4. Sinakovski E J et al. *Phys. Plasmas* **4** 1736 (1997)
5. Razumova K A et al. *Plasma Phys. Control. Fusion* **42** 973 (2000)
6. Razumova K A et al., in *27th EPS Conf. on Controlled Fusion and Plasma Phys.* P2039 (2000) p. 205

PACS numbers: 83.10.Dd, 83.80.Nb, 92.10.Rw
DOI: 10.1070/PU2001v044n03ABEH000934

Model for the formation of hummocks in a drifting ice cover

A V Marchenko

1. Introduction

Hummocks constitute a characteristic feature of the sea ice cover. They are produced by the deformations caused by the compression and shearing of the ice cover generated by wind and sea currents. The hummocks are formed in the open ocean and in the vicinity of the shores and greatly affect navigation in the ice-covered sea of the Arctic regions. The hummocks produced in the sea-shelf regions near the hydrotechnical structures greatly affect the distribution of the loads exerted by the ice on these structures.

The hummocks are pieces of ice pushed out under and over the surface of the surrounding flat ice cover. The above-water part of a hummock, the sail, may be several meters high while the height of the underwater part, the keel, may be tens of meters. Hummocks are fairly often extended horizontally [1]. Hummocks have a significant influence on the rheological properties of the ice cover and make it spatially inhomogeneous and anisotropic.

Theoretical modeling of hummock formation (the ridging process) may be classified into two types of analyses. In the studies of the first type (see, for instance, [2]) the ridging processes are taken into account in the large-scale simulation of the ice cover dynamics. Ridging is treated as the main mechanism for evolution of the ice cover thickness profile. The simulation yields the evolution of the thickness distribution for the ice cover under plastic strain. The structure and evolution of an individual hummock are ignored in the simulation process.

The first model of hummock formation was developed by Parmeter and Coon [3] in 1973. Parmerter and Coon analyzed the observational data and put forward a hypothesis that there was a maximum hummock height depending on the thickness of the ice sheets making up the hummock. According to the hypothesis, a hummock grows in height and width if its vertical dimension is smaller than the maximum size and after its height has reached the maximum size only the hummock width grows. The maximum hummock height is determined by the bending load breaking down the edge of the floe pushing against the hummock owing to the lack of balance between gravity and the lifting force acting on the hummock edge in water. Parmeter and Coon estimated the compression stress required for the hummock formation from the equations for conservation of mass and energy.

Hopkins and co-workers [4, 5] used a different approach to modeling the ridging process. They treated a hummock as a pile of ice blocks of a given shape with viscous elastic forces acting between them. The motion of each ice block is described by a separate equation. New ice blocks are produced in the model when the floe edge pushes against the hummock. High-capacity computer simulations involved calculations of the motion for the large number of ice blocks making up the hummock yielding a realistic representation of the ridging process and confirming the hypothesis of the maximum hummock height.

It was only in 1998 that a hummock was produced under laboratory conditions [6] in the ice basin of the Technological University in Helsinki. The thickness of the artificially frozen ice was not more than 10 cm. The experimental results demonstrated that the growth of the hummocks under compression was accompanied by floes being pushed under hummocks so that these two processes cannot be monitored separately in practice. The results of the laboratory experiments are corroborated by the data of observations conducted in northern Baltic Sea which demonstrated that ice hummocks were largely composed of flat floes piled up on each other.

The objective of the present study was to develop a model of the ridging process that would make it possible to analyze the formation of hummocks in the ice cover consisting of an arbitrary number of floes. It is assumed that the hummocks are formed at the lines of contact between floes driven by winds. The suggested mechanism is valid for the sea ice cover in which the regions of flat and ridged ice can always be identified. A flat ice cover region is broken down under compression so that hummocks are produced while the flat ice regions are displaced with respect to each other. It will be demonstrated that the displacements are periodic owing to the self-sustained oscillations accompanying shifting of the drifting ice [7].

2. Basic equations

Let us consider the conservation of mass, momentum, and energy for an ice layer floating on a liquid surface. The appropriate differential equations for the one-dimensional case are

$$\frac{\partial m}{\partial t}+\frac{\partial mv}{\partial x}=0\,,$$
$$\frac{\partial mv}{\partial t}+\frac{\partial mv^2}{\partial x}=\frac{\partial \sigma}{\partial x}+f,$$
$$\frac{\partial E}{\partial t}+\frac{\partial Ev}{\partial x}=\frac{\partial \sigma v}{\partial x}+fv\,. \tag{2.1}$$

Here m is the mass of the ice floating on the unit surface area of the ocean, v is the ice drift velocity, σ are the internal stresses in the ice, E is the surface energy density of the ice, f is the friction force of the atmosphere and the ocean acting on the ice, x is the horizontal coordinate, and t is the time.

