High angular resolution observations of the central regions (nuclei) of many galaxies (M87, NGC4621, NGC7052, etc.) made with the Hubble Space Telescope allowed measurements of the rotational velocity distribution of the central gas structures and mass determination of the underlying BH: $M = 10^8 - 10^9\,M_\odot$.

Very-long-baseline interferometric observations (VLBI) of maser sources in near-nuclear regions permitted reliable BH-mass determinations in the case of NGC4258 ($M = 3.9 \times 10^7\,M_\odot$) and in five other galaxies.

Measurements of the velocities and accelerations of individual stars near the center of our Galaxy allowed their orbits to be determined at distances $r < 0.005$ pc ($\cong 10^4 r_{\rm g}$) from the central BH. Its mass, as derived from the distribution of individual star velocities in our Galaxy, is $2.6 \times 10^6\,M_\odot$. An independent estimate, based on the accelerations of individual stars, yields a BH mass of $3 \times 10^6\,M_\odot$.

The X-ray luminosity of the Galactic center is very low ($L_{\rm x} = 10^{37}$ erg s^{-1}) and is $\sim 10^{-8}$ of the critical Eddington luminosity. Using data for ~ 45 galaxies, a correlation has been found between the mass of the galactic bulge and that of the central BH: $M_{\rm BH} \cong (0.2\% - 0.5\%) M_{\rm bulge}$.

In two cases (our Galaxy and galaxy NGC4258), the observed matter density inside the measured nuclear region of the galaxy ($r < 0.005\ {\rm pc} \cong 1.5 \times 10^{16}\ {\rm cm} \cong 10^4 r_{\rm g}$ for $M = 10^6\,M_\odot$) is $\rho > 10^{12} - 10^{13}\,M_\odot/{\rm pc}^3 = 10^{-10} - 10^{-9}$ g cm^{-3} (in the solar neighborhood $\rho \approx 0.1\,M_\odot/{\rm pc}^3$, in the most dense stellar clusters $\rho \sim 10^5\,M_\odot/{\rm pc}^3$). At $\rho > 10^{12} - 10^{13}\,M_\odot/{\rm pc}^3$, the characteristic time scale of evaporation of individual dark bodies is $T_{\rm dyn} < 10^8 - 10^7$ yr for galactic ages of $\sim 10^{10}$ yr. So massive dark bodies in the nuclei of our Galaxy and NGC4258 must be single compact objects.

The width and profile of the X-ray line FeXXV at 6.4 keV in the spectra of some galactic nuclei correspond to hot-gas ($T \cong 10^7$ K) velocities of $V_{\rm rot} = 100\,000$ km s^{-1} at distances of $r = (3 - 6) r_{\rm g}$.

All these data strengthen our assurance of the existence of supermassive BH in galactic nuclei.

6. Prospects for finding sufficient criteria of BH existence

In future, the following experiments are envisaged which will provide sufficient criteria of BH existence.

(1) Space-based radio and X-ray interferometers with an angular resolution of $10^{-6} - 10^{-7}$ arcsec. These facilities will provide a possibility to directly observe processes near the event horizon, which has an angular size of $\sim 10^{-6}$ arcsec in nearby galactic nuclei.

(2) Gravitational wave bursts from binary BH mergings.

(3) Detection of a radio pulsar in a binary system with a BH (one pulsar in pair with a BH is expected per ~ 1000 single pulsars).

(4) Detailed studies of X-ray line profiles and rapid variability of X-ray fluxes from accreting BHs.

(5) Studying the gravitational microlensing of galactic nuclei by stars in intervening galaxies — gravitational lenses (angular resolution of up to 10^{-6} arcsec).

(6) Routine accumulation of reliable mass estimations of BH candidates and statistical comparisons of the observed NS and BH properties.

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Superfluidity in neutron stars

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1. Introduction

Neutron stars (NSs) are the most compact of all stars. Their masses are around $1.4\,M_\odot$, where $M_\odot$ is the solar mass, and their radii are about 10 km. Accordingly, the mean density of the NS matter is a few times ρ_0, where $\rho_0 = 2.8 \times 10^{14}$ g cm^{-3} is the density of the matter in atomic nuclei. Atomic nuclei are almost incompressible in laboratory conditions. NSs are often called natural laboratories of supranuclear-density matter.

