 

*REVIEWS OF TOPICAL PROBLEMS*

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# Selected problems of supersymmetry phenomenology

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## Contents



**Abstract.** **Several problems of phenomenological supersymmetry are considered, including the hierarchy problem; gauge coupling unification; proton decay in Grand Unification Theory; Higgs boson spectrum and the upper bound on the lightest Higgs boson mass; and dark matter. The bounds on the superpartner masses and the possibility of their discovery using future accelerators are also discussed.**

## 1. Introduction

The term supersymmetry phenomenology sounds paradoxical as we can talk about phenomenology only if the relevant particle at least occurs in nature. Bosons and fermions that can exist in the same supermultiplet have not yet been found, unfortunately. There is a paucity of indirect evidence confirming the predictions of the phenomenological (or low-energy) supersymmetric theory. Nevertheless, the detailed phenomenology of new particles has been developed over the last twenty years, on the one hand, because the mathematics of the theory are so elegant and, on the other hand, driven by the hierarchy problem in the electroweak theory.

A large number of supersymmetric models have been put forward. Their predictions have been analyzed in detail. An intensive search for direct and indirect supersymmetry manifestations is being conducted in various experiments. One recent example is how the possible deviation of the anomalous magnetic moment of a muon from the value predicted by the Standard Model (SM) was immediately interpreted in dozen of studies as a manifestation of low-energy supersymmetry.

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The present review covers a very wide range. Several reviews discussing supersymmetry phenomenology were published in the mid-eighties [1 – 4]. The progress in the field has been fast and it would be hardly feasible to cover the field known as supersymmetry phenomenology in a single review paper.

This is why we have used an alternative approach by identifying several issues, which seem to be most interesting to us. In Section 2 we start with the Grand Unification theories because the numerical values of the gauge coupling constants $\alpha_3$, $\alpha_2$, and $\alpha_1$ are such that they are unified on a larger scale in the framework of a low-energy supersymmetry. This is the only area in which phenomenology is valid, indeed, as the experimentally measured $\alpha_i$ values at low energies plus running of constants in the supersymmetric model yield successful results, in contrast to the SM in the absence of supersymmetry.

The next issue we discuss in the paper is proton decay in the unified supersymmetry theories (Section 3). In the simplest Grand Unification theories the probability of proton decay proves to be too large and does not agree with the available experimental constraints. After the lowest limit for the mass of the lightest Higgs boson had gone up to 110 – 115 GeV the situation grew much worse as the minimal model with $\tan\beta \approx 1$ was ruled out.

Then we discuss (Section 4) the Higgs sector of the minimal supersymmetric model (MSSM). The low-energy supersymmetric model predicts a fairly small mass for the lightest Higgs boson and the theory is confronted with serious trouble because the Higgs boson was not observed at the LEP II accelerator — as the lower limit for $m_{\rm h}$ increases the range of possible values of the parameters in the MSSM shrinks swiftly. The recent indication on $m_{\rm h} \approx 115$ GeV would have saved the theory, therefore. Section 5 treats expansions of the minimal model that make it possible to increase the value of $m_{\rm h}$.

That the bulk of matter in the universe has a nonbaryonic origin is confirmed by a variety of observational and predicted evidence. The most popular candidate for the role is the lightest supersymmetric particle, which is absolutely stable in many models. Section 6 presents this aspect of the supersymmetry phenomenology. The concluding section of the review is devoted to the prospects for discovering superpartners at new accelerators.

## 2. Problem of hierarchies; Grand Unification and unification of the gauge constants

The electroweak theory successfully provides a description for the experimental data in the tree approximation. The radiative electroweak correction terms must be added in order to describe the parameters of the intermediate vector bosons measured with accuracy $10^{-3}$. The Higgs boson mass is the only free parameter of the theory. All the available measurements can be described satisfactorily with the use of the one-loop terms and leading two-loop correction terms [5, 6]; the fit quality is determined by the parameter $\chi^2/\text{n.d.f.} = 21.1/14$. The electroweak theory has a logical weakness in the Higgs sector, however.

The point is that in the theory the mass scale is fixed by the (negative) squared mass of the Higgs scalar field. The radiative correction term for this quantity diverges as the squared cut-off momentum $\Lambda$ (by the self-action of the Higgs field, and the gauge bosons and fermions). The coefficient at $\Lambda^2$ is the sum of the terms of the series in the coupling constants; for instance the gauge sector yields $(c_2 g^2 + c_4 g^4 + \ldots)\Lambda^2$. Therefore the bare mass must also include a term proportional to $\Lambda^2$ with a negative sign in addition to the final balance required for the generation of the 'observed' vacuum average Higgs field $\eta \approx 246$ GeV.

The scale of the expectation value of Higgs field $\eta$ is not the largest scale in physics, there exists the Planck scale $M_{\text{Pl}} \approx 10^{19}$ GeV; the gauge constants perhaps may be unified into a single one on a scale $M_{\text{GUT}} \approx 10^{16}$ GeV. If $\Lambda^2$ is regarded not as a formal mathematical parameter but as a number of the order of $M_{\text{GUT}}$ or $M_{\text{Pl}}$, then the loop contributions must be highly accurately compensated with the bare mass $(\eta/M)^2 \sim 10^{-28} - 10^{-34}$ (the situation is much worse for the formal quantity $\Lambda$ as it equals infinity). 'Incomplete compensation' results in drawing the expectation value of Higgs field $\eta$ and the masses of the W and Z bosons, quarks and leptons together with it to the Grand Unification scale or to the Planck scale $M_{\text{Pl}}$. This is the essence of the problem of hierarchies as presented back in the year 1976 [7].

It was suggested that the problem of hierarchies could be resolved with the use of theories which lack fundamental scalar fields where the mass of the gauge bosons is generated at the expense of the nonzero expectation value of product of the fields of new quarks, the so-called technicolor theories. These theories however cannot reproduce the results of the SM for the loop electroweak correction terms, which have been verified in experiments to a good accuracy — the additional correction terms given by the new quarks are too large. Meanwhile, the experimental results apply significant constraints on the possible existence of additional fermion generations [8].

The alternative approach is given by the supersymmetric theories. The Higgs boson acquires a fermion partner, referred to as higgsino. Their masses are identical owing to the supersymmetry (which is conserved when the radiative correction terms are taken into account). The fermion mass does not include correction terms with quadratic divergences as it contains only a logarithmic divergence,

$$m_{\text{f}} = m_{\text{f}}^0 \left[ 1 + \tilde{c}_2 g^2 \ln \frac{\Lambda}{m_{\text{f}}^0} + \tilde{c}_4 g^4 \ln^2 \frac{\Lambda_0}{m_{\text{f}}^0} + \ldots \right].$$

This is why the need for fine tuning disappears even for $\Lambda \sim M_{\text{Pl}}$.

All particles must have superpartners in order to prevent the appearance of a correction term with a quadratic divergence for the Higgs boson mass in the higher loops. This is how gauginos (photinos, gluinos, W and Z bozinos), sleptons, (or leptinos), and squarks (or quarkinos) are produced. In addition to the above particles, MSSM includes another additional Higgs doublet. A doublet pair is required both for the generation of the masses of the upper ($T_3 = +1/2$) and the lower ($T_3 = -1/2$) fermions in a way which does not roughly violate supersymmetry and for compensating the triangular anomalies (produced by the higgsino loops).

The quadratic divergence is eliminated in the diagrams in the following way: the loop with the gauge boson and the Higgs is compensated with the negative contribution of the loop with the gaugino and higgsino (the closed fermion loops contain the minus sign), the loop with quarks is compensated with a quarkino loop, and so on (Fig. 1). The masses of the superpartners must be exactly identical to achieve exact compensation but that does not happen in nature. It is allowed though to have a slight difference in the masses (on the $\Lambda$ scale). It should be introduced in such a way, however, that the equation for the mass shift must include the squared difference between the masses of the superpartners instead of $\Lambda^2$.

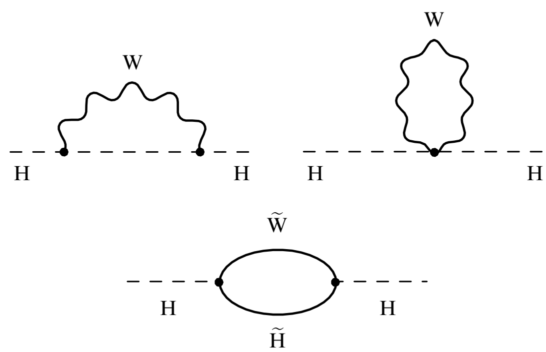


**Figure 1.** Compensation of the quadratic divergences in the gauge sector of the supersymmetric models.

Therefore, exact tuning is not required if

$$\frac{g^2}{16\pi^2}\,\Delta m^2 \lesssim \eta^2 ,$$

and we obtain the requirement, which is the most important one for the entire scenario, namely, that the masses of the superpartners should not be much larger than 1 TeV (the particles known at present have masses which are smaller at least by an order of magnitude). The fact that the highest-energy accelerators do not produce superpartners determines the lower limit of their possible masses. The quarkinos and gluinos are not seen at the Tevatron. Hence, they must have a mass greater than 200 – 300 GeV. The charged leptinos and gauginos which are not involved in strong interactions are not seen at the LEP II $e^+e^-$ collider and therefore they must be heavier than 100 GeV. The experimental constraints on the superpartner masses are discussed in detail in Ref. [9].