The ice concentration on the ocean surface is assumed to be unity and we can write $m=\rho_{\mathrm{i}}h(x,t)$ where $h(x,t)$ is the ice thickness and $\rho_{\mathrm{i}}\approx 930$ kg m^{-3} is the sea ice density. The surface energy density of the ice cover is given by the equation

$$E=K+P+W\,, \tag{2.2}$$

where $K=\rho_{\mathrm{i}}hv^2/2$ is the surface density of the kinetic energy, and P and W are the surface densities of the potential and internal energies.

The surface density of the potential energy of the floating ice is given by the equation

$$P=\rho_{\mathrm{i}}g\int_{z_-}^{z_+}z\,\mathrm{d}z-\rho_{\mathrm{w}}g\int_{z_-}^{0}z\,\mathrm{d}z \tag{2.3}$$

It equals the difference between the potential energy of the ice taken from the level $z = 0$ of the unperturbed liquid surface and the potential energy of the liquid displaced by the ice. Here $\rho_w \approx 1020$ kg m^{-3} is the seawater density. The upper and lower surfaces of the ice cover are determined by the equations $z = z_-(x, t)$ and $z = z_+(x, t)$. The thickness of the ice cover is $h = z_+ - z_-$.

A further simulation step included the processes of irreversible ice cover compression accompanied with variation of its thickness and dissipation of the mechanical energy. The variation of the internal energy is determined by the energy dissipation

$$\mathrm{d}W = \mathrm{d}D \geqslant 0 \,. \tag{2.4}$$

The variation of the ice thickness under compression is determined by ridging and rafting processes. Ridging gives rise to hummocks produced by ice rubble. Rafting is the piling up of flat ice sheets. Sometimes compression is determined by a combination of ridging and rafting.

As the ice thickness grows the potential energy of the ice increases irreversibly,

$$\mathrm{d}P \geqslant 0 \,. \tag{2.5}$$

Let us assume that a flat ice cover includes a ridged ice region with the boundaries $x = x_-(t)$ and $x = x_+(t)$. For $x < x_-$ and $x > x_+$ the ice thickness is constant and equals $x = x_-$ and $x = x_+$, respectively.

$$\frac{\mathrm{d}U_\mathrm{r}}{\mathrm{d}t} = [hQ] \,, \tag{2.6}$$

$$\frac{\mathrm{d}I_\mathrm{r}}{\mathrm{d}t} = [\sigma] + \rho_\mathrm{i}[hvQ] + F_\mathrm{r} \,, \tag{2.7}$$

$$\frac{\mathrm{d}E_\mathrm{r}}{\mathrm{d}t} = [\sigma v] + [EQ] + A_\mathrm{r} \,, \tag{2.8}$$

where U_r, I_r and E_r are the linear density of volume, impulse and energy of the ice in the region $x \in (x_-, x_+)$, and $[\lambda] = \lambda_+ - \lambda_-$ for any symbol λ or combination of symbols

The flows $Q_+ \geqslant 0$ and $Q_- \leqslant 0$ are defined by the formulae

$$Q_\pm = \frac{\mathrm{d}x_\pm}{\mathrm{d}t} - v_\pm \,, \tag{2.9}$$

where v_+ and v_- are the ice drift velocities in the regions $x > x_+$ and $x < x_-$.

The values σ_+ and σ_- are equal to the stress in the flat ice covers for $x = x_+$ and $x = x_-$; E_+ and E_- are the ice surface energy densities in the region $x > x_+$ and $x < x_-$, F_r is the external drag force, acting on the ridge by $x \in (x_-, x_+)$, and A_r is the power of this force.

The system of equations (2.6) – (2.8) is a generalization of the relationships at a discontinuity [8] for the case when material is built up at the fracture. Similar relationships have been considered in the theory of gas diffusion with an admixture of dispersive particles [9], where the necessity of their inclusion was connected with the overturning of compression waves. Such a surface has been called 'sheet'. Physically, the sheeting corresponds to those regions where one may not neglect the collisions between particles of the admixture. Sheet fractures have been used to describe the formation and drift of bands of unbroken ice in a dispersive ice cover [10, 11]. In this case, the properties of the material also change on the surface of the fracture since the ice cover inside the hummock is made up of lumps of ice.

3. Evaluation of the characteristic scale of the problem

The friction force acting between the ice cover and the water determines a rather low velocity of ice drift under natural conditions. Therefore, we can take $V = 0.1$ m s^{-1} as a characteristic scale of the ice drift velocity [12].

In the conditions under consideration the characteristic time scale for the ice drift is determined by the self-sustained oscillations of the ice cover. The observational data demonstrate that for a constant wind acting on the ice, the ice deformation proceeds in the form of quasi-periodic shifts produced by the relative displacements of the floes [7]. The period of such shifts can be as long as several minutes [13]. This is why $T = 1$ min is selected as a characteristic time scale.

