directed momenta of equal amplitude, but also by the existence of an electric field in the p – n junction of the laser structure, which is responsible for electron and hole currents whereby electrons and holes have oppositely sensed momenta.

Therefore, for the first time observations were made of the superradiance of electrons and holes in a bulk semiconductor at room temperature. The coherent interaction of the optical field with an e – h system was attended with oscillations at a frequency of over 1 THz with a change of sign of the field amplitude. The superradiance mode in the semiconductor was accompanied by the formation of a cooperative e – h state (the domains of macroscopic polarization). The lifetime of this cooperative state is shorter than 1 ps. In this case, the coherence of interaction of the electromagnetic field with the e – h system is retained throughout periods much longer than the T_2 time. This may be caused by the pairing of electrons and holes residing in the cooperative state, their condensation, and the formation of a state similar to the BCS state of Cooper pairs in superconductors. In this case, the scattering of the pairs by each other does not result in a loss of coherence.

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Bose condensate from the standpoint of laser physics

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1. Coherence of Bose condensates and the inversion condition

Bose condensate has long been the object of keen interest of researchers. It has drawn their attention primarily in the context of the problems of superconductivity and superfluidity [1, 2]. The coherent state of a laser-generated electromagnetic field with a specific frequency and spatial configuration can also be considered as a Bose condensate of photons. Relatively recently, a new wave of interest in Bose condensate research was generated in connection with the pursuance of successful experiments to cool atoms to record breaking low temperatures of the order of 10^{-7} K [3 – 5]. For so low a temperature it has been possible to obtain a Bose condensate of atoms captured in a trap [6]. A Bose condensate of atoms is primarily of general physical interest. In the state of a Bose condensate, the wave nature of matter is much pronounced and an ensemble of particles large enough in number behaves like a classical field which possesses an amplitude and a phase.

The Bose condensate of particles has always *a priori* been assumed to be a coherent state of matter. In this case, the Bose condensation of particles was silently implied to form this coherent state automatically. But for a researcher with the mentality of a laser physicist this statement is hard to accept without proof. To take one example, the accumulation of photons in a single resonator mode in an 'underexcited' laser is possible due to spontaneous transitions, but this state of the electromagnetic field will not be coherent. The coherent state of the electromagnetic field (photons) in a laser is formed by *induced* transitions when the *self-excitation* condition is fulfilled.

Laser. According to the self-excitation condition, the emission of electromagnetic energy by the active medium of a laser should exceed the losses arising from possible absorption and dissipation inside the laser and the emergence of radiation from the laser for subsequent use. In the context of a two-level model of the laser active medium, this condition is of the form

$$\frac{N_2}{g_2} - \frac{N_1}{g_1} > \Delta N_{\mathrm{th}} \,, \qquad (1)$$

where ΔN_{th} is the threshold value of the population difference, which depends on the total loss of electromagnetic radiation inside the laser and the transparency of the output mirror. Clearly the fulfillment of similar conditions is also necessary to obtain the *coherent* state of any Bose particles. Condition (1) is sufficient for laser excitation. The necessary condition is the inequality

$$\frac{N_2}{g_2} > \frac{N_1}{g_1} \,, \qquad (2)$$

which is referred to as the 'inverse population condition.'

For interband transitions in a semiconductor laser, the condition equivalent to inequality (2) is of the form [9]

$$\mu_{\mathrm{e}} - \mu_{\mathrm{h}} > \hbar\omega \,, \qquad (3)$$

where $\mu_{\mathrm{e,h}}$ are the respective chemical potentials of electrons and holes, and $\hbar\omega$ is the energy of emitted photons. Inequality (3) testifies to the fact that the electron and hole states should be degenerate, i.e., their densities should be substantial.

