

METHODOLOGICAL NOTES

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Electric field strength of charged conducting balls and the breakdown of the air gap between them

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Contents

Abstract. **Field amplification factors at the surfaces of two charged conducting balls are calculated numerically. It is shown that as the balls are brought closer together, except when their potentials are equal, the amplification factors go to infinity, and in the case of like-charged balls the field at the surface of one of them changes sign. Breakdown field strengths for the air gap between balls of a different diameter are calculated using the experimental data of other authors as the base. The results suggest that the minimum breakdown field strength is 26 kV cm^{-1}. The author's earlier results on the interaction force between the balls are revised.**

1. Introduction

Electric field strength at the surfaces of two charged spherical conductors is in many respects an important characteristic of their state and interaction. We can mention numerous laboratory experiments on the spark breakdown in the air gap between conducting balls [1 – 4]. Usually, such experiments are designed to identify and measure only the breakdown voltage, although the strength of the electric field as its differential characteristic appears to be a more important physical quantity. It is clear, however, that its measurement and computation encounter a great deal of difficulty. Another application area of the problem of determining charged ball field strengths is related to the interaction between charged water drops in the air, which is critical for the formation of precipitation, thunderstorms, and other atmospheric phenomena [5 – 10]. Hence, comprehensive studies are necessary. Examples of well-known processes involving droplets cover electrization (charging), coagulation, fragmentation, corona discharge, and sparkover in the air gap between droplets. In this case it is important to know the role of different factors in the development of a given phenomenon. Evidently, the electric field at the drop surface is one of the main factors underlying processes that occur between the drops.

One frequently cited work [11] (see also Refs [8, 12, 13]) describes a method for the calculation of the field strength at the surface of two charged spherical particles placed in an external uniform electrical field and the electrostatic force of their interaction. This rather cumbersome method [11] is based on the solution of the Laplace equation for the potential in bispherical coordinates. In the present paper, as in earlier Refs [14, 15], a simpler method is applied to calculate interaction forces and field strength in the absence of an external field using the electrical image technique [16]. Also, it is worth noting that our computations described below show that some calculated results of work [11] prove to be incorrect for small separations between the balls, probably because of a large error in the numerical evaluation of sums of the series.

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2. Mathematical formulation of the problem

Let us consider two conducting spheres of radii R_1, R_2 and distance l between their centers, bearing charges q_1 and q_2 (see Fig. 1). Electrostatic induction results in that the charge of one sphere creates an electrical image (charge Q_{11}) in the other. This image is in turn a source of a secondary image q_{12} in the first sphere. It follows that the field strength at point A is generated by an infinite number of charge-images formed in the two spheres. If only the first ball is charged, the expression for the field strength at point A can be represented in the form

$$E_{A1} = k\left(\sum_{n=1}^{\infty}\frac{q_{1n}}{r_{1n}^2} - \sum_{n=1}^{\infty}\frac{Q_{1n}}{R_{1n}^2}\right), \qquad k = \frac{1}{4\pi\varepsilon_0}\,. \tag{1}$$

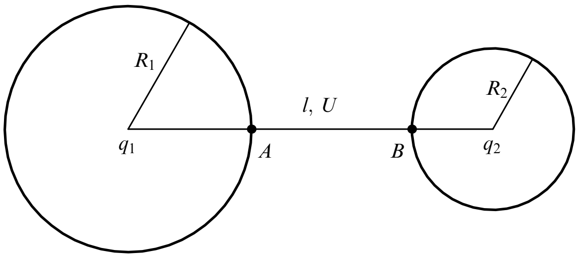

Figure 1. Layout of the ball arrangement.

Using the results of monograph [17], the following expressions can be written down:

$$q_{1n} = q_1 \frac{\gamma \sinh \beta}{\sinh n\beta \left[\gamma + \sinh (n-1)\beta / \sinh n\beta\right]} ,$$
$$Q_{1n} = -q_1 \frac{\gamma \sinh \beta}{r(1+\gamma) \sinh n\beta} , \tag{2}$$

$$r_{1n} = \frac{1}{1+\gamma} - r + \frac{r\gamma}{\gamma + \sinh (n-1)\beta/\sinh n\beta} , \tag{3}$$

$$R_{1n} = r - \frac{1}{1+\gamma} - \frac{r\gamma\left[\gamma + \sinh (n-1)\beta/\sinh n\beta\right]}{1+\gamma^2 + 2\gamma \cosh \beta} . \tag{4}$$

