), which makes an angle  $\delta$  with the  $\xi$  axis; and  $\gamma$  is the angle between the  $\xi$  and  $X$  axes.

Let us assume that the elastic displacement at the entrance,  $\mathbf{u}_0$ , makes an angle  $\varphi_0$  with the  $\xi$  axis (see Fig. 3), so that  $u_\xi = u_0 \cos \varphi_0$  and  $u_\eta = u_0 \sin \varphi_0$ . Then, the tip of the vector of the resulting displacement  $\mathbf{u}(z, t) = \mathbf{u}_\xi + \mathbf{u}_\eta$  at the exit ( $z = d$ ) describes an ellipse. The major axis of the ellipse makes an angle  $\delta$  with the  $\xi$  axis, and this angle can be found using the formula

$$\tan 2\delta = \tan 2\varphi_0 \cos(\Delta kd), \quad (53)$$

while the semiaxes are given by the formulas

$$r_{1,2}^2 = \frac{1}{2} u_0^2 \left[ 1 \pm (1 - \sin^2 2\varphi_0 \sin^2(\Delta kd))^{1/2} \right], \quad (54)$$

where  $\Delta k = k_1 - k_2$ . The axial ratio is the ellipticity:

$$\frac{a}{b} \equiv \frac{r_2}{r_1} = \left| \frac{1 - (1 - \sin^2 2\varphi_0 \sin^2(\Delta kd))^{1/2}}{\sin 2\varphi_0 \sin(\Delta kd)} \right|. \quad (55)$$

Let us use formulas (53)–(55) in specific cases.

(1) If at the entrance the wave is polarized along vibrations belonging to one of the normal modes,  $\mathbf{u}_0 \parallel \xi$  ( $\varphi_0 = 0$ ) or  $\mathbf{u}_0 \parallel \eta$  ( $\varphi_0 = \pi/2$ ), at the exit it remains linearly polarized, retaining its former direction of polarization.

(2) Now let  $\mathbf{u}_0$  be directed along the bisector of the angle between  $\xi$  and  $\eta$ . In this case the polarization becomes

elliptical, so that

$$r_1 = u_0 \left| \cos \left( \frac{\Delta kd}{2} \right) \right|, \quad r_2 = u_0 \left| \sin \left( \frac{\Delta kd}{2} \right) \right|. \quad (56)$$

The major axis of the ellipse is directed along the bisector ( $\delta = \varphi_0 = \pi/4$ ), and the ellipticity is

$$\frac{a}{b} = \frac{r_2}{r_1} = \left| \tan \left( \frac{\Delta kd}{2} \right) \right|. \quad (57)$$

Here, we have an interesting special case mentioned earlier, precisely, when the polarization of the transmitted wave becomes circular. Indeed, at

$$|\Delta kd| = (2n + 1) \frac{\pi}{2} \quad (n \text{ are integral numbers}) \quad (58)$$

we find, from (56) and (57), that  $r_1 = r_2 = u_0/\sqrt{2}$  and  $a/b = 1$ .

(3) Now we turn to the general case where a linearly polarized wave enters the sample at an arbitrary angle  $\varphi_0$ , travels a distance  $d$ , and exits the sample still remaining linearly polarized. How can this be and what will be the angle  $\Theta(d)$  (see Fig. 3) at the exit? Formulas (53)–(55) provide the following answer: the condition

$$|\Delta kd| = (2n + 1)\pi \quad (59)$$

must be met, with the rotation angle obeying the condition (to within  $\pi$ )

$$\Theta(d) = -2\varphi_0. \quad (60)$$

(4) Another interesting specific case directly follows from condition (60). This is the possibility of rotation of the polarization plane through an angle  $\Theta = \pm\pi/2$  (to within  $\pi$ ) if the angle at the entrance is  $\varphi_0 = \pm\pi/4$ . Of course, one must not forget that, first of all, condition (59) must be met, which results in the same situation as the particular case with (56).

The most intriguing of the above effects is the transformation of linear polarization into circular polarization and the rotation of the linear polarization vector (with respect to the polarization of the incident wave  $\mathbf{u}_0$ ) through an angle  $\pm\pi/2$ . Such effects are related, respectively, to conditions (58) and (59), which can be achieved by selecting proper values of the field strength  $B$  and the thickness  $d$ . This is possible if (at least in order of magnitude)

$$\Delta kd = \frac{2\pi d}{\Lambda} \left( \frac{\beta LB}{C'_{44}} \right) \approx 1, \quad (61)$$

where  $\Lambda$  is the sound's wavelength. To check the possibility of this condition being valid, we must estimate the phenomenological parameter  $\beta$ , which is possible if we adopt an approach based on the coupled equation of magnetoelastic dynamics. This is the topic of Section 3.

### 3. Description of BR based on the coupled equations of magnetoelastic dynamics (centrosymmetric EMS)

#### 3.1 Thermodynamic potential and equations of motion

To examine the coupled equations of magnetoelastic dynamics, we must write the expression for the thermodynamic potential density  $\Phi$ , which incorporates three terms: the magnetic ( $\Phi_m$ ), the elastic ( $\Phi_e$ ), and the magnetoelastic

( $\Phi_{me}$ ), i.e.,

$$\Phi = \Phi_m + \Phi_e + \Phi_{me}.$$

Limiting ourselves to the lowest-order terms in magnetic and elastic variables and taking into account the fact that  $\Phi$  is invariant with respect to the crystallochemical group  $G_F1'$  (see below), we obtain

$$\Phi_m = \frac{1}{2} \mathcal{E} m^2 + \frac{1}{2} K_a l_x^2 + \frac{1}{2} K_c l_z^2 - d_s(l_x m_z + l_z m_x) - d_a(l_x m_z - l_z m_x) - 2\mathbf{M}_0 \mathbf{m} \cdot \mathbf{B}, \quad (62)$$

$$\Phi_e = \frac{1}{2} C_{ijkl} e_{ij} e_{kl}, \quad (63)$$

$$\Phi_{me} = B_{ijkl} l_i l_j e_{kl} + \Pi_{ijkl} m_i l_j e_{kl}. \quad (64)$$

Here, we have introduced the relative values of the vectors of magnetization  $\mathbf{m} = \mathbf{M}/2M_0$  and antiferromagnetism  $\mathbf{l} = \mathbf{L}/2M_0$ . If we adopt the model of equal-modulus sublattice magnetizations, then  $\mathbf{M}_1^2 = \mathbf{M}_2^2 = \mathbf{M}_0^2$ , where  $\mathbf{M}_0$  is the nominal constant magnetization vector. This implies that

$$\mathbf{m} \cdot \mathbf{l} = 0, \quad m^2 + l^2 = 1. \quad (65)$$

In equation (62), the potential  $\Phi_m$  is written in explicit form, which can be used for all EMSs discussed above. The general formula (62) corresponds to the  $\bar{1}(+)2_x(-)2_y(+)$  EMS of an orthorhombic crystal (structure  $G$ ). For other orthorhombic EMSs,  $A$  and  $C$  [23],  $\Phi_m$  can be found by a cyclic permutation of the coordinate axes. In particular, for the permutation (33) we have the potential  $\Phi_m$  for EMS of the  $A$  type.<sup>5</sup> The particular cases of potential (62) include the magnetic potentials for uniaxial antiferromagnets if we set  $K_a = K_c \equiv K$ , and  $d_a = 0$  for the  $\bar{1}(+)4_z(-)2_d(+)$  EMS or  $d_s = 0$  for the  $\bar{1}(+)N_z(+)2_x(-)$  ( $N = 3, 4, 6$ ) EMS. The constants in  $\Phi_m$  (62) have the following meaning:  $\mathcal{E}$  is the exchange interaction (homogeneous exchange), nonhomogeneous exchange is characterized by the terms with the derivatives  $\partial \mathbf{l} / \partial x_i$  and  $\partial \mathbf{m} / \partial x_i$  (we ignore these terms because they are small for the wavelengths of interest to us), the constants  $K$  represent the magnetic anisotropy, and  $d_s$  and  $d_a$  reflect the symmetric and antisymmetric Dzyaloshinski interaction responsible for weak ferromagnetism (17).

The elastic (63) and magnetoelastic (64) parts of  $\Phi$  are written in general form for an arbitrary EMS with two sublattices. Their explicit form for the EMS of interest to us can be found in textbooks on the theory of elasticity and magnetism (e.g., see the book by Belov et al. [33] and the reviews in Refs [34, 35]). The first term in  $\Phi_{me}$  (64) reflects the (antiferro)magnetoelastic interaction, and the second term is responsible for piezomagnetism (or magnetostriction linear in  $\mathbf{m}$ ). In equations (62) and (64), we discarded the small terms quadratic in  $\mathbf{m}$ , with the exception of the term in (62) responsible for exchange interaction.

