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Entangled quantum states of atomic systems

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Contents

Abstract. The current status of studies in the physics of entangled states of atomic systems — an interdisciplinary field involving quantum optics, quantum information, and the foundations of quantum mechanics — is reviewed. In the first part of the review, an introduction to the theory of entangled states is given, their properties and applications are described. In the second part, experiments on the creation and detection of entangled states in atomic systems are discussed along with associated experimental proposals for their refinement. Today's most advanced experimental technique for creating entangled ion states in an ion trap is considered, and promising methods focussed on the analogous states of neutral atoms are analyzed.

1. Introduction

The ways of scientific thought are often unpredictable. Some ideas quickly flourish and fade into the background, whereas others slowly develop for decades and then enjoy an explosion of new interest. About ten years ago, such a metamorphosis occurred in the field of physics devoted to *entangled quantum states*.

The first attempts to analyze the properties of entangled states were made by Einstein et al. [1] and Schrödinger [2] as early as the 1930s. At that time, quantum theory was still in the making, so unusual properties of new objects of quantum theory stirred doubts about the adequacy of the quantum-mechanical picture of the world. [1] In particular, the purpose of the famous paper by Einstein, Podolsky, and Rosen [1] was to prove the incompleteness of the quantum-mechanical description of the world using the example of entangled states. For the several decades since, entangled states have been intrinsically connected to discussions on the foundations of quantum mechanics [4]. For example, many papers have been devoted to the possibility of describing the world with local hidden-variable theories which provide an alternative to the conventional quantum theory. Contrary to Einstein's expectations, entangled states proved to be the primary objects speaking *in favor* of quantum theory. Notice, however, that this conceptual argument has not yet received an unconditional experimental resolution [5].

Other authors, in contrast, stress the specific nonlocal nature of entangled states, which even resulted in a certain mystification of these states. Thus, the unusual (from the classical point of view) properties of entangled states, combined with the projection postulate of quantum mechanics, led to speculations about 'action at a distance', which contradict the postulates of special relativity. Superficial statements of this kind were, of course, fallacious (see, e.g., Refs [6, 7]). However, a more thorough analysis of this problem, respecting the restrictions of special relativity, can easily trigger serious discussions (see, for example, Refs [8 – 10]).

Despite the traditional fundamental significance of entangled states, the present surge of interest in them was caused by entirely new physical problems. During the last few years, entangled states have become an important object for

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[1] A popularized account of the early arguments on the essence of quantum mechanics can be found in recent book [3].

many new applied disciplines: quantum cryptography [11], quantum information theory [12], and the physics of quantum computing [13]. From time to time, novel uses of entanglement are proposed, such as the idea of employing entangled states to synchronize spatially separated clocks [14, 15]. Accordingly, scientists now tend to view entangled states as a special *nonlocal quantum resource* which has many applications. In this situation, one naturally faces the question of how to produce and store this new resource.

The processes of cascade decay of atomic excitations [16] and spontaneous parametric down-conversion of light in nonlinear crystals [17] are the traditional sources of entangled states. Using these processes, one can create photon pairs with entangled polarization states. The created pairs can then be easily controlled with the simplest devices of linear optics — mirrors and polarizers (see, e.g., Ref. [18]). However, these sources of entangled states have a number of drawbacks.

On the one hand, the decay of a high-frequency photon into a photon pair and the relaxation of atomic excitations are both random processes because they are triggered by quantum fluctuations in the medium under study. On the other hand, the created photon pairs propagate at the speed of light, which makes them difficult to localize and store for later use. This is why so much attention has been paid recently to so-called deterministic methods for creating entangled states of *massive* particles — primarily of individual atoms and ions held in traps of corresponding types. The word 'deterministic' here denotes the possibility of creating the required entangled state of given particles at any preassigned moment of time [19].

Note that the two above-mentioned 'carriers' of entangled states — photons and atoms or ions — in principle nicely complement each other [20]. Indeed, if the current research efforts produce a reliable experimental technique for creating and manipulating entangled atomic states and for controllably coupling these atoms to light, a perfect experimental base for all applications of entangled states will emerge. Atoms will then enable long-term storage of specific quantum information, while photons will provide a means for transmitting this information over virtually any distances (see, e.g., Ref [21]). If the possibility of realizing quantum computers is added to this picture, one can speak about the creation of the 'quantum Internet' [22].

The subject of entangled states has been touched upon in many reviews of modern quantum theory (see, for example, Refs [4, 6 – 8, 23, 24] and the relevant chapters in books [3, 9, 12, 25, 26]). However, these treatments are rather general, whereas the development of the applications of entangled states invites a more in-depth analysis of their properties and methods for their production. At the same time, the problems related to the creation of photon pairs in nonlinear conversions and cascade decay processes have already attracted a good deal of attention (see, e.g., Refs [16, 17, 27 – 29]). Entangled states of atoms and ions are, on the contrary, a relatively new subject, almost neglected until the beginning of 1990s. Therefore, we considered it necessary to devote this review exactly to atomic systems. In doing so, we will limit ourselves to entangled states of systems of individual (i.e. individually addressable) atoms and ions as these systems are currently considered the best candidates for implementing the practical applications of entangled states [20, 30].

For these reasons, the scope of this review excludes the actively studied technique for creating pseudoentangled states of nuclear spins in organic molecules, based on methods of nuclear magnetic resonance (NMR) (see review [31]), as well as various techniques for creating entangled states in solid-state systems, which are most promising in the long-term perspective [32 – 34].

Most papers on entangled states can be found in the Quantum Physics section of the Los Alamos National Laboratory e-print archive [35]. This is why some references in this review, to papers yet unpublished, have the form quant-ph/$*$, where the asterisk stands for the identification number of the paper in this e-print archive.

In conclusion we would like to make a linguistic remark. In the context of this review, the term 'entangled' refers to the intercorrelation of physical systems in these states, their specific entwinement or interconnection. Interestingly enough, the word 'entangled' also bears connotations of muddle and confusion. This is rather symbolic because new readers often acquire a similarly jumbled impression of this popular field. We hope that this review will allow the reader to form adequate ideas about the physics of entanglement.

2. Theory of entangled states

In many fields of physics, theoretical studies far outpace experiment. This is also the situation in virtually all fields of physics related to quantum information (see reviews [23, 36] and book [25]), including the physics of entanglement. While new theoretical results appear almost every day, significant experimental advances are much rarer. In addition, the burgeoning theory of entangled states is not homogeneous. One can distinguish in it at least two directions: (1) mathematical studies of the properties of entangled states, and (2) experimental proposals for the creation and utilization of entangled states. In the first two sections of this review, we consider the formal aspect of the theory and applications of entanglement; the following sections are devoted to experiments and experimental proposals.

2.1 Quantum correlations

As noted above, entanglement is simply a special, quantum form of correlation, which however has a number of specific features distinguishing it from classical correlations. It is convenient to consider these differences using some illustrative example; we will take it to be a system of two spin-1/2 particles in the state with zero total spin (the singlet state). After Einstein, Podolsky and Rosen, who introduced a state with similar properties more than 60 years ago [1], this state is sometimes called the EPR state. Accordingly, a system of spin-1/2 objects in the state with zero total spin is termed an EPR pair, and the corresponding correlations are referred to as EPR correlations.

The EPR state is given by

$$|\psi_{\mathrm{EPR}}\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle_1 \otimes |\downarrow\rangle_2 - |\downarrow\rangle_1 \otimes |\uparrow\rangle_2\right), \qquad (1)$$

where $|\uparrow\rangle_i$ and $|\downarrow\rangle_i$ with $i = 1, 2$ are the wave functions of the ith spin aligned 'up' and 'down', respectively. The $\otimes$ symbol denotes the operation of direct (tensor) product, which is necessary to describe compound quantum systems [37].

Since the total spin of the system in the EPR state is zero, the spins of individual particles are always antiparallel, i.e. anticorrelated. At first sight, there is nothing particularly quantum in such a state: one can give many examples of

similarly anticorrelated states of classical systems. The difference arises when we begin to describe the properties of EPR correlations using the rules of quantum mechanics: the operator representation of physical quantities, which implies noncommuting physical variables, and the corresponding formula for quantum averages:

$$\langle \hat{A} \rangle_{\psi} = \langle \psi | \hat{A} | \psi \rangle \,. \tag{2}$$

Here, $\hat{A}$ is the operator of the quantity in question, and ψ is the wave function of the system considered. The superposition principle for quantum wave functions results in the specific property of *coherence*, i.e. the ability of quantum states to interfere. At the same time, the noncommutativity of physical variables leads to the fact that if some physical quantities are definite in a given state, then others should contain fluctuations necessitated by the internal indeterminacy of quantum states.

The anticorrelation of the spins of individual particles in state (1) is manifested in the condition

$$\langle \hat{\sigma}_1^{(z)} \otimes \hat{\sigma}_2^{(z)} \rangle_{\psi_{\mathrm{EPR}}} = \langle \psi_{\mathrm{EPR}} | \hat{\sigma}_1^{(z)} \otimes \hat{\sigma}_2^{(z)} | \psi_{\mathrm{EPR}} \rangle = -1 \,, \tag{3}$$

where $\sigma_i^{(z)}$ $(i = 1, 2)$ are the Pauli operators (matrices) proportional to the projection operator of the ith spin onto the chosen quantization axis z. Since the total spin of the system is zero, the spin projections of the particles onto any other directions in space should be anticorrelated as well. In particular, this should hold for the x- and y-axes of the chosen Cartesian system of coordinates:

$$\left\langle \hat{\sigma}_1^{(x)} \otimes \hat{\sigma}_2^{(x)} \right\rangle_{|\psi\rangle} = \left\langle \hat{\sigma}_1^{(y)} \otimes \hat{\sigma}_2^{(y)} \right\rangle_{|\psi\rangle} = -1 \,. \tag{4}$$

The similarity of equations (3) and (4) arises from the fact that the system states with vanishing total spin are invariant with respect to any rotations of the frame of reference. (In the case of the EPR state, this can be easily verified by explicit calculations.) Taken together, these equations express a characteristic feature of quantum correlations — the simultaneous (anti)correlation of complementary (noncommuting) observables of the constituent subsystems.

For a system of two spin-1/2 objects, there is only one quantum states with vanishing total spin and the properties described above — the EPR state. Therefore any attempts to construct a quasi-classical analogue of the EPR state, which would have the same properties, are bound to fail. Indeed, by analogy with coherent quantum superposition (1), let us consider a classical statistical mixture of two states with antiparallel particle spins. Like any statistical mixture, this state is described by a density matrix (statistical operator)

$$\hat{\rho}_{\mathrm{cl}} = \frac{1}{2} \left(|\uparrow\downarrow\rangle \langle\uparrow\downarrow| + |\downarrow\uparrow\rangle \langle\downarrow\uparrow| \right) = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1/2 & 0 & 0 \\ 0 & 0 & 1/2 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}. \tag{5}$$

Here, the matrix is written in the $\{|\uparrow\uparrow\rangle, |\uparrow\downarrow\rangle, |\downarrow\uparrow\rangle, |\downarrow\downarrow\rangle\}$ basis, where for simplicity we omitted the tensor product signs and the subscripts numbering individual spins. The diagonal elements of matrix (5) govern the joint probability distribution of two completely anticorrelated classical random variables, which can assume two equiprobable values relevant to the 'up' and 'down' states of the spin.

Writing down the density matrix corresponding to the $|\psi_{\mathrm{EPR}}\rangle$ pure state, viz.

$$\hat{\rho}_{\mathrm{EPR}} = |\psi_{\mathrm{EPR}}\rangle\langle\psi_{\mathrm{EPR}}| = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1/2 & -1/2 & 0 \\ 0 & -1/2 & 1/2 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \tag{6}$$

we see that the difference between matrices (5) and (6) is limited to two off-diagonal matrix elements equal to $-1/2$. However, these very elements give rise to the fact that, unlike $\hat{\rho}_{\mathrm{EPR}}$, the $\hat{\rho}_{\mathrm{cl}}$ state is not rotation-invariant and does not exhibit the anticorrelation of the spin projections onto the x- and y-axes:

$$\left\langle \hat{\sigma}_1^{(x)} \hat{\sigma}_2^{(x)} \right\rangle_{\hat{\rho}_{\mathrm{cl}}} = \left\langle \hat{\sigma}_1^{(y)} \hat{\sigma}_2^{(y)} \right\rangle_{\hat{\rho}_{\mathrm{cl}}} = 0 \,, \tag{7}$$

as easily verified by an explicit calculation. Therefore, the quantum nature of the correlations in the EPR state is determined by the off-diagonal elements of the density matrix (6), which make this state coherent.