Here, $r = l/(R_1 + R_2)$, $\gamma = R_2/R_1$, and the parameter β is related to the distance between the ball centers by the equation

$$\cosh \beta = \frac{r^2(1+\gamma)^2 - (1+\gamma^2)}{2\gamma} . \tag{5}$$

Now, let the first ball be uncharged ($q_1 = 0$), and the second one bear charge q_2. Then, by analogy, the field strength at the point A is given by

$$E_{A2} = k\left(- \sum_{n=1}^{\infty} \frac{q_{2n}}{r_{2n}^2} + \sum_{n=1}^{\infty} \frac{Q_{2n}}{R_{2n}^2} \right) , \tag{6}$$

$$q_{2n} = q_2 \frac{\sinh \beta}{\sinh n\beta \left[1 + \gamma \sinh (n-1)\beta/\sinh n\beta\right]} ,$$
$$Q_{2n} = -q_2 \frac{\sinh \beta}{r(1+\gamma) \sinh n\beta} , \tag{7}$$

$$r_{2n} = -\frac{1}{1+\gamma} + \frac{r}{1+\gamma \sinh (n-1)\beta/\sinh n\beta} , \tag{8}$$

$$R_{2n} = \frac{1}{1+\gamma} - \frac{r\left[1 + \gamma \sinh (n-1)\beta/\sinh n\beta\right]}{1 + \gamma^2 + 2\gamma \cosh \beta} . \tag{9}$$

If the two balls have nonzero charges, the field strength at point A is

$$E_A = E_{A1} + E_{A2} = \frac{kq_1}{R_1^2} K_1 ,$$
$$K_1 = K_1(r, \gamma, \alpha), \qquad \alpha = \frac{q_2}{q_1} . \tag{10}$$

The field strength at point B (see Fig. 1) is calculated using the same formulas with substitutions $\alpha \to 1/\alpha$, $\gamma \to 1/\gamma$. Accordingly, it may be represented as

$$E_B = \frac{kq_2}{R_2^2} K_2 . \tag{11}$$

Quantities K_1 and K_2 found in this way are actually amplification factors of the proper field at each spherical surface. These factors were derived by computer-assisted calculations using the above formulas. In this way, all terms in the sums were expressed through the parameter $z = \exp(-\beta)$ [where β was defined by relation (5), as before]. Calculation of the sums was terminated as soon as the parameter z dropped to computer null, i.e., 10^{-38} in the routine calculation regime; check-up calculations were made in a double accuracy regime up to 10^{-80}.

3. Numerical results of field strength calculations

Special attention in calculations was drawn to two cases that appear to be most frequently realized in practice: the first case corresponds to equal potentials of the balls charged from a single voltage source, $\alpha = \gamma$, and the second relates to inductive ball charging when the charges on the balls are proportional to their radius squared, $\alpha = \gamma^2$ [18] (it should be recalled that $\alpha = q_2/q_1$, $\gamma = R_2/R_1$). Characteristic results of numerical calculations are presented in Fig. 2 depicting the dependences of amplification factors on the dimensionless distance between the ball centers. At equal ball potentials $\alpha = \gamma$, the field amplification factors monotonically vanish with decreasing distance between the balls (curve *1* in Fig. 2, $\alpha = \gamma = 1$). Physically, this is understandable because the potential difference between points A and B remains zero as they are brought closer together. In the remaining cases, the field strength at small interball distances increases, so that the amplification factors tend to infinity on approaching the balls. Hence, there is the possibility of a spark breakdown in the air gap between similarly charged balls, with the field strength at the surface of the smaller one being inward-directed toward its center (i.e., having a negative value) when the balls are positioned close enough and $\alpha > \gamma > 1$. In