A remark concerning magnetoelastic interaction is in order. Expressions (62) and (64) are written in a coordinate system linked to the axes of the crystal. However, under acoustic strains these axes experience local rotations deter-

mined by the antisymmetric tensor

$$\omega_{ij} = \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} - \frac{\partial u_j}{\partial x_i} \right) = -\omega_{ji}. \quad (66)$$

The so-called rotation-invariant theory of magnetoelastic phenomena requires that this fact be taken into account [36, 37]. The simplest variant of this theory (and the very idea of such a theory) was first introduced by Vlasov (see Ref. [36]) and amounts to the following. In  $\Phi_m$  and  $\Phi_{me}$  one must go from the local coordinate system to the laboratory coordinate system via the local rotation transformation

$$l_i \rightarrow l_i - \omega_{ij} l_j, \quad m_i \rightarrow m_i - \omega_{ij} m_j. \quad (67)$$

Here,  $\Phi_m$  acquires magnetoelastic terms proportional to  $\omega_{ij} e_{kn}$  and  $\omega_{ij} \omega_{kn}$  and  $\Phi_{me}$  acquires the terms  $l_i l_j \omega_{kn}$  and  $l_i m_j \omega_{kn}$ . What is characteristic here is that no new constants of the theory emerge in the process. Most often the matter boils down to renormalizing the constants that already exist in (62)–(64), with the corrections being so small that we ignore them here. Note, however, that there are cases where the corrections of the rotation-invariant theory must, probably, be taken into account (see Ref. [35]).

We mention here one more nonreciprocal effect related to the presence in the thermodynamic potential of this theory of terms with  $\omega_{ij}$ . The effect consists in changing the velocity of the wave (due to the antisymmetry of  $\omega_{ij}$ ) when the directions in which the wave propagation and its polarization are interchanged. For instance, if the geometry of the experiment  $\mathbf{k} \parallel Z$  and  $\mathbf{u} \parallel X$  is replaced with  $\mathbf{k} \parallel X$  and  $\mathbf{u} \parallel Z$ , the relative velocity difference for these two geometries is  $\Delta v/v \approx B_{44}/C_{44} \approx 10^{-3} - 10^{-4}$  ( $\text{MnF}_2$ ), which is an extremely small quantity. The effect has been corroborated by Melcher's experiments [34]. Of course, this type of nonreciprocity differs from that related to the term  $(Lk)$  in  $\Delta \hat{C}^s$  (4), which has been discussed earlier and will be discussed in greater detail in Section 4.3.

The stress tensor  $t_{ij}$  is defined differently in the rotation-invariant theory. It is calculated in terms of the thermodynamic potential (with allowance for terms containing  $\omega_{ij}$  and the fact that  $\partial u_i / \partial x_j = e_{ij} + \omega_{ij}$ ) in the following manner [35]:

$$t_{ij} = \frac{\partial \Phi}{\partial (\partial u_i / \partial x_j)} = \frac{1}{2} \left( \frac{\partial \Phi}{\partial e_{ij}} + \frac{\partial \Phi}{\partial \omega_{ij}} \right). \quad (68)$$

The tensor  $t_{ij}$  is nonsymmetric because  $\omega_{ij}$  is antisymmetric.

After we have found  $t_{ij}$ , we can use the former equations (16) of elastic dynamics. The latter prove to be coupled (because of the magnetoelastic interaction) with the equations of magnetic (spin) dynamics, so that one is forced to solve them simultaneously.

In the Landau–Lifshitz variant, these equations (without allowance for dissipation) have the form

$$\begin{aligned} \dot{\mathbf{M}} &= \gamma \left( \mathbf{M} \times \frac{\partial \Phi}{\partial \mathbf{M}} + \mathbf{L} \times \frac{\partial \Phi}{\partial \mathbf{L}} \right), \\ \dot{\mathbf{L}} &= \gamma \left( \mathbf{M} \times \frac{\partial \Phi}{\partial \mathbf{L}} + \mathbf{L} \times \frac{\partial \Phi}{\partial \mathbf{M}} \right), \end{aligned} \quad (69)$$

where  $\gamma = g e \hbar / 2mc$  is the gyromagnetic ratio.

Note that here the parameter  $\gamma$  is a scalar quantity, so that it would seem that Eqs (69) are invalid if  $\gamma$  is a tensor that

<sup>5</sup> The  $\bar{1}(+)2_x(+)2_y(-)$  EMS. Lanthanum cuprate  $\text{La}_2\text{CuO}_4$  has such a structure and an orientation state with  $\mathbf{L} \parallel Z$  in fields  $B \equiv B_y > 30$  kG (see Ref. [32] and Appendix 2 in Ref. [7]).

takes into account the anisotropy. In Ref. [28] it was shown that this is not the case. If we replace the sublattice magnetizations by the spin densities  $\mathbf{S}_{1,2} = -\mathbf{M}_{1,2}/\gamma$ , instead of (69) we obtain a system of equations in terms of the sum and the difference of the spin densities,  $\boldsymbol{\mu} = \mathbf{S}_1 + \mathbf{S}_2$  and  $\mathbf{v} = \mathbf{S}_1 - \mathbf{S}_2$ , where  $\gamma$  is not present explicitly. Equations (69) written in terms of  $\boldsymbol{\mu}$  and  $\mathbf{v}$  contain the gyromagnetic ratio only in the Zeeman energy, where, as it has been established, the anisotropy of the tensor  $\hat{\gamma}$  can be taken into account. What we just said is true for ferromagnetism, too.

Of course, other types of equations of magnetic dynamics (due to the limits of applicability of the Landau–Lifshitz equations) can also be used (e.g., see Ref. [28]). However, for our purposes the Landau–Lifshitz equations are quite sufficient. Incidentally, the relationships in (65) follow from (69) as constants of motion.

After these facts have been established, the procedure is as follows. The Landau–Lifshitz equations are used to find the dynamical components  $\Delta\mathbf{l}$  and  $\Delta\mathbf{m}$  caused by acoustic strains  $e_{ij}$  and  $\omega_{ij}$  as functions of frequency  $\omega$ , field  $\mathbf{B}$ , etc. Substituting these values  $\Delta\mathbf{l}$ ,  $\Delta\mathbf{m} \propto e_{ij}, \omega_{ij}$  into (68) and using (15) and (66), we find the stress tensor  $t_{ij}$  expressed in terms of the derivatives  $\partial u_i / \partial x_k$ . Plugging this tensor into equations (16) yields a system of homogeneous linear equations for the elastic displacements  $\mathbf{u}$ . The solution of this system in the form  $\mathbf{u} \propto \exp(i\mathbf{k} \cdot \mathbf{r} - i\omega t)$  determines the natural wave numbers and polarizations (amplitude ratios) at the given frequency  $\omega$  for the sought normal modes.

The reader must bear in mind the following. Generally, the coupling coefficients in the relationships for  $\Delta\mathbf{m}$ ,  $\Delta\mathbf{l} \propto e_{ij}$ ,  $\omega_{ij}$  found from equations (69) have resonant denominators of the type  $\omega^2 - \omega_{\text{AFMR}}^2$ , in view of which the coupling coefficients grow without limit near  $\omega_{\text{AFMR}}$ , the antiferromagnetic resonance frequency. This means that, strictly speaking, the normal modes are magnetoacoustic waves (the literature devoted to the study of such waves is vast). We, however, will proceed differently and examine fairly low frequencies, such that

$$\omega^2 \ll \omega_{\text{AFMR}}^2. \quad (70)$$

Then, the term  $\omega^2$  in the resonance denominator can be ignored in comparison to  $\omega_{\text{AFMR}}^2$ . This approximation does not mean that we are using the quasi-static approach (equilibrium coupling between magnetic and elastic vibrations), since the coupling coefficients retain terms linear in frequency  $\omega$  thanks to the presence of the first time derivatives on the right-hand sides of equations (69). Such an approximation ensures a finite value of the terms in  $\Delta\hat{C}^a$  (5) and the corresponding CBR effect and the validity of the rule (13) obtained earlier.

With the quasi-equilibrium approach, one must set

$$\dot{\mathbf{M}} = \dot{\mathbf{L}} = 0, \quad (71)$$

in equations (69), but the CBR effect in this case vanishes. The physics of this is that the acoustic activity is related to the precession of magnetic moments, which, of course, vanishes at  $\omega = 0$ .

