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The physics of planetary rings

D ter Haar

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Fridman A M, Gor'kavyi N N *Physics of Planetary Rings — Celestial Mechanics of Continuous Media* (New York: Springer-Verlag, 1999)

"*... rings are — well, magical; and they are rare and curious.*"
J R R Tolkien *The Lord of the Rings*

The book by Fridman and Gor'kavyi which is the subject of the present review is an English edition of the Russian monograph by the same authors, which was published in 1994. It would be misleading to call it a translation, as the English edition is about thirty per cent longer than the Russian original and many parts of the latter have been completely rewritten. The reason for this is partly the explosion of observational data due to the Voyager programme, and partly further developments in the pioneering theories of the two authors. Rather than summarizing the various topics which are discussed in the individual chapters of the book under review, we shall give an overview of several aspects of the physics of planetary rings, which are discussed in much more detail in the monograph. In fact, we shall follow the exposition of the book under review quite closely, without explicitly mentioning that these topics are considered in the book. We hope that in this way the reader of the present article will get an idea of the flavor and contents of this volume which, in the opinion of the present reviewer, gives a fascinating and comprehensive picture of what we know at this moment about the beautiful world of planetary rings.

1. A brief historical survey

Before Lippershey invented the telescope early in the seventeenth century, only four planets — Mars, Venus, Jupiter, and Saturn — and one satellite, our own Moon, were known. This, however, changed dramatically when, using a telescope constructed by himself, in 1610 Galilei discovered the four large Jovian satellites and also the Saturnian rings — or rather an indication of these rings; it was left to Huygens to find that Galilei's 'Saturn's children' composed, in fact, a ring system. Huygens also discovered the Saturnian largest satellite. Over the next four centuries a few more planets were found and several more satellites, so that in 1946, when the present reviewer was writing his doctoral thesis about the origin of the solar system [1], the number of known planets was nine, that of known satellites was 29: our own Moon, the two Martian satellites, a single Neptunian satellite, four Uranian satellites, nine Saturnian satellites, and eleven Jovian satellites, and only a single ring system was known — the Saturnian rings. The situation changed dramatically in the late seventies: first of all, in 1977 several groups [2 – 4] observed the occultation of the star SAO 158687 by the Uranian rings and this was followed by the occultation of a star by the Neptunian rings in 1978, and the direct observation in 1979, in the television survey by Voyager 1, of the Jovian rings. The discovery of the Jovian rings was only one of many made by the Voyager spacecraft. These observations led not only to the discovery of many satellites, but also provided detailed information about the structure of the various ring systems. When the English edition of Fridman and Gor'kavyi's book was sent to the printer in 1999, we knew of 67 satellites — apart from our own Moon, the two Martian satellites and the single Plutonian satellite, there were 16 Jovian satellites, 22 Saturnian satellites (four more than there were when the Russian edition of the book was sent to the printer in 1994), 17 Uranian satellites (two more than in 1994), and 8 Neptunian satellites. The large amount of detailed information about the satellite and ring systems of the four large planets imposes considerable restrictions on theorists who want to give an explanation of these data. The book by Fridman and Gor'kavyi presents us not only with an excellent survey of the observational data but also with a detailed analysis and, in many cases, an explanation of them, with special emphasis on the Uranian and Neptunian systems. For this the authors combine classical mechanics, in the form often called celestial mechanics, with contemporary ideas about collective phenomena — hence the subtitle *Celestial Mechanics of Continuous Media*.

2. Introduction

Let us briefly summarize those aspects of the observational data which have to be explained and which are relevant to our later discussion. The first fact which needs an explanation is why there exist rings at all and not just systems of satellites. This is a problem which has been studied for nearly three hundred years and the questions to be answered are essentially the following three: (i) What is the source of the matter from which the rings are formed? (ii) Why do the rings not agglomerate into satellites? and (iii) What determines the outer boundary of the ring? In the next section we shall answer these questions. Once these questions have been answered there remains a whole family of questions about the structure of the rings such as: Why are there gaps in the rings? What is the relation between the ring structure and the many satellites of the parent planet? Most of our discussion in the present review will be devoted to a consideration of the answers given by Fridman and Gor'kavyi to many of the questions of this nature.