If we use the language of quantum optics and, namely, consider formula (1) to be the joint state of two two-level atoms, we can illustrate the quantum nature of the corresponding correlations in another way. Condition (3) then signifies the anticorrelation between the diagonal elements of the density matrices of the atoms, i.e. populations, whereas condition (4) implies the anticorrelation between the off-diagonal elements of the density matrices, i.e. the so-called coherences which are responsible for the quantum interference effects and do not commute with populations.

Thus, the rule of averaging in quantum mechanics (2), involving wave functions and implying interference, and all relations arising from the operator representation of physical quantities result in the need to describe some correlated states of compound quantum systems by coherent superpositions of type (1), which have no classical counterpart.

In conclusion we note that the notions of entangled states and quantum correlations apply not only to quantum systems that consist of physically identical particles (two photons, two spins, etc.) but also to heterogeneous compound systems, such as an atom and a photon (Fig. 1) or to different degrees of freedom of the same physical object (for example, internal and translational degrees of freedom of an atom).

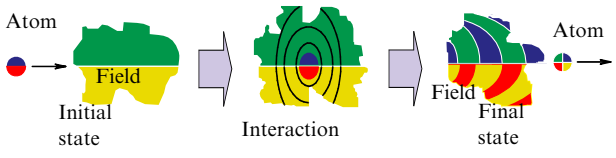

Figure 1. Illustration to the process of creating entanglement between an atom and electromagnetic field. Initially independent states of the atom and the field become entangled in the interaction area and remain entangled even after the cessation of the interaction and the spatial separation of the atom and the field. Adapted from Ref. [23].

2.2 Pure entangled states

As follows from Section 2.1, quantum correlations are related to coherent superposition states of compound quantum systems. For this reason, we will formulate the first and the simplest definition of entangled states for *pure* quantum states.

A pure state of a compound quantum system $\mathcal{Q}=\mathcal{A}+\mathcal{B}+\ldots$ is called entangled if its wave function cannot be factored into a tensor product of the wave functions of its constituent parts:

$$\Psi_{\mathcal{Q}} \neq \Psi_{\mathcal{A}} \otimes \Psi_{\mathcal{B}} \otimes \ldots \tag{8}$$

Obviously, wave functions that can be factored to form (8) contain no correlations whatsoever because any operator is then averaged independently over each constituent part.

So, pure quantum states can be either quantum-correlated (entangled) or altogether uncorrelated. At the same time, the EPR state introduced in Section 2.1 is not just entangled (the impossibility to factor this state as the tensor-product state follows from its invariance with respect to reference system rotations), but *maximally entangled*. That is, the EPR state exhibits the additional property

$$\mathrm{Tr}_2\,\hat{\rho}_{\mathrm{EPR}} = \mathrm{Tr}_1\,\hat{\rho}_{\mathrm{EPR}} = \frac{1}{2}\,\hat{1}\,, \tag{9}$$

where Tr_i denotes the partial trace over the degrees of freedom of the ith spin.

By definition, a pure state of a *bipartite* quantum system $\mathcal{Q}=\mathcal{A}+\mathcal{B}$ is called maximally entangled if the density matrix that results from performing the partial trace (partial averaging) over any of the constituent parts of the quantum system is proportional to the identity matrix. The resulting partial density matrices $\hat{\rho}_{\mathcal{A}} = \mathrm{Tr}_{\mathcal{B}}\,|\psi\rangle\langle\psi|$ and $\hat{\rho}_{\mathcal{B}} = \mathrm{Tr}_{\mathcal{A}}\,|\psi\rangle\langle\psi|$ describe the states of the parts of the quantum system, considered separately.

In the case of entangled states, the partial density matrices $\hat{\rho}_{\mathcal{A}}$ and $\hat{\rho}_{\mathcal{B}}$ have nonzero quantum entropy $S(\hat{\rho}) = \mathrm{Tr}\,\hat{\rho}\log\hat{\rho}$, whereas the entropy of the pure state of the compound quantum system is zero: $S(|\psi\rangle\langle\psi|)=0$. This testifies that the fluctuations of the parts of the compound system are interrelated. The degree of their correlation is higher, the more random they are considered separately, because the fluctuations in both individual parts arise from the same source — the purely quantum fluctuations of the compound system. One can say that the purely quantum fluctuations in the pure entangled state $|\psi\rangle$ of the compound quantum system reduce to the classical fluctuations of the separately considered subsystems. Therefore the original, quantum fluctuations can be described by the entropy of the resulting marginal probability distributions (partial density matrices).

In general, one can rigorously show [38] that the entanglement of a bipartite pure entangled state can be quantified by a simple measure

$$\mathcal{E}\bigl(|\psi\rangle\bigr) = S(\hat{\rho}_{\mathcal{A}}) = S\bigl(\mathrm{Tr}_{\mathcal{B}}\,|\psi\rangle\langle\psi|\bigr)\,. \tag{10}$$

Since for pure states the entropies of the partial density matrices are identically equal: $S(\hat{\rho}_{\mathcal{A}}) = S(\hat{\rho}_{\mathcal{B}})$ (see, for example, Ref. [12]), definition (10) can as well apply to matrix $\hat{\rho}_{\mathcal{B}}$. In the case of tensor-product states, measure (10) obviously vanishes because the partial trace then yields a pure state. The measurement unit of entanglement is determined by the base of the logarithm function that appears in the entropy definition, similarly to measurement units of information. The term *e-bit* (entanglement bit) is sometimes used in the scientific literature for the case of the binary logarithm.

The definitions of entangled states and the entanglement measure introduced above will be consistent only if they satisfy a number of conditions that qualify entanglement as a nonlocal quantum resource [39]. In particular, these definitions should be invariant with respect to local quantum operations and classical communication (LQCC). Let us consider LQCC operations in more detail.

Suppose that the two parts of an entangled quantum system (for example, the two spins of an EPR pair) are distributed among two different persons. Following the established tradition, we will call them Alice and Bob. Assume further that Alice and Bob are spatially separated and can communicate using classical means only, i.e. they cannot realize direct coherent interaction between the parts of the compound quantum system or hand them over to each other.

At the same time, Alice and Bob can subject their own parts of the compound quantum system to any local quantum operations, namely, unitary or superoperator transformations [40] and quantum measurements, including nonorthogonal ones [41]. Finally, they can transmit the results of these measurements, as well as any other classical information, over classical information channels. The mathematical structure of thus defined LQCC operations is very complex [42]; however, it is completely justified from the practical point of view.

It was shown in paper [39] that LQCC operations can never increase, on average, the entanglement measure (10) of a given entangled state. Therefore, entanglement cannot be created using local means only, which agrees with the significance of entanglement as a nonlocal quantum resource.

Although entanglement of pure states cannot be created out of nothing, it can be diluted and concentrated [38]. More rigorously, this property is formulated as follows. Let N copies of the bipartite quantum systems (for example, pairs of spin-1/2 objects) be distributed between Alice and Bob. Each of these systems is in an entangled pure state $|\psi\rangle$, so that the total entanglement equals $N\mathcal{E}(|\psi\rangle)$. Then, there exist such LQCC operations that can transform N original bipartite systems into M bipartite systems [if $N\neq M$, the physical objects (spin pairs) themselves are added or discarded as necessary] each of which will be in the state $|\psi'\rangle$, and the total entanglement $M\mathcal{E}(|\psi'\rangle)$ will be conserved in the $N\to\infty$ limit:

$$\lim_{N\to\infty}\frac{M\mathcal{E}\bigl(|\psi'\rangle\bigr)}{N\mathcal{E}\bigl(|\psi\rangle\bigr)} = 1\,. \tag{11}$$

Therefore, asymptotically as the number of original objects tends to infinity, entanglement can be reversibly transformed into more or less entangled states, as quantified by measure (10). In particular, one can transform original nonmaximally entangled physical objects into a smaller number of maximally entangled objects, and thereby make them more useful for practical applications, while preserving the total amount of entanglement. This property of asymptotic reversibility distinguishes measure (10) among all possible entanglement measures of pure states. Notice that if one considers irreversible protocols of entanglement transformation, other measures of entanglement become adequate as well [43, 44].

2.3 Multipartite entangled states

Pure entangled states of bipartite systems, considered above, represent the simplest case of quantum-correlated systems. Generalizations of the introduced notions and properties to the case of mixed states or multipartite systems are nontrivial and often ambiguous. This subject is currently being actively investigated, and its conceptual framework is not yet fully

established. For these reasons, we will restrict ourselves to the generally accepted results from the scientific literature on this subject, whose volume increases almost daily. [2]

The most popular entangled state of tripartite systems is the GHZ state

$$|\psi_{\mathrm{GHZ}}\rangle = \frac{1}{\sqrt{2}}\left(|000\rangle + |111\rangle\right) \tag{12}$$

named after the authors (Greenberger, Horne, and Zeilinger) of the paper [45] in which the properties of this state were first considered. In the case of multipartite quantum systems, the role analogous to the EPR and GHZ states is played by the so-called Schrödinger-cat states [2] (see also Refs [7–9]). An example of the Schrödinger-cat state is given by

$$|\psi_{\mathrm{cat}}\rangle = \frac{1}{\sqrt{2}}\left(|\underbrace{000\ldots0}_{N}\rangle + |\underbrace{111\ldots1}_{N}\rangle\right). \tag{13}$$

It is implied that, due to the large number N of involved particles, states $|000\ldots0\rangle$ and $|111\ldots1\rangle$ describe different states of a macroscopic object. Consequently, state (13) applies to a coherent superposition of macroscopically different states of a large object, in much the same way as the superposition of 'dead' and 'alive' states of a cat that E Schrödinger introduced in Ref. [2]. Recently, such states of systems that consist of relatively few particles have become the object of many studies owing to the appearance of experimental possibilities for their creation (see Section 3).

One could think that multipartite pure entangled states are just as equivalent, i.e. reversibly transformable to each other, as different bipartite states are. For example, one may conjecture that two GHZ triplets are equivalent to three EPR pairs. However, this is not true. There are no LQCC operations reversibly transforming these systems into each other, even asymptotically as the number of copies tends to infinity [46]. Generally, an n-partite quantum system has such entangled states that cannot be reversibly transformed into entangled states of systems that involve a lower number of particles, $m < n$. Therefore, we may say that n-particle entanglement cannot be reduced to lower order entanglement [46]. Schrödinger-cat states (13) provide the very one example of such irreducible n-particle entangled states.

2.4 Mixed entangled states

As noted above, in the case of pure states all correlations are quantum, i.e. they correspond to entanglement. On the contrary, mixed states which are described by density matrices are to some extent analogues with classical statistical ensembles because they can also include classical correlations.

To separate classical correlations from quantum, one introduces *separable*, i.e. classically correlated, quantum states. By definition, separable states $\hat{\rho}_{\mathrm{sep}}$ can be represented in the form

$$\hat{\rho}_{\mathrm{sep}} = \sum_i \hat{\rho}_{\mathcal{A}}^{(i)} \otimes \hat{\rho}_{\mathcal{B}}^{(i)} \otimes \ldots, \tag{14}$$

where $\hat{\rho}_{\mathcal{S}}^{(i)}$ are some density matrices of the constituent parts of the quantum system considered ($i = 1, \ldots, N$; $\mathcal{S} = \mathcal{A}, \mathcal{B}, \ldots$). Each term in sum (14) describes statistically independent (multiplicative) states of subsystems $\mathcal{A}, \mathcal{B}, \ldots$ of the compound quantum system $\mathcal{Q}$. The summation over index i (incoherent mixing) gives rise to the classical correlations.

Density matrices (14) can be prepared locally — that is, using local quantum operations along with classical communication and without having any entangled states from the start. Exactly in this sense separable states are not quantum-correlated. [3] States that cannot be represented in form (14) are called nonseparable or *mixed entangled* states.

Many different entanglement measures can be introduced for mixed entangled states of bipartite systems (see, e.g., Refs [39, 48–50]). All of them are justified in some way or another, but there is no such universal entanglement measure as in the case of pure states. From the practical point of view, *entanglement of formation* and *distillable entanglement* are the most interesting entanglement measures for mixed states [39].

The entanglement of formation is defined as the minimum average entanglement of pure-state ensembles that realize the given mixed state:

$$\mathcal{E}_{\mathrm{form}}(\hat{\rho}) = \min_{\hat{\rho}=\sum_i p_i|\psi_i\rangle\langle\psi_i|} \sum_i p_i\, \mathcal{E}\left(|\psi_i\rangle\right). \tag{15}$$

Here, $\mathcal{E}(|\psi_i\rangle)$ is the entanglement (10) of pure states $|\psi_i\rangle$ that compose the quantum ensemble $\{p_i, |\psi_i\rangle\}$ which realizes the mixed state $\hat{\rho}$. The minimization in formula (15) is performed over all possible realizations of the state $\hat{\rho}$ by ensembles of pure states.