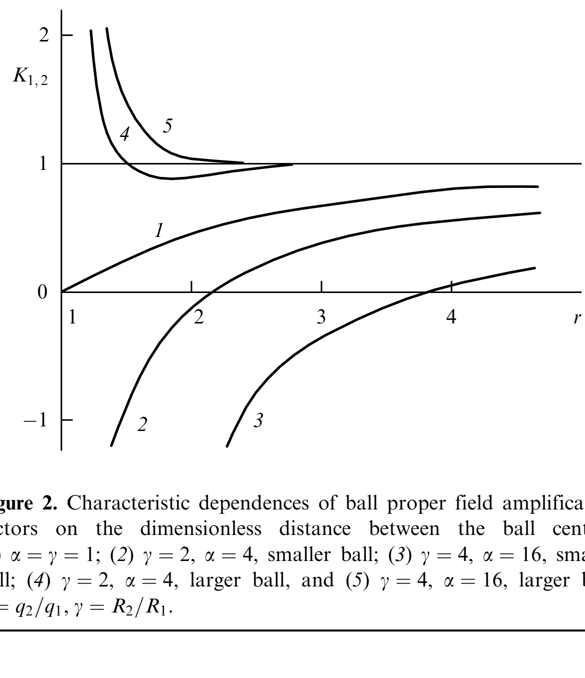

Figure 2. Characteristic dependences of ball proper field amplification factors on the dimensionless distance between the ball centers: (*1*) $\alpha = \gamma = 1$; (*2*) $\gamma = 2$, $\alpha = 4$, smaller ball; (*3*) $\gamma = 4$, $\alpha = 16$, smaller ball; (*4*) $\gamma = 2$, $\alpha = 4$, larger ball, and (*5*) $\gamma = 4$, $\alpha = 16$, larger ball; $\alpha = q_2/q_1$, $\gamma = R_2/R_1$.

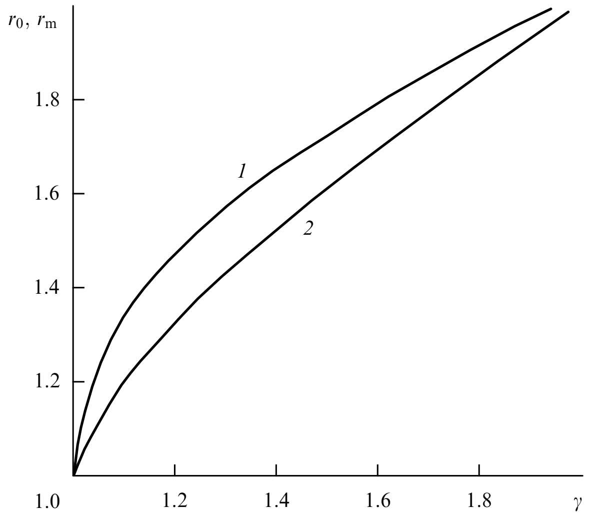

Figure 3. Plots of the dimensionless distance between the balls, at which the field amplification factor of the smaller ball vanishes (curve *1*), and the distance at which the field amplification factor of the larger ball passes through the minimum, versus the ball radius ratio.

Fig. 2, field amplification factors are plotted against distances between the ball centers [curves *2* ($\gamma = 2, \alpha = 4$), and *3* ($\gamma = 4, \alpha = 16$)]. Field strength at the larger ball surface is always directed outward (curves *4* and *5* in Fig. 2, also corresponding to $\gamma = 2, \gamma = 4$). Such a situation takes place in all cases when $\alpha > \gamma > 1$. In contrast, if $\gamma > \alpha > 1$, then the plots of field amplification coefficients versus distances between the ball centers become qualitatively opposite. Specifically, the field strength changes its sign at the larger ball surface, while that at the smaller ball surface retains its initial direction. Also, it can be seen that, when γ decreases down to unity, the plots of field amplification factors versus the interball distance tend to the dependence $K(r)$ for the case of $\alpha = \gamma = 1$. Figure 3 shows the dependence of the distance between the balls, at which the field amplification factor of the smaller one vanishes (curve *1*), and the same distance at which the field amplification factor of the larger one passes through the minimum (curve *2*), on the ratio between ball radii; here, $\alpha = \gamma^2$.