Bose condensate of atoms. Let us consider the process of Bose condensation from the viewpoint of formation of a coherent state. The relationship equivalent to the inversion condition in a laser should follow from the requirement that the formation of a Bose condensate under the action of the condensate itself (induced production of coherent particles) should exceed the condensate decay rate. The result is the relationship

$$n(\varepsilon) > \left[\exp\left(\frac{\varepsilon}{kT}\right) - 1\right]^{-1} . \qquad (4)$$

The right-hand side of inequality (4) is nothing but the equilibrium distribution function of the particles outside of the Bose condensate. Consequently, even a weak disturbance of the equilibrium distribution towards the increase of the number of particles with some specific energy ε results in the formation of a state which can be termed inverse, if advantage is taken of laser terminology. Condition (4) is considerably less stringent than condition (3), since the Bose condensate is produced with a zero energy. Its coherence now becomes comprehensible: as soon as some particle escapes from the Bose condensate owing to fluctuations, it immediately restores itself by virtue of induced, i.e. coherent, transitions.

Superconducting Bose condensate. The interaction of electrons responsible for their pairing is most efficient for the electrons whose energy is close to the Fermi energy [1, 13–15], to which there corresponds a surface (the Fermi surface) in the momentum space. The interaction efficiency lowers with depth away from the Fermi surface, so that the effective number of interacting electrons is appreciably smaller than the total number of electrons. The energy diagram of a superconductor can therefore be represented as follows (Fig. 1). The superconducting condensate of Cooper pairs N lies on the pillow of unpaired electrons m. Owing to the interaction with some agent (primarily with phonons), the Cooper pairs may decompose. As a consequence, unpaired quasi-particles n form separated from the condensate of Cooper pairs by a superconducting energy gap Δ in the energy space. Bearing this diagram in mind and assuming the subsystem of free quasi-particles in the semiconductor to be quasi-equilibrium with chemical potential μ, it is possible to represent the 'inverse population condition' in the following form:

$$\mu > 0\,. \tag{5}$$

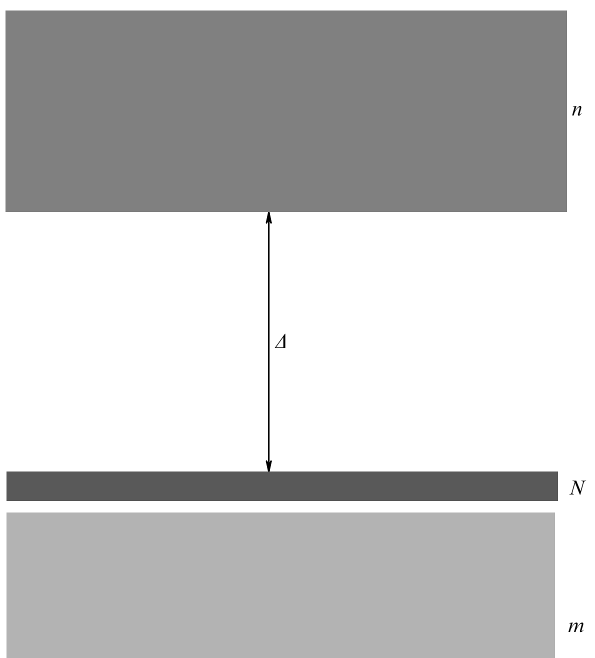

Figure 1. Energy diagram of a superconductor. Condensate of Cooper pairs (N); unpaired electrons below the Fermi level (m); free quasi-particles (n).

The condition $\mu = 0$ corresponds to the equilibrium between the condensate of Cooper pairs and the ensemble of quasi-particles, and therefore an arbitrarily weak disturbance of the equilibrium in favor of the quasi-particles results in the 'inversion' condition. We compare condition (5) with the similar condition (3) for a laser. Condition (3) is much more severe: a weak disturbance of the thermodynamically equilibrium distribution does not result in the inversion condition in lasers. The point is that laser-generated photons carry away a significant amount of energy stored in the active medium, whereas the condensate of Cooper pairs is formed with a zero energy as is the Bose condensate of atoms.

We now pass on to the description of the dynamics of Bose condensates.