The parameters of the intermediate Z and W bosons measured with a high accuracy are highly sensitive to the contributions made by new particles owing to the radiative correction terms. If there were a large number of superpartners with masses of the order of the masses of the Z and W bosons we would have obtained large correction terms thus undermining the successful description of the experimental data provided by the SM [5, 6].

At the same time, the contributions of heavy particles are suppressed in the supersymmetric models: if the superpartner masses are greater than $M_Z$ their contribution decreases rapidly:

$$\delta \frac{M_W}{M_Z} \sim g^2 O\left(\frac{M_Z}{M_{SUSY}}\right)^2 .$$

In view of the direct constraints on the superpartner masses we can conclude that, essentially, their contribution to the radiative correction terms is negligibly small. One possible exception is the contribution of the stop quarks magnified as $m_t^4/(M_Z M_{SUSY})^2$ (for more detail, see, for instance, Ref. [10]).

The superpartners that cannot be observed because of their high masses change the evolution of the gauge coupling constants under virtual momenta that are greater than their masses. The gauge constants in the one-loop approximation 'run' according to the famous equation

$$\hat{\alpha}_i(M_{GUT}) = \frac{\hat{\alpha}_i(M_Z)}{1 - (b_i/2\pi)\,\hat{\alpha}_i(M_Z)\ln(M_{GUT}/M_Z)} , \qquad (1)$$

where $b_i$ are the coefficients of the Gell-Mann – Low function, the subscript $i = 1, 2, 3$ corresponds to the groups U(1), $SU(2)_L$, and $SU(3)_c$, and $\hat{\alpha}_i(\mu)$ are the constants in the subtraction scheme $\overline{MS}$ where the contribution of the particles with masses $m > \mu$ has been subtracted. (It should be recalled that the coefficients $b_i$ calculated within the $\overline{MS}$ scheme depend on all particles available in the theory, including the very heavy particles, and this fact presents significant problems for discussing Grand Unification theories. This is why a modification of the $\overline{MS}$ scheme has been put forward [11] which consists in subtracting the contribution of particles with masses greater than $\mu$.) Figure 2 illustrates the behavior of the gauge constants in the MSSM.

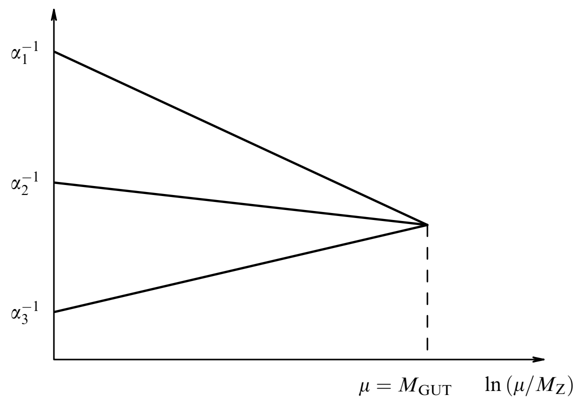


**Figure 2.** Gauge coupling constants in the one-loop approximation.

There may be two approaches to Eqn (1): one is to use the values of $\hat{\alpha}_i(M_Z)$ known from the 'low-energy' physics and to verify whether the three lines intersect at the same point. As $\hat{\alpha}_3(M_Z)$ is known with the poorest accuracy, the other approach is to determine $M_{GUT}$ as the point at which the values of $\hat{\alpha}_1$ and $\hat{\alpha}_2$ coincide and then to find $\hat{\alpha}_3(M_Z)$. A comparison of the calculated result with the experimental data demonstrates whether the theory has unification. This is why we shall use the second approach.

The initial values of $\hat{\alpha}_1(M_Z)$ and $\hat{\alpha}_2(M_Z)$ are given by

$$\hat{\alpha}_1(M_Z) = \frac{5}{3}\frac{\hat{\alpha}(M_Z)}{\hat{c}^2} , \qquad \hat{\alpha}_2(M_Z) = \frac{\hat{\alpha}(M_Z)}{\hat{s}^2} , \qquad (2)$$

where $\hat{\alpha}(M_Z)^{-1} = 127.93(6)$, $\hat{s}^2 = 0.2311(2)$ are the values [8][1] of the current fine structure constant and the electroweak mixing angle in the $\overline{MS}$ scheme with the subtracted particles whose masses are greater than $M_Z$.

For the coefficients $b_i$ we have

$$\begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} 0 \\ -6 \\ -9 \end{pmatrix} + N_H \begin{pmatrix} 3/5 \\ 1 \\ 0 \end{pmatrix} + 2N_G , \qquad (3)$$

where the first term is the contribution of the vector supermultiplet, the second term is the contribution of the chiral doublets of the Higgs bosons (here $N_H$ is the number of doublet pairs; in the MSSM $N_H = 1$), and the third term is the number of quark – lepton generations. As the quark – lepton generation gives rise to full SU(5) multiplets the number of generations does not affect the values of $M_{GUT}$ and $\hat{\alpha}_3(M_Z)$.

For $N_H = 1$ Eqns (1) – (3) yield

$$\hat{\alpha}_3(M_Z) = \frac{b_2 - b_1}{(3/5)\hat{c}^2(b_2 - b_3) + \hat{s}^2(b_3 - b_1)}\,\hat{\alpha}(M_Z)$$
$$= \frac{7}{15\hat{s}^2 - 3}\,\hat{\alpha}(M_Z) = 0.117(1) , \qquad (4)$$

which is in excellent agreement with the value derived from the total set of exact measurement data [6] (primarily from the Z boson decays): $\hat{\alpha}_s(M_Z) = 0.118(3)$.

In the case of the Standard (non-supersymmetric) Model we have

$$\begin{pmatrix} \tilde{b}_1 \\ \tilde{b}_2 \\ \tilde{b}_3 \end{pmatrix} = \begin{pmatrix} 0 \\ -22/3 \\ -11 \end{pmatrix} + \tilde{N}_H \begin{pmatrix} 1/10 \\ 1/3 \\ 0 \end{pmatrix} + \frac{4}{3} N_G , \qquad (5)$$

where $\tilde{N}_H$ is the number of Higgs boson doublets. For $\tilde{N}_H = 1$ we obtain

$$\tilde{\hat{\alpha}}_3(M_Z) = \frac{71}{135\hat{s}^2 - 24}\,\hat{\alpha}(M_Z) = 0.077 , \qquad (6)$$

which differs radically from the experimental results.

When we return to the supersymmetric model we shall note that, in addition to the leading logarithmic approximation, the two-loop additional terms have been determined [12, 13] which cause an approximately 10% increase in the value of $\hat{\alpha}_3(M_Z)$. In the same approximation we must take into account the fact that since the masses of the superpartners are higher than $M_Z$ (at least for most new particles) they enter the

[1] We employ here a more cautious estimate for the error in $\hat{\alpha}$ than the authors of Ref. [8].

'run' for higher virtualities (the so-called threshold effects). This results in a decrease in the value of $\hat{\alpha}_3(M_Z)$. The superpartners must have masses of TeV order [12] to obtain an agreement with the value of $\hat{\alpha}_3(M_Z) = 0.118(3)$.

## 3. Proton decay

We see that the only 'numerically' successful result of the supersymmetric models until now has been the unification of the gauge constants. Therefore, we inevitably arrive at the conclusion that unified (supersymmetric) theories are needed. The most vivid prediction of the unified theories is the nonconservation of the baryon and lepton charges and, as a consequence, the proton decay.

The mechanism of proton decay in the supersymmetric unified theories is the same as in the conventional unified theories, namely, an exchange of a heavy gauge boson with a mass equal to the unification mass $M_{\mathrm{GUT}}$. We obtain then the operator with dimensionality 6:

$$O_6 \approx \frac{1}{M_{\mathrm{GUT}}^2}(\mathrm{qq})(\mathrm{ql}) . \tag{7}$$

The numerical value of $M_{\mathrm{GUT}}$ is found from the identity $\alpha_1(M_{\mathrm{GUT}}) = \alpha_2(M_{\mathrm{GUT}})$:

$$\ln\frac{M_{\mathrm{GUT}}}{M_Z} = \frac{3-8\hat{s}^2}{14}\,\pi\big(\hat{\alpha}(M_Z)\big)^{-1} , \tag{8}$$
$$M_{\mathrm{GUT}} = 2\times 10^{16}\ \mathrm{GeV} .$$

For such a large unification mass the lifetime of the proton, which decays primarily via the channel $\mathrm{p}\to \mathrm{e}^+\pi^0$, was estimated at $\tau_{\mathrm{p}} \approx 10^{34}$ years; the figure is outside the range of experimental observations because of the background problems.