Apart from the planetary rings there exist many other ring systems in the Universe, such as protoplanetary discs, accretion discs, and galactic stellar and gaseous discs. However, the planetary discs are an extreme example of

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such discs. Not only are they the ones which have been studied in most detail — the Voyager spacecraft produced thousands of photographs of the Jovian, Saturnian, Uranian, and Neptunian rings with a resolution of a few kilometers and there are also observations from ground-based and satellite observatories and they have been probed by radio and stellar occultations — but if one examines the ratio of the thickness of such systems to their radius, they are easily the most oblate: in their case this ratio is 10^{-6}, which one can compare with a ratio of, say, 10^{-4} for a piece of paper. If one looks at the age of such systems in terms of their revolution period, they have the longest lifetime. However, the longevity of planetary rings is a feature of only the so-called primary rings, i.e. the dense rings consisting of rather large particles (from micrometers to 10 and even to 20 meters in size). Of the planetary rings, the classical A, B, and C rings of Saturn, the nine Uranian dense rings, the main Neptunian rings, and the main Jovian ring certainly belong to the primary class, whereas the E and G rings of Saturn, the dusty Uranian rings, and the rarefied Jovian rings belong to the secondary class. The outer radius of the primary rings is sharply limited and equal to about two radii of the mother planet. As a rule, the band of satellites starts beyond the outer boundary of the primary rings. The secondary rings, which may occur at any distance from the planet, are the rarefied gaseous dusty rings which need a constant influx of matter for their continued existence; the particles in these rings are micrometric or submicrometric in size.

3. Why are there rings?

Let us return to the three questions we put at the beginning of the previous section. We shall start by assuming that the initial situation from which the rings (and the satellites) originate is a disc of gas and dust surrounding the planet. In that case the first question has been answered automatically. In order to answer the second question we must look at the balance of forces acting on the particles, especially after they have grown to a certain size. The dynamic forces acting upon a particle are the centrifugal force, the tidal force, that is, the gravitational force exerted by the planet, and the self-gravitational force. The tidal force decreases with increasing distance from the planet, whereas the centrifugal force increases, which means that near the planet the tidal forces will break up the growing particle, but at a sufficient distance from the planet this will not happen. This argument was presented in 1848 by Roche and the resulting limit was called the Roche limit. However, nearly a century later Jeffreys [5] pointed out that molecular cohesion for small particles is more important than self-gravitation. If one takes that into account, it turns out that the break-up due to the centrifugal force only occurs for particles of kilometer size and such particles are not found in the rings. For particle sizes up to 10 m, which are the ones found in the planetary rings, the centrifugal forces are insufficient to overcome the tensile strength of the particles, and another solution must be found. This was achieved by Gor'kavyi and Fridman [6, 7] who studied the collisional break-up of ring particles. They suggested that in a collision between two particles fragments are sheared off and in a zone close to the planet these fragments will not return to the parent particles, whereas further from the planet the fragments will reunite with their parents. The boundary between the zones is close to the Roche limit, which is not surprising as in this event the competition is also observed between the self-gravitation — in this case, in fact, the gravitational pull of the parent — and the gravitational force exerted by the planet — in this case not the centrifugal force, but the magnitude of the shear velocity, that is, the difference in rotational velocities specific of particles on orbits with slightly different radii. In a detailed analysis by Gor'kavyi and Taĭdakova [8], in which the four-body problem of the motion of a fragment in the gravitational field of the two colliding particles and the planet was studied, they not only found a limiting radius of the zone in which primary rings are situated, but they also determined the particle size distribution which agreed with the mass spectrum obtained from the radio-occultation of the rings. It looks therefore as if the three questions put at the beginning of the previous section have been satisfactorily answered by Fridman and Gor'kavyi.

4. Collective effects in planetary rings

Having found an answer to the first three questions in Section 2, we now must consider the other questions posed there. There are two kinds of effects which appear important in determining the structure and the various features of the planetary rings. On the one hand, there occur resonance effects which are connected with the presence of satellites. The most important and most impressive achievements of the theories developed by Fridman and Gor'kavyi are connected with these effects and we shall discuss them in later sections. On the other hand, various collective effects manifest themselves and we shall consider those in the present section.