According to current views [1], a NS consists of four main layers. The outer layer, which extends to a density of 4×10^{11} g cm^{-3}, is *the outer crust*, which consists of degenerate electrons (e) and ions (nuclei). Deeper, to $\sim 0.5\rho_0$, there is the *inner crust*, composed of nuclei, electrons, and free neutrons (n). Further, to $\sim 2\rho_0$, there is the *outer core* containing a Fermi liquid of neutrons with a small admixture of degenerate electrons and protons (p) and, possibly, muons. Finally, the NS central region constitutes the *inner core*, whose composition is largely unknown. A reliable theory of the superdense NS matter is still absent. The main difficulty is in the description of strong interactions of various particles accounting for many-body effects. Instead of a strict theory, there are many different theoretical models. Some models predict the appearance of hyperons in the NS cores, while others predict either pion or kaon condensation or the appearance of light, almost free quarks (u, d, and s). One cannot exclude mixtures of different phases, for instance, hyperons and quarks. Some models give a fairly stiff equation of state and, therefore, rather high maximum allowed masses of NSs, $(2-3)\,M_\odot$. According to other theories, the equation of state is moderate or soft, and the maximum masses are lower, $(1.5-2)\,M_\odot$. The nature of matter in the NS cores is the main mystery of NSs. Its solution would be of fundamental importance for physics and astrophysics.

Let us mention a hypothesis of Witten [2], according to which a plasma of almost free quarks is the most stable state of matter not only at high pressures but also at zero pressure. If so, instead of NSs, so-called *strange stars* should exist, almost fully composed of quark matter of density $\gtrsim \rho_0$.

NSs are observed in all ranges of the electromagnetic spectrum, from radio to hard gamma rays. They may be either isolated objects or components of binary systems. They manifest themselves as radio and X-ray pulsars, X-ray bursters, X-ray transients, soft-gamma repeaters, or anomalous X-ray pulsars. The birth of NSs in supernova explosions leads to powerful neutrino outbursts. NSs may be powerful sources of gravitational radiation.

2. Superfluid gaps

One of the main features of NSs is *superfluidity* of the baryon component of their matter. It is believed that superfluidity is produced by Cooper pairing of baryons with opposite momenta under the action of the attractive part of the strong interaction of particles. Superfluidity is switched on as the temperature T falls below a critical temperature, $T_{\rm c}$, leading to the appearance of a gap Δ in the baryon dispersion relation near the Fermi level. The gap has practically no effect on the equation of state of matter, i.e., on the NS masses and radii.

One can expect superfluidity of free neutrons and superfluidity of nucleons in atomic nuclei in the inner NS crust. The superfluidity of n, p, and other particles is possible in an NS core. The superfluidity of charged particles, for instance, p, means superconductivity.

The superfluidity of neutrons in NSs was predicted by Migdal [3]. Wolf [4] showed, that singlet-state ($^1S_0$) pairing of neutrons can occur in the NS crust but disappears in the NS core because the nuclear attraction of n in the singlet state turns into repulsion with increasing density. However, as pointed out by Hoffberg et al. [5], one can expect triplet-state ($^3P_2$) pairing of n with an anisotropic gap in the NS core. Owing to a rather low fraction of p in the NS core, their pairing usually occurs in the singlet state. The critical temperatures $T_{\rm cn}$ and $T_{\rm cp}$ in NSs have been calculated by many authors (see, e.g., [6] for references). The results are sensitive to the nuclear interaction model and many-body theory employed; they are affected by kaon or pion condensations.

There may be pairing of hyperons [8] in hyperonic matter and pairing of quarks [9] in quark matter.

In all the cases mentioned above, microscopic calculations give critical temperatures $T_{\rm c} \lesssim 10^{10}$ K and lower. Recently, a new type of quark superfluidity was proposed [9], which consists in pairing of unlike quarks (ud, us, ds). For a typical Fermi energy of quarks of ~ 500 MeV, one can expect critical temperatures of $T_{\rm c} \sim 50$ MeV $\sim 5 \times 10^{11}$ K. However, this pairing is suppressed by the difference in the Fermi momenta of unlike quarks. If the difference is large enough, the pairing should disappear.

3. Vortices and fluxoids

For simplicity, consider first the core of a rotating NS composed of n, p, and e. The rotation of a neutron superfluid is realized [10,11] in the form of quantized vortices (Feynman-Onsager vortices) parallel to the NS spin axis. The total number of vortices in a NS is $\sim 2 \times 10^{16}/P$, where P is the spin period in seconds. The overall vortex motion of the neutron superfluid reproduces a solid-body rotation with the period P. The vortices also penetrate into the inner NS crust, where the superfluidity of n is present. The slowing-down of the neutron superfluid, which accompanies the NS spin-down, is associated with the outward drift of vortices and their disappearance at the boundary of the superfluid region.