Thus, entanglement of formation is the minimum amount of pure entanglement necessary to create the given state with the help of local quantum operations and classical communication. States that have zero entanglement of formation are separable and vice versa. Note that the minimization in definition (15) is nontrivial — so far, an analytical formula for the entanglement of formation of arbitrary mixed states has been derived only for the case of two entangled qubits (two-level systems) [51].

The distillable entanglement $\mathcal{E}_{\mathrm{dist}}(\hat{\rho})$, in its turn, is defined as the maximum amount of pure entanglement that can, in principle, be extracted (distilled) from the given mixed state with the help of LQCC operations asymptotically as the number of copies of the original state tends to infinity. This process of extracting the pure entanglement from mixed entanglement is called entanglement purification [39]. Since this definition of distillable entanglement is based on some hypothetical optimal purification protocol, the mathematical formula for this measure remains to be derived. At present, several protocols for distillation of pure entanglement from mixed states are known [39, 52–54]. However, there are no reasons to believe that any of them is optimal.

The above-mentioned entanglement measures are intended to be of use in practical creation and utilization of mixed quantum-correlated states. A totally different approach is taken when one first postulates some axiomatic properties to which any entanglement measure should satisfy, and then selects an appropriate function. Like the entanglement measure of pure states, any entanglement measure $\mathcal{E}(\hat{\rho})$ of mixed states should comply with a number of conditions that qualify mixed-state entanglement as a nonlocal resource [39, 55]:

• $\mathcal{E}(\hat{\rho}) = 0$ for separable states $\hat{\rho}$ and $\mathcal{E}(\hat{\rho}) \geqslant 0$ otherwise;

[2] One can always find the latest results in the e-print archive of the Los Alamos National Laboratory [35].

[3] Note that in some cases this interpretation of quantum correlations is questionable [42, 47].

• all entanglement measures should be invariant with respect to local unitary transformations;
• no entanglement measure can increase, on average, as a result of physically realizable (i.e. superoperator) local operations and, consequently, any LQCC operations.

These conditions already place serious restrictions on the choice of entanglement measures [56]. If they are complemented by other nontrivial requirements, new interesting theoretical results may follow. For example, adding the conditions of asymptotic additivity and continuity of distillable entanglement, Horodecki et al. [55] have shown that any 'good' entanglement measure should be bounded by the entanglement of formation from above, and the distillable entanglement from below.

Notice that apart from the search for adequate entanglement measures of mixed states, there exists a less general approach to the problem of characterizing mixed entangled states. This is the search for criteria of nonseparability [$\mathcal{E}_{\rm form}(\hat{\rho}) > 0$] and distillability [$\mathcal{E}_{\rm dist}(\hat{\rho}) > 0$] of mixed states $\hat{\rho}$. Of special interest here is the structure of *bound entangled states*, which have $\mathcal{E}_{\rm form}(\hat{\rho}) > 0$ but $\mathcal{E}_{\rm dist}(\hat{\rho}) = 0$ (see, e.g., Ref. [57]). Many interesting results have already been produced in this field [58 – 60], but the ultimate answers to the posed questions are as yet unknown.

Thus, the problem of adequate entanglement measures and criteria is still unsolved for mixed states. The situation becomes even more difficult in the case of various generalizations of the systems considered, such as entangled states of continuous-variable systems [61] or multipartite systems [62 – 64]. The recently posed problem of nonlocal properties (entanglement) of *quantum operations* [65] provides yet another nontrivial extension of the theory.

3. Applications of entangled states

The rebirth of the physics of entangled states over the last few years derives solely from the new practical applications of entanglement. In particular, such applications as quantum computing and improved frequency standards stimulate the development of new methods for creating entangled states of atoms and ions, which are the subject of the subsequent sections of this review. Below, we briefly discuss the presently known fields of applications of entangled states. A more detailed description can be found in the cited literature as well as in reviews [23, 36] and book [25].

3.1 Studies of the physical foundations of quantum mechanics

The experimental verification of the Bell inequalities grew the first serious application of entangled states. Since the paper of Einstein, Podolsky and Rosen [1], it became clear that the laws of quantum mechanics are incompatible with some habitual classical conceptions, such as the notion of locality of physical effects. However, it took 30 years to elucidate this discrepancy mathematically, which was achieved by John Bell with his famous inequalities [66]. The Bell inequalities were designed to test the so-called weak nonlocality of quantum mechanics, which states that the outcome of some physical experiment can in some sense influence the outcome of another experiment even if the two events are separated by a spacelike interval (for details, see reviews [4, 7, 8]).

After the appearance of the first Bell inequalities, many similar relations have been derived, and many experiments confirming the weak nonlocality of quantum mechanics have been carried out [4]. For example, it was shown theoretically that any bipartite pure entangled state violates some Bell type inequality [12], and that in the case of GHZ states one can formulate an equality, rather than an inequality, with similar properties [45]. The correlations corresponding to GHZ states have also been detected in experiment [67, 68].

Unfortunately, in real experiments, especially in those with atoms, one usually creates only mixed entangled states which can only approximate the corresponding pure states. Therefore, the traditional Bell inequalities formulated for pure states are often inadequate for detecting actual nonlocal correlations in experiment. A more promising approach in this respect is to measure the averages of special operators known as entanglement witnesses. One can show that for any nonseparable state there exists an entanglement witness detecting its nonlocality [69]. At the same time every entanglement witness corresponds to some Bell inequality which can be verified experimentally, thereby falsifying a certain class of local hidden variable theories [70].

Note that the laborious work of experimenters and theoreticians on uncovering the distinctions between the quantum-mechanical and local-hidden-variable pictures of the world continues to this day. The former constantly set up new, improved experiments for testing Bell inequalities (see, e.g., Ref. [71]), whereas the latter disclose the implicit assumptions made in such experiments [5] or invent ever more sophisticated local hidden variable theories (see, e.g., Ref. [72]).

Apart from the tests of Bell inequalities and the like, yet another fundamental application of entangled states consists in the studies of decoherence in quantum systems. Decoherence is the process in which quantum superpositions lose their coherence owing to the interaction with the environment (see reviews [7, 73 – 75]). Decoherence is actually responsible for the fact that we did not observe (at least so far) superpositions of dead and alive cats in reality. However, experiments in creating small 'Schrödinger kittens' consisting of several photons and atoms are already feasible [76 – 78]. Therefore, it is very interesting to compare decoherence times measured in experiment with those predicted theoretically. In most cases, the theory predicts that the lifetime of Schrödinger kittens decreases exponentially with their size. The result of this exponential dependence is that the lifetime of coherent superpositions for macroscopically different states of objects that are commensurate with real cats should be extremely small.

Thus, decoherence is the main factor determining how quantum or classical the behavior of a given physical object will be. Furthermore, according to one viewpoint [73], natural decoherence is the solution to the quantum measurement problem [41] (see also Refs [7 – 9]). Hence the interest in experimentation with the decoherence of mesoscopic systems which involve a relatively small number of atoms, photons, phonons, etc. and therefore stand on the border between classical and quantum domains. The first experimental confirmation of the mentioned exponential dependence for states of type (13) has already been reported [78].

It should be noted that entangled states of multipartite systems are not the only example of superpositions of macroscopically different states. Quantum superpositions of different coherent states of a quantum oscillator present another popular example. Such states have already been realized in experiment using both the vibrational degrees of

freedom of trapped ions [79] and the electromagnetic field inside a high-Q cavity [80].

3.2 Metrological applications

Metrological applications of entangled states are probably the most respectable of all, as they are based on one of the most traditional and authoritative fields of physics. For example, the EPR state considered in the beginning of this review presents a particular case of spin-squeezed states [81, 82] which can be very useful in ultrahigh-resolution spectroscopy. The special role of such states arises from the fact that entanglement suppresses quantum fluctuations of some variables.

Creating spin-squeezed states, one can considerably reduce quantum projection noise which presently is the main factor limiting the precision of optical frequency standards (atomic clocks) [81]. For example, the use of entangled states can increase the precision of the frequency standards that are based on the Ramsey spectroscopy of trapped ions by a factor of $\sqrt{N}$, where N is the number of trapped ions [83, 84].

Other metrological applications of entangled states include special methods for the absolute calibration of photodetectors, absolute brightness measurements, and high-precision measurements of polarization dispersion in birefringent media [85], as well as the new methods for synchronizing spatially separated clocks [14, 15].

3.3 Applications to the physics of quantum information

Unlike the metrological uses, the applications of entanglement to quantum computing (see a set of lectures [12], review [13] or monograph [26]) are still considered wishful thinking by many. Indeed, the attempts to construct a full-scale quantum computer capable of realizing the advantages of quantum algorithms over classical ones will hardly succeed in the next decade. Nevertheless, the tremendous progress made in this field over the last few years allows us to hope that this problem will be solved eventually.

Strictly speaking, quantum computing cannot be considered an application of entangled states because entanglement is not absolutely necessary to perform quantum computations. Instead, as a genuine quantum computer executes almost any quantum algorithm, it creates, in passing, a complexly distributed entangled state of its memory registers (qubits). Even one of the simplest quantum gates, known as a controlled-NOT (CNOT), creates an entangled state of two qubits when it acts on a superposition state of the controlled qubit. The close interconnection between methods for implementing quantum gates and methods for creating entangled states results in their often being easily transformable into each other in experiment (see, e.g., Refs [86, 87]).

However, the great significance of entangled states for quantum computations has entirely different roots. The fact is that without using entangled states, one cannot make quantum algorithms faster than their classical counterparts. This was rigorously proven for the cases of Shor's quantum algorithm for factoring large numbers [88] and Grover's algorithm for searching in an unsorted database [89], which are presently the most serious potential applications of quantum computing. One can expect that this holds true for other quantum algorithms as well. Thus, the statement that entanglement underlies the advantages of quantum computers over classical ones appears reasonably justified.

It is appropriate to mention here so-called pseudopure qubit states used in NMR-based quantum computers [31]. A pseudopure state corresponding to a pure state $|\psi_{\text{pure}}\rangle$ has the following form

$$\hat{\rho}_{\text{ps. pure}} = \frac{1-\epsilon}{N}\,\hat{1} + \epsilon\,|\psi_{\text{pure}}\rangle\langle\psi_{\text{pure}}|\,,$$

where N is the dimensionality of the quantum system considered, $\hat{1}$ is the identity matrix, and parameter $\epsilon \ll 1$. These are the only states that can be used in present NMR-based computers because at temperatures currently achievable in the laboratory, systems of nuclear spins are 'very hot'. The temperatures corresponding to characteristic transitions in such systems are below 1 mK. At present, it is technically infeasible to cool the used organic molecules to such low temperatures in order to obtain pure states.

Pseudoentangled states corresponding to pure entangled states $|\psi_{\text{pure}}\rangle$ are very close to the maximally random states $\hat{1}/N$. It is important that the maximally random state is not only separable but also devoid of any correlations, including classical ones. [4] If $\epsilon \ll 1$ (in current experiments $\epsilon \sim 10^{-6}$), all pseudoentangled states, including the seven-qubit state considered in Ref. [90], are separable. Therefore, they do not carry *quantum* correlations and can be described by a purely classical probability distribution [91]. It then follows from the above discussion that this technique of quantum computations, presently the most advanced of all, will never realize the advantages of quantum algorithms over classical ones and will only remain a model demonstration of the possibilities for coherent manipulation of quantum states.

The situation with quantum-informational applications of entangled states is paradoxical in that entanglement not only allows one to break the widely used RSA scheme for public-key cryptography with the aid of Shor's algorithm [12] but also makes possible entirely new methods of *quantum cryptography* [92, 93]. Quantum cryptography protocols offer absolute security in the sense that one can always detect any attempt at eavesdropping. This feature of quantum cryptography derives from the fact that qubits, unlike classical bits, cannot be copied [94] because any measurement of a quantum state inevitably changes the measured state itself. As a result, eavesdropping can never remain unnoticed in quantum cryptography protocols. Note that not all protocols of quantum cryptography proceed from employing entangled states [11], but those that do offer some additional advantages [95].

The first protocol of quantum cryptography was proposed as early as 1982 [96], which explains why quantum cryptography is currently the maturest field of the physics of quantum information. After a series of papers reporting the experimental realization of quantum cryptography with the aid of various entangled states of photons [97–99], it even became standard to say that quantum cryptography is now commercially available.

Yet another field of physics of quantum information, where entangled states are of fundamental importance, is *quantum information theory*. It deals with various generalizations of classical information theory [100] to the quantum case and, in particular, with the problem of capacities of quantum information channels. This problem has at least two aspects. First, one can consider the capacity of quantum channels with respect to the transmission of classical informa-

[4] To avoid confusion, we note that in contrast to Eqn (9) the matrix $\hat{1}/N$ here describes the state of the entire quantum system rather than the state of some of its parts.

tion. Classical messages are then encoded in quantum states of carriers, such as two-level systems (qubits). However, owing to the specific properties of the quantum measurement, the recorded information cannot always be retrieved in its entirety, even if the information channel does not introduce any noise to the transmitted message.