2. Dynamic laser equations

To describe the laser dynamics, the following system of equations [11, 12] is commonly taken advantage of:

$$\frac{\mathrm{d}A}{\mathrm{d}t} + \left[\gamma_c + \mathrm{i}(\omega_c - \omega)\right]A = \mathrm{i}2\pi\omega B\,, \tag{6a}$$

$$\frac{\mathrm{d}B}{\mathrm{d}t} + \left[\frac{1}{\tau_2} + \mathrm{i}(\omega_0 - \omega_c)\right]B = -\mathrm{i}\,\frac{|\mu|^2}{\hbar}\,NA\,, \tag{6b}$$

$$\frac{\mathrm{d}N}{\mathrm{d}t} + \frac{1}{\tau_1}\,N = J + \frac{\mathrm{i}}{2\hbar}(AB^* - A^*B)\,, \tag{6c}$$

where A is the amplitude of the field in the resonator, B is the polarization amplitude, and $N = N_2/g_2 - N_1/g_1$, γ_c^{-1}, τ_1 and τ_2 are the relaxation times of the dynamic quantities A, N, and B, respectively.

The system of equations (6) and a modification of it has been used validly to interpret, predict, and analyze different dynamic laser regimes. The most remarkable of them is the regime of nonperiodical pulsations (dynamic chaos, Fig. 2) first discovered by Grasyuk and Oraevskiĭ [13]. Subsequently these theoretical predictions were experimentally borne out [14].

3. System of equations for the Bose condensate of atoms

To describe the dynamics of an atomic Bose condensate, many authors take advantage of the equation [1]

$$\mathrm{i}\hbar\,\frac{\partial}{\partial t}\,\psi(\mathbf{r},t) = \left[-\frac{\hbar^2}{2m}\,\Delta + V(\mathbf{r},t) + U\left|\psi(\mathbf{r},t)\right|^2\right]\psi(\mathbf{r},t)\,, \tag{7}$$

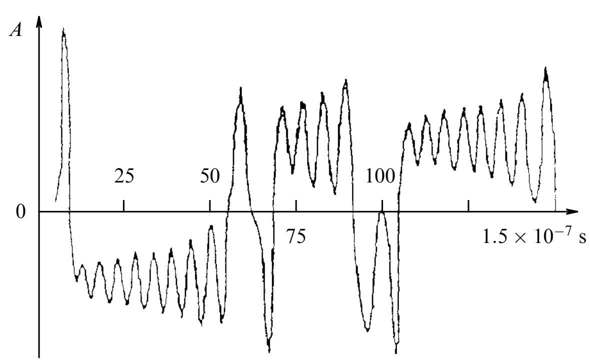

Figure 2. Temporal variation of the laser field amplitude in the mode of chaotic pulsations (calculation) [13]. Plotted on the axes are the quantities in relative units.

where $V(\mathbf{r}, t)$ is the energy of interaction of the condensate with an external field, e.g., with the fields of the trap in which the atoms are confined. U is determined by the energy of pair interaction of the particles with each other: $U = 4\pi\hbar^2 a/m$, where a is the scattering length. By its sense, the dynamic variable $\psi(\mathbf{r}, t)$ is the wave function of the subsystem of atoms which have 'fallen out' as the Bose condensate. In the limit of a sufficiently large number of atoms in the condensate, it is treated as a classical quantity and is referred to as the order parameter.

In the context of Eqn (7), the dynamics of the particles which remain beyond the Bose condensate (incoherent particles) are left 'behind the curtain.' However, the dynamics of incoherent particles become fundamentally important when atoms can flow into the trap and escape from it. It is also fundamentally important when the fields in the trap are perturbed, with the effect that the equilibrium particle distribution is disturbed. In other words, a dynamic relationship between Eqn (7) and the equation for the atoms remaining beyond the Bose condensate is called for. To unify the Bose condensate and incoherent particles in a single dynamic system, the following system of equations was proposed [8]:

$$\frac{\partial}{\partial t}\psi(\mathbf{r}, t) + \frac{\mathrm{i}}{\hbar}\left\{-\frac{\hbar^2}{2m}\Delta + V(\mathbf{r}, t) + U\Big[\big|\psi(\mathbf{r}, t)\big|^2 + N(\mathbf{r}, t)\Big]\right\}\psi(\mathbf{r}, t) = \frac{1}{2}S^{(\mathrm{R})}(\mathbf{r}, t)\,\psi(\mathbf{r}, t) + \eta(\mathbf{r}, t)\,, \tag{8a}$$