The measurements made under natural conditions for the ice cover [14] yielded the highest stresses of the order of 10^5 N m^{-1}. Stresses of the order of 10^4 N m^{-1} correspond to the initial stages of the ridging process. This is why we have selected $\Sigma = 10^4$ N m^{-1} as a characteristic stress.

Let us make an order-of-magnitude estimate of the terms in equations (2.6) – (2.8). The friction force with which the water is acting on the hummock keel is estimated as

$$F_\mathrm{r} = \rho_\mathrm{w} C_\mathrm{w} h_\mathrm{k} \delta v^2 \,,$$

where $C_\mathrm{w} \approx 1$ is the resistance coefficient for non-streamlined bodies, δv is the difference between the velocities of the water and the ice, and h_k is the distance from the point of the hummock which is the deepest in the water to the lower ice surface. For the sake of assessments we shall take $h_\mathrm{k} \approx 10$ m and $\delta v \approx 0.1$ m s^{-1}. Then the assessment yields $F_\mathrm{r} \approx 100$ N m^{-1}.

Let us evaluate the inertial term in equation (2.7) assuming that the hummock has a triangular sail shape with side edge angles of the sail and keel being 30° (see Fig. 1) [1]. Under such conditions the hummock volume per unit length is $U_\mathrm{r} \approx 2h_\mathrm{k}^2 = 200\,\mathrm{m}^2$. The inertial term is of the order of $\rho_\mathrm{i} U_\mathrm{r} V T^{-1} \approx 300$ N m^{-1}.

Let the characteristic ice thickness be $h = 1$ m. The penultimate term on the right-hand side of equation (2.7) is of the order of $\rho_\mathrm{i} h V^2 \approx 10$ N m^{-1}. Hence we obtain the estimate $[\sigma] \ll \Sigma$. In other words, the difference between stresses on two sides of a triangular hummock is much smaller than the stresses themselves. Therefore, we can take

$$[\sigma] = 0 \,. \tag{3.1}$$

in the stress calculations. Note that this equation is inapplicable to hummocks of trapezoid shape with a fairly large

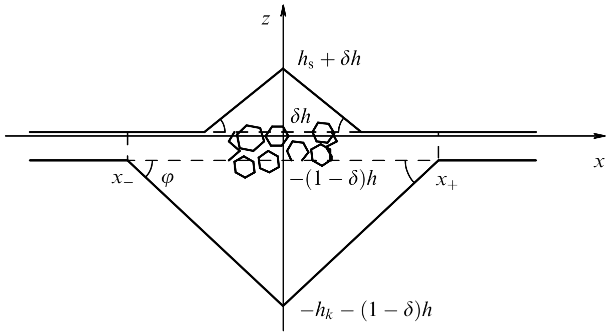

Figure 1.

volume U_r. Let us estimate the width of a trapezoid-shaped hummock for which the inertial term is of the order of 10^5N m^{-1} (the extreme stresses generated with the formation of large hummocks). Let us take $U_r \approx (h_k + h_s)L_r$ where h_s is the sail height and $h_k + h_s = 20$ m. The condition $\rho_i U_r V T^{-1} \approx 10^4$ N m^{-1} yields $L_r \approx 300$ m.

Using similar estimates we can demonstrate that in equation (2.8) for the energy the characteristic values of both the kinetic energy of the hummock and the work A_r are much smaller than other terms of the equation. Therefore, we shall assume below that

$$dE_r = dP_r + dD_r . \tag{3.2}$$

Let us estimate the characteristic values of the densities of the potential P_f and kinetic K_f energies of a flat ice cover. These densities are given by

$$2P_f = \delta\rho_i g h^2 , \quad 2K_f = \rho_i h v^2 , \tag{3.3}$$

where v is the drift velocity and h is the ice thickness. Assuming $v \approx V$ and $h \approx 1$ m we obtain $K_f \ll P_f$. Using this estimate we assume below that

$$E_f = P_f . \tag{3.4}$$

4. Hypotheses on the self-similarity of the hummock shape and energy dissipation

The hypothesis that the hummock shape is self-similar consists in the assumption that the variation of the shapes of the underwater and above-water parts of the hummock (the sail and the keel) is determined by the variation of the hummock volume during the ridging process. The sail and the keel of the hummock are at hydrostatic equilibrium in the course of the process. Hence we obtain the equations

$$dP_r = \frac{dP_r}{dU_r}\, dU_r , \quad dL_r = \frac{dL_r}{dU_r}\, dU_r , \tag{4.1}$$

where $L_r = x_+ - x_-$ is the hummock width.