### 3.2 Piezomagnetism and LBR in CoF<sub>2</sub>

As noted earlier, the theory based on magnetoelastic dynamics makes possible a quantitative description of the magnetoacoustic phenomena of interest to us by expressing,

in particular, the phenomenological parameters of the symmetry theory [e.g.,  $\beta$  in (36)] in terms of the magnetic, elastic, and magnetoelastic parameters determined through studies of other independent phenomena.

**3.2.1 Estimating the parameter  $\beta$ .** To demonstrate the validity of the above statement, we first study a situation in which the antisymmetric contribution (5) to  $\Delta\hat{C}$  is zero. According to Section 2.6.1, this happens for the  $\bar{1}(+)4_z(-)2_d(+)$  EMS in the state with  $\mathbf{L} \parallel Z$  for, say CoF<sub>2</sub>. Here, we can express the above-mentioned constant  $\beta$  in (46), which determines the LBR, in terms of known parameters. The situation occurs not only in the low-frequency range (70). We do, however, use the quasi-static approximation (13).

As a result, the earlier procedure of calculating the LBR of the (LB) type leads us in the current case where  $\mathbf{k} \parallel \mathbf{B} \parallel \mathbf{L} \parallel Z$  to the following relationship:

$$\frac{\Delta k}{k} = \frac{2B_{44}\mathcal{E}}{(\mathcal{E}K - d_s^2)C_{44}} WB_z \equiv -\Delta. \quad (72)$$

Here,  $W$  is the parameter that determines the piezomagnetic effect (Borovik-Romanov [39])

$$M_x = 2We_{xy} = \frac{2W}{C_{44}} t_{xz},$$

where [6]

$$W \simeq -\frac{2M_0(B_{44}d_s + \Pi_{44}K)}{\mathcal{E}K - d_s^2}. \quad (73)$$

Formulas (72) and (73) are approximate, since the conditions

$$B \ll H_{sf} = \sqrt{\mathcal{E}K} - |d_s| \approx 10^5 \text{ Oe (for CoF}_2) \quad (74)$$

and  $K \ll B_{44}$  have been included. The complete formulas are given in Ref. [6], which, among other things, allow for all contributions to the energy that are related to the tensor  $\omega_{ij}$ .

From the experiment involving CoF<sub>2</sub> [39] it was found that  $|W|/C_{44} = 2 \times 10^{-9} \text{ G}^{-1}$ . The following parameters in expressions (72) and (73) are known [40] (in  $\text{erg cm}^{-3}$ ):  $\mathcal{E} = 1.4 \times 10^9$ ,  $K = 3.8 \times 10^7$ ,  $d_s = 1.7 \times 10^8$ ,  $C_{44} = 10^{12}$ , and, finally,  $2M_0 = 10^3 \text{ G}$ . Thus, to estimate  $\Delta$  by formula (72), we should know the magnetoelastic constant  $B_{44}$ . As a rough approximation it can be assumed [6] that the contribution to piezomagnetism determined by the constant  $W$  of (73) is evenly divided between the magnetostriction constant  $B_{44}$  and the piezomagnetism constant  $\Pi_{44}$ . Then, formula (73) yields  $B_{44} \approx -1.4 \times 10^8 \text{ erg cm}^{-3}$ .

Now, if we compare (72) with the phenomenological formula (46) obtained earlier, we find the ratio  $|\Delta/B_z| \approx 3 \times 10^{-5} \text{ kG}^{-1}$  and then the sought constant

$$\beta = \left| \frac{\Delta}{B_z} \right| \frac{C_{44}}{2M_0} \approx 30. \quad (75)$$

**3.2.2 Oscillations of the transmitted sound in the EA state.** Due to the dependence of  $\Delta k$  on the field  $B_z$ , the amplitude of the wave at the exit from the sample must oscillate as  $B_z$  increases. According to (47), the oscillation period  $\Delta B$  is determined by the equation

$$\Delta k(B + \Delta B) - \Delta k(B) = \frac{2\pi}{d}, \quad (76)$$

which yields

$$\Delta B = \left(\frac{\Delta}{B}\right)^{-1} \frac{A}{d}. \quad (77)$$

There is reason to speak about oscillations if  $\Delta B$  is at least several times smaller than the fields  $B$  in which the experiment is conducted:

$$\Delta B \ll B. \quad (78)$$

According to (77) and (78), the samples must be so thick that  $d \gg Q\lambda/B_z$  (where  $Q = |\Delta/B_z|^{-1}$  is independent of  $B_z$  in accordance with (72) for the field strengths considered here, i.e.,  $B_z \ll H_{sf}$ ), which corresponds to large values of  $B_z$  [obeying, however, condition (74)], and the frequencies  $\omega$  must be sufficiently high. In the case at hand, at frequencies in the vicinity of  $\omega = 10^9 \text{ s}^{-1}$  and a velocity  $v = 10^5 \text{ cm s}^{-1}$  and with equation (77) combined with the fact that  $Q \approx 3 \times 10^4 \text{ kG}$ , inequality (78) corresponds to the condition  $d \gg 2 \text{ mm}$  even for fields as strong as  $B \approx 10 \text{ kG}$ . In this respect,  $\text{CoF}_2$  is not a very suitable object for observing the oscillations in question, since this requires very thick samples (transparent to microwave sound).

### 3.3 Effects of mixed ( $L$ ) and ( $LL$ , $LB$ ) type for the $\bar{1} (+) 3_2 (-) 2_x (-)$ EMS

**3.3.1 LBR of AF nature in an EP state.** The interest in this case lies in the fact that the AF CBR [the ( $L$ )-type effect] is superimposed on LBR of the same AF nature [the effect of the ( $LL$ ,  $LB$ ) type]. An important factor here is the ratio of these two effects, so it is advisable to begin with the coupled equations of magnetoelastic dynamics, which provide this ratio [5, 7]. More than that, an experiment has been carried out that reflects this situation (for  $\text{MnCO}_3$  and  $\text{FeBO}_3$ ), and a quantitative interpretation of it is desirable.

In centrosymmetric antiferromagnets (which we have been studying so far) that are in the easy-plane state ( $\mathbf{L} \perp 3_z \| Z$ ), when there is a strong field  $\mathbf{B} \| X \perp \mathbf{L} \| Y$  ( $B \gtrsim 10^3 \text{ G}$  for hematite), LBR is related to the AF terms ( $LL$ ) and ( $LB$ ) in  $\Delta \hat{C}^s$  (4). We examine this effect in the absence of CBR, i.e., at  $\Delta \hat{C}^a = 0$ . Suppose that again  $\mathbf{k} \| Z$ .

In the adopted system of coordinates (see Fig. 2) with the azimuthal angle  $\varphi_B$  for  $\mathbf{B} \equiv \mathbf{B}_\perp$  at low frequencies  $\omega \ll \omega_{\text{AFMR}} \equiv \omega_f$  ( $\omega_f$  is the frequency of the quasi-ferromagnetic branch of AFMR, see below), the elastic constants  $\Delta \hat{C}$ , including  $\Delta \hat{C}^a$  for the time being, have the form

$$\begin{aligned} \Delta C_{55}^s &\equiv \Delta C_{55} = -B_{14} l_y^4 U \cos^2 3\varphi_B, \\ \Delta C_{44}^s &\equiv \Delta C_{44} = -B_{14} l_y^4 U \sin^2 3\varphi_B, \\ \Delta C_{54}^s &= \frac{1}{2} B_{14} l_y^4 U \sin^6 \varphi_B, \\ \Delta C_{54}^a &= i \left( \frac{\omega}{4\gamma H_E} \right) \tilde{H}_{74} l_y^3 U \sin 3\varphi_B \equiv i \Delta C. \end{aligned} \quad (79)$$

Here  $l_y \simeq l_0 = 1$ ,

$$U = 4B_{14} \frac{H_E}{M_0 H_f^2} \quad (80)$$

is what is known as the exchange enhancement factor of magnetoelastic interaction [35, 41],  $H_E = \mathcal{E}/4M_0$  is the

exchange field, and

$$H_f^2 = \frac{\omega_f^2}{\gamma^2} \approx B(B + H_D) + 2H_E H_\Delta, \quad (81)$$

with  $H_\Delta$  the effective field, which includes the basic anisotropy (usually very small) and magnetostrictive strain (magnetoelastic gap) [34]. The presence of the coefficient  $U$  in the symmetric components  $\Delta C_{\alpha\beta}$  plays an important role in determining the size of the effects in question. In fields  $B = 100 - 1000 \text{ Oe}$ , the coefficient  $U$  reaches, according to formula (80), values in the  $10^3 - 10^4$  range (for  $\alpha\text{-Fe}_2\text{O}_3$  and  $\text{FeBO}_3$ ).<sup>6</sup>

Note that, according to the formula for  $\Delta \hat{C}^a$  in (79), there is no exchange enhancement, since the field  $H_E$  in the numerator (in  $U$ ) cancels out with the same field in the denominator. The reason for this is that the AF correction  $\Delta \hat{C}^a$  emerges because the vector  $\mathbf{M}$  precesses with a frequency  $\omega$  and thus leaves the  $XY$  plane, a situation also hindered by the exchange field  $H_E$ . We also note that in the case at hand  $\Delta \hat{C}^a$  is related to piezomagnetism and weak ferromagnetism. Accordingly, the constant

$$\tilde{H}_{74} = \Pi_{xyz} - H_D M_0, \quad (82)$$

where the field  $H_D$  emerges due to the terms with  $\omega_{ij}$  in the thermodynamic potential of the rotation-invariant theory.