4. Spark breakdown

Experimental data on the spark breakdown in gases most frequently contain information about the breakdown voltage, and only rarely about the electric field strength near the electrodes, at which the breakdown begins to develop. In all probability, such a situation is due to two factors, one being the difficulty of field strength measurement in an experiment, the other the difficulty of its precise theoretical calculation at the electrode surface (except in the case of a plane electrode). Not infrequently, experiments on the spark breakdown in the air gap are carried out using identical spherical electrodes. Let us therefore consider two similarly sized but oppositely charged conducting balls of diameter D, maintained at potential difference U, with the distance between their centers l. The field strength at the ball surfaces could be calculated as before using formulas (1)–(11). However, a simpler formula may be used for the case under consideration, such as proposed in Ref. [17]. Using the accepted notation, the maximum field strength in the air gap between the balls (as attained at their surfaces) can be written in the form

$$E = E_0 \left[1 + \frac{2r+1}{(2r-1)^2} + \sinh\beta \sum_{n=1}^{\infty} \frac{1}{\sinh 2n\beta} \left(\frac{1 + r + \sinh\beta \coth 2n\beta}{(1 - r - \sinh\beta \coth 2n\beta)^2} + \frac{\cosh 2\beta + r + (2r+1)\sinh\beta \coth 2n\beta}{(\cosh 2\beta - r + (2r-1)\sinh\beta\coth 2n\beta)^2} \right) \right], \quad E_0 = \frac{U}{D}\,. \tag{12}$$

Here, U is the voltage between the balls, $r = l/D$ is the dimensionless distance between the ball centers, and the parameter β is related to r by the formula $\cosh\beta = r$.

Equation (12) was used to calculate breakdown field strengths in experiments conducted by different authors and described in Refs [1–3]. These experiments allowed breakdown field voltages to be determined at normal atmospheric pressure for air gaps of different sizes between pairs of identical balls having different diameters. The results of these calculations are presented in Fig. 4. The six solid circles correspond to the mean breakdown strengths computed from the experimental findings obtained using an alternating voltage of 60 Hz and reported in Ref. [3] (with British standards for breakdown voltage measurements in discharge gaps between balls). The results were averaged for each pair of balls based on the computations for different distances between them at the instant of breakdown. The cross mark stands for the mean breakdown strength obtained from Eqn (12) in similar experiments reported in Ref. [2]. All mean strength values were obtained with an error of less than 3% which was due to experimental scatter. Vertical line segments *1* and *2* depict the range of breakdown field strengths in experiments [3] where constant voltage was applied to balls 2 and 5 cm in diameter, respectively. Line section *3* shows the range of breakdown strengths in experiments with plane electrodes and constant voltage,

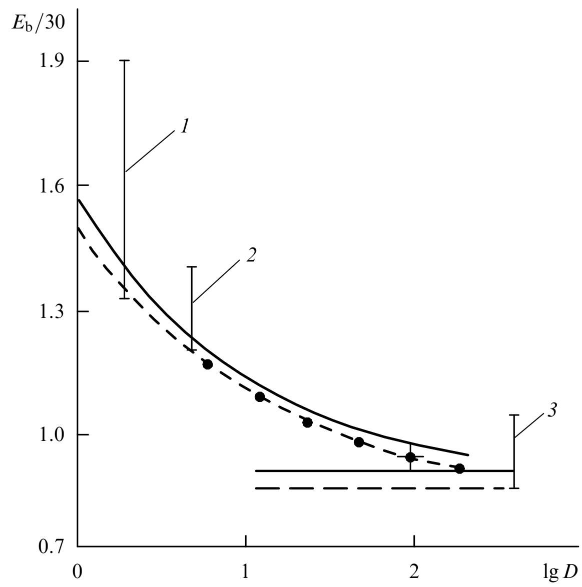

Figure 4. Dependence of the breakdown field strength for the air gap between balls on the logarithm of their radius (in centimeters) deduced from experimental data of different authors. Points and vertical line segments correspond to alternating and constant voltage, respectively.

reported in Ref. [1]. The lower ends of lines *1* and *2* correspond to the breakdown at a maximum separation of the balls. Figure 4 shows that the breakdown strength asymptotically tends to ~ 26 kV cm^{-1} with increasing ball diameter (horizontal dashed line); evidently, this value corresponds to the minimal breakdown field strength for plane electrodes.

The solid curve in Fig. 4 displays the dependence of the breakdown field strength on the logarithm of ball diameter, found from the known semiempirical formula [4]

$$E_{\mathrm{b}} = 27.2\left(1 + \frac{0.734}{\sqrt{D}}\right) \ \mathrm{kV\ cm^{-1}}\,. \tag{13}$$

The horizontal solid line corresponds to the minimum asymptotic value of 27.2 kV cm^{-1} (plane electrodes). Comparison of the results obtained using formulas (12) and (13) indicates that Eqn (13) yields somewhat overestimated breakdown strengths for balls of a relatively large and small diameter. The six experimental points corresponding to mean breakdown field strengths in Fig. 4 were utilized to find an interpolational polynomial having the form

$$\begin{aligned} \frac{E_{\mathrm{b}}}{30} &= 1.51 - 0.826\,t + 1.04\,t^2 - 0.912\,t^3 \\ &\quad + 0.377\,t^4 - 0.0563\,t^5\,, \\ 0 &\leqslant t \equiv \lg D \leqslant 2.3\,. \end{aligned} \tag{14}$$

Here, D is measured in centimeters as above. This polynomial is represented by the dashed curve in Fig. 4.