The observational data indicate that the shape of the hummock sail and keel are often close to a triangular or trapezoid shape [1]. For a hummock of triangular shape with the same angle φ of the side edges of the sail and the keel produced in a flat ice cover of the height h (see Fig. 1) the self-similarity hypothesis yields the following equations

$$U_r = hL_r + AL_r^2 ,$$

$$2P_r = \rho_i g L_r \left[\delta h^2 + 2L_r(\delta A h + BL_r)\right] ,$$

$$2h_k = L_r \operatorname{tg}\varphi , \quad h_s = \gamma h_k , \tag{4.2}$$

where the coefficients A, B, and γ are given by the equations

$$4A = (1 + 2\gamma^2) \operatorname{tg}\varphi ,$$

$$24B = (1 + 2\gamma)(\gamma \operatorname{tg}\varphi)^2 ,$$

$$\gamma = \sqrt{\frac{\delta}{1-\delta}} , \quad \delta = \frac{\rho_w - \rho_i}{\rho_w} .$$

The self-similarity hypothesis is satisfied in the process of rafting of two floes (see Fig. 2). Under these conditions the

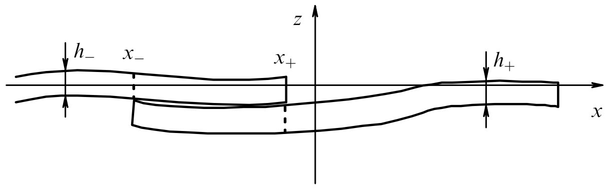

Figure 2.

following equations are satisfied:

$$U_r = L_r(h_+ + h_-) , \quad 2P_r = \rho_i \delta g L_r (h_+ + h_-)^2 . \tag{4.3}$$

To determine the dissipative function we shall take

$$dD_r = (\sigma_d^+ Q_+ - \sigma_d^- Q_-)\, dt \geqslant 0 , \tag{4.4}$$

in accordance with the general principles of thermodynamics. Here $\sigma_d^\pm$ are the generalized thermodynamic forces which are determined by the scenario of the ridging process.

The following scenario of the ridging process is quite well-known [1]. The ice cover from the region with $x > x_+$ pushes against the right-hand edge side of the hummock sail. In the process the flat ice sheet is broken into pieces of rubble which fall on both sides of the sail. The edge of the ice sheet pushing into the hummock from the region with $x < x_-$ is broken under the weight of the hummock sail. The hummock keel is formed of the ice rubble pushed down into the water by the weight of the hummock sail.

The energy dissipation is determined primarily by the friction of the ice sheet slipping over the hummock sail on the right-hand side of the hummock sail. It is assumed, therefore, that $\sigma_d^- = 0$ while σ_d^+ is given by the law of dry friction according to the equation

$$\sigma_d^+ = \mu\rho_i g h h_s \operatorname{ctg}\varphi , \tag{4.5}$$

where μ is the friction coefficient. According to equations (4.2), the generalized force σ_d^+ is a function of the hummock volume.

In the process of rafting the mechanical energy is dissipated by means of friction between the ice floes. Assume that the ice sheet from the region with $x > x_+$ is under the ice sheet from the region with $x > x_-$ (see Fig. 2). Assuming that the interaction between the ice sheets is described by the law of dry friction we obtain the equation

$$dD_r = \mu\rho_i g h_-\, L_r\, dL_r , \tag{4.6}$$

where μ is the friction coefficient.

Equations (4.4) and (4.6) yield

$$\sigma_d^\pm = \mu\rho_i g h^- x^\pm . \tag{4.7}$$

Equations (4.4) imply that dD_r is a function of the volume variation dU_r.

5. Ridging stress

Let us analyze several simple mathematical models of ridging in which the stresses required for the ice ridging process are found from the system of equations (2.6)–(2.8) as functions of the hummock volume and the thickness of the ice making up the hummock.

5.1 Rafting

It can be readily seen that the following equations are satisfied in the rafting process:

$$\frac{\mathrm{d}x_{\mp}}{\mathrm{d}t} = v_{\pm}\,, \quad Q_+ = -Q_- = -[v]\,. \tag{5.1}$$

The volume and the potential energy of the ice in the rafting region are given by equations (4.3). Equation (4.6) determines the dissipation of the mechanical energy. Using equations (3.1), (3.2), (3.4), and (5.1) we can rewrite equations (2.6) and (2.8) as

$$\frac{\mathrm{d}U_{\mathrm{r}}}{\mathrm{d}t} = -(h_+ + h_-)[v]\,, \tag{5.2}$$

$$\frac{\mathrm{d}P_{\mathrm{r}}}{\mathrm{d}t} + \frac{\mathrm{d}D_{\mathrm{r}}}{\mathrm{d}t} = \sigma_{\mathrm{r}}[v] - (P_{\mathrm{f}}^+ + P_{\mathrm{f}}^-)[v]\,.$$