In view of what we have said, it is advisable to first study LBR, assuming  $\Delta C_{54}^a$  small compared to the components  $\Delta \hat{C}^s$  in (79). Here, we again can use Fig. 3, in which we should now set  $X \| \mathbf{B}$  and  $Y \| \mathbf{L}$ . The emerging normal transverse modes are polarized along the axes

$$\xi = X \cos \gamma - Y \sin \gamma, \quad \eta = X \sin \gamma + Y \cos \gamma,$$

which are rotated about  $Z$  with respect to the  $X$  and  $Y$  axes by the angle  $\gamma = (\pi n/2) - 3\varphi$  ( $n$  is an integer) and, hence, by the angle  $\varphi_\xi = (\pi n/2) - 2\varphi_B$  with respect to the twofold symmetry axis  $2(-)$ . The corresponding phase velocities for these modes are

$$v_\xi = \frac{\omega}{k_\xi} = \left( \frac{C_\xi}{\rho} \right)^{1/2}, \quad v_\eta = \frac{\omega}{k_\eta} = \left( \frac{C_\eta}{\rho} \right)^{1/2} \equiv v, \quad (83)$$

where  $C_\xi = C_{44} - B_{14}U$  and  $C_\eta = C_{44}$ . We repeat: in this approximation ( $\Delta \hat{C}^a = 0$ ) the normal modes are linearly polarized with the above directions of polarization ( $\xi$  and  $\eta$ ) and phase velocities ( $v_\xi$  and  $v_\eta$ ).

<sup>6</sup> The coefficients  $U$  appear in the magnetoacoustics of EP antiferromagnets because the acoustic strains  $e_{ij}$  ( $e_{xz}$  and  $e_{yz}$  in the given case for  $\mathbf{k} \| Z$ ) act on the system ( $\mathbf{M}, \mathbf{L}$ ) through the antiferromagnetism vector  $|\mathbf{L}| \approx 2M_0$  (the first term in  $\Phi$  (67)) and tend to rotate the system (vector  $\mathbf{L}$ ) in the easy plane  $XY$ , while the Zeeman energy  $-\mathbf{M} \cdot \mathbf{B}$ , where  $M = M_0 B/H_E \ll 2M_0$ , hinders this rotation. As a result, the rotation angle  $\delta\varphi$  (see Fig. 2) is determined by the ratio of the magnetoelastic energy  $B_{14}e_{yz}$  (or  $e_{xz}$ ) to the Zeeman energy  $MB = M_0 B^2/H_E$ , i.e., by  $(B_{14}H_E/M_0 B^2)e_{yz}(e_{xz})$ . It is the first factor (in parentheses) that gives the order of magnitude of the enhancement factor  $U$  (80) if we ignore the anisotropy in EP and the Dzyaloshinski field  $H_D$ . If we allow for them,  $B^2 \rightarrow H_f^2$ . The rotation angle  $\delta\varphi$  in EP antiferromagnets increases by a factor of  $H_E/B$  in comparison, say, to a ferromagnet, where instead of the small magnetization  $M = \chi_\perp B$  we have the magnetization  $M_0$ . It is the fact that the numerator in (80) contains the factor  $H_E$  (the exchange field) that led to the term 'exchange enhancement' [41].

**3.3.2 Allowance for the antisymmetric component  $\Delta C_{54}^a$ .** Now, let us assume that  $\Delta C_{54}^a \neq 0$  in (79), which brings acoustic AF activity into the picture. Solving Eqns (16) together with (68) and allowing for (79), we arrive at elliptical BR with the ellipses elongated along the  $\xi$  and  $\eta$  axes. The ellipticity is determined by the polarization ratios

$$\left(\frac{u_\eta}{u_\xi}\right)_1 = \left(\frac{u_\xi}{u_\eta}\right)_2 = \frac{\Delta C_{54}^a}{C_\eta - C_\xi} = i \frac{\Delta C}{C_\eta - C_\xi} \equiv iA, \quad (84)$$

where in the given case

$$A = \frac{\omega \tilde{\Pi}_{74} \sin 3\varphi_B}{2\omega_E B_{14}}, \quad (85)$$

with  $\omega_E = 2\gamma H_E$  the exchange frequency. Here, we again have a superposition of CBR and LBR [corresponding to (84)]. Only now the LBR is related to antiferromagnetism (just as the CBR is). Nevertheless, the estimate (85) shows that the parameter determining the ellipticity,  $|A|$ , is much smaller than unity. This fact has already been taken into account in (84).

If one allows for the fact that  $A \neq 0$ , the velocities and wave numbers of the normal modes change, of course. However, these corrections prove to be quadratic in  $A$ , with the result that we can ignore them and leave the approximate expressions (83):  $v_1 \simeq v_\xi = (C_\xi/\rho)^{1/2}$  and  $v_2 \simeq v_\eta = (C_{44}/\rho)^{1/2} \equiv v$ . Here,

$$\frac{\Delta k}{k} \equiv \frac{k_1 - k}{k} = -\frac{v_1 - v}{v} \simeq \frac{UB_{14}}{2C_{44}}. \quad (86)$$

The approximate part of these relations allows for the fact that  $UB_{14}/2C_{44} \ll 1$ . However, this is not always the case. For instance, for hematite, according to the estimates made in Ref. [6], at  $B = 1$  kG we have  $\Delta k/k \approx B_{14}U/2C_{44} \approx 20\%$ . But for fields  $B \approx 2$  kG, in which saturation is reached (in the sense that the domain structure ceases to exist), we already have  $\Delta k/k \approx 10\%$ .

Apparently, the simplest way to detect, in experiments, the gyrotropic BR effect (AF activity) related to  $\Delta C_{54}^a$  of (79) is to proceed as follows. If the sound with  $\mathbf{k}||Z$  is polarized along one of the axes,  $\xi$  or  $\eta$  (i.e., at an angle  $\varphi_\xi = (\pi/2) - 2\varphi_B$  to the twofold symmetry axis  $2(-)$ ), at  $\Delta C_{54}^a = 0$  the transmitted wave has the same linear polarization with  $\mathbf{u}||\xi$  or  $\eta$ . At  $\Delta C_{54}^a \neq 0$ , similar to the case represented by (26) and (25), ellipticity emerges, and there is rotation of the polarization plane (the major axis of the ellipse), with both determined by the above formulas with the parameter  $A$  taken from (85) and  $\Delta k$  taken from (86). The CBR effect (against the background of LBR) is strongly dependent on the angle  $\varphi_B$ : it disappears at  $3\varphi_B = 0, \pm\pi$  and is at its maximum at  $3\varphi_B = \pm\pi/2$ . More than that, for the effect to be at its maximum,  $\Delta kd$  must be either  $\pi/2$  or  $\pi$  for the angle  $\Theta$  and the ellipticity  $a/b$ , respectively.

**3.3.3 Transmitted-sound oscillations in  $B$  in the EP state.** Since experiments have been carried out for the situation examined here, i.e., for the  $\bar{1}(+)3_z(+)2_x(-)$  structure and the state with  $\mathbf{L}||Y$ ,  $\mathbf{B}||\mathbf{M}||X$ , and  $\mathbf{k}||Z$  (see Ref. [42] for  $\text{MnCO}_3$  and Ref. [43] for  $\text{FeBO}_3$ ), we will focus on the power oscillations of the transmitted sound related to the dependence of  $\Delta k$  on  $B$ . Bearing in mind that in the given case, according to (86), (80), and (81), the  $\Delta k$ -vs.- $B$  dependence is nonlinear, it is convenient to write the oscillation period  $\Delta B$  from (76) in the

form

$$\Delta B = \frac{2\pi}{d} \left( \frac{\partial \Delta k}{\partial B} \right)^{-1}, \quad (87)$$

where  $\Delta B$  must still satisfy condition (78). The above small effects of ellipticity and CBR related to  $\Delta C_{54}^a$  can be ignored.