$$\frac{\partial}{\partial t}n_p(\mathbf{r}, t) + \frac{\partial}{\partial \mathbf{r}}n_p(\mathbf{r}, t)\frac{\partial \varepsilon}{\partial \mathbf{p}} - \frac{\partial}{\partial \mathbf{p}}n_p(\mathbf{r}, t)\frac{\partial \varepsilon}{\partial \mathbf{r}} = -S_p^{(\mathrm{R})}(\mathbf{r}, t) + S_p^{(\mathrm{C})}(\mathbf{r}, t) + Q_{\mathrm{in}}(\mathbf{r}, \mathbf{p}, t) - Q_{\mathrm{out}}(\mathbf{r}, \mathbf{p}, t)\,, \tag{8b}$$

where $N(\mathbf{r}, t) = \int n(\mathbf{r}, \mathbf{p}, t)\,\mathrm{d}^3p/(2\pi\hbar)^3$ is the density of incoherent quasi-particles [15] and $n_p(\mathbf{r}, t)$ is their momentum distribution. $S_p^{(\mathrm{R})}$ and $S^{(\mathrm{R})} = \int S_p^{(\mathrm{R})}\,\mathrm{d}^3p/(2\pi\hbar)^3$ are recombination integrals which describe the dynamic exchange between the subsystems of coherent and incoherent particles, and $\eta(\mathbf{r}, t)$ is a random Langevin force caused by spontaneous transitions.

The system of equations (8) can serve as the basis for writing the dynamic system of equations for a so-called atomic laser, i.e., the source of a beam of coherent particles emanating from a trap [8]. Under amply justified assumptions, the system of equations for an atomic laser can be shown to reduce to the form [16]

$$\frac{\mathrm{d}A}{\mathrm{d}t} = \big[\sigma_0(N - N_0) - \gamma\big]A\,, \tag{9a}$$

$$\frac{\mathrm{d}N}{\mathrm{d}t} = -2\sigma_0(N - N_0)A^2 - vN + Q\,, \tag{9b}$$

where A is the modulus of the order parameter averaged over the trap volume, N and N_0 are the average and critical densities of incoherent particles, σ_0 is the constant for the interaction between incoherent particles and the Bose condensate, and γ and v are the attenuation coefficients arising from the escape of coherent and incoherent particles from the trap.

The system of equations (9) for an 'atomic laser' is isomorphous with the system of equations for a laser with a broad amplification line [12]. The theory of relaxation laser pulsations [12, 17] is therefore fully applicable to the 'atomic laser.' In particular, by varying the amplitude of the radio-frequency field in trap, it is possible to excite deep pulsations of the atomic order parameter. The manifestation will be the emergence of a coherent atomic beam outside the trap in the form of regular repetitive bunches. It is also possible to bring the 'atomic laser' to the regime of dynamic chaos, as was previously done with a CO_2 laser [18]. The experimental investigation of dynamic chaos in an atomic trap is a fascinating problem. It would be highly instructive to observe how the chaotic pulsations of the order parameter inside the trap show up in its spatial configuration.

In summary we note that in the context of the system of equations (8) it is possible to describe not only the dynamics of a previously formed Bose condensate, but also its formation process during the cooling of atoms in a trap.

4. Dynamic system of equations for a superconductor

By taking advantage of the model given in Fig. 1 and of the particle conservation law, it is possible to write the equations which unite the order parameter, free quasi-particles, and phonons in a single dynamic system [19]. The equation for the complex order parameter is of the form

$$\frac{\partial \Psi(\mathbf{r}, t)}{\partial t} - \frac{1}{\tau_m}\left(1 - 2\frac{n(\mathbf{r}, t)}{N_s} - 4\frac{\big|\Psi(\mathbf{r}, t)\big|^2}{N_s}\right)\Psi(\mathbf{r}, t) + D\left(\nabla - \mathrm{i}\frac{2e}{\hbar c}\mathbf{A}(\mathbf{r}, t)\right)^2\Psi(\mathbf{r}, t) = \frac{1}{2}S^{(\mathrm{R})}\Psi(\mathbf{r}, t) + \Xi(r, t)\,, \tag{10}$$

$$S^{(\mathrm{R})}(\mathbf{r}, t) = \iint W(\mathbf{p}, \mathbf{p}')\big[n_p n_{p'}(M_{p-p'} + 1) - (1 - n_p)(1 - n_{p'})M_{p-p'}\big]\,\mathrm{d}^3p'\,\mathrm{d}^3p\,, \tag{11}$$