Hence we obtain the following expression for the stress $\sigma_{\mathrm{r}} = \sigma_+ = -\sigma_-$:

$$\sigma_{\mathrm{r}} = -(h_+ + h_-)\left(\frac{\mathrm{d}P_{\mathrm{r}}}{\mathrm{d}U_{\mathrm{r}}} + \frac{\mathrm{d}D_{\mathrm{r}}}{\mathrm{d}U_{\mathrm{r}}}\right) + P_{\mathrm{f}}^+ + P_{\mathrm{f}}^-\,. \tag{5.3}$$

5.2 Ice ridging at a stationary wall

Assume that the ridge is at $x = 0$. The left-hand side of the hummock coincides with the wall while the right-hand side is at $x = x_+$. As the hummock shape is self-similar we obtain

$$\mathrm{d}x_+ = \frac{\mathrm{d}x_+}{\mathrm{d}U_{\mathrm{r}}}\,\mathrm{d}U_{\mathrm{r}}\,. \tag{5.4}$$

As $Q_- = 0$ we can rewrite equations (2.6) and (2.8) in the following form:

$$\frac{\mathrm{d}U_{\mathrm{r}}}{\mathrm{d}t} = h\left(\frac{\mathrm{d}x_+}{\mathrm{d}t} - v_+\right), \tag{5.5}$$

$$\frac{\mathrm{d}P_{\mathrm{r}}}{\mathrm{d}U_{\mathrm{r}}}\,\frac{\mathrm{d}U_{\mathrm{r}}}{\mathrm{d}t} + (\sigma_{\mathrm{d}}^+ - P_{\mathrm{f}}^+)\left(\frac{\mathrm{d}x_+}{\mathrm{d}t} - v_+\right) = \sigma_+ v_+\,.$$

Equations (5.4) and (5.5) yield the stress σ_{r} required for ice ridging at the wall:

$$\sigma_+ = -\left(h\frac{\mathrm{d}P_{\mathrm{r}}}{\mathrm{d}U_{\mathrm{r}}} + \sigma_{\mathrm{d}}^+ - P_{\mathrm{f}}^+\right)\left(1 - h\frac{\mathrm{d}x_+}{\mathrm{d}U_{\mathrm{r}}}\right)^{-1}. \tag{5.6}$$

The friction stress σ_{d}^+ is a function of the hummock volume with the thickness h equal to the flat ice sheet thickness and depends on the scenario of hummock development. If a hummock is triangular in shape then the stress σ_{d}^+ is given by equation (4.5).

5.3 Development of a triangular hummock in the uniform ice cover

Assume that the ice cover has the same properties on both sides of the hummock and that $h_+ = h_- = h$. Under these circumstances we can assume that the flows of ice into the hummock are identical at both sides:

$$Q_+ = -Q_- = Q\,. \tag{5.7}$$

According to equation (5.7), the midpoint of the hummock moves at the mean velocity of the ice floes making up the hummock and we have

$$2Q = \frac{\mathrm{d}L_{\mathrm{r}}}{\mathrm{d}t} - [v]\,. \tag{5.8}$$

Using equation (5.7) we can rewrite equations (2.6) and (2.8) as

$$\frac{\mathrm{d}U_{\mathrm{r}}}{\mathrm{d}t} = 2hQ\,, \tag{5.9}$$

$$\frac{\mathrm{d}P_{\mathrm{r}}}{\mathrm{d}U_{\mathrm{r}}}\,\frac{\mathrm{d}U_{\mathrm{r}}}{\mathrm{d}t} + (\sigma_{\mathrm{d}}^+ + \sigma_{\mathrm{d}}^- - 2P_{\mathrm{f}})\,Q = \sigma_{\mathrm{r}}[v]\,.$$

Using equation (5.8) we can find the ridging stress

$$\sigma_{\mathrm{r}} = -\left(h\frac{\mathrm{d}P_{\mathrm{r}}}{\mathrm{d}U_{\mathrm{r}}} + \frac{\sigma_{\mathrm{d}}^+ + \sigma_{\mathrm{d}}^-}{2} - P_{\mathrm{f}}\right)\left(1 - h\frac{\mathrm{d}L_{\mathrm{r}}}{\mathrm{d}U_{\mathrm{r}}}\right)^{-1}. \tag{5.10}$$

If the ridging process proceeds via the scenario described in Section 4 then the stress $\sigma_{\mathrm{d}}^- = 0$ and σ_{d}^+ is given by equation (4.5).

6. Self-sustained oscillations of the ice cover caused by ridging

Let us consider the development of triangular hummocks at the boundaries between three ice floes under the effect of wind-generated stresses. The wind velocity is directed towards the sea shore. The hummocks and ice floes are numbered as shown in Fig. 3. floe *3* is stopped by the shore and, therefore, is stationary. The drift velocities for floes *1* and *2* are v_1 and v_2. At the initial moment small hummocks are assumed to exist between the ice floes.