Superconductivity of p in the NS core is described by the Ginzburg – Landau theory. The proton coherence length (2 – 6 fm) is typically much smaller than the London screening length (100 – 300 fm), i.e., a type-II superconductor forms. If the NS core initially contained a quasi-uniform magnetic field B, then after the superconducting transition this field splits into fluxoids (Abrikosov vortices), thin magnetic flux tubes parallel to the initial field. The total number of fluxoids is $\sim 10^{31}\,(B/10^{12}$ G).

Neutron vortices may be pinned by atomic nuclei or by defects of the crystal of atomic nuclei in the NS crust and also by fluxoids in the NS core. The pinning may be accompanied by vortex-creep effects. Owing to the drag of proton superfluid by neutron vortices (similar to the drag of superfluid ^{3}He by the motion of superfluid ^{4}He [12]), neutron vortices in the NS core acquire magnetic moments. Electrons (which are non-superfluid) scatter off the magnetic fields of fluxoids and vortices [13], producing a strong coupling of electrons with superfluid n and p (so-called *mutual friction*). Vortices and fluxoids are also exposed to other forces (the Magnus force, buoyancy force etc.).

The superconductivity of quark matter is of a different character. According to estimates, a type-I superconductor forms in the case of any pairing of quarks (like or unlike). This superconductor expels the magnetic field (see Section 5).

4. Pulsar glitches

Isolated radio pulsars spin down slowly, transforming their rotational energy into electromagnetic radiation. Some pulsars show glitches. Within a very short time interval (probably, several minutes) the NS spin period decreases and then increases smoothly (relaxes), returning to the regime of slow permanent growth (NS spin-down) within a month or so. The most frequent and powerful glitches are exhibited by the Vela and Crab pulsars. The relative jumps of their periods are $\sim 10^{-6}$ and $\sim 10^{-8} - 10^{-7}$, respectively.

The most popular model of glitches was developed by Alpar et al. (e.g., [14]) and exploited the idea of n superfluidity. Owing to mutual friction (see Section 3), the pulsar core rotates almost as a rigid body. It is connected tightly with the non-superfluid part of the crust and rotates together with it. Only the n superfluid in the pulsar crust is relatively independent. The spin-down torque acts on the non-superfluid part of the crust. In the regime of persistent spin-down, the rotation of the n superfluid in the crust adjusts to the spin-down of the other part of the NS by means of the permanent unpinning of vortices from some pinning centers and pinning them to others. However, the lag of the spin-down of the neutron superfluid in the crust can be accumulated, leading finally to a glitch — the unpinning of numerous vortices at once and subsequent relaxation (pinning to new centers). The theory requires the moment of inertia of the neutron superfluid in the crust to be $\sim 10^{-2}$ of the total moment of inertia. The pinning of vortices in the pulsar core is difficult to calculate, which introduces great uncertainties into the theory.

5. Magnetic field evolution in an NS core

The core of a newly born NS may possess a strong magnetic field. First we describe its evolution in a model of the core composed of n, p, and e.

If superfluidity is absent then the magnetic field experiences normal ohmic decay. In a rather weak field ($B \lesssim 10^{11}$ G), the electrical resistance is isotropic and so weak that the decay time exceeds the age of the Universe [11]. In a stronger field, the resistance across the field may substantially increase owing to magnetization of e and p, which accelerates the decay to $10^6 - 10^8$ yr [15].

In the presence of (type-II) proton superconductivity the magnetic field evolution is determined by the drift of fluxoids. Simple estimates yield the typical time of fluxoid expulsion from a superconductor by ohmic diffusion $\sim 10^8$ yr [11]. Subsequent works took into account other mechanisms of interaction of fluxoids with the medium. These mechanisms have not been studied in full detail; the inclusion of all mechanisms together is a complicated task, and the results are not definite. For instance, Srinivasan et al. [16] took into account the interpinning of fluxoids and vortices. Vortices drift outwards following the pulsar spin-down and drag fluxoids, i.e., the fluxoid expulsion operates on spin-down time scales ($\sim 10^8 - 10^{10}$ yr). Ding et al. [17] included the consideration of creep of fluxoids and vortices, as well as mutual friction. Hsu [18] incorporated the force that acts on fluxoids, being proportional to the gradient of the proton superfluid gap. In his model, fluxoids can drift to the NS center in the central region of the NS.