$n(\mathbf{r}, t) = \int n_p(\mathbf{r}, t)\,\mathrm{d}^3p/(2\pi\hbar)^3$, $N_s/2$ is the density of superconducting electrons at absolute zero, τ_m is the temporal parameter determined by interelectron interactions, and $\Xi(r, t)$ is a random Langevin force caused by spontaneous transitions.

The spectral density of free quasi-particles $n_p(\mathbf{r}, t)$ satisfies the kinetic equation [20]

$$\frac{\partial}{\partial t}n_p(\mathbf{r}, t) + \frac{\partial}{\partial \mathbf{r}}n_p(\mathbf{r}, t)\frac{\partial \varepsilon}{\partial \mathbf{p}} - \frac{\partial}{\partial \mathbf{p}}n_p(\mathbf{r}, t)\frac{\partial \varepsilon}{\partial \mathbf{r}} = -S_p^{(\mathrm{C})}(\mathbf{r}, t) - S_p^{(\mathrm{R})}(\mathbf{r}, t)\,. \tag{12}$$

The energy of quasi-particles in this equation is defined by the well-known BCS relationship. $S_p^{(\mathrm{C})}$ is the collision integral and $S_p^{(\mathrm{R})}$ is the recombination integral which describes the transitions between the subsystems n and $(N + m)$ (see Fig. 1) [20]. The recombination integrals involve, along with the density of free quasi-particles, the phonon density M_{q}. If the phonon density is nonequilibrium, it also necessitates a dynamic equation. It is quite frequently written in the following simple form [20]:

$$\frac{\partial M_{\mathrm{q}}}{\partial t} = -\frac{1}{\tau_{\mathrm{q}}}(M_{\mathrm{q}} - M_{\mathrm{q}0}) + S_{\mathrm{q}}^{(\mathrm{R})} + S_{\mathrm{q}}^{(\mathrm{C})} + Q_{\mathrm{q}}\,. \tag{13}$$

The first term on the right-hand side of this equation describes the transfer of phonons from the superconducting sample to the substrate, the second one describes the production of phonons in the recombination of free quasi-particles, the third one arises from the scattering of quasi-particles, and the fourth one describes the excitation of phonons by an external source. M_{q0} is the equilibrium phonon density for a given temperature.

If required, it is possible to write a recombination integral in which photons would appear in lieu of phonons (or along with them). Clearly the system of equations (10), (12), and (13) should be complemented with the Maxwell equations, which will be connected to it by the superconducting current [1] and the density of photons, if the latter act on the superconducting sample.

Eventually there results a comprehensive dynamic system with a multidimensional phase space. That is why a wide diversity of dynamic regimes are possible in a semiconductor, which are of interest both from theoretical and practical viewpoints.

5. Concluding remarks

We emphasize once again that the decisive role in the production of a Bose condensate of any nature is played by induced transitions. We believe that they are the universal mechanism responsible for the violation of symmetry in Nature. In order for this to happen, conditions are necessary whereby the induced process of production of some object with an inherent feature would exceed its decay process. It seems likely that induced processes have played a crucial role in the formation of our Universe with the symmetry broken in favor of electrons and protons. A similar supposition can be made as to the origin of life with the left chirality of protein molecules. Suchlike suppositions have been discussed in the literature.

The processes of self-organization occurring in nonlinear systems have been discussed in the literature for about 20 years. A special term — synergetics — was invented to unify a diversity of self-organization processes in a common realm. But this is a unification of processes by a superficial feature. In our opinion, induced transitions are the mechanism which unifies self-organization processes.

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