It is worthwhile to note here that the literature contains conflicting data on the minimal breakdown strength for the dry air gap under normal conditions. For example, graphs in reference book [19] suggest that the breakdown field strength for the air gaps between plane electrodes spaced 1 and 10 cm apart is 31 and 27 kV cm^{-1}, respectively. Raĭzer [1] estimated the minimum breakdown field strength at 26 kV cm^{-1}. Both the number and the quality of experimental data processed for the purpose of the present paper also give reason to accept ~ 26 kV cm^{-1} as the minimal breakdown field strength for the dry air gap under normal atmospheric pressure (horizontal dashed line in Fig. 4).

Line sections *1*, *2*, and *3* in Fig. 4 indicate that the breakdown strength increases with decreasing distance between the electrodes maintained at a constant potential difference. It may be supposed that this effect, absent in case of an alternating voltage (closed circles in Fig. 4) and significantly reduced after the removal of the electrodes, is related to the overlap in volume charge areas of closely spaced corona-producing electrodes, and as a consequence gives rise to a through current that tends to compensate for the potential difference between the electrodes [2]. When alternating voltage is applied, the corona effect, if any, is much smaller. Therefore, the breakdown strength is equal to the minimal one observed at a constant voltage, as in the case of maximally spaced electrodes.

It should be emphasized that the exact but complicated formula (12) for the calculation of the field strength at the surfaces of two identical spheres bearing charges of unlike signs may be substituted by Pick's approximate formula (see, for instance, Ref. [20])

$$E = \frac{E_0}{4S}\left(1 + 2S + \sqrt{(1+2S)^2 + 8}\right), \qquad S = r - 1\,. \tag{15}$$

It is noted in book [20] that Pick's formula holds at $0 < S < 1$. However, the comparison of the results of highly accurate numerical calculations using formulas (1)–(11) and (12) with those obtained by the Pick formula indicated that it yields fairly exact field strength values at the ball surfaces over the entire range of $0 < S < \infty$ with an error of less than 0.5%.

Let us now compare the results of our field strength computations and evaluations of interaction forces between the balls [14] with those reported in Ref. [11]. It should be emphasized from the very beginning that the results of all these calculations are qualitatively consistent. The largest quantitative difference arises at small distances between the balls, that is to say, where series of the types (1), (6), and (12) begin to poorly converge.

The minimal distance between ball centers calculated thus far is $r = 1.0005$ [11]. This value will be used for comparison. For oppositely charged identical balls with $\gamma = 1$, $\alpha = -1$ ($\alpha = q_2/q_1$, $\gamma = R_2/R_1$), formulas (1)–(11) and (12) yield the field amplification factor $K_1 = K_2 = 2000.812$, and Eqn (15) gives $K_1 = K_2 = 2000.667$ compared with $K_1 = K_2 = 423.400$ in Ref. [11]. For the same case, Refs [14] and [11] estimate the interaction force between the balls as $F/|F_{\mathrm{mC}}| = -89.449$ and -89.456, respectively (F_{mC} is the maximum force calculated in the Coulomb approximation, i.e., in the event of point charges at the ball centers). Comparable results of interaction force computations were also obtained in Refs [14] and [11] for balls with characteristics other than the above (the difference does not exceed tens of percent), while the results reported in Refs [15] and [11] differ severalfold.