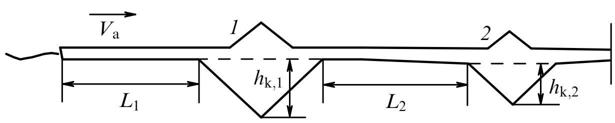

Figure 3.

The complete system of equations describing the process under consideration is

$$\frac{\mathrm{d}L_1}{\mathrm{d}t} = -Q_1\,, \quad \frac{\mathrm{d}L_2}{\mathrm{d}t} = -Q_1 - Q_2\,, \tag{6.1}$$

$$\rho_{\mathrm{i}}\,hL_1\,\frac{\mathrm{d}v_1}{\mathrm{d}t} = f_1 + \sigma_{\mathrm{r},1}(U_{\mathrm{r},1})\,, \tag{6.2}$$

$$\rho_{\mathrm{i}}\,hL_2\,\frac{\mathrm{d}v_2}{\mathrm{d}t} = f_2 - \sigma_{\mathrm{r},1}(U_{\mathrm{r},1}) + \sigma_{\mathrm{r},2}(U_{\mathrm{r},2})\,, \tag{6.3}$$

$$\frac{\mathrm{d}U_{\mathrm{r},1}}{\mathrm{d}t} = 2hQ_1\,, \quad \frac{\mathrm{d}U_{\mathrm{r},2}}{\mathrm{d}t} = 2hQ_2\,. \tag{6.4}$$

Equations (6.1) describe the variation of the ice flow size with the consumption of ice for building up the hummocks. According to the definitions (5.8), the flows Q_1 and Q_2 are given by

$$2Q_1 = \frac{\mathrm{d}L_{\mathrm{r},1}}{\mathrm{d}U_{\mathrm{r},1}}\,\frac{\mathrm{d}U_{\mathrm{r},1}}{\mathrm{d}t} - v_2 + v_1\,,$$

$$2Q_2 = \frac{\mathrm{d}L_{\mathrm{r},2}}{\mathrm{d}U_{\mathrm{r},2}}\,\frac{\mathrm{d}U_{\mathrm{r},2}}{\mathrm{d}t} + v_2\,. \tag{6.5}$$

Equations (6.2) and (6.3) describe the momentum balance for the ice floes *1* and *2*. Equation (5.10) determines the ridging stresses $\sigma_{r,1}(U_{r,1})$ and $\sigma_{r,2}(U_{r,2})$ as functions of the hummock volumes $U_{r,1}$ and $U_{r,2}$. The stresses generated by the friction of wind and water at the surfaces of floes *1* and *2* are denoted as f_1 and f_2 and given by

$$f_{1,2} = (\rho_a C_a V_a^2 - \rho_w C_w v_{1,2}^2) L_{1,2} , \qquad (6.6)$$

where ρ_a is the air density, C_a and C_w are the respective friction coefficients, and V_a is the wind velocity.

Equations (5.9) for the ice mass balance yield equations (6.4). The system of six equations (6.1)–(6.4) includes six unknown functions of time L_1, L_2, $U_{r,1}$, $U_{r,2}$, v_1, and v_2 and is closed. Equations (4.2) relate the hummock volumes $U_{r,1}$ and $U_{r,2}$ to the sail heights $h_{s,1}$ and $h_{s,2}$ and the keel drafts $h_{k,1}$ and $h_{k,2}$.

In numerical simulations we assumed that $h = 1$ m, $V_a = 15$ m s^{-1}, $C_a = 0.003$ (see [15]), $C_w = 0.005$ (see [16]), and $\mu = 0.3$ (see [17]). At the initial moment $t = 0$ we took $L_1(0) = 20$ km, $h_{k,1}(0) = 0.5$ m, $h_{k,2}(0) = 0.1$ m, $v_1(0) = 0.3$ m s^{-1}, $v_2(0) = 0$. Figure 4 presents the numerical simulation results for $L_2(0) = 500$ m (a, b), $L_2(0) = 1$ km (c, d), and $L_2(0) = 3$ km (e, f).

The initial conditions indicate that the hummock size is small at the initial moment. The small hummocks can withstand low compression stresses. This is why in the model under consideration they are regarded simply as the zones where the ice cover is weakened and where ridging occurs.