In a star with a superconducting quark core (type-I superconductor) the magnetic field is completely expelled from the core by the Meissner effect. Simple estimates [19] yielded the time of expulsion due to ohmic diffusion $\sim 10^4$ yr. Chau [20] noted that the expulsion may be accompanied by the enhancement of magnetic-field nonuniformities and prolonged to 10^7 yr. Alford et al. [21] considered an expulsion due to pairing of like and unlike quarks and obtained expulsion times larger than the age of the Universe in both cases.

The evolution of magnetic fields in NS cores requires further consideration. It may be coupled to the thermal evolution of NSs, evolution of the magnetic field in their crusts, and to the possible accretion of matter from the companion to the NS in a binary system.

6. Damping of stellar pulsations

NS pulsations can be generated by different mechanisms. Of particular interest are the so-called r-modes [22], associated with density waves, which are capable of emitting gravitational radiation and growing under the action of this radiation. It is assumed that the r-modes are generated in young rapidly rotating NSs born in supernova outbursts and live for a sufficiently long time (from several days to about one year), slowing down NS rotation and producing powerful gravitational radiation. Gravitational detectors of the new generation, e.g., LIGO II and LIGO III, could detect several such events per year from a distance of up to 20 Mpc.

However, pulsations can be damped by the viscosity of the matter in the NS core. In the absence of superfluidity, the typical damping time controlled by shear viscosity is $\tau \sim 10\, T_9^2$ yr, where T_9 is the internal temperature expressed in 10^9 K. In a hot, non-superfluid NS the damping due to bulk viscosity appears to be stronger. Here, three cases can be distinguished.

(1) If the NS core contains neither hyperons nor quarks, and beta equilibrium is supported by modified Urca processes, then the damping of pulsations with frequency $\omega \sim 10^4$ s^{-1} due to bulk viscosity [23] dominates at $T \gtrsim 10^9$ K and yields $\tau \sim 10/T_9^6$ yr. For a typical value $T \sim 10^9$ K, the time scale τ may be several years.

(2) If beta equilibrium is realized through direct Urca process, the bulk viscosity and neutrino cooling rate will be enhanced by several orders of magnitude [24]. The bulk viscosity will dominate at $T \gtrsim 10^8$ K, giving $\tau \sim 1/T_9^4$ min. In this case the temperature will decrease to $\sim 10^8$ K within several days, and τ will also be several days.

(3) In the presence of hyperons or quarks, the bulk viscosity is determined by neutrinoless reactions involving these particles [25, 26] and will be so high as to be able to dominate at $T \gtrsim 10^7$ K, giving $\tau \sim 1/T_9^2$ s.

Regardless of the neutrino cooling rate, we will have τ of a few minutes for a young and hot star. Thus, the existence of r-modes is critically sensitive to the presence of hyperons or quarks.

The effect of superfluidity on pulsation damping is twofold. On the one hand, the gap in the dispersion relation of superfluid particles can strongly suppress bulk viscosity and slow down the damping of pulsations [27]. On the other hand, mutual friction is equivalent to a large effective viscosity, so that $\tau \lesssim 10^4$ s [28]. The onset of superfluidity in a cooling NS depends on the cooling regime and the values of T_c. It is possible that a young and hot NS remains non-superfluid and r-modes be unaffected by superfluidity. Even so, superfluidity may strongly damp pulsations of older NSs.

7. Neutron star cooling

The decrease of surface temperature in time in a cooling NS is governed, in particular, by the NS neutrino luminosity and heat capacity (see, e.g., [6]). The presence of the gap in the dispersion relation of superfluid particles affects the heat capacity and suppresses neutrino reactions involving these particles. Moreover, a new specific mechanism of neutrino emission appears due to Cooper pairing of particles. In the absence of superfluidity, we have either the so-called *slow* cooling, controlled by neutrino reactions of modified Urca process, or *fast* cooling due to direct Urca process. Superfluidity may affect the cooling process in such a way that fast cooling will look like slow cooling, and vice versa. Thus, superfluidity in the NS core may become a powerful cooling regulator. This allows one, in principle, to determine the critical temperatures in the NS cores by comparing cooling theories with observations of thermal radiation from NSs.