If we are concerned with the processes possessing large length and time scales compared to the mean free path and the mean free time of the particles in the rings, we can treat the ring as a continuous system, so that we must consider a hydrodynamic approach to the ring description. The first task is to obtain the relevant equations for the bulk properties of the system. One derives them in the same way as one derives the Navier – Stokes equation or the magnetohydrodynamic equations by starting from the kinetic equation for the particle distribution function and taking suitable averages of its moments. The kinetic equation for the particle distribution function $f(r, v, t)$ has the form

$$\frac{\partial f}{\partial t} + \left(\mathbf{v} \cdot \frac{\partial f}{\partial \mathbf{r}}\right) + \left(\frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} \cdot \frac{\partial f}{\partial \mathbf{v}}\right) = C(f)\,, \qquad (1)$$

where $C(f)$ is the collision integral describing the evolution of the distribution function due to collisions, which in the case of the disc particles are mostly inelastic. Before using this equation one substitutes the expression for $\mathrm{d}\mathbf{v}/\mathrm{d}t$ from the equation of motion of a ring particle. The equation of motion of a particle moving in a disc uniformly rotating with a constant angular velocity $\mathbf{\Omega}$ and in a gravitational field described by a potential Ψ_G in a frame of reference rotating with the angular velocity $\mathbf{\Omega}$ takes the form

$$\frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} = 2[\mathbf{v} \times \mathbf{\Omega}] + \big[\mathbf{\Omega} \times [\mathbf{r} \times \mathbf{\Omega}]\big] - \frac{\partial \Psi_G}{\partial \mathbf{r}}\,, \qquad (2)$$

where $\mathbf{v}$ is the velocity of the particle, and $\mathbf{r}$ its position. If we introduce a quantity $\mathbf{W}$ through the relation $\mathbf{W} \equiv [\mathbf{\Omega} \times \mathbf{r}]$, we can rewrite Eqn (2) in the form

$$\frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} = -\nabla\left(\Psi_G - \frac{1}{2}\,\mathbf{W}^2\right) + [\mathbf{v} \times \mathrm{rot}\,\mathbf{W}]\,. \qquad (3)$$

If we now define $\mathbf{E}$ and $\mathbf{H}$ through the relationships $\mathbf{E} \equiv -\nabla[\Psi_G - (1/2)\mathbf{W}^2]$ and $\mathbf{H} \equiv \mathrm{rot}\,\mathbf{W}$, Eqn (3) becomes

$$\frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} = \mathbf{E} + [\mathbf{v} \times \mathbf{H}] \,. \tag{4}$$

One can see that this equation of motion is the same as that of the charged particle moving in an electromagnetic field, being written in a Hartree system of units where the particle charge e, its mass m, and the velocity c of light *in vacuo* are equal to unity: $e = m = c = 1$. This means that the hydrodynamic equations governing the collective properties of planetary rings will have the same form as the hydrodynamic equations describing a plasma or as the equations of magnetohydrodynamics, and one can therefore avail oneself of the results from plasma physics to illuminate features of the planetary rings. However, we should be careful, since we have so far only considered the case of a uniformly rotating disc, whereas the planetary discs exhibit differential rotation. The hydrodynamic equations are equations for the hydrodynamic velocity $\mathbf{V}$, the number density n, and the temperature T. By integrating over the coordinate along the axis of rotation we can essentially reduce the problem to a two-dimensional one where the bulk variables are now the surface density σ, the radial velocity V_r, the tangential velocity V_ϕ, and the temperature T.

We proceed as follows. Let us first find the stationary solutions for our set of equations: σ_0, $V_{r0}(=0)$, $V_{\phi 0}(=\Omega r)$ and T_0, where Ω will be a function of r. Next for a moment restrict our discussion to the case where we consider solely the effect of the parent planet on the ring and to axisymmetric perturbations of the stationary solution. We now substitute the expressions

$$\sigma = \sigma_0 + \sigma_1 \exp(\gamma t + \mathrm{i}kr)\,, \qquad V_r = V_{r1} \exp(\gamma t + \mathrm{i}kr)\,,$$
$$V_\phi = \Omega r + V_{\phi 1} \exp(\gamma t + \mathrm{i}kr)\,, \qquad T = T_0 + T_1 \exp(\gamma t + \mathrm{i}kr) \tag{5}$$

into the equations discussed above. The resultant equations are formally the same as those studied by Braginskiĭ [9] when considering the transport properties of plasmas. Linearizing these equations with respect to the deviations from the stationary solution, we get a set of homogeneous linear algebraic equations for σ_1, V_{r1}, $V_{\phi 1}$, and T_1, and hence a dispersion equation to determine γ as a function of k. We are interested in finding instabilities, that is, solutions of the dispersion equation with a positive real γ. There are two kinds of instabilities which appear: the gravitational or Jeans instability, and the diffusion instability occurring under circumstances where the particle velocity increases with decreasing surface density. However, while these instabilities are capable of explaining some features of the rings, they produce only structures with length scales of up to a few kilometers.