As was discovered by Weinberg, Sakai, and Yanagida [14, 15] almost twenty years ago, the supersymmetric models provided a new mechanism of proton decay by means of the exchange of heavy spinor partners of the heavy Higgs bosons which enter into the same multiplets of the Grand Unification group as the Higgs doublets giving mass to the particles of the SM. Violation of the baryon and lepton numbers in this mechanism is described by the operators which are quadratic in the fermion (quarks and leptons) and boson (quarkinos and leptinos) fields. These operators have a dimensionality of 5 and their form is

$$O_5 = \frac{1}{M_{\mathrm{GUT}}}(\mathrm{qq})(\tilde{\mathrm{q}}\tilde{\mathrm{l}}) . \tag{9}$$

The proton lifetime decreases significantly under these conditions because the decay probability is now proportional to $M_{\mathrm{GUT}}^{-2}$ (in contrast to $M_{\mathrm{GUT}}^{-4}$ in the case of the operator of dimensionality 6).

The proton decay proceeds via the one-loop diagram in which the scalar superpartners are transformed into quarks and leptons. Two types of operators of dimensionality 5 are possible:

$$O_5^L = \varepsilon_{abc} Q_L^a Q_L^b Q_L^c L_L ,$$
$$O_5^R = \varepsilon_{abc} (U_R^a)^* (U_R^b)^* (D_R^c)^* (E_R)^* . \tag{10}$$

Here the capital letters denote the chiral supermultiplets, and the subscripts $a$, $b$, and $c$ denote the color multiplets. The $F$ terms of the operators in Eqn (10) are reduced to operators of the type (9) expressed in terms of the spinor and scalar fields. Until recently it was assumed that the operators $O_5^L$ dominate, and therefore we shall start our analysis with them.

Identical quark fields are not allowed owing to the antisymmetry in color while the presence of the Higgs triplet enhances the contributions of the second and third generation quarks. Since the c and t quarks are not involved in proton decay their scalar superpartners propagate in the loop. Then two lower quarks exit from the loop resulting in the domination of the decay mode $\mathrm{p}\to\mathrm{K}^+\nu$. The contemporary experimental limit for this decay mode is $\tau(\mathrm{p}\to\mathrm{K}^+\nu) > 2\times 10^{33}$ years.

The decay amplitude is proportional to the product of the Yukawa coupling constants of the up and down fermions. For high values of $\tan\beta \equiv v_2/v_1$ (here $v_1$ and $v_2$ are the vacuum averages of the Higgs doublets) this proportionality results in an increase in the decay amplitude. The experimental constraint on the lifetime $\tau_{\mathrm{p}}$ can be satisfied within the framework of the minimal unified SU(5) theory for $\tan\beta \approx 3$ if we assume that the masses of the scalar quarks (which are inversely proportional to the decay amplitude) are large ($m_{\tilde{\mathrm{q}}} \gtrsim 10$ TeV) and that simultaneously the diagonal element of the mass matrix of the chargino is small ($m_{\widetilde{\mathrm{W}}} \lesssim 200$ GeV).

Even such extreme assumptions do not resolve the problem, however. The point is that the mass of the lightest Higgs boson for $\tan\beta \approx 1$ finds itself in the experimentally forbidden range (see Section 5). For $\tan\beta \gg 1$ the Higgs boson proves to be heavier. In particular, the $(b-\tau)$ unification (the identity of the masses of the b quark and the $\tau$ lepton on the Grand Unification scale) yields $\tan\beta \approx 50-60$ [16] and the probability of proton decay via the operator $O_5^L$ proves to be higher by a factor of approximately 100 than for $\tan\beta \approx 1$. This is not all, however.

It has been noted recently [17, 18] that for such high values of $\tan\beta$ the operator $O_5^R$ proves to be the dominant one because its contribution to the amplitude grows as $\tan^2\beta$ (in contrast to the operator $O_5^L$ whose contribution is proportional simply to $\tan\beta$). The ratio between the respective amplitudes is given by (see Fig. 3)

$$\frac{RRRR}{LLLL} \sim \frac{V_{\mathrm{ts}}}{V_{\mathrm{cd}}V_{\mathrm{us}}}\,\frac{\mu}{m_{\widetilde{\mathrm{W}}}}\,\frac{m_{\mathrm{b}}m_{\mathrm{t}}^2 m_{\mathrm{d}}}{g^2 m_\mu m_{\mathrm{c}} v_1 v_2} \approx \frac{\tan\beta}{4}\,\frac{\mu}{M_{\widetilde{\mathrm{W}}}} . \tag{11}$$

Here the probability of the decay $\mathrm{p}\to\mathrm{K}^+\nu$ via the operator $O_5^R$ for $\tan\beta\approx 40$ proves to be higher by approximately a factor of $10^4$ than the probability of decay via the operator $O_5^L$ for $\tan\beta \approx 3$ (even for $\mu/m_{\widetilde{\mathrm{W}}} \approx 1$).

The discovery of proton decay would have been the simplest way out of the current situation. The absence of proton decay can be explained if $\tan\beta$ proves to be small (this

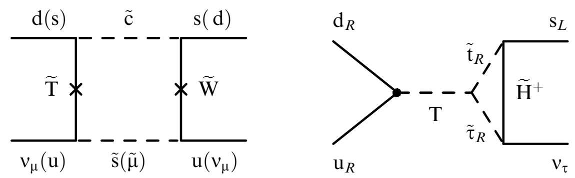


**Figure 3.** Diagrams describing proton decay via the operators of dimensionality 5: (a) $O_5^L$, (b) $O_5^R$.

opportunity is provided by the nonminimal Higgs sector for low energies; see Section 5) or the Higgs sector in the Grand Unification scale differs from the minimal version [2].

## 4. Higgs MSSM sector

At present the central area of study in the MSSM is its Higgs sector including two doublets of the Higgs fields: $H_1$ and $H_2$. The $H_1$ doublet generates the masses of the charged leptons and the 'down' quarks while the doublet $H_2$ provides the masses to the 'up' quarks.

The effective potential of the Higgs fields in the MSSM can be written as

$$V(H_1, H_2) = \mu_1^2|H_1|^2 + \mu_2^2|H_2|^2 - \mu_3^2(H_1 H_2 + \text{c.c.})$$
$$+ \frac{1}{8}\,\bar{g}'^2\left(|H_2|^2 - |H_1|^2\right)^2$$
$$+ \frac{1}{8}\,g^2(H_1^+\sigma_a H_1 + H_2^+\sigma_a H_2)^2 + \Delta V(H_1, H_2)\,, \qquad (12)$$

where $\Delta V(H_1, H_2)$ are the loop correction terms for the potential under consideration, and $g'$ and $g$ are the constants of the U(1) and SU(2) gauge interaction.

The effective potential (12) does not include unknown constants $\lambda_i$ of self-action of the Higgs fields. Its form is determined by the symmetry of the initial MSSM Lagrangian. In the limit of non-violated supersymmetry the first two terms in Eqn (12) are the contributions of the $F$ terms, there is no third term, and the next two terms are due to the contributions of the $D$ terms to the potential of interaction of the Higgs doublets. We have here $\mu_1^2 = \mu_2^2 = \mu^2$ where $\mu$ is the only dimensional parameter in the MSSM superpotential which is responsible for the mixing of superfields $\hat{H}_1$ and $\hat{H}_2$.

A soft violation of the supersymmetry causes a change in the mass terms of the Higgs fields in the MSSM Lagrangian. In addition, mixing between the Higgs doublets ($\mu_3^2 \neq 0$) is generated. As a result, the parameters $\mu_1$, $\mu_2$, and $\mu_3$ have the following relationships with the parameters of the soft supersymmetry violation:

$$\mu_1^2 = m_1^2 + \mu^2\,, \qquad \mu_2^2 = m_2^2 + \mu^2\,, \qquad \mu_3^2 = B\mu\,. \qquad (13)$$

The following identity is satisfied for the minimum choice of the fundamental parameters on the Grand Unification scale $M_{\text{GUT}}$: $m_1^2(M_{\text{GUT}}) = m_2^2(M_{\text{GUT}}) = m_0^2$.

In the MSSM the $\text{SU}(2)\times\text{U}(1)$ symmetry is violated spontaneously for $(\mu_3^2)^2 > \mu_1^2\mu_2^2$; the inequality is obviously satisfied if $\mu_1^2$ or $\mu_2^2$ prove to be smaller than zero. In contrast to the SM, however, it is not a necessary condition in the MSSM that the potential (12) must included negative $\mu_1^2$ or $\mu_2^2$ for the Higgs fields to acquire the nonzero expectation values. In addition, the MSSM develops an elegant mechanism of radiative violation of the gauge symmetry [19]: owing to the large mass of the t quark the parameters $\mu_1$ and $\mu_2$, which coincide in the scale $M_{\text{GUT}}$, differ significantly in the electroweak scale.