5. Some remarks on the ball interaction force

Review article [14] presented a detailed investigation of interaction forces between charged spherical conductors spaced at different distances. In particular, it was noticed [14] that the interaction force at $\alpha = \gamma^2$[i.e., $q_2/q_1 = (R_2/R_1)^2$] is repulsive regardless of the distance between the spheres. Indeed, this inference comes from the plots of dimensionless force (measured in units of maximum force calculated in the Coulomb approximation) versus dimensionless distance between the ball centers (measured in units of $R_1 + R_2$ and calculated with a step of 0.02 on the axis r at different γ values) (Fig. 5). However, thorough calculations in later studies with as small a step as $\Delta r \leqslant 0.00001$ and a highly accurate evaluation of series (10^{-80}) have shown that the interaction force does change sign and turns into an attractive force at small interball distances (its dependence on the separation between the balls for such a case is qualitatively illustrated in Fig. 6). The area of attraction between the balls at any γ lies below the curve in Fig. 7 (r_0 is the distance between the balls at which the interaction force vanishes). Figure 8 displays force maxima and the relative position of the balls in which these maxima are attained. The arrow indicates the direction of rising γ. It is worthwhile to note that in the limit $\gamma \to \infty$ near the peak of the curve the situation corresponds to a charged ball and a practically uncharged material point; therefore, each of the forces F_{m} and F_{mC} tends to zero but their interaction is defined by Coulomb's law (the $F_{\mathrm{m}}/F_{\mathrm{mC}}$ ratio tends to unity).

Thus, the improvement of numerical calculations done in paper [14] leads to the conclusion that a case of identical similarly charged spheres ($\alpha = \gamma = 1$) is the only case in which like-charged conducting balls repulse each other regardless of the distance between them.

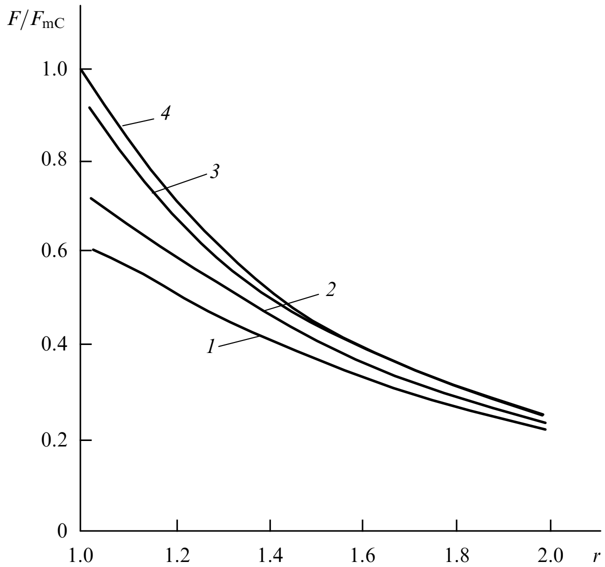

Figure 5. Plots of dimensionless force acting on each ball versus dimensionless distance between their centers for $\alpha = \gamma^2$ and different γ values calculated with a step $\Delta r = 0.02$: (*1*) $\gamma = 1.6$; (*2*) $\gamma = 6$; (*3*) $\gamma = 50$, and (*4*) the curve corresponding to the Coulomb interaction; $\alpha = q_2/q_1$, $\gamma = R_2/R_1$.

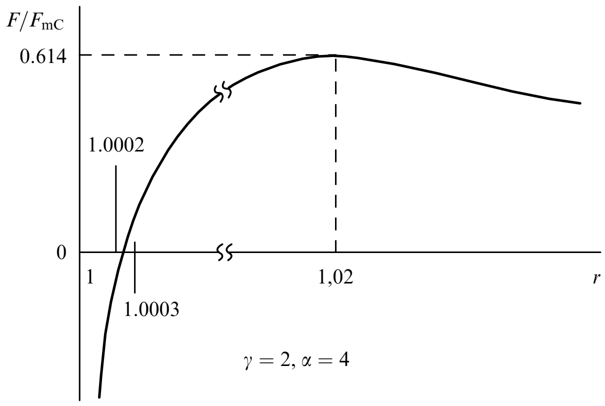

Figure 6. Qualitative aspect of the same dependences as shown in Fig. 5 but at small distances between the balls and being calculated with a step $\Delta r = 0.00001$.