Of course, the previous analysis will also not lead to non-axisymmetric instabilities which could produce the eccentric structure shown, for instance, by some of the narrow Saturnian rings. In order to look for such instabilities we must generalize the form of the deviations from the stationary solution and allow perturbations with an azimuthal mode $m = 1$. It is interesting to note that Maxwell [10] considered just this kind of instabilities in his study concerning the stability of the Saturnian rings. Although Maxwell had used the unrealistic — for the Saturnian rings — approximation of an absolutely rigid body that led him to the mistaken conclusion about the collapse of a continuous ring onto the planet [11], he made a correct inference about the meteoritic structure of the Saturnian rings. Fridman, Morozov and Polyachenko [11] showed that a ring consisting of water ice does not collapse onto the planet but quite the contrary disintegrates into lumps due to the small-scale bending instability in the plane of the ring. Later Fridman and Gor'kavyi [12] found another instability (they called it the ellipse instability) which deforms a real circular ring into an elliptical one under circumstances where the particles in the ring will decelerate (accelerate) particles which move closer to (further away from) the planet. We note in passing that for an initially axisymmetric ring this so-called ellipse instability is an example of spontaneous symmetry breaking.

There are still several structural features which have not yet been explained. Most of them are due to the presence of satellites, but there is one, the large-scale structures with length scales of 1000 km, which can be explained without invoking satellites. Fridman and Gor'kavyi showed that these structures may be due to what they called the accretion instability. This instability arises when apart from the disc of the rings and the planet there is an influx of particles, which means that it develops in the early stages of the evolution of the ring system. It occurs because a 'particle wind' will be stopped preferentially by regions where there is a fluctuation producing a higher density, and hence a greater stopping power. These fluctuations will necessarily have a greater length scale since for smaller fluctuations diffusion will spread out the structure.

5. Resonance effects in planetary rings

We shall now consider how satellites affect the structure of the rings. Satellites do this through their gravitational fields: the trajectories of ring particles are changed if a satellite is present. One might expect that the effect would be most important in the immediate vicinity of a satellite and that its importance would diminish when one gets further away from the satellite. However, there are certain regions where resonance effects come into play and the effect is again important. These regions are those where the ratio of the orbital frequency of the particle, Ω, to the orbital frequency of the satellite, Ω_0, equals the ratio of two integers, viz. $\Omega_0/\Omega = n/m$. Special regions are those of the so-called Lindblad resonances, where either $n = m + 1$ (outer Lindblad resonance) or $n = m - 1$ (inner Lindblad resonance). If such a region coincides with the edge of a ring, one might expect the edge to oscillate in the m mode. Indeed, it is found that the edge of the Saturnian B ring oscillates in the $m = 2$ mode due to a 2:1 resonance with Mimas, and the outer edge of the Saturnian A ring oscillates in the $m = 7$ mode due to the 7:6 resonance with Janus. If, on the other hand, the resonance occurs inside a continuous ring, it will produce a spiral density wave with the same azimuthal number. The effect of the spiral wave of density depends on such parameters as the pressure and the viscosity. For instance, in the Saturnian disc the 1:2 resonance with Mimas has led to a radial flow of matter between the A and B rings, leading to the formation of an extended gap, the Cassini Division, as shown by Fridman, Khoruzhii, and Gor'kavyi [13]. On the other hand, in the A and B rings themselves, resonances with Janus, Prometheus, and Pandora have led to the appearance of the density waves observed.

Whereas the Saturnian ring system has been known and observed for a long time, the Uranian system was only discovered in 1977, and while in the case of Saturn theorists were able in most cases to explain observed features, in the case of Uranus the fact that there was a ring system at all gave the opportunity to use the observations which were available to make predictions about what still might be observed. This opportunity was brilliantly grasped by Fridman and Gor'kavyi [7]. They suggested that beyond the outer boundary of the Uranian rings there would be a series of as yet unobserved satellites, that the positions of the rings would be determined by 1:2, 2:3, and 3:4 Lindblad resonances with those satellites, and the orbits of those satellites were predicted, which determine — by Lindblad resonances — the positions of two rings simultaneously. A few months after the publication of the results following from these hypotheses, Voyager 2 found four of the predicted five satellites with an orbital radius within 1% of the predicted value, and the fifth one within 3%. This remarkable prediction has been compared by Ginzburg with that of Le Verrier and Adams of the existence of Neptune, while Arnol'd compared this finding with that predicting atomic properties on the basis of Mendeleev's periodic table of the elements.