As a result of a spontaneous symmetry violation at the electroweak scale each of the doublets $H_1$ and $H_2$ acquires the vacuum average:

$$H_1 = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v_1\end{pmatrix}, \qquad H_2 = \frac{1}{\sqrt{2}}\begin{pmatrix}v_2\\ 0\end{pmatrix}. \qquad (14)$$

The case of a 'valley' when $v_1 = v_2$ or $v_1 = -v_2$ must be analyzed separately. Under such conditions the contribution of the $D$ terms to Eqn (12) vanishes and the potential of interaction of the Higgs fields proves to be limited from below only under the condition $2|\mu_3^2| < \mu_1^2 + \mu_2^2$. Otherwise, the MSSM does not have a stable vacuum state, that is, the minimum of the interaction potential (12) is obtained for $|v_1| = |v_2| \to \infty$.

The expectation values $v_1$ and $v_2$ are typically replaced in the analysis with their ratio and a sum of the squares:

$$\tan\beta = \frac{v_2}{v_1}\,, \qquad v^2 = \frac{1}{\sqrt{2}\,G_{\text{F}}} = v_2^2 + v_1^2 = (246\ \text{GeV})^2\,.$$

Here $G_{\text{F}}$ is the Fermi constant. Using two equations determining the minimum of the interaction potential of the Higgs fields ($\partial V/\partial v_1 = 0, \partial V/\partial v_2 = 0$) for the sum of squared expectation values of the Higgs fields $v^2$ and the angle $\beta$ we obtain

$$\sin 2\beta = \frac{2\mu_3^2}{\mu_1^2 + \mu_2^2 + \varDelta_\beta}\,,$$
$$M_Z^2 = \frac{g^2 + g'^2}{4}\,v^2 = \frac{2(\mu_1^2 - \mu_2^2\tan^2\beta + \varDelta_Z)}{\tan^2\beta - 1}\,. \qquad (15)$$

The loop correction terms $\varDelta_\beta$ and $\varDelta_Z$ in Eqn (15) are given by

$$\varDelta_\beta = \frac{2}{v^2\tan 2\beta}\,\frac{\partial\Delta V}{\partial\beta} + 4\,\frac{\partial\Delta V}{\partial(v^2)}\,,$$
$$\varDelta_Z = \frac{1}{\cos^2\beta}\left\{2\,\frac{\partial\Delta V}{\partial(v^2)}\cos 2\beta - \frac{1}{v^2}\,\frac{\partial\Delta V}{\partial\beta}\sin 2\beta\right\}. \qquad (16)$$

The spectrum of the Higgs MSSM sector consists of three neutral particles (two CP-even and one CP-odd), and one charged particle. The determinants of the mass matrices ($2\times 2$) of the CP-odd and charged Higgs bosons vanish, corresponding to the appearance of Goldstone bosons $\eta^0$ and $\eta^\pm$ in the theory spectrum. The latter are the linear superpositions of the CP-odd ($\text{A}_1 = \sqrt{2}\,\text{Im}\,\text{H}_1^0$, $\text{A}_2 = \sqrt{2}\,\text{Im}\,\text{H}_2^0$) and charged components of the Higgs doublets:

$$\eta^0 = \text{A}_1\cos\beta + \text{A}_2\sin\beta\,,$$
$$\eta^\pm = (\text{H}_1^-)^*\cos\beta + \text{H}_2^+\sin\beta\,, \qquad (17)$$

which are absorbed by the vector Z and $\text{W}^\pm$ bosons under a spontaneous violation of the gauge symmetry.

The equations for the masses of the other two Higgs bosons (the CP-odd boson $m_{\text{A}}$ and the charged boson $m_{\chi^\pm}$), which are linear combinations orthogonal to Eqns (17), have been derived in Refs [20–25] and [23–26], respectively, where the one-loop correction terms have been taken into consideration. They have the following form:

$$m_{\text{A}}^2 = \mu_1^2 + \mu_2^2 + \varDelta_{\text{A}}\,,$$
$$m_{\chi^\pm}^2 = m_{\text{A}}^2 + m_{\text{W}}^2 + \varDelta_{\chi^\pm}\,. \qquad (18)$$

[2] D I Kazakov has noted that a nonminimal Higgs sector is required for reaching an agreement between the theoretical predictions and the experimental constraints on the probability of the proton decay $\text{p}\to\text{K}^+\nu$.

Here $\Delta_{\rm A}$ and $\Delta_{\chi^\pm}$ represent the contributions of the loop correction terms.

A significant distinctive feature of the supersymmetric models is the presence of the light Higgs boson in the CP-even sector. The CP-even mass matrix of the Higgs MSSM sector has the simplest form in the basis of the fields:

$$\begin{aligned} \chi_1 &= h_1\cos\beta + h_2\sin\beta\,, \\ \chi_2 &= -h_1\sin\beta + h_2\cos\beta\,, \end{aligned} \tag{19}$$

where $h_1 = \sqrt{2}\,{\rm Re}\,{\rm H}_1^0$ and $h_2 = \sqrt{2}\,{\rm Re}\,{\rm H}_2^0$. The dependence on the constants $\mu_1^2$, $\mu_2^2$, and $\mu_3^2$ whose values are determined by the scale of supersymmetry violation and can vary over a fairly wide range, enters only into the matrix element $M_{22}^2$ in this field basis:

$$M_{ij}^2 = \begin{pmatrix} M_Z^2\cos^2 2\beta + \Delta_{11} & -\dfrac{1}{2}M_Z^2\sin 4\beta + \Delta_{12} \\ -\dfrac{1}{2}M_Z^2\sin 4\beta + \Delta_{12} & m_{\rm A}^2 + M_Z^2\sin^2 2\beta + \Delta_{22} \end{pmatrix}. \tag{20}$$

The loop correction terms $\Delta_{ij}$ for the CP-even Higgs MSSM sector have been analyzed in Refs [20–25, 27–34]. They are expressed in terms of the partial derivatives of $\Delta V(H_1, H_2)$ with respect to $v$ and $\beta$:

$$\begin{aligned} \Delta_{11} &= 4v^2\,\frac{\partial^2\Delta V}{\partial(v^2)^2}\,, \\ \Delta_{12} &= 2\left(\frac{\partial^2\Delta V}{\partial\beta\,\partial(v^2)} - \frac{1}{v^2}\,\frac{\partial\Delta V}{\partial\beta}\right), \\ \Delta_{22} &= 4\,\frac{\partial\Delta V}{\partial(v^2)} + \frac{1}{v^2}\,\frac{\partial^2\Delta V}{\partial\beta^2} - \Delta_{\rm A}\,. \end{aligned} \tag{21}$$

The squared masses of the CP-even Higgs bosons are determined as

$$m_{\rm H,h}^2 = \frac{1}{2}\left(M_{11}^2 + M_{22}^2 \pm \sqrt{(M_{11}^2 - M_{22}^2)^2 + 4M_{12}^4}\right), \tag{22}$$

where $M_{11}^2$, $M_{22}^2$, and $M_{12}^2$ are the matrix elements (20). If the mass of the CP-odd Higgs boson is small ($m_{\rm A}^2 \ll M_Z^2$) in the tree approximation the mass of the lightest CP-even Higgs boson is $m_{\rm h}^2 \leqslant m_{\rm A}^2\cos^2 2\beta$. As the scale of supersymmetry violation increases the mass of the lightest Higgs boson in the CP-even sector grows and for $m_{\rm A}^2 \gg M_Z^2$ approaches its upper theoretical limit $M_{\rm Z}|\cos 2\beta|$ which coincides with the matrix element $M_{11}^2$.

We see thus that the mass of one of the CP-even Higgs bosons is always smaller than $m_{\rm A}$. The trace of the mass matrix (20) is not changed as a result of the unitary transformations and hence we have $m_{\rm h}^2 + m_{\rm H}^2 = m_{\rm A}^2 + M_Z^2$. Since the mass of the lightest Higgs boson $m_{\rm h} \leqslant M_{\rm Z}|\cos 2\beta|$ the mass $m_{\rm H}$ of the heavier CP-even Higgs boson is always greater than $m_{\rm A}$.

In the tree approximation the upper limit of the mass of the lightest Higgs boson in the MSSM has been determined in Ref. [35]. When the loop correction terms for the effective potential of interaction of the Higgs fields are taken into consideration the spectrum of the Higgs bosons is not changed qualitatively but the upper limit of $m_{\rm h}$ in the MSSM is significantly increased:

$$m_{\rm h} \leqslant \sqrt{M_Z^2\cos^2 2\beta + \Delta_{11}}\,. \tag{23}$$

Here the loops containing the t quark and its superpartners make the main contribution to $\Delta_{ij}$.

In the supersymmetric models the t quark has two scalar superpartners, namely, the right one $\tilde{\rm t}_R$ and the left one $\tilde{\rm t}_L$ with the masses $m_{\rm Q}$ and $m_{\rm U}$, respectively. These states are mixed as a result of a spontaneous symmetry violation in the electroweak scale giving rise to two charged scalar particles with the masses $m_{\tilde{\rm t}_1}^2$ and $m_{\tilde{\rm t}_2}^2$:

$$\begin{aligned} m_{\tilde{t}_1,\tilde{t}_2}^2 &= \frac{1}{2}\left[m_{\rm Q}^2 + m_{\rm U}^2 + 2m_{\rm t}^2 \pm \sqrt{(m_{\rm Q}^2 - m_{\rm U}^2)^2 + 4m_{\rm t}^2 X_{\rm t}^2)}\,\right], \\ X_{\rm t} &= A_{\rm t} + \frac{\mu}{\tan\beta}\,, \qquad m_{\rm t} = \frac{1}{\sqrt{2}}\,h_{\rm t} v\sin\beta\,. \end{aligned} \tag{24}$$

Here $A_{\rm t}$ is the constant of interaction between $\tilde{t}_R$ $(\tilde{t}_L, \tilde{b}_L)$ and the Higgs doublet $\rm H_2$ which violates the supersymmetry.