A fundamental theorem on the capacity of quantum information channels with respect to classical messages was recently proven in paper [101] and generalized in Ref. [102]. For the purposes of this review it is interesting to note that the maximum capacity is reached when one uses *collective* measurements at the channel output. Such *entangled measurements* do not factor into tensor-producted measurement results of separate qubits transmitted along the channel.

Second, one can consider the problem of the genuine quantum capacity of information channels — that is, of their ability to reliably transmit entire *quantum states* — the quantum information proper (see review [103]). The subtlety is that the transmitted states should preserve any correlations they may have with other systems, i.e. their entanglement [104]. This requirement is particularly important for the quantum-informational applications of entanglement, discussed in this section. Indeed, if anyone can produce lots of entanglement then only quantum channels can distribute this resource in space or time.

Unfortunately, no such powerful theorems have been proved so far for the case of the transmission of quantum states along the quantum channels as for the case of classical messages. Some progress was recently made in this field owing to the introduction of the concept of coherent information [105 – 107] and the development of methods for calculating its quantity in various physical systems [108]. However, the final solutions to these problems are still far away.

In connection with the problem of quantum information channels, we should say a few words about the so-called *quantum teleportation*. In its physical essence, quantum teleportation consists in the transfer of a quantum state from one physical object to another without direct interaction between them [109]. A pre-existing entangled quantum state, such as an EPR pair, and an ordinary classical communication channel are necessary to accomplish this transfer. At a certain stage of the quantum teleportation procedure, the essential information about the teleported state is encoded in a classical message which is transmitted along the classical information channel. At the last stage of the procedure, this information (two classical bits per each teleported qubit) is used to reconstruct the original quantum state. Hence the analogy between quantum teleportation and teleportation as we known it from science fiction stories. To date, the phenomenon of quantum teleportation has been demonstrated in several experiments for various states of electromagnetic field [110 – 113].

From the viewpoint of quantum information theory, quantum teleportation is just a noiseless quantum channel allowing perfect transmission of qubits (quantum information) from a one physical system to another. The connection between quantum teleportation and quantum information theory made it possible to prove many important theorems on quantum capacities of information channels by applying to gedanken experiments wherein quantum teleportation protocols are utilized [39]. Note also that quantum teleportation has a sort of antipode — *dense quantum coding* which allows one to transmit an increased amount of classical information along a quantum channel if an entangled state is available [114, 115].

Entanglement swapping [116] is another interesting generalization of quantum teleportation. The purpose of entanglement swapping consists in creating entanglement between two particles that never interacted. This is achieved by manipulating in a special way the pre-existing quantum correlations in the system. Therefore, entanglement swapping is close to the idea of teleporting entanglement, but should nevertheless be distinguished from actual teleportation of entangled states [117 – 119]. Entanglement swapping has been realized in experiment for polarization states of photons [120].

Finally, *the quantum generalizations of games theory* [121] are probably the most speculative application of entangled states. As in the case of quantum computing, an expansion in the set of permissible physical operations (from the classical to the quantum one) allows one to solve problems inaccessible to classical approaches. In particular, one can reach the Nash equilibrium in some games by utilizing quantum game strategies [122]. The well-known prisoner dilemma, for example, can be solved with the help of a quantum strategy that employs entangled states [123]. Whether such quantum games have any practical significance remains an open question, but at the moment researchers are quite happy with the fundamental interest in such quantum generalizations of various classical fields of science.

4. Entangled states of ions

The applications of entangled states, described in Section 3, allow us to judge entanglement a specific quantum resource. The problem of producing this resource, i.e. of methods for creating entangled states, then becomes naturally important. This is the subject of the following sections of this review. Below, we consider today's most advanced experimental technique which employs metastable internal states of ions held in a linear radio-frequency ion trap.

4.1 Internal states of ions and atoms as carriers of quantum information

The atoms of alkali metals (Na, K, Rb, Cs) and the ions of alkali-earth metals (Be^+, Ca^+, etc.) isoelectronic to them [124] are probably the most popular objects of study in modern atomic physics experiments. This is mostly due to the relative simplicity of their spectra: these atoms and ions have a single *s*-electron in the outer electron shell, so in most cases their spectra are governed by only one-electron transitions. The alkali atoms and alkali-earth ions turned out to be very convenient for studies in the physics of quantum information, too. The hyperfine splitting sublevels of their $^2S_{1/2}$ ground state offer almost ideal conditions for long-term storage of quantum information.

Consider for definiteness the $^9Be^+$ ion which was used in all experiments at the National Institute of Standards and Technology (Boulder, USA) (see review [125]). In these experiments, the internal state of each $^9Be^+$ ion carried an elementary quantum memory register — a qubit. The two-level quantum system of this qubit was formed by the following internal states

$$|\downarrow\rangle = {}^2S_{1/2}|\,F=2, M_F=2\rangle\,, \quad |\uparrow\rangle = {}^2S_{1/2}|\,F=1, M_F=1\rangle\,.$$

Here, F is the total angular momentum of the atom, including the nuclear spin, and M_F is the projection of the angular

momentum onto the quantization axis coincident with the direction of a weak applied magnetic field (Fig. 2). The transition frequency between the levels $|\downarrow\rangle$ and $|\uparrow\rangle$ was $\omega_{\uparrow\downarrow} \approx 1.25$ GHz, and the coherence lifetime between these levels exceeded 10 min in some experiments [125].

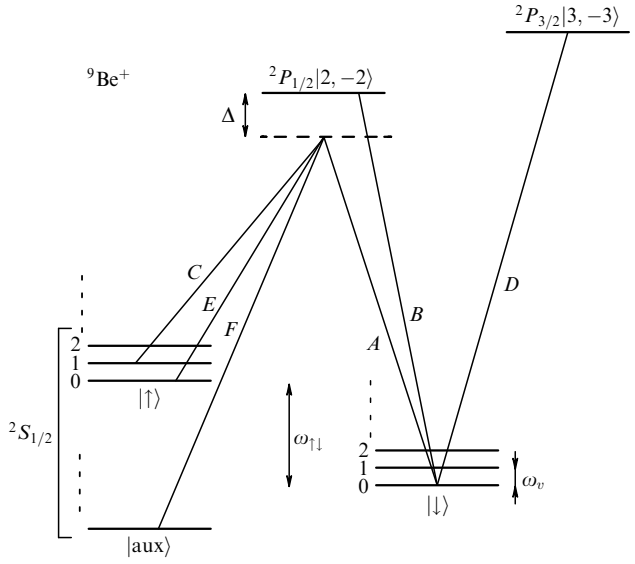

Figure 2. Energy levels of a $^9Be^+$ ion in an ion trap and the transitions used in experiment (see text for details).

Apart from a long lifetime, memory registers should have the ability to initialize, i.e. to come to some predefined initial state, as well as to record and read out information. It turned out that in ion qubits all these operations can be realized with the help of appropriate laser pulses.

A qubit can be conveniently initialized using the standard technique of optical pumping [126]. To this end, one excites a transition connecting one of the qubit states with some upper level. For the sake of definiteness, let this be the $|\downarrow\rangle \leftrightarrow {}^2P_{1/2}$ transition (transition B in Fig. 2). If the atom was initially in the $|\uparrow\rangle$ state, nothing happens. If, on the contrary, the atom was in the $|\downarrow\rangle$ state subjected to the applied field, it starts to make transitions between the initial state $|\downarrow\rangle$ and the upper state $^2P_{1/2}$. However, owing to the fact that the $^2P_{1/2}$ state sometimes decays into the opposite qubit state $|\uparrow\rangle$ rather than to the original state $|\downarrow\rangle$, the atom eventually ends up in the $|\uparrow\rangle$ state and stays there. Thus, regardless of the initial state of the atom, one can always prepare it in the $|\uparrow\rangle$ state, which is necessary for subsequent experiments. In actual experiments, optical pumping is usually realized by choosing an appropriate configuration of the cooling lasers.

To record quantum information in an ion qubit, i.e. to prepare it in an arbitrary superposition state $|\psi\rangle = \alpha|\uparrow\rangle + \beta|\downarrow\rangle$, one may use Raman pulses. To this end, one nonresonantly excites the $^2P_{1/2} \leftrightarrow |\uparrow\rangle$ and $^2P_{1/2} \leftrightarrow |\downarrow\rangle$ transitions with a biharmonic laser field satisfying Raman resonance conditions, when the detuning Δ of the laser field components from their respective transitions is the same (Raman transition AE in Fig. 2). Such laser radiation excites coherent oscillations of the population (Rabi oscillations) between the qubit levels $|\uparrow\rangle$ and $|\downarrow\rangle$, while the upper level $^2P_{1/2}$ remains almost unpopulated [127]. Selecting appropriate detuning, duration, phase, and intensity of the pulse, one can create an arbitrary superposition state $|\psi\rangle$ of the qubit.

Finally, information can be read out from the quantum register of an individual ion by measuring its state in the original basis of states $|\uparrow\rangle$ and $|\downarrow\rangle$. An efficient way to perform this measurement is to detect the resonance fluorescence on a so-called cycling transition. The $^2P_{3/2} \leftrightarrow |\downarrow\rangle$ transition in the $^9Be^+$ ion (transition D in Fig. 2) is a cycling transition in the sense that, owing to the selection rules, the upper state $^2P_{3/2}$ can decay exclusively back to the $|\downarrow\rangle$ level. If upon the measurement the ion's state $|\psi\rangle = \alpha|\uparrow\rangle + \beta|\downarrow\rangle$ collapses to the $|\downarrow\rangle$ state, the atom starts to make cycling transitions $^2P_{3/2} \leftrightarrow |\downarrow\rangle$, scattering about 10^6 photons per second, a fraction of which can be collected by a photodetector. If, on the other hand, the initial state of the ion collapses to the $|\uparrow\rangle$ state, nothing happens to the atom and there is no resonance fluorescence. Therefore, the absence or presence of the resonance fluorescence, as registered by a photodetector, indicates to the experimenter the state in which the ion is upon the measurement. The actual accuracy of this qubit measurement technique approaches 90% in experiment [125].

These features of qubit initialization, and recording and readout of quantum information apply almost unchanged to other alkali-earth ions and neutral alkali atoms.

4.2 Ions in radio-frequency ion traps

As regards experiments in the physics of quantum information, all the advantages of alkali-earth ion qubits, described in Section 4.1, are significant only if the ions can be individually addressed. For this to be true, the ions should be fixed in space and the distance between neighboring ions should be greater than the wavelengths of the optical transitions used (about 0.5 μm). Only in this case can quantum information be recorded in and retrieved from the desired ion without disturbing the others.

It turned out that such a spatial ion configuration can be realized in modern radio-frequency (rf) ion traps. Owing to their nonzero electric charge, ions efficiently interact with external electric fields. The Earnshow theorem, however, prohibits the trapping of charged particles in any electrostatic potential in vacuum. Therefore, apart from a static electric potential, rf ion traps also employ a radio-frequency quadrupole electromagnetic field. After the name of their inventor, rf ion traps are sometimes called the Paul traps [128].

For certain conditions, the effective potential which the ions 'feel' in the rf trap is harmonic in all three dimensions. The depth of the potential well of the ion trap and, consequently, the initial energy of ions caught in this potential is of the order of 1 eV, and so as to order the ions in space, it is necessary to cool them using different laser methods to low temperatures ($T < 1$ K) [129].

At temperatures satisfying the condition $T < e^2/150k_B d$, where e is the electron (ion) charge, k_B is the Boltzmann constant, and d is the characteristic dimension of the ion trap, the ions arrange themselves in a strict spatial configuration called a Coulomb crystal [130]. For example, for $d \approx 10$ μm, the temperature of such a sort of phase transition is around 10 mK. As a result of the joint action of the effective central potential of the trap and the repulsive Coulomb potential arising between the positively charged ions, the latter take on a certain equilibrium positions relative to one another and from then on execute merely small oscillations about these equilibrium positions.

If the effective harmonic potential of an ion trap increases in some direction outward from the center much more slowly

than in others, the Coulomb crystal lines up in an ion string along the direction of the greatest softness of the trapping potential (Fig. 3). This arrangement of ions is particularly favorable for individual addressing. For example, in experiment [131] the distance between neighboring ions of the string exceeded 5 μm. Under these conditions, a laser beam can be focused on any ion in such a way that the probability to excite other ions is below 1%. Then, one can efficiently control the internal state of each ion in the string with the help of appropriate laser pulses, as described in Section 4.1.