6. Genesis of the Uranian ring system

There are now so many observational data about the Uranian ring system and the diverse processes which can proceed in such a system during the various stages of its development have been studied in so much detail that it is possible to give a detailed account of how the system in its present form most probably came into being, starting from Uranus surrounded by an extended disc of gas and dust. We shall briefly sketch in the present section how the protodisc developed into the system as we observe now. For more details we must refer to the book by Fridman and Gor'kavyi.

We saw earlier that in a gas disc surrounding a planet there are two zones: a close zone in which particles break up through collisions, and a second, far zone where they are more likely to stick together than break up. The first zone, which in the case of Uranus stretches to about 50 Mm from the planet, will be the ring zone, and the second zone is that of the satellites. We now want to split the satellites into two categories, depending on whether or not they resonantly interact with the rings. The inner Lindblad resonances of a satellite occur at the distances from the planet which are at most about two thirds of the orbital radius R of the satellite: the 1:2 resonance occurs at $0.63R$, and the other resonances occur closer to the satellite. As the ring zone extends to 50 Mm from the planet, we can split the satellites into those which can resonantly interact with the rings, i.e. those that are not more than $50/0.63(\simeq 80)$ Mm from Uranus (which we shall call the close satellites) and those that are too far away to interact resonantly (the far satellites).

We can now look at the development of the protodisc around Uranus. We first of all note that in a differentially rotating, turbulent, viscous gaseous disc with dust there should be drift motions. In the earliest stages of its development, there could be an aerodynamic positive drift, that is, gas motion away from the planet. This may have led to a disc of gas and dust with a radius of something like 1 Gm and a surface density which at the present position of Oberon is in the region of 400 g/cm^2, having a maximum of about 1000 g/cm^2 in the region of Ariel, and falling off to densities of 10 to 100 g/cm^2 in the region of the inner satellites.

The next stage should be the formation of some of the satellites. One would expect that the growth rate for satellites should have been the greater the further away from Uranus they were, because of the dominance of break-up processes over sticking processes nearer the planet, and the greater initial density in the disc. Indeed, the first satellites to be formed were the far satellites Ariel, Umbriel, Miranda, Titania, and Oberon as well as the close satellites (where there was high disc density) Portis, Juliet, Cressida, and Desdemona. The other satellites, namely, Puck, Rosalind, Bianca, Belinda, Cordelia, and Ophelia were formed at a later stage.

Once some of the close satellites had been formed, resonance effects became important since they had Lindblad resonances inside the ring zone. However, it is only lower Lindblad resonances, that is, resonances with $n = 1$, 2, or 3, which were important, since resonances with larger n, in general, lie outside the ring zone. We have seen in the previous section that due to resonances, the gas drifts away from the resonance. This negative drift is counteracted by the positive drift which we discussed a moment ago, and a ring is formed a little distance away from the resonance. Once rings are formed, a positive drift starts which is due to the exchange of angular momentum between the gas in the ring and the matter arriving from outside of the ring zone. This is the so-called ballistic drift which is, in general, stronger than the aerodynamic drift.

It is now possible to sketch the sequence of events in the Uranian system. The first close satellite to be formed is Portia and she starts to form the 4 ring through her 1:2 resonance, and the ϵ ring through her 2:3 resonance. The latter resonance, lying closer to Portia, is stronger and the ϵ ring becomes more massive. The second close satellite, Juliet, also forms a ring at the position of its 2:3 resonance, which lying at the edge of the ring zone condenses much later into Cordelia. While drifting outwards, the ϵ ring partly condensed into the two satellites Desdemona and Cressida which, in turn, through their 2:3 and 3:4 resonances created the η, α, and δ rings as well as the $1986U1R$ ring, and reinforced the ϵ ring. The α and β rings were formed later at the 1:2 resonance of Rosalind and the 2:3 resonance of Bianca, two satellites which were formed quite some time after Portia, Juliet, Desdemona, and Cressida.

Once the rings were formed, they were kept in stable positions through a balance between the positive and negative drifts arising partly from the forces in the gas and partly from the resonances with the satellites.

7. Conclusion

We have tried to give some idea of the wealth of physical problems presented by the ring systems of the giant planets and of the solutions found for these problems in recent years. The book by Fridman and Gor'kavyi contains detailed discussions not only of the topics touched upon here but also of many other, related subjects, and it can be most highly recommended to anybody who wants to become acquainted with this fascinating field which straddles astronomy and physics.

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