Since the masses $m_{\tilde{\rm t}_1}^2$ and $m_{\tilde{\rm t}_2}^2$ must be positive, the mixing between the left and right superpartners of the t quark, which is determined by the value of $X_{\rm t}$, cannot be too high:

$$X_{\rm t}^2 < \frac{(m_{\rm Q}^2 + m_{\rm t}^2)(m_{\rm U}^2 + m_{\rm t}^2)}{m_{\rm t}^2}\,.$$

Otherwise, the squark fields acquire nonvanishing expectation values while gluons and photons become massive.

The contribution of the one-loop correction terms induced by the t quark and its superpartner to the effective potential of interaction of the Higgs fields can be expressed only in terms of their masses:

$$\begin{aligned} \Delta V({\rm H}_1, {\rm H}_2) = \frac{3}{32\pi^2}\Bigg[& m_{\tilde{t}_1}^4\left(\ln\frac{m_{\tilde{t}_1}^2}{q^2} - \frac{3}{2}\right) \\ &+ m_{\tilde{\rm t}_2}^4\left(\ln\frac{m_{\tilde{\rm t}_2}^2}{q^2} - \frac{3}{2}\right) - 2m_{\rm t}^4\left(\ln\frac{m_{\rm t}^2}{q^2} - \frac{3}{2}\right)\Bigg]\,, \end{aligned} \tag{25}$$

where $q^2 \approx m_{\rm t}^2$.

The sum of the one-loop and two-loop correction terms for the upper limit for the mass of the lightest Higgs boson was described with the following formula derived in the leading logarithmic approximation in Ref. [21]:

$$\begin{aligned} \Delta_{11} \approx{}& \frac{3}{2\pi^2}\,\frac{m_{\rm t}^4}{v^2} \\ &\times\left[\frac{1}{2}\,U_{\rm t} + L + \frac{1}{16\pi^2}\left(3\,\frac{m_{\rm t}^2}{v^2} - 32\pi\alpha_3\right)(U_{\rm t}L + L^2)\right] \\ &- \frac{3}{4\pi^2}\,\frac{m_{\rm t}^2}{v^2}\,(M_{\rm Z}\cos 2\beta)^2 L\,, \end{aligned} \tag{26}$$

where

$$L = \ln\frac{M_{\rm S}^2}{m_{\rm t}^2}\,, \quad U_{\rm t} = \frac{2X_{\rm t}^2}{M_{\rm S}^2}\left(1 - \frac{X_{\rm t}^2}{12M_{\rm S}^2}\right), \quad M_{\rm S} = \sqrt{m_{\tilde{t}_1}m_{\tilde{t}_2}}\,.$$

The two first terms in the square brackets in Eqn (26) represent the contribution of the one-loop correction terms which reaches its highest value for $X_{\rm t} = \pm\sqrt{6}M_{\rm S}$ when $U_{\rm t} = 6$. Equation (26) demonstrates that the loop correction terms for $m_h$ are proportional to $m_t^4$; they logarithmically depend on the masses of the superpartners of the t quark and are practically independent of the choice of $\tan\beta$. The sum of the one-loop and two-loop correction terms for $m_h^2$ is of the order of $M_Z^2$ in magnitude.

As follows from Eqn (23) the upper bound on the mass of the lightest Higgs boson in the MSSM is largely determined by $\tan\beta$. The bound grows with an increase in $\tan\beta$ and for $\tan\beta \gg 1$ it may be as high as 125 – 128 GeV in the realistic supersymmetric models with $M_S \leqslant 1000$ GeV.

## 5. Mass of the lightest Higgs boson

When we start discussing possible values of the mass of the lightest Higgs boson in the MSSM and its extensions we must note that in a number of realistic scenarios the mass proves to be much lower than the absolute upper limit.

From the viewpoint of the renormalization group analysis in the MSSM the simplest case corresponds to small values of $\tan\beta$, that is, $\tan\beta \ll 50-60$ when the Yukawa constants $h_b$ and $h_\tau$ of the b quark and $\tau$ lepton can be ignored. Under such conditions an analytic solution of the system of equations of the renormalization group in the MSSM can be found in Ref. [37]. Then the boundary conditions are specified on the Grand Unification scale $M_{GUT}$.

The solution of the renormalization group equations for the Yukawa constant of the t quark has the form

$$Y_t(t) = \frac{E(t)}{6F(t)}\left(1 + \frac{1}{6Y_t(0)\,F(t)}\right)^{-1}, \qquad F(t) = \int_0^t E(t')\,dt', \tag{27}$$

$$E(t) = \left[\frac{\tilde{\alpha}_3(t)}{\tilde{\alpha}(0)}\right]^{16/9}\left[\frac{\tilde{\alpha}_2(t)}{\tilde{\alpha}(0)}\right]^{-3}\left[\frac{\tilde{\alpha}_1(t)}{\tilde{\alpha}(0)}\right]^{-13/99},$$

where

$$Y_t(t) = \frac{h_t^2(t)}{(4\pi)^2}, \qquad \tilde{\alpha}_i(t) \equiv \frac{\hat{\alpha}_i(t)}{4\pi} = \frac{g_i^2(t)}{(4\pi)^2}, \qquad t = \ln\frac{M_{GUT}^2}{q^2}.$$

On the electroweak scale when $t = t_0 = 2\ln(M_{GUT}/M_t^{pole})$ for $h_t^2(0) \geqslant 1$ the second term in the denominator of the expression for $Y_t(t)$ proves to be much smaller than unity:

$$\frac{1}{6Y_t(0)\,F(t)} \approx \frac{1}{10h_t^2(0)}.$$

This is the reason why the dependence of $Y_t(t)$ on the initial conditions is weak and the solutions of the equations of the renormalization group tend to the quasi-fixed point $Y_t^{QFP}(t_0)$ [38]:

$$Y_t^{QFP}(t_0) \approx \frac{E(t_0)}{6F(t_0)} \approx \frac{(1.26)^2}{(4\pi)^2}. \tag{28}$$

Together with the Yukawa constant of the t quark reaching the infrared quasi-fixed point as $Y_t(0)$ increases, it is also reached by the solutions of the equations of the renormalization group for the corresponding trilinear constant $A_t$ of coupling for the scalar fields and the combination of masses of the scalar particles

$$\mathfrak{M}_t^2 = m_Q^2 + m_U^2 + m_2^2,$$

in which $m_Q$ and $m_U$ are the masses of the superpartners of the doublet of the left and right upper quarks of the third generation and $m_2^2$ determines the parameter $\mu_2^2$ [see Eqn (13)] in the interaction potential (12) of the Higgs fields.

An analytic solution for $A_t(t)$ and $\mathfrak{M}_t^2$ can be written in the form

$$A_t(t) = A_t(0)\,\frac{\epsilon_t(t)}{E(t)} + M_{1/2}\left[\frac{tE'(t)}{E(t)} - \frac{tE(t) - F(t)}{F(t)}\left(1 - \frac{\epsilon_t(t)}{E(t)}\right)\right], \tag{29}$$

$$\begin{aligned}\mathfrak{M}_t^2(t) = {} & \left[\mathfrak{M}_t^2(0) - A_t^2(0)\right]\frac{\epsilon_t(t)}{E(t)} \\ & + \left[A_t(0)\,\frac{\epsilon_t(t)}{E(t)} - M_{1/2}\,\frac{tE(t) - F(t)}{F(t)}\left(1 - \frac{\epsilon_t(t)}{E(t)}\right)\right]^2 \\ & + M_{1/2}^2\left[\frac{d}{dt}\left(\frac{t^2E'(t)}{E(t)}\right) - \frac{t^2E'(t)}{F(t)}\left(1 - \frac{\epsilon_t(t)}{E(t)}\right)\right],\end{aligned}$$

where $\epsilon_t(t) = Y_t(t)/Y_t(0)$ and $M_{1/2}$ is the gaugino mass on the Grand Unification scale.

The quasi-fixed point (28) formally corresponds to the limit $Y_t(0) \to \infty$. As we approach this point the value of $\epsilon_t(t)$ decreases and the solutions (29) become entirely independent of the initial conditions on the scale $M_{GUT}$. In the vicinity of the infrared quasi-fixed point $A_t(t_0)$ is proportional to $M_{1/2}$ and $\mathfrak{M}_t^2(t_0) \sim M_{1/2}^2$. The solutions of the system of the renormalization group equations and the particle spectrum in the quasi-fixed point mode for $\tan\beta \approx 1$ have been analyzed in Refs [39 – 43].