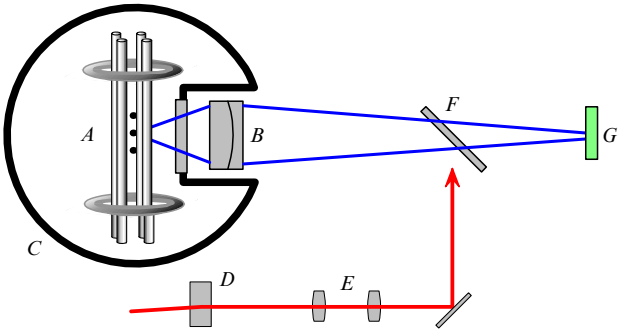

Figure 3. Scheme of the experimental setup used for manipulating trapped ions in a Paul trap: (*A*) linear rf (Paul) trap with a string of trapped ions, (*B*) imaging optics used for focusing the laser beam on individual ions and collecting their fluorescence, (*C*) vacuum chamber, (*D*) acousto-optical deflector used for guiding the focused laser beam to a desired ion, (*E*) telescope, (*F*) beam splitter, (*G*) photodetector detecting the fluorescence of the ions. Adapted from Ref. [131].

However, no manipulation of individual ions can create an entangled state of ions in the trap. An interaction is needed to create any correlations, including quantum ones. In ion traps, the repulsive Coulomb interaction between positively charged ions can serve this need. This interaction gives rise to collective ion vibrational modes in a Coulomb crystal. For instance, the normal vibrational mode with the lowest frequency corresponds to the situation when all ions translationally displace in the same direction from their respective equilibrium positions. This mode is called the center-of-mass (COM) mode because its description is equivalent to that of the center of mass of the Coulomb crystal oscillating in the effective potential of the trap. Depending on the ion used and other parameters of the trap, the frequency of the COM-mode can vary between 100 kHz and 10 MHz.

To apply the normal vibrational modes arising from the interionic Coulomb interaction to the creation of entangled states and other experiments in the physics of quantum information, one should cool these vibrations to their ground quantum states, where vibrational phonons are absent. To this end, one first cools the ions using the Doppler methods and then employs the sideband cooling technique [132]. In the latter case, the trapped ions are irradiated by a laser whose frequency is detuned to the red from some dipole transition in the ion species used by exactly the frequency of the cooled vibrational mode. Therefore, upon an excitation to an upper level and the subsequent reverse decay, the number of phonons in this mode decreases by one. After a large number of such transitions, the cooled mode loses virtually all its phonons. This cooling results in the best localization of ions about their equilibrium positions. Notice that the cooling of high-frequency vibrational modes is less important because at the same temperature, the phonon population and, accordingly, the ion displacement amplitudes of these modes are correspondingly lower.

The sideband cooling technique is effective exclusively in the Lamb – Dicke limit: $\eta = k\Delta z \ll 1$, where Δz is the maximum displacement amplitude of the ions, and k is the projection of the wave vector of the laser beam onto the axis of the ion string. Only in this case the vibrational sidebands due to different normal modes do not mix up in the excitation spectra.

The coupling between the internal degrees of freedom of the ions, the quantum oscillator of the selected vibrational mode and the Raman-resonant biharmonic laser field is described by the Jaynes – Cummings model [125]:

$$\hat{H}_I = \hbar\eta\Omega\,(\hat{S}_+\hat{a} + \hat{S}_-\hat{a}^\dagger)\,. \tag{16}$$

Here, η is the Lamb – Dicke parameter, Ω is the effective Rabi frequency of the driven Raman transition, $\hat{S}_\pm$ are the transition operators between internal states $|\uparrow\rangle$ and $|\downarrow\rangle$, and $\hat{a}^\dagger$ and $\hat{a}$ are the phonon creation and annihilation operators of the vibrational mode used.

However, the main interest in cooling the normal vibrational modes in ion traps arises from the fact that it is necessary to implement the scheme for quantum computations in ion traps, proposed by Cirac and Zoller in 1995 [133]. Since then this scheme has remained one of the most popular designs for a quantum computer. In the scheme, one of the normal vibrational modes of trapped ions, cooled to the ground vibrational state, serves as a quantum computer bus, i.e. transmits quantum information between individual ions.

Consider the cooling process applied to a vibrational mode of trapped $^9Be^+$ ions. Due to the energy level pattern of the selected vibrational mode (usually, the COM mode), the ground state sublevels $|\uparrow\rangle$ and $|\downarrow\rangle$ split into manifolds of equidistant vibrational levels separated by the energy of the vibrational phonon $\hbar\omega_v$ (Fig. 2). In experiment [134], the vibrational frequency of the COM mode was $\omega_v/2\pi \approx 11$ MHz, so that individual vibrational levels were well resolved in the Raman excitation spectra [125].

If we consider single excitation of the vibrational mode at most, the ground state $|0\rangle$ and the first excited state $|1\rangle$ of the vibrational mode form a two-level system — another qubit in addition to the one formed by internal states $|\uparrow\rangle$ and $|\downarrow\rangle$. In this system of one ion but two qubits one can already perform two-qubit quantum gates. The simplest one can be performed by applying a Raman 2π-pulse to the transition between the $|1\rangle|\uparrow\rangle$ level and some lower lying auxiliary level $|0\rangle|\text{aux}\rangle$ (see Raman transition *CF* in Fig. 2). As is well known, the effect of a 2π-pulse is to change the phase of the involved states by π, while preserving the amplitude coefficients of the wave function. Therefore, in a system of two qubits formed by states $|0\rangle|\downarrow\rangle$, $|1\rangle|\downarrow\rangle$, $|0\rangle|\uparrow\rangle$, and $|1\rangle|\uparrow\rangle$, only the state $|1\rangle|\uparrow\rangle$ will reverse its sign, whereas other states will not be affected.

In this scheme we therefore face *conditional dynamics* — the situation in which the dynamics of one qubit depend on the state of another. It can be shown that the presence of conditional dynamics and the ability to apply arbitrary unitary operations to individual qubits are sufficient to execute any quantum algorithm [86]. In particular, if we complement the controlled phase incursion operation described above by two Raman $\pi/2$-pulses of frequency difference $\omega_{\uparrow\downarrow}$ (i.e. in resonance with $|n\rangle|\uparrow\rangle \leftrightarrow |n\rangle|\downarrow\rangle$ Raman

transitions), the resulting pulse sequence will implement the CNOT quantum gate [134].

In the above scheme, the conditional dynamics and quantum gates were applied to the system formed by the internal qubit of one $^9Be^+$ ion (states $|\uparrow\rangle$ and $|\downarrow\rangle$) and the qubit of the first two Fock states of the quantum oscillator corresponding to the collective ion vibrational COM mode. Most real applications require, however, that the conditional dynamics be applied to exclusively internal qubits of two or more ions.

In order to apply quantum logic gates to the internal qubits of two arbitrary trapped ions, one can perform the following sequence of operations [133]. First, the internal state of one ion is transferred to the qubit of the vibrational COM mode (to accomplish this, the phonon mode should initially be in the ground quantum state). This is done by applying a Raman π-pulse to the $|0\rangle|\downarrow\rangle \leftrightarrow |1\rangle|\uparrow\rangle$ transition (Raman transition AC in Fig. 2). As a result, the vibrational qubit and the internal qubit of the first ion exchange their internal states (Fig. 4a), which is a well-known feature of the interaction Hamiltonians of type (16). Then, one realizes the conditional dynamics in the system of the vibrational qubit and the internal qubit of the second ion, as described above (Fig. 4b).

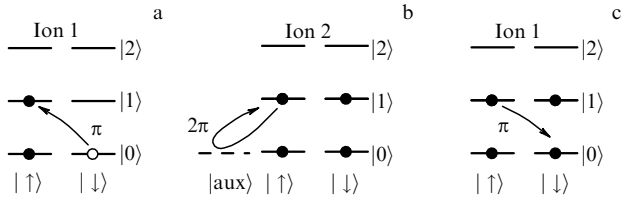

Figure 4. Scheme illustrating the realization of the conditional dynamics in a system of two ions with the help of a phonon of a common vibrational mode (see text for details).

Finally, one performs the reverse exchange of the quantum states between the vibrational qubit and the internal qubit of the first ion (Fig. 4c). The final state of the vibrational qubit, resulting from the conditional dynamics, is therefore transferred to the internal state of the first ion, whereas the vibrational mode returns to its ground quantum state. It is important that the vibrational COM mode is common for all trapped ions, i.e. all ions access the same vibrational qubit. Therefore, the COM mode really plays the role of a computer bus, mediating the coherent interaction between arbitrary trapped ions — the exchange of quantum information.

Notice that by sequentially applying quantum logic gates to the vibrational qubit and the internal qubits of trapped ions, one can create arbitrary entangled states of the internal degrees of freedom of the ions. Importantly, this procedure is fully deterministic (i.e. contains no operations with random outcome) and creates very stable entangled states of ions. The stability of such entangled states derives from the low sensitivity of the $|\uparrow\rangle$ and $|\downarrow\rangle$ ion internal states to all noise present in the ion traps [125].

4.3 Experiments in the physics of quantum information

For many years, the group of Wineland et al. at the National Institute of Standards and Technology (Boulder, USA) has retained its leadership in experiments with trapped ions. To date, this group has already carried out several experiments in the physics of quantum information.

Almost immediately after the appearance of the quantum computer proposal [133], Wineland's group performed an experiment [134] implementing the CNOT quantum logic gate in a system of the vibrational qubit and the internal qubit of a trapped ion, as described in Section 4.2. True enough, the ion trap contained only one ion, and so this experiment only illustrated the possibility of realizing the quantum computing scheme proposed by Cirac and Zoller [133].

The implementation of quantum gates in a system of at least two ions met with serious difficulties. First, the trap in question could hold ions and cool them down to the ground vibrational state only if the distance between neighboring ions did not exceed 2 μm. At such distances, a laser beam cannot be focused on one ion without disturbing the other. Of course, one could increase the distance between the ions by reducing the effective trapping potential, but the vibrational frequency of the COM mode then became smaller than the linewidth of the working dipole transitions. In this situation, the vibrational sidebands merged with the principal dipole transition line in the excitation spectrum, and one could no longer apply the sideband cooling technique to drive the COM mode to the ground quantum state.

Second, the heating of oscillator vibrational modes was anomalously fast in these experiments. With the cooling turned off, the COM mode warmed up significantly from the ground state in about 1 ms, which is several orders of magnitude smaller than the theoretical value of 500 s. The source of this fast heating has not yet been pinpointed [125].

Limiting themselves to working with a single ion, Wineland et al. created a Schrödinger-cat state in their next experiment [79]. It was a superposition of two coherent states of the quantum oscillator that described the vibrations of the trapped ion about its equilibrium position. To create this state, they cooled the vibrations of the trapped ion to the ground state, and prepared the ion in the $|\psi_{\pi/2}\rangle = (1/\sqrt{2})\,(|\uparrow\rangle + |\downarrow\rangle)$ superposition state with the help of a resonant Raman $\pi/2$-pulse. Then, the ion was driven on the blue vibrational sideband, which corresponded to $|n\rangle|\uparrow\rangle \leftrightarrow |n+1\rangle|\uparrow\rangle$ transitions (e.g., Raman transition EC in Fig. 2), in order to create different states of motion for each internal state of the ion.

The resulting entangled state of internal and vibrational degrees of freedom of the ion was given by

$$|\psi_{\text{ent}}\rangle = \frac{1}{\sqrt{2}}\left[\left|\alpha \exp\left(-i\varphi/2\right)\right\rangle|\uparrow\rangle + \left|\alpha \exp\left(i\varphi/2\right)\right\rangle|\downarrow\rangle\right], \quad (17)$$

where $|\alpha \exp(-i\varphi/2)\rangle$ and $|\alpha \exp(i\varphi/2)\rangle$ were coherent states [148] of the ion's motion, corresponding to different internal states of the ion and differing by their vibration phases $-\varphi/2$ and $\varphi/2$. To observe the interference of the coherent states, another resonant $\pi/2$-pulse was applied to the system, bringing it to the state

$$|\psi_{\text{ent}}\rangle = \frac{1}{\sqrt{2}}\left(|S_+\rangle|\uparrow\rangle - i|S_-\rangle|\downarrow\rangle\right). \quad (18)$$

Here,

$$|S_\pm\rangle = \frac{1}{\sqrt{2}}\left[\left|\alpha \exp\left(i\varphi/2\right)\right\rangle \pm \left|\alpha \exp\left(-i\varphi/2\right)\right\rangle\right] \quad (19)$$

are the superpositions of different coherent states of the quantum oscillator.