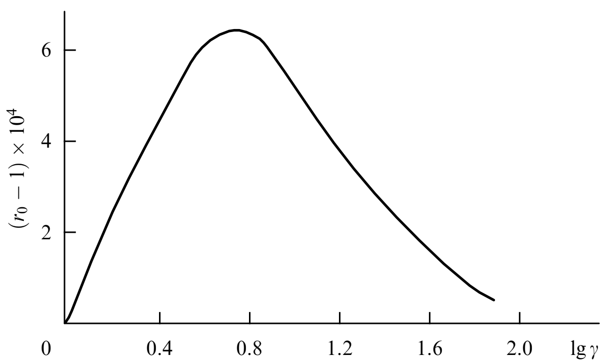

Figure 7. Range of parameter values under the curve corresponds to the case of attraction between like-charged balls for $\alpha = \gamma^2$; r_0 — dimensionless distance between the balls at which the interaction force changes sign; $\alpha = q_2/q_1$, $\gamma = R_2/R_1$.

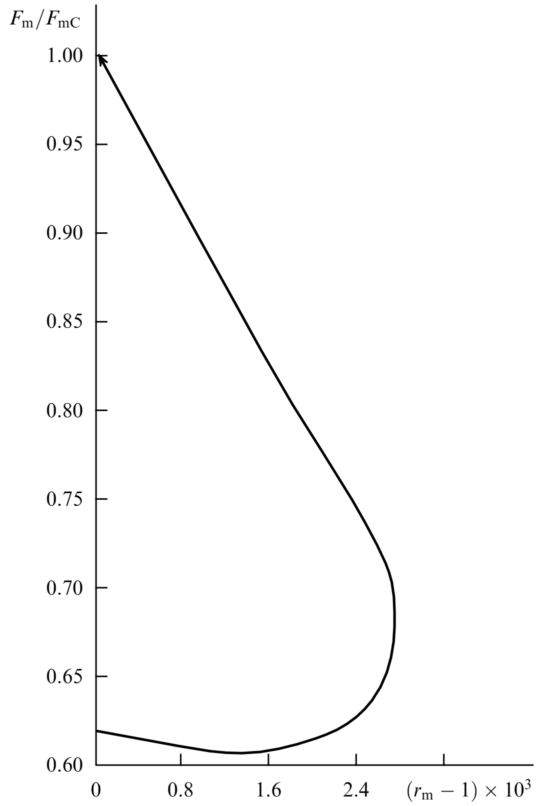

Figure 8. Points in the curve giving force maxima and their positions at different ball radius ratios γ; r_{m} — dimensionless distance between the balls at which the interaction force attains its maximum.

6. Conclusions

We have derived relations to evaluate proper-field amplification factors at the surface of each of the two closely spaced charged spherical conductors. Numerical calculations of the amplification factors have demonstrated that in the case of equal ball potentials $\alpha = \gamma$ these factors show qualitatively similar behavior regardless of their radius ratio γ, and that they monotonically decrease from 1 to 0 as the balls are brought closer together. In other cases of closely spaced similarly charged balls, amplification factors begin to grow infinitely, with the amplification factor of the smaller ball changing its sign at $\alpha > \gamma > 1$. In other words, the field strength at the surface of the smaller positively charged ball positioned close to the larger one is inward-directed towards its center. Conversely, when $1 < \alpha < \gamma$, bringing the balls closer together results in a change of sign of the amplification factor for the larger ball. Physically, it is quite understandable. Imagine, for example, similarly charged balls of markedly different radii ($\alpha < \gamma$). As such balls are brought closer together, the field strength at the smaller ball surface prevails and exceeds the proper field strength of the larger one. In such a situation, i.e., at a small separation between the balls bearing charges of like sign, there is a great probability of sparkover in the air gap between them.

Calculations of the electric field strength at the surfaces of identical but oppositely charged spheres indicate that Pick's approximate formula (15) can be used efficiently for the purpose. The numerical processing of experimental data reported by different authors revealed graphical and analytical dependences of a minimal breakdown field strength for the air gap between spheres on their radii. The asymptotic property of large radii allows for the conclusion that the minimum breakdown field strength for the dry air gap at normal atmospheric pressure is ~ 26 kV cm^{-1}.

It was concluded in Ref. [14], using the results of numerical computations, that the interaction force between similarly charged spherical conductors for which the condition $\alpha = \gamma^2$ is satisfied is repulsive, regardless of the distance between them. However, more accurate calculations presented in this paper are at variance with this conclusion. It turns out that at very small distances between the spheres [smaller than $7 \times 10^{-4}(R_1 + R_2)$], the interaction force changes sign and becomes attractive. Therefore, it may be inferred that a case of identical similarly charged spheres $\alpha = \gamma = 1$ is the only case where the interaction force between like-charged balls is repulsive, regardless of the distance between them.

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