It can be seen that the ridging process goes on for approximately 1.3 hours. In this period the hummock keels grow to approximately 13 m. The hummock sail size is close to 4 m. The final dimensions of the hummock produced on the right-hand side of ice floe *2* are somewhat larger than the dimensions of the left-hand hummock. The motion of the large floe *1* is practically monotonic. The motion of floe *2* exhibits oscillations whose period depends on the floe size. The oscillation periods are approximately 5, 10, and 20 min for $L_2(0) = 500$ m, $L_2(0) = 1$ km, and $L_2(0) = 3$ km, respectively. These oscillations can be regarded as self-sustained oscillations as they are generated only under a steady-state wind load and are determined only by the internal structure of the ice cover which depends on the dimensions of the floes and hummocks. The period of the self-sustained oscillations decreases as the floe size diminishes.

The self-sustained oscillations are generated owing to the non-uniform dissipation of the mechanical energy at the floe edges which is caused by the process of ridging. If at the initial moment both hummocks have identical dimensions, the motion of ice floe *2* does not exhibit oscillations. At the moment when the ridging process is discontinued the hummocks have similar dimensions irrespective of their original sizes.

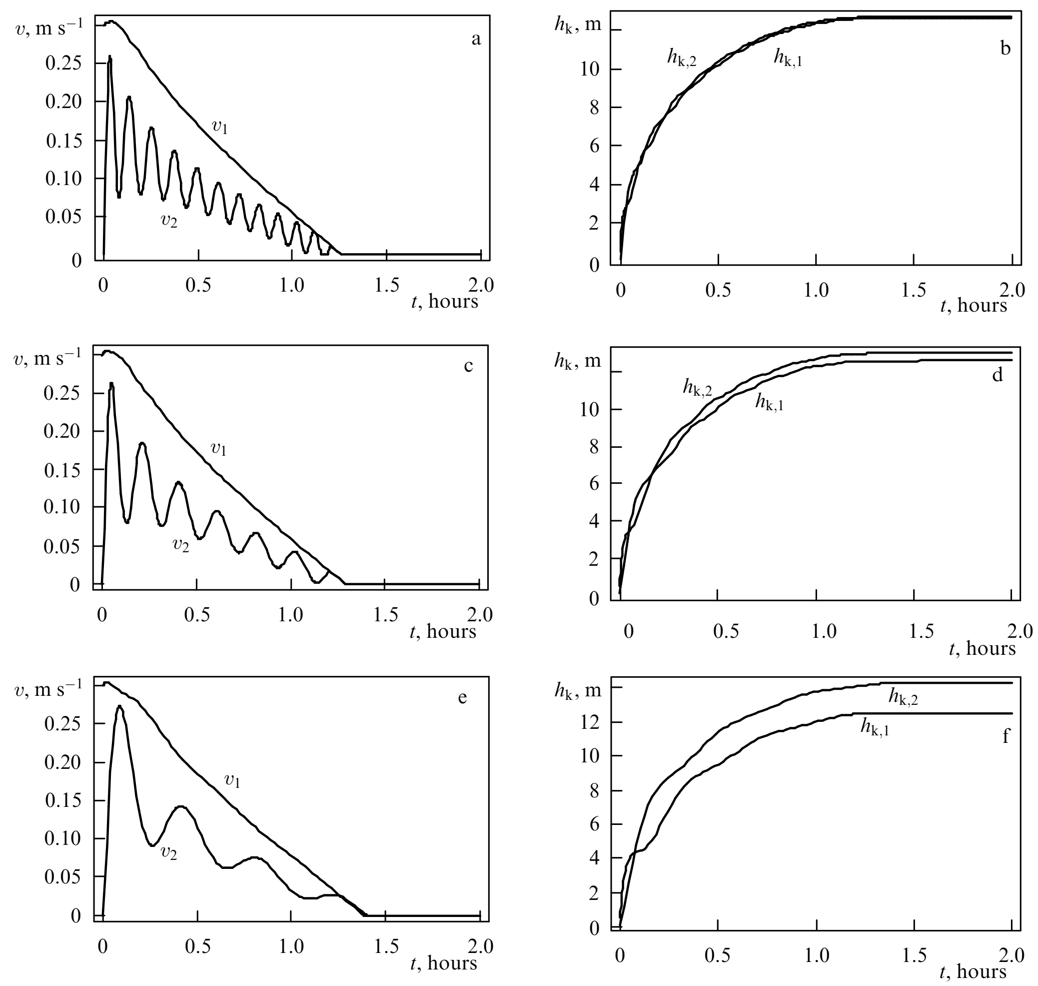

Figure 4.

7. Conclusion

The paper presents a new approach to the modeling of the ridging process in a drifting ice cover based on the representation of a hummock by a discontinuity line in the equations of the ice cover dynamics. The conservation of mass, momentum, and energy yield the relations at the discontinuity line. The stresses required for hummock formation can be calculated from the relations at the discontinuity line as functions of the hummock volume if the hypotheses on the self-similarity of the hummock shape and the energy dissipation are satisfied.