When detecting the resonance fluorescence on the cycling $|\downarrow\rangle \leftrightarrow {}^2P_{3/2}$ transition, Monroe et al. [79] observed characteristic interference fringes in the curve of the population $P_\downarrow$ of the $|\downarrow\rangle$ state versus the phase separation φ (Fig. 5). Note that in the case of $\varphi = \pi$, the created superposition states of the ion's motion involved antiphase vibrational coherent states with amplitudes of up to 18 nm. Given that the spatial dimensions of the corresponding wave packets did not exceed 10 nm, one could truly speak about creating a coherent superposition of different quasi-classical states of a quantum oscillator.

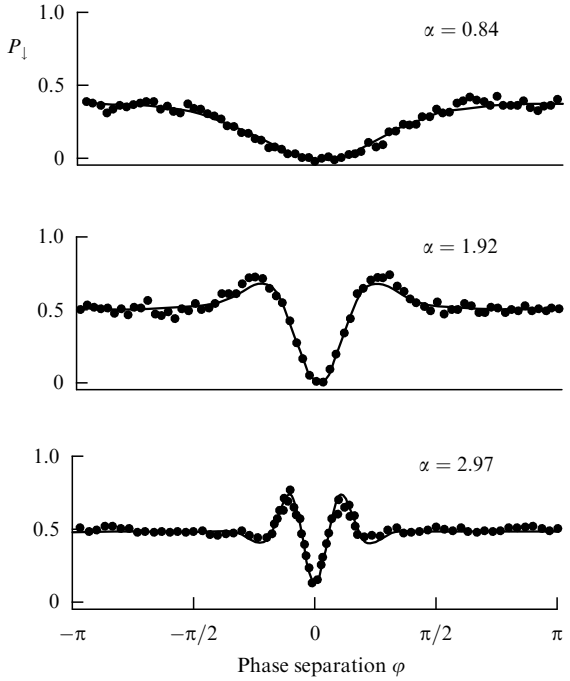

Figure 5. Population of the detected level, $P_\downarrow = \left|\langle\alpha \exp(\mathrm{i}\varphi/2)| - |\alpha \exp(-\mathrm{i}\varphi/2)\rangle\right|^2/4$, as a function of the phase difference φ for various amplitudes α of the interfering coherent states. Adapted from Ref. [79].

The next experiment ran by Wineland's group was the deterministic entanglement of two trapped ions [19]. To avoid the problem of individual addressing, these authors employed the technique of differential excitation of Raman transitions. This means that they excited the blue sideband transitions in both ions simultaneously but with different Rabi frequencies. For example, if the effective Rabi frequencies influencing the ions differed by a factor of two, then one could apply a π-pulse to one ion, whilst simultaneously applying a 2π-pulse to the other ion, which was practically equivalent to applying a π-pulse to the first ion only.

Displacing the atoms along the trap axis, one could vary the effective Rabi-frequency fields acting on individual ions. The optimal selection of experimental parameters allowed the creation of the following entangled state of two ions:

$$|\psi_2\rangle = \frac{3}{5}\,|\uparrow\downarrow\rangle - \frac{4}{5}\,|\downarrow\uparrow\rangle\,, \tag{20}$$

which is a good approximation to the EPR state (1). According to measure (10), the entanglement of the pure $|\psi_2\rangle$ state is $\mathcal{E}(|\psi_2\rangle) \approx 0.94$.

Instead of the COM mode, in experiment [19] Wineland and co-workers utilized the *stretch mode*, which corresponded to the situation when the ion string uniformly stretched and contracted in a periodic fashion. They preferred this mode because it could be cooled better, with the population of the ground vibrational state being 99% versus 95% in the case of the COM mode. Moreover, this mode heated up at a much lower rate than the COM mode.

To verify that state (20) was actually created in the experiment, Wineland and his group made use of the fact that the EPR state is invariant to reference frame rotations or, equivalently, to simultaneous identical rotations of the quasi-spins of both the ions described by internal ion states $|\uparrow\rangle$ and $|\downarrow\rangle$ [5]. Applying identical Raman pulses to both ions, one could rotate their quasi-spins by the same angle and then detect the total resonance fluorescence on the cycling ${}^2P_{3/2} \leftrightarrow |\downarrow\rangle$ transitions in two ions. Only in the case of the EPR state or the approximating state (20), is the average detected fluorescence intensity independent of the duration of the applied pulses.

Analyzing the obtained experimental curves of the total fluorescence intensity versus the duration of applied pulses, the authors concluded that the fidelity of creating the desired entangled state (i.e. its actual population in experiments) was about 70%. The imperfection of the experimental procedure was explained by laser intensity fluctuations and the residual nonzero population of the excited states pertaining to the stretch mode [19].

Finally, in experiment [77] Wineland's group created the four-ion entangled state

$$|\psi_4\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\uparrow\uparrow\uparrow\rangle + \mathrm{i}|\downarrow\downarrow\downarrow\downarrow\rangle\right). \tag{21}$$

They employed a novel scheme for creating entangled states due to Sørensen and Mølmer [136]. In this scheme, two Raman pulses which virtually excite the levels of the vibrational mode used are simultaneously applied to all trapped ions. The heating of the vibrational mode and its imperfect initial cooling are insignificant in this situation.

However, the authors still did not manage to create a perfect $|\psi_4\rangle$ state. The following mixed state was created instead:

$$\rho_{\mathrm{exp}} = 0.13|\psi_4\rangle\langle\psi_4| + 0.87\rho_R\,, \tag{22}$$

where ρ_R is a mixture of reducible states, i.e. states that can be factored into tensor products of one-, two- or three-particle states and therefore do not carry genuine four-particle entanglement (see Section 2.3). Nevertheless, the state created in the experiment still contained a significant portion of the irreducible four-particle entangled state $|\psi_4\rangle$, as seen from expression (22). According to the authors' opinion, the experimental imperfections were mostly due to intensity fluctuations of the laser fields used.

Further advances in Wineland's group experimentation are hindered primarily by the two technical problems mentioned: intensity fluctuations of the laser fields and the anomalously fast heating of vibrational modes in trapped

[5] Notice that the characterization of entangled states actually created in experiments is an important separate problem which only recently attracted the attention of theorists [60, 135].

ions. At present, these authors are actively investigating these problems [77, 137]. Anomalous heating — the more unpleasant and puzzling problem of the two — can probably be solved by employing larger traps or by simultaneously trapping two different ion species and continuously cooling both of them through the mechanism of sympathetic cooling [138].

Many experimental groups all over the world are currently attempting to implement quantum logic gates in linear rf ion traps but, apart from Wineland's group, only the group of Blatt and co-workers from the Innsbruck University (Innsbruck, Austria) has made significant progress. These experimenters used $^{40}Ca^{+}$ ions, which have the advantage that all necessary radiative transitions can be easily excited with the help of semiconductor or solid-state lasers [131]. In addition, the experimental setup they created does not exhibit anomalously fast heating of vibrational states. Thanks to this fact, Roos et al. [139] managed to put 99.9% of the population into the ground vibrational state in one recent experiment.

Moreover, using the cooling techniques based on the electromagnetically induced transparency [140], Blatt's group simultaneously cooled down two vibrational modes to their ground states. Since the sideband cooling technique was not employed in this case, the requirement that the vibrational bands be resolved in excitation spectra was redundant. This allowed one to build up the vibrational frequencies of the trapped ions and thereby increase the distance between neighboring ions to 5 μm. At such distances trapped ions can in fact be individually addressed for the recording and readout of quantum information by laser beams, as required by the Cirac – Zoller scheme [133]. In the near future, these authors plan to demonstrate the application of a quantum CNOT logic gate to the internal states of *two* ions, and to create entangled states of several (up to five) trapped ions [131].

The serious difficulties faced by experimenters in trying to implement the Cirac – Zoller scheme impelled many theoreticians to search for alternative schemes aimed at effecting quantum logic gates and creating entangled states in a system of trapped ions. We have already mentioned the scheme proposed by Sørensen and Mølmer [136], which in principle allows one to create entangled states of any number of trapped ions.

Other proposals for implementing quantum logic gates and creating entangled states in a system of trapped ions are discussed in review [141] and recent papers [142 – 144]. Steane et al. [145] presented a rather general theoretical and experimental analysis of the limitations on the speed of quantum logic gates in ion traps. These studies show that novel cooling techniques and novel methods for implementing quantum logic gates should allow one to realize quantum logic gates in Coulomb crystals consisting of a few ions in the near future.

5. Entangled states of neutral atoms

Despite its conceptual clarity and comparative technical simplicity, the scheme for quantum computation in ion traps described in Section 4 is still unrealized. This encourages many researchers to consider physical systems that would be free from the drawbacks of trapped ions. In this connection, a lot of attention has been paid recently to methods for creating entangled states of neutral atoms in various traps.

5.1 Neutral atoms in dipole traps

As compared with ions, neutral atoms are much less sensitive to low-frequency electromagnetic fields. On the one hand, this makes them less susceptible to electromagnetic noise; on the other hand, it does not allow utilization of such deep and efficient traps as in the case of ions. Today, several types of neutral atom traps are known, each employing its own configuration of static and/or optical electromagnetic fields (see review [146]).

However, for experiments in the physics of quantum information, of greatest applicability are so-called optical dipole traps and especially their standing-wave variant — *optical lattices* (see review [147]). This is because the dipole traps are purely potential (nondissipative) in a very good approximation, just like the ion traps. Therefore, they preserve any correlations, including quantum ones, between trapped atoms. Moreover, in contrast to other nondissipative traps, such as magnetic ones, the potential of dipole traps can be easily changed by turning on or off additional lasers.

The physical aspect underlying optical dipole traps is very simple. The electric component of the trapping laser field polarizes the atom, and the induced atomic dipole moment couples to the same optical field, giving rise to some effective potential $U_{\rm dip}$. Within the quantum description, this potential corresponds to the time-averaged shift of the energy levels, caused by the interaction with the laser field of the dipole trap (optical Stark shift) [148].

In actual experiments, the frequency detuning of the trapping laser field from the dipole transitions in trapped atoms is usually much larger than the natural width of lines at atomic transitions [147]. This is because of the need to minimize laser photon scattering by trapped atoms, which leads to the heating and escaping of atoms from the trap as well as to the loss of the internal state coherence in trapped atoms.

In the relevant approximation, the effective trapping potential reads as

$$U_{\rm dip} \approx \frac{3\pi c^2}{2\omega_{\rm l}^3}\,\frac{\Gamma}{\Delta_{\rm l}}\,I_{\rm l}(\mathbf{r})\,. \tag{23}$$

Here, $\Delta_{\rm l} = \omega_{\rm l} - \omega_{\rm A}$ is the detuning of the frequency $\omega_{\rm l}$ of the trapping lasers from the closest transition frequency $\omega_{\rm A}$ of trapped atoms, Γ is the dephasing rate of this transition, and $I_{\rm l}(\mathbf{r})$ is the intensity of the trapping laser field. One can see from expression (23) that depending on the sign of the frequency detuning, the potential of a dipole trap can be either attractive or repulsive. Thus, there exist dipole traps with blue and red frequency detunings [147], in which potential minima are located at the minima and maxima of the laser field intensity, respectively.

If a dipole trap is formed by counterpropagating laser beams, the resulting optical field is a standing wave, and the $U_{\rm dip}$ potential represents a system of microscopic potential wells, whose minima are located at the nodes or antinodes of the standing wave depending on the sign of the frequency detuning. These micropotential wells are regularly set out in all three spatial directions and have dimensions of the order of the wavelength of trapping lasers (Fig. 6). By analogy with crystal lattices, such configurations of a laser field are called *optical lattices*.

The main advantage of optical lattices over other dipole traps is their ability to trap atoms in individual micropotential wells and thereby localize them in regions smaller than the

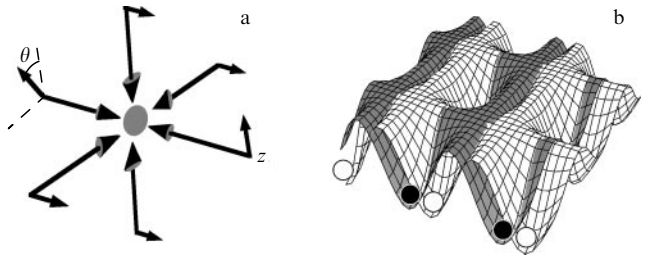

Figure 6. (a) Laser beam configuration forming an optical lattice. A variation in the angle θ of polarization results in the translational displacement of the micropotential wells along the z-axis. (b) Structure of the effective trapping potential for two kinds of atoms with different spin orientations, which are trapped in the corresponding micropotential wells of the optical lattice. Adapted from Ref. [184].

wavelength of the trapping lasers. In addition, varying the phase and polarization of the trapping laser beams, one can shift the nodes and antinodes of the standing wave, translationally displacing the micropotential wells along with the trapped atoms. This allows effective control of atoms' positions in the trap as well as individual addressing of trapped atoms, similarly to the case of ion traps.