If we take into account the weak dependence of the Yukawa constants on their initial values on the Grand Unification scale, take the value of the current mass of the t quark to be $m_t(M_t^{pole}) = 165 \pm 5$ GeV calculated in the $\overline{MS}$ renormalization scheme, and make use of the equation relating $h_t(t_0)$ to $m_t(M_t^{pole})$,

$$m_t(M_t^{pole}) = \frac{h_t(M_t^{pole})}{\sqrt{2}}\,v\sin\beta, \tag{30}$$

we can determine the range of permissible values of $\tan\beta$ in the vicinity of the quasi-fixed point (28).

The theoretical analyses performed in Refs [42 – 45] have demonstrated that the value of $\tan\beta$ varies in the range between 1.3 and 1.8. For such comparatively low values of $\tan\beta$ the mass of the lightest Higgs boson is not greater than $94 \pm 5$ GeV [42 – 44] which is lower by 25 – 30 GeV than the absolute upper limit in the MSSM. The available LEP II experimental data [46] have practically ruled out this range for the Higgs boson masses.

In order to meet the existing experimental constraints on the lightest Higgs boson mass we must either analyze the solutions leading to large $\tan\beta$ values in the MSSM or expand the Higgs sector of the MSSM. It has been shown in Refs [41, 42, 47] that in the range $\tan\beta \approx 50-60$ the solutions of the renormalization group equations for the Yukawa constants and the parameters of weak supersymmetry violation also reach the infrared quasi-fixed point.

Moreover, it is only near such quasi-fixed points for $\tan\beta \approx 1$ and $\tan\beta \approx 50-60$ that the unification of the Yukawa constants of the b quark and $\tau$ lepton occurs in the MSSM which naturally occurs in the minimal schemes of unification of the gauge interactions such as SU(5), $E_6$ and SO(10) [48]. The opportunity for such $(b-\tau)$ unification in the MSSM was analyzed in detail in Refs [40, 41, 47, 49].

Though the upper limit for the mass of the lightest Higgs boson grows with an increase in $\tan\beta$, at the same time more difficulties are encountered with the too rapid proton decay in the supersymmetric Grand Unification theories which is caused by the operators of dimensionality 5 (see Section 3).

The nonminimal supersymmetric standard model (NMSSM) is the simplest expansion of the MSSM which makes it possible to preserve the unification of the gauge constants and to increase the upper limit for the lightest Higgs boson mass [50–52]. According to its structure the NMSSM superpotential [51] is invariant with respect to the discrete transformations $\hat{s}'_\alpha = \hat{s}_\alpha \exp(2i\pi/3)$ of the group $Z_3$. The term $\mu\hat{H}_1\hat{H}_2$ of the MSSM superpotential does not meet this requirement and must therefore be ignored ($\mu = 0$). $Z_3$ symmetry typically appears in the 'string' models in which all the observed fields remain massless in the limit of exact supersymmetry.

In addition to the doublets $H_1$ and $H_2$ the Higgs sector of the NMSSM contains an additional field $Y$ which has a singlet character with respect to the gauge interactions. For $\tan\beta \approx 1$ all the Yukawa constants are small with the exception of the Yukawa constant $h_t$ for the t quark, the self action constant $\varkappa$ of the neutral scalar field $Y$, and the constant $\lambda$ which describes the interaction between the field $Y$ and the doublets $H_1$ and $H_2$.

If we ignore all the Yukawa constants with the exception of $h_t$, $\lambda$, and $\varkappa$ we can write the full NMSSM superpotential as

$$W_{\text{NMSSM}} = \lambda\hat{Y}(\hat{H}_1\hat{H}_2) + \frac{\varkappa}{3}\,\hat{Y}^3 + h_t(\hat{H}_2\hat{Q})\hat{U}_R^c\,. \tag{31}$$

Under a spontaneous violation of the $SU(2)\times U(1)$ symmetry the field $Y$ acquires the nonzero expectation value ($\langle Y\rangle = y/\sqrt{2}$) and the effective $\mu$ term is generated ($\mu = \lambda y/\sqrt{2}$). This results in mixing between the doublets $H_1$ and $H_2$ which is required for the Higgs doublet $H_1$ to acquire the vacuum average $v_1$ without which the lower quarks and charged leptons remain massless.

When the neutral superfield $\hat{Y}$ is introduced into the NMSSM superpotential a respective $F$ term appears in the potential of interaction between the Higgs fields. A result is an increase in the upper limit for the lightest Higgs boson mass in comparison with the MSSM limit:

$$m_h \leqslant \sqrt{\frac{\lambda^2}{2}\,v^2\sin^2 2\beta + M_Z^2\cos^2 2\beta + \varDelta_{11}}\,. \tag{32}$$

Equation (32) has been derived in Ref. [52] in the tree approximation ($\varDelta_{11} = 0$).

When $\lambda = 0$ the expressions for the upper limits in the MSSM and NMSSM coincide. The contribution of the loop correction terms to the upper limit for $m_h$ remains approximately the same in the NMSSM as in the MSSM. In particular, we can obtain expressions in the NMSSM for the contributions of the t quark and its superpartners to $\varDelta_{11}$ by replacing the parameter $\mu$ with $\lambda y/\sqrt{2}$ in the appropriate MSSM equations. The Higgs sector and the one-loop correction terms in the NMSSM have been analyzed in Refs [53, 54].

The upper limit for $m_h$ in the NMSSM increases with an increase in $\lambda(t_0)$. Here the limit differs significantly from the respective limit in the MSSM only in the range of small $\tan\beta$ values. If $\tan\beta \gg 1$, $\sin 2\beta$ vanishes and the upper limits for the lightest Higgs boson mass in the MSSM and NMSSM practically coincide. The case of small $\tan\beta$ is realized only for fairly large values of the Yukawa constants of the t quark on the electroweak scale $h_t(t_0)$. The growth of the Yukawa constants on the electroweak scale is accompanied with an increase in $h_t(0)$ and $\lambda(0)$ at the scale $M_{\text{GUT}}$.

We see that in the framework of theoretical analysis the most attractive case is the limit of strong Yukawa coupling when $h_t^2(0)$ and $\lambda^2(0) \gg g_i^2(0)$. For a certain ratio between $h_t$ and $\lambda$ in the limit under consideration the Yukawa constants of the b quark and $\tau$ lepton prove to be identical at the Grand Unification scale [55, 56]. It is precisely in this area of the parametric space that the upper limit for the lightest Higgs boson mass in the NMSSM reaches its highest value which is greater by 7–10 GeV than the respective limit in the MSSM [57, 58]. The upper limit for the lightest Higgs boson mass in the NMSSM has been compared in Ref. [36] with the limits for $m_h$ in the minimal standard model and the MSSM.

As the Yukawa constants grow the solutions of the NMSSM renormalization group equations are drawn towards the quasi-fixed (Hill) line ($\varkappa = 0$) or surface ($\varkappa \neq 0$) in the space of Yukawa constants [56] as well as towards some straight lines or planes in the space of the parameters of weak violation of supersymmetry [59]. In the limit of $h_t(0), \lambda(0), \varkappa(0) \to \infty$ all solutions at the electroweak scale are concentrated in the vicinity of the quasi-fixed points [56–59].

Unfortunately, under the conditions of a strong Yukawa coupling in the framework of the NMSSM with a minimal set of fundamental parameters it is impossible to obtain a self-consistent solution of the system of algebraic equations

$$\frac{\partial V(v_1, v_2, y)}{\partial v_1} = 0\,, \qquad \frac{\partial V(v_1, v_2, y)}{\partial v_2} = 0\,, \qquad \frac{\partial V(v_1, v_2, y)}{\partial y} = 0\,, \tag{33}$$

which determines the position of the physical minimum of the effective potential of interaction of the Higgs fields $V(v_1, v_2, y)$. This solution is found only for $\lambda^2(0)$ and $\varkappa^2(0) \leqslant 0.1$ (see Refs [54, 60]) when the upper limits for $m_h$ in the MSSM and NMSSM practically coincide.

Moreover, three degenerate vacuums occur in the NMSSM owing to the $Z_3$ symmetry which makes it possible to resolve the problem of the $\mu$ term. After the phase transition on the electroweak scale the universe is filled up with three degenerate phases so that the domain walls are produced. The cosmological observational data do not confirm the domain structure of the vacuum. An attempt to break the $Z_3$ symmetry and the domain structure of the vacuum by introducing nonrenormalized operators into the NMSSM Lagrangian gives rise to quadratic divergences, that is the hierarchy problem [61].

The NMSSM must be modified in order to avoid the domain structure of the vacuum and to derive a self-consistent solution for the conditions of strong Yukawa coupling. The simplest way to modify the NMSSM is to introduce additional terms $\mu\hat{H}_1\hat{H}_2$ and $\mu'\hat{Y}^2$ into the superpotential of the Higgs sector [62], which are not prohibited by gauge symmetry. The introduction of additional bilinear terms into the NMSSM superpotential destroys the $Z_3$ symmetry and no domain walls are produced in such a theory.