The modeling approach was implemented for the case of flat ice cover when the discontinuity line was straight and the velocities of the floes making up the hummock were perpendicular to the discontinuity line. Equations have been derived for the stresses generated in the process of rafting, in the case of hummock formation at a solid wall, and in the case of compression of a uniform ice cover.

Numerical simulations were conducted for the case of two hummocks formed at the line of contact of three floes of identical thickness and different lengths. The left-hand side of the left-hand floe was assumed to be free, the right-hand floe was stationary and the length of the middle floe was smaller than the length of the left-hand floe. The wind acting on the ice surface causes compression of the ice cover. The computer simulation results demonstrate that after the termination of the ridging process both hummocks have approximately identical dimensions. During the ridging process the velocity of the middle floe exhibits oscillations determined by non-uniform dissipation of the mechanical energy in the hummocks. The oscillation period decreases with a decrease in the length of the middle floe. The calculated period varied from 5 min to 20 min. Observations of drifting ice cover under natural conditions exhibited self-sustained oscillations with such periods [7].

The equations derived for the ridging stresses can be used for developing large-scale rheological ice cover models for appropriate climatic conditions in which the ice cover is treated as a continuous medium with plastic properties. The ridging stresses determine the limiting compression stresses describing the plastic properties of the ice cover.

It is important to calculate the frequencies of the oscillatory motion of the ice cover accompanying ridging near the hydrotechnical structures as they should be known for assessing the effects of ice dynamics on these structures. In particular, it would be useful to analyze a possible resonance between the ice oscillations and the natural frequencies of the structures. Any resonance will enhance the danger of a catastrophic failure of the structure.

The study was supported by the Russian Fund for Fundamental Research (projects 99-01-01150 and 99-02-17005) and by the Norwegian Science Consulate (project 128087/730).

References

1. Kovacs A, Sodhi S *Cold Region Technol.* (2) 209 (1980)
2. Pritchard R, in *Ice in Surface Waters* (Ed. H T Shen) (Rotterdam: Balkema, 1999) p. 1041
3. Parmerter R R, Coon M D *J. Geophys. Res.* **77** 6565 (1972)
4. Hopkins M A *J. Geophys. Res.* **103** (C10) 21883 (1998)
5. Hopkins M A, Tuhkuri J, Lensu M *J. Geophys. Res.* **104** (C6) 13605 (1999)
6. Tuhkuri J, Lensu M, Saarinen S "Laboratory and field studies of ice ridge formation", in *Proc. of the 15th Int. Conf. on Port and Ocean Eng. under Arctic Conditions* (*POAC'99*) Vol. 3 (Helsinki: HUT, 1999) p. 1118
7. Smirnov V N *Dinamicheskie Protsessy v Morskikh L'dakh* (Dynamical Processes in Sea Ice) (St. Petersburg: Gidrometeoizdat, 1996)
8. Sedov L I *Mekhanika Sploshnoĭ Sredy* (Mechanics of a Continuous Medium) Vol. 1 (Moscow: Nauka, 1983) [Translated into English: *A Course in Continuum Mechanics* (Groningen: Wolters-Noordhoff, 1971)]
9. Kraĭko A N *Prikl. Mat. Mekh.* **43** 500 (1979)
10. Marchenko A V *Prikl. Mat. Mekh.* **56** 419 (1992)
11. Makshtas A P, Marchenko A V, in *Zakonomernosti Krupnomasshtabnykh Protsessov v Norvezhskoĭ Energoaktivnoĭ Zone i Prilegayushchikh Raĭonov* (Large-Scale Processes in the Norwegian Energy-Active Zone and Adjacent Regions) (St. Petersburg: Gidrometeoizdat, 1994) p. 150
12. Appel' I L, Gudkovich Z N *Chislennoe Modelirovanie i Prognoz Evolyutsii Ledyanogo Pokrova Arkticheskikh Moreĭ v Period Tayaniya* (Numerical Simulation and Prediction of Ice Cover Evolution during the Melting Period in the Arctic Seas), (St. Petersburg: Gidrometeoizdat, 1992) p. 144
13. Martin S, Drucker R *J. Geophys. Res.* **96** (C6) 10567 (1991)
14. Coon M D, Echert D C, Knoke G S, in *Ice in Surface Waters* (Ed. H T Shen) (Rotterdam: Balkema, 1999) p. 1049
15. Andreas E L, in *The Atmospheric Boundary Layer over Polar Marine Surfaces* (CRREL Monograph 96-2) (Philadelphia: American Society for Testing and Materials, 1996) p. 38
16. Lepparanta M, in *Physics of Ice Covered Sea* Vol. 1 (Ed. M Lepparanta) (Helsinki: Helsinki University Printing House, 1994) p. 305
17. Mellor M *Mechanical Behavior of Sea Ice* (CRREL Monograph 83-1) (Philadelphia: American Society for Testing and materials, 1983)