In the experiments with dipole traps, the loading of atoms into the trap and the preparation of their initial state are problems in their own right. Since the potential of a dipole trap is relatively shallow, usually corresponding to a temperature of about 1 mK, it can hold only atoms pre-cooled by various laser techniques [149]. Magnetooptical traps are a convenient source of such pre-cooled atoms [150]; the temperature of loaded atoms is then about 0.1 mK.

Ordinary magnetooptical traps usually hold about 10^7 atoms which continuously move over the entire volume of the trap (see, e.g., Ref. [151]). Upon loading into an optical lattice, these atoms therefore randomly distribute over the micropotential wells. Although an optical lattice containing many atoms should allow simultaneous interactions between multiple atomic pairs and therefore parallel quantum computations [152], the present experimenters in the physics of quantum information are mostly interested in the simplest configurations involving only a few atoms. To load a small controlled number of atoms into a dipole trap, one therefore has to either use special magnetooptical traps [153] or intentionally induce the loss of some of the trapped atoms [154].

For implementing a controlled interaction between atoms, it is desirable that atoms be regularly distributed over the volume of the optical dipole trap. Particularly interesting are optical lattices containing one atom in each micropotential well. A similar configuration was recently realized with the aid of cold collisions [154].

Another interesting approach to creating a regular spatial distribution of atoms is to load an optical lattice with a Bose – Einstein condensate [156, 157]. In this case, the condensate atoms automatically distribute themselves in a regular and coherent fashion over the ground vibrational states of all micropotential wells in the optical lattice. Certain conceptual difficulties, however, arise from the fact that each micropotential well then holds a fractional number of atoms in general.

Apart from the efficient loading of atoms into the trap, one should ensure the maximum controllability of their positions in quantum information experiments. As noted above, dipole traps offer this possibility when standing waves, i.e. optical lattices, are used. Upon the loading into an optical lattice, each atom predominantly oscillates within a single micropotential well. To regularize the atomic motion even further, one can cool the atomic oscillations to their ground quantum state using the sideband cooling technique [155]. In this way one achieves the maximum localization of atoms, which is limited solely by the Heisenberg uncertainty relations.

Finally, we should mention methods for detecting internal states of neutral atoms in dipole traps. In this respect, dipole traps are inferior to ion traps. Due to the shallowness of their effective trapping potential, an attempt to detect internal states using laser resonance fluorescence on cycling transitions results in the escape of atoms from the trap, as photon scattering quickly heats trapped atoms. One can minimize these losses by using short detecting pulses; however, the small quantum efficiency of existing photodetectors will then allow state measurements of many atoms only.

Equally destructive are the detection methods based on a variant of the Stern – Gerlach experiment, and various time-of-flight methods for detecting the state of motion of the expanding atomic cloud, obtained by turning off the trapping potential [147]. By this means the presently known methods are hardly applicable to experiments in the physics of quantum information, and the search for nondestructive methods for measuring atomic wave functions of the internal and spatial variables in dipole traps should go on.

5.2 Realization of conditional dynamics in a system of neutral atoms

The main problem with experiments on creating entangled states of neutral atoms is how to realize the controlled coherent interaction between trapped atoms, which should introduce conditional dynamics to the system. Unlike ions, neutral atoms do not couple through the strong electrostatic interaction which allows an efficient coherent interaction even at large distances. Therefore, one usually couples neutral atoms with the help of optical fields — that is, through the exchange of photons. Three proposals for implementing coherent interaction between trapped neutral atoms are currently being discussed in the scientific literature.

The first one is to use the coupling arising during a direct collision between two atoms that are in metastable excited states [158]. During the last few years, experiments on cold atomic collisions have indeed reached the level where one can treat collisions as coherent and, to some extent, controllable processes [159]. The authors of Ref. [158] propose to carry out controlled collisions between atoms trapped in micropotential wells of an optical lattice. Precisely controlling the positions of two trapped atoms, one can adiabatically bring them together to a distance where their spatial wave functions begin to overlap. In this way, one can control the phase incursion between wave function components corresponding to different internal states of the atoms, thus realizing conditional dynamics.

Despite its conceptual simplicity, this method probably imposes too stringent requirements on the control of atomic positions, infeasible in existing experimental setups. In any case, to the best of our knowledge no attempts have been made to implement this scheme in experiment.

The second method for effecting the coherent interaction between neutral atoms employs a high-Q cavity. Systems

consisting of a high-Q cavity and atoms inside it have long since been the subject of *cavity quantum electrodynamics*, and a lot of interesting results have already been produced in this field [160].

Since the intracavity electromagnetic field is strictly correlated over the entire volume of the cavity and individual atoms can very efficiently interact with the same cavity mode [148], the cavity can serve as a kind of mediator in realizing the coherent interaction between the atoms. Of course, the theoretical treatment of such a mediated interaction is complicated by the need to consider the cavity's own degrees of freedom. However, the use of a cavity in experiment relaxes the requirements on the control of the atomic positions and offers new opportunities for observing the states of intracavity atoms as, for example, by monitoring the cavity transmittance with respect to a weak external laser field [161].

The simplest way to realize such a cavity-mediated interaction is to transfer the internal state of one atom to the state of the cavity, then perform a direct interaction between the second atom and the cavity, and finally transfer the cavity state back to the first atom [162]. Thus, one endorses here the same idea as in the case of quantum logic gates between ions in an ion trap, with the distinction that the intracavity electromagnetic field now acts as the quantum bus.

However, this method requires that the cavity contains no more than one atom at each time instant. To apply quantum logic gates to several atoms, one therefore would have to put atoms in and out of the cavity repeatedly, which is technically problematic. The situation would be more favorable if the atoms were permanently trapped inside the cavity, with pairwise interactions turned 'on' and 'off' by an external laser field, similarly to the case of ion traps.

This is exactly what is proposed in another frequently cited method for creating entangled atomic states in a cavity [163]. This method employs the stimulated Raman adiabatic transition technique [164] in the situation when one leg of the Raman transition couples to the fundamental mode of the cavity. An additional advantage of this approach is that the cavity remains virtually unpopulated during the adiabatic transition, which eliminates the primary source of relaxation — the decay of cavity field states. The experimental progress towards implementing cavity-based schemes for creating entangled atomic states is discussed in Section 5.3.

The resonant dipole-dipole interaction is the last widely discussed candidate for the controlled coupling between trapped neutral atoms. It is the quantum counterpart of the classical electromagnetic interaction between two oscillating dipoles and is closely related to the collective relaxation effects in a system of dipole-interacting atoms — *superradiance* and *subradiance* [165, 166].

The first proposal to use the dipole-dipole interaction for realizing conditional dynamics came as early as 1995 [167]; however, more serious treatments of this possibility were offered only recently in Refs [87, 168–170]. Unlike controlled collisions which require nanoscopic distances between the interacting atoms, the resonant dipole-dipole interaction becomes significant at a distance of the order of the wavelength of the chosen working transition, which can vary between 1 μm in the case of dipole transitions from the alkali-atom ground state [168] and a few millimeters in the case of Rydberg atoms [169].

Consider for example how one can create maximally entangled states in the simplest model system of two dipole-interacting two-level atoms [168]. The appropriateness of this example follows from the fact that the resonant dipole-dipole interaction is often of crucial importance in both cold interatomic collisions [159] and interatomic interactions inside a cavity [171, 172].

Consider two identical two-level atoms fixed at a distance R from each other; for example, they can be trapped in neighboring micropotential wells of an optical lattice. Owing to the resonant dipole-dipole interaction, the energy eigenstates of this system then coincide with the Dicke basis [148]: the doubly excited state, the ground state, and the two singly excited (symmetric and antisymmetric) maximally entangled Dicke states given by

$$|\Psi_e\rangle = |e\rangle_1 \otimes |e\rangle_2\,, \quad |\Psi_g\rangle = |g\rangle_1 \otimes |g\rangle_2\,,$$

$$|\Psi_s\rangle = \frac{1}{\sqrt{2}}\left(|g\rangle_1 \otimes |e\rangle_2 + |e\rangle_1 \otimes |g\rangle_2\right),$$

$$|\Psi_a\rangle = \frac{1}{\sqrt{2}}\left(|g\rangle_1 \otimes |e\rangle_2 - |e\rangle_1 \otimes |g\rangle_2\right),$$

respectively. Here, $|g\rangle_i$ and $|e\rangle_i$ $(i = 1, 2)$ are the ground and excited states of the original two-level atoms.

It is easy to verify that the singly excited Dicke states $|\Psi_s\rangle$ and $|\Psi_a\rangle$ are maximally entangled according to the definition introduced in Section 2.2. The radiative relaxation in this system consists in the decay to/from the antisymmetric state $|\Psi_a\rangle$ at the rate of $[1-f]\gamma$, and the decay to/from the symmetric state $|\Psi_s\rangle$ at the rate of $[1+f]\gamma$, where γ is the radiative decay rate in the original two-level atoms, and f is the correction to the radiative decay rates due to the collective relaxation effects.

As shown in Fig. 7, the dipole-dipole atomic interaction splits the singly excited Dicke states $|\Psi_a\rangle$ and $|\Psi_s\rangle$, which initially are degenerate in energy. The amount of this splitting coincides with the constant of the dipole-dipole interaction, $\chi \sim \gamma\lambda^3/R^3$, where λ is the transition wavelength of the original two-level atoms. If the distance R between the atoms is smaller than the wavelength λ of the working radiative transition, then the splitting χ between the levels exceeds the total natural width of the singly excited Dicke

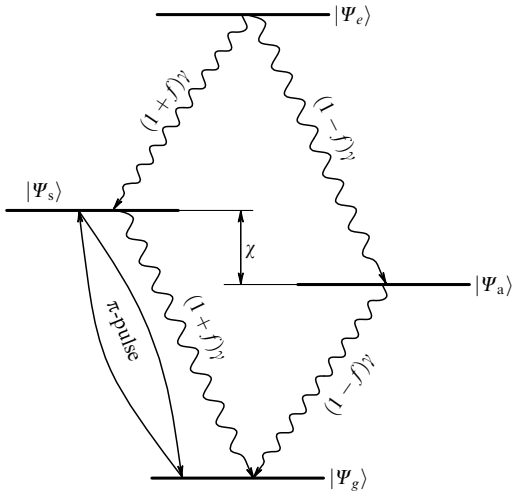

Figure 7. Energy levels of a system of two dipole-interacting two-level atoms. Shown are the channels and rates of the relaxation decay in the system as well as the exciting π-pulse that creates the maximally entangled state $|\Psi_s\rangle$.\

energy levels and, therefore, these levels can be resolved by a laser excitation. Consequently, one can selectively transfer the population from the ground state, where the total population is assumed to be initially concentrated, to one of the maximally entangled states $|\Psi_a\rangle$ or $|\Psi_s\rangle$, by applying a resonant laser π-pulse [148] to the $|\Psi_g\rangle \to |\Psi_a\rangle$ or $|\Psi_g\rangle \to |\Psi_s\rangle$ transition, respectively. It was shown in Refs [87, 170] that in the case of $R/\lambda < 0.1$, one can transfer virtually all the system's population to the chosen maximally entangled state by selecting the optimal parameters of the π-pulse.

The model scheme considered above is deficient in that the created entangled states $|\Psi_a\rangle$ and $|\Psi_s\rangle$ are radiatively unstable (see Fig. 7). The further analysis [173] showed, however, that one can generalize the proposed scheme to the case of multilevel atoms, where radiatively stable entangled states can be created, increasing their significance for practical applications.

For the sake of completeness, we mention here some other proposals for creating entangled states of atoms [174 – 176], which, however, are hardly feasible in our opinion.

5.3 Experiments on the creation of entangled states of neutral atoms

As we already mentioned, theory by far outpaces experiment in virtually all the fields of the physics of quantum information. Methods for creating entangled states of neutral atoms are no exclusion. Here the number and complexity of experimental proposals exceed all capabilities of the existing experimental setups. Therefore we will limit ourselves to considering only several experimental setups which appear most promising as regarding the creation of entangled states in neutral atoms.

We should start with the most advanced experimental technique developed by Haroche with collaborators at École Normale Supérieure (Paris, France), although strictly speaking this technique does not pertain to the subject-matter of entangled states in trapped atoms. In the experiments of Haroche's group the rubidium atoms in Rydberg states consecutively pass through a high-Q cavity, which mediates the interaction between the atoms, at a high speed (Fig. 8). The wavelength of the fundamental mode of this cavity is about 6 mm, while the relaxation times of the utilized Rydberg states and the intracavity electromagnetic field are 30 ms and 160 μs, respectively [68].

Rydberg atoms feature extremely large orbital dimensions (about 125 nm) and, accordingly, a huge dipole transition moment which efficiently couples to single photons even in relatively big cavities with a volume of about 1 cm^3. By employing the atomic velocity selection, one can control the time of flight through the cavity and therefore the time of atomic interactions with the cavity in experiment.