When we analyze the modified NMSSM (MNMSSM) it is reasonable to assume that the lightest Higgs boson mass has the largest value precisely for $\varkappa = 0$. Hence, the total

MNMSSM superpotential for $\tan\beta \approx 1$ can be written as

$$W_{\mathrm{MNMSSM}} = \mu(\hat{H}_1\hat{H}_2) + \mu'\hat{Y}^2 + \lambda\hat{Y}(\hat{H}_1\hat{H}_2) + h_t(\hat{H}_2\hat{Q})\hat{U}_R^c \,. \tag{34}$$

Within the framework of the supergravitational models terms of the superpotential (34) which are bilinear in the superfields may be produced owing to the additional term $(Z(H_1H_2)+Z'Y^2+\text{h.c.})$ in the Käler potential [63, 64] or the nonrenormalized interaction between the fields of the observed and 'hidden' sectors [64, 65].

The theory has not lost its predictive power even though the parameter space of the model under study has been expanded considerably. The particle spectrum in the MNMSSM framework has been analyzed under the conditions of strong Yukawa coupling [62]. Even for comparatively small values of $\tan\beta \geqslant 1.9$ the lightest Higgs boson mass can be as high as 125–127 GeV in this model. The highest upper limit for the boson mass is obtained for $\tan\beta \approx 2.2-2.4$. The lightest Higgs boson mass in the MNMSSM is not greater than $130.5 \pm 3.5$ GeV.

In the supersymmetric model the upper limit for the lightest Higgs boson mass can be significantly increased if several $5 + \bar{5}$ matter multiplets are introduced in addition to the singlet. The introduction of new particles causes a change in the evolution of the gauge constants. In particular, in the models under consideration the strong interaction constant which decreases with an increase in $q^2$ in the MSSM grows on approaching the Grand Unification scale. At the same time, for $t$ in the range from zero to $t_0$ all gauge constants increase in comparison with the respective values in the MSSM. This behavior of $\tilde{\alpha}_i(t)$ causes the growth of the upper limits for the Yukawa constants which exist under the assumption that the solution of the renormalization group equations have no Landau pole up to the scale $M_{\mathrm{GUT}}$.

The upper limit for the lightest Higgs boson mass grows owing to the extension of the range of admissible values for the Yukawa constants. For instance, the upper limit for $m_h$ in the NMSSM has been shown [58] to increase to 155 GeV when four–five additional $5 + \bar{5}$ matter multiplets have been introduced. When a large number of matter multiplets is introduced the perturbation theory proves to be inapplicable for $q^2 \sim M_{\mathrm{GUT}}^2$.

The upper limit for the lightest Higgs boson mass in the more complicated versions of the MSSM has also been intensely discussed recently [66–68]. For instance, three SU(2) triplets can be introduced into the Higgs sector of the supersymmetric models in addition to the singlet. The introduction of triplets breaks the gauge constant unification at high energies. In order to restore this unification we must add several matter multiplets which carry the $\mathrm{SU(3)_c}$ color but are not involved in the electroweak interactions. Numerical analysis [67] demonstrated that under such conditions unification of the gauge constants takes place on the scale $\widetilde{M}_{\mathrm{GUT}} \sim 10^{17}$ GeV and the lightest Higgs boson mass is not greater than 190 GeV.

The upper limit for $m_h$ is also increased significantly by the occurrence of fourth-generation particles in the MSSM [68], which is rather questionable from the viewpoint of the current limitations on additional fermion generations [8] (see, however, Ref. [69]). We see that an increase in the upper limit for the lightest Higgs boson mass in the supersymmetric models is typically accompanied by a significant growth of the number of particles in the models which must be regarded as a serious shortcoming of such models.

## 6. Dark matter

The bulk of the matter in the universe does not consist of neutrons and protons. This conclusion is inferred from the fact that the contemporary mean density of matter in the universe is close to the critical value $\rho_c = 3H_0^2/8\pi G_N = 5.2 \times 10^{-6}$ GeV cm$^{-3}$ (here we have used the numerical value of the Hubble constant $H_0 = 70$ km s$^{-1}$ Mpc$^{-1}$) and from the nucleosynthesis theory which does not allow such a high density of nucleons. It is currently assumed that about 30% of the density in the universe is due to relic particles or the so-called hidden mass. Supersymmetric models provide a natural solution to the hidden mass problem [70]: in most realistic models the lightest superpartner (LSP) is absolutely stable owing to the conservation of the so-called $R$ parity.

The superparticles are generated in pairs at the early stages of the universe evolution and decay rapidly giving rise to LSP in addition to the conventional particles. The LSPs must be electrically neutral and not involved in strong interactions. Otherwise, the contemporary density of the anomalous isotopes proves to be unacceptably high. (Since the LSP is neutral it does not capture an electron on a Coulomb orbit and it cannot be 'glued onto' nuclei as it does not have strong interactions.) These requirements are satisfied by the lightest neutralino (a mixture of the superpartners of photon, Z boson, and two neutral Higgs bosons) which is described by a Majorana spinor and denoted by the subscript $\chi$. Let us make an order-of-magnitude estimate for the contemporary density of the relic neutralinos.

At a universe temperature exceeding the neutralino mass the reactions of neutralino generation and annihilation proceed at a fast rate and therefore the neutralino density is equal to its equilibrium value. During expansion of the universe its temperature becomes smaller than the neutralino mass, the equilibrium density of neutralino starts to decrease exponentially and the characteristic time of the annihilation reaction exponentially increases. When the annihilation time becomes equal to the universe lifetime describing the expansion rate quenching occurs; annihilation is stopped and subsequently the neutralino density decreases only owing to the expansion of the universe.

The quenching moment is given by the equation

$$t_u = \frac{M_{\mathrm{Pl}}}{T_{\mathrm{fr}}^2} = \frac{1}{\sigma_{\mathrm{ann}} v n_\chi} = t_{\mathrm{ann}} \,, \tag{35}$$

where $M_{\mathrm{Pl}} = 1/\sqrt{G_N} = 10^{19}$ GeV is the Planck mass,

$$n_\chi = \frac{2}{(2\pi)^{3/2}} (m_\chi T_{\mathrm{fr}})^{3/2} \exp\left(-\frac{m_\chi}{T_{\mathrm{fr}}}\right)$$

is the equilibrium neutralino density, and $\sigma_{\mathrm{ann}} v$ is the total cross section for neutralino annihilation multiplied by velocity. Using the equilibrium photon density $n_\gamma = (2.4/\pi^2)T^3$ we obtain the following expression for the neutralino density at the present moment:

$$n_{\chi_0} = \frac{T_0^3}{M_{\mathrm{Pl}}\sigma_{\mathrm{ann}} v T_{\mathrm{fr}}} \,, \qquad \rho_\chi = \frac{m_\chi T_0^3}{M_{\mathrm{Pl}} T_{\mathrm{fr}}(\sigma_{\mathrm{ann}} v)} \,. \tag{36}$$

Here $T_0 = 2.7$ K is the contemporary temperature of the universal background.

Over a fairly wide range of $\sigma_{\rm ann}v$ values Eqn (35) yields $m_\chi/T_{\rm fr} \approx 20$. (The point is that the dependence on this parameter is exponential so a small variation causes a large variation of $\sigma_{\rm ann}v$. It is the exponential dependence that provides a high accuracy of the evaluation of the residual density of the relict particles.) Dividing $\rho_\chi$ by $\rho_{\rm c}$ we obtain

$$\Omega_\chi = \frac{8\pi}{3}\,\frac{20T_0^3}{\sigma_{\rm ann}vH_0^2M_{\rm Pl}^3}\,, \tag{37}$$

where $H_0 = 70$ km s$^{-1}$ Mpc$^{-1}$ $= (14\times10^9\ \text{yr})^{-1}$.

The next question is: what is the cross section of neutralino annihilation? We shall limit the analysis to the case of photinos which annihilate yielding pairs of charged leptons and quarks (Fig. 4). Since the cross section is proportional to the fourth power of the charge we can ignore the contribution of quarks and limit the analysis to six possible annihilation channels: $\mathrm{e}_L\bar{\mathrm{e}}_L$, $\mathrm{e}_R\bar{\mathrm{e}}_R$, $\mu_L\bar{\mu}_L$, $\mu_R\bar{\mu}_R$, $\tau_L\bar{\tau}_L$, and $\tau_R\bar{\tau}_R$.