Suppose that at the entrance (at  $z = 0$ ) into a plate of thickness  $d$  the incident wave is linearly polarized, has an amplitude  $u_0$ , and a polarization vector  $\mathbf{u}$  making an angle  $\varphi_0$  with the  $\xi$  axis. It can be shown (see Ref. [6]) that at the exit from the plate ( $z = d$ ) the intensities of the wave with the same polarization as at the entrance ( $I_l$ ) and of the transversely polarized wave at an angle  $\varphi_0 + \pi/2$  ( $I_t$ ) are given by the following formulas:

$$\frac{I_t}{u_0^2} = \sin^2 2\varphi_0 \sin^2 \left( \frac{\Delta kd}{2} \right), \quad \frac{I_l}{u_0^2} = 1 - \frac{I_t}{u_0^2}. \quad (88)$$

Clearly, the oscillations are related to the factor  $\sin^2(\Delta kd/2)$ .

Condition (78), which specifies that the oscillations are exhibited with sufficient clarity, can be written as the condition imposed on the thickness  $d$  of the sample:

$$d \gg \frac{AC_{44}}{2B_{14}U}, \quad (89)$$

where, as the reader will recall,  $A$  is the sound wavelength. Note that, for a rough estimate, condition (89) is written in the approximation in which  $H_\Delta = 0$  in  $H_\Delta^2$  of (81).

Let us estimate  $d$  for  $\text{FeBO}_3$  using (89). Suppose that  $v \equiv \omega/2\pi = 200$  MHz,  $B = 400$  G,  $C_{44} = 9.2 \times 10^{11}$  erg cm $^{-3}$ ,  $B_{14} = 14 \times 10^6$  erg cm $^{-3}$ ,  $M_0 = 500$  G,  $H_E = 2.9 \times 10^6$  G,  $H_D = 10^5$  G, and  $v = 4.6 \times 10^5$  cm s $^{-1}$  (the temperature is 77 K and we use formula (80) for  $U$  [43, 44]). Condition (89) yields  $d \gg 0.4$  mm.

Let us briefly discuss the experiment conducted with  $\text{FeBO}_3$  in Korolyuk's laboratory in the Ukraine (see Refs [43, 44]). The above value  $B = 400$  G corresponds to the middle of the interval of field strengths used in that experiment. The thickness of the sample used, 1.2 mm, clearly does not agree very well with condition (89). Incidentally, in lower fields, e.g.,  $B = 100$  G, the situation is better, since (89) yields  $d \gg 0.1$  mm, but the researchers found that in these lower fields the agreement between the experimental results and those produced by formula (88) for  $I_t$  (which they were verifying) was the worst.

According to Mitsai et al. [44], the essence of the discrepancy between experiment (Ref. [43]) and theory (Ref. [44]) probably lies in the method by which the samples are mounted. In this method an additional magnetic anisotropy is induced in the basal plane  $XY$ , and this anisotropy is nonuniform over the sample thickness  $z$ . Bearing all this in mind, Mitsai et al. [44] were able to describe the experiment. In analyzing the experimental data, it is probably necessary to allow for the real crystal structure of the samples (their block nature) [45].

The situation with  $\alpha\text{-Fe}_2\text{O}_3$  proved to be somewhat more favorable. Using the Appendix to Ref. [46], which gives the values of the necessary parameters taken from the original papers (room temperature), e.g.,  $C_{44} = 8.5 \times 10^{11}$  G,  $2B_{14} = 27 \times 10^6$  erg cm $^{-3}$ ,  $H_E = 9.2 \times 10^6$  Oe,  $H_D = 22 \times 10^3$  Oe, and  $M_0 = 870$  G at  $\omega/2\pi = 200$  MHz and the field strength  $B = 2000$  G, we find for  $d$  in (89) that

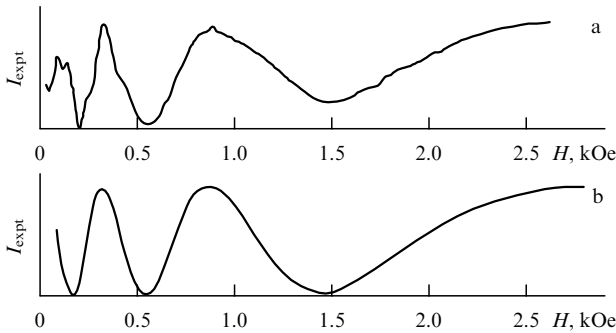
$$d \gg 0.2 \text{ mm}.$$

Hence, in fields  $B$  that are sufficiently high for saturation to set in, condition (89) is satisfied sufficiently well already for a sample  $d = 2$  mm thick.

As far as we know, an experimental study of oscillations in  $B$  of the transmitted sound intensity for hematite has yet to be conducted.

Gakel' [42] was the first to experimentally detect the oscillations of the amplitude of sound transmitted through an AF plate made of  $\text{MnCO}_3$  ( $T_N = 37$  K) as a function of the field  $B \perp 3 \parallel Z$ . Theoretical investigations made it possible for one of the present authors (E A T) [6] to quantitatively describe this experiment.

Figure 4 depicts the experimental and theoretical  $I_1(H)$  curve for a geometry corresponding to formula (88). (Unfortunately, the angle  $\varphi_0$  was not specified in Ref. [42]). The theoretical curve was built with the help of two fitting parameters, one of which was  $H_\Delta$  (81). In the estimates (89) of  $d$  given above for  $\text{FeBO}_3$  and  $\alpha\text{-Fe}_2\text{O}_3$ , it was assumed that  $H_\Delta = 0$ . Bearing in mind future studies of the effect in  $\text{FeBO}_3$  and especially in  $\alpha\text{-Fe}_2\text{O}_3$ , one should, of course, allow for the fact that  $H_\Delta \neq 0$  in the quantitative description of the experiment. Incidentally, the value of this quantity is known for these materials from other AFMR measurements. We repeat: the most favorable situation for such experiments probably involves  $\alpha\text{-Fe}_2\text{O}_3$ . The experiments can be conducted at room temperature and involve single-crystal plates 1 to 2 mm thick in fields of 2 to 10 kOe at frequencies in the 100 to 200 MHz range.



**Figure 4.** Intensity  $I(H)$  of the transmitted sound: (a) the experimental curve according to Ref. [42] and (b) the theoretical curve corresponding to formula (88).

**3.3.4 The EA state.** The above EP state with  $\mathbf{L} \perp Z$  at  $\mathbf{B} \perp Z$  is realized in hematite at temperatures higher than the Morin point ( $T > T_M$ ), so that we need only examine, in terms of the coupled equations (16) of magnetoelastic dynamics with allowance for (61) and (69), the EA state with  $\mathbf{L} \parallel Z$ . Suppose that again  $\mathbf{B} \perp Z$  and  $\mathbf{k} \parallel Z$ . This has been done within the symmetry approach in Section 2.6.2, so that now the problem reduces to finding the phenomenological constants  $\alpha_1$  and  $\beta_1$  in (48) and (49). To this end, we use equations (69), as we did in Section 3.2.1 for  $\text{CoF}_2$ , to express the magnetic dynamical variables  $\Delta l_i$  and  $\Delta m_j$  in terms of the elastic variables  $e_{ij}$  and  $\omega_{ij}$  and then to exclude the former from equations (16) and (68). We then compare the renormalized elastic constants  $\tilde{C}_{44}$ ,  $\tilde{C}_{55}$ , and  $\tilde{C}_{45}$  obtained in this way to (48) and thus find the constants  $\alpha_1$  and  $\beta_1$ . Here, we adopt the following approximations for

frequencies and fields:

$$\omega \ll \omega_{\text{AFMR}} \equiv \gamma H_0,$$

$$BH_D \ll H_0^2 = H_A(2H_E + H_A) - H_D^2. \quad (90)$$

The introduced effective field  $H_0$  determines the AFMR frequency as  $B \rightarrow 0$  in the situation at hand. As a result, for the sought constants we obtain