Invoking the idea of the cavity-mediated interaction between atoms [162], Haroche et al. [68] have already performed several experiments in the physics of quantum information: the creation of EPR and GHZ states of atoms and Schrödinger-cat states of the intracavity field as well as the nondestructive measurement of single photons.

Although the experimental technique of Haroche and his associates is currently the most advanced for applications in the physics of quantum information, it is subject to some serious restrictions. In particular, the technique and its experimental implementation require that the cavity contain no more than one atom at each time instant. Since the lifetime

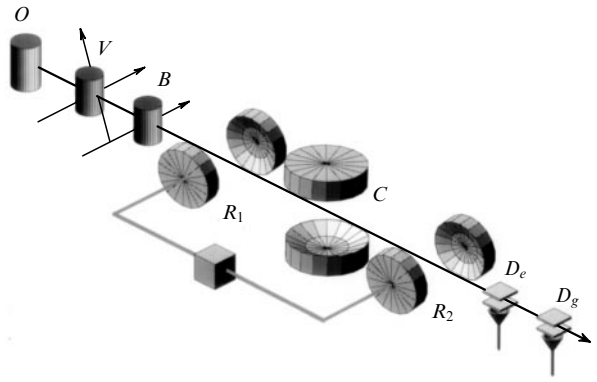

Figure 8. Scheme of the experimental setup for creating entangled states of Rydberg atoms: (O) thermal source of rubidium atoms; (V) velocity selection zone; (B) Rydberg atom preparation unit; (R_1) auxiliary cavity preparing the initial state of Rydberg atoms; (C) high-Q cavity interacting with traversing Rydberg atoms; (R_2) auxiliary cavity specifying the measurement basis of internal states, and (D_e, D_g) detectors of the final states of Rydberg atoms. Adapted from Ref. [177].

of the created atomic and field states is still rather short, this technique can be effectively applied to at most several atoms [177]. Therefore, one cannot speak about creating a quantum computer based on these principles. Like the NMR-based quantum computing experiments in molecules, these experiments will probably remain only a model demonstration of the possibilities for coherent engineering of quantum states.

To avoid the above-mentioned difficulties, the experimental group of Kimble and co-workers at the California Institute of Technology (Pasadena, USA) is making efforts to trap traversing atoms inside a cavity for a long time by utilizing the dipole trap formed by the cavity itself. In these experiments, caesium atoms are preliminarily trapped in a magnetooptical trap at a small distance above the cavity. When the trapping field is turned off, the atoms freely gravitate towards the cavity, some of them entering the space between the cavity mirrors (Fig. 9). Continuously monitoring the cavity transmittance, one can detect the appearance of an atom inside a cavity and turn on the potential of the optical trap at that moment [181]. The atom is then trapped by the intracavity field which is induced by only a few photons. Further monitoring the cavity transmit-

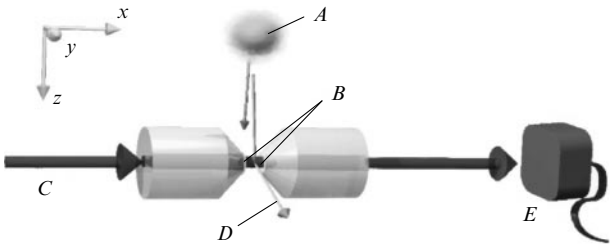

Figure 9. Scheme of the experimental setup for trapping neutral atoms inside a high-Q cavity: (A) cloud of about 10^4 neutral atoms held in a magnetooptical trap; (B) mirrors of the high-Q cavity; (C) laser beams used to form the intracavity dipole trap and monitor the state of the cavity field; (D) trajectory of a single atom scattered by the cavity field, and (E) photodetector. Adapted from Ref. [178].

tance, one can even reconstruct the trajectory of the atom inside the cavity [178].

A similar experiment was recently performed by the group of Rempe with collaborators at the Max Plank Institute for Quantum Optics (Garching, Germany) [179]. The only significant difference was that they used an atomic fountain of rubidium atoms to insert them into the cavity. Therefore, the atoms flew into the cavity at a small speed from below rather than from above. In a recent experiment of this group [180], traversing rubidium atoms underwent the process of stimulated Raman adiabatic transition in the situation when one leg of the Raman transition is coupled to the fundamental mode of the cavity. This is a significant step towards realizing the above-mentioned scheme for creating entangled states [163].

The main objective in both Kimble's and Rempe's groups is currently the long-term trapping of atoms inside a cavity. (In present experiments, the trapping time is only a few milliseconds.) To this end, one should develop a technique for the laser cooling of intracavity atoms, which is a complicated theoretical and experimental problem [182]. In a longer perspective, both groups intend to simultaneously trap two and more atoms inside a cavity, which should allow them to implement the methods for realizing controlled coherent coupling between trapped atoms, described above.

To avoid the problem of effective intracavity trapping and cooling, the group of Meschede and his co-workers at Bonn University (Bonn, Germany) intends to first trap and cool a small number of atoms in a dipole trap and only then inject them into the space between cavity mirrors. In the already existing dipole trap, atoms can be stored for about 30 s [153]. After cooling the atoms in an optical lattice, one can then translationally displace them into the cavity by varying the phase of the trapping laser beams [183]. This also should allow Meschede's group to start experimentation with methods for creating entangled states of atoms inside a cavity.

The group of Deutsch and his associates at the University of New Mexico (Albuquerque, USA) follows its own proposals [168] for creating entangled states of trapped atoms in an optical lattice with the help of the resonant dipole-dipole interaction. So far, these authors have theoretically examined the spatial configurations of dipole-interacting atoms that are optimal for implementing the coherent interatomic coupling [184]. In a recent experiment, Haycock et al. [185] observed coherent tunneling of atoms between neighboring micropotential wells in an optical lattice, which indicates a high degree of spatial coherence of the atomic wave functions and the effectiveness of the control methods used. Still, the implementation of a controlled coherent interatomic coupling in this system will probably require long-term research efforts.

Finally, the group of Polzik with collaborators at Aarhus University (Aarhus, Denmark) has recently succeeded in creating a nonseparable state of a macroscopic atomic ensemble [82]. They created a spin-squeezed atomic state by irradiating a cloud of atoms held in a magnetooptical trap with squeezed light [148]. Although the atoms of the cloud could not be individually addressed and therefore were hardly suitable for quantum computing experiments, such quantum-correlated states are of interest for improving the precision of frequency standards (atomic clocks).

Notice that Sorensen et al. [186] recently proposed to create similar states (albeit with a higher degree of spin squeezing) in Bose–Einstein condensates by making use of interatomic collisions. In contrast to the case of controlled cold collisions [158], considered above, there is no need to further control the positions of atoms in a Bose–Einstein condensate because all atoms reside in the same quantum state which can be calculated and measured quite accurately [187].

Thus, the current experiments on creating entangled states in neutral atoms face serious difficulties in both preparing the initial spatial configuration of atoms and realizing the coherent interatomic coupling that should build up quantum correlations in the system. Today's most advanced technique employing a Rydberg atom passing through a high-Q cavity will probably soon develop to its fundamental limit, due to the lifetime of the atomic Rydberg states and construction specifics of the experimental setup.

Within other experimental approaches, the problem of the efficient control of atomic positions and interatomic interaction has been addressed only preliminarily. Given the present technical bottlenecks, we believe that the most promising approaches are those involving the trapping of neutral atoms in a dipole trap and the interatomic coupling via exchange of optical photons through a high-Q cavity or directly through the mechanism of the resonant dipole-dipole interaction.

6. Conclusions

Let us sum up the above discussion of entangled states of atomic systems. The purpose of this review was to introduce the reader to one of the most promising directions of modern quantum theory. Trying to present the complete picture of this field of physics, we have attempted to reflect the most significant results in both the general theory of entangled states and the experimental methods for creating entangled states in atomic systems.

The present burgeoning studies in the physics of entangled states were triggered by the appearance of new applications of entanglement in the 1980s–1990s. Most of these applications belong to different fields of the physics of quantum information, such as quantum cryptography, quantum computing, quantum information theory, etc. The physics of entangled states apparently forms a hidden basis in all these fields. Although entanglement is never an end in itself, it often determines the advantages of new, quantum approaches over their classical counterparts.

It is worthy of note, however, that not all applications of entangled states are related to quantum information. There is also a great interest in the fundamental implications of entanglement for the foundations of quantum mechanics as well as in various metrological applications of entangled states. Altogether, the convergence of such different fields of physics at a single point — the physics of entanglement — is remarkable, but it only supports the long since expressed opinion of E Schrödinger that the permissibility of entangled states constitutes the essence of quantum mechanics, with all its conceptual difficulties and amazing possibilities [188].

In recent years the new capabilities offered by entangled states have greatly changed the attitude of researchers to this field of physics. Now entanglement is predominantly viewed as a specific quantum resource promising many applications. Accordingly, the development of technologies that would allow large-scale production of various forms of this resource has become an important problem of physics.

For a long time, photon pairs created in the processes of cascade decay of atomic excitations and parametric down-

conversion of light in nonlinear crystals have remained the main source of entangled states. Even now they are the primary carrier of entangled states in, for example, quantum cryptography protocols and will apparently play an important role in the transmission of quantum information along quantum networks in the future.

However, the ever expanding range of requirements put forward by new applications of entangled states stimulates the search for better physical systems to realize these states. In particular, such 'ultimately exact' sciences as metrology and computer science find it difficult to tolerate the intrinsic randomness of the appearance processes of entangled photon pairs. Hence the idea to search for deterministic methods for creating entanglement. It turned out that atoms and ions held in modern electromagnetic traps are the most convenient experimental basis for implementing such methods. One can easily control the positions of these atoms in experiment, while their metastable internal states offer almost ideal conditions for storing quantum states, including their possible quantum correlations.

The first proposal for a quantum computer that utilized internal and vibrational states of ions held in a linear rf ion trap has greatly contributed to the interest in entangled states of atomic systems. Based on this idea, experimenters have created entangled states of two and four ions and performed other experiments in the physics of quantum information in ion traps. The further development of this experimental approach, however, met with serious difficulties which still remain largely unresolved.

As a result, many researchers turned to neutral atoms which are free of the discovered drawbacks of ions although they have their own, related to the preparation of initial atomic states in the trap, the realization of the coherent interatomic interaction, and the detection of final states. To date, significant theoretical and experimental progress has been made in this field, but most actual experiments are only approaching the stage of development when one will be able to speak about the controlled creation of entangled states of two or more atoms.

Further progress in the methods for creating entangled states of atomic systems will probably involve overcoming the anomalous heating of vibrational modes in traps with beryllium ions or developing experiments with other ion species. The solution of these problems would allow the simultaneous trapping of several (up to 10) ions and the application of arbitrary quantum operations to them. However, a further increase in the number of trapped ions will inevitably lead to new experimental problems, such as the nonharmonicity of the effective trapping potential, nonlinear coupling between different vibrational modes of a Coulomb crystal and, finally, the breakdown of the linear structure of Coulomb crystals.

These problems will probably require a change in the design of the ion traps used to implement quantum logic gates. One possible approach is to employ multiplexing schemes [125], in which individual ions are only briefly introduced into the 'processor trap' to perform computations and remain in auxiliary 'storage traps' for the rest of the time. Another way out is offered by ion matrices, each cell of which can be used to apply quantum logic gates to a few ions, with the operation results later exchanged between individual cells [20, 21, 142].

As for neutral atoms, the existing variety of experimental approaches to realizing the coherent interatomic coupling does not warrant any fundamental restrictions on the number of atoms used. In our opinion, it is currently most promising to develop the experimental approaches employing high-Q cavities and the resonant dipole-dipole interaction.

In conclusion we note that this review cannot be complete if only because the number of papers devoted to various aspects of entanglement exceeds several thousand (see bibliography [189]). Moreover, according to some observations the number of such papers increases with every year in geometric progression [190].

The interest in entangled states apparently follows the standard pattern of classical positive-feedback dynamics. On the one hand, the widening range of applications stimulates the development of the general theory of entangled states and relevant experimental techniques. On the other hand, the ever increasing popularity of this field attracts more and more researchers, which in turn facilitates the discovery of new applications of entangled states.

Nevertheless, despite the efforts to popularize the applications of entangled states, the physics of entanglement so far remains mostly theoretical and largely speculative, which probably constitutes its 'growing pains'. Even the staunch apologists of the physics of quantum information admit that the creation of such devices as a full-scale quantum computer remains a rather distant goal. For the time being, however, physicists have a unique opportunity to draw novel ideas from this interdisciplinary field of science and to set up new experiments to reveal new unexpected aspects of quantum physics. We can only wish quantum theory, a recent centenarian, to remain in the future a source of inspiration in both fundamental and applied fields of physics.

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