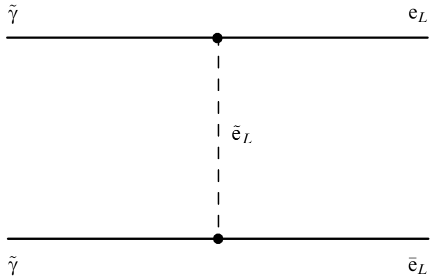


**Figure 4.** Diagram describing the annihilation of the relic photinos.

According to the Pauli principle for identical Majorana particles the total spin of two photinos equals zero in the *s* wave and unity in the *p* wave. Since the helicity of the lepton pair is $\pm1$ annihilation occurs in the *p* wave (the small contribution of the *s* wave is proportional to the mass of the final fermion). The total cross section of annihilation is

$$\sigma_{\rm ann}v_{\tilde{\gamma}} = 6\,\frac{\alpha^2(4\pi)^2}{6\pi}\,\frac{m_{\tilde{\gamma}}^2}{m_{\tilde{\rm l}}^4}\,v_{\tilde{\gamma}}^2\,, \tag{38}$$

where

$$v_{\tilde{\gamma}}^2 = \frac{3T_{\rm fr}}{m_{\tilde{\gamma}}} \approx 0.15\,.$$

For $(m_{\tilde{\gamma}}/50\,\text{GeV})(100\,\text{GeV}/m_{\tilde{\rm l}})^2 \approx 10$ the value of $\Omega_\chi$ proves to be equal to 0.3 which must be regarded as a very lucky coincidence.

A large number of studies have been concerned with a more accurate calculation of the dependence of $\Omega_\chi$ on the parameters of the supersymmetric models and currently neutralino with a mass of several tens of gigaelectronvolts is regarded as the main candidate for the role of the cold dark matter while the requirement $\Omega_\chi = 0.3$ is used for determining the constraints for the parameters of the supersymmetric models.

An intense experimental search is currently under way for the dark matter consisting of neutralinos. If the halo of our galaxy consists of neutralinos (which is a natural assumption) the local density $n_\chi \sim 0.3/m_\chi[\text{GeV}]$ cm$^{-3}$ is higher by a factor of approximately $10^5$ than the average density in the universe.

The experiments can be classified into two types: (1) the search for the products of annihilation of neutralinos from the galactic halo (photons, positrons, antiprotons), or the neutrinos produced by annihilation of the neutralinos accumulated in the core of the earth (or the sun); and (2) the search for the recoil nuclei produced on elastic scattering of neutralinos. In the latter case the characteristic energy of the heavy nuclei $E \approx m_\chi v^2/2$ is in the range of hundreds of kiloelectronvolts for the velocity $v \approx 600$ km s$^{-1}$ similar to the velocity with which the earth travels with respect to the center of the galaxy. Owing to the rotation of the earth around the sun the detector count should exhibit an annual periodicity with an amplitude of about 10%. Some experimental results have indicated such a periodicity [71].

Note that the calculated neutralino scattering cross sections are typically smaller than the sensitivity thresholds of contemporary detectors by several orders of magnitude. Experiments are planned in which the detector sensitivity will be sufficient for studying a significant range of parameters of the supersymmetric models [72].

## 7. Search for superpartners at the existing and future accelerators

No direct evidence of the existence of superparticles has yet been obtained even though the search for them is one of the principal tasks of the operations with existing and planned accelerators. The superpartners of the observed particles can be produced only in pairs owing to the conservation of *R* parity. The products of their decay must include the lightest stable supersymmetric particle the role of which is typically played by the lightest neutralino in realistic models. Since the neutralino interacts with matter only via weak interactions it is not detected in accelerator experiments removing a significant proportion of the energy and momentum of the colliding particles. This is why 'nonconservation' of energy and momentum is a signal indicating the generation of superparticles in a given event.

The lower limits have been determined for the masses of the superpartners of the observed particles since such events have not been recorded with the existing accelerators. The measurements of the decay width for the Z boson made in the LEP I and SLAC experiments [73] ruled out the existence of superparticles with masses smaller than $M_{\rm Z}/2$ into which the Z boson can decay with an appreciable probability. Subsequently, the experimental data obtained with the $\mathrm{e^+e^-}$ LEP II collider made it possible to improve the lower limits for the masses of the superparticles which are not involved in strong interactions.

The superpartners of the charged Higgs bosons and $\mathrm{W}^\pm$ bosons, known as charginos, are produced in $\mathrm{e^+e^-}$ collisions owing to the exchange of the virtual electron sneutrino $\tilde{\nu}_{\rm e}$ or the decay of the virtual vector $\gamma^*$ and $\mathrm{Z}^*$ bosons. If $M_{\tilde{\chi}^\pm} - M_{\tilde{\chi}^0} \geqslant 3$ GeV and $M_{\tilde{\nu}} > 500$ GeV then the experimental data rule out the existence of charginos with masses smaller than 94 GeV [74]. If the sneutrino can have an arbitrary mass the lower limit for $M_{\tilde{\chi}^\pm}$ decreases to 56 GeV [75].

The sleptons and squarks are generated during $\mathrm{e^+e^-}$ annihilation owing to the exchange of the intermediate vector $\gamma^*$ and $\mathrm{Z}^*$ bosons. Under such conditions an important additional contribution to the selectron generation cross section is made by the diagram with neutralino exchange in the t channel which causes a significant increase in the cross section of the process under consideration. The limits on the masses of the sleptons and squarks prove to be somewhat smaller than the kinematical limit $\sqrt{s}/2$ which is explained by a strong suppression of the cross section over the phase volume near the threshold.

The OPAL collaboration data have demonstrated that the lower limits for $\tilde{e}_R$, $\tilde{\mu}_R$, and $\tilde{\tau}_R$ are 89, 82, and 81 GeV, respectively [76]. The lower LEP II limits on the masses of the superpartners of the t and b quarks vary in accordance with the angles of mixing ($\theta_{\tilde{t}}$ and $\theta_{\tilde{b}}$) between the left and right squarks [77]. For the lightest superpartner $\tilde{t}_1$ of the t quark this limit varies between 89 and 91 GeV. The lower limit for the mass of the lightest superpartner $\tilde{b}_1$ of the b quark has been found to be somewhat weaker (from 75 to 90 GeV) because its charge is half the charge of $\tilde{t}_1$ in magnitude.

The Tevatron (FNAL) studies of the process of annihilation of protons with antiprotons have yielded much more stringent limits for the squark and gluino masses. If a squark has a greater mass than a gluino then the gluino mass $M_{\tilde{g}} \geqslant 180$ GeV [9]. If the squarks and the gluinos have identical masses then their mass cannot be smaller than 260 GeV [9] according to the D0 collaboration results. Finally, if the squark mass is considerably smaller than the gluino mass the limit derived by UA1 and UA2 groups is applicable: $M_{\tilde{g}} \geqslant 300$ GeV [78]. The new experiments conducted on the modernized Tevatron are expected to deliver a significant impulse to the search for new particles.

After completion of the LEP II operations the main hopes for discovering new particles in CERN are attached to new experiments planned on the large hadron collider (LHC). The total energy of the colliding proton beams in this accelerator is planned to reach 14 TeV and its luminosity will be as high as $10^{34}$ cm$^{-2}$ s$^{-1}$. The LHC accelerator will make it possible to detect squarks and gluinos with the masses of up to 2.5 TeV as well as sleptons with masses below 340 – 350 GeV [79, 80].

According to preliminary estimates, the LHC luminosity may be increased up to $10^{35}$ cm$^{-2}$ s$^{-1}$ and the total beam energy up to 15.2 TeV. The 'new physics' capacity of the modernized LHC (SLHC) will be enhanced by approximately 20%. In particular, the SLHC will make it possible to detect squarks and gluinos with masses of up to 3 TeV [79, 80]. The accuracy of high-precision measurements will be significantly enhanced by means of increasing the luminosity by an order of magnitude, and, as a result, improving the event statistics. Though these improvements in the LHC will not be radical in character they may prove to be very significant for fully describing the superparticle spectrum.

CERN requested the ATLAS and CMS collaborations to analyze the prospects for operating a hadron collider with a center-of-mass energy of 28 TeV (LrHC) [79, 80] and the analysis is underway while a group of US physicists have put forward a design for developing a 100-TeV hadron collider (VLHC) [80]. Both accelerators are expected to have luminosities of the order of $10^{34}$ cm$^{-2}$ s$^{-1}$. These colliders, in particular the VLHC, will undoubtedly have a much greater capacity than that of the LHC. For instance, the VLHC will make it possible to investigate the energy range up to 30 TeV and detect squarks and gluinos with masses up to 10 TeV.

Even though the colliders have an impressive potential for discovering squarks, gluinos, and Higgs bosons, they have only a very limited capacity for detecting the superparticles which are not involved in strong interactions. The lacking data on the spectrum of sleptons, charginos, and neutralinos, can be obtained from the experiments on linear $e^+e^-$ colliders the plans for whose constructions are being intensively discussed in Germany, USA, and Japan (TESLA, NLC, and JLC, respectively). The center-of-mass energy of the colliding electron and positron beams in these accelerators is planned to vary between 0.5 and 1 TeV.

A 35-km linear collider project (CLIC) planned for $\sqrt{s} \approx 3-5$ TeV is being discussed at CERN [81]. The CLIC will make possible high-precision measurements of the parameters of the Higgs sector and the superparticles. For instance, the self-action constant $\lambda$ of the Higgs fields can be measured to an accuracy of 10% with this accelerator. The CLIC linear collider will be able to record squarks and sleptons with masses not exceeding 1.5 – 2.5 TeV.

It should be expected, therefore, that in the first decades of the 21st century the supersymmetry concepts will be either brilliantly confirmed or the new experimental data will banish 'phenomenological supersymmetry'.

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