$$-\alpha_1 = \frac{H_E \tilde{B}_{44}^2}{M_0^3 H_0^2}, \quad \beta_1 = 4 \frac{H_E H_D \tilde{B}_{44} B_{14}}{M_0^2 H_0^4}, \quad (91)$$

where  $\tilde{B}_{44} = B_{44} + H_A M_0/2$  is the magnetoelastic constant renormalized because of local rotations  $\omega_{ij}$  which lead, with allowance for (67), to an energy term related to magnetic anisotropy,  $K_a = K_c = 2M_0 H_A$ , that is added to  $\Phi_m$  (62). [Here one must bear in mind the cyclic permutation of coordinates (33).] In this calculation, we also allowed for the fact that

$$L_z \simeq 2M_0 \left[ 1 - \frac{1}{2}(l_\perp^2 + m_\perp^2) \right] \approx 2M_0,$$

where

$$l_\perp \equiv \frac{L_\perp}{2M_0} = \frac{BH_D}{H_0^2} \quad \text{and} \quad m_\perp = \frac{M_\perp}{2M_0} = \frac{BH_A}{H_0^2} \quad (92)$$

are the relative projections of the AF and ferromagnetic vectors onto the basal plane [47]. These are small quantities, but their emergence in a field  $\mathbf{B} \perp Z$  points to the beginning of a phase transition of the type of rotation of  $\mathbf{L}$  about this field.

Using explicit formulas (91) for the constants  $\alpha_1$  and  $\beta_1$  and condition (61) at  $\beta = \beta_1$ , we can estimate the distance  $z = d$  over which one can observe the circular polarization in hematite at  $T < T_M$  and the rotation of the polarization plane by  $\pi/2$  [for which a condition of type (61) must be met]; both phenomena have been described in Section 2.6.3. Assuming, following Refs [47, 46], that  $H_A = 0.54$  kOe,  $H_D = 30$  kOe,  $H_0 = 63$  kOe,  $H_E = 4.51 \times 10^6$  Oe,  $M_0 = 870$  G,  $\tilde{B}_{44} \approx B_{44} = 26.5 \times 10^6$  erg cm $^{-3}$ , and  $2B_{14} \approx 27 \times 10^6$  erg cm $^{-3}$ , we obtain

$$d \approx \frac{3 \times 10^{12}}{vB}, \quad (93)$$

where, according to (90), the values of the field  $B$  must satisfy the inequality  $B \ll 132$  kG. Equation (93) shows that achieving circular polarization in a sample with  $d \approx 3$  mm requires the use of high frequencies  $v \approx 10^9$  Hz and sufficiently high fields  $B \approx 10$  kG, while at higher frequencies ( $v \approx 10^{10}$  Hz) the field strength can be reduced to  $B \approx 1$  kG.

In conclusion of this section, we will briefly touch on an AF effect predicted in Ref. [6] that manifests itself in the acoustics of crystals with a threefold symmetry axis and, in particular, with rhombohedral symmetry, such as  $\text{FeBO}_3$  and  $\alpha\text{-Fe}_2\text{O}_3$ . The thing is that in such crystals an acoustic beam sent along the axis 3 experiences, in the absence of antiferromagnetism, what is known as conical refraction (e.g., see Section 56 in Ref. [8]): the beam intensity is evenly distributed over a circular cone whose axis is the specified symmetry axis. What antiferromagnetism does here is that in the EP state conical refraction is transformed into bilinear refraction: instead of being conical, the beam splits into two beams

corresponding to two radial (group) velocities for two normal acoustic modes with phase velocities given by Eqn (83) with  $\mathbf{k} \parallel \mathbf{Z}$ . The latter are only projections of the radial velocities onto the  $Z$  axis. The divergence angle of these beams depends on the magnetic field strength, and for hematite may vary from  $10^\circ$  to  $16^\circ$  (in fields  $B$  in the 5 to 200 kG range).

#### 4. Centroantisymmetric EMS: effects of the ( $LE$ ) and ( $Lk$ ) types

##### 4.1 How to write the invariant AF contributions to the elastic constants?

As noted in Section 2.1, the magnetic symmetry group of CAS antiferromagnets possesses anti-inversion  $\bar{1}' \equiv \bar{1} \cdot 1'$ , where (as the reader will recall)  $1'$  is the time reversal operation:  $t \rightarrow -t$ . From the viewpoint of crystal symmetry elements, this means in the EMS code the inversion  $\bar{1}$  is an odd element,  $\bar{1}(-)$ . This suggests that in the AF parts  $\Delta\hat{C}^s$  (4) and  $\Delta\hat{C}^a$  (5) of all the linear-in- $\mathbf{L}$  (and generally odd) terms, the only invariant terms are ( $Lk$ ) in  $\Delta\hat{C}^s$  and ( $LE$ ) in  $\Delta\hat{C}^a$ , which determine the effects characteristic precisely of CAS antiferromagnets. Some analogs of these terms were first predicted and studied in optics (see Refs [19–21]). As the data listed in Table 1 and Section 2.1 imply, the effects of both types are nonreciprocal, with ( $Lk$ ) belonging to nongyrotropic phenomena, and ( $LE$ ), to gyrotropic phenomena.

What is interesting here is that a peculiar analog of the effect of the ( $LE$ ) type has been predicted in kinetics [48]. This is a nonlinear (quadratic in current  $J$ ) antiferromagnetic-electric Hall effect (the field  $E_\perp \propto LJ^2$  is transverse to the current) caused by antiferromagnetism (the vector  $\mathbf{L}$ ) and can exist in the absence of a magnetic field ( $\mathbf{B}$ ).

The effects in antiferromagnetoacoustics discussed below are related to the AF contributions to the elastic constants of the form

$$\Delta C_{\alpha\beta}^a = i\Delta C_{\alpha\beta}'' = ia_{\alpha\beta kn}(\omega)L_k E_n, \quad (94)$$

$$\Delta C_{\alpha\beta}^s = \Delta C_{\alpha\beta}' = b_{\alpha\beta kn}(\omega)L_k k_n, \quad (95)$$

which are determined from the criterion of invariance under crystallochemical symmetry transformations. Formulas (3), (8), and (9) (Onsager relations, hermiticity, and the requirement that the observables be real) imply that in the present case the tensors  $\hat{a}(\omega)$  and  $\hat{b}(\omega)$  we are interested in are, as functions of  $\omega$ , odd in  $\omega$  and vanish as  $\omega \rightarrow 0$ . Their form is again determined from the requirement that (94) and (95) be invariant under transformations of the crystallochemical group, the elements present in the EMS code. For instance, for  $\text{Cr}_2\text{O}_3$  (the space group  $R\bar{3}c \equiv D_{3d}^6$ ), this is  $\bar{1}(-)3_2(+)\bar{2}_x(-)$  (e.g., see the fairly recent paper [49] concerning the symmetry description of  $\text{Cr}_2\text{O}_3$  based on the crystallochemical approach). The procedure of directly establishing the form of the tensors  $\hat{a}$  and  $\hat{b}$  is extremely cumbersome (tensors of rank six); however, one can use tables that exist in the literature. For the terms  $L_k k_n$  in  $\Delta C_{\alpha\beta}^s$ , they are similar to those for the terms of type  $L_k B_n$  for EMS with a symmetry center  $\bar{1}(+)$  (except for  $\bar{1}$ , which have the same code) described in Sections 2.6 and 3.3. One should simply replace  $\mathbf{B}$  with  $\mathbf{k}$ .

As for the terms of type ( $LE$ ) in  $\Delta\hat{C}^a$  (5), they can be found by using the following simple approach. Their form (in symmetry) coincides with that obtained from terms linear in

the components of the vector  $\mathbf{M}$  if in them the  $M_i$  are replaced by

$$M_i^E = \lambda_{ijk} L_j E_k, \quad (96)$$

i.e., by the relation that determines the magnetoelectric effect (e.g., see Chapter 2 in Ref. [7]). This does not mean, however, that the effects of the ( $LE$ ) type (which are of interest to us) are directly related to the magnetization (96) caused by the electric field  $\mathbf{E}$ . In addition to this trivial channel for which the magnetoelectric effect is responsible, the contribution to  $\Delta\hat{C}^a$  obtained in this manner,

$$\Delta C_{\alpha\beta}^a = ia_{\alpha\beta jk}^M L_j E_k, \quad (97)$$

generally contains an independent effect (94) (the  $LE$  channel), which coincides with that just mentioned only in symmetry and whose specific mechanisms can be found only from the microscopic theory of this phenomenon.<sup>7</sup>

Below, when discussing the effects of both types, ( $LE$ ) and ( $Lk$ ), we present (without going into details) the corresponding terms in  $\Delta\hat{C}^a$  and  $\Delta\hat{C}^s$  for specific trigonal ( $\text{Cr}_2\text{O}_3$ ) and tetragonal (trirutiles  $\text{Fe}_2\text{TeO}_6$ , etc.) antiferromagnets of interest to us (discussions about their magnetic structure can be found, for instance, in Refs [51, 49]).

##### 4.2 Antiferromagnetic-electric (AF-E) CBR effect in the easy-axis state ( $\mathbf{L} \parallel \mathbf{Z}$ )

We begin with the  $\bar{1}(-)3_2(+)\bar{2}_x(-)$  and  $\bar{1}(-)4_2(+)\bar{2}_d(-)$  EMS with the even principal symmetry axes  $3_2(+)$  and  $4_2(+)$ , respectively, in the EA state ( $\mathbf{L} \parallel \mathbf{Z}$ ). This corresponds to the ground state of the oxide  $\text{Cr}_2\text{O}_3$  ( $T_N = 308$  K) and the trirutile  $\text{Fe}_2\text{TeO}_6$  ( $T_N = 210$  K).

Let  $\mathbf{k} \parallel \mathbf{E} \parallel \mathbf{Z}$ . Then the type ( $LE$ ) contribution to the relevant components of tensor  $t_{ij}$  (14) is given by the following relationships:

$$\begin{aligned} \Delta t_{zx} &= -\Delta t_{xz} = 2iaL_z E_z e_{yz}, \\ \Delta t_{yz} &= -\Delta t_{zy} = 2iaL_z E_z e_{zx}. \end{aligned} \quad (98)$$

The equations of motion (16) combined with (98) yield circularly polarized waves with wave numbers and polarization ratios of the type

$$k_\pm^2 = k_0^2 \left( 1 \pm \frac{a}{C_{44}} L_z E_z \right), \quad \left( \frac{u_x}{u_y} \right)_\pm = \pm i. \quad (99)$$

<sup>7</sup> Here, we are dealing with a situation where a general rule of the physics of AF phenomena manifests itself in the given case. We call this rule the *independent-channel rule* and formulate it as follows: if in statics there exists a certain AF effect, i.e., an effect related to vector  $\mathbf{L}$  (weak ferromagnetism (17), magnetoelectric effect (96), piezomagnetism, etc.), in kinetics, acoustics, and optics there always are, in addition to the trivial effect directly related to the specified phenomenon, independent channels (with independent constants) with the same symmetry. This was first discovered in the spontaneous (AF) Hall effect when the experiment described in Ref. [4] showed without doubt that in the Hall field, in addition to contributions proportional to  $\mathbf{B}$  and  $\mathbf{M}$ , there is an independent (and, as was found, principal) contribution proportional to  $\mathbf{L}$  (despite the relationship (17) between  $\mathbf{M}$  and  $\mathbf{L}$ ). A similar situation exists in magneto-optics. Krichevskiy et al. [50] found that the resulting Faraday effect can be described only if one allows for three independent contributions:  $B$ ,  $M$  and  $L$ . The rule, however, manifests itself most vividly in NMR in the problem of the effect of  $\mathbf{E}$  on the frequency spectrum. This effect provided by the channel related to terms of type  $L_i E_j$  in a hyperfine field yields results that differ dramatically from those provided by the trivial channel directly through  $\mathbf{M}^E$  (96). The first (compared to the second) even produces additional splitting of the NMR spectrum caused by  $\mathbf{E}$  [49].

The angle of the quasi-Faraday rotation (the antiferromagnetic-electric effect) referred to one wavelength ( $z = \lambda$ ) is

$$\varphi_A = \frac{1}{2} (k_+ - k_-) \lambda = \pi \frac{a}{C_{44}} L_z E_z. \quad (100)$$

We know of no attempts to detect this effect in experiments, although an analog in optics has been observed by Krichevtsov et al. [52].

#### 4.3 Antiferromagnetic-electric effect in the easy-plane state ( $\mathbf{L} \perp \mathbf{Z}$ )

We now turn to the  $\bar{\Gamma}(-)4_z(\pm)2_d(-)$  EMS in the EP state ( $\mathbf{L} \perp \mathbf{Z}$ ), and let  $\mathbf{E} \perp \mathbf{k} \parallel \mathbf{Z}$ . Calculations for these two cases ( $4_z(+)$  and  $4_z(-)$ ) produce somewhat different results.

For the  $\bar{\Gamma}(-)4_z(+ )2_d(-)$  structure, we have formulas (99) and (100) with the following substitution:

$$L_z E_z \rightarrow L_x E_x + L_y E_y. \quad (101)$$

This situation refers to the trirutile  $\text{Cr}_2\text{TeO}_6$  [51].

What we have just said is also true for the  $\bar{\Gamma}(-)4_z(-)2_d(-)$  structure, the only difference being that in (99) and (100) instead of (101) we must introduce the substitution

$$L_z E_z \rightarrow L_x E_y + L_y E_x. \quad (102)$$

It is important to note, however, that, in the real EP state to which (99) with the substitution (101) and (102) corresponds, the CBR effect mixes with an AF LBR effect of the ( $LL$ ) type. Only in the case of the EA state ( $\mathbf{L} \parallel \mathbf{Z}$ ) can formulas (99) and (100) be used in the original (pure) form.

#### 4.4 Nonreciprocal LBR effect in the easy-axis state of $\text{Cr}_2\text{O}_3$

The authors of the present review know of no data from the literature concerning the effects of spatial dispersion in acoustics (the more so if these effects are of the AF origin). Here, we give only one example of the possible effect of ( $Lk$ ) terms in  $\Delta\hat{C}^s$  (4) on LBR in  $\text{Cr}_2\text{O}_3$  in the ground state  $\mathbf{L} \parallel \mathbf{Z}$ , although there have been many discussions in optics concerning this problem (e.g., see the reviews in Refs [21, 22] and the book by one of the present authors [7]; in the latter, the acoustic effects of the ( $Lk$ ) type are also treated in greater detail).

Let  $\mathbf{k} \parallel \mathbf{X}$ . Here, the effect of the terms of type ( $Lk$ ) on the components  $t_{xy}$  and  $t_{xz}$  in the equations of elastic dynamics (16) reduces to renormalizing the elastic constants,

$$C_{44} \rightarrow C_{44} - b_2 L_z k_x, \quad C_{14} \rightarrow C_{44} + b_1 L_z k_x, \quad (103)$$

where  $k_x \approx k_0 s_x$ , and  $k_0$  is the natural wave number of the problem in the absence of the ( $Lk$ ) effect ( $b_1 = b_2 = 0$ ).

Fundamentally (and we believe only such an approach to be proper here), the natural modes produced by the elastic constants (103) are two transverse waves with polarization vectors  $\mathbf{u}_1 \perp \mathbf{u}_2$  lying in the  $YZ$  plane. The velocities of these waves and the angles that the vectors  $\mathbf{u}_1$  and  $\mathbf{u}_2$  make with the  $Y$  and  $X$  axes depend on the  $C_{44}$ -to- $C_{14}$  ratio. Hence, the renormalization (103) of these constants leads to a situation in which the velocity and the angles change somewhat under sign reversal of  $s_x = k_x/k$ . This constitutes the nonreciprocity effect.

And yet, by introducing one more approximation (if only to avoid complex formulas) we can see how this actually

happens. Precisely, let us put  $C_{14} = 0$ , bearing in mind that  $C_{14}$  is much smaller than  $C_{44}$  and  $C_{66}$ , which are other constants of the problem (e.g., for  $\alpha\text{-Fe}_2\text{O}_3$ ,  $C_{14}$  is almost ten times smaller than  $C_{44}$  and  $C_{66}$ ). In this approximation, equations (16) produce the following normal modes:

$$\begin{aligned} k_{x1}^2 &= \frac{\rho\omega^2}{C_{66}} & \text{for } \mathbf{u}_1 \parallel Y, \\ k_{x2}^2 &= k_{02}^2 \left( 1 + \frac{b_2}{C_{44}} L_z k_{02} s_x \right) & \text{for } \mathbf{u}_2 \parallel Z. \end{aligned} \quad (104)$$

In deriving (104), instead of  $|k_x|$  we took its value in the zeroth approximation  $|k_x| \equiv k_{02} = (\rho\omega^2/C_{44})^{1/2}$  and plugged it into the small term  $b_2 L_z k_x$  in equations (16) (as in (103)). Thus, in these approximations, the nonreciprocity manifests itself only in one of the acoustic modes corresponding to  $\mathbf{k} \parallel \mathbf{X}$ , the mode with polarization  $\mathbf{u} \parallel \mathbf{Z}$ , whose phase velocity

$$v_{(xz)} = \frac{\omega}{k_{x2}} \quad (105)$$

changes on sign reversal of  $s_x$ .

## 5. Conclusion

Notwithstanding the considerable size of this review, the authors, unfortunately, were unable to cover all the new and interesting aspects of magnetoacoustics that appeared in the last decade. Some of these aspects were only mentioned briefly with reference to the original work. Among such phenomena are, for instance, the transformation of conical refraction (in the absence of antiferromagnetism) into bilinear refraction (see the end of Section 3.3.4 and Ref. [6]). The problem requires not only further experiments but also theoretical research.

We did not mention, in connection with BR, the problem of acoustooptic diffraction of AF origin; in particular, we have not discussed a new mechanism of this diffraction related to acoustic modulation of polarized light [53, 54].

We also found no place in this review for the peculiar phenomena of acoustic BR near the antiferromagnetism–ferromagnetism phase transition point [55], where exchange–striction interaction plays a significant role and where, thanks to the increase in magnetization  $\mathbf{M}$ , magnetoacoustic activity manifests itself more vividly.

Finally, for understandable reasons (greater complexity and almost total lack of corresponding research) we did not study exchange–noncollinear magnetic structures (commensurable as well as noncommensurable). Still, one example comes to mind immediately: a highly noncollinear (‘quadratic’) structure characterized by an exchange doublet whose components differ in chirality (neodymium cuprate is a possible representative). The effect of chirality on BR has been studied in Ref. [56], whose results we were unable to present here.

And yet, despite all these gaps and the selective nature of the material of our review taken from the existing original work (with the addition of new results), the content of the review envelopes a broad set of different AF effects in acoustics corresponding to various crystal systems, exchange magnetic structures, orientation states, and geometries of the external magnetic and electric fields. As a result, even an interested reader will find it not easy to find the principal and most interesting topic for setting up a fairly simple, from our theoretical viewpoint, experiment that would promise the

discovery of a new effect. Therefore, we thought it proper to give in this final section a subjective list of topics that merits, in our belief, priority in experiments (we, of course, will in no way be surprised if the attentive reader makes his or her own list that may differ from ours). So, here is our choice.

(1) The spontaneous AF effect of acoustic activity (against the background of LBR of crystallochemical nature) in the orthoferrite  $\text{YFeO}_3$  in its ground state with  $\mathbf{L} \parallel \mathbf{X} \parallel \mathbf{a}$  and  $\mathbf{M} \parallel \mathbf{B} \parallel \mathbf{Z} \parallel \mathbf{c}$ . The new effect, in comparison to optics, occurs for a wave vector  $\mathbf{k} \parallel \mathbf{L} \parallel \mathbf{X}$ , when the longitudinal and transverse waves mix, so that there is polarization-plane rotation (25) and ellipticity in the plane with the vector  $\mathbf{k}$  ('the planar Faraday effect and ellipticity'). Since here and in what follows we mean effects that are linear in antiferromagnetism vector, a thermal treatment that abolishes the equal status of  $+\mathbf{L}$  and  $-\mathbf{L}$  must be introduced.

(2) A similar planar Faraday effect and ellipticity in hematite  $\alpha\text{-Fe}_2\text{O}_3$  at  $T > T_M = 260$  K in the situation where  $\mathbf{M} \parallel \mathbf{X} \parallel \mathbf{B}_\perp$  and  $\mathbf{Y} \parallel \mathbf{L}$  (see Fig. 2). The effect is isotropic (in contrast to the previous case) in the sense that in fields  $\mathbf{B}_\perp \perp \mathbf{Z}$  sufficiently high for overcoming basic anisotropy and removing the domain structure ( $B_\perp \gtrsim 2$  kG) and when  $\mathbf{L} \perp \mathbf{B}_\perp$ , the size of the effect does not vary with the change in the direction of  $\mathbf{B}_\perp$  (the angle  $\varphi_B$ ) and is still determined by formulas (25) and (26).

(3) Acoustic AF activity (circular BR) quadratic in  $\mathbf{B}$  in  $\text{CoF}_2$ ,  $\text{MnF}_2$  and the like ( $\mathbf{L} \parallel \mathbf{Z}$  and  $\mathbf{B} \perp \mathbf{Z}$ ) at  $\mathbf{k} \parallel \mathbf{Z}$ . (Note that here, in contrast to the ordinary Faraday effect, the effect is quadratic in  $\mathbf{B}$ , with this field being perpendicular rather than parallel to the wave vector  $\mathbf{k}$ ). The quasi-Faraday angle is given by formula (40). There is an optical analog, which has been detected experimentally. Theoretically, the effect also exists when  $B_x B_y$  is replaced by  $E_x E_y$  or  $e_{xy}$ .

(4) Linear BR in  $\text{CoF}_2$ ,  $\text{MnF}_2$ , etc. and also in hematite in the easy-axis state (i.e., at  $T < T_M$ ). In both cases  $\mathbf{k} \parallel \mathbf{L} \parallel \mathbf{Z}$ , but the directions of the magnetic field are different:  $\mathbf{B} \parallel \mathbf{Z}$  for  $\text{CoF}_2$ , etc., and  $\mathbf{B} \perp \mathbf{Z}$  for  $\alpha\text{-Fe}_2\text{O}_3$ . For different values of the parameters and magnetic field strengths, the following phenomena are possible: (a) transformation of a linearly polarized wave into a circularly polarized wave, and (b) the rotation of the polarization plane through  $90^\circ$  (or another fixed angle) (see the end of Section 2).

(5) Acoustic activity in hematite at  $T > T_M$  ( $\mathbf{L} \perp \mathbf{Z}$ ) related to piezomagnetism [the term  $\Pi_{xyz}$  in (82)] and local rotations in the rotation-invariant theory (the terms with  $H_D$  in (82)). The measured effects, the rotation angle  $\Theta$  and the ellipticity, are again given by formulas (25) and (26) with  $A$  taken from (85).

(6) Oscillations of the intensity of the transmitted sound as a function of the magnetic field strength  $B$ . (a) For  $\text{FeBO}_3$  it is advisable to use another method (different from that used by Korolyuk et al. [43]) of mounting the sample, so that to minimize as much as possible the nonuniform and anisotropic strains that constitute the instrumental factor. (b) For  $\alpha\text{-Fe}_2\text{O}_3$  in the EP state ( $\mathbf{L} \perp \mathbf{Z}$ ). The desirable parameters are: sample thickness  $d \approx 2$  mm, field strength  $B \equiv B_\perp \gtrsim 2$  kG, and sound frequency  $\nu \approx 100\text{--}200$  MHz.

(7) The antiferromagnetic-electric effect of acoustic CBR in the situation where  $\mathbf{k} \parallel \mathbf{L} \parallel \mathbf{E} \parallel \mathbf{Z}$  and the principal symmetry axis  $N_z$  is even. Examples are the rhombohedral crystal  $\text{Cr}_2\text{O}_3$  with  $N_z \equiv 3_z(+)$  and the tetragonal antiferromagnet trirutile  $\text{Fe}_2\text{TeO}_6$  with  $N_z \equiv 4_z(+)$ . The effect is caused by the electric field  $\mathbf{E}$  and exists in the absence of a magnetic field ( $B = 0$ ). However, a magnetic field is needed for thermal electromag-

netic treatment of the sample in order to remove the AF domain structure (domains with  $\pm L_z$ ). The size of the effect (the quasi-Faraday rotation angle) is determined by equation (100).

(8) The nonreciprocal effect of linear BR in  $\text{Cr}_2\text{O}_3$  caused by spatial dispersion related in a centroantisymmetric antiferromagnet to the antiferromagnetism vector  $\mathbf{L} \parallel \mathbf{Z}$  (the terms of type  $(Lk)$  in the AF part  $\Delta C^s$  of elastic constants). A fairly simple situation for an experiment:  $\mathbf{k} \parallel \mathbf{X} \parallel 2_x(-)$  (the odd twofold symmetry axis), and the polarization vector of elastic displacements in the wave  $\mathbf{u} \parallel \mathbf{Z}$ . The results are given by formulas (104) and (105).

We repeat: the proposed list of effects has in view only the beginning of research in the large and fairly new area of antiferromagnetoacoustics. Further development of this research is needed not only in experiments but also in theory (especially on the basis of appropriate microscopic models).

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