In particular, this has been done [6] for simplified models of NSs whose cores consist only of n, p, and e, assuming the critical temperatures $T_{\rm cn}$ and $T_{\rm cp}$ to be constant over the core. We have chosen NS models with masses 1.3 or 1.48 $M_\odot$ corresponding to the turned-off and turned-on direct Urca process. The results have been compared with the surface temperatures of several middle-aged NSs ($10^4 - 10^6$ yr), inferred using the interpretation of the observed spectra with the black-body spectrum or hydrogen-atmosphere models. In all the four cases (two masses and two spectral models) we have determined the values $T_{\rm cn}$ and $T_{\rm cp}$, which agree with the majority of observational data. In all the cases, we have obtained $T_{\rm cn} \approx 3 \times 10^8$ K. These results are preliminary. At the next stage, more realistic models of superfluidity should be considered, with $T_{\rm c}$ variable over the NS core.

8. Summary

Superfluidity of the baryon component of matter (nucleons, hyperons, quarks) in NSs is predicted by almost all microscopic theories. However, the theoretical values of $T_{\rm c}$ are very uncertain.

Superfluidity strongly affects many processes in NSs (glitches, magnetic field evolution, pulsation damping, cooling); however, theoretical description of many superfluid effects is still far from perfect.

In principle, superfluidity can be studied by confronting theoretical models with observations of NSs. One can expect that these studies will clarify the properties of superfluidity and help in resolving the main mystery of NSs, i.e. in determining the composition and equation of state of matter in their cores.

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References

1. Shapiro S L, Teukolsky S A *Black Holes, White Dwarfs, and Neutron Stars* (New York: John Wiley & Sons, 1983) [Translated into Russian (Moscow: Mir, 1985)]
2. Witten E *Phys. Rev. D* **30** 272 (1984)

3. Migdal A B *Nucl. Phys.* **13** 655 (1959)
4. Wolf R A *Astrophys. J.* **145** 834 (1966)
5. Hoffberg M et al. *Phys. Rev. Lett.* **24** 775 (1970)
6. Yakovlev D G, Levenfish K P, Shibanov Yu A *Usp. Fiz. Nauk* **169** 825 (1999) [*Phys. Usp.* **42** 737 (1999)]
7. Balberg S, Barnea N *Phys. Rev. C* **57** 409 (1998)
8. Bailin D, Love A *Phys. Rep.* **107** 325 (1984)
9. Alford M, Rajagopal K, Wilczek F *Phys. Lett. B* **422** 247 (1998)
10. Ginzburg V L, Kirzhnits D A *Zh. Eksp. Teor. Fiz.* **47** 2006 (1964) [*Sov. Phys. JETP* **20** 1346 (1965)]
11. Baym G, Pethick C, Pines D *Nature* **224** 673 (1969)
12. Andreev A F, Bashkin E P *Zh. Eksp. Teor. Fiz.* **69** 319 (1975) [*Sov. Phys. JETP* **42** 164 (1976)]
13. Alpar M A, Langer S A, Sauls J A *Astrophys. J.* **282** 533 (1984)
14. Alpar M A, in *The Lives of the Neutron Stars* (Eds M A Alpar et al.) (Dordrecht: Kluwer, 1995) p. 185
15. Haensel P, Urpin V A, Yakovlev D G *Astron. Astrophys.* **229** 133 (1990)
16. Srinivasan G et al. *Current Sci.* **59** 31 (1990)
17. Ding K Y, Cheng K S, Chau H F *Astrophys. J.* **408** 167 (1993)
18. Hsu S D H *Phys. Lett. B* **469** 161 (1999)
19. Bailin B, Love A *Nucl. Phys. B* **205** 119 (1982)
20. Chau H F *Astrophys. J.* **479** 886 (1997)
21. Alford M, Berges J, Rajagopal K *Nucl. Phys. B* **571** 269 (2000)
22. Andersson N, Kokkotas K D *Int. J. Mod. Phys. D* **10** 381 (2001); gr-qc/0010102
23. Sawyer R F *Phys. Rev. D* **39** 3804 (1989)
24. Haensel P, Schaeffer R *Phys. Rev. D* **45** 4708 (1992)
25. Jones P B *Proc. Roy. Soc. London A* **323** 111 (1971)
26. Madsen J *Phys. Rev. D* **46** 3290 (1992)
27. Haensel P, Levenfish K P, Yakovlev D G *Astron. Astrophys.* **357** 1157 (2000)
28. Lindblom L, Mendell G *Phys. Rev. D* **61** 